QCD IN THE LARGE N_C LIMIT AND ITS APPLICATIONS TO NUCLEAR PHYSICS

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27th Annual Hampton University Graduate Studies Program (HUGS 2012)

LECTURE I: FOUNDATIONS

- QCD in a nutshell
- Hydrogen atom in N dimensions
- Large Nc limit of QCD
- Mesons, baryons and its interactions

LECTURE II: APPLICATIONS

- NN interaction
- Baryon masses
- Axial current

Bibliography

1. *Baryons in the 1/N Expansion*. E. Witten. Nucl. Phys. B160 (1979) 57.

2. *Phenomenology of large Nc QCD*. R. F. Lebed. Czech.J.Phys. 49 (1999) 1273-1306. e-Print: nucl-th/9810080.

3. *Large N QCD.* A.V. Manohar. Lecture Notes. e-Print: hep-ph/9802419.

4. *Large Nc baryons*. E. E. Jenkins. Ann. Rev. Nucl. Part. Sci. 48 (1998) 81-119. e-Print: hep-ph/9803349.

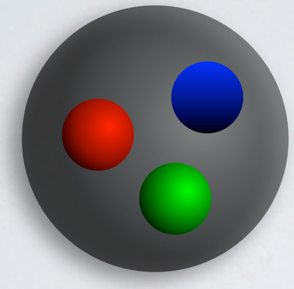
5. *Aspects of Symmetry* (chapter 8). Sidney Coleman (1988). Cambridge University Press.

