Nuclear Structure and Short-Range Correlations (Day 2)

Or Hen – MIT

Hampton University Graduate School (HUGS),
June 7th 2017, JLab, Newport-News VA.

Course Outline

Day I: Overview of Nuclear Systems and EM Probes.

Day II: Nuclear Structure.

(Short / Long Range)

(Experiment / Theory)

Day III: Cross Connections.

(QCD in Nuclei: Modification and Transparency)

(Contact Formalism and Short-Range Universality)

(Neutrino Physics)

(Neutron Stars)

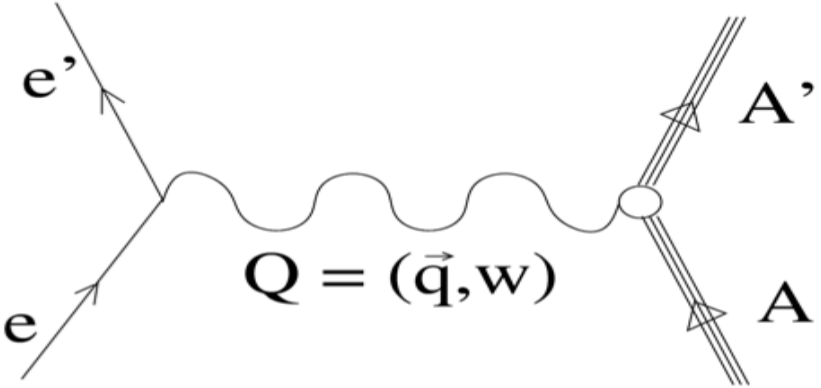
Reminder From Yesterday

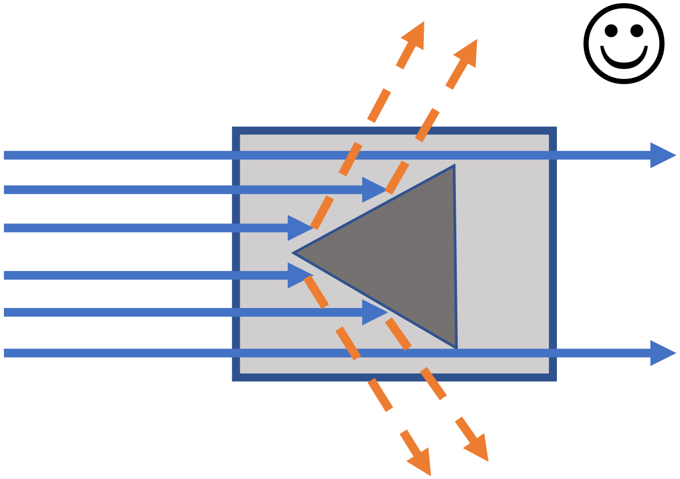
Goal: Study the internal structure (and dynamics) of complex objects

Means: using high energy lepton scattering

Reaction determined by two variables:

- $Q^2 = -q^2$ Interaction-Scale
- $x_B = Q^2/(2m_p\nu)$ Dynamics



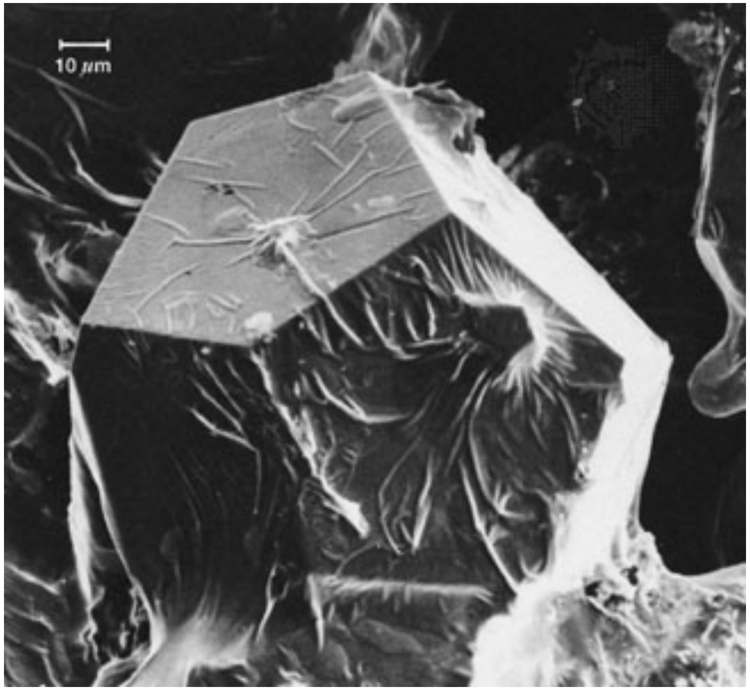


Reminder From Yesterday

Goal: Study the internal structure (and dynamics) of complex objects

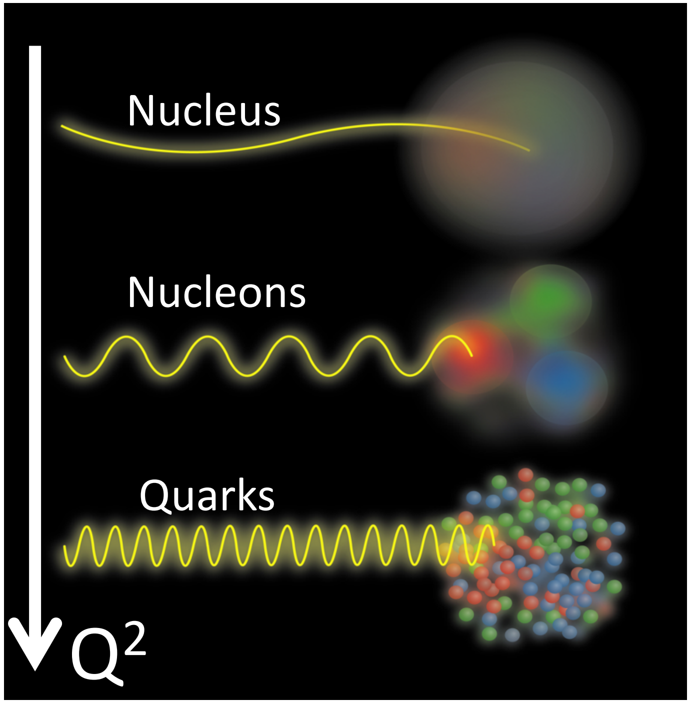
Means: using high energy lepton scattering

100s eV – 100s keV:
Material structure

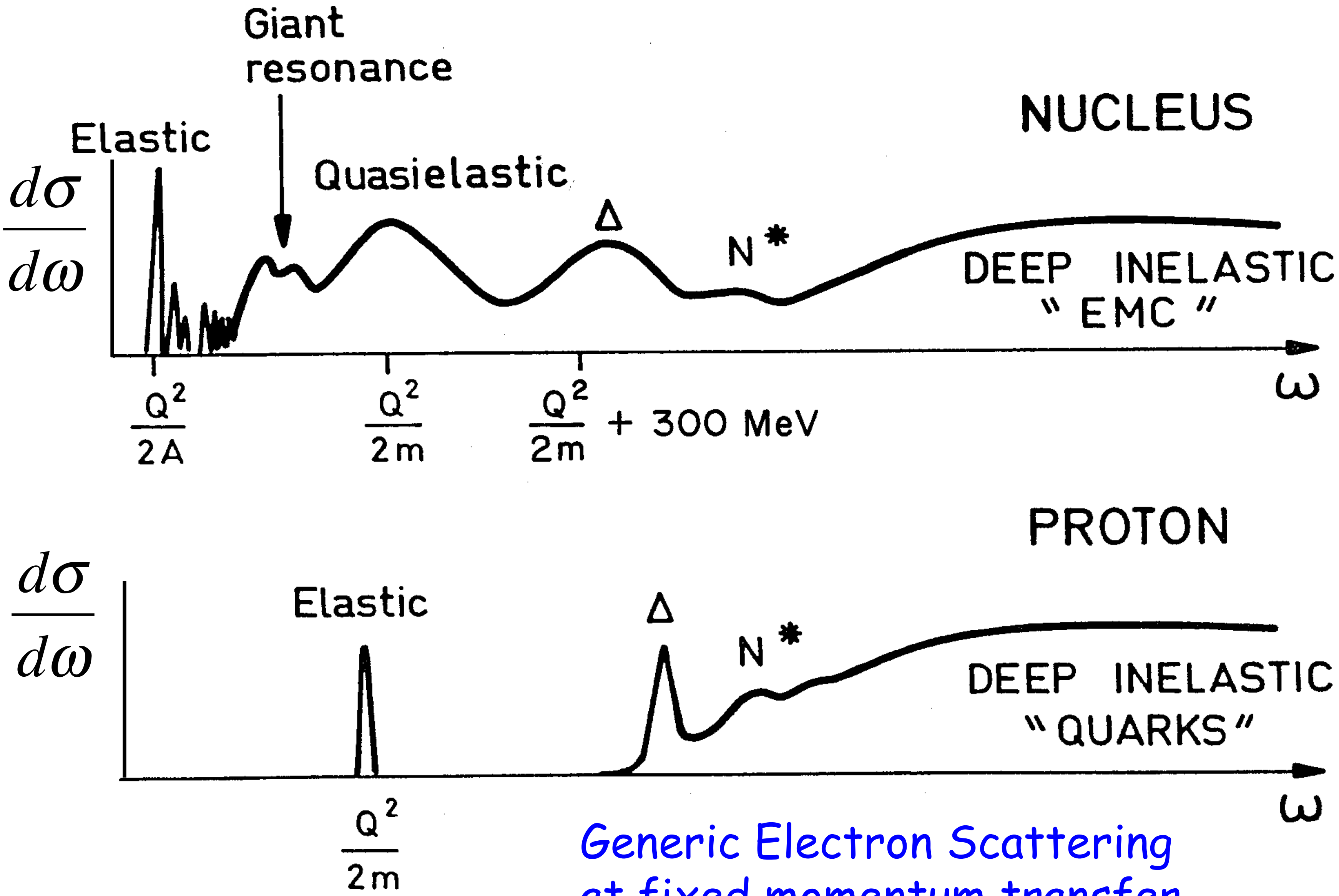


Energy
=
Resolution !

