Nuclear Structure and Reactions - III

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U.S. DEPARTMENT OF ENERGY | Office of Science

Outline of my six lectures

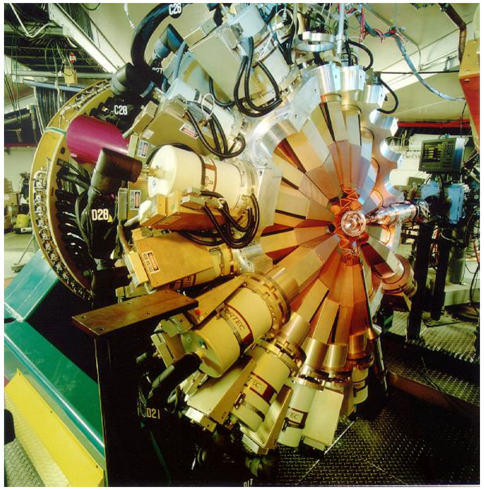
- One of HUGS goals is to introduce students to "topics of interest in nuclear physics". My lectures will attempt to describe what is interesting in the study of nuclei.
- Lecture 1: Search for the limits of nuclear binding and production of new isotopes
- Lecture 2: Attempts to model atomic nuclei I
- **Lecture 3: Attempts to model atomic nuclei II**
- Lecture 4: Nuclear Reactions
- Lecture 5: The origin of atoms – Nuclear Astrophysics I
- Lecture 6: The origin of atoms – Nuclear Astrophysics II

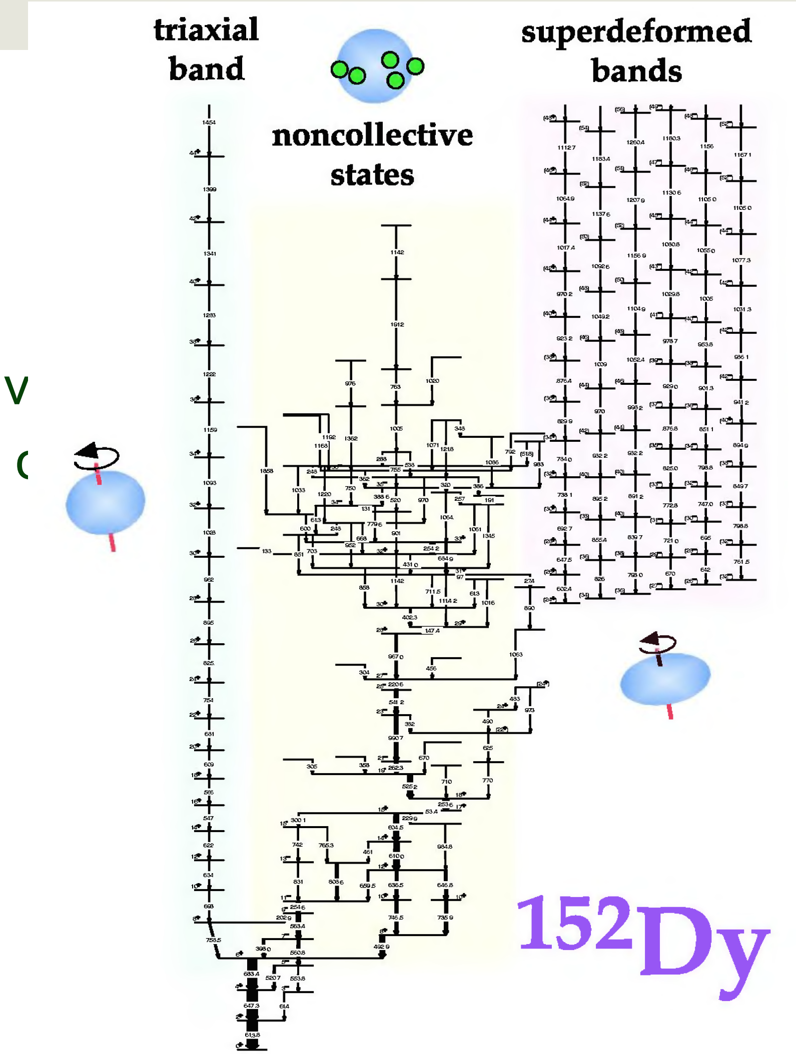
It is important for you to ask questions.

Nuclear Spectroscopy

- Incredible variety of excited states!
- Regular bands
  - I(I+1) behavior: rotational
  - nħΩ behavior: vibrational
- Regular bands signatures of nuclear collectiv
- Bands with no visible patterns are signature (
effects)

- GAMMASPHERE





From W. Nazarewicz, in *An Advanced Course in Modern Nuclear Physic*s, J.M. Aria, M. Lozano (eds.), Springer (2001)

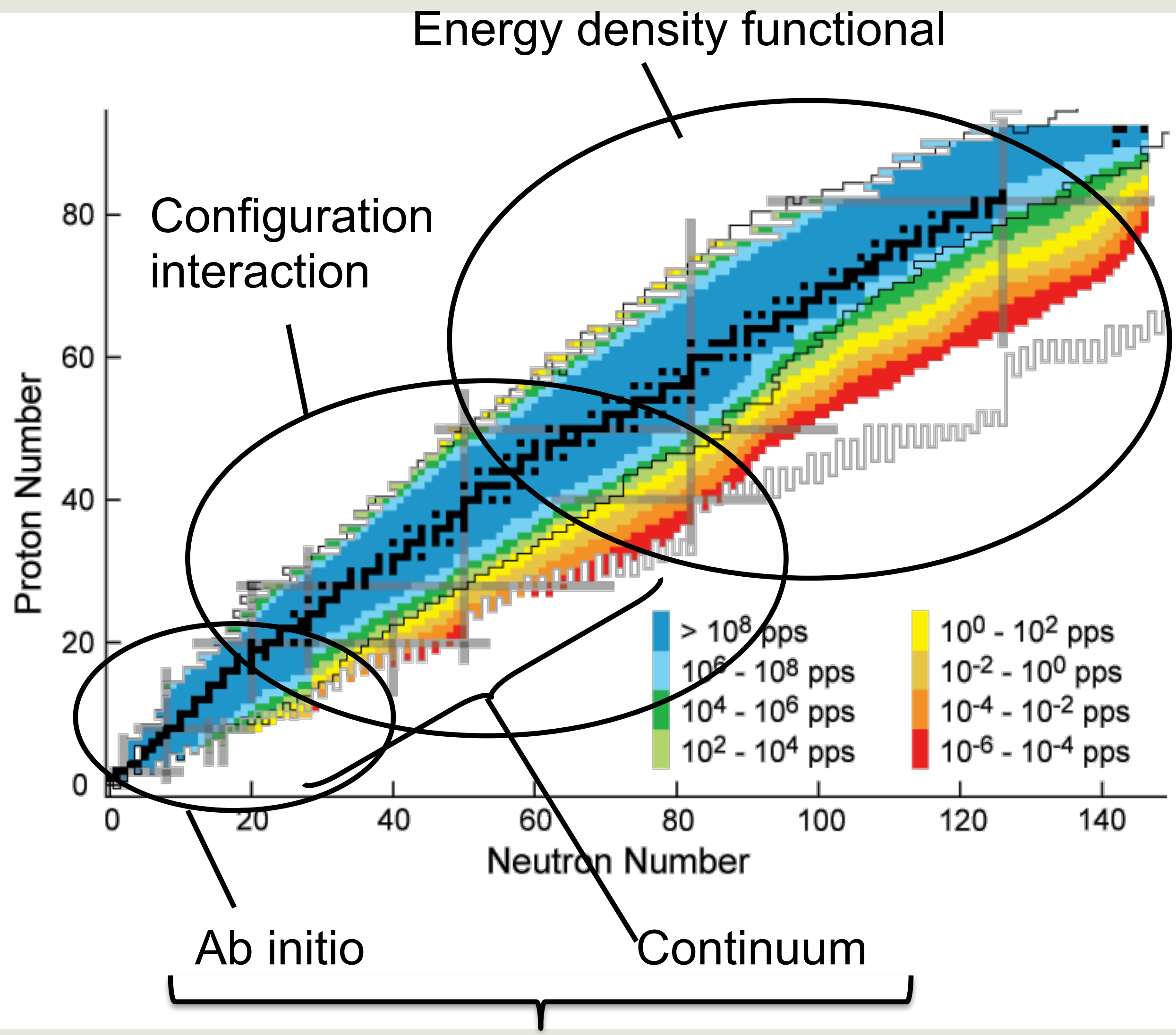
The Road Map: Understanding the Stability of Atomic Nuclei

- Step 1: Use *ab initio* theory and study of exotic rare isotopes to determine the interactions of nucleons in light nuclei and connect these to QCD by comparison to lattice calculations of NN and NNN forces
- Step 2: For mid-mass nuclei use configuration interaction models. The degrees of freedom and interactions must be determined from exotic nuclei
- Step 3: Use density functional theory to connect to heavy nuclei. Exotic nuclei help determine the form and parameters of the DFT.

The last step is the one that may answer the question of the limits of nuclei.

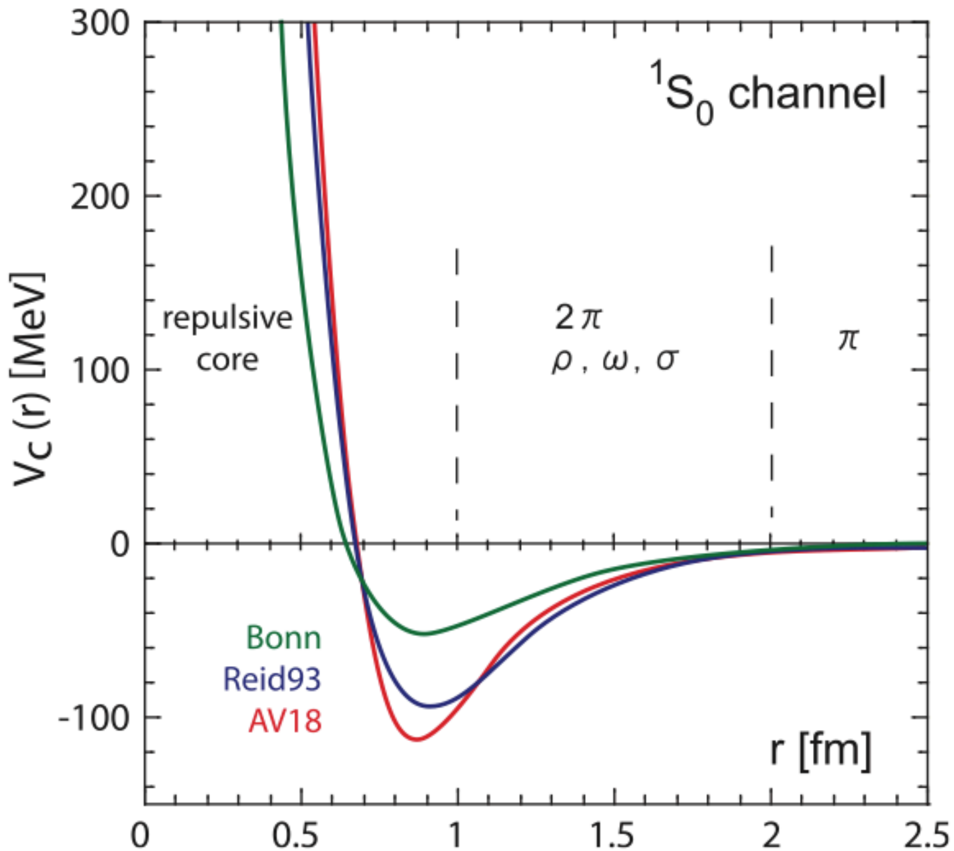
Theory Road Map: Comprehensive Model of Nuclear Structure and Reactions