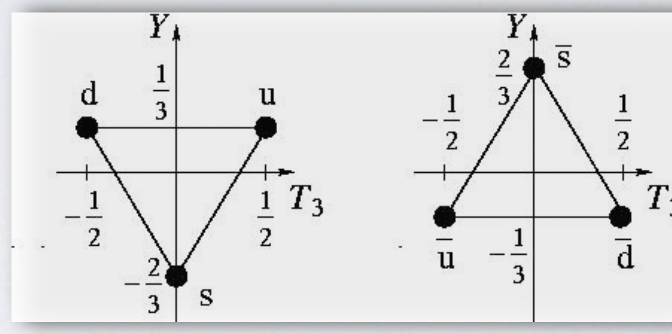
Hadron spectrum & Quark Model

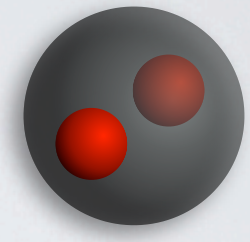
Gell-Mann & Ne'eman, 1964

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

SUF(3) Flavor





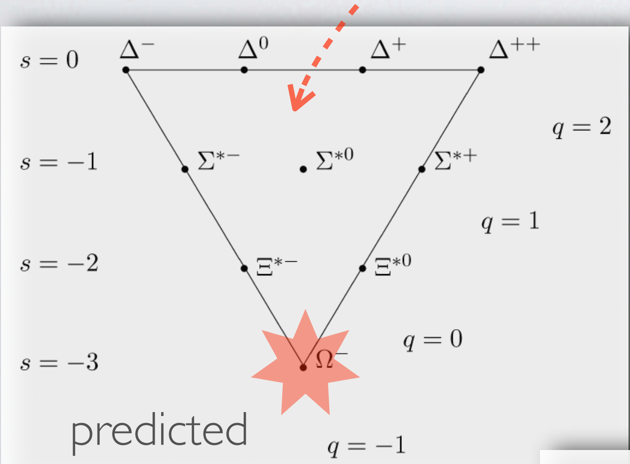


Approximate symmetry

$m_u \neq m_d \neq m_s$

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

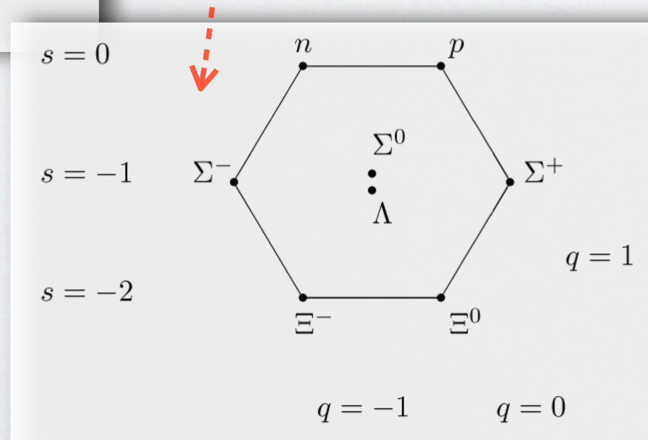
$$3 \otimes \bar{3} = 8 \oplus 1$$



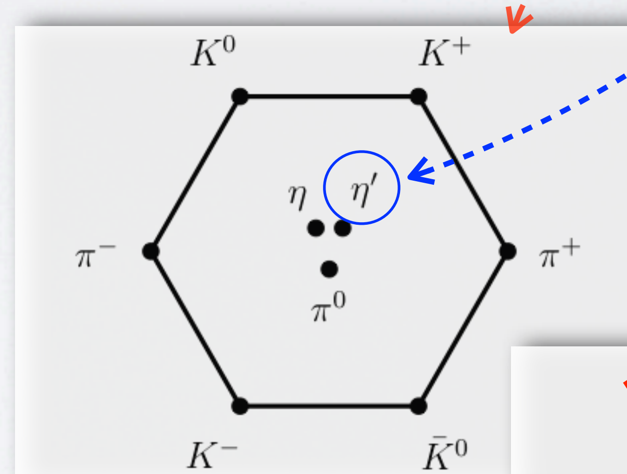
$J^P = \frac{3}{2}^+$

$\Lambda(1405)$

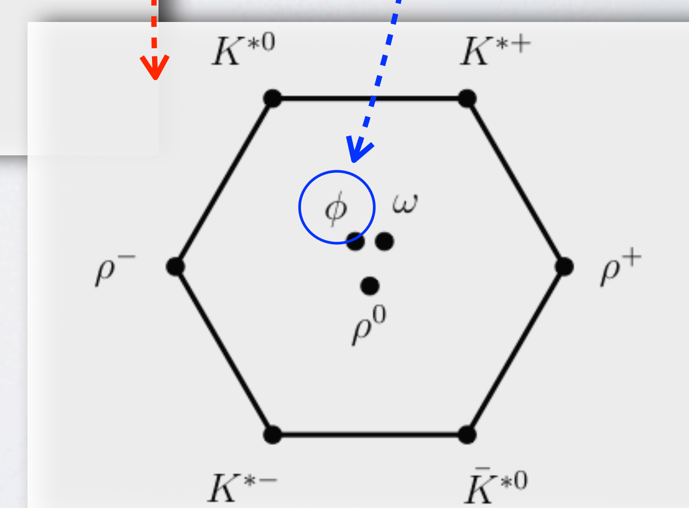
$J^P = \frac{1}{2}^-$



$J^P = \frac{1}{2}^+$



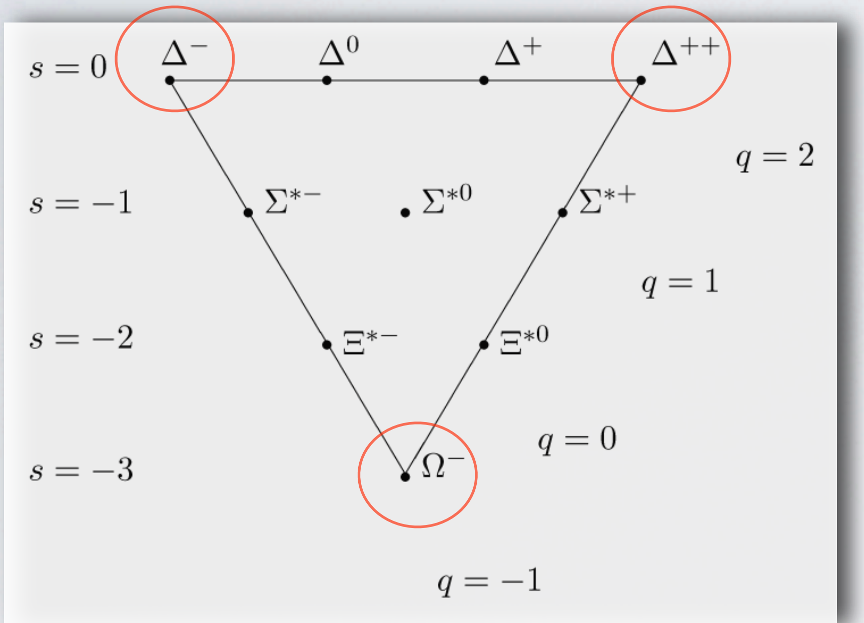
$J^P = 0^-$



$J^P = 1^-$

Hadron spectrum & Quark Model

Baryon octect



Baryon	Wave function
Δ^{++}	uuu
Δ^{+}	$(uud+udu+duu)/\sqrt{3}$
Δ^{0}	$(udd+dud+ddu)/\sqrt{3}$
Δ^{-}	ddd
Σ^{*+}	$(uus+usu+suu)/\sqrt{3}$
Σ^{*0}	$(uds+usd+dus+dsu+sud+sdu)/\sqrt{6}$
Σ^{*-}	$(dds+dsd+sdd)/\sqrt{3}$
Ξ^{*0}	$(uss+sus+ssu)/\sqrt{3}$
Ξ^{*-}	$(dss+sds+ssd)/\sqrt{3}$
Ω^{-}	sss

$$\Delta^{++} = u\uparrow u\uparrow u\uparrow$$

Pauli's principle

$$\psi_{\Delta^{++}} = \psi(space)\ \psi(spin)\ \psi(flavor)\ \boxed{\psi(color)}$$

A S S S A

$N_c = 3$

$$\psi(color) = (rgb - rbg + gbr - grb + brg - bgr)\,/\sqrt{6}$$

Particles in Nature are color singlets

QCD in a nutshell

Quantum Chromo Dynamics = gauge theory of the strong interactions

Fritzsch, Gell-Mann & Leutwyler

$f = u, d, s, c, b, t$

SU$_c$(3) gauge group

$$\mathcal{L}_{QCD} = \sum_f \bar{q}_f \left(i D_\mu \gamma^\mu - m_f \right) q_f - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} \qquad q'_{f,\alpha} = U_{\alpha\beta}\, q_{f,\beta}$$

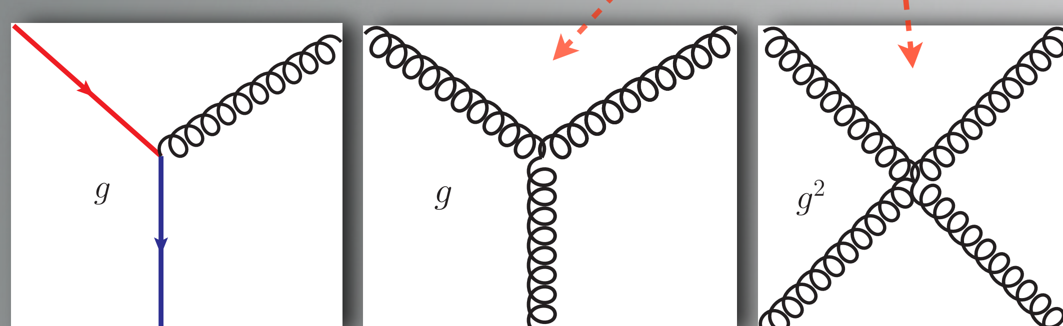
$$U \equiv \exp\left(-i\ \theta_a \frac{\lambda_a}{2} \right)$$

$$D_\mu q_f \equiv \left(\partial_\mu + i g \mathcal{A}_\mu \right) q_f \qquad \mathcal{A}_\mu = \mathcal{A}^a_\mu \frac{\lambda^a}{2}$$

Eight non-commuting generators

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = i f^{abc} \frac{\lambda_c}{2}$$

$$F^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu - g f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu$$



$$\mathcal{A}_\mu(x) \to \mathcal{A}_\mu(x) - \frac{1}{e} \partial_\mu \theta(x) \qquad \text{(QED)}$$

$$\mathcal{A}^a_\mu(x) \to \mathcal{A}^a_\mu(x) + \frac{1}{g} \partial_\mu \theta^a(x) + f^{abc} \mathcal{A}^c_\mu(x) \theta^b(x) \qquad \text{(QCD)}$$

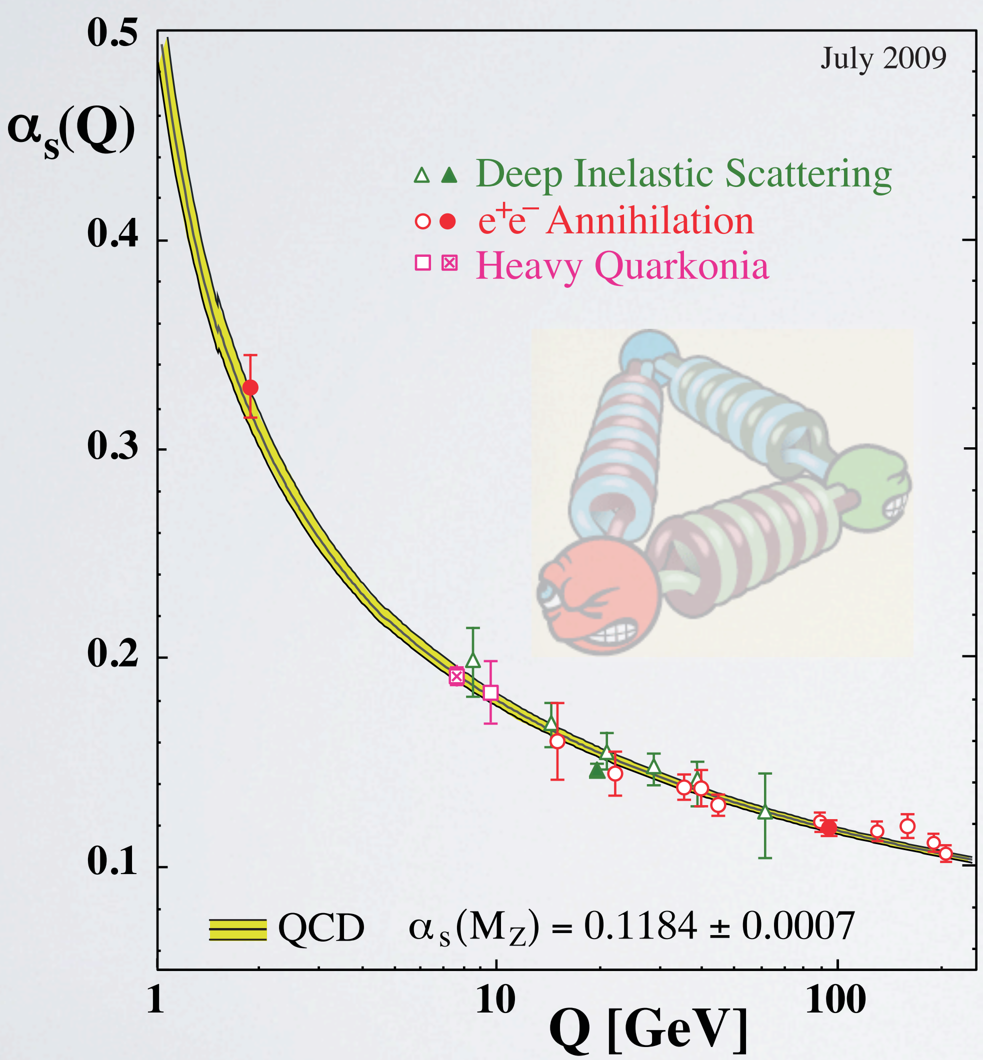
Generators of the adjoint representation

