Hadron Spectroscopy Using the Stochastic LapH Method

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June 22, 2012

QCD

- Start with the (Euclidean) Lagrangian for QCD
- $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \sum_f \overline{\psi_f} (\gamma_\mu D_\mu + m_f) \psi_f$
- Where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$
- and $D_\mu = \partial_\mu + igA_\mu$
- Path integral formalism

$$\mathcal{Z} = \int \mathcal{D}A_\mu \int \mathcal{D}\psi \mathcal{D}\overline{\psi}\, e^{-(\overline{\psi} M \psi + S_G)}$$

Lattice

- QCD is hard! Lets use computers to do physics for us!
- Discretize space and time onto a grid e.g. $24^3 \times 128$ and $32^3 \times 256$
- Compute gauge configurations on the lattice to perform Monte Carlo integration of path integrals.

- Challenge is in the details.
- Time and space are actually continuous so this introduces discretization errors, minimized using a series of clever tricks.
- Grid is very large so requires huge computing resources and would want many configurations to get good statistics
- Sophisticated approximation methods required to reduce computing resources.

Interested in the low energy spectrum

- Introduce "smearing" of the fields suppress high frequency modes easing the calculation of lower energy spectrum.
- $f = \{u, d, s\}$ ignoring the heavy quarks c, b, t
- Set the light quark masses equal $m_u = m_d$ giving perfect $SU(2)$ isospin symmetry
- However we must set parameters to give nonphysical masses so we have a heavier pion. This allows us to calculate low energy resonances without dealing with multihadron states which are still challenging.

Extract Energy/Mass

- Construct hadron operators couple to specific symmetry channel of hadron states. A simple example $\Rightarrow$ single-site meson operator:

$$O(t) = \sum C^{AB}_{\alpha\beta} \delta_{ab} \overline{\tilde{\psi}}^{A}_{a\alpha}(\mathbf{x}, t) \gamma_4 \tilde{\psi}^{B}_{b\beta}(\mathbf{x}, t)$$

  with appropriate choice of spin α β and flavor A B combinations in $C^{AB}_{\alpha\beta}$ to represent different particles.

- Construct the correlation function by inserting complete set of states:

$$C(t) = \left\langle 0 | O(t) \overline{O}(0) | 0 \right\rangle = \sum_n \left\langle O(t) | n \right\rangle \left\langle n | \overline{O}(0) \right\rangle e^{-E_n t}$$

  For large t, E_0 term dominate. Fit $C(t)$ to obtain E_0.

- Define effective mass as:

$$M_{eff}(t) = \frac{1}{dt} \ln \left[\frac{C(t)}{C(t+dt)} \right] \longrightarrow E_0 \quad \text{for large } t.$$

Computing with lattice QCD

- Physical observables can be expressed as:

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \int \mathcal{D}\psi \mathcal{D}\overline{\psi}\, O\, e^{-(\overline{\psi} M \psi + S_G)}$$

- The A_μ integral done using Monte Carlo, the ψ integrals done using Gaussian integration.
- Example with common operators

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi}\, \psi(x)\overline{\psi(y)}\, e^{-\overline{\psi} M \psi} = M^{-1}(x, y)\ \det(M)$$

$$\int \mathcal{D}\psi \mathcal{D}\overline{\psi}\, \psi_a \psi_b \overline{\psi_x}\, \overline{\psi_y}\, e^{-\overline{\psi} M \psi} = \left(M^{-1}_{ay} M^{-1}_{bx} - M^{-1}_{ax} M^{-1}_{by} \right) \det(M)$$

- Need to compute M^{-1} but M is very large!