100s MeV – 10s GeV:
Nuclear structure

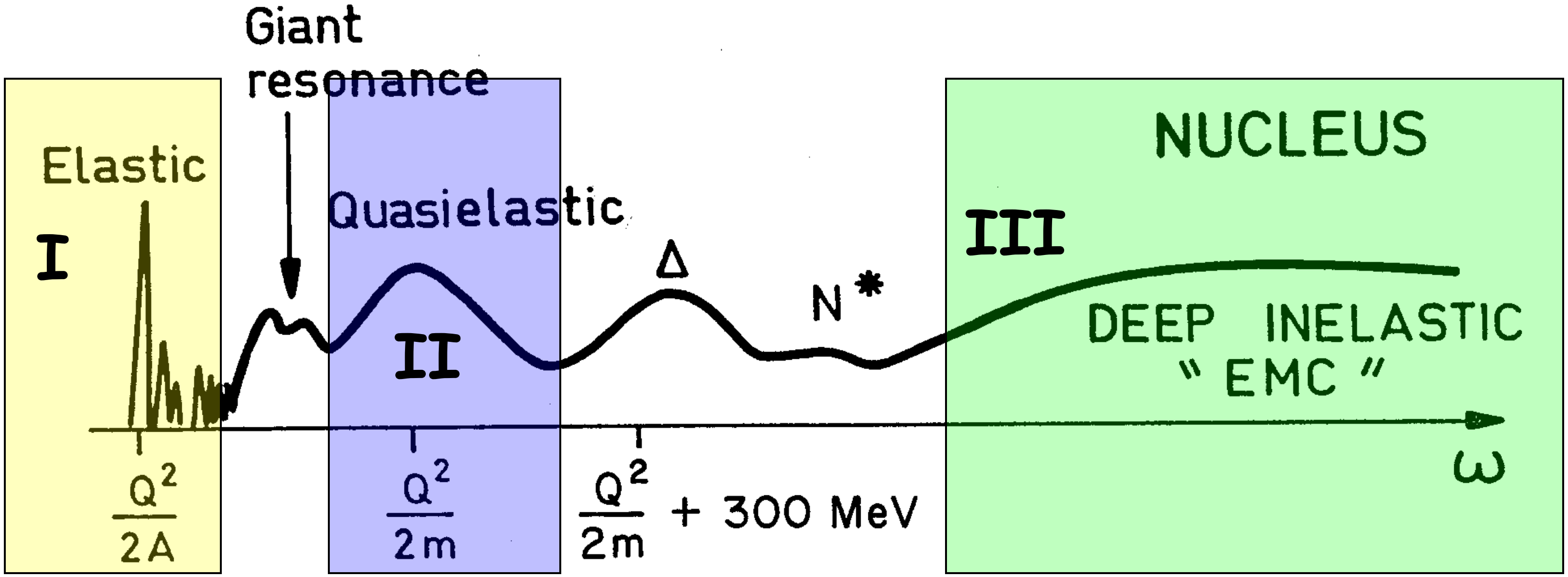


Reminder From Yesterday



Generic Electron Scattering
at fixed momentum transfer

Reminder From Yesterday



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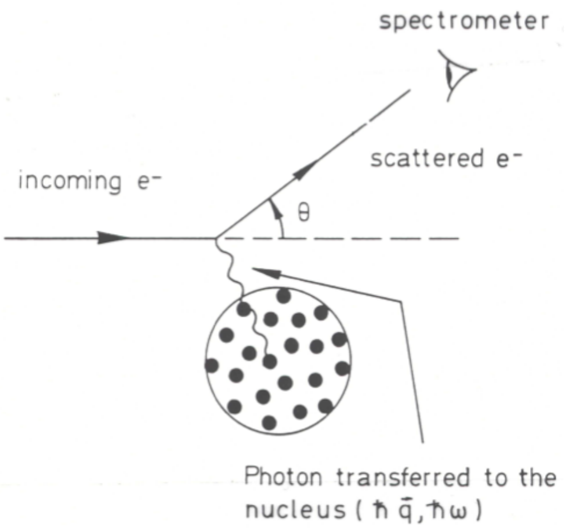
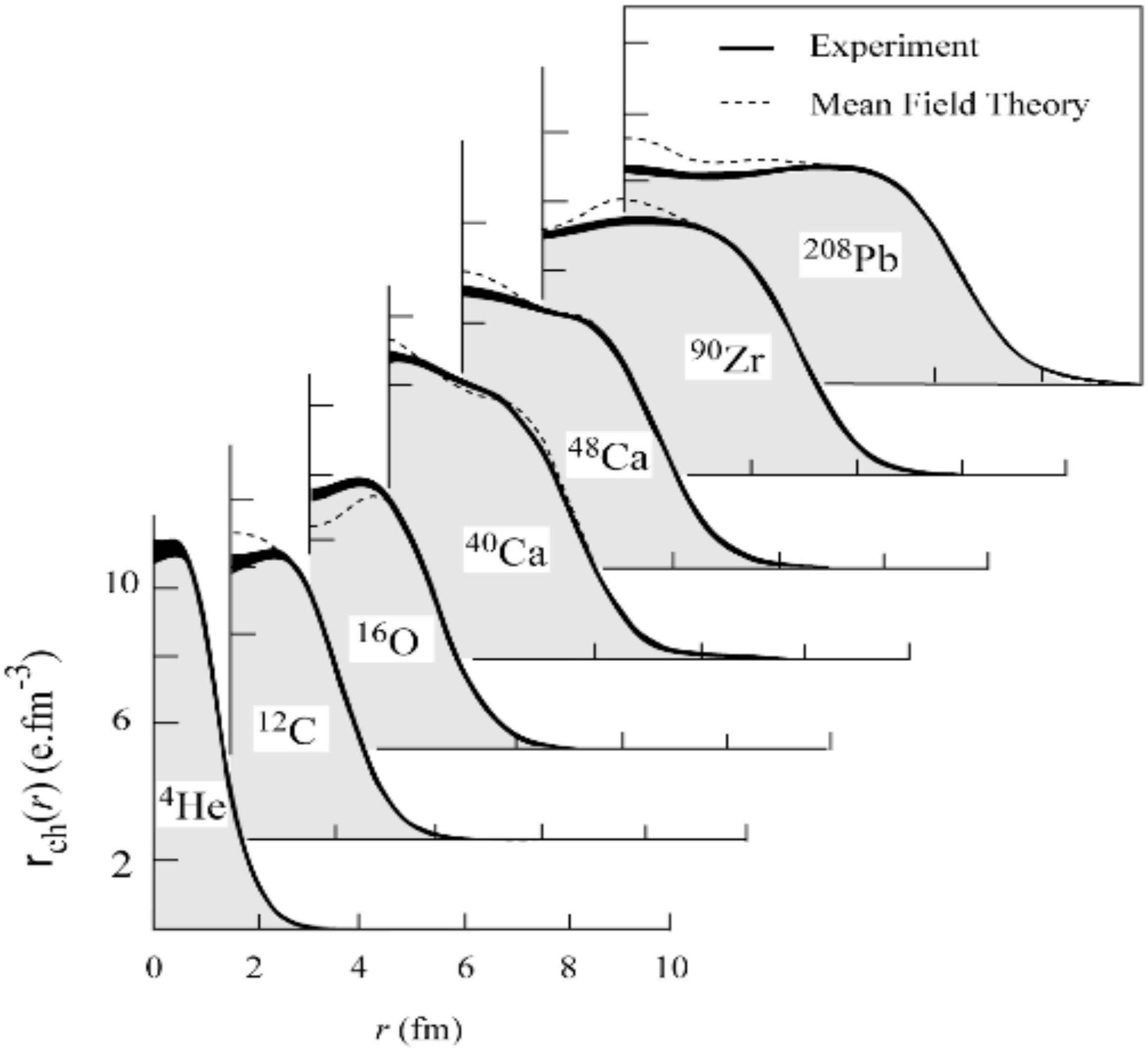
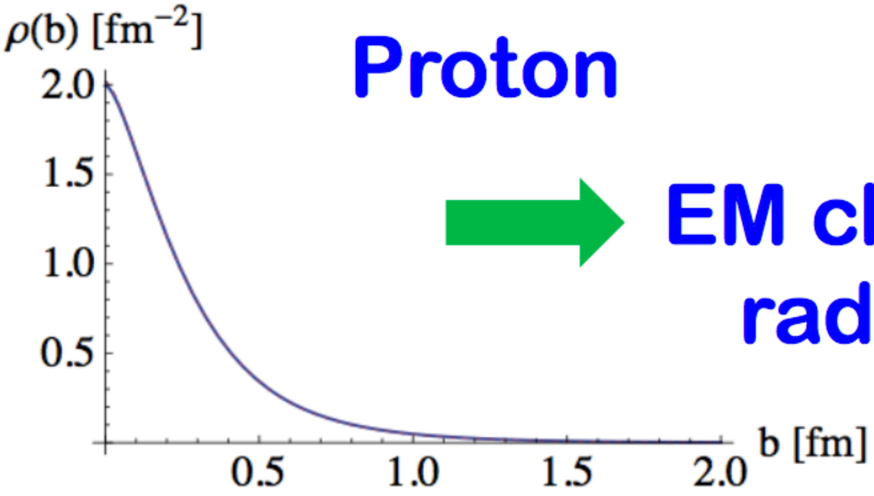


Charge Distribution, $r_{CH}(r)$, is a Fourier Transform of the Charge Form Factor, F(q)