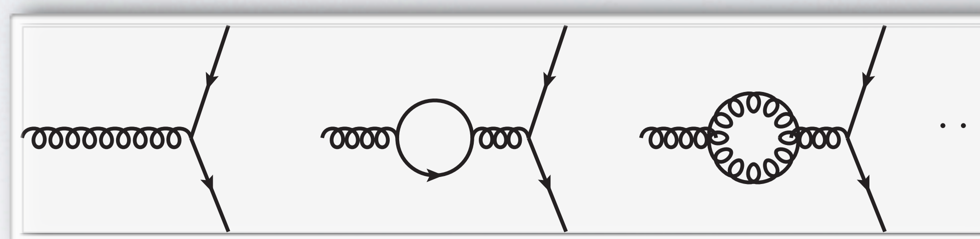
Gluons are in the adjoint representation

QCD in a nutshell

Asymptotic Freedom

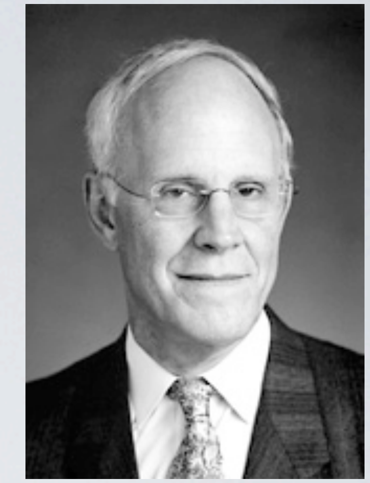
David J. Gross, H. David Politzer & Frank Wilczek, 2004





$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}N_f)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

$$\alpha_s \equiv g^2/4\pi$$



Non-perturbative methods

- Chiral Perturbation Theory

  Too much to say about for a two-hours lecture

- Lattice QCD

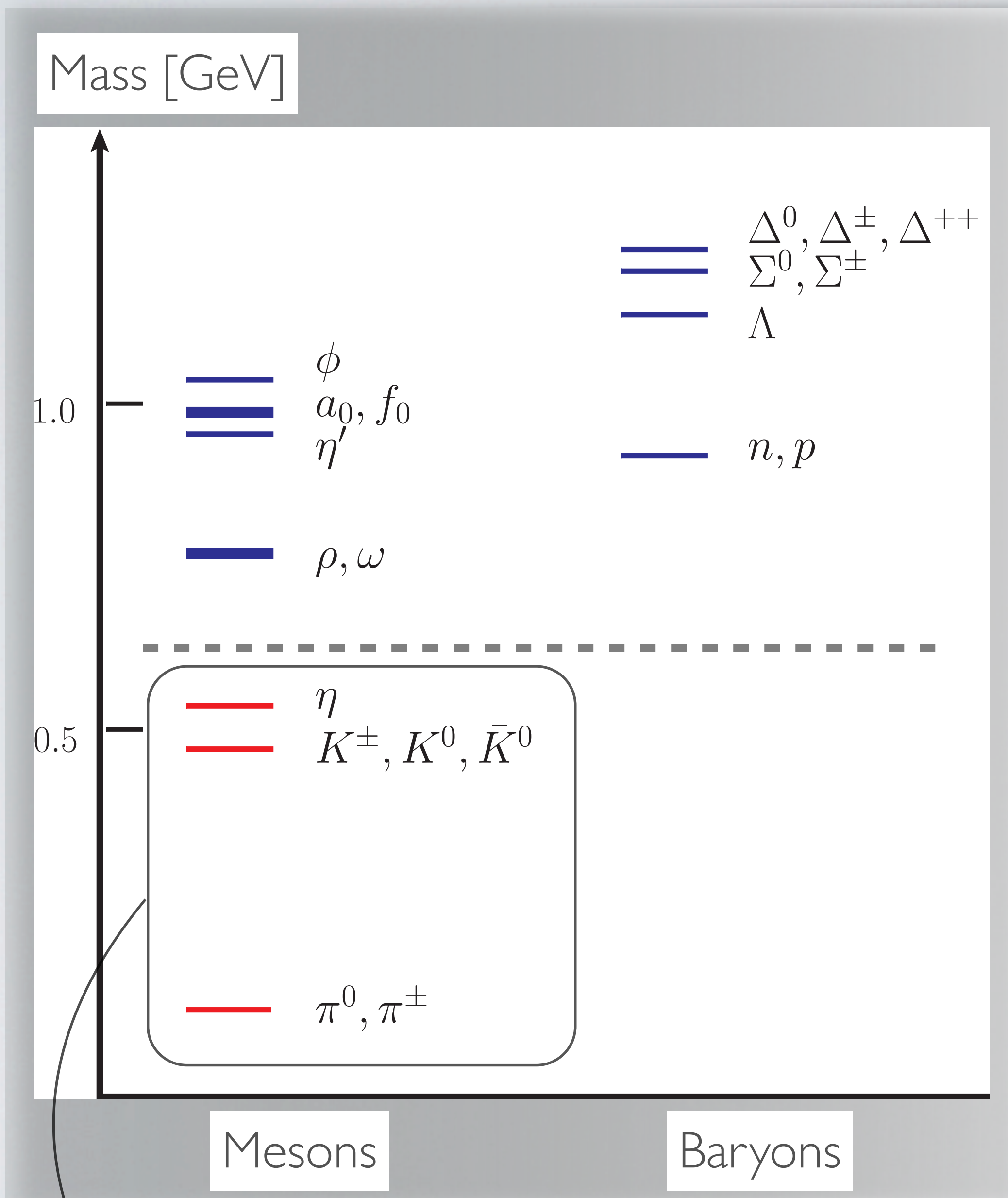
  Jo Dudek lectures next week !

- Large Nc limit of QCD

  I'll try to give you an overview

Chiral Perturbation Theory

Effective Field Theory of QCD in the chiral limit



Three (very) light ps mesons
Eight light ps mesons

$$m_u \sim 2.0 \pm 1.0 \text{ MeV} \quad m_d \sim 4.5 \pm 0.5 \text{ MeV} \quad m_s \sim 100 \pm 30 \text{ MeV} \quad \ll 1 \text{ GeV} \lesssim \quad m_c \sim 1.29 \pm 0.11 \text{ GeV} \quad m_b \sim 4.19 \pm 0.18 \text{ GeV} \quad m_c \sim 172.9 \pm 1.5 \text{ GeV}$$

chiral limit $\quad m_u = m_d = m_s = 0 \quad SU_L(3) \times SU_R(3)$

$$\mathcal{L}^0_{QCD} = \sum_{l=u,d,s} \left(\bar{q}_{L,l}\, i\not{D}\, q_{L,l} + \bar{q}_{R,l}\, i\not{D}\, q_{R,l} \right)$$

$$q_L = \frac{1}{2}(1-\gamma_5)q \qquad q_R = \frac{1}{2}(1+\gamma_5)q$$

$$SU_c(3) \quad \begin{pmatrix} u_L \\ u_L \\ u_L \end{pmatrix} \begin{pmatrix} d_L \\ d_L \\ d_L \end{pmatrix} \begin{pmatrix} s_L \\ s_L \\ s_L \end{pmatrix} \qquad SU_c(3) \quad \begin{pmatrix} u_R \\ u_R \\ u_R \end{pmatrix} \begin{pmatrix} d_R \\ d_R \\ d_R \end{pmatrix} \begin{pmatrix} s_R \\ s_R \\ s_R \end{pmatrix}$$

$$SU_f(3) \qquad\qquad SU_f(3)$$

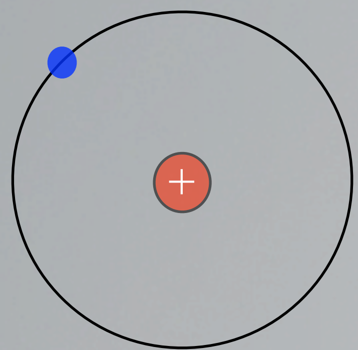
Doublets of opposite parity ?