Stochastic Inversion with Dilution

- Solve the inversion stochastically! Introduce N_r of Z_4 random noise vectors η^r, with expectation value: $E(\eta^r \eta^{r\dagger}) = 1$.
- Solve $MX^r = \eta^r$ for X so that $X = M^{-1}\eta$ then

$$E(X_i \eta_j^*) = \sum_k M_{ik}^{-1} E\left(\eta_k \eta_j^*\right) = M_{ij}^{-1}$$

- Projected onto dilution subspace:

$$\eta^{r[a]} = P^{(a)} \eta^r$$

- Solve $Mx^{r[a]} = \eta^{r[a]}$ separately in each subspace, M^{-1} can be approximated as:

$$M^{-1} \approx \boxed{\frac{1}{N_r} \sum_{r=1}^{N_r} \sum_a x^{r[a]} \eta^{r[a]\dagger}}$$

Laplacian Heaviside (LAPH) Smearing

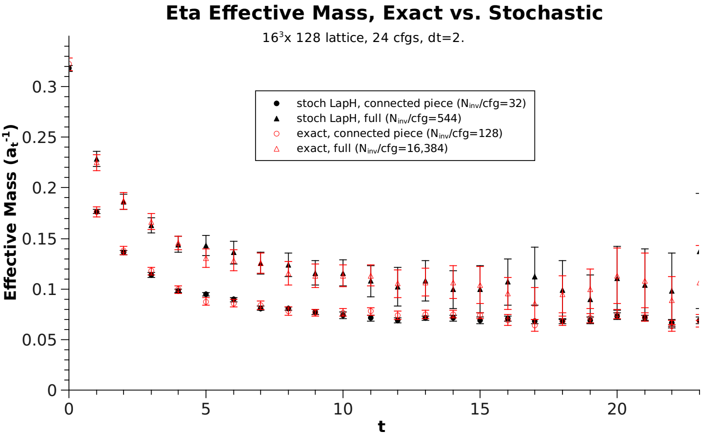
- Define the covariant Laplacian operator

$$\Delta_{ab} = \sum_k \left[U_k^{ab}(x)\delta(y, x+\hat{k}) + U_k^{\dagger\, ab}(y)\delta(y, x-\hat{k}) - 2\delta(x,y)\delta^{ab} \right]$$

- Δ is self adjoint so the eigenvalues are real. Eigenvalues are also gauge invariant.
- Laph quark field smearing picks out low-frequency modes:

$$\tilde{\psi}(x) = \Theta(\sigma_s^2 + \Delta)\psi(x)$$

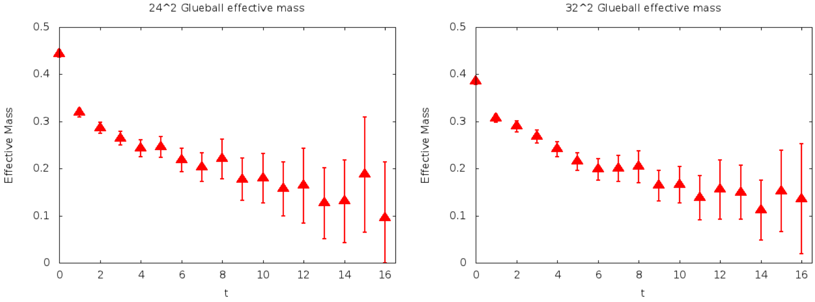
  Θ is the heaviside function so this picks off the first N Laplacian eigen-modes up to an eigenvalue cutoff σ_s.
- Perform dilution on spin-time-Laph eigenspace rather than spin-color-space-time.



Comparison of eta isoscalar meson effective masses, using exact Dirac matrix inversion (black) versus stochastic inversion with dilution in LapH space (red). Especially when the “full” correlator is considered (including Wick contractions of fermion fields on same time), the stochastic method allows much greater statistics per computation time.

Scalar Glueball

- One of the benefits of computing the covariant Laplacian eigenvalues is the we can use the eigenvalues directly as an operator.
- The eigenvalues of the Laplacian are invariant under rotations and gauge transformations. Since any quantity with the same transformation properties works as an operator the eigenvalues can be used for a scalar Glueball.
- After testing it was found that any of the eigenvalues worked equally well and what was used is linear combinations of all the eigenvalues below the cutoff.



- Results for the effective mass of the Glueball operator on $24^3 \times 128$ and $32^3 \times 256$ lattices.
- This channel will couple with other scalar multihadron operators (e.g. two pion states.) To extract the spectrum of the scalar sector it is required to mix and diagonalize our Glueball operator with other scalar operators.

Thank you