- Theory Road Map – Goal is a "standard model" for understanding nuclei
- The comprehensive description will allow us to explore for other physics



"Ab Initio" start with NN forces

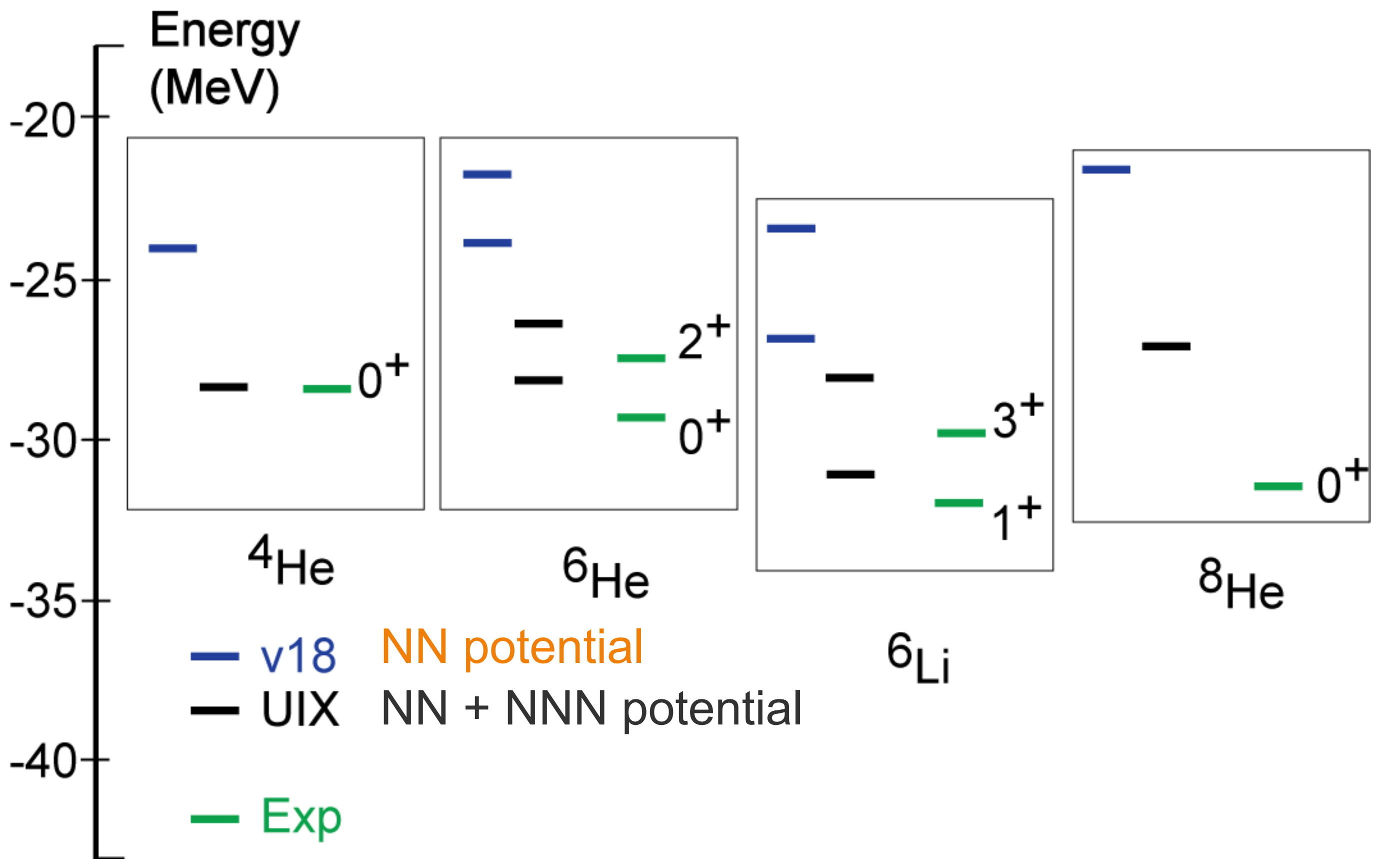
- Approach: Construct NN potentials based on neutron and proton scattering data and properties of light nuclei (Bonn, Reid, Illinois AV18, Nijmegen, etc.)
- More recent approach is to construct the potentials some more fundamental theory
  - QCD Inspired EFT
  - String Theory Inspired – Hashimoto et al
  - Lattice QCD



N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007)

Comparison of Calculated and Measured Binding Energies with NN models

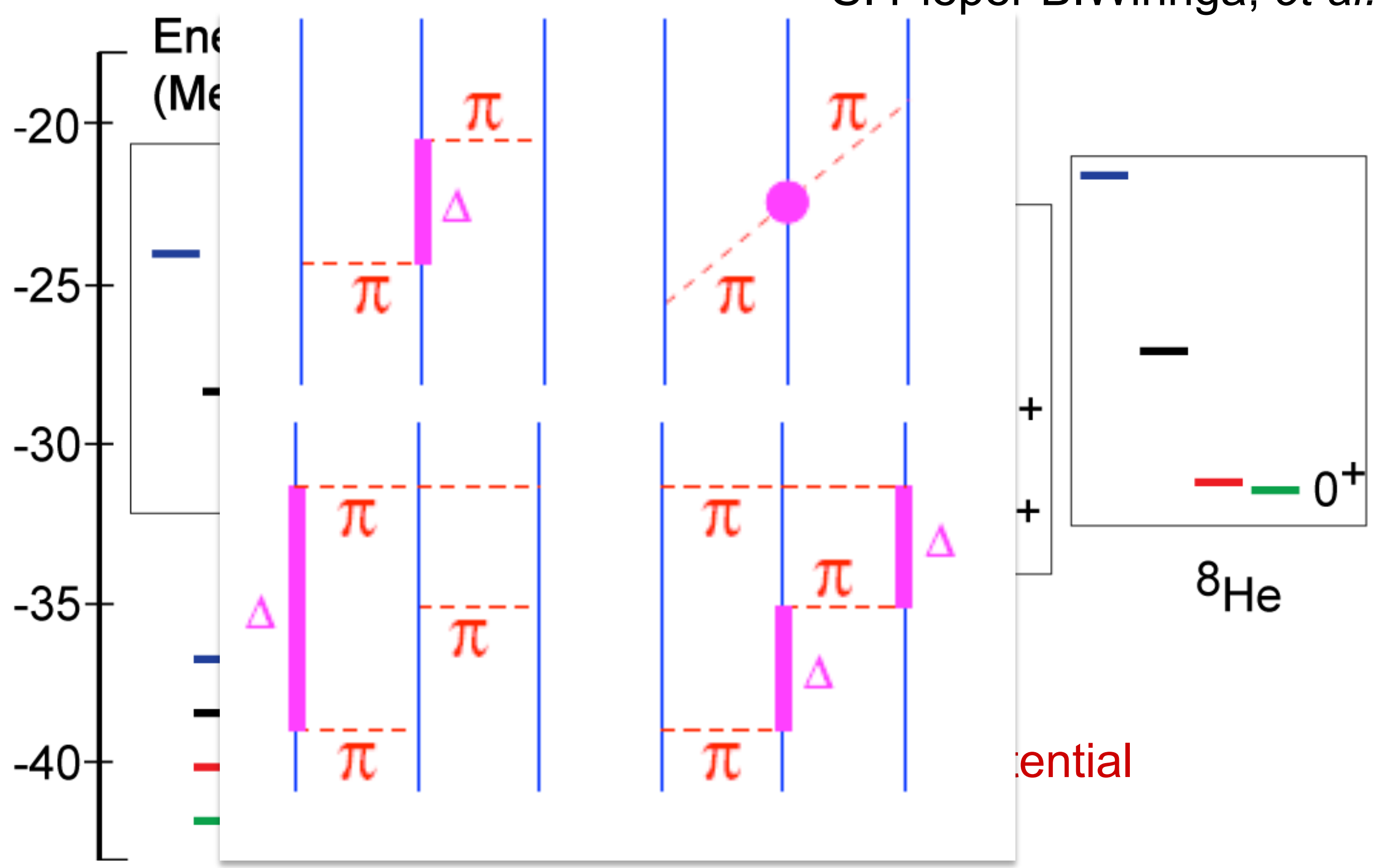
- Greens Function Monte Carlo techniques allow up to mass number 12 to be calculated
- Example blue 2-body forces V_{18}
- S. Pieper B.Wiringa, *et al.*