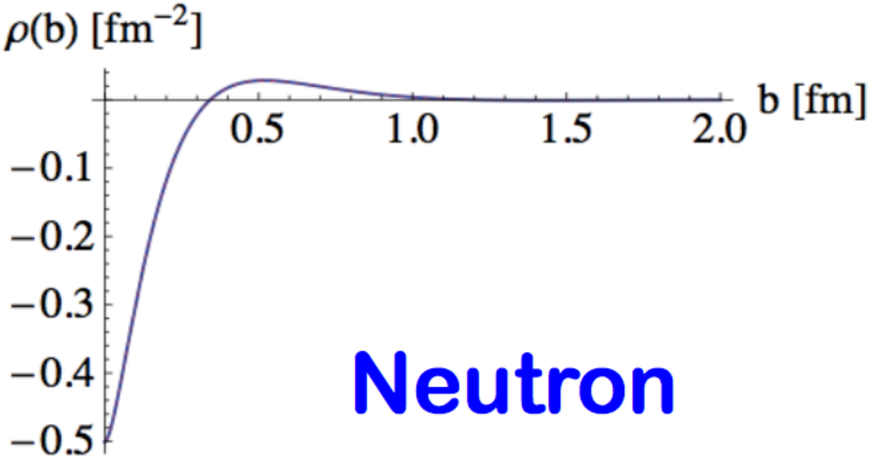
Reminder From Yesterday

Electric charge distribution



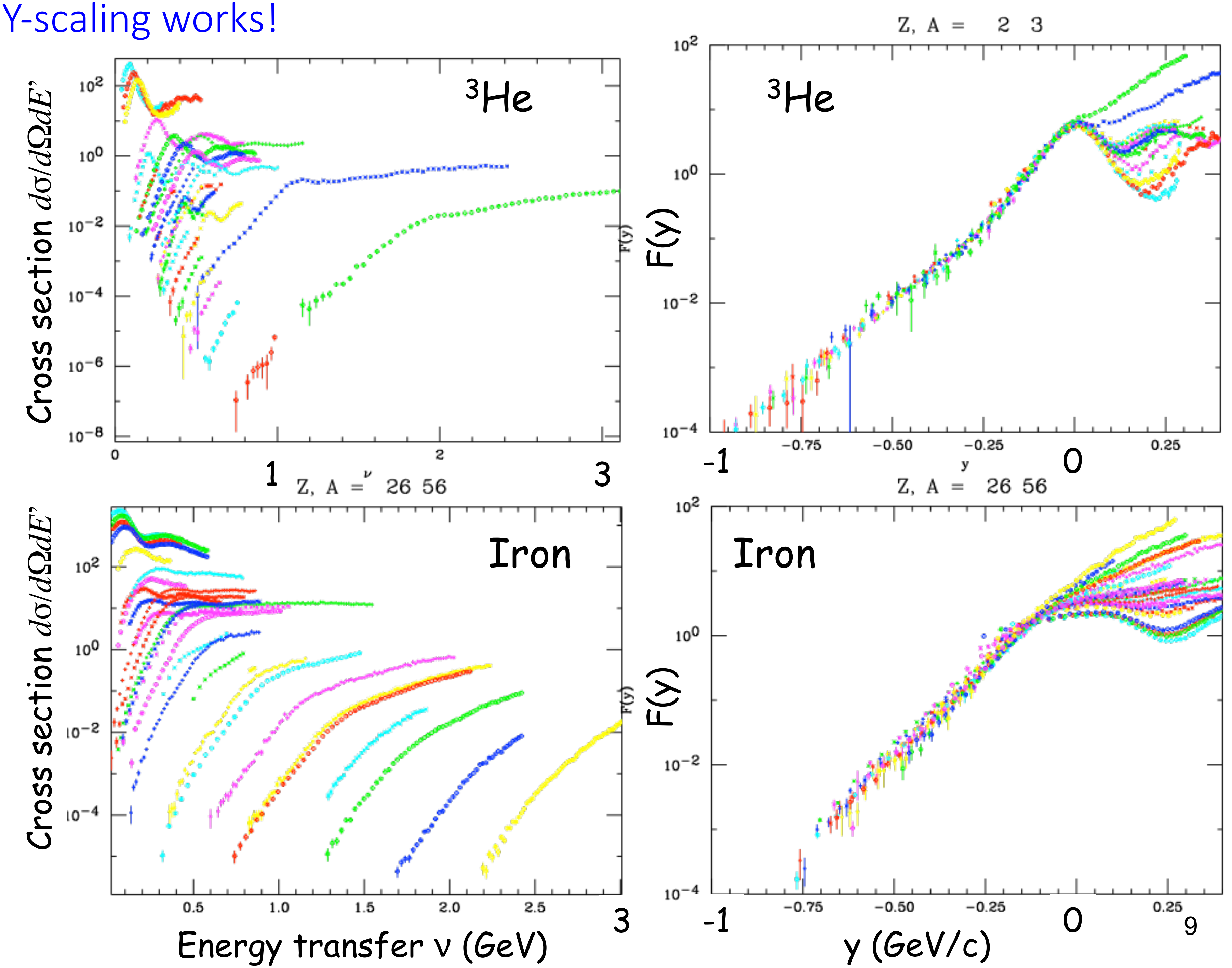
Proton

→ **EM charge radius!**



Neutron

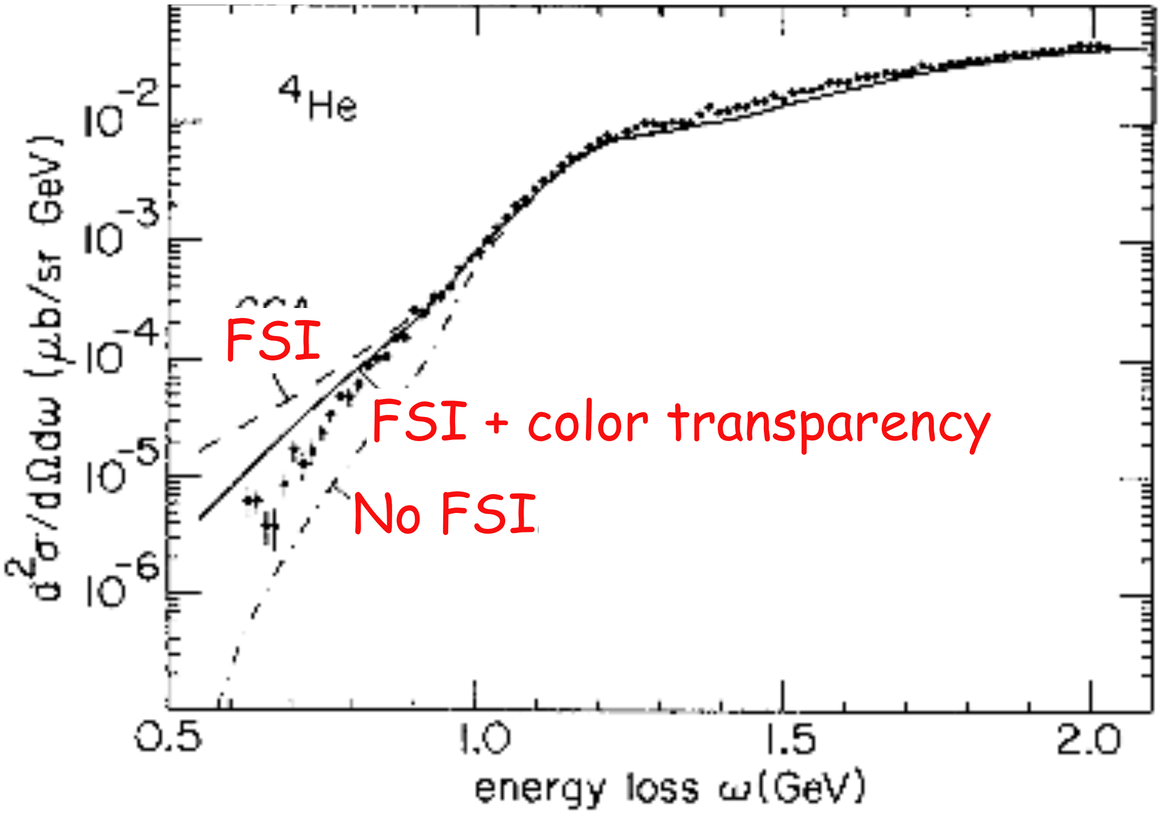
Y-scaling works!



Reminder From Yesterday

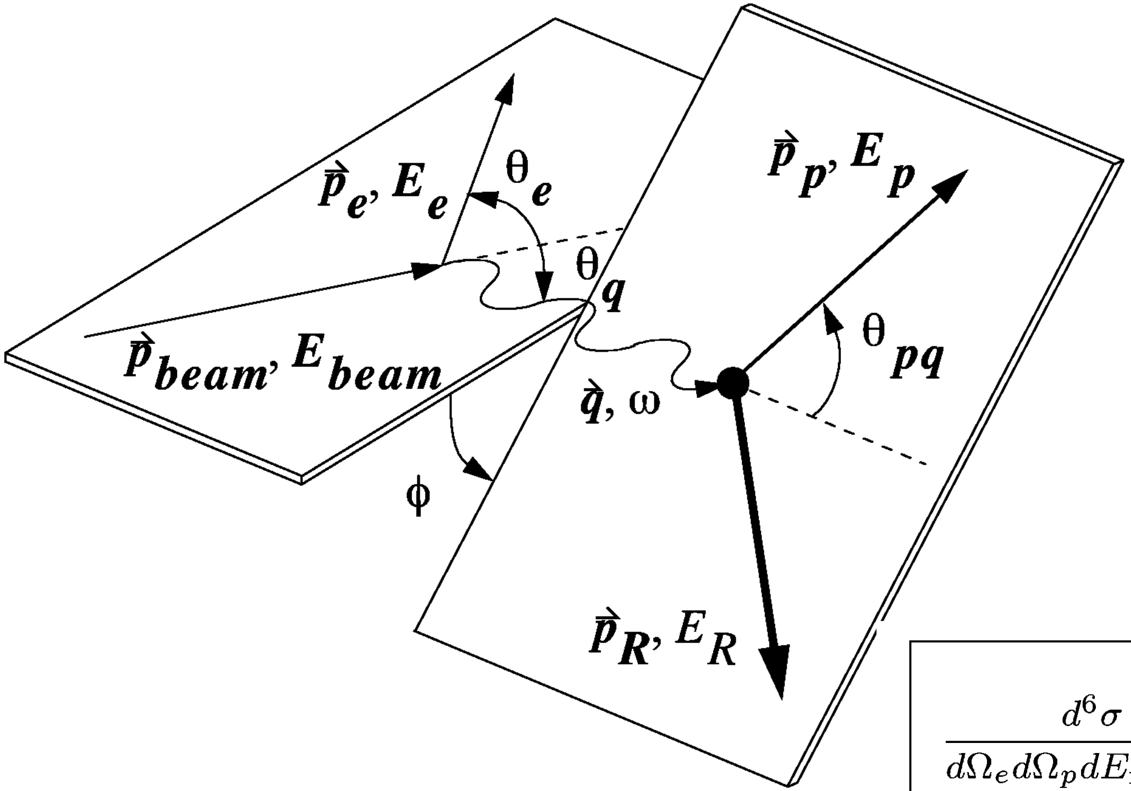
Final State Interactions (FSI) complicate this simple picture

^{4}He(e,e') at 3.595 GeV, 30^o



Benhar et al. PRC 44, 2328
Benhar, Pandharipande, PRC 47, 2218
Benhar et al. PLB 3443, 47

(e,e’p) Spectroscopy



$$\frac{d^6\sigma}{d\Omega_e d\Omega_p dE_{\mathrm{miss}} d\omega} = K\sigma_{\mathrm{Mott}}\,[\, v_L \mathbf{R_L} + v_T \mathbf{R_T} + v_{LT}\mathbf{R_{LT}}\cos(\phi) + v_{TT}\mathbf{R_{TT}}\cos(2\phi)\,]$$

where

K = (phase space)

σ_{Mott} = (relativistic Rutherford scattering)

$v = v\ (q, \omega)$ (electron kinematics)

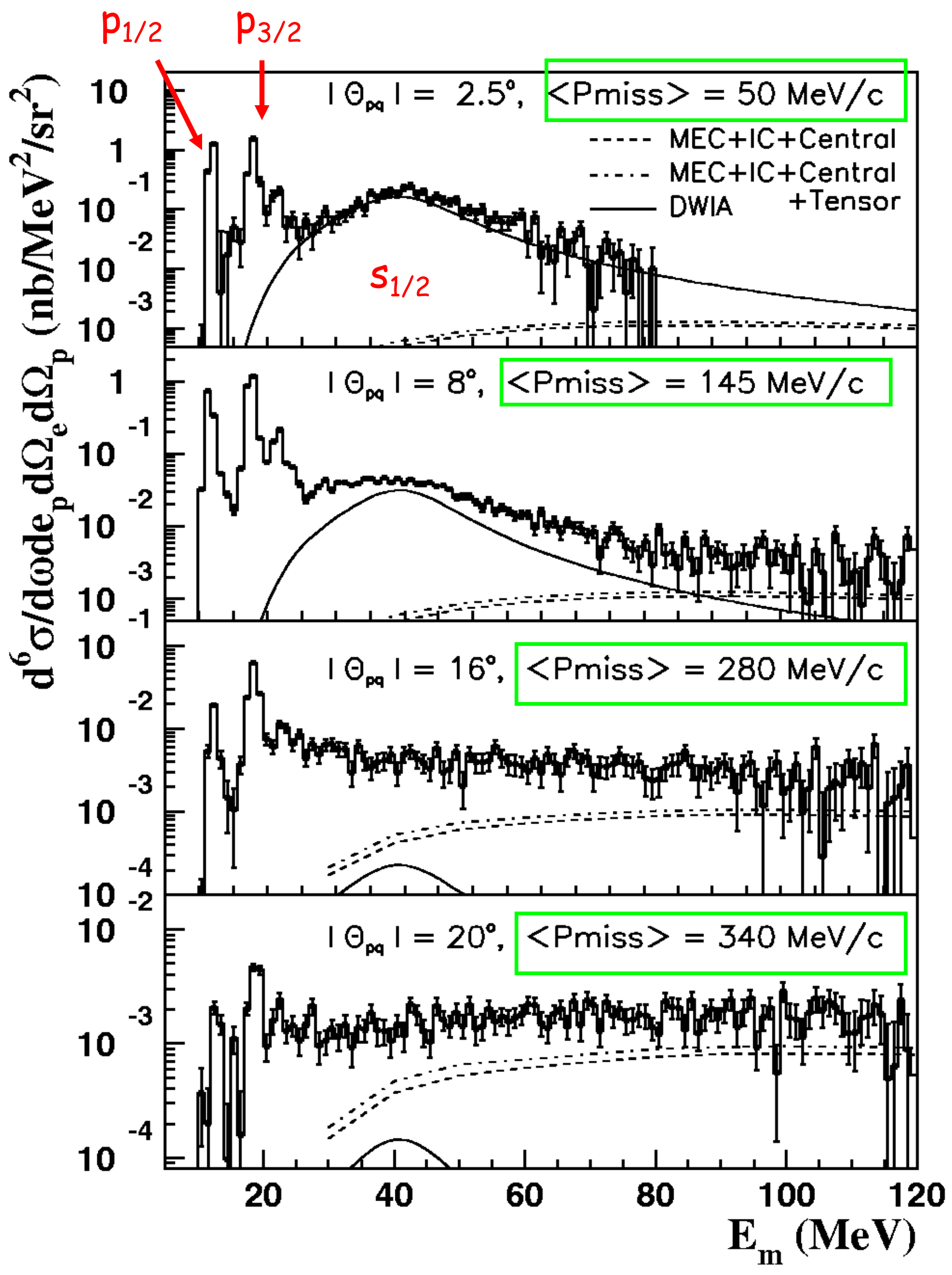
And then there were four (response functions, that is)