Spontaneous symmetry breaking

Nambu-Goldstone bosons

Eight (three) massless pseudo scalar particles !

Hydrogen atom in N-dimensions



$$H = \frac{p^2}{2m} - \frac{\alpha}{r}$$

$$\alpha = 1/137 = 0.0073$$

Is perturbation theory applicable? NO !

$$r \to \bar{r}t, \;\; p \to \bar{p}/t, \;\; t \equiv 1/(m\alpha^2)$$

$$H \to \frac{H}{m\alpha^2} \equiv \bar{H} = \frac{\bar{p}^2}{2} - \frac{1}{\bar{r}}$$

The expansion parameter disappears, we can absorb the overall energy by redefining the temporal scale

A hidden expansion parameter is the space dimension

Hydrogen atom in N-dimensions with N very large

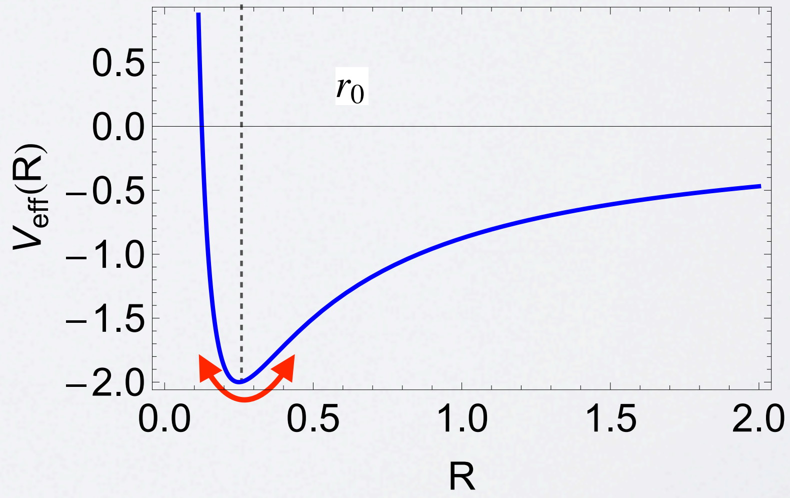
$$\left[-\frac{1}{2m}\left(\frac{\partial^2}{\partial r^2} + \frac{N}{r}\frac{\partial}{\partial r}\right) - \frac{\alpha}{r}\right]\Psi = E\,\Psi$$

Rescaling $\quad \Psi = r^{-N/2}\bar{\Psi}\,, \;\; r = N^2 R$

$$H = \frac{1}{N^2}\left[-\frac{1}{2mN^2}\frac{\partial^2}{\partial R^2} + \frac{1}{8mR^2} - \frac{\alpha}{R}\right]$$

$$M_{eff} = mN^2 \qquad V_{eff}(R) = 1/(8mR^2) - \alpha/R$$

To lowest order in 1/N $\quad E_0 = V_{eff}(r_0)/N^2 = -2m\alpha^2/N^2$



$$E_0\Big|_{N=3} = -2/9\; m\alpha^2$$

$$E_0\Big|_{exact} = -1/2\; m\alpha^2$$

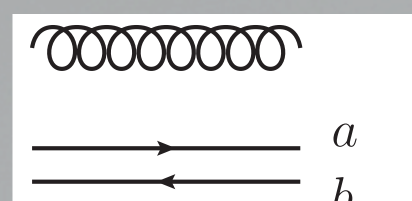
Large Nc limit of QCD

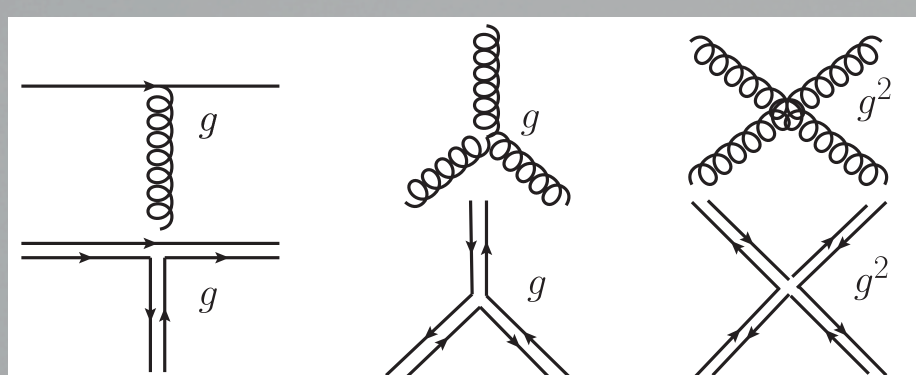
G. t'Hooft, 1974

$$SU(3) \to SU(N_c)$$

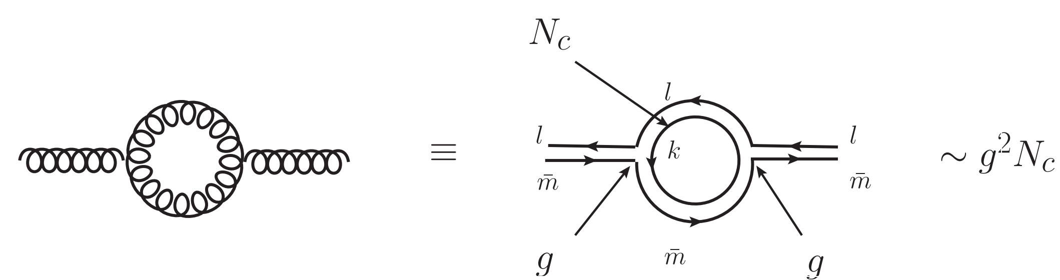
t'Hooft double-line notation

$$A^a_{\mu b} \sim q^a \bar{q}_b$$





t'Hooft limit



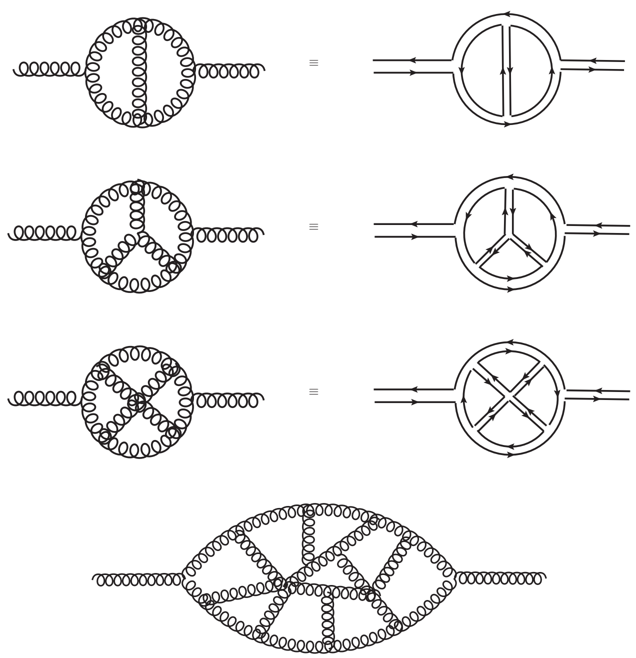
$$\lim_{N_c \to \infty} g^2 N_c = \text{constant}$$