New information from exotic isotopes

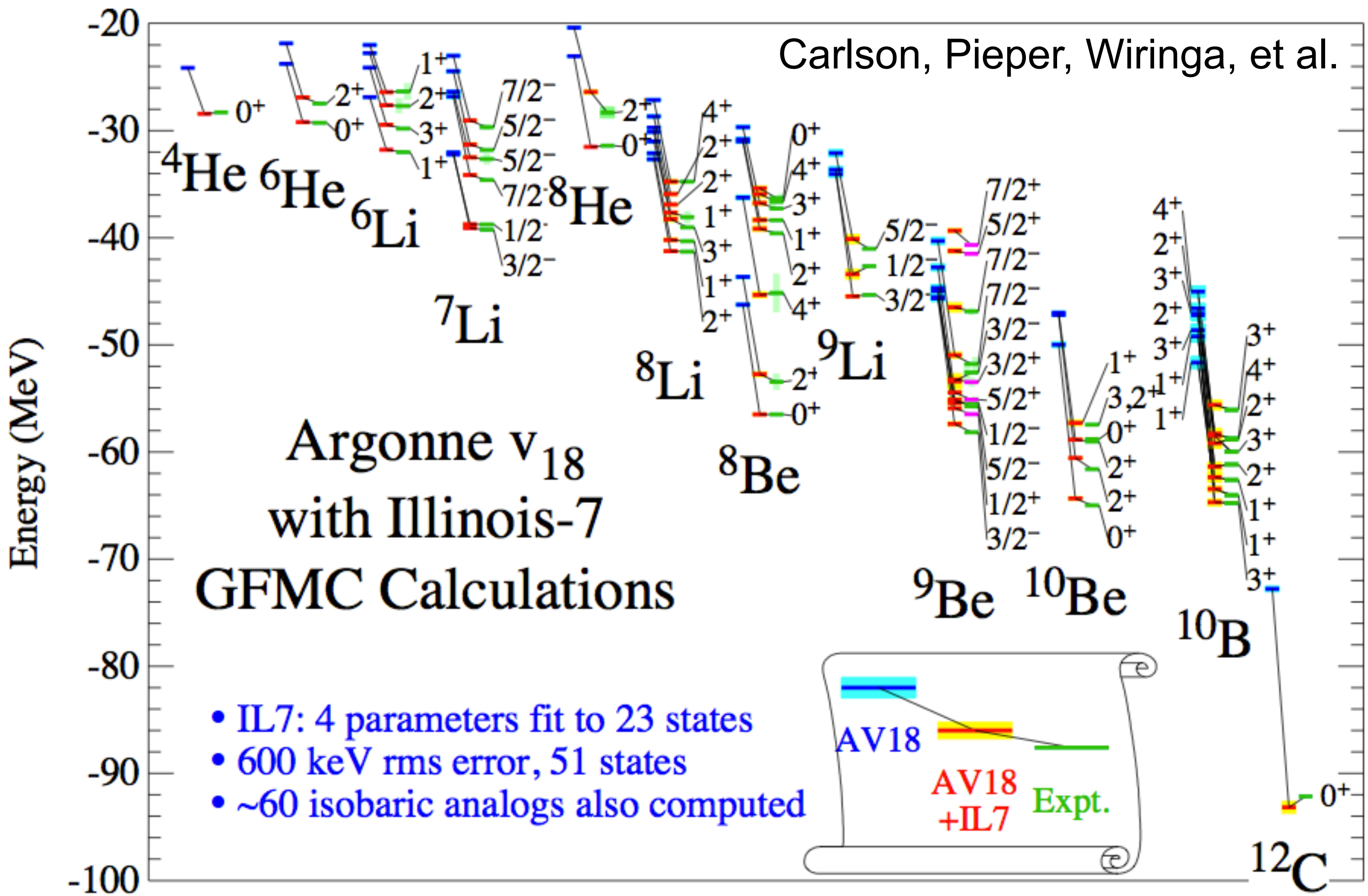
S. Pieper B.Wiringa, *et al.*

- Neutron rich nuclei were key in determining the isospin dependence of 3-body forces and the development of IL-2R from UIX
- New data on exotic nuclei continues to lead to refinements in the interactions



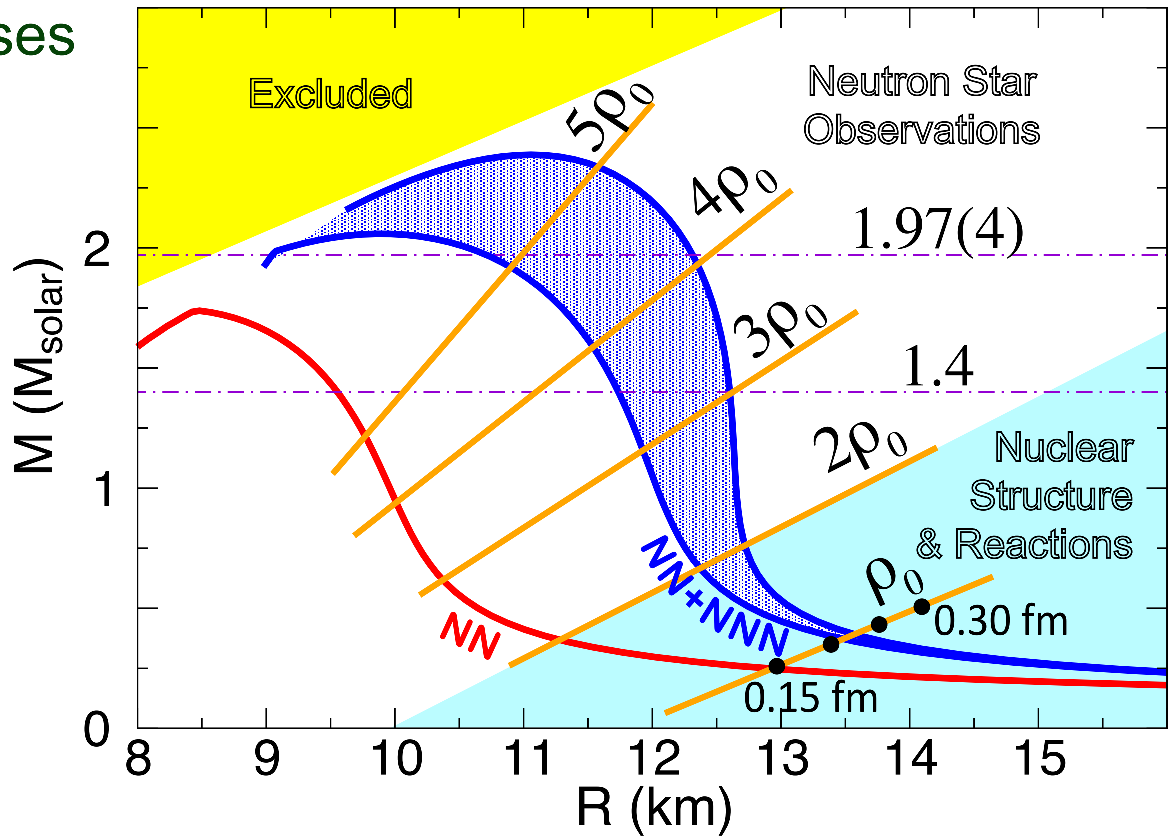
Properties of exotic isotopes are essential in determining NN and NNN potentials

Current status of the GFMC calculations



Influence of 3N forces

- Big Bang Nucleosynthesis: Calculate all key reactions
- Neutron star masses



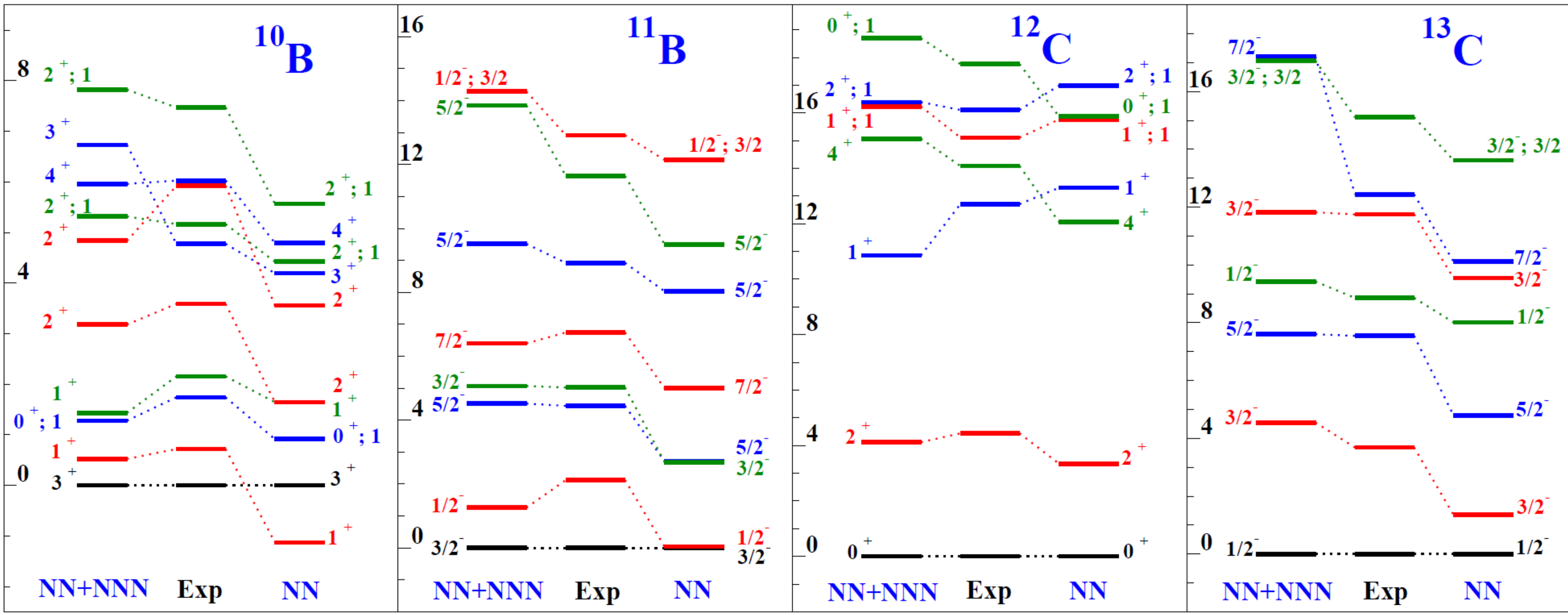
Gandolfi et al., PRC85, 032801 (2012)

A Gerzelis, Guelph

- Half-life of ^{14}C (Maris, Navratil *et al.* PRL), structure of stable calcium isotopes, etc.

“No Core” Shell Model

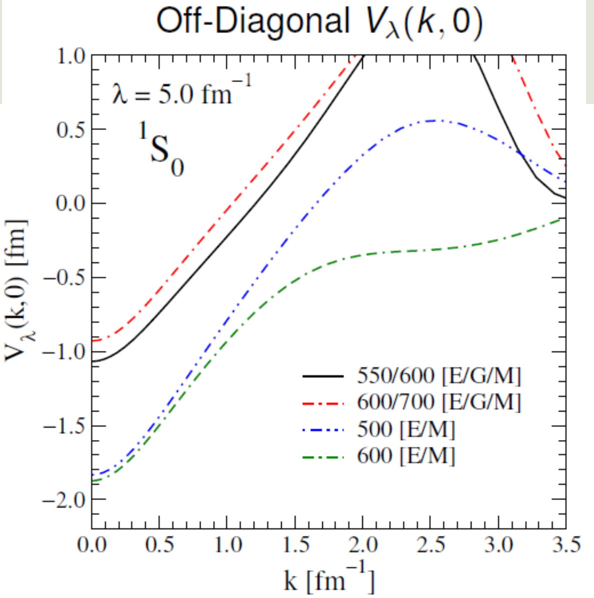
- Another approach that people call *ab initio* is to use the interactions in a shell model
- Start with a realistic interaction and then and diagonalizable in a large basis of many-body states