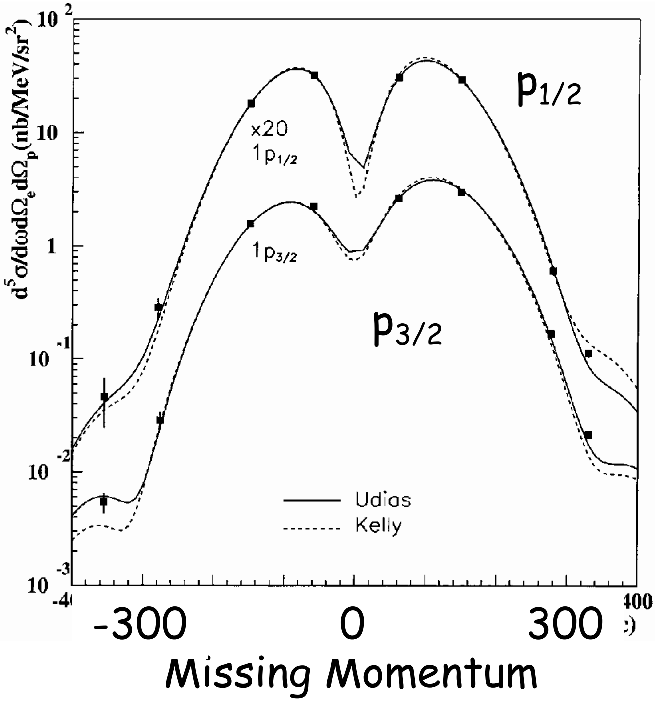
Each R now depends on more variables
R = R(q, ω, p_{miss}, E_{miss})

(When you include electron and proton spin, there are 18!)

(And if you scatter from a polarized spin-1 target, there are 41. Double Yikes!!)

^{16}O(e,e'p) and shell structure



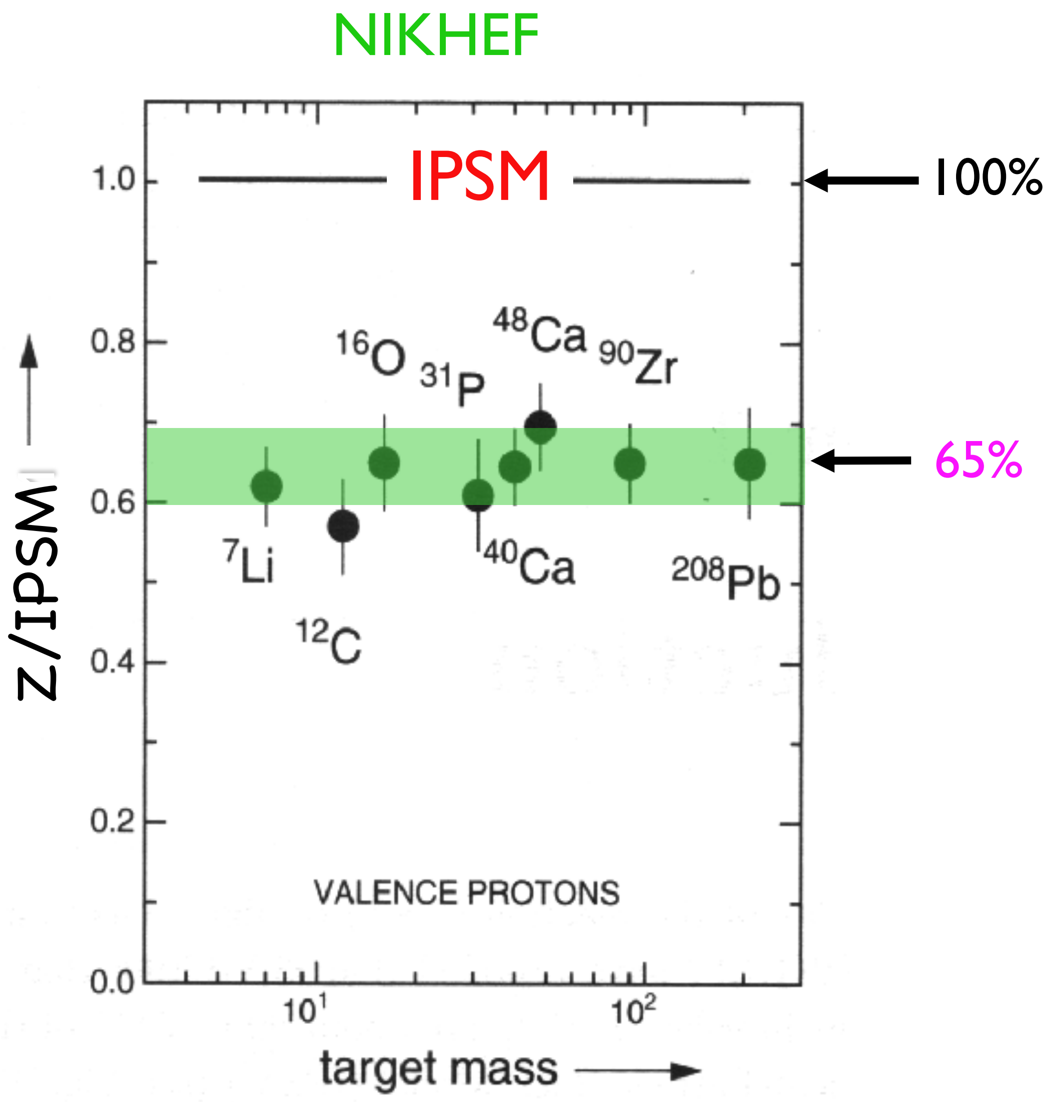


$1p_{1/2}$, $1p_{3/2}$ and $1s_{1/2}$ shells visible

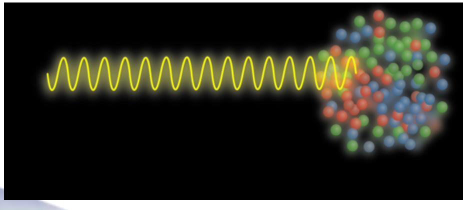
Momentum distribution as expected for l = 0, 1

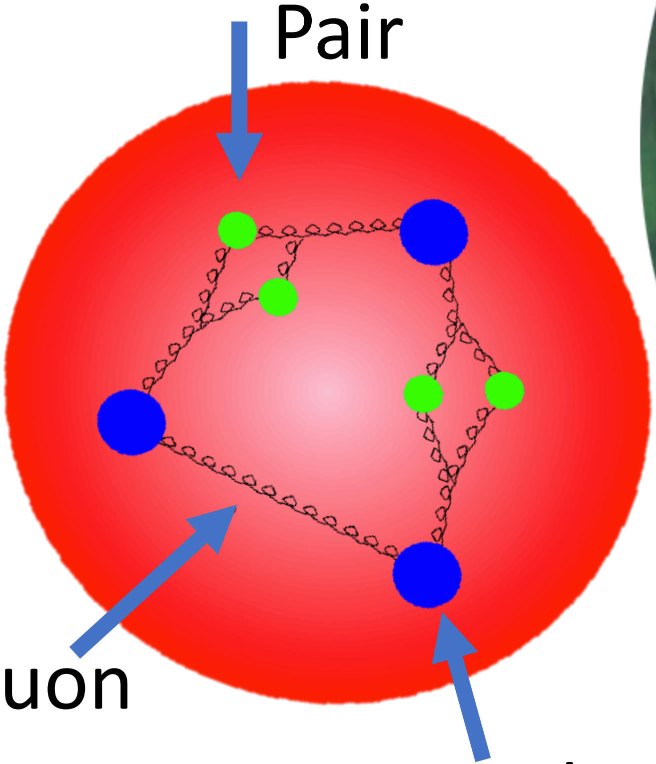
Fissum et al, PRC 70, 034606 (2003)