$$g \sim 1/\sqrt{N_c}$$

Large Nc limit of QCD

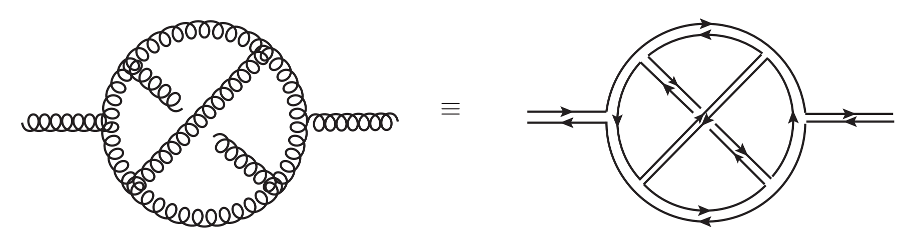
Planarity

Planar diagrams



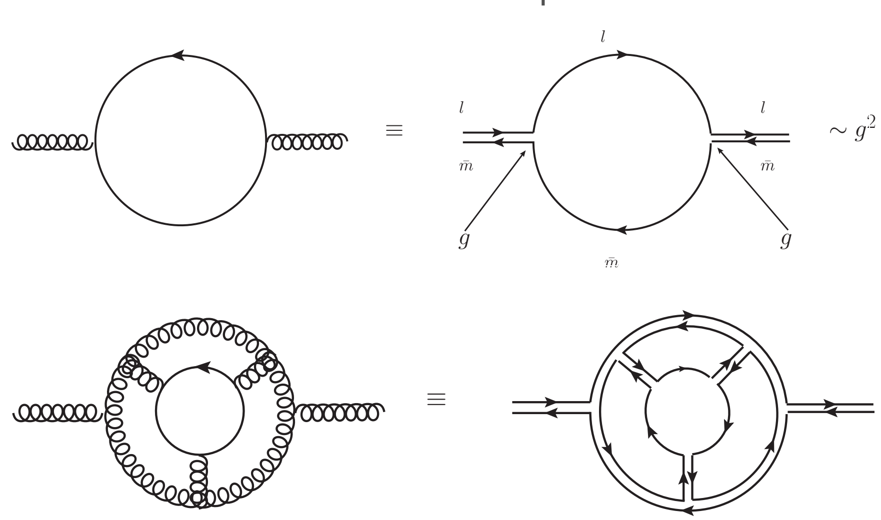
$\sim \mathcal{O}(1)$

Non-planar diagrams



$\sim 1/N_c^2$

Quark loops



$\sim 1/N_c$

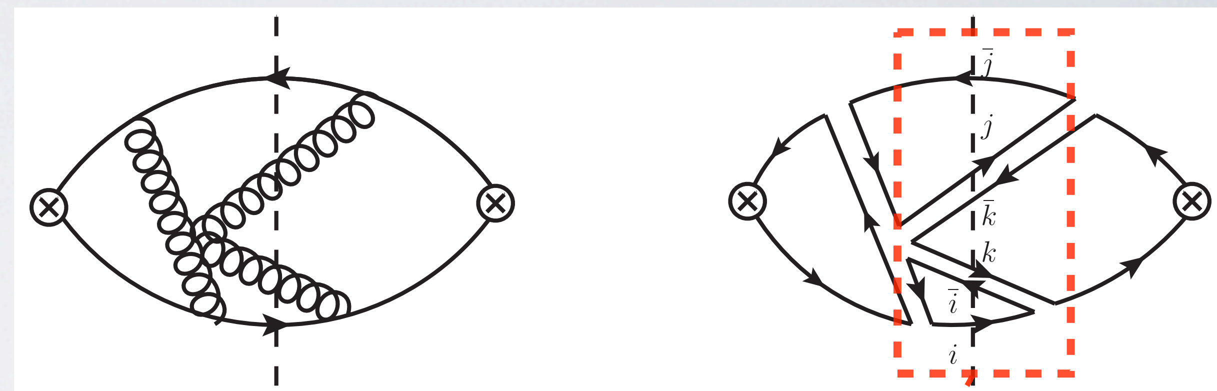
Leading order Feynman diagrams in the large Nc limit are planar with a minimum number of quark loops

Mesons in the large Nc limit

Basic assumption: QCD in the large Nc limit is a confining theory and quarks, anti-quarks and gluons must combine in order to give a colorless state.

Large Nc meson

$$|1\rangle_c = \frac{1}{\sqrt{N_c}}\left(q_{c_1}\bar{q}_{c_1} + \cdots + q_{c_{N_c}}\bar{q}_{c_{N_c}}\right)$$

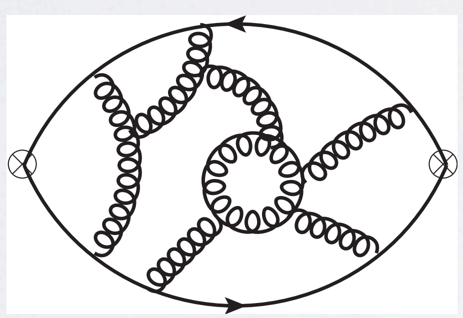


only one color singlet state is allowed as intermediate state

$$\bar{q}_j A^j_k A^k_i q^i \sim \bar{q}q$$

$$\cancel{(\bar{q}_k A^k_l q^l)(A^j_m A^m_j) \sim \bar{q}qg}$$

$\mathcal{O}(N_c)$



$= \sum_n$  $= \sum_n \frac{f_n^2}{k^2 - M_n^2}$

$$f_n \sim \mathcal{O}(\sqrt{N_c})$$

$$M_n \sim \mathcal{O}(1)$$

Mesons are stable and light

Baryons in the large Nc limit

$\Big\}$ N_c quarks

antisymmetric color singlets

$$\epsilon_{i_1 i_2 \cdots i_{N_c}} q^{i_1} q^{i_2} \ldots \; q^{i_{N_c}}$$

E. Witten, 1979

$$H_B = N_c\, m + N_c\, t + V = N_c\,(m + t + v) = N_c\, h$$

current quark mass

quark kinetic energy

potential energy

$$V = N_c^2 \left(\frac{1}{N_c}\, v \right)$$

# of quark pairs

two-quarks interaction

baryons are heavy

$$M_B \sim \mathcal{O}(N_c)$$

Interaction between quarks negligible $\sim \mathcal{O}(1/N_c)$

Potential felt by any individual quark $\sim \mathcal{O}(1)$

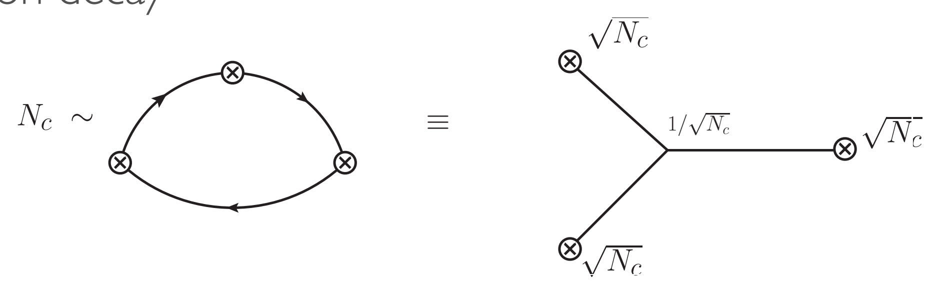
Hartree picture: each quark move independently in background potential