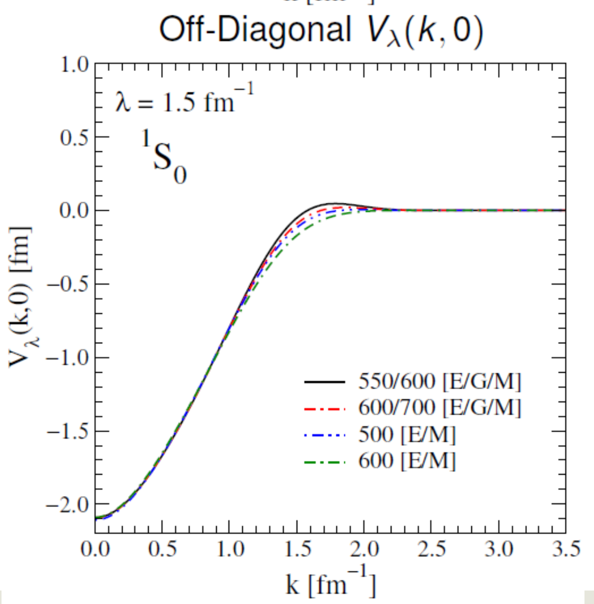


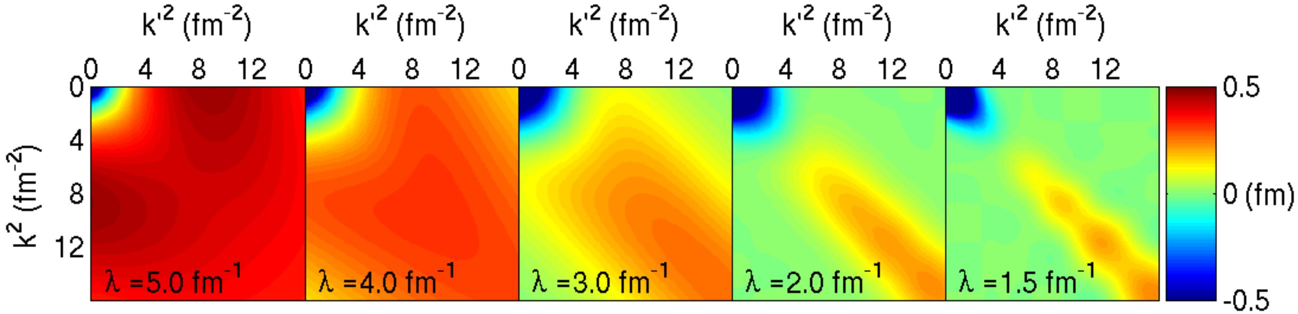
From A. Poves, International School on Exotic Beams, Santiago de Compostela, September 4-11 2010 (see also J Vary, etc.)

Renormalization

- N-N interactions may not be useful for calculating nuclear structure
  - Hard-core is not described by theory based on meson exchange
  - Pauli exclusion limits the interactions that are important
- Solution: renormalization using similarity renormalization group (SRG) or Lee-Suzuki schemes
- Results in so-called V_{lowK} interactions



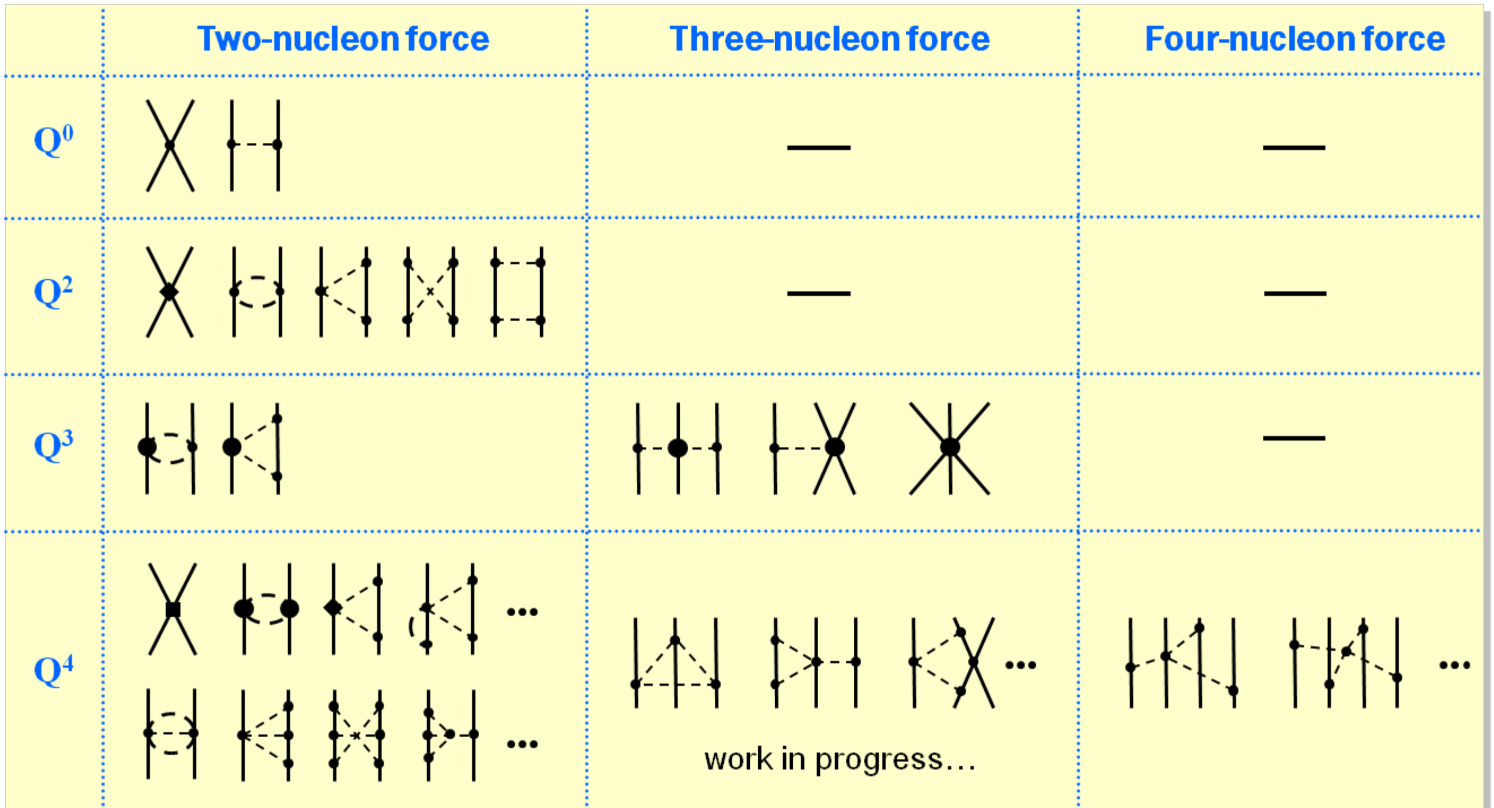




From R. Furnstahl, workshop on Perspectives of the Ab Initio No-Core Shell Model, TRIUMF (2011)

Other Approach: EFT based on QCD Symmetries – "Chiral"

- Use the features of the pion in constructing an effective theory



Cut-off parameter $\Lambda \cong 500$ MeV

Contact interactions have constants that are fit to experiment

Picture from E. Epelbaum

Effective Field Theory, EFT, based on QCD Symmetries (Weinberg,Epelbaum ,Furnstahl, Machleidt, van Kolck, Navrátil,…)

Nuclear mean field: Wood-Saxon Potential

- Nuclear properties can be calculated from the Schrödinger equation: HC=EC (C is the many-body wavefunction)
- H = T + V (kinetic + potential)
- What do we use for V?
- Woods-Saxon potential is a simple approximate V that can be used to infer basic properties
- Note: Like most of what has been done in nuclear structure, this is an empirical approach

$$f_o(r) = \frac{1}{1 + [\exp(r - R_o)/a_o]}$$

$$V(r) = V_o(r) + V_{so}(r)\ \vec{\ell} \cdot \vec{s} + V_c(r)$$

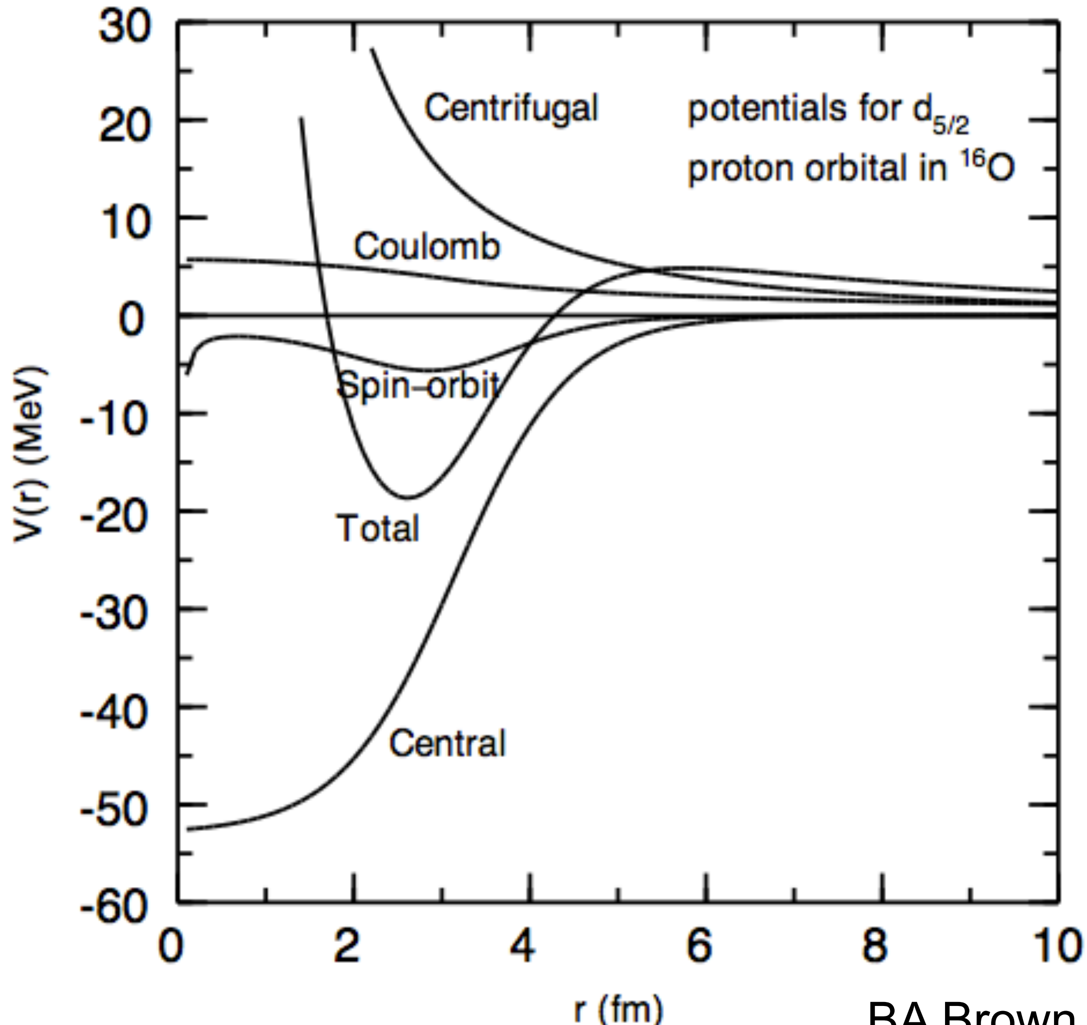
$$V_o(r) = V_o\ f_o(r)$$

$$V_{so}(r) = V_{so}\frac{1}{r}\frac{df_{so}(r)}{dr}$$

BA Brown

Example: The $d_{5/2}$ Orbit in ^{16}O

- Nucleons in the potential obey the Pauli principle
- Number of neutrons or protons per orbit is N = 2 *j* + 1
- Woods-Saxon potential for a $0d_{5/2}$ (refers to n *l* j) proton with an ^{16}O core
- The total and parts of the potential are shown