Reminder From Yesterday



Partonic – Nucleonic Interplay

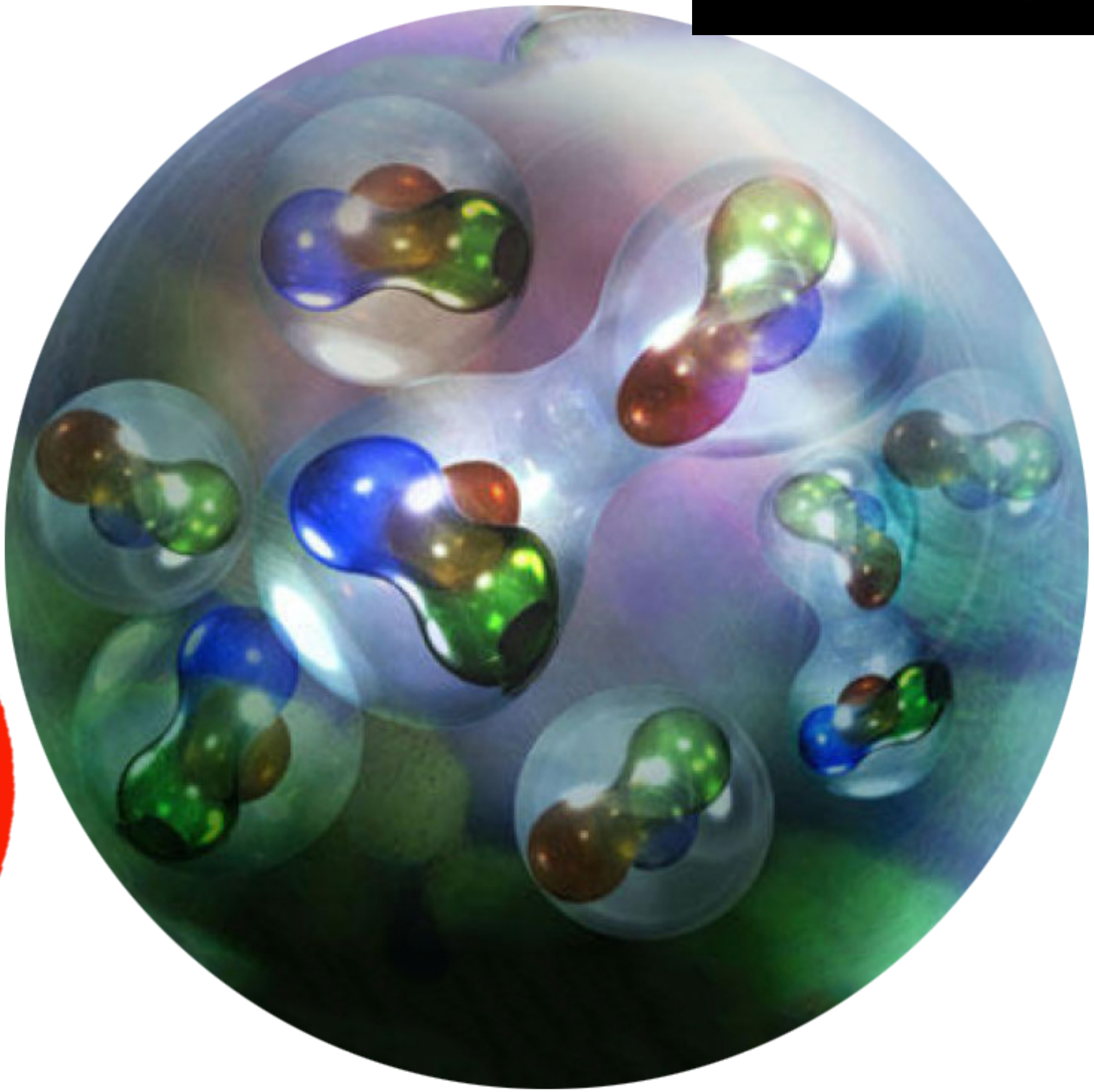




Quark – Anti-quark Pair

Gluon

Quark



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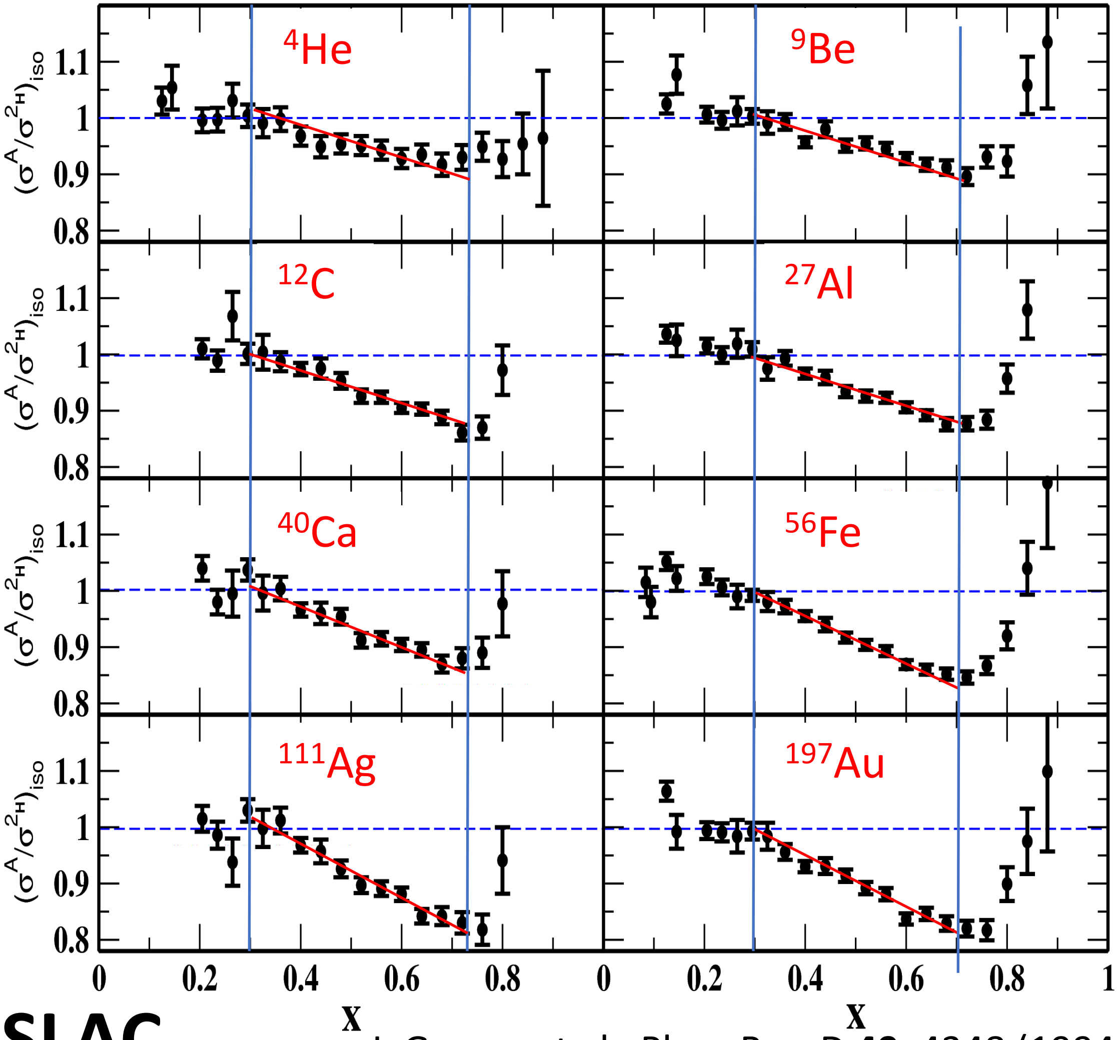
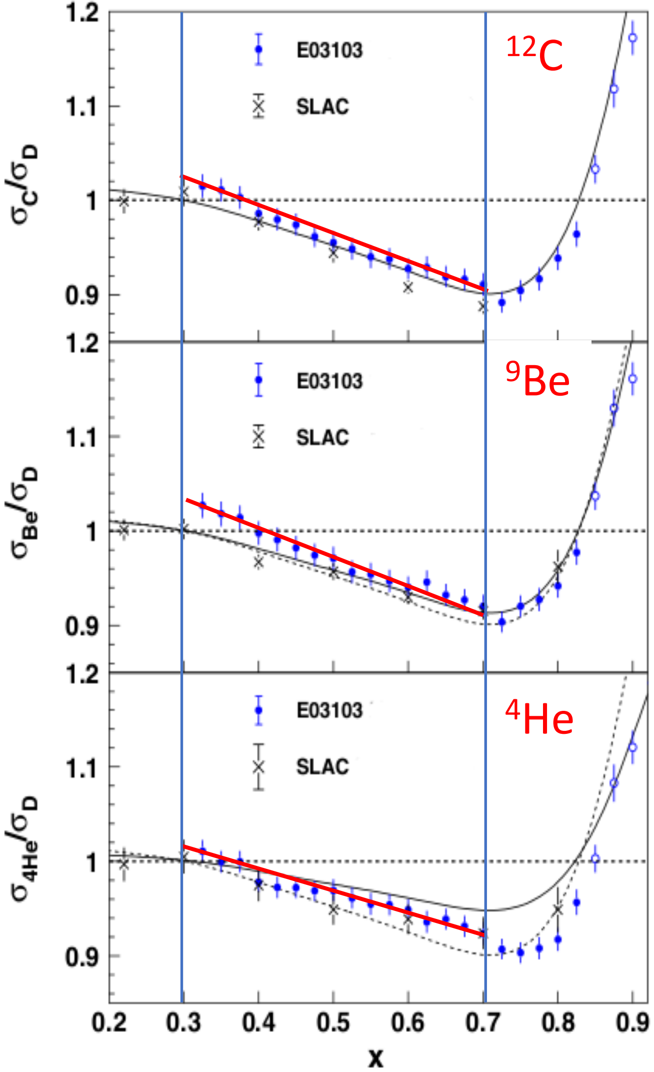


SLAC

J. Gomez et al., Phys. Rev. D **49**, 4348 (1994).

JLab



J. Seely et al., Phys. Rev. Lett. **103**, 202301 (2009).

Today: Short-Range nuclear Structure

Theory:

1. Beyond the mean-field: NN Correlations,
2. Effective vs. ab-initio calculations
3. Phase-equivalent NN interactions
4. Reaction theory: confronting theory and experiment.

Experiment:

1. (e,e'), (e,e'N), (e,e'NN) => Details of NN correlations,
2. Correlations in asymmetric nuclei,
3. NN interactions at short distances.

Contact Formalism: Effective theory for short-distance.

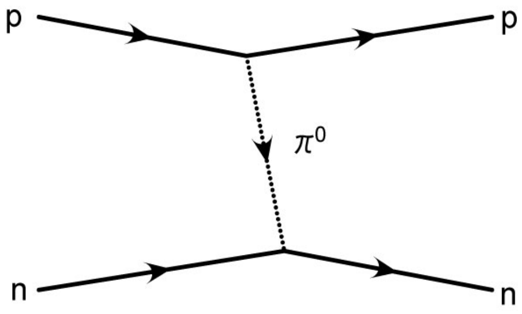
Nuclear *Many-Body* Challenge

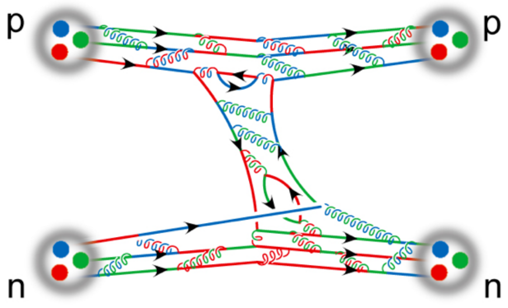
Many-body Schrödinger Equation

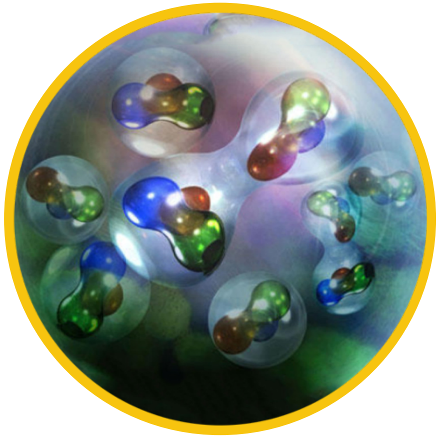
$$\sum_i \left\{-\frac{\hbar^2}{2m_i}\nabla_i^2\Psi(\vec{r}_1,\ldots,\vec{r}_N,t)\right\} + U(\vec{r}_1,\ldots,\vec{r}_N)\Psi(\vec{r}_1,\ldots,\vec{r}_N,t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r}_1,\ldots,\vec{r}_N,t)$$

Main Challenges:

1. No ‘fundamental’ Interaction.
2. Complex phenomenological parametrizations (e.g. over 18 operators)

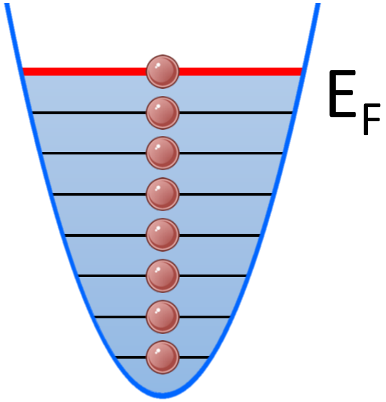






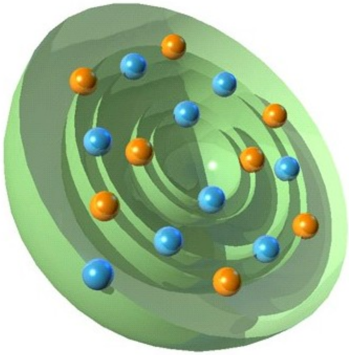
Solution: Effective Theories

Fermi Gas Model

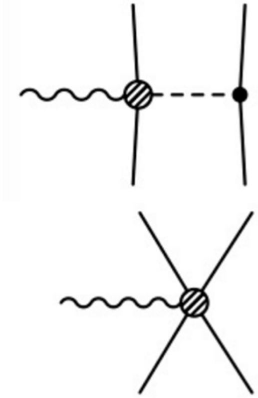


Liquid Drop Model

Shell Model

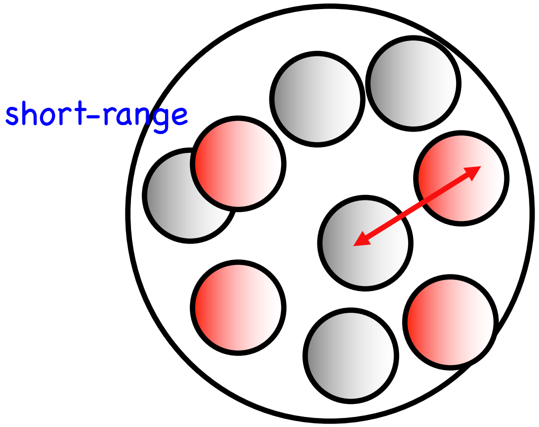


Chiral Perturbation Theory*



* Should converge to exact solution

Nuclear Structure in More Detail

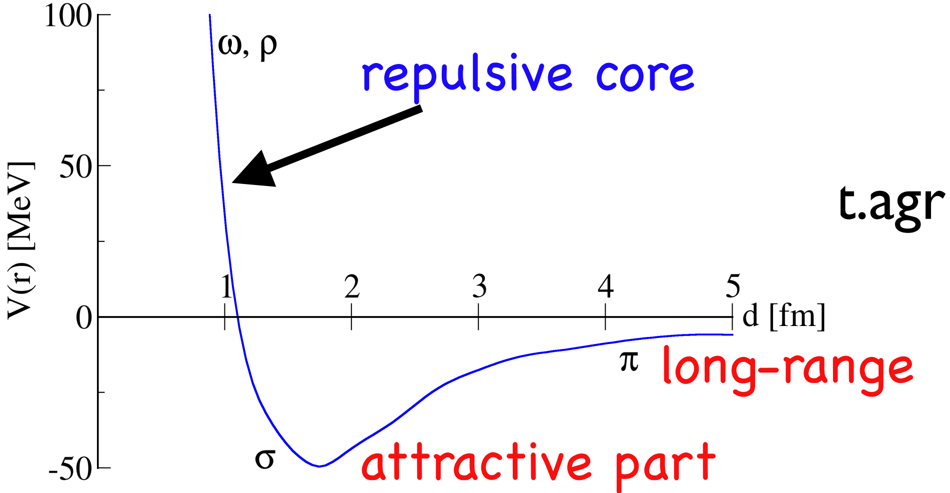


- nucleons are bound
  - energy (*E*) distribution
  - shell structure
- nucleons are not static
  - momentum (*k*) distribution

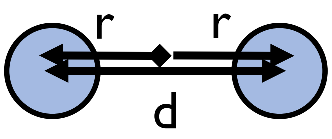
=> Need a spectral function!

$$S(\vec{p},E)=\sum_i|\Phi_a(p)|^2\,\delta(E+\epsilon_a)$$

Determined by the N-N potential:

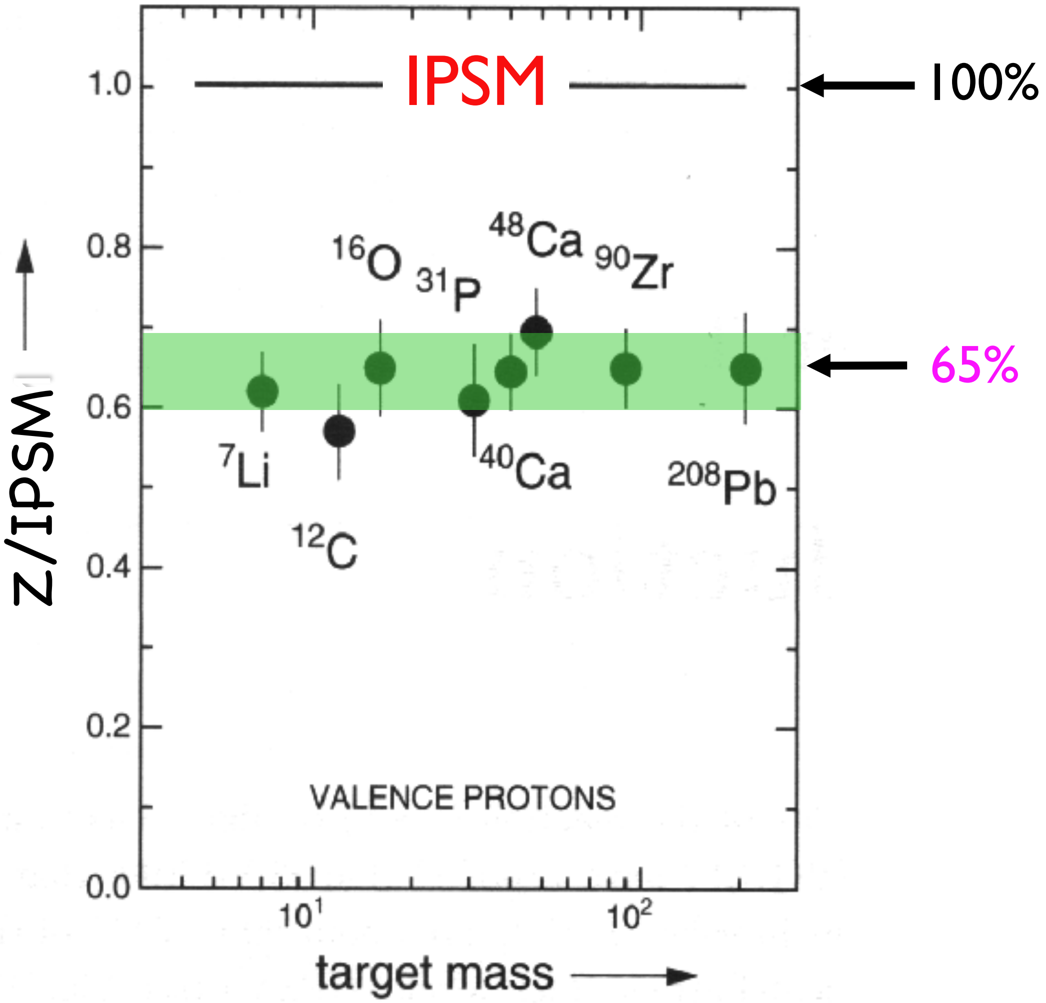


on average:
Net binding energy: ≈ 8 MeV
distance: ≈ 2 fm



Spectroscopic Factors

NIKHEF



Modern *Ab-Initio* Calculations

Use smart algorithms and fast computers to solve the Many-body Schrödinger Equation:

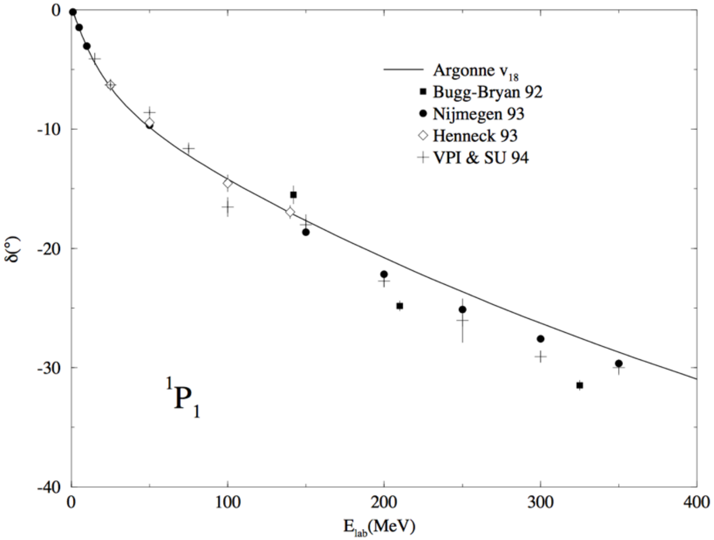
$$\sum_i \left\{ -\frac{\hbar^2}{2m_i} \nabla_i^2 \Psi(\vec{r}_1, \dots, \vec{r}_N, t) \right\} + U(\vec{r}_1, \dots, \vec{r}_N) \Psi(\vec{r}_1, \dots, \vec{r}_N, t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}_1, \dots, \vec{r}_N, t)$$

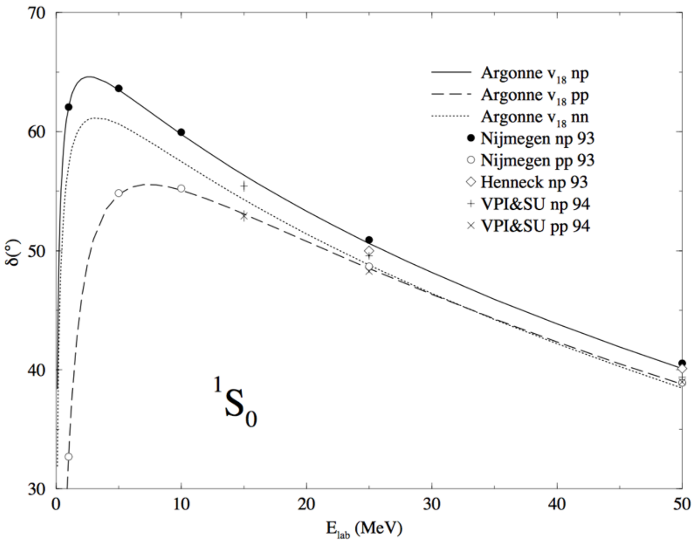
Need to determine the NN interaction -> phase shifts fits.