Baryons are heavy in the large Nc limit but their size and shape are finite

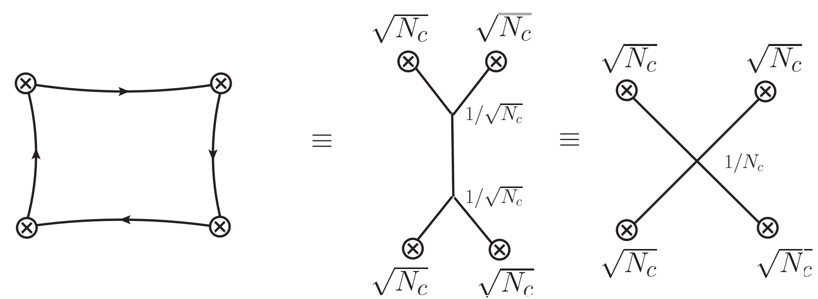
$$\psi_B(x_1, \ldots, x_N) = \Pi_{i=1}^{N_c} \phi(x_i)$$

Meson and baryon interactions

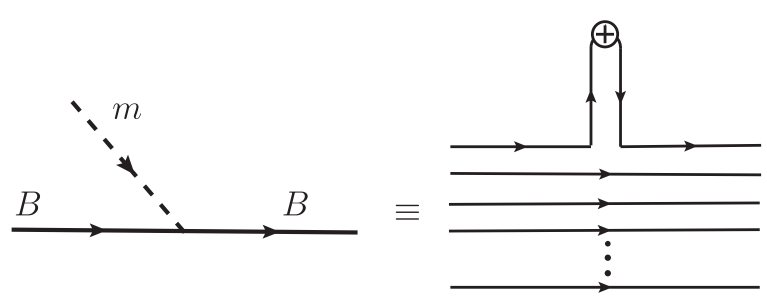
Meson decay



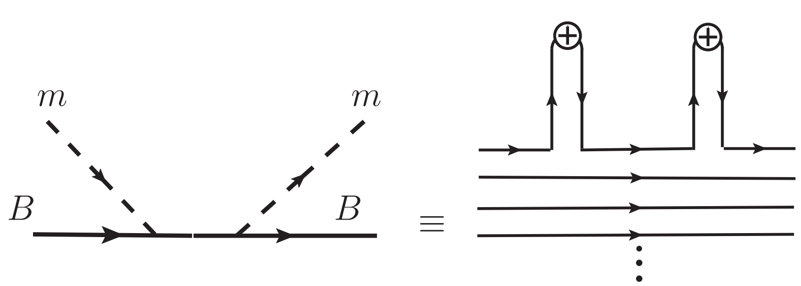
Meson-meson interaction



Meson-baryon interaction



Meson-baryon scattering



Witten's counting rules

Mesons are very narrow

$$\Gamma_{\text{meson}} \sim \mathcal{O}(1/N_c)$$

Mesons are free and non-interacting

$$V_{\text{meson}} \sim \mathcal{O}(1/N_c)$$

Meson-baryon coupling constant

$$g_A \frac{N_c}{F_\pi}\, \partial_i \pi^a X^{ia} \sim \mathcal{O}(\sqrt{N_c})$$

Meson-baryon scattering

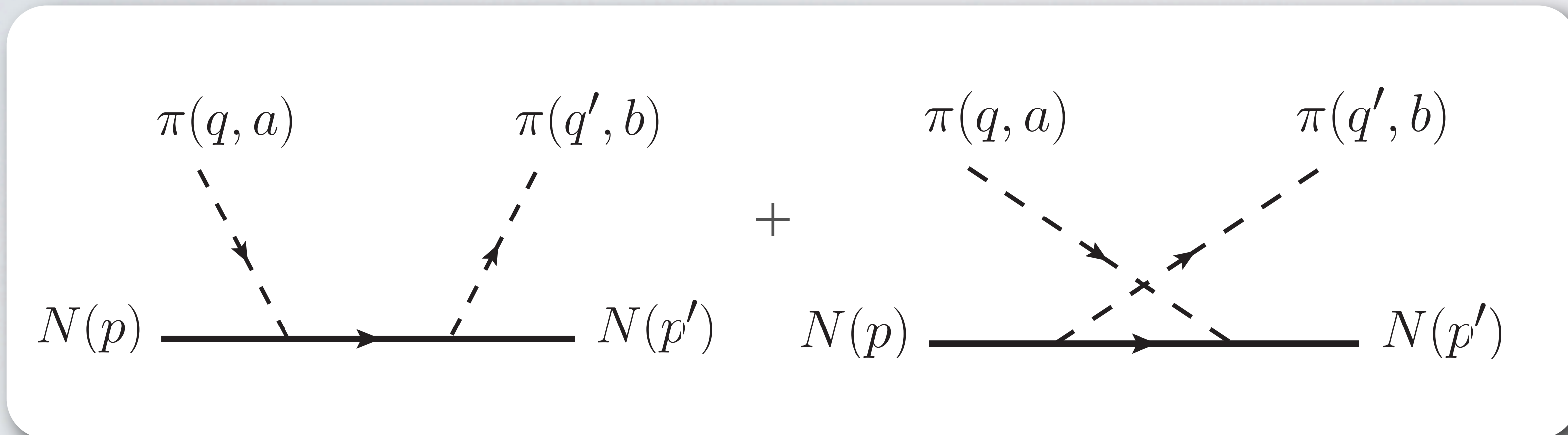
$$\sim \mathcal{O}(1)$$

mesons are scattered by baryons

Spin-Flavor symmetry

Pion-nucleon scattering amplitude is $\sim \mathcal{O}(1)$

but the pion-nucleon vertex is $\sim \mathcal{O}(\sqrt{N_c})$



cancellation between diagrams

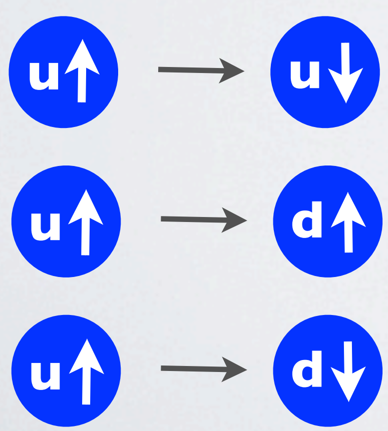
$$\propto \frac{N_c^2 g_A^2}{F_\pi^2}\left[X^{ia}, X^{jb}\right] = \mathcal{O}(Nc^0)$$