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Nuclear Many-Body Problem

- H×C=E C
- The energy of a system |C> of n particles can be written

$$E(C) = < C \mid H \mid C > = \sum_{\alpha} < \alpha \mid T \mid \alpha > + \frac{1}{2} \sum_{\alpha\beta} < \alpha\beta \mid V \mid \alpha\beta >$$

- Where the labels α, β are over the states of the n particles
- For one particle in state i above the Fermi level

$$E(Ci) = < Ci \mid H \mid Ci >$$
$$= E(C) + < i \mid T \mid i > + \sum_{\alpha} < i\alpha \mid V \mid i\alpha >$$

- For two particles in state i and j

$$E(Cij) = E(C) + < i \mid T \mid i > + \sum_{\alpha} < i\alpha \mid V \mid i\alpha >$$
$$+ < j \mid T \mid j > + \sum_{\alpha} < j\alpha \mid V \mid j\alpha > + < ij \mid V \mid ij >$$
$$= E(C) + e_i + e_j + < ij \mid V \mid ij >$$

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Hartree Fock Technique

- Minimize E(C) for a trial wave function C(Φ_α(r)…..)

$$\frac{\partial}{\partial \phi_i^*(\vec{r})}\left\{E(C)-\sum_\alpha \lambda_\alpha \int \mid \phi_\alpha(\vec{r}_1)\mid^2 \, d\tau_1\right\}=0$$

$$=T\phi_i(\vec{r})-\lambda_i\phi_i(\vec{r})+\frac{1}{2}\left\{\sum_\beta \int \phi_\beta^*(\vec{r}_2)\,V(\vec{r},\vec{r}_2)\,\phi_i(\vec{r})\,\phi_\beta(\vec{r}_2)\,d\tau_2\right.$$

$$+\sum_\alpha \int \phi_\alpha^*(\vec{r}_1)\,V(\vec{r}_1,\vec{r})\,\phi_\alpha(\vec{r}_1)\phi_i(\vec{r})\,d\tau_1-\sum_\beta \int \phi_\beta^*(\vec{r}_2)\,V(\vec{r},\vec{r}_2)\,\phi_\beta(\vec{r})\,\phi_i(\vec{r}_2)\,d\tau_2$$

$$\left.-\sum_\alpha \int \phi_\alpha^*(\vec{r}_1)\,V(\vec{r}_1,\vec{r})\,\phi_i(\vec{r}_1)\,\phi_\alpha(\vec{r})\,d\tau_1\right\},$$

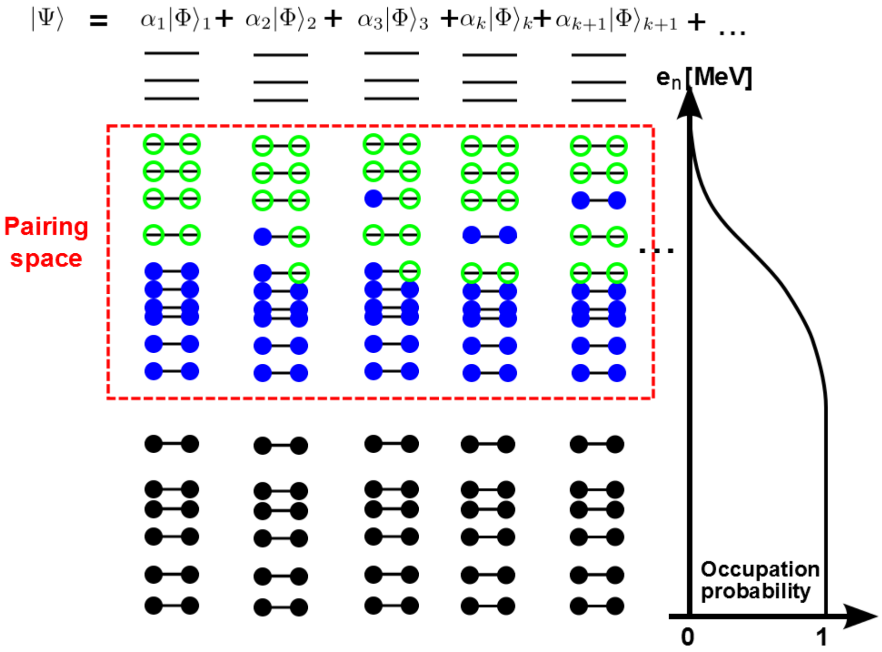
- Lagrange multipliers, λ, to conserve quantum numbers

$$< i \mid T \mid i > + \sum_\alpha < i\alpha \mid V \mid i\alpha > = \lambda_i = e_i$$

- Steps: Guess ϕ, solve for ϕ and λ, calculate integrals, solve, iterate

Hartree-Fock-Bogoliubov Theory

- Hartree-Fock does not universally give ground states of $J^{\pi} = 0^{+}$
  $\Rightarrow$ put particles in pairs of opposite spins
- Way to ensure this is the case is to change the basis to particles to quasi-particles: superposition of particles and holes
- Deformed HFB theory is the cornerstone of the modern nuclear mean-field



N. Schunck

Density Functional Theory

- Widely used in Chemistry (calculation of molecular properties as good as experiment) – based on Hohenberg-Kohn (Phy Rev **136** B864)
- Relies on the variation concept where observables are treated as variational parameters, e.g. local density ρ(r)
- Minimize the variational equation $\delta(E(\rho) - \int V(r)\rho(r)\,dr) = 0$, $E=\langle C|\hat{H}|C\rangle$
- Two step procedure
  - Equation ensures that the total energy is minimized at a fixed ρ(r)
  - Minimization of E(ρ(r)) with ρ(r) gives the exact ground state energy and the exact value of ρ(r) for the ground-state wave function
- **We don't know the correct form.** Example: Skyrme functional

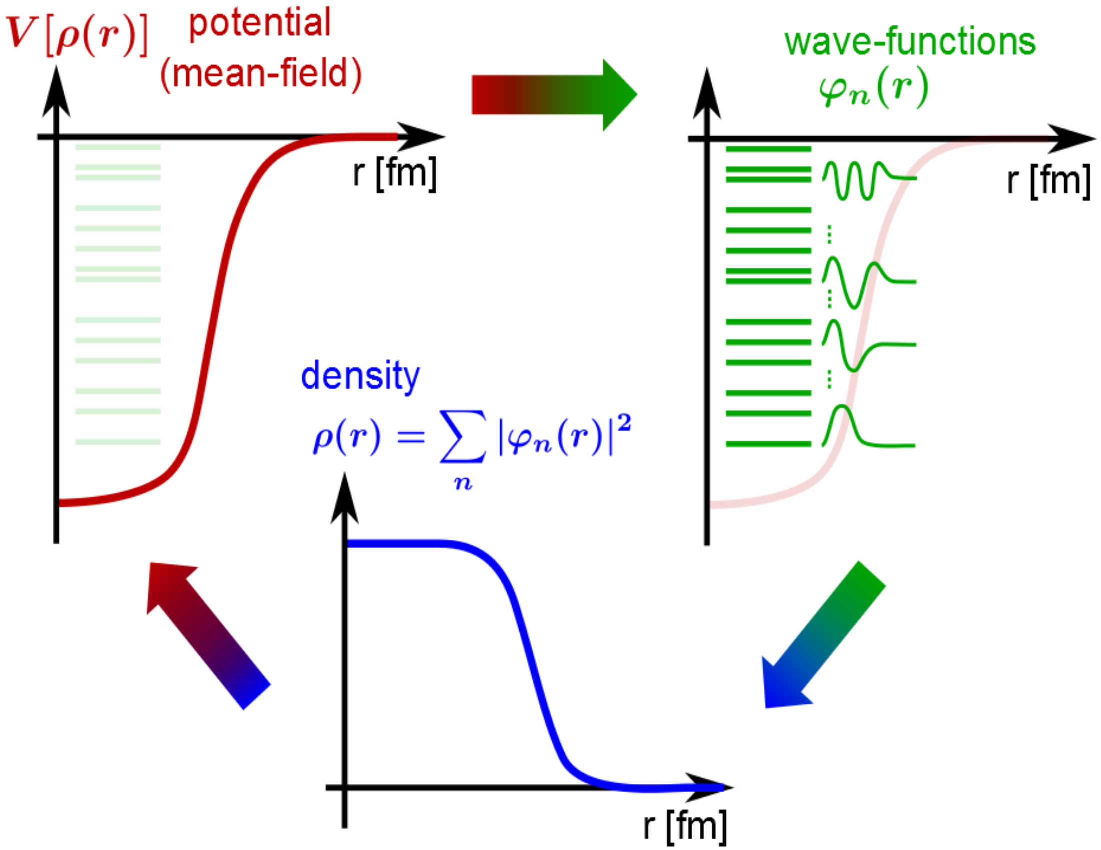
$$
\begin{aligned}
\mathcal{E}[\rho, \tau, \mathbf{J}] \; = \; & \frac{1}{2M}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^{2+\alpha} + \frac{1}{16}(3t_1 + 5t_2)\rho\tau \\
& + \frac{1}{64}(9t_1 - 5t_2)(\nabla\rho)^2 - \frac{3}{4}W_0\rho\nabla\cdot\mathbf{J} + \frac{1}{32}(t_1 - t_2)\mathbf{J}^2
\end{aligned}
$$

- where $\rho(x) = \sum_i |\phi_i(x)|^2$ and $\tau(x) = \sum_i |\nabla\phi_i(x)|^2$ (and $\mathbf{J}$)

