Form Factors with Electrons and Positrons

Part 2: Proton form factor measurements

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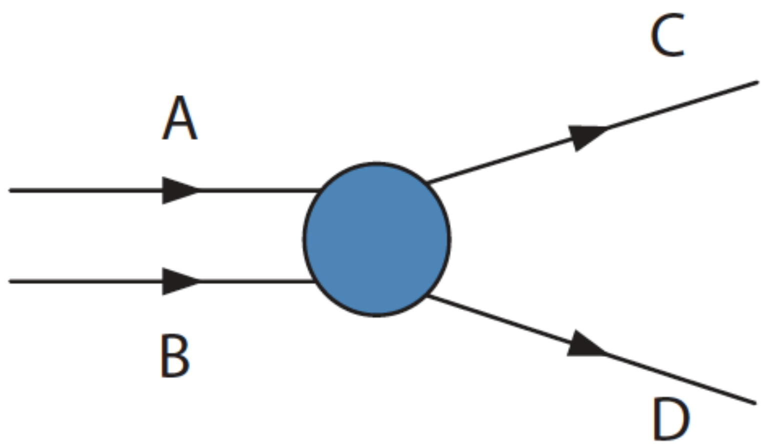
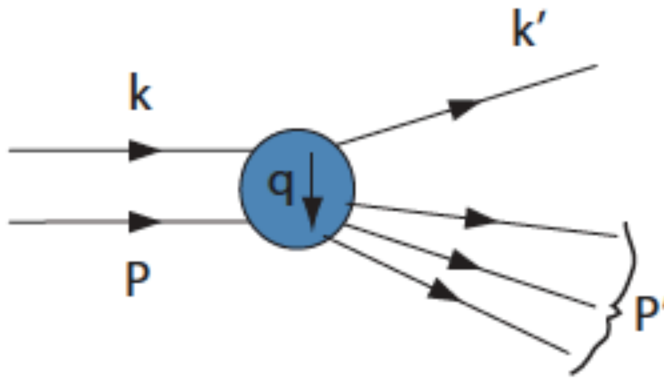
General scattering A + B → C + D



- **Fermi's Golden Rule**

$$\sigma = \frac{(2\pi)^4}{\sqrt{(P_{\mu A} P_B^{\mu})^2 - (m_A m_B c^2)^2}} \int |\mathcal{M}|^2 \mathcal{S}$$
$$\left[\delta^4 (P_A^{\mu} + P_B^{\mu} - p_1^{\mu} \ldots . p_N^{\mu}) \prod_i \delta(p_i^{\mu} p_{i\mu} - m_i^2 c^4) \frac{d^4 p_i}{(2\pi)^3} \right]$$

Lorentz invariants



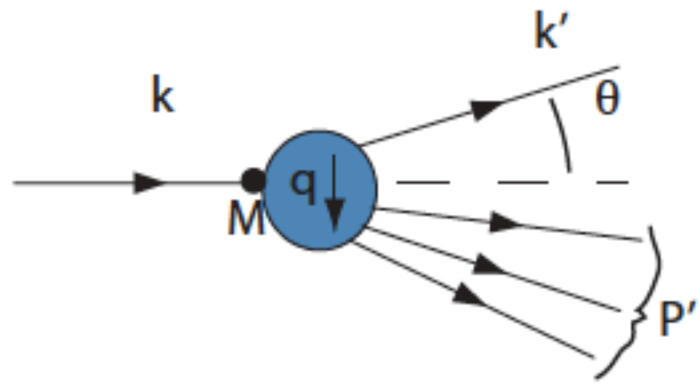
The 5 invariant masses $k^2 = m_\ell^2$, $k'^2 = m_{\ell'}^2$, $P^2 = M^2$, $P'^2 \equiv W^2$, $q^2 \equiv -Q^2$ are invariants. In addition you can define 3 Mandelstam variables:
$s = (k+P)^2$, $t = (k-k')^2$ and $u = (P-k')^2$.
$s+t+u = m_\ell^2 + M^2 + m_{\ell'}^2 + W^2$. There are also handy variables
$\nu = (p \cdot q)/M$, $x = Q^2/2M\nu$ and $y = (p \cdot q)/(p \cdot k)$.

Energy transfer

Bjorken scaling var.

Inelasticity

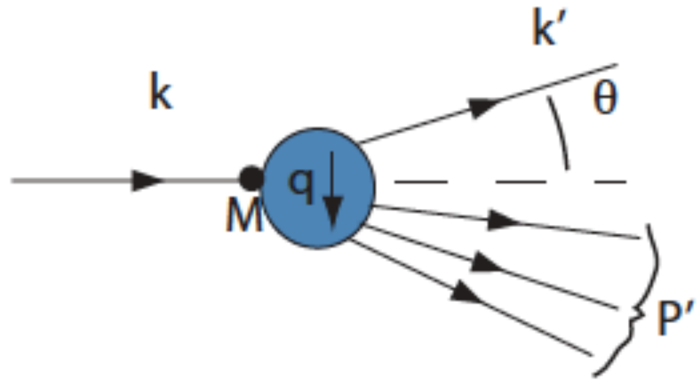
LAB frame kinematics



The beam k is going in the z direction. Confine the scatter to the $x-z$ plane.

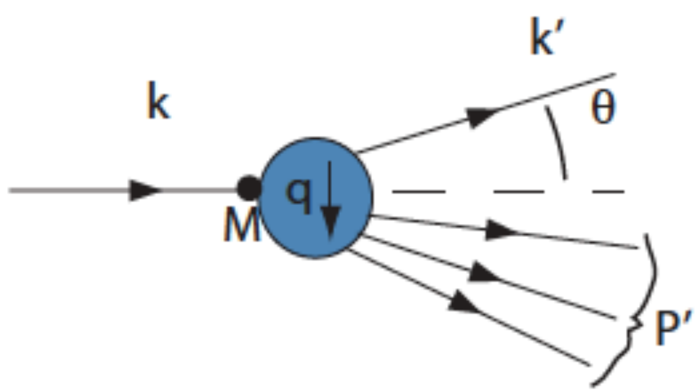
$$\begin{aligned} k^\mu &= (E_k, 0, 0, k) \\ P^\mu &= (M, 0, 0, 0) \\ k'^\mu &= (E'_k, k'\sin\theta, 0, k'\cos\theta) \\ q^\mu &= k^\mu - k'^\mu \end{aligned}$$

LAB frame kinematics



$$
\begin{aligned}
s &= E_{CM}^2 = 2E_k M + M^2 - m^2 \\
t &= -Q^2 = -2E_k E_k' + 2kk' \cos\theta + m_k^2 + m_k'^2 \\
\nu &= (p \cdot q)/M = E_k - E_k' \qquad \text{energy transfer to target} \\
y &= (p \cdot q)/(p \cdot k) = (E_k - E_k')/E_k \qquad \text{the inelasticity} \\
P'^2 &= W^2 = 2M\nu + M^2 - Q^2 \quad \text{invariant mass of } P'^\mu
\end{aligned}
$$

LAB frame kinematics



Final States

- Elastic scattering: W = M, $\nu = Q^2/2M$
- Inelastic scattering: W > M + m_π
- Resonance scattering: W = M_R

$$
\begin{aligned}
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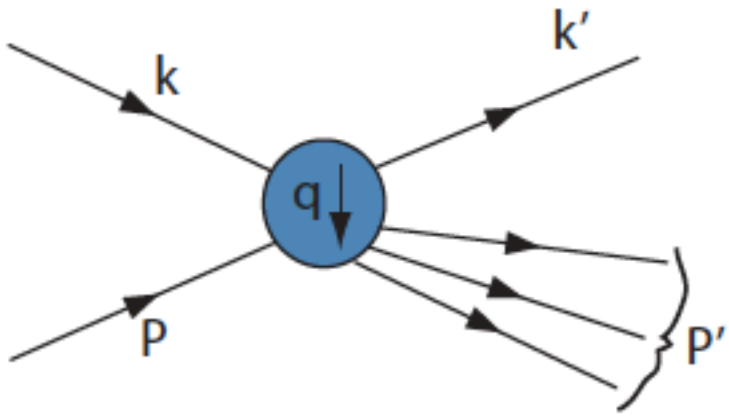
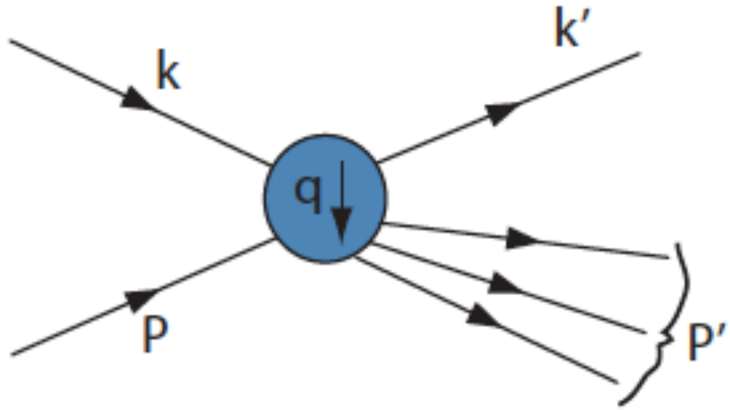
CM frame kinematics



The beam k is going in the z direction. Confine the scatter to the $x-z$ plane.

$$
\begin{aligned}
k^\mu &= (E_k, 0, 0, k) \\
P^\mu &= (E_M, 0, 0, -k) \\
k'^\mu &= (E'_k, k'\sin\theta, 0, k'\cos\theta) \\
q^\mu &= k^\mu - k'^\mu
\end{aligned}
$$

CM frame kinematics



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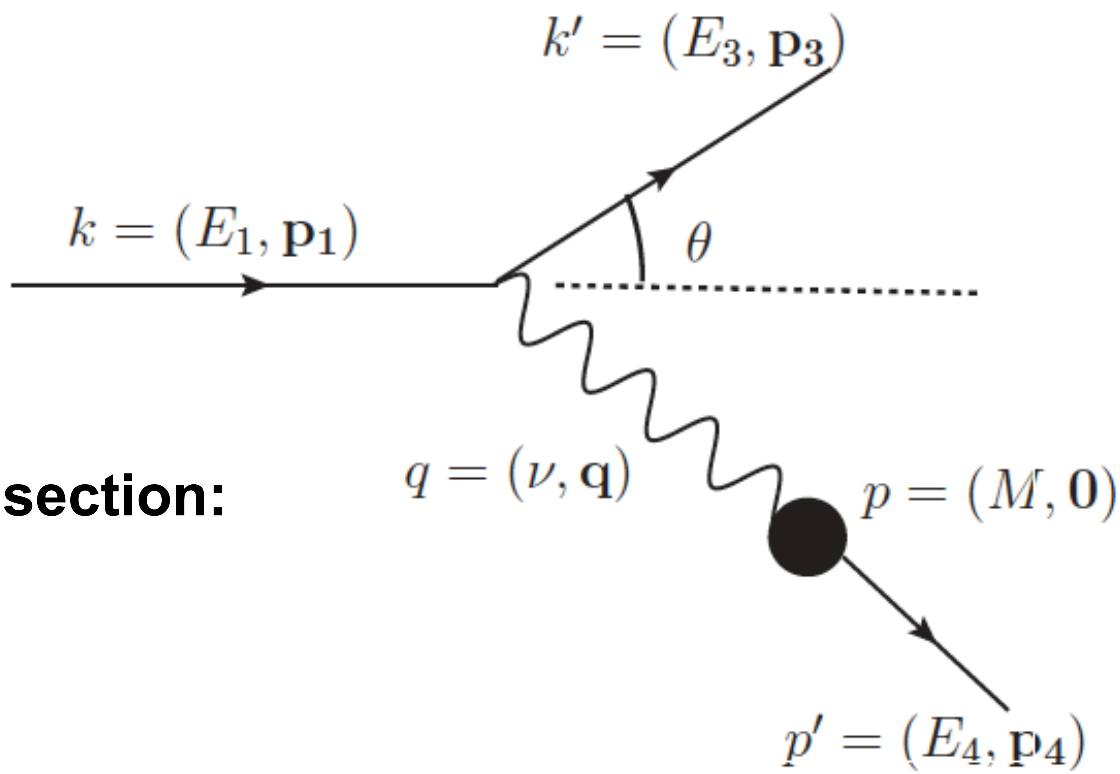
\begin{aligned}
s &= E_{CM}^2 = (E_k + E_M)^2 \\
t &= -Q^2 = m_k^2 + m_k'^2 - 2(E_k E_k' - kk' \cos\theta_{CM}) \\
\nu &= (p \cdot q)/M = \frac{(E_k - E_k')E_M + k(k - k'\cos\theta)}{M} \\
y &= (p \cdot q)/(p \cdot k) = \frac{(E_k - E_k')E_M + k(k - k'\cos\theta)}{E_k E_m + k^2}
\end{aligned}
$$

Lepton-lepton scattering



- **Unpolarized scattering cross section:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{ME_1}\right)^2 |\bar{M}|^2$$

- **Lepton-lepton scattering (Feynman, Dirac):**

$$-iM = \bar{u}(k')(ig_e\gamma^\mu)u(k)\left(-i\frac{g_{\mu\nu}}{q^2}\right)\bar{u}(p')(-ig_e\gamma^\nu)u(p)$$

- **The matrix element accounts for the strength of the interaction $\alpha = 2\pi e^2/hc = 1/137$, higher orders suppressed by powers of α**

  → One-photon exchange approximation (Born approximation)

Lepton-lepton scattering

- **Matrix element: average over initial, sum over final spins**

$$\langle |\bar{M}|^2 \rangle = \frac{8e^4}{(k-k')^4}[(k.p)(k'.p')+(k.p')(p.k')-(k.k')M^2-(p.p')m^2+2m^2M^2]$$

$$= \frac{e^4 M^2}{E_1 E_3 \sin^4\theta/2}\left(\cos^2\theta/2 - \frac{q^2}{2M^2}\sin^2\theta/2\right) \qquad k \gg m \quad \text{(ERL)}$$

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- **Dirac cross section (pointlike spin-1/2 target)**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\theta/2}\frac{E_3}{E_1}\left(\cos^2\theta/2 - \frac{q^2}{2M^2}\sin^2\theta/2\right)$$

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- **Mott cross section (pointlike spin-0 target)**

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)}\frac{E_3}{E_1}\cos^2(\theta/2) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{rec}^{-1}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{\alpha}{2E_1}\right)^2 \left(\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\right) \qquad f_{rec} = \frac{E_1}{E_3} = 1 + \frac{2E_1}{M}\sin^2(\theta/2)$$

Lepton-nucleon scattering

- **Lepton-lepton scattering:**

$$-iM = \bar{u}(k')(ig_e\gamma^\mu)u(k)\left(-i\frac{g_{\mu\nu}}{q^2}\right)\bar{u}(p')(-ig_e\underline{\gamma^\nu)u(p)}$$