consistency conditions

Gervais & Sakita, 1984
Dashen & Manohar, 1993

$SU(2N_f)$ spin-flavor symmetry

$$J = I = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{N_c}{2}$$

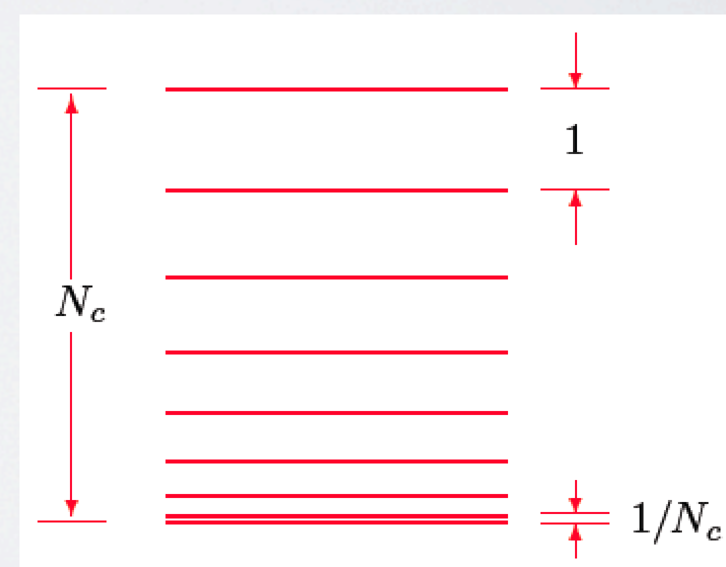


Spin

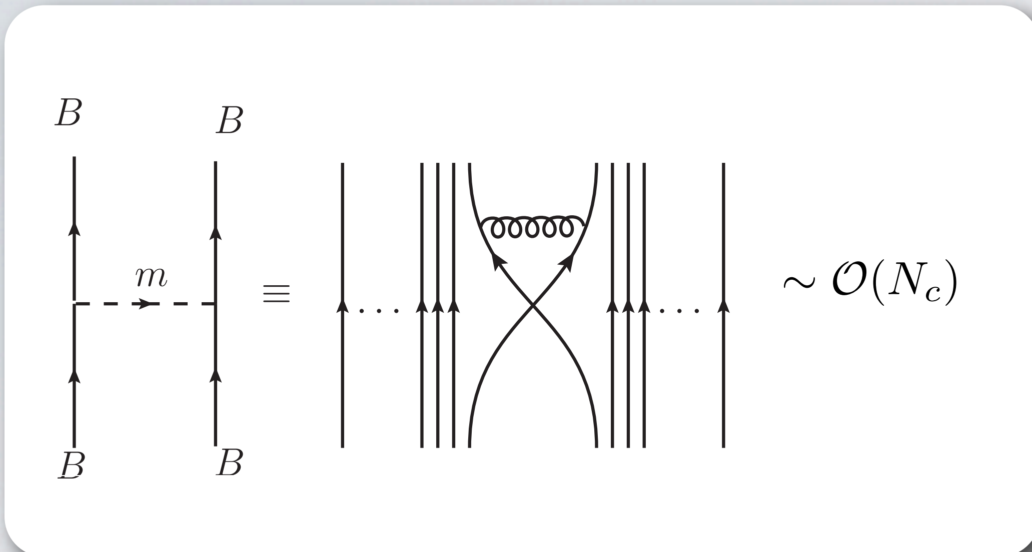
Flavor

Spin-Flavor

$$B = \begin{pmatrix} N \\ \Delta \\ \Delta_{5/2} \\ \vdots \\ \Delta_{N_c/2} \end{pmatrix}$$



Baryon-baryon interaction



Spin-flavor structure of the NN potential

Kaplan, Savage & Manohar, 1996

Box and cross box diagram cancellations

Banerjee, Cohen & Gelman, 2002

$$V(r) = V_C(r) + (\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) W_S(r) + (\tau_1 \cdot \tau_2) W_T(r) S_{12} \sim N_c$$

OBE potential at leading Nc

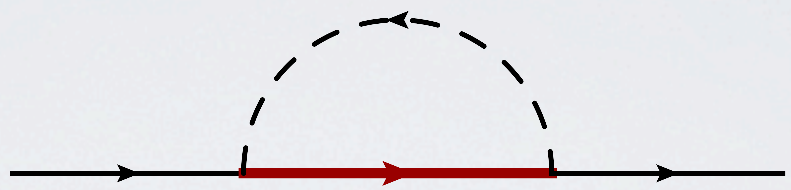
		Meson	Coupling	Scaling	Order
Scalar	$I=0$	σ	$B^\dagger B\phi$	$\sqrt{N_c}$	LO
	$I=1$	a_0	$B^\dagger I^a B\phi^a$	$1/\sqrt{N_c}$	NLO
Pseudo-scalar	$I=0$	η	$B^\dagger J^i B\partial^i\phi$	$1/\sqrt{N_c}$	NLO
	$I=1$	π	$B^\dagger G^{ia} B\partial^i\phi^a$	$\sqrt{N_c}$	LO
Vector	$I=0$	ω^0	$B^\dagger B V^t$	$\sqrt{N_c}$	LO
		ω	$B^\dagger \epsilon_{ijk} J^k B\partial^i V^j$	$1/\sqrt{N_c}$	NLO
	$I=1$	ρ^0	$B^\dagger I^a B V^{ta}$	$1/\sqrt{N_c}$	NLO
		ρ	$B^\dagger \epsilon_{ijk} G^{ka} B\partial^i V^{ja}$	$\sqrt{N_c}$	LO
Axial	$I=0$	f_1	$B^\dagger J^i B A^i$	$1/\sqrt{N_c}$	NLO
	$I=1$	a_1	$B^\dagger G^{ia} B A^{ia}$	$\sqrt{N_c}$	LO