S Bogner

Pictorial version of DFT

- N. Schunck, Exotic Beam Summer School 2012

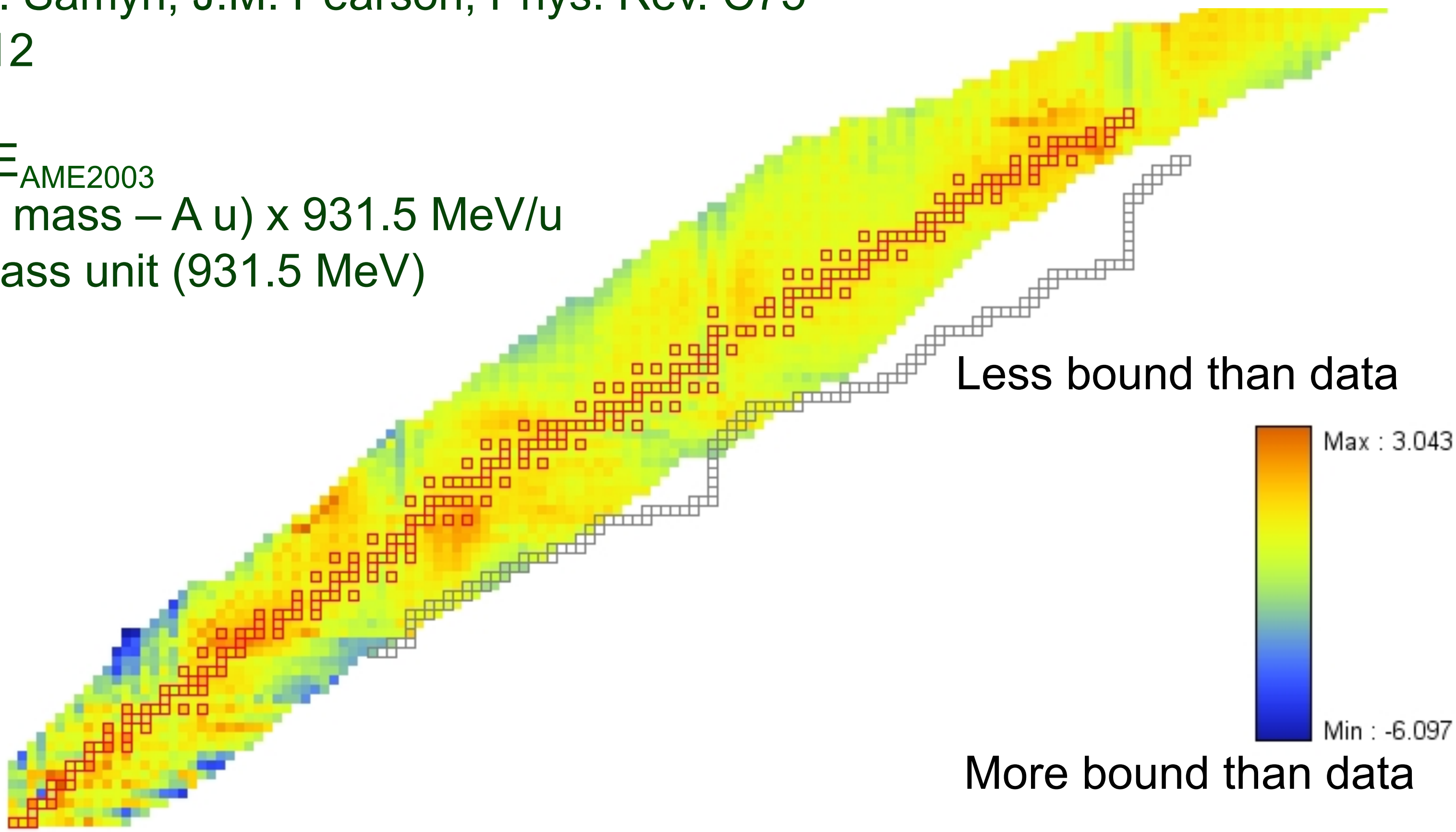


Self-consistent mean-field theory

New Physics from Mass Model Comparison to Data

HFB-14: *Hartree-Fock-Bogoliubov w/delta pairing force*
S. Goriely, M. Samyn, J.M. Pearson, Phys. Rev. C75 (2007) 064312

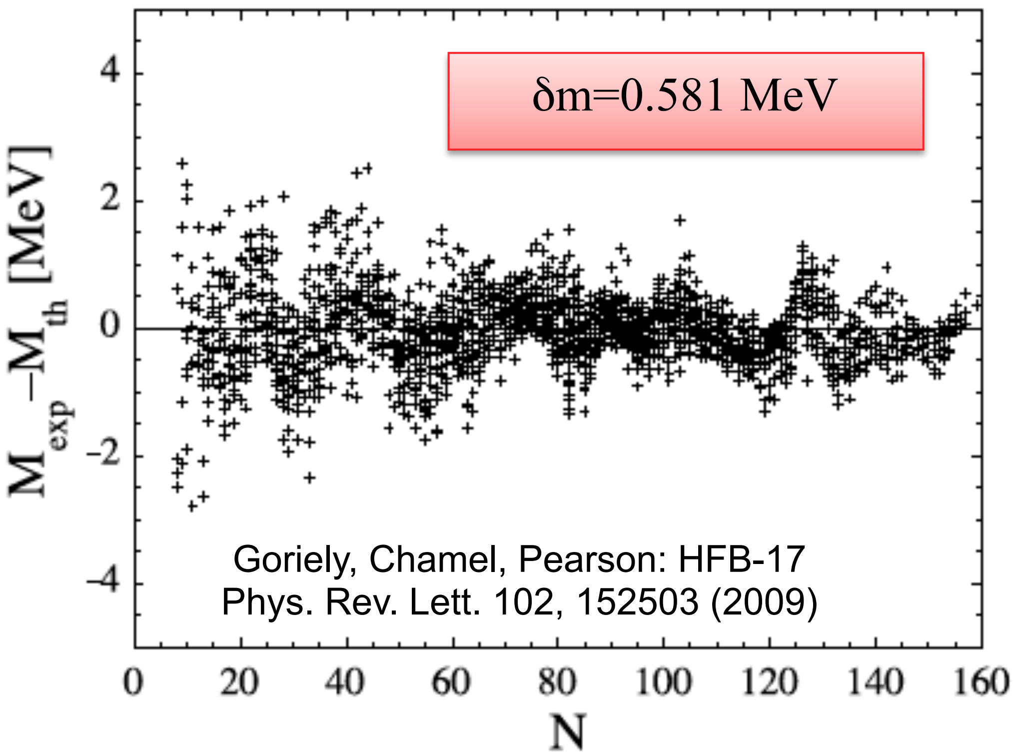
$ME_{HFB14} - ME_{AME2003}$
ME = (Actual mass – A u) x 931.5 MeV/u
u = atomic mass unit (931.5 MeV)



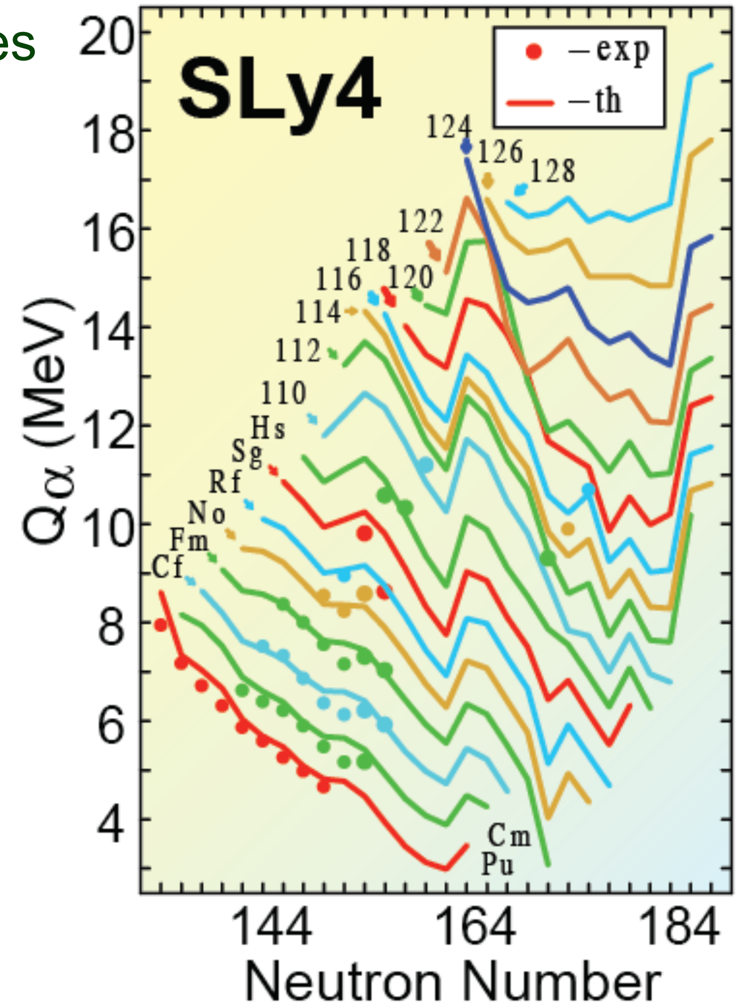
www.nuclear masses.org

Nuclear Density Functional Theory: applications

Mass table

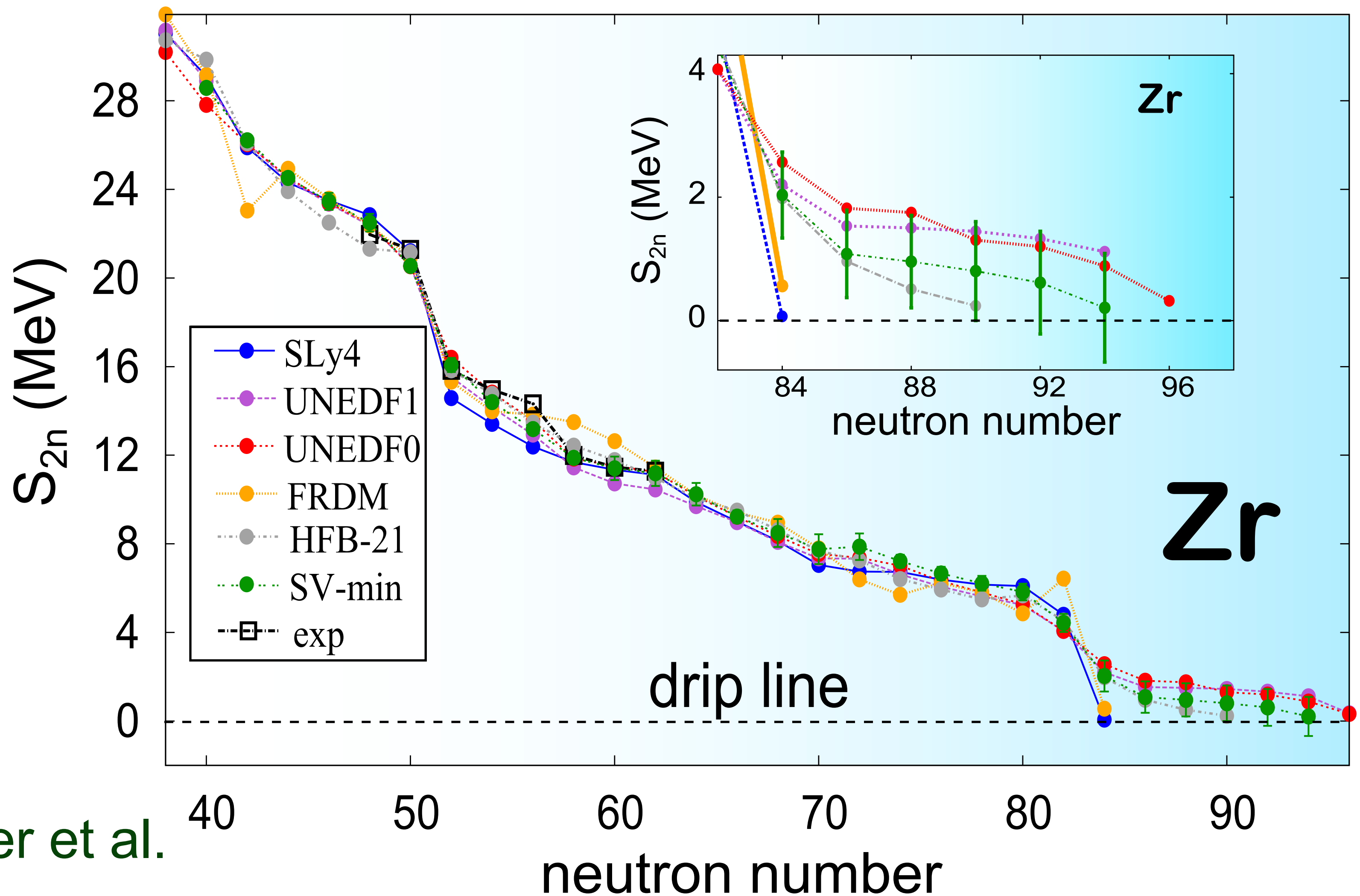


BE differences



Cwiok et al., Nature, 433, 705 (2005)