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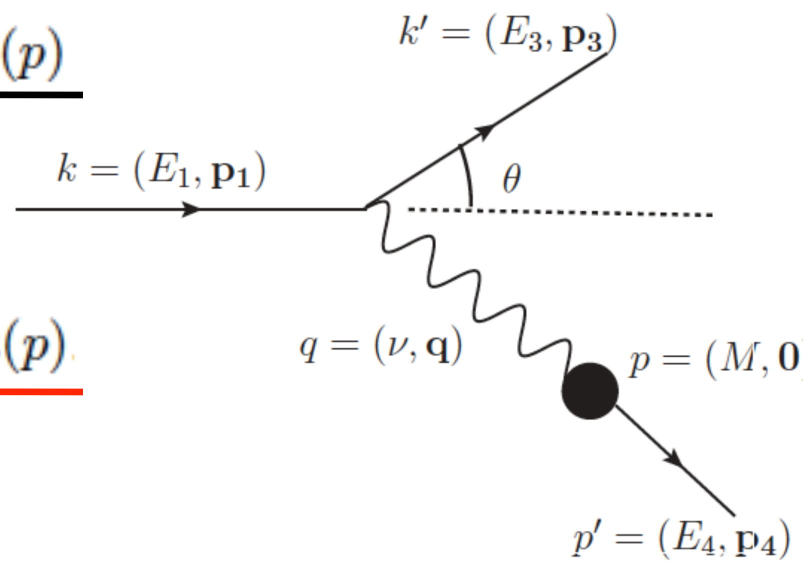

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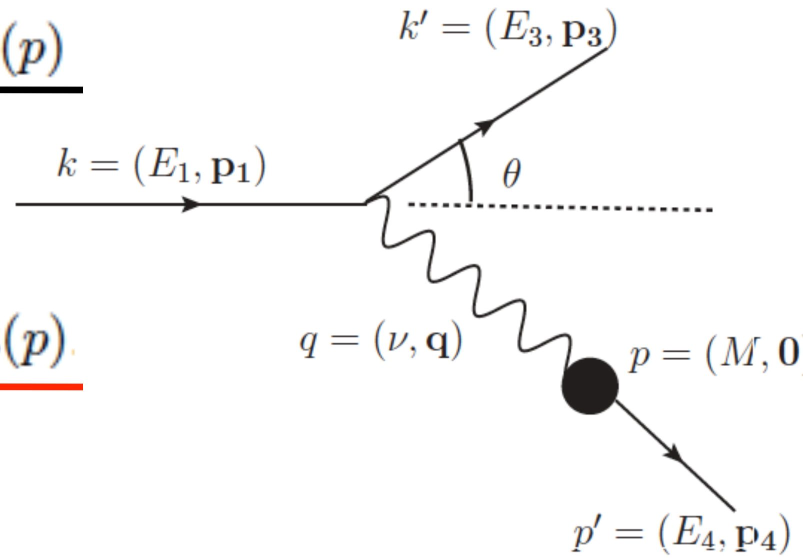

- **Nucleon vertex factor (current)**

$$\Gamma^\nu = \gamma^\nu F_1(q^2) + i\sigma^{\nu\alpha}\frac{q_\alpha}{2M}F_2(q^2)$$

Dirac (F_1) and Pauli (F_2) “form factors”

$$Q^2 = -q^2 \qquad \tau = \frac{Q^2}{4M^2}$$

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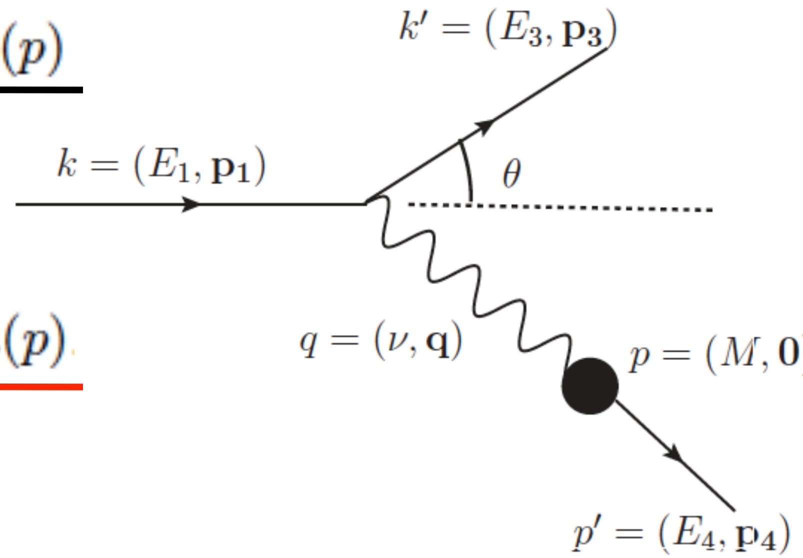

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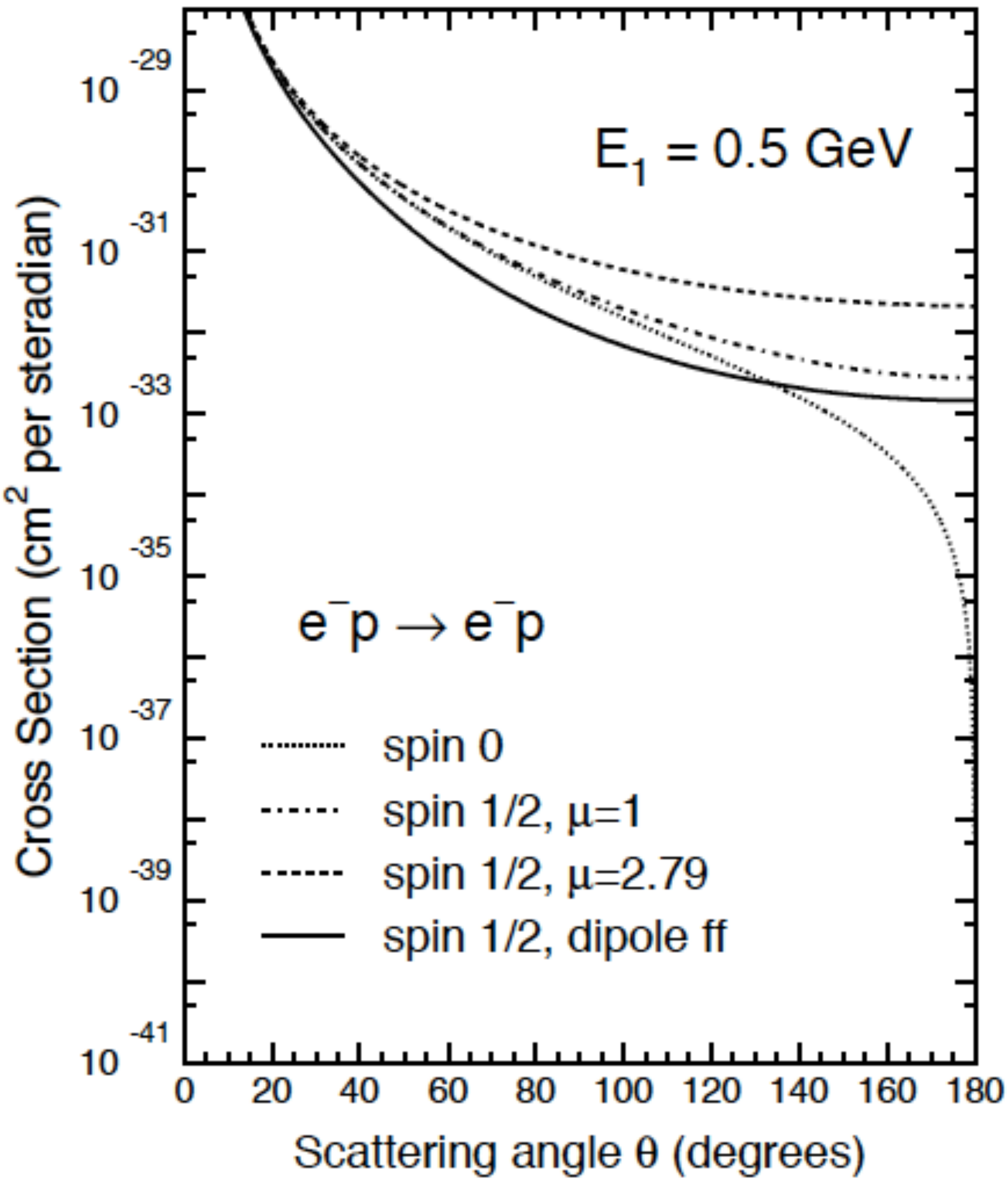
- **Differential (Rosenbluth) cross section**

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} f_{rec} = A(Q^2) + B(Q^2)\tan^2\frac{\theta_e}{2}$$

Rosenbluth formula

$$= F_1^2(Q^2) + \tau\left[F_2^2(Q^2) + 2\left(F_1(Q^2) + F_2(Q^2)\right)^2 \tan^2(\theta_e/2)\right]$$

Elastic cross section affected by structure



Sachs form factors

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} f_{rec} = A(Q^2) + B(Q^2)\tan^2\frac{\theta_e}{2}$$

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- **Sachs form factors can be obtained from Dirac and Pauli form factors**

$$G_E(q^2) = F_1(q^2) - \tau F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

and vice versa

$$F_1(q^2) = \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$F_2(q^2) = \frac{G_M(q^2) - G_E(q^2)}{1+\tau}$$

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$$F_2(q^2) = \frac{G_M(q^2) - G_E(q^2)}{1+\tau}$$

- **Normalization at $Q^2 = 0$**

$$F_1^p(0) = G_E^p(0) = 1; \quad F_1^n(0) = G_E^n(0) = 0;$$

$$F_2^p(0) = G_M^p(0) - G_E^p(0) = \boxed{\kappa_p = \mu_p - 1 = 1.79\,\mu_N}$$

Proton

$$F_2^n(0) = G_M^n(0) - G_E^n(0) = \boxed{\kappa_n = \mu_n = -1.91\,\mu_N}$$

Neutron

Sachs form factors

- **With Sachs form factors the Rosenbluth formula becomes simpler**

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} f_{rec} = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta_e}{2}$$

Only squares of Sachs form factors occur

Sachs form factors

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- **Introducing the "virtual photon polarization"**

$$\epsilon = [1 + 2(1+\tau)\tan^2(\theta_e/2)]^{-1}$$

Forward angle: θ = 0° → ε = 1 Backward angle: θ = 180° → ε = 0

Sachs form factors

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Forward angle: θ = 0° → ε = 1 Backward angle: θ = 180° → ε = 0

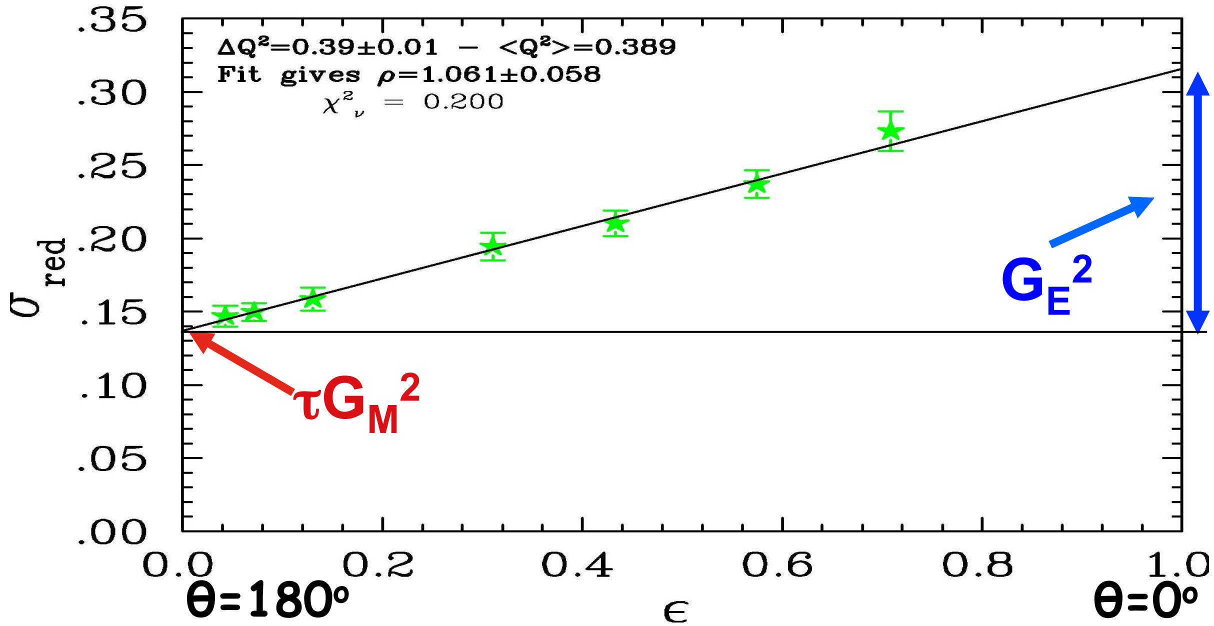
- **Obtain simple expression for a "reduced cross section"**

$$\sigma_r := \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{red}} = \epsilon(1+\tau) f_{rec} \left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Mott}$$

$$\boxed{= \epsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}$$

$$\tau = \frac{Q^2}{4M^2}$$

Rosenbluth method



$\sigma_{red} = \varepsilon G_E^2 + \tau G_M^2$

➔ **Determine**

$|G_E|$, $|G_M|$, $|G_E/G_M|$

- **In One-photon exchange, form factors are related to radiatively corrected elastic electron-proton scattering cross section**

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} = S_0 = A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}$$