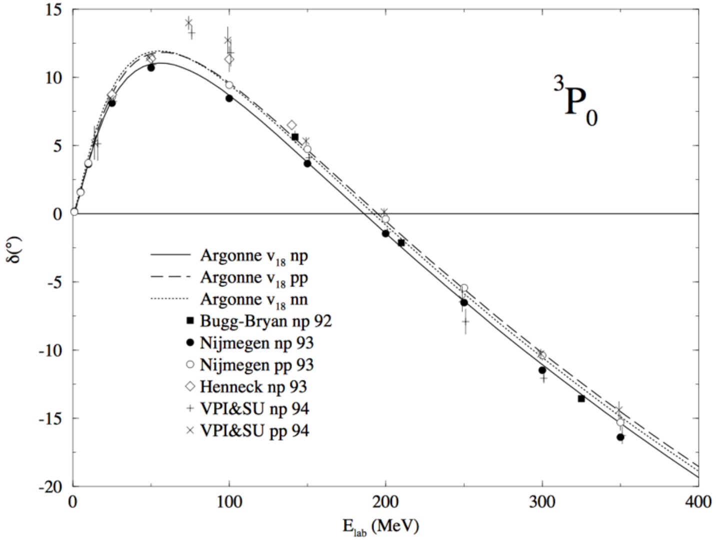
Various models on the market. We start with the ‘traditional’ AV18 and then discuss the others…

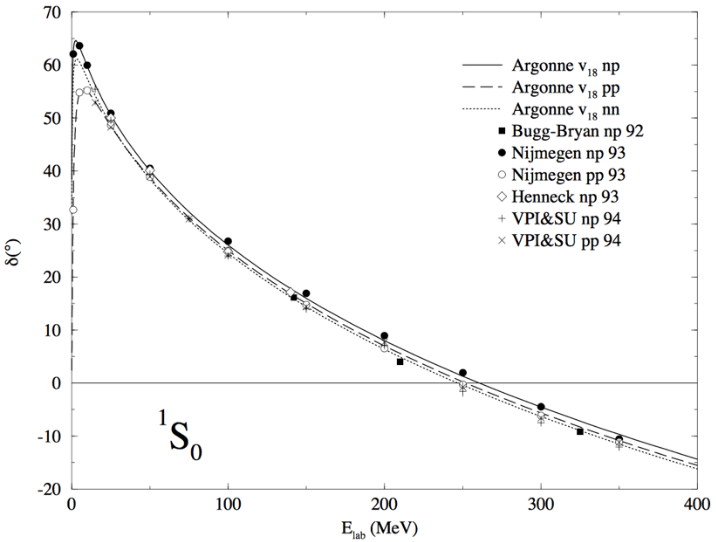
Resulting in one and two body densities in coordinate and momentum space

AV18 Phase Shifts Fits









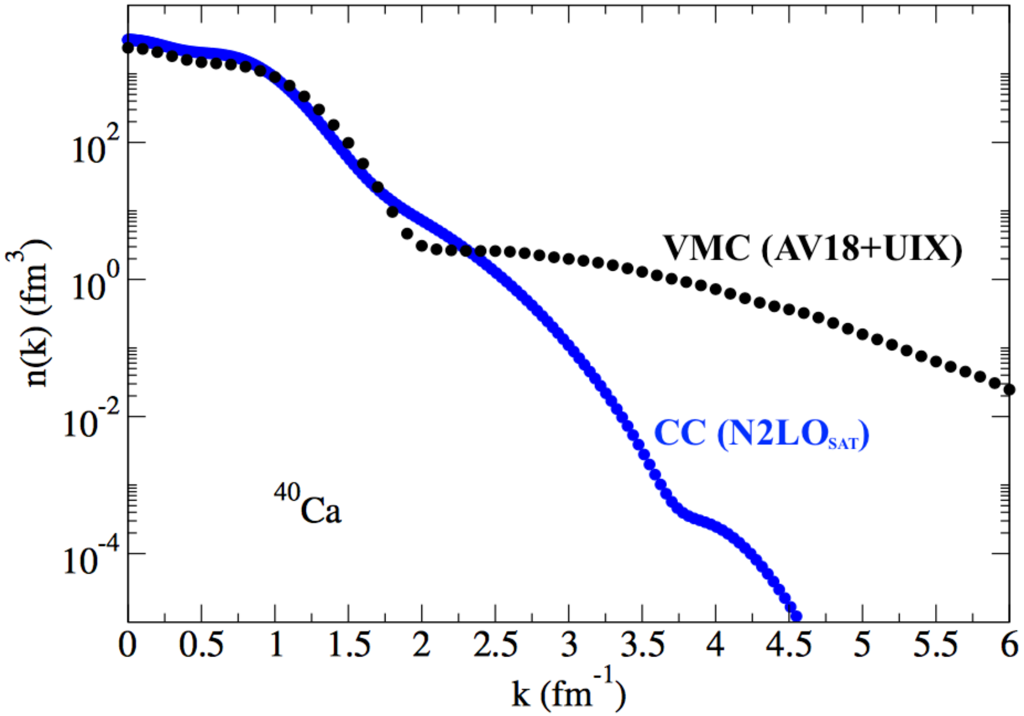
Interlude: Different NN Interactions

- Observables are products of wave-function times operators: $O|\varphi\rangle$.
- Can always ‘shift’ complexity from the wave-function to the operators.
- Specifically, using unitary transformations we can build many ‘phase equivalent’ interactions AND describe experiments using a series of many-body operators and simple wave functions.

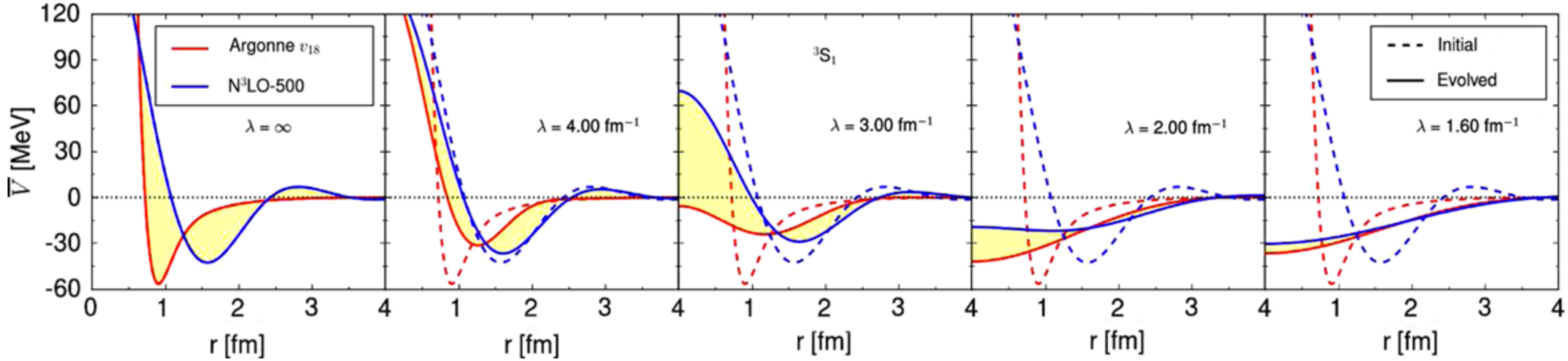
$$O|\varphi\rangle \rightarrow OUU^{+}|\varphi\rangle$$

Very useful for low-energy reactions where wave functions are complicated and operators are simple.

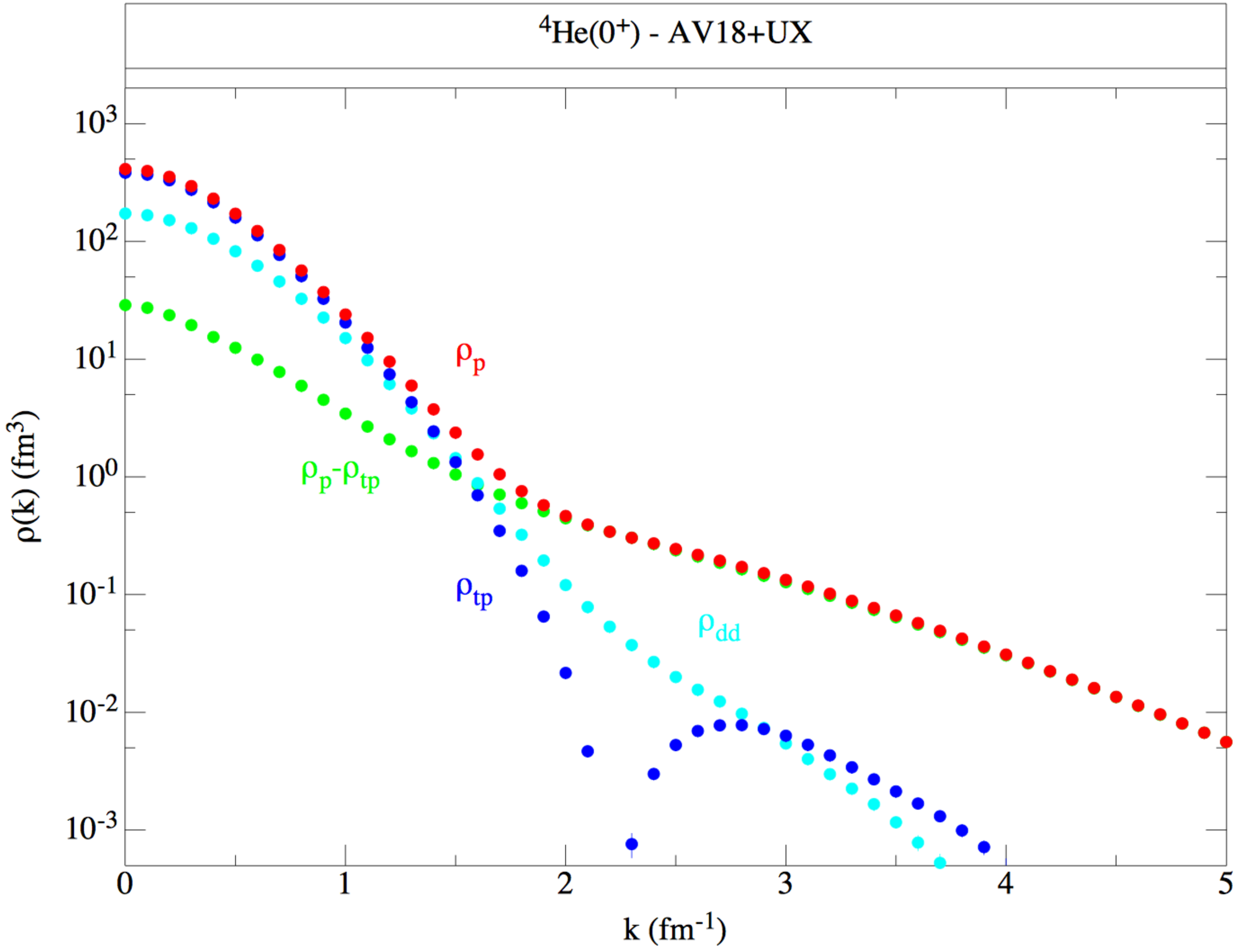
Interlude: Different NN Interactions



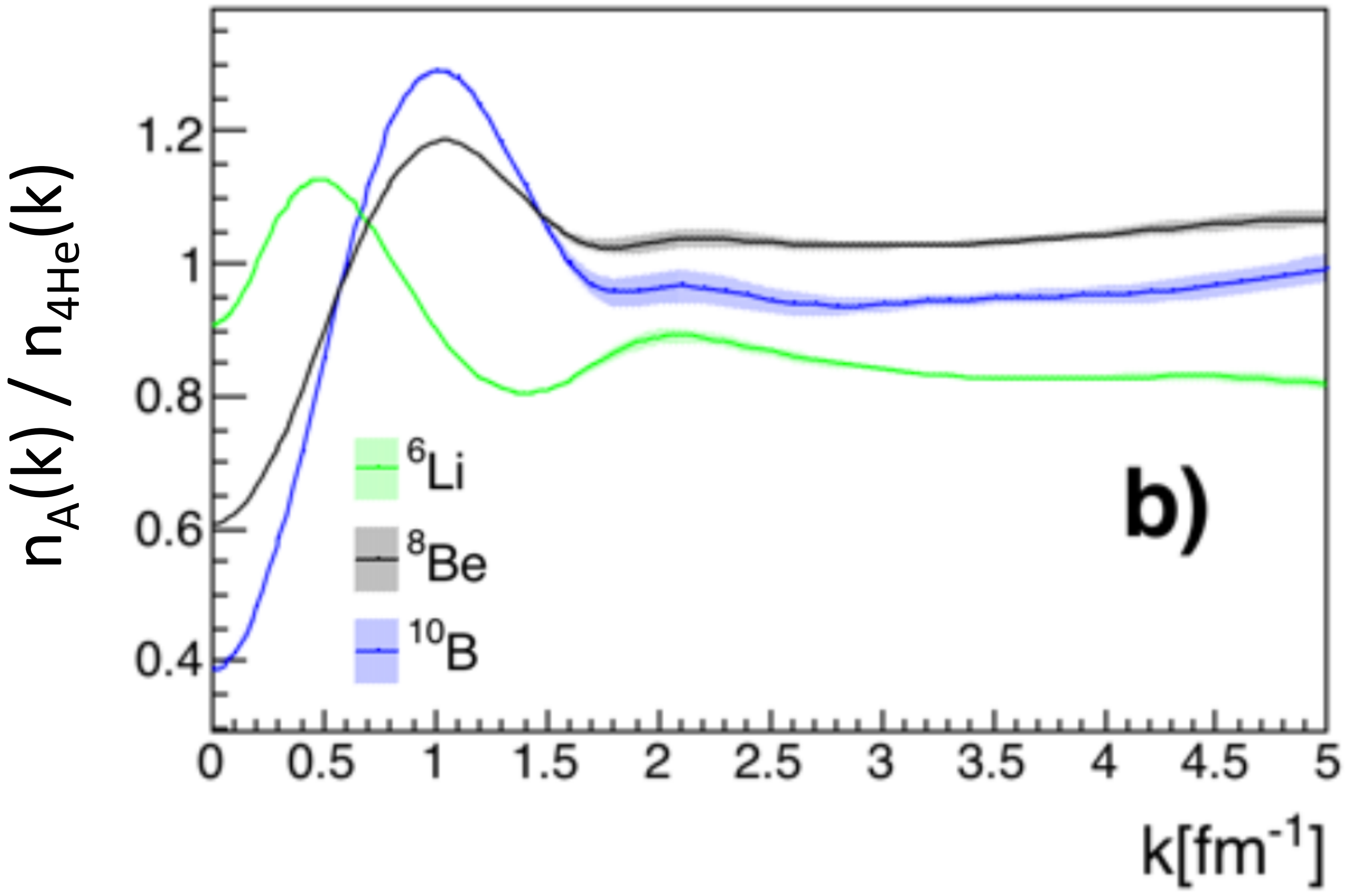
Differences at high-momentum due to cutoff used in the calculations. Not a problem for many observables! But not all…



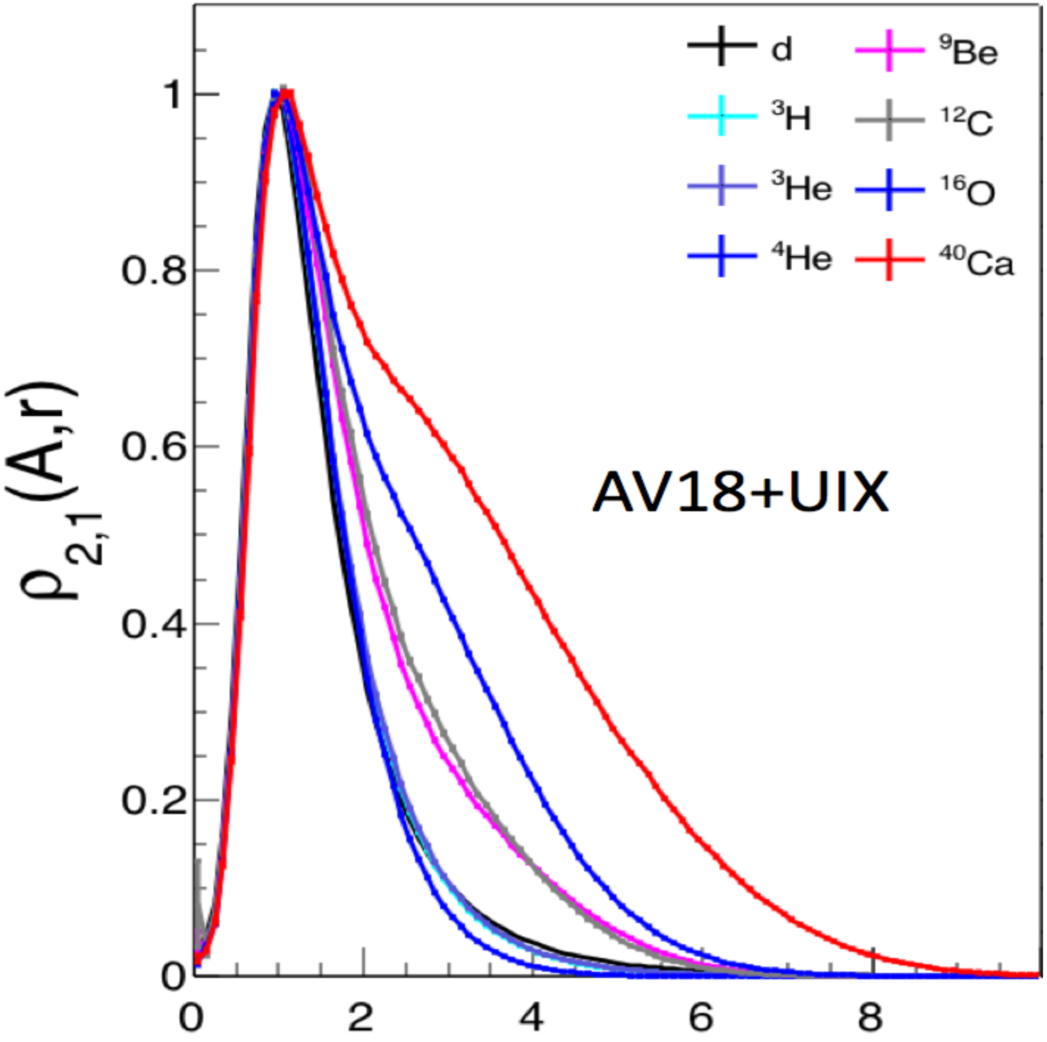
Single-Nucleon Momentum Distribution



High-Momentum Scaling!



Pair Density Distribution

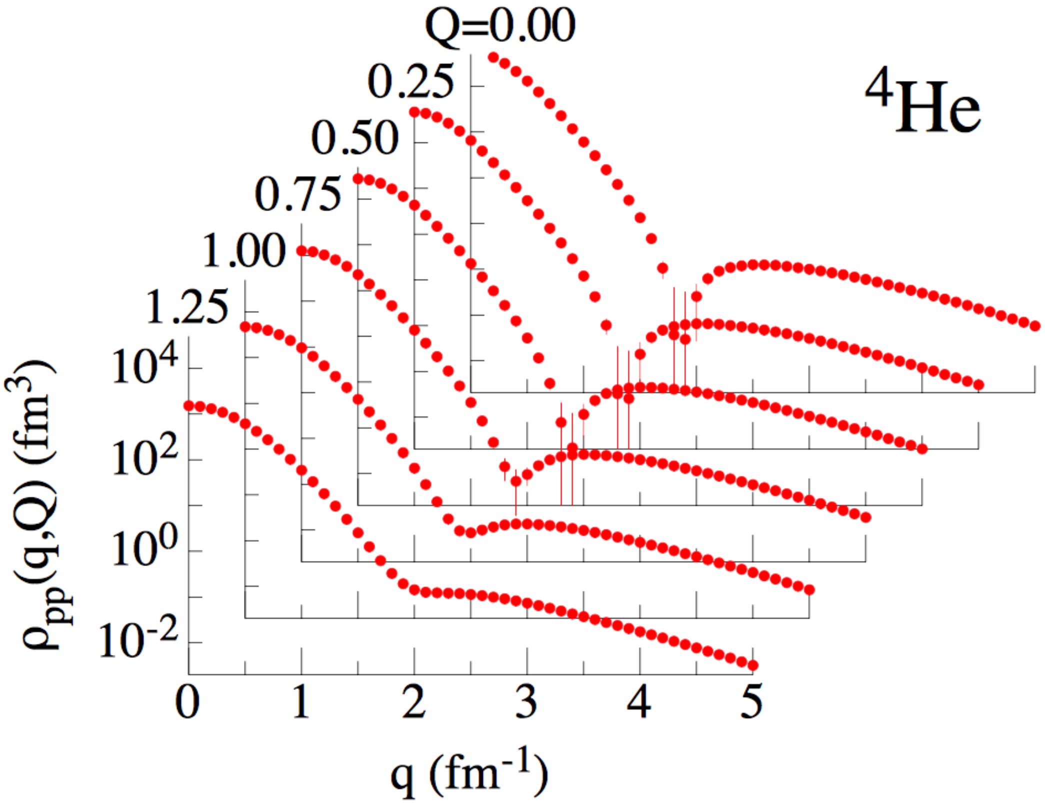


Probability to find 2 nucleons at a given distance, r [fm]

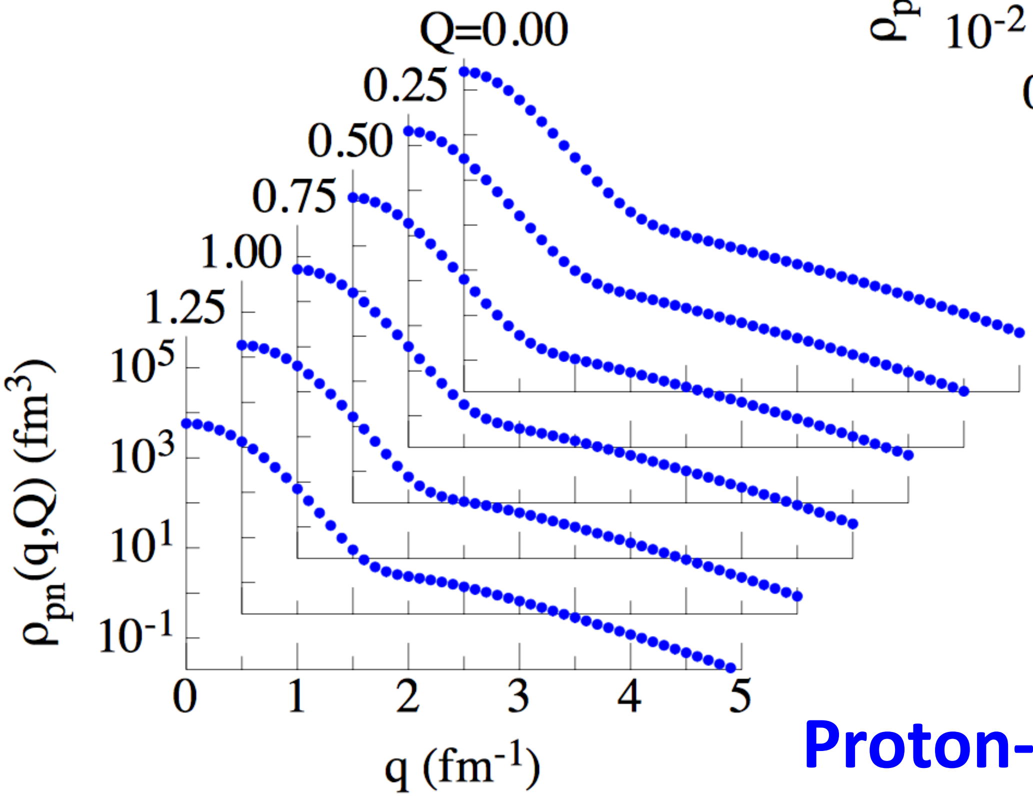
=> Short Distance Scaling!

Is the a connection between the short-distance and high-momentum scaling?

Pair Momentum Distribution