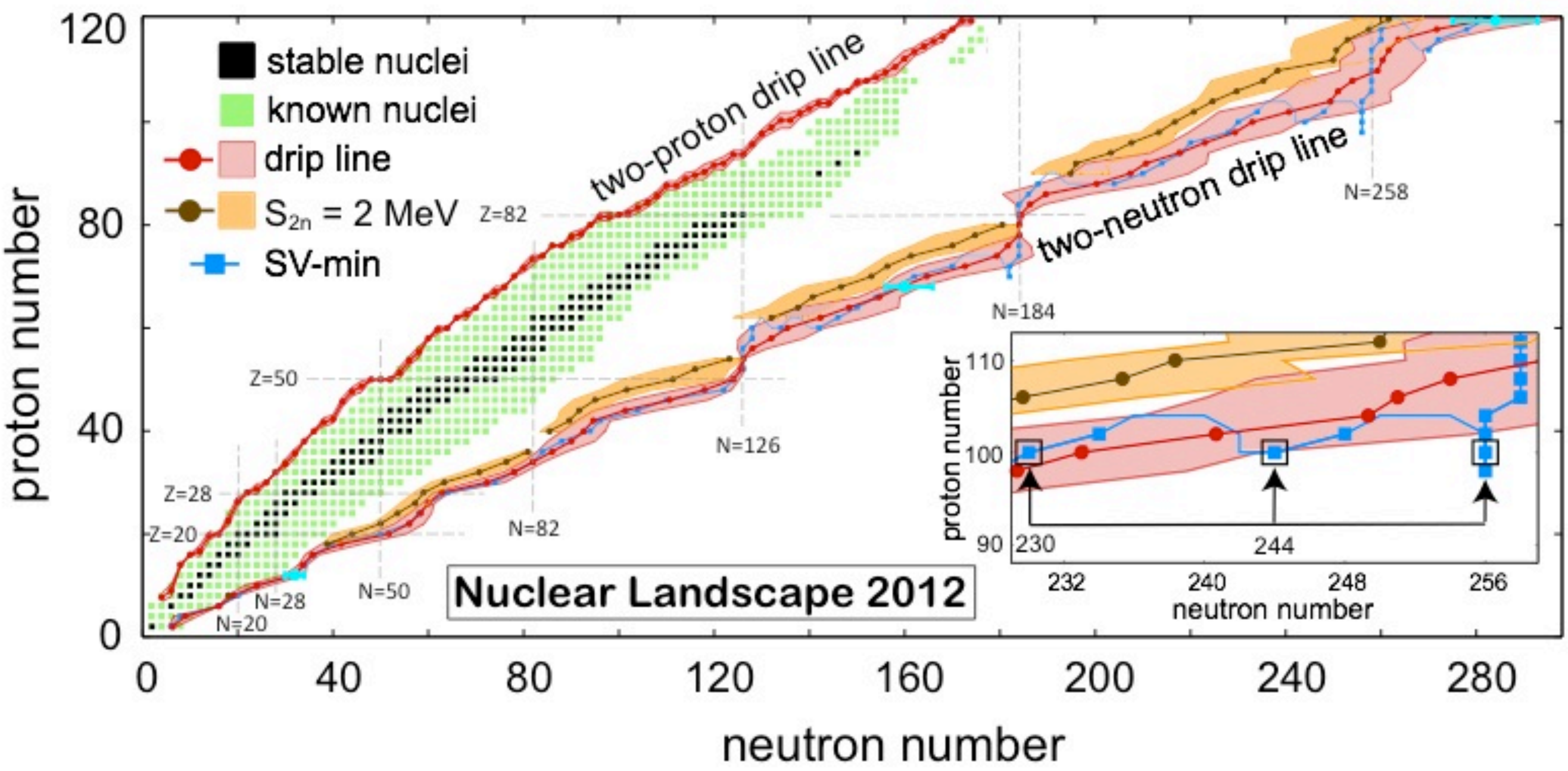
DFT Prediction for Binding of Heavy Zr Isotopes



Erler et al.

Prediction of the limits of the nuclear landscape

265 stable isotopes, 3100 observed, more like 2000 "known"

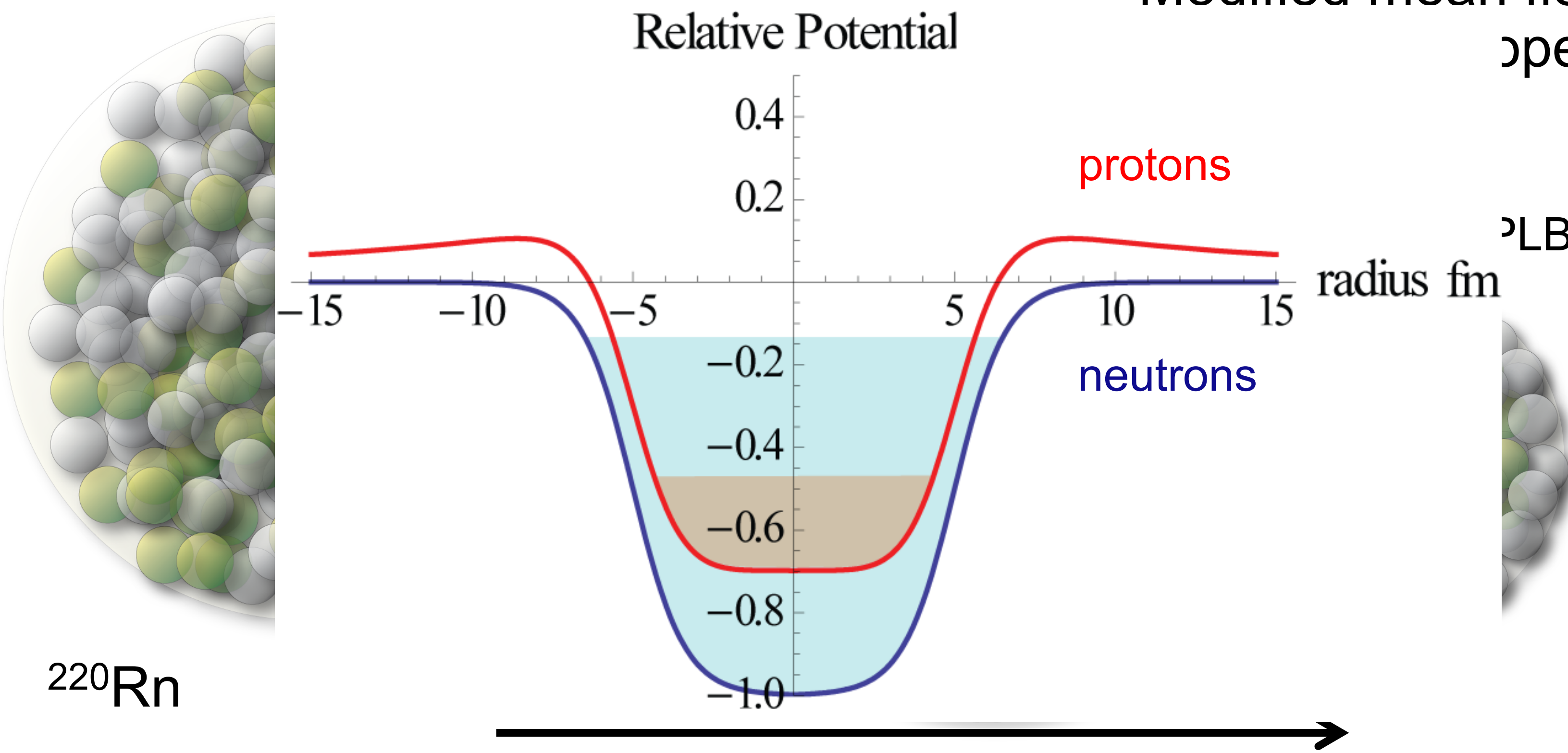


J. Erler et al., Nature 486, 509 (2012)

Weakly bound isotopes have unique features

- Large neutron skins
- Modified mean field

“Normal”



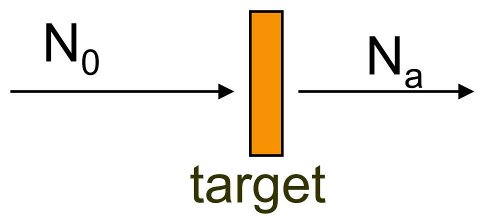
^{220}Rn

^{80}Ni

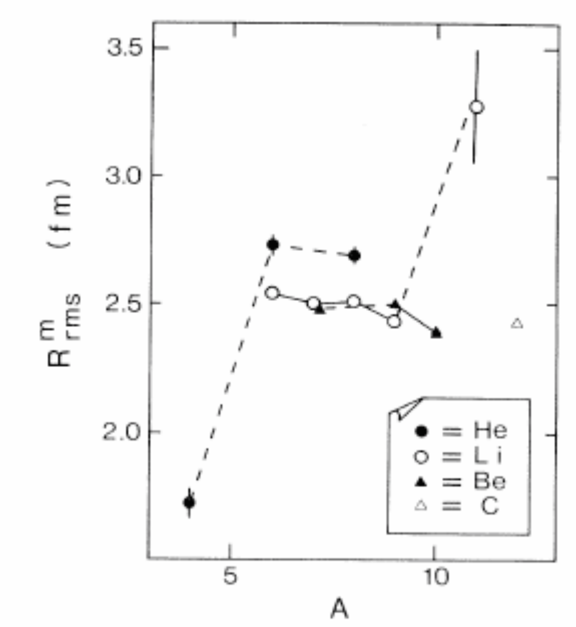
Science: Pairing in low-density material, new tests of nuclear models, open quantum system, interaction with continuum states - Efimov States - Reactions

Experimental "Discovery"

- Evidence for the size of ^{11}Li was found from total interaction cross section measurements (Tanihata PRL1985)