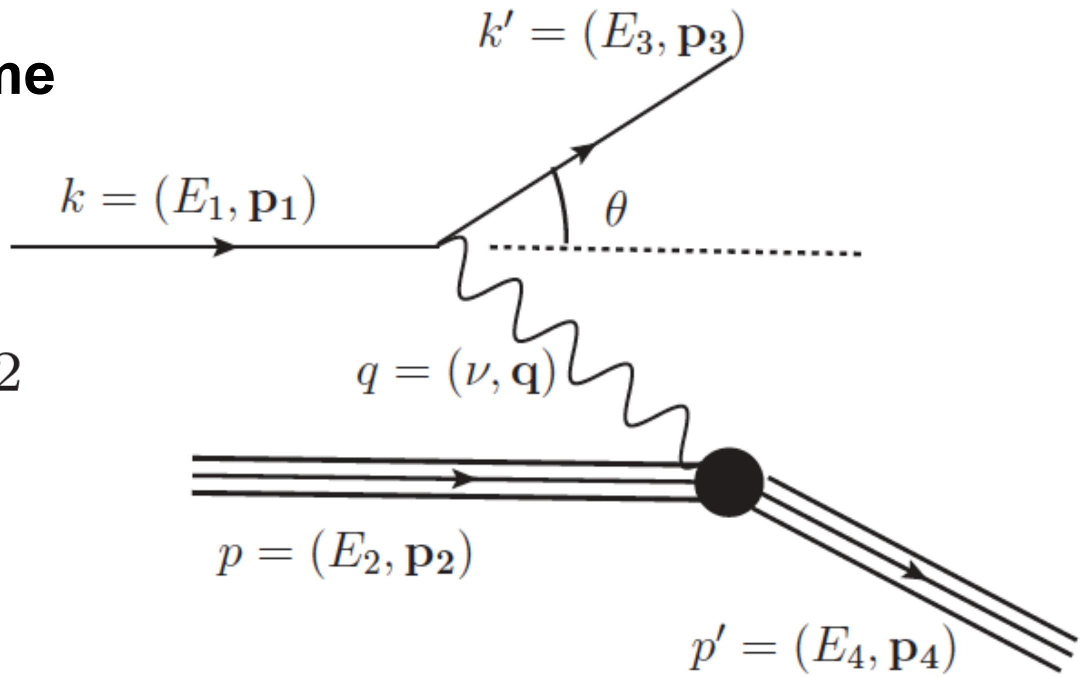
$$= \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}$$

$$= \frac{\epsilon\, G_E^2 + \tau G_M^2}{\epsilon\,(1+\tau)}, \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Fourier transform

- **Breit frame: defined as frame in which no energy is transferred (ν=0):**

$$\vec{p}_2 = -\vec{p}_4 \;\rightarrow\; Q^2 = \vec{q}^{\,2}$$

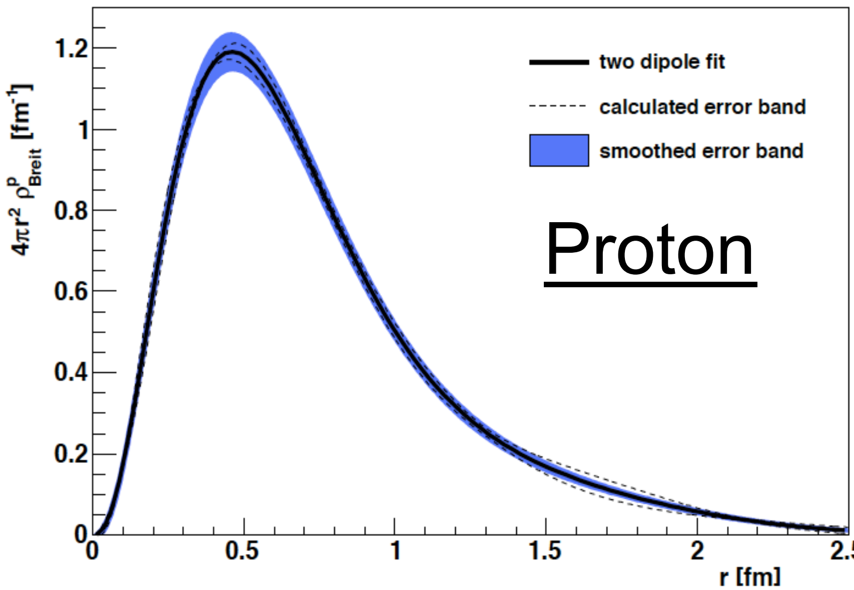


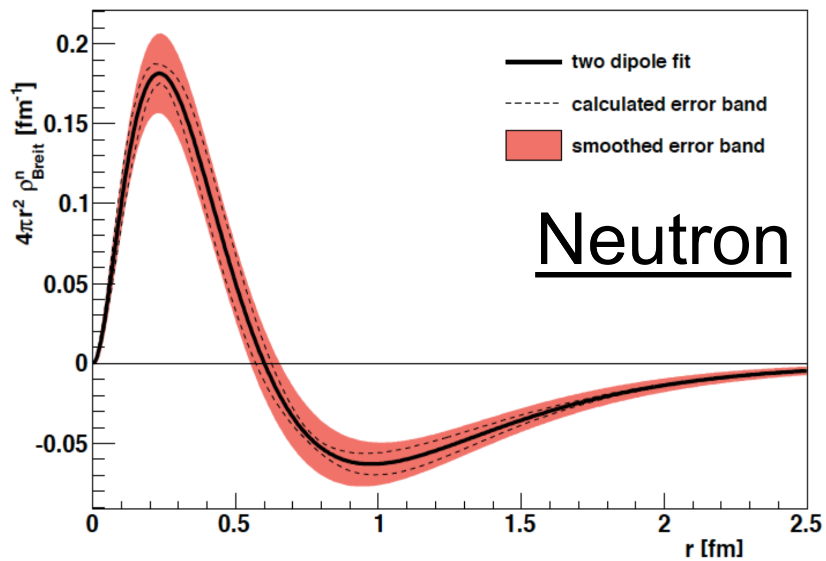
- **The Sachs form factors can be interpreted as spatial charge and magnetization density distributions in the Breit frame obtained via Fourier transforms**

$$G_E = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r} \qquad G_M = \int \mu(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$$

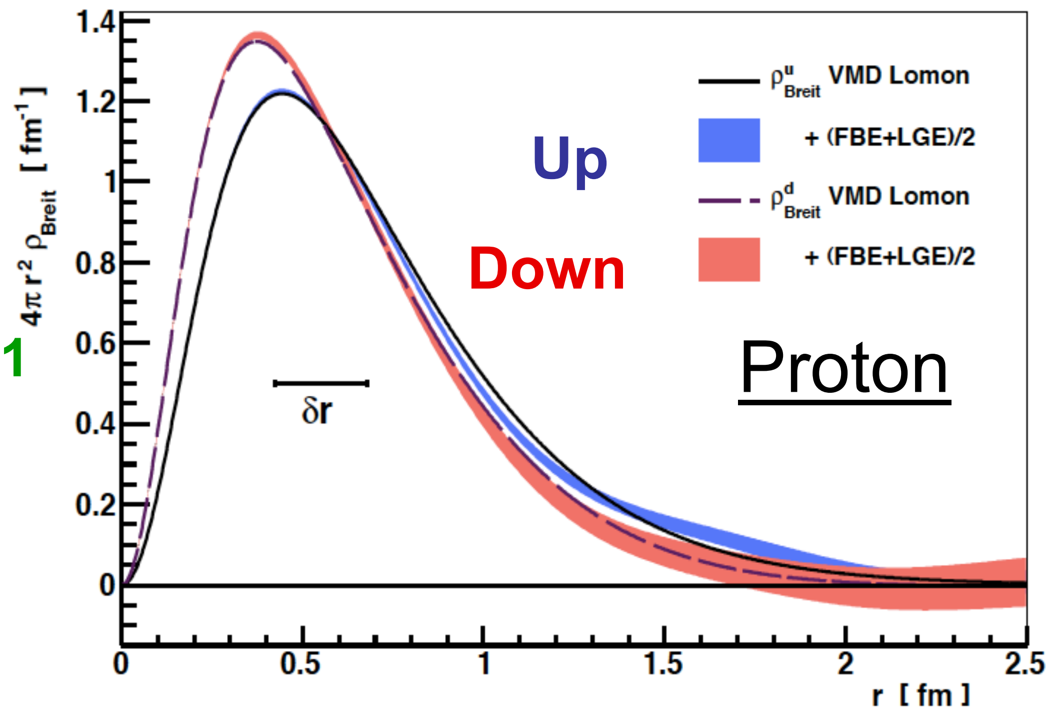
- **However, the effect of recoil prohibits the interpretation as rest frame distributions!**

Spatial distributions in the Breit frame



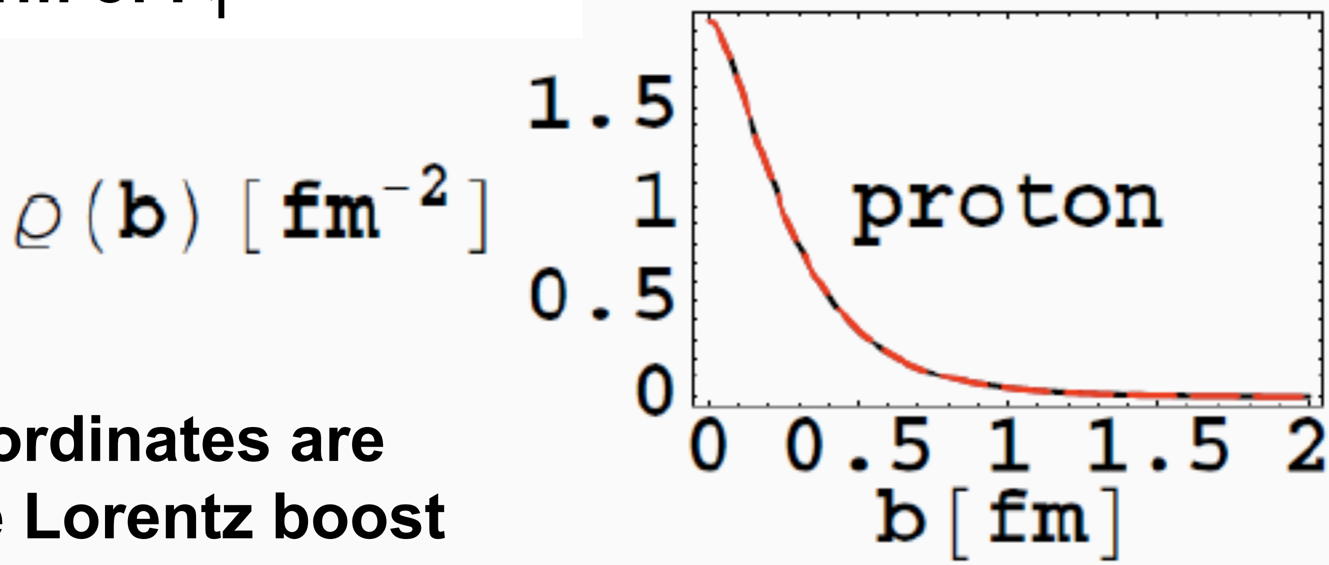


C. Crawford et al.
PRC 82 (2010) 045211

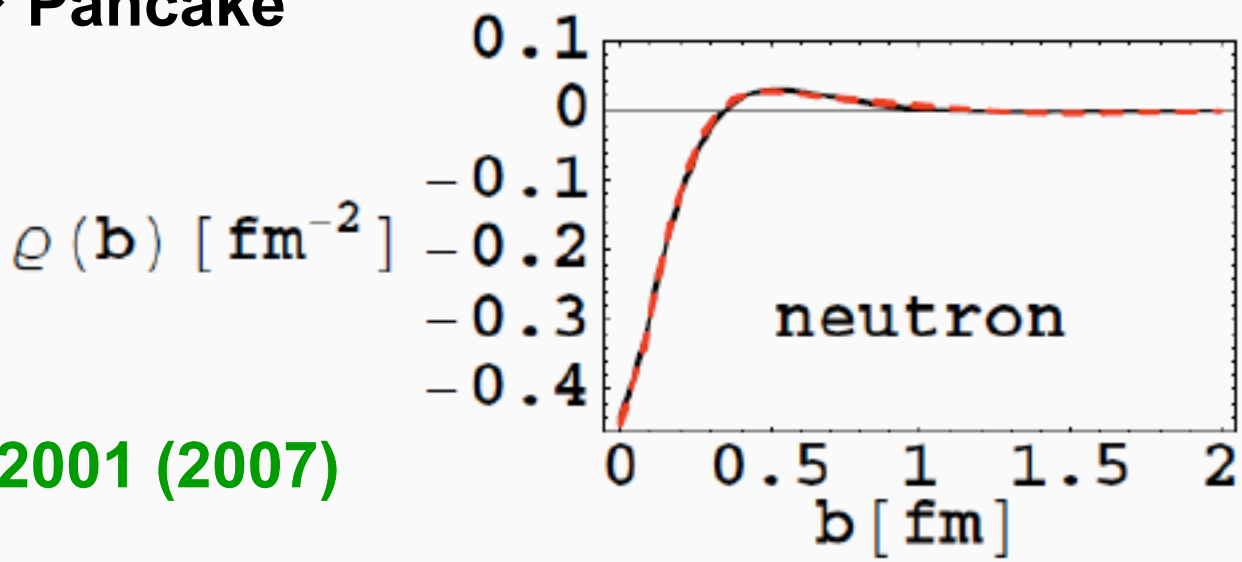


Spatial distributions in transverse plane

Charge density ρ(b) in the transverse plane of the infinite momentum frame is a 2D Fourier transform of F_1



The transverse coordinates are not affected by the Lorentz boost
Lorentz contraction → Pancake



G.A. Miller,
Phys. Rev. Lett. 99, 112001 (2007)

Multipole expansion and charge radius

$$G_E(Q^2) \cong G_E(\mathbf{q}^2) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3\mathbf{x}$$

$$G_E(\mathbf{q}^2) = \int_0^\infty \rho(r) r^2\, dr \int_0^\pi \sin\theta\, d\theta \left(1 + i|\mathbf{q}| r\cos\theta - \frac{1}{2}\mathbf{q}^2 r^2 \cos^2\theta + \ldots\right)$$

$$G_E(\mathbf{q}^2) = 1 - \frac{1}{6}\mathbf{q}^2 \int |\mathbf{x}|^2 \rho(|\mathbf{x}|)\, d^3\mathbf{x} + \ldots$$

$$= 1 - \frac{1}{6}\mathbf{q}^2 \langle r^2 \rangle + \ldots$$

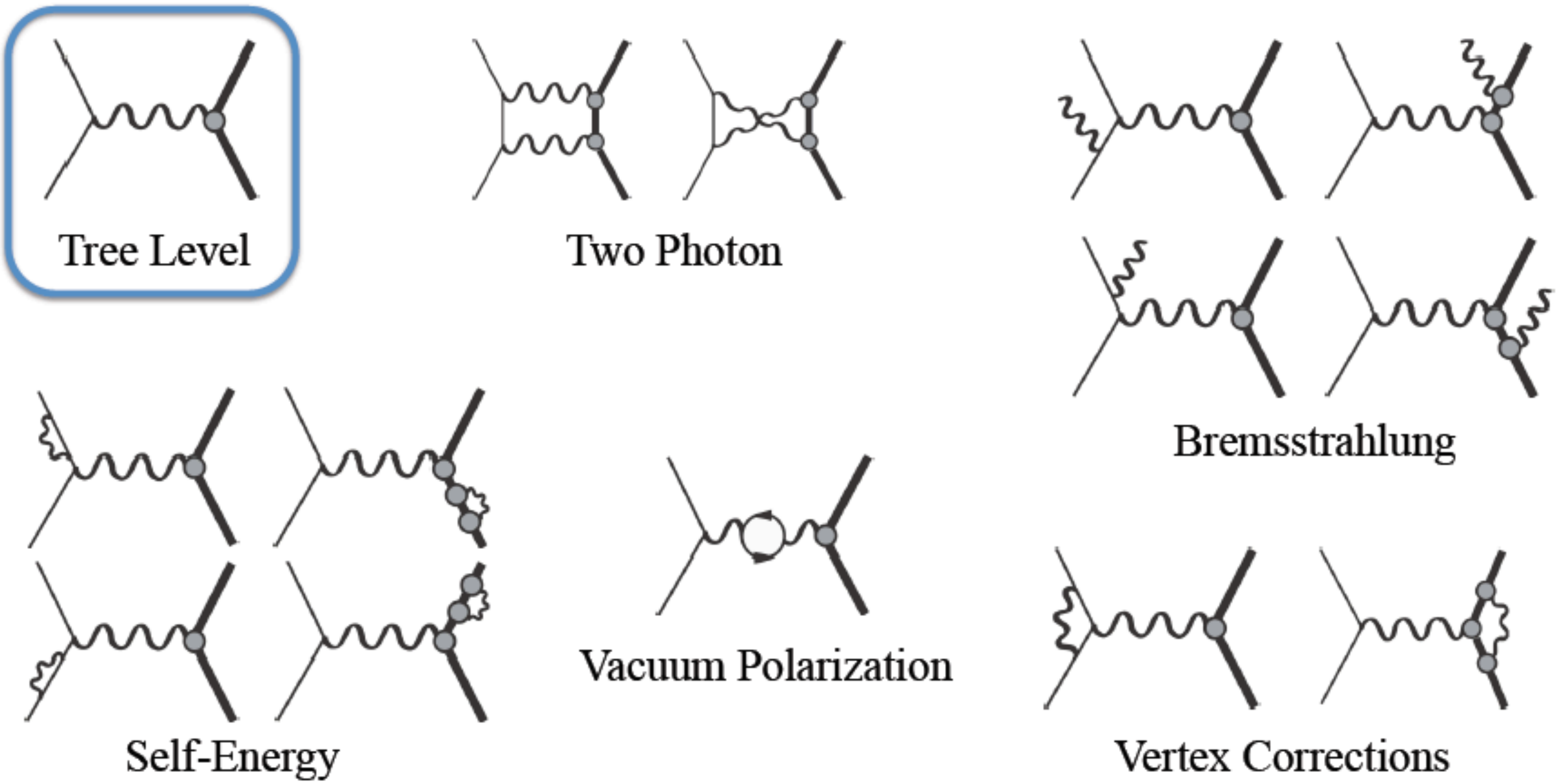
- **The proton charge radius is obtained as the slope of electric form factor at $Q^2 = 0$**

$$\langle r^2 \rangle = -6 \frac{dG_E}{dQ^2}|_{Q^2=0}$$

- **Likewise for the magnetic radius**

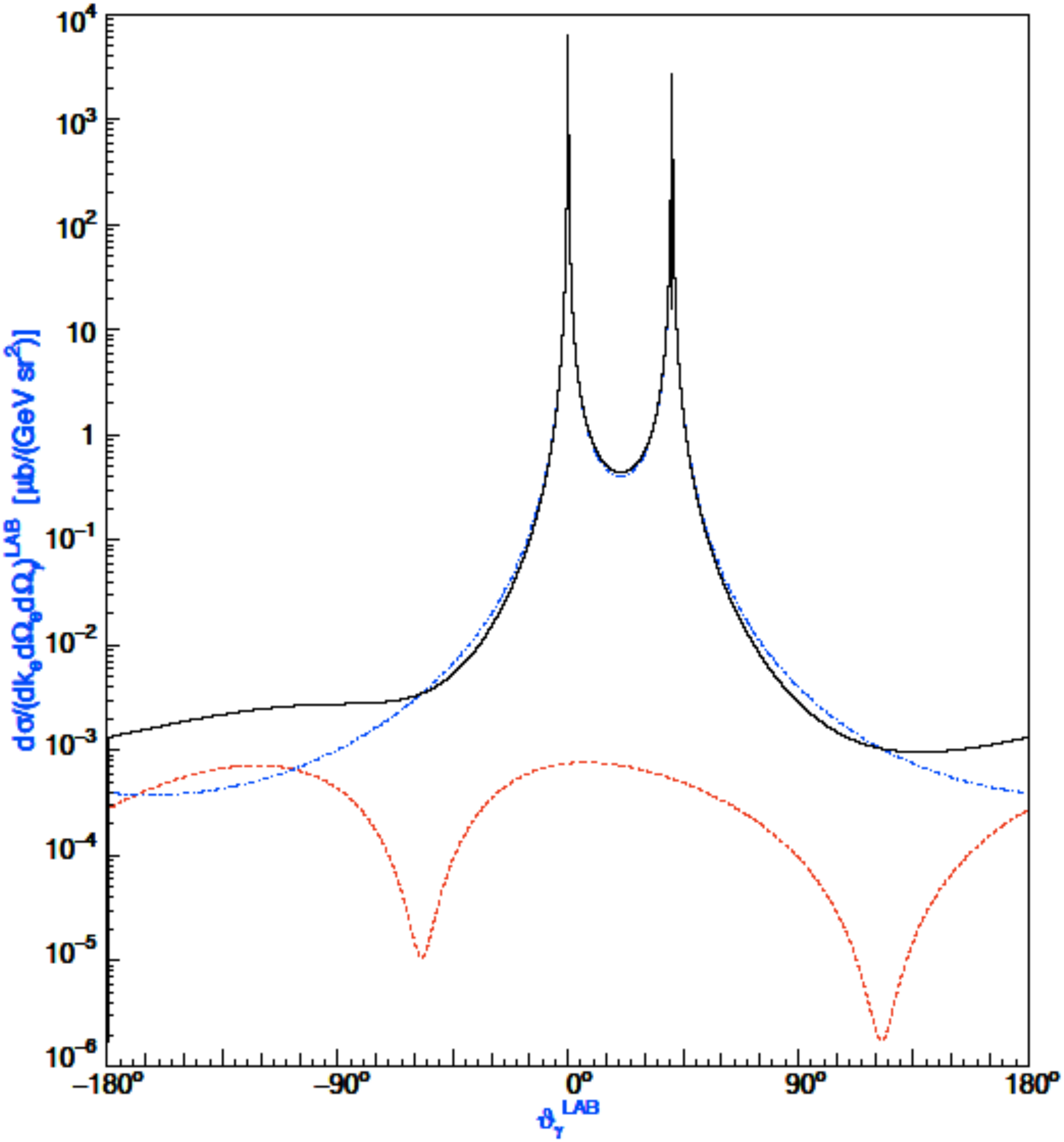
Radiative corrections

- Feynman rule: higher-order diagrams contribute to same final state
- Suppressed by powers of α – corrections of order % to tens of %



L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969)
L. C. Maximon and J. A. Tjon, Phys. Rev. C 62, 054320 (2000)
M. Vanderhaeghen et al. Phys. Rev. C 62, 025501 (2001)

Radiative corrections



Angular distribution of the emitted photon

- **Mainz VCS**

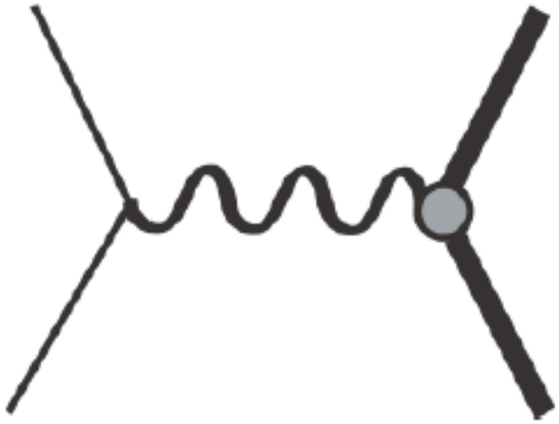
E = 705 MeV

θ = 40°

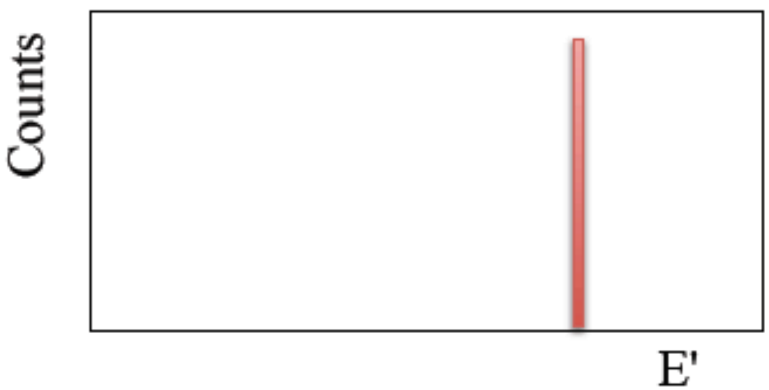
Bethe-Heitler (e)

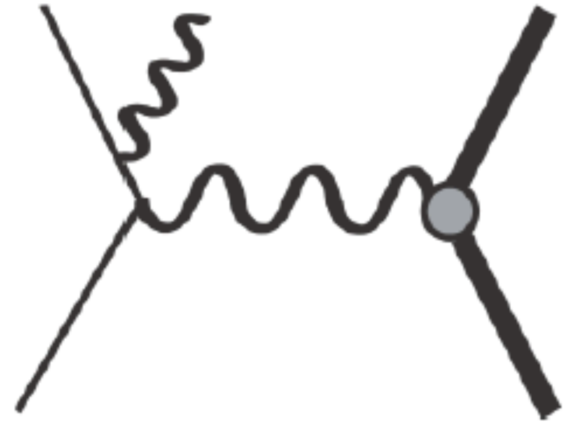
Born (p)

Radiative corrections

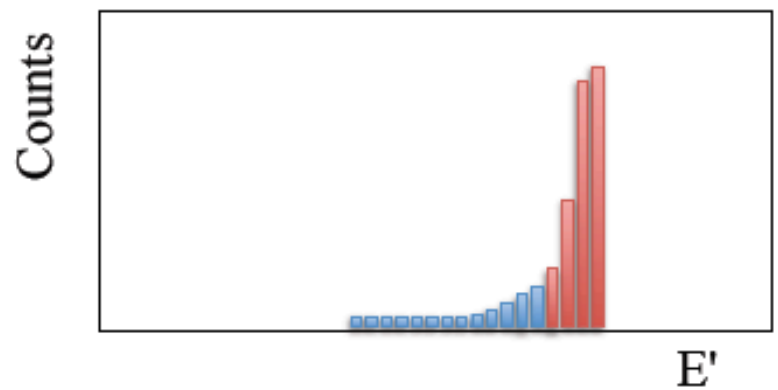


Born-level elastic scattering

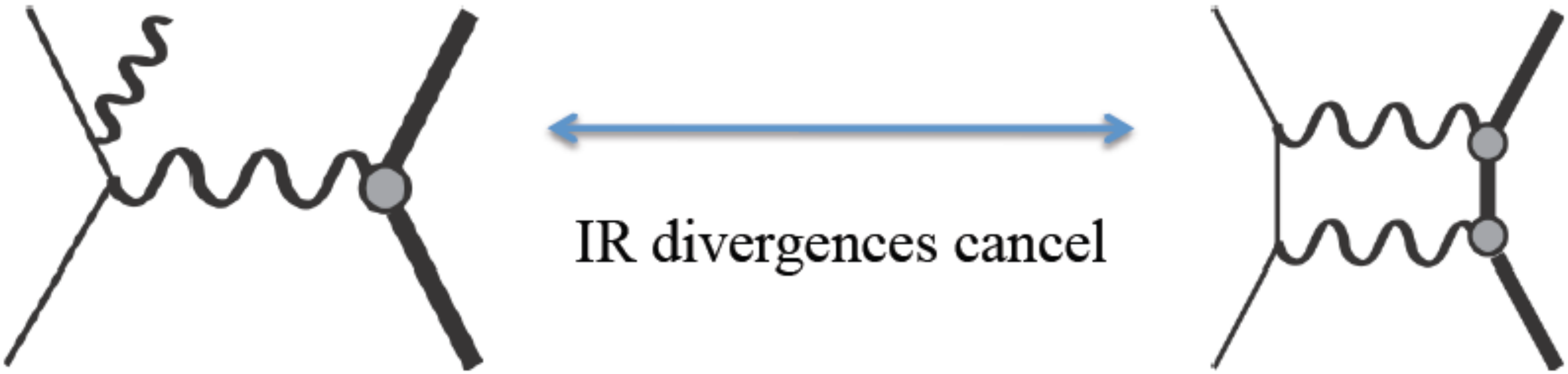




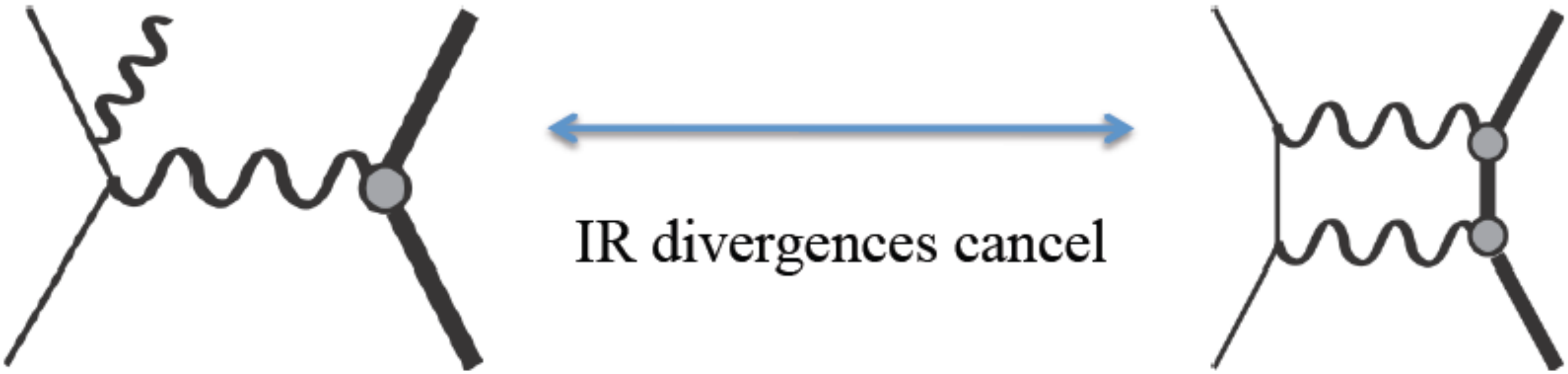
Soft bremsstrahlung $\propto Log\left(\frac{Q^2}{m^2}\right)$

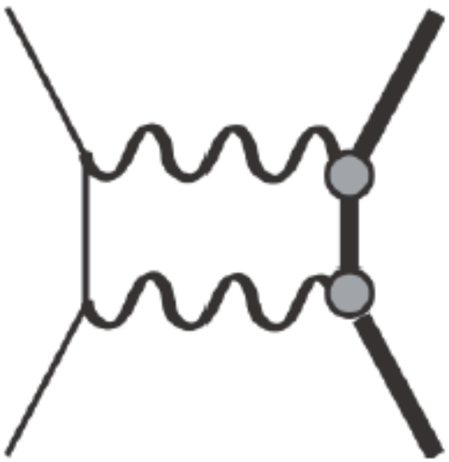


Radiative corrections

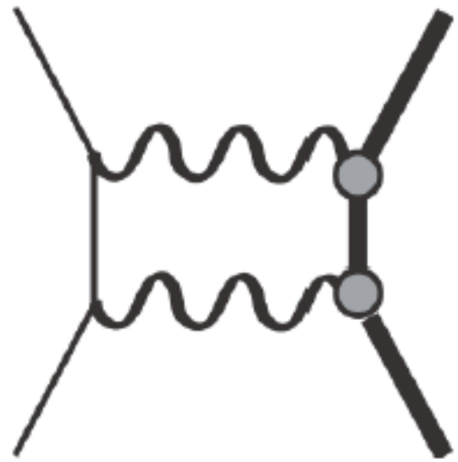


Radiative corrections





Soft

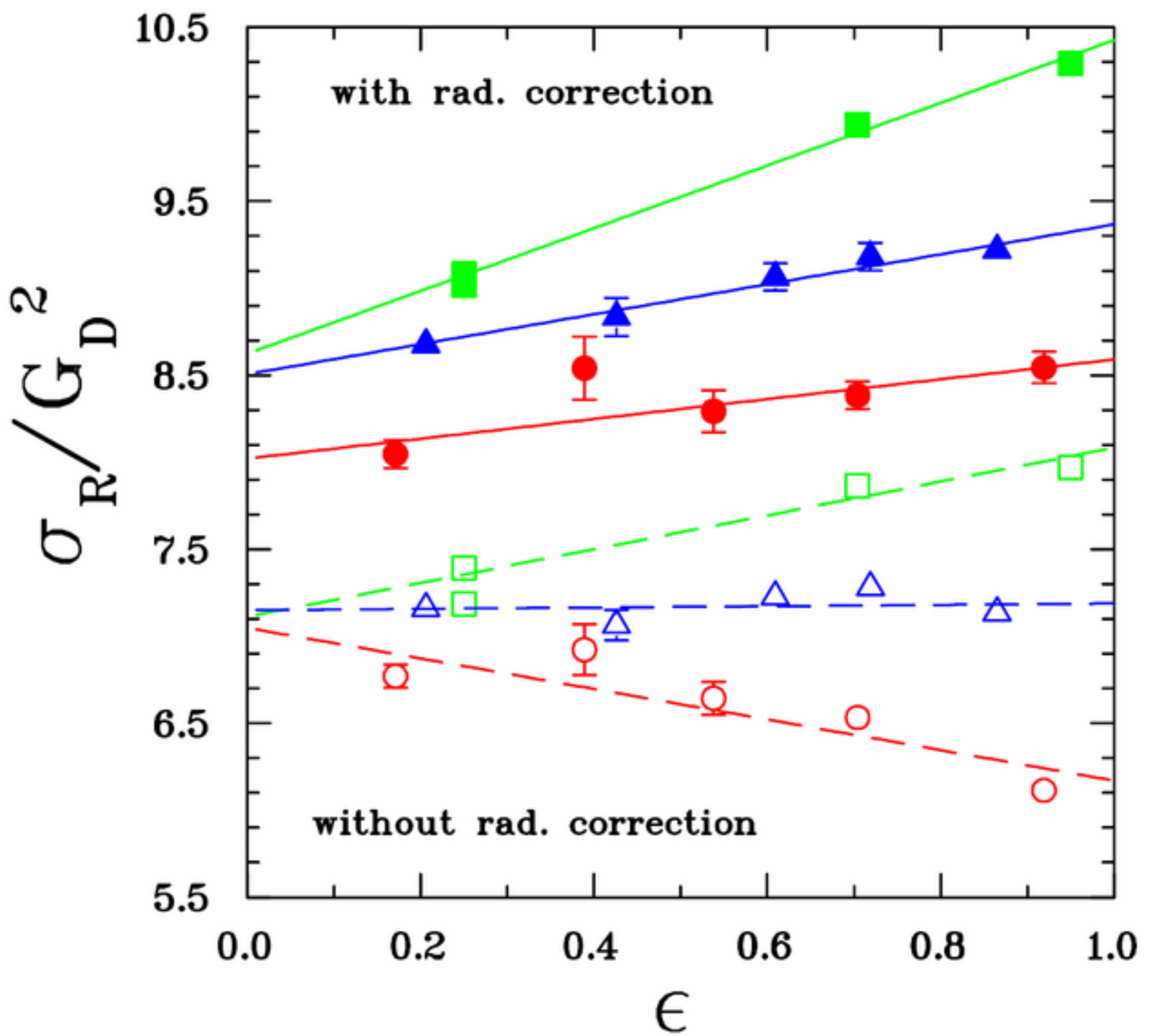


Hard

- One photon carries little momentum
- IR divergent
- Needed to cancel brem. IR divergence

- Both photons carry momentum
- Proton current off-shell
- Intermediate state?

How radiative corrections matter at high Q^2

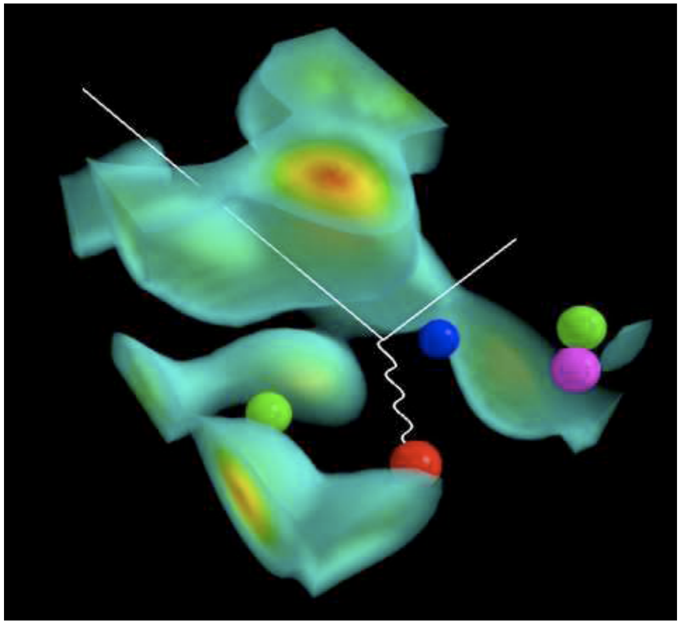


Q^2 = 1.75, 3.25 and 5 $(GeV/c)^2$

Motivation

Proton electric and magnetic form factors G_E and G_M

- Unpolarized and polarized methods
- Traditional and new techniques
- Overview of experimental data



A. Thomas, W. Weise,
The Structure of the Nucleon (2001)

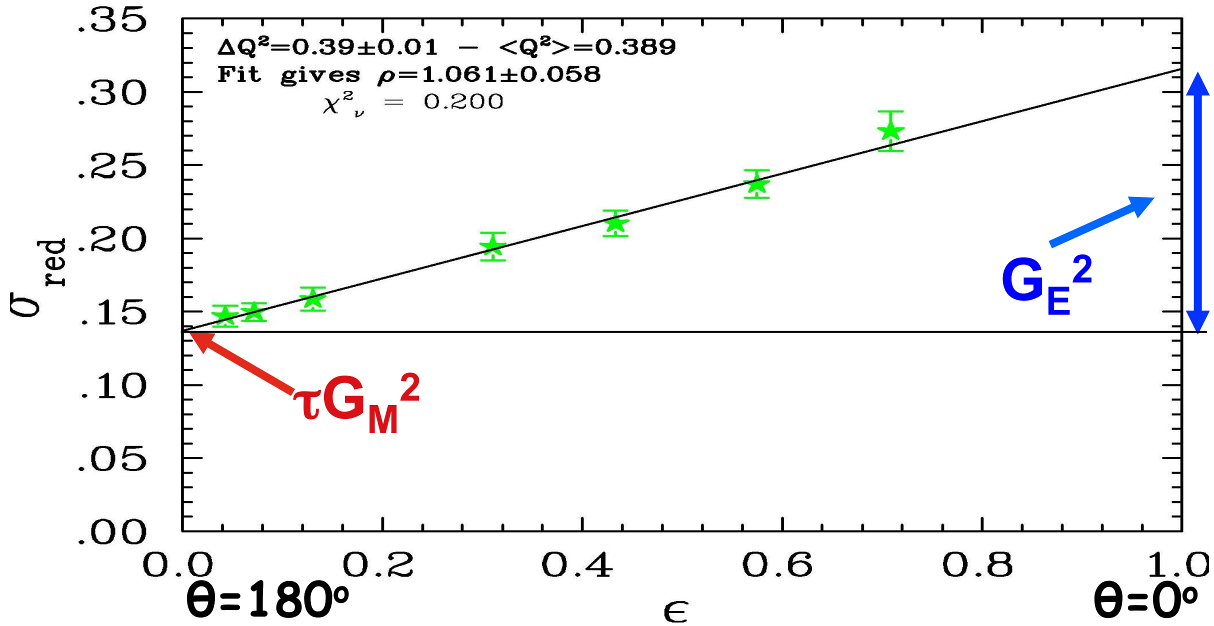
High Q^2: Energy frontier

- Proton form factor ratio
- Transition to pQCD
- Two-photon exchange: $G_E(Q^2)$ uncertain

Low Q^2: Precision frontier

- Pion cloud effect
- Deviations from dipole form
- The Proton Radius Puzzle: 7σ discrepancy

Form factors from Rosenbluth method



$\sigma_{red} = \varepsilon G_E^2 + \tau G_M^2$

→Determine $|G_E|$, $|G_M|$, $|G_E/G_M|$

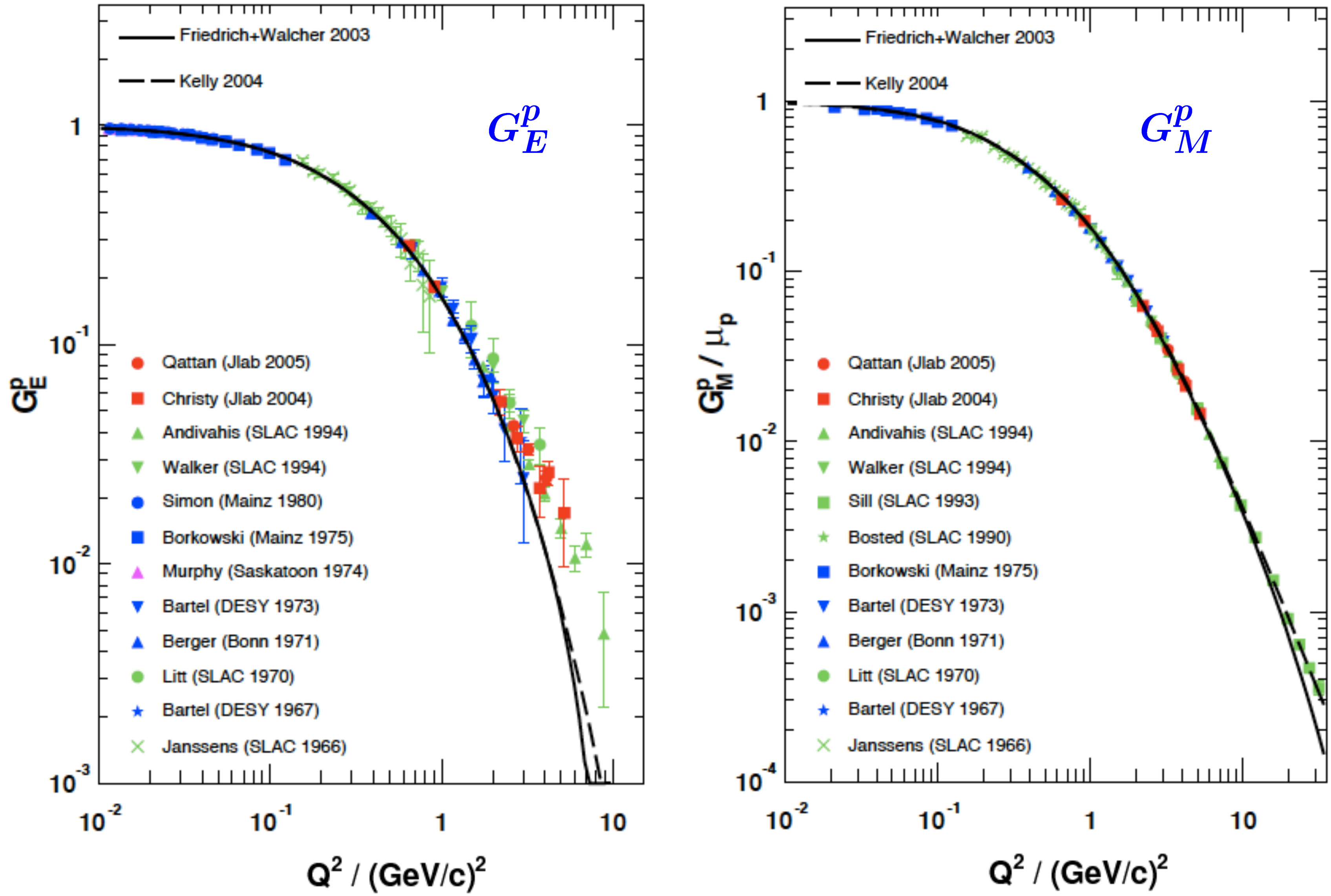
- In One-photon exchange, form factors are related to radiatively corrected elastic electron-proton scattering cross section

$$\frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} = S_0 = A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2}$$

$$= \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}$$

$$= \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1+\tau)}, \qquad \epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

G^p_E and G^p_M from unpolarized data



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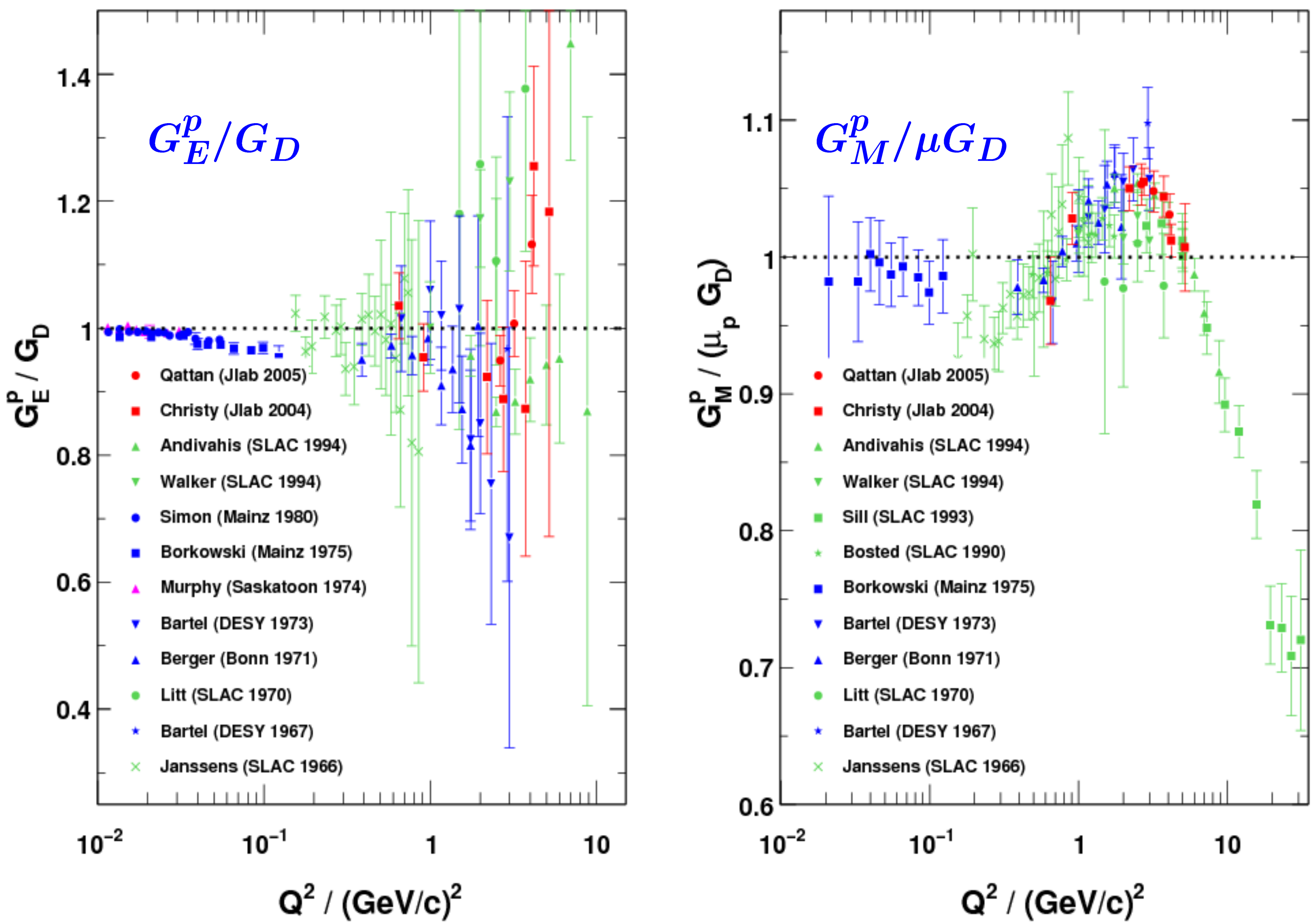

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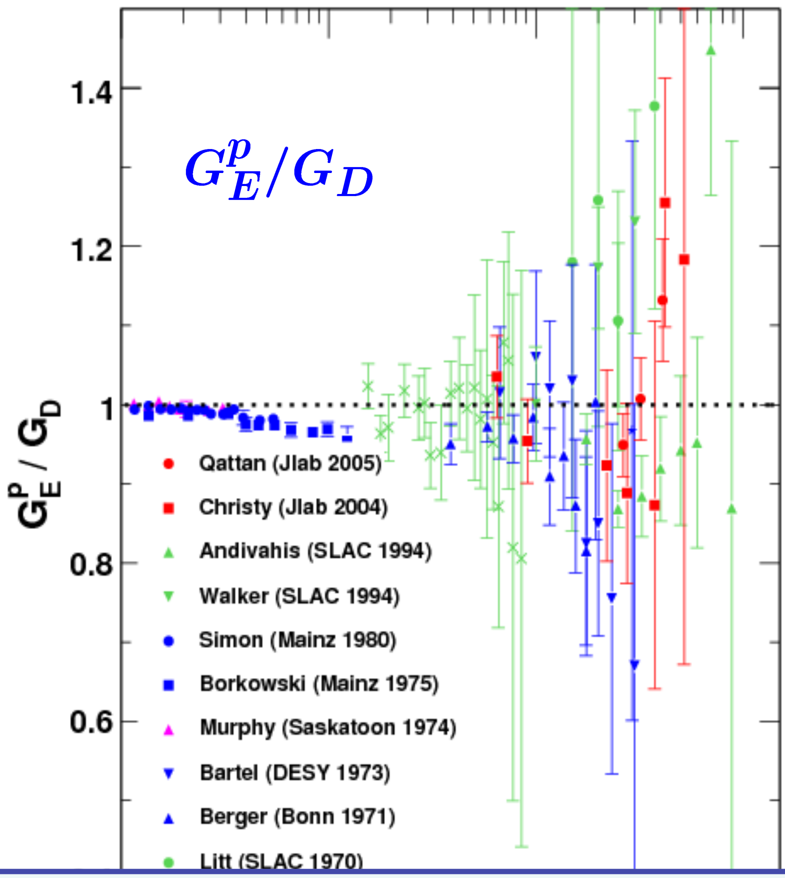
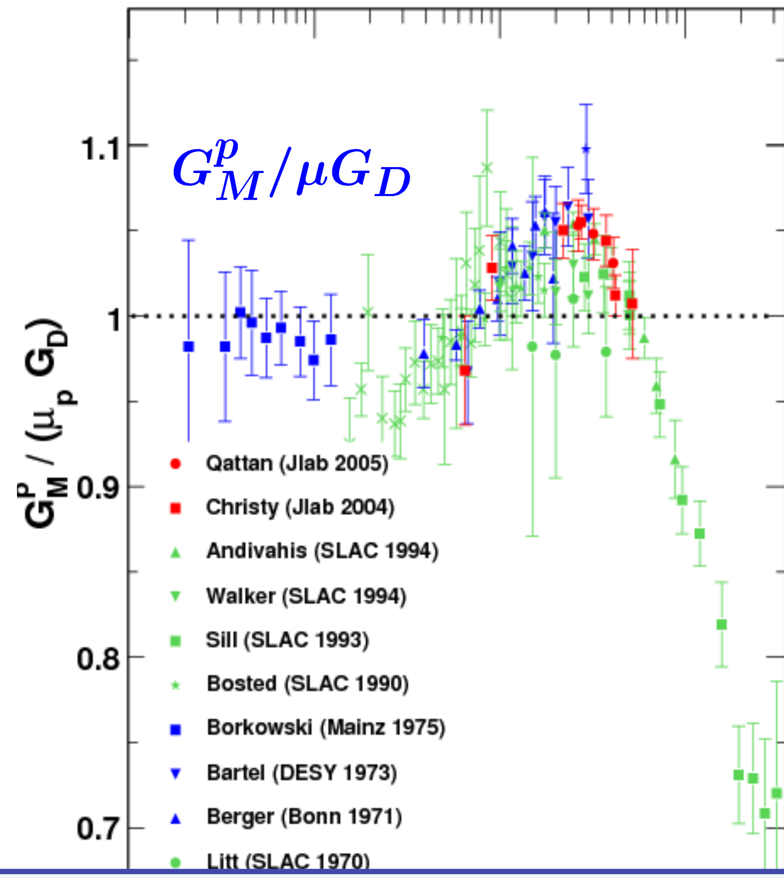
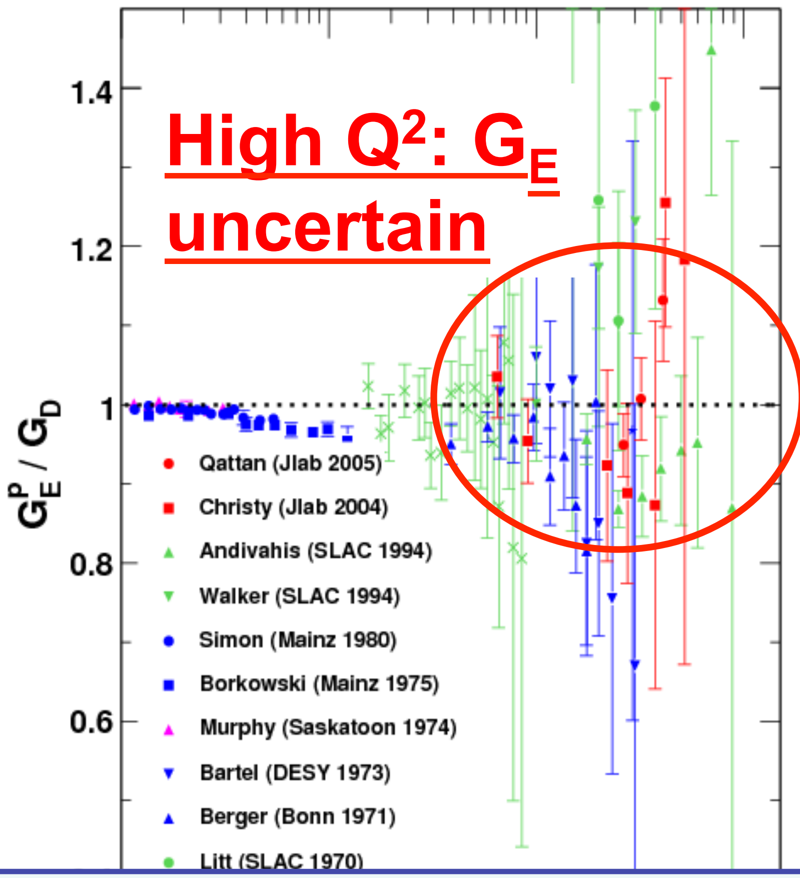


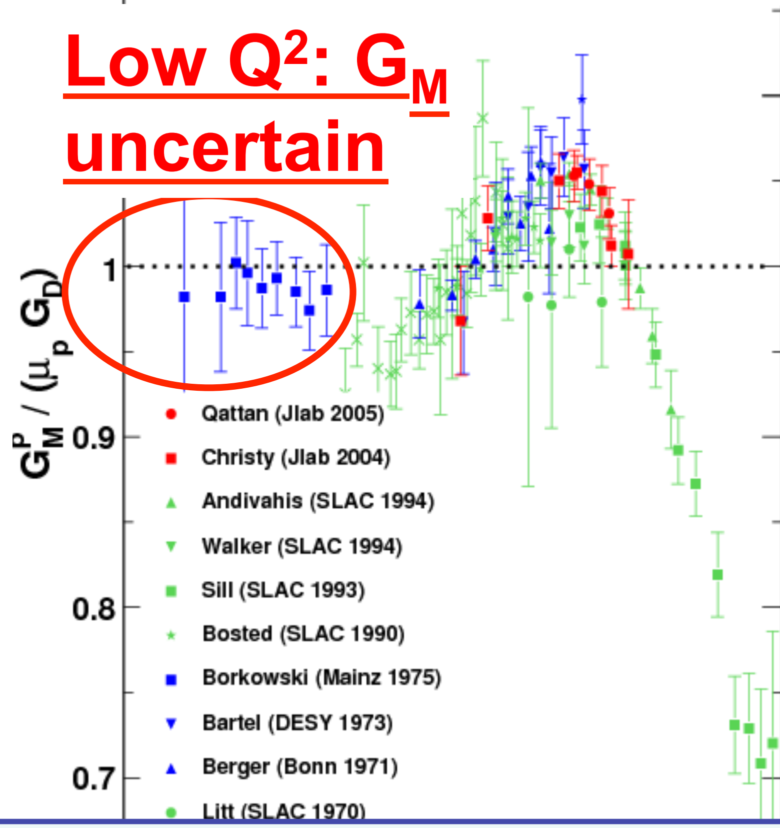


- $G(Q^2) \overset{\text{Fourier}}{\longleftrightarrow} \rho(r)$ **charge and magnetization density (Breit fr.)**
- **Dipole form factor** $G_D = \frac{1}{\left(1+\frac{Q^2}{0.71}\right)^2} \leftrightarrow \rho_D(r) = \rho_0\, e^{-\sqrt{0.71}r}$
- $G^p_E \approx G^p_M/\mu_p \approx G^n_M/\mu_n \approx G_D$ **within 10% for $Q^2 < 10$ (GeV/c)^2**

G^p_E and G^p_M from unpolarized data



$\sigma_{red} = \varepsilon G_E^2 + \tau G_M^2$



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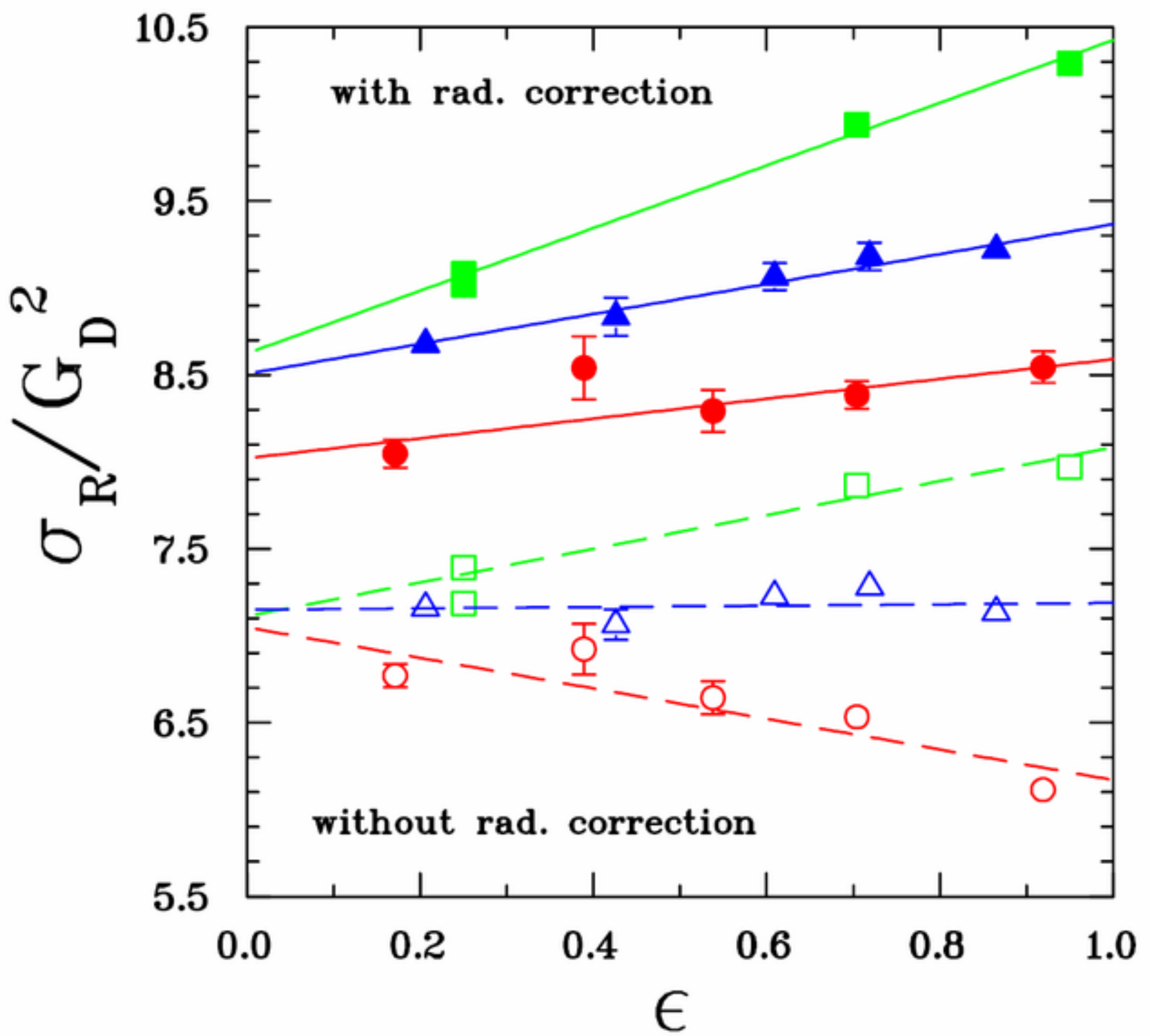
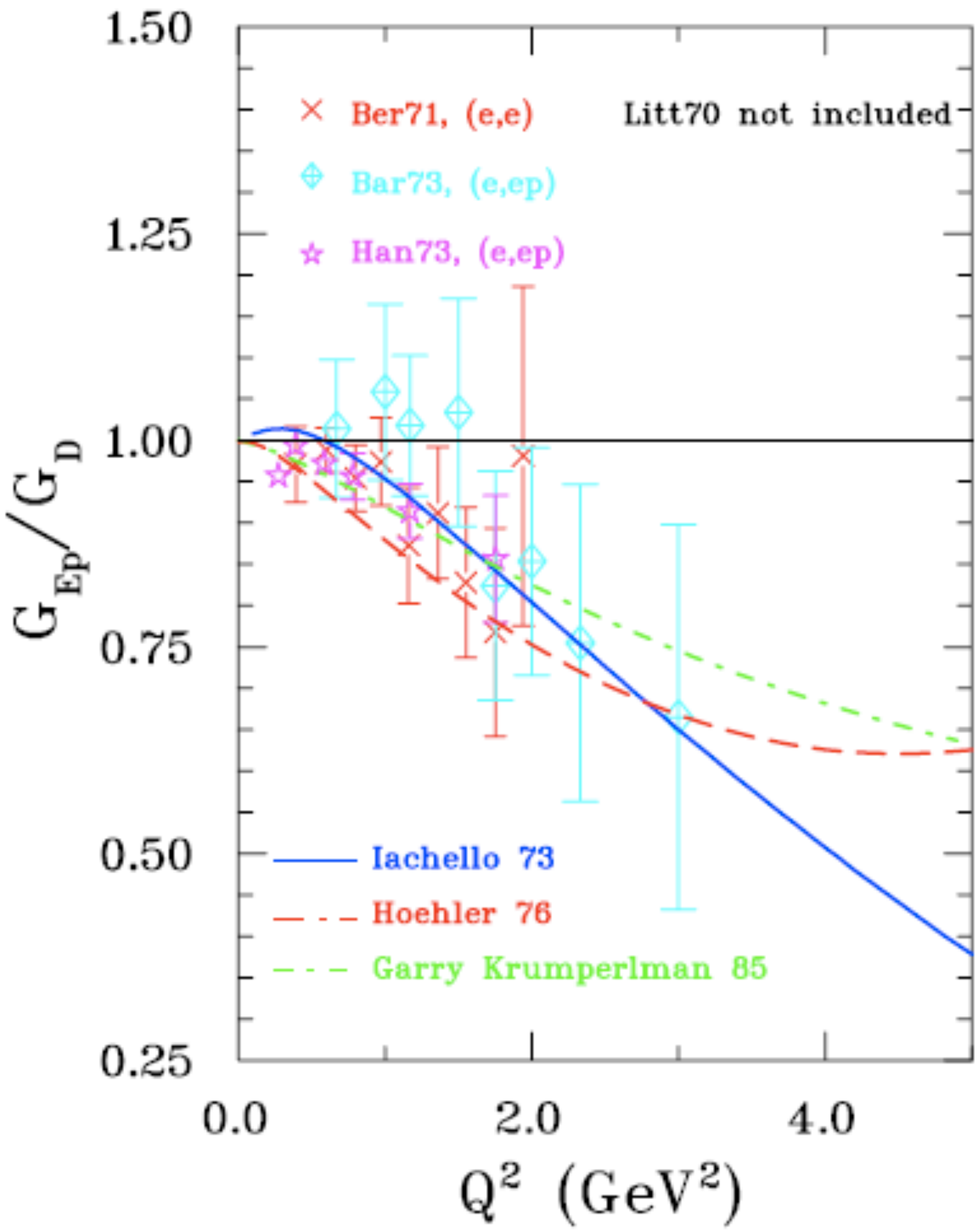
How radiative corrections matter …



Rosenbluth data and form factor scaling

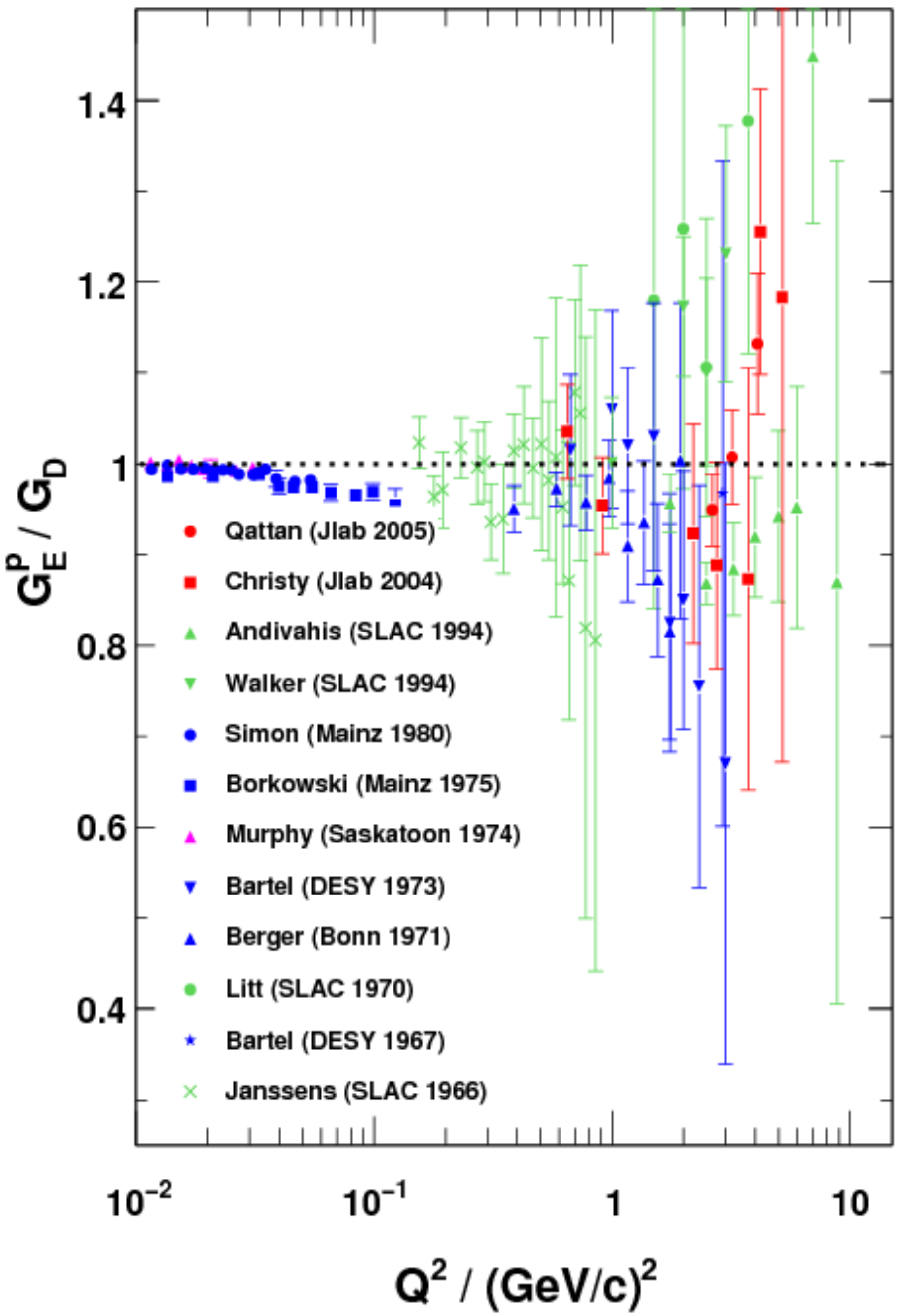


Early Rosenbluth data from 1970's from DESY and Bonn showed onset of for decline of GE faster than standard dipole

However, data from SLAC favored scaling (Litt 1970, Walker 1994, Andivahis 1994)

Scaling confirmed at JLab by Christy 2004 and Qattan 2005

Rosenbluth data and form factor scaling



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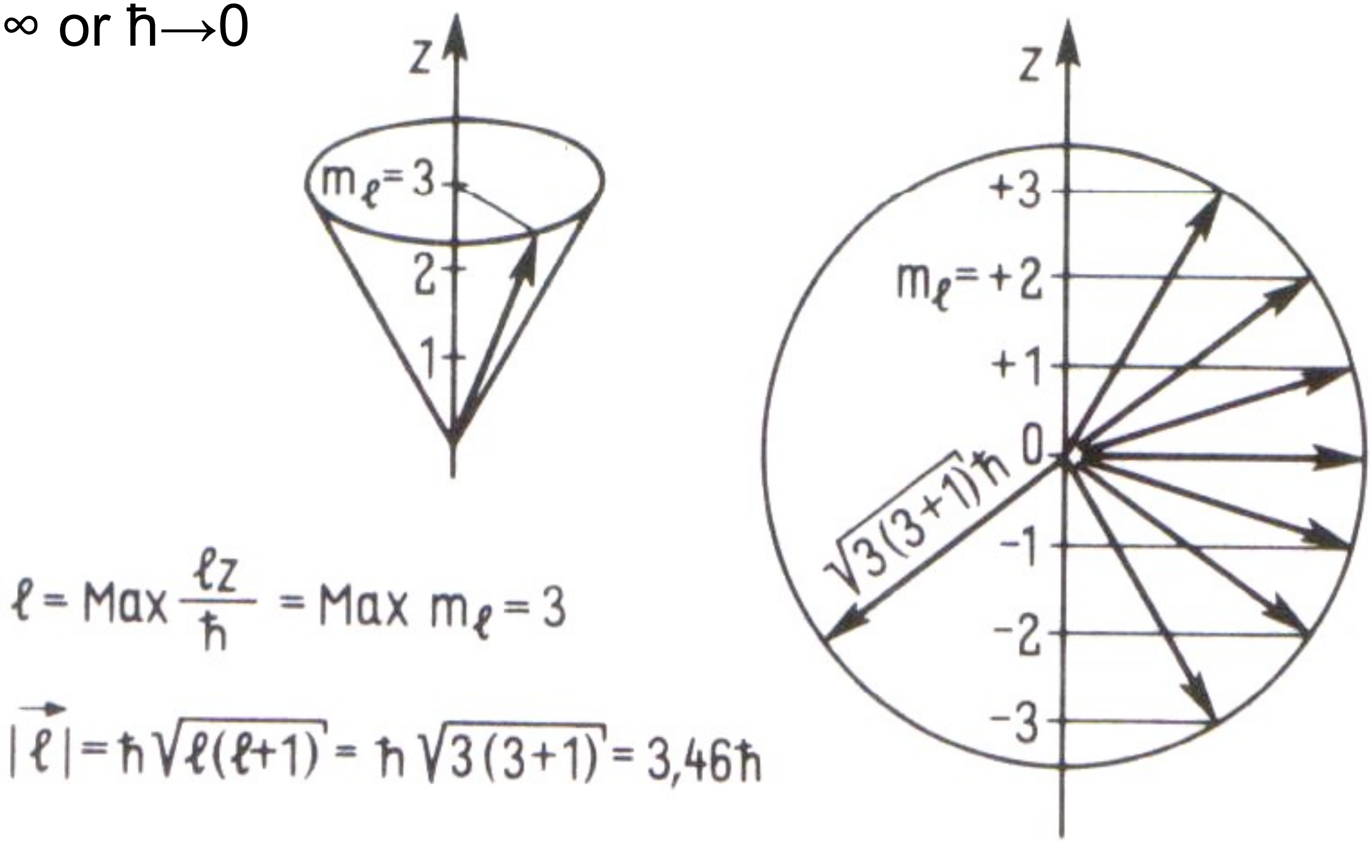
Concept of spin and polarization

Quantum mechanics without spin?

- **Early QM: energy quantized**
  → Planck: black-body radiation, Einstein: photo-electric effect
- **Orbital angular momentum quantized**
  Bohr, Sommerfeld: Atomic shell model
- **Correspondence principle**
  QM observables have classical analogon for N→∞ or ħ→0



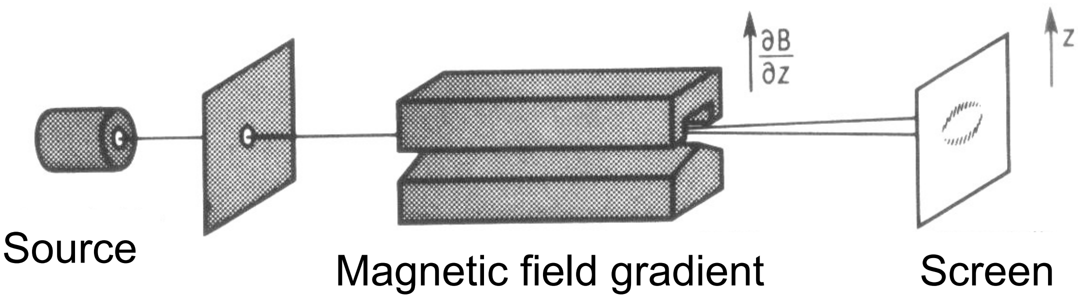
Quantum mechanics without spin?

- **Early QM: energy quantized**
  $\rightarrow$ Planck: black-body radiation, Einstein: photo-electric effect
- **Orbital angular momentum quantized**
  Bohr, Sommerfeld: Atomic shell model
- **Correspondence principle**
  QM observables have classical
  analogon for $N \rightarrow \infty$ or $\hbar \rightarrow 0$

- **But incomplete! Existing puzzles of the**
  - Zeeman effect (1896)
  - Fine structure and hyperfine structure (Goudsmit+Uhlenbeck 1925)
  - Stability of atoms (Pauli 1924, Dirac 1928)

  explained by concept of **electron spin ½**

Discovery of electron spin

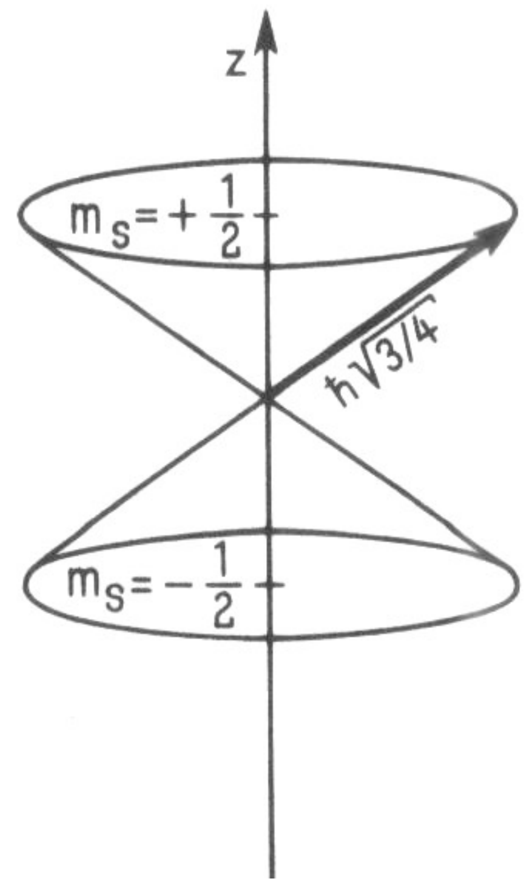


Stern-Gerlach Experiment (1922)

- **Stern and Gerlach 1922:**
  Ag atoms through a non-uniform magnetic field
- **Phipps and Taylor 1927:**
  same observation with H atoms (L=0!)

The beam split in two. This marked the discovery of the electron spin. A new type of internal angular momentum, with a quantum number that can take on only two values, **$s=\frac{1}{2}$, $m_s=\pm\frac{1}{2}$**

Spin is fundamentally different from orbital momentum! No classical analogon!



Pauli exclusion principle

- From spectra of complex atoms, Wolfgang Pauli (1925) deduced a new rule:

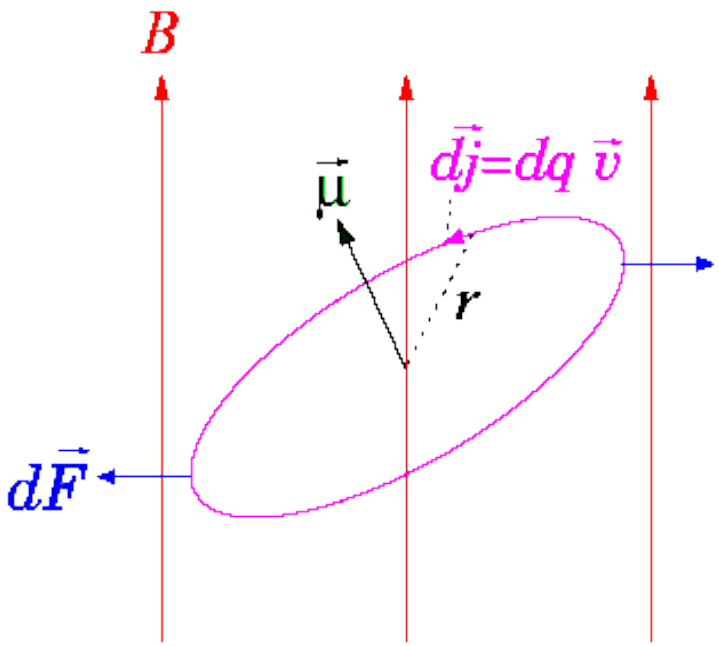
"Pauli Exclusion Principle"

"In a given atom, no two electrons* can be in the same quantum state, i.e. they cannot have the same set of quantum numbers n, l, m_ℓ , m_s"

- → every "atomic <u>orbital</u> with n,l,m_l" can hold 2 electrons: (↑↓)
- Systems of bosons have symmetric, fermions have antisymmetric wave function
- → Fermi and Bose statistics
- → explains stability of atoms

*Note: More generally, no two identical fermions (any particle with spin of ħ/2, 3ħ/2, etc.) can be in the same quantum state.

Orbital motion and magnetic moment



$\mu = i\,\pi r^2$

$\vec{Torque} = \vec{\mu} \times \vec{B}$

$Energy = -\vec{\mu} \cdot \vec{B}$

- Classical ring current
- Angular momentum $\vec{l} = \vec{r} \times \vec{p}$
- Torque causing precession
- Magneton: $\mu_{B(N)} = \dfrac{e\hbar}{2m_{e(p)}c}$

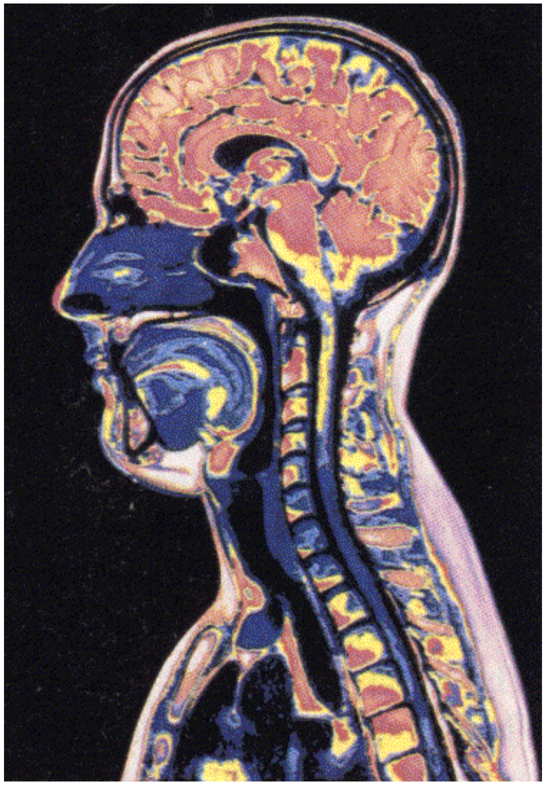
  =magnetic moment of a uniformly charged sphere with $l = \hbar$
- Magnetic moment $\vec{\mu} = g\,\mu_B \dfrac{\vec{l}}{\hbar}$

- Magnetic moment proportional to angular momentum remains true with relativity and QM.
- Orbital angular momentum and magnetic moment: g = 1

Spin and magnetic moment

- Magnetic moment proportional to spin $\vec{\mu} = g_s \, \mu_{B(N)} \frac{\vec{s}}{\hbar}$
- The ratio of magnetic moment to "spin" involves a "g" factor
  - Electron g_s~2
  - Proton g_s~5.59
  - Neutron g_s~3.83

  } Deviation from 2 is "anomalous"
- **Electron g_s=2 predicted by Dirac Eq.!** spin is a property of spacetime
- Spin precession: Larmor frequency $\omega_L = \frac{g \, \mu_{B(N)} B}{\hbar}$ → MRI



- $\longrightarrow$ **Spin manipulation through magnetic interaction**

Spin ½ and polarization

- **Single spin "measurement":**
  choose quantization axis (e.g. magnetic field) with arbitrary but fixed orientation

  $\longrightarrow$ **s=½, m_s=±½**

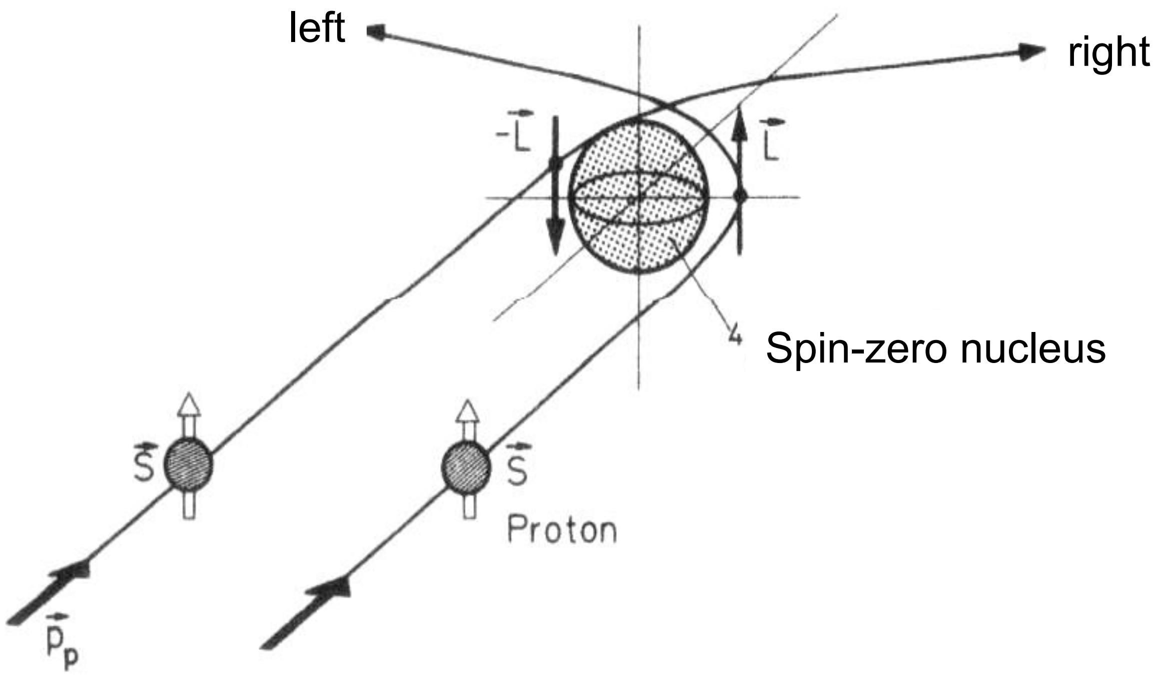
- **N "measurements" (ensemble of spins):**

  Probability $p(m_s)$ to find m_s $\longrightarrow$ "Polarization"

- **Spin asymmetry $A = (N_+ - N_-) / (N_+ + N_-)$**

- **Polarization vector can be controlled externally**
  "Spin orientation" is "jargon"

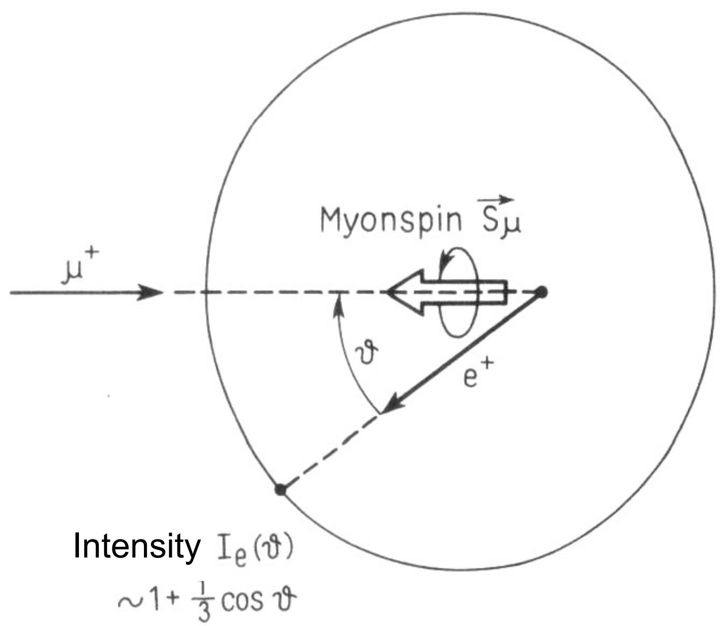
Polarimetry



- **Strong LS interaction energy allows hadron polarization measurements with large analyzing powers**

- **Electron polarization measured with Compton (eγ) or Moller (ee) scattering**

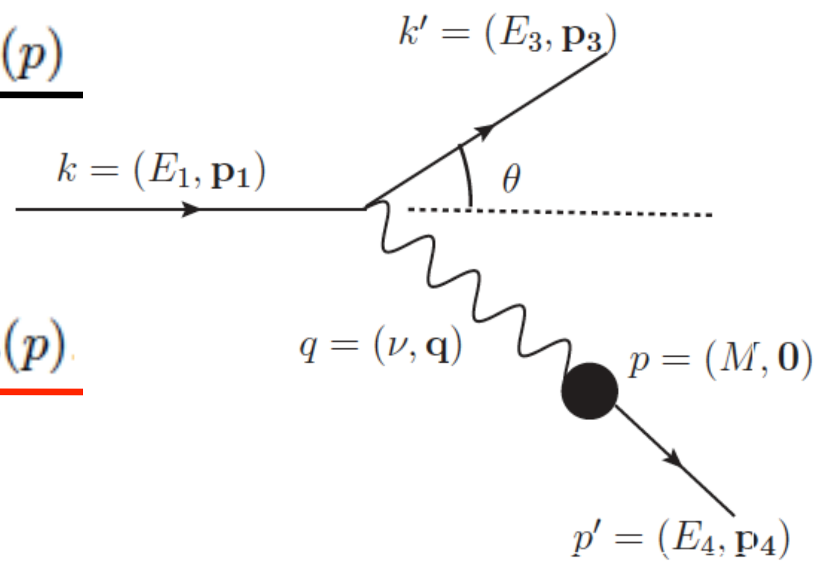
- **Muon polarization from angular distribution of decay $\mu \to e\nu\bar{\nu}$**



Lepton-nucleon scattering

- **Lepton-lepton scattering:**

$$-iM = \bar{u}(k')(ig_e\gamma^\mu)u(k)\left(-i\frac{g_{\mu\nu}}{q^2}\right)\bar{u}(p')(-ig_e\underline{\gamma^\nu)u(p)}$$



- **Lepton-nucleon scattering:**

$$-iM = \bar{u}(k')(ig_e\gamma^\mu)u(k)\left(-i\frac{g_{\mu\nu}}{q^2}\right)\bar{u}(p')(-ig_e\underline{\Gamma^\nu)u(p)}$$

- **Nucleon vertex factor (current)**

$$\Gamma^\nu = \gamma^\nu F_1(q^2) + i\sigma^{\nu\alpha}\frac{q_\alpha}{2M}F_2(q^2)$$

Dirac (F_1) and Pauli (F_2) “form factors”

$$Q^2 = -q^2 \qquad \tau = \frac{Q^2}{4M^2}$$

- **Spin dependent, polarized cross section:**
  no more averaging over initial and summing over final spins in the matrix element