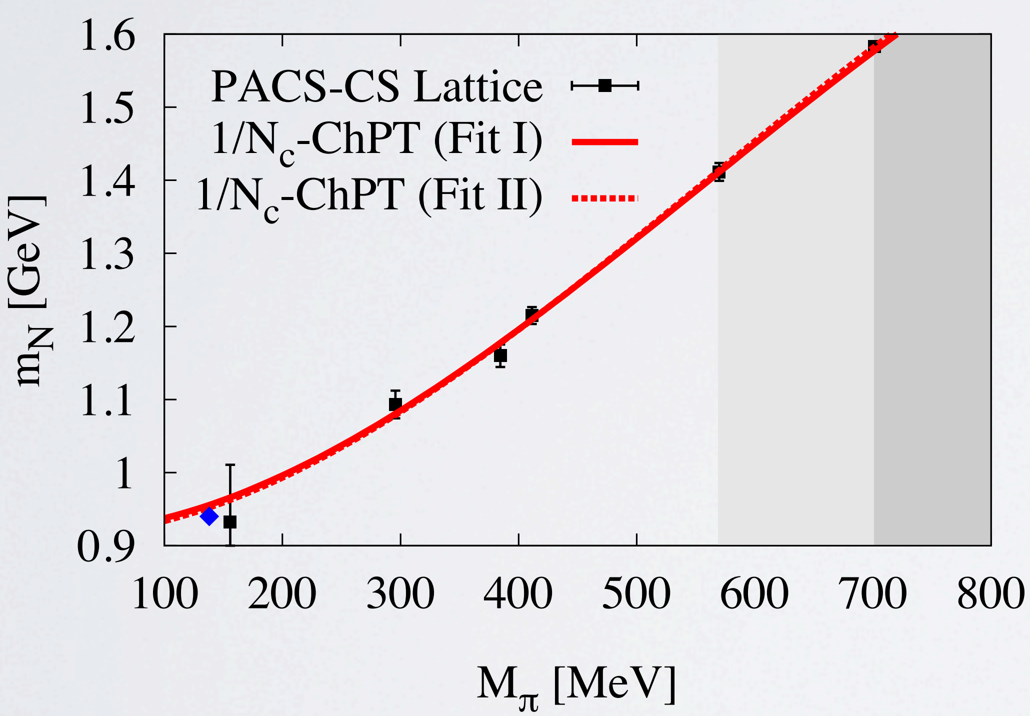
ACC & E. Ruiz Arriola, 2010

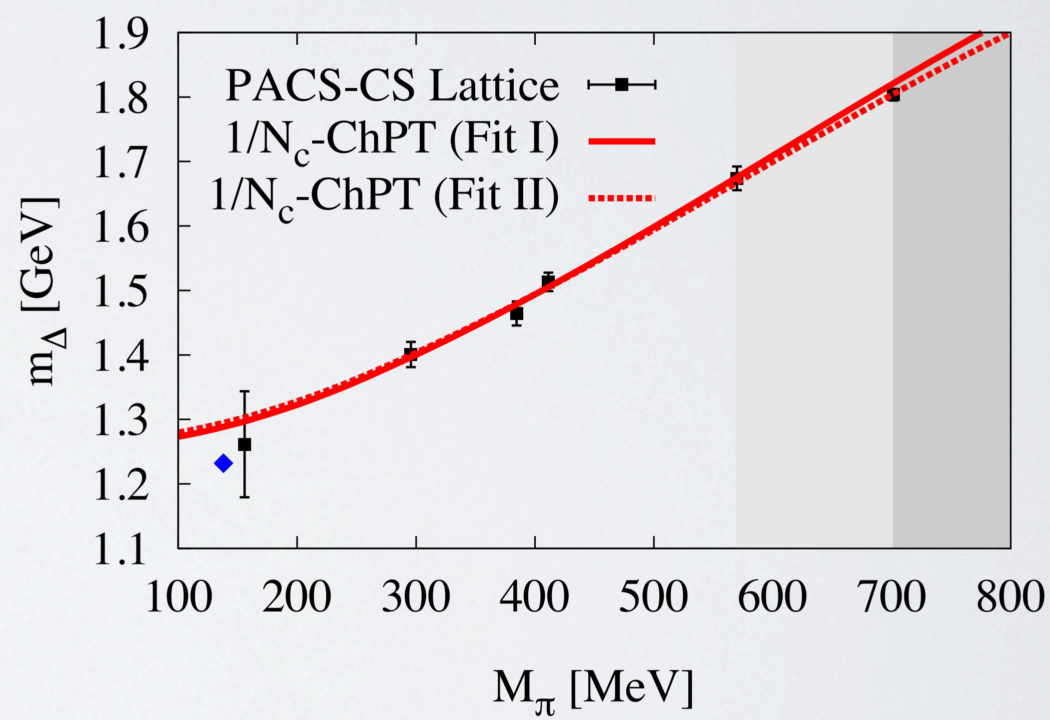
$$\begin{aligned}
V_C(r) &= -\frac{g^2_{\sigma NN}}{4\pi}\frac{e^{-m_\sigma r}}{r} + \frac{g^2_{\omega NN}}{4\pi}\frac{e^{-m_\omega r}}{r}, \\
W_S(r) &= \frac{1}{12}\frac{g^2_{\pi NN}}{4\pi}\frac{m_\pi^2}{\Lambda_N^2}\frac{e^{-m_\pi r}}{r} + \frac{1}{6}\frac{f^2_{\rho NN}}{4\pi}\frac{m_\rho^2}{\Lambda_N^2}\frac{e^{-m_\rho r}}{r}, \\
W_T(r) &= \frac{1}{12}\frac{g^2_{\pi NN}}{4\pi}\frac{m_\pi^2}{\Lambda_N^2}\frac{e^{-m_\pi r}}{r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] \\
&\quad - \frac{1}{12}\frac{f^2_{\rho NN}}{4\pi}\frac{m_\rho^2}{\Lambda_N^2}\frac{e^{-m_\rho r}}{r}\left[1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right],
\end{aligned}$$

1/Nc - Chiral Perturbation Theory

Nucleon and Delta masses



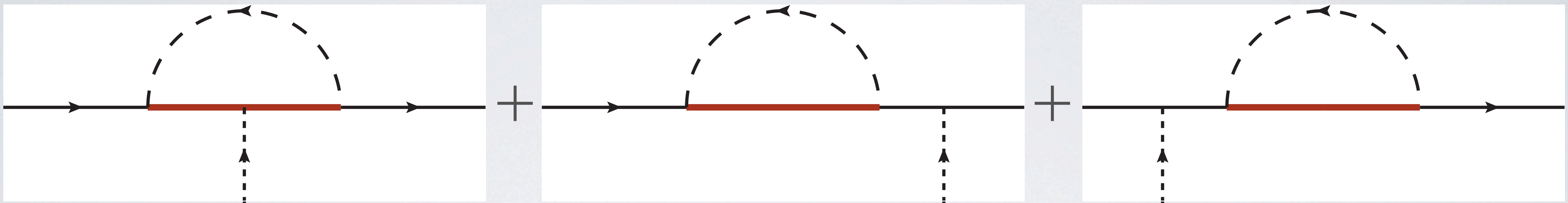




ACC & Goity, 2012

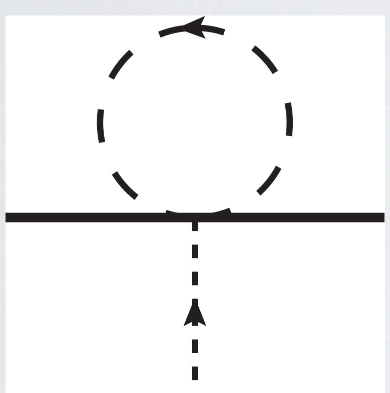
1/Nc - Chiral Perturbation Theory

Axial current g_A



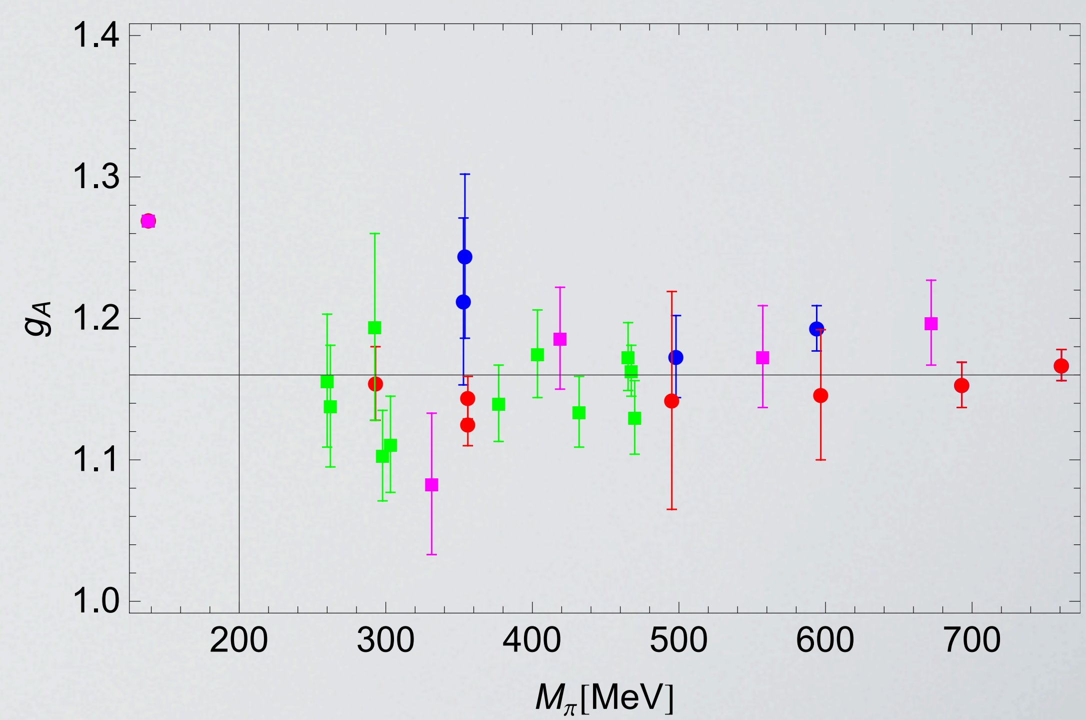
ACC & Goity, 2012

Exact cancellation in the large Nc limit and almost exact for Nc=3



All the pion mass dependence of g_A is dominated by the tad-pole diagram and and the counter-terms

The result is a mild dependence of g_A with the pion mass



Summary

✤ Large Nc limit is a fundamental feature of QCD

✤ It is not restricted to low or high energies, so it may be considered a non-perturbative method to solve QCD at low-energies

✤ There are a lot of situations where quantities in the large Nc limit are not far from the real world Nc=3

✤ The combination of the large Nc limit & ChPT looks promising to explain recent lattice QCD results for hadronic quantities