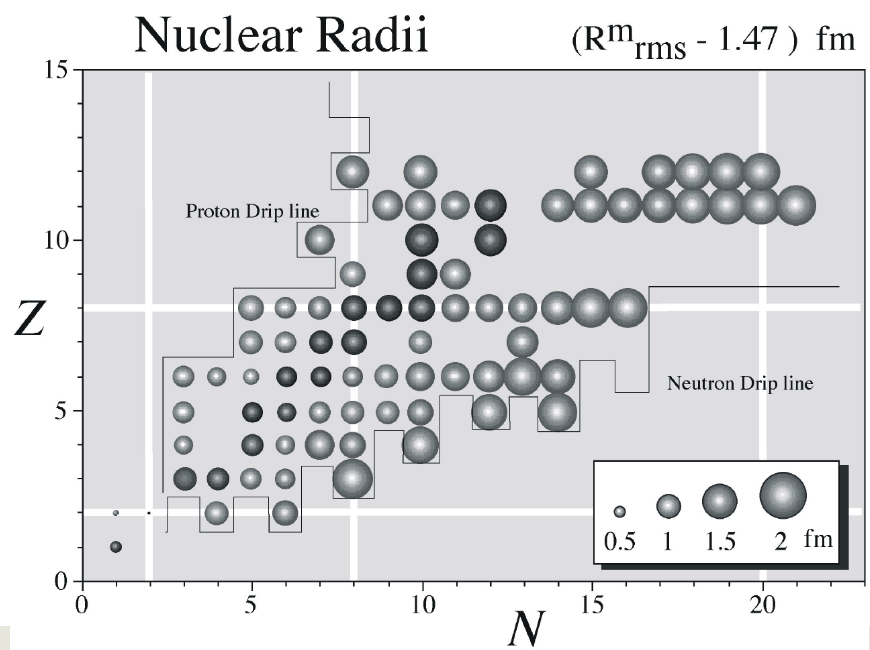
$\sigma_{interaction} = -\mu \ln(N_a/N_0)$



- One of the first thing we learn about nuclei is that Nuclear radii follow the formula: $r = r_0 A^{1/3}$ (Equation 1.2 Wong *Introductory Nuclear Physics*

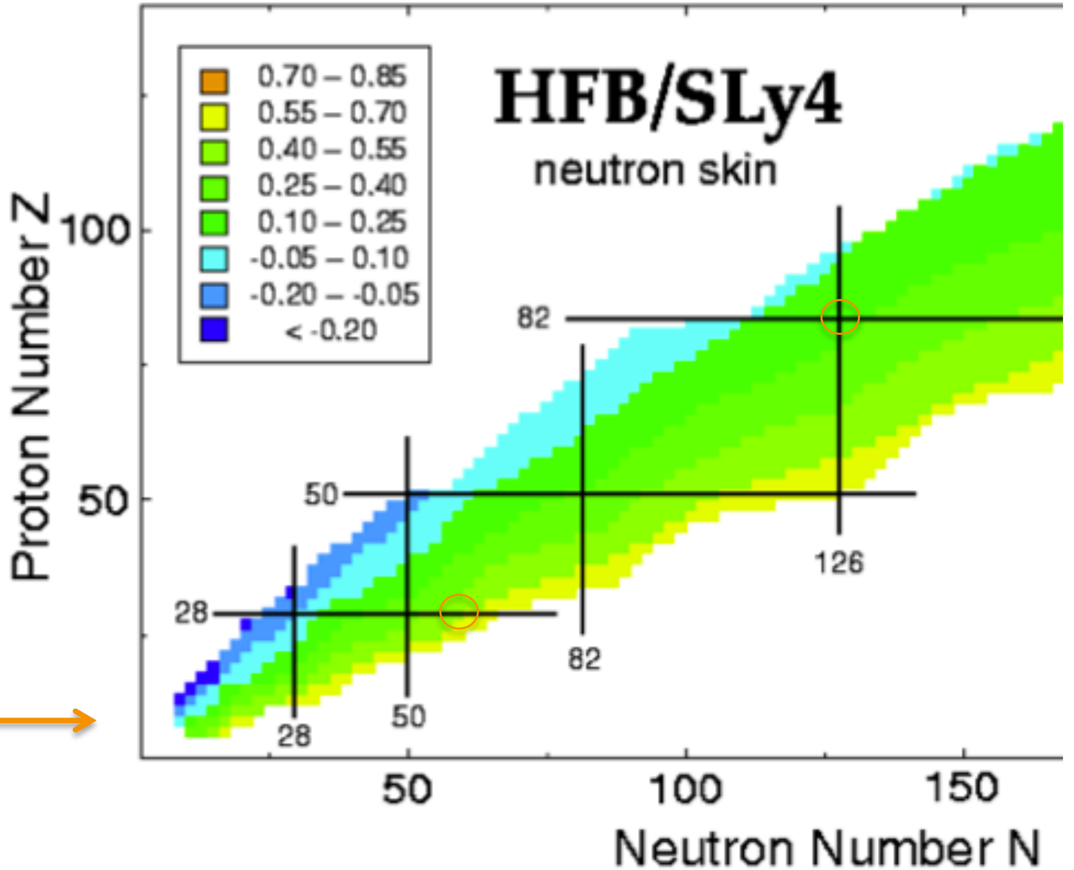
- This is incorrect

I. Tanihata, OSAKA



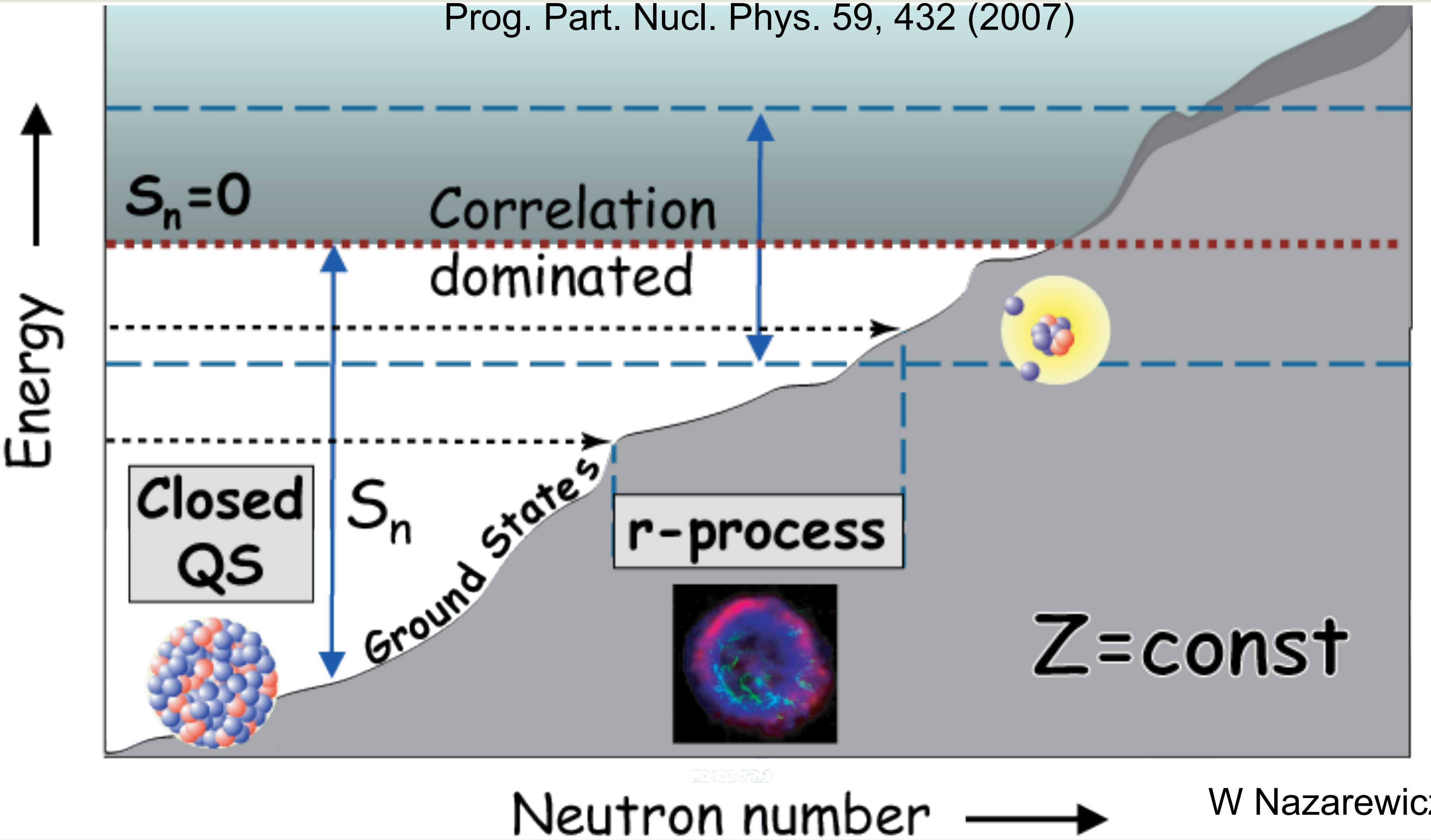
Neutron Skins

- Measurement of the Neutron Radius of ^{208}Pb through Parity Violation in Electron Scattering; S. Abrahamyan et al. (PREX Collaboration) Phys. Rev. Lett. 108, 112502 (2012)
- Result: R_n-R_p=$0.33_{-0.18}^{+0.16}$ fm
- Predictions for ^{84}Ni range from 0.5 – 0.8 fm
- See S. Mizutori et al., Phys.Rev. C61 (2000) 044326



Weakly Bound Nuclei are Open Quantum Systems

Prog. Part. Nucl. Phys. 59, 432 (2007)



W Nazarewicz

What we now know about atomic nuclei

- Nuclear can be approximated by protons, neutrons, and their pair-wise interactions
- Nuclear radii follow the formula $r = r_0 A^{1/3}$ (Equation 1.2 Wong *Introductory Nuclear Physics*)
- The nuclear force has a saturation property where each nucleon can only interact with a few of its neighbors and the total binding energy increases linearly with A.
- Nuclei obey a shell model with magic numbers 2, 8, 20, 28, 50, etc.
- Resonance properties, etc.

- Three body forces are important
- Only true for N~Z nuclei; ^{11}Li has valence orbits as large as ^{208}Pb
- This is only true for the stable isotopes found in nature. Some heavy isotopes of mid-mass nuclei may accept 20+ nuclei with no change in BE
- Magic numbers change depending on relative A/Z
- Neutron number can dramatically change the values away from stability

Cool Questions

- How good is the approximation of neutrons and protons in the nucleus?
- How much are neutrons and protons modified in the nucleus and how is this reflected in nuclear structure?
- What are the interactions at pay in a nucleus and how do we understand them from the underlying QCD?
- Is there a standard model for nuclear structure and what is it? Are there forces and interactions beyond this nuclear standard model?

References

- Overview of nuclear structure - http://folk.uio.no/mhjensen/phy981/alex.pdf
- Shell Model - http://ejc2011.sciencesconf.org/conference/ejc2011/EJC2011version_finale_poves.pdf
- Nuclear Theory Road Map – http://fribusers.org/8_**THEORY**/3_DOCUMENTS/Blue_Book_FINAL.pdf
- Effective Field theory for nuclei - Evgeny Epelbaum, arXiv:1302.3241v1 [nucl-th]
- Density functional theory of nuclei - J.E. Drut, R.J. Furnstahl, L. Platter Prog.Part.Nucl.Phys.64:120-168,2010
- Physics of weakly-bound nuclei - K. Hagino, I. Tanihata, H. Sagawa, arXiv:1208.1583 [nucl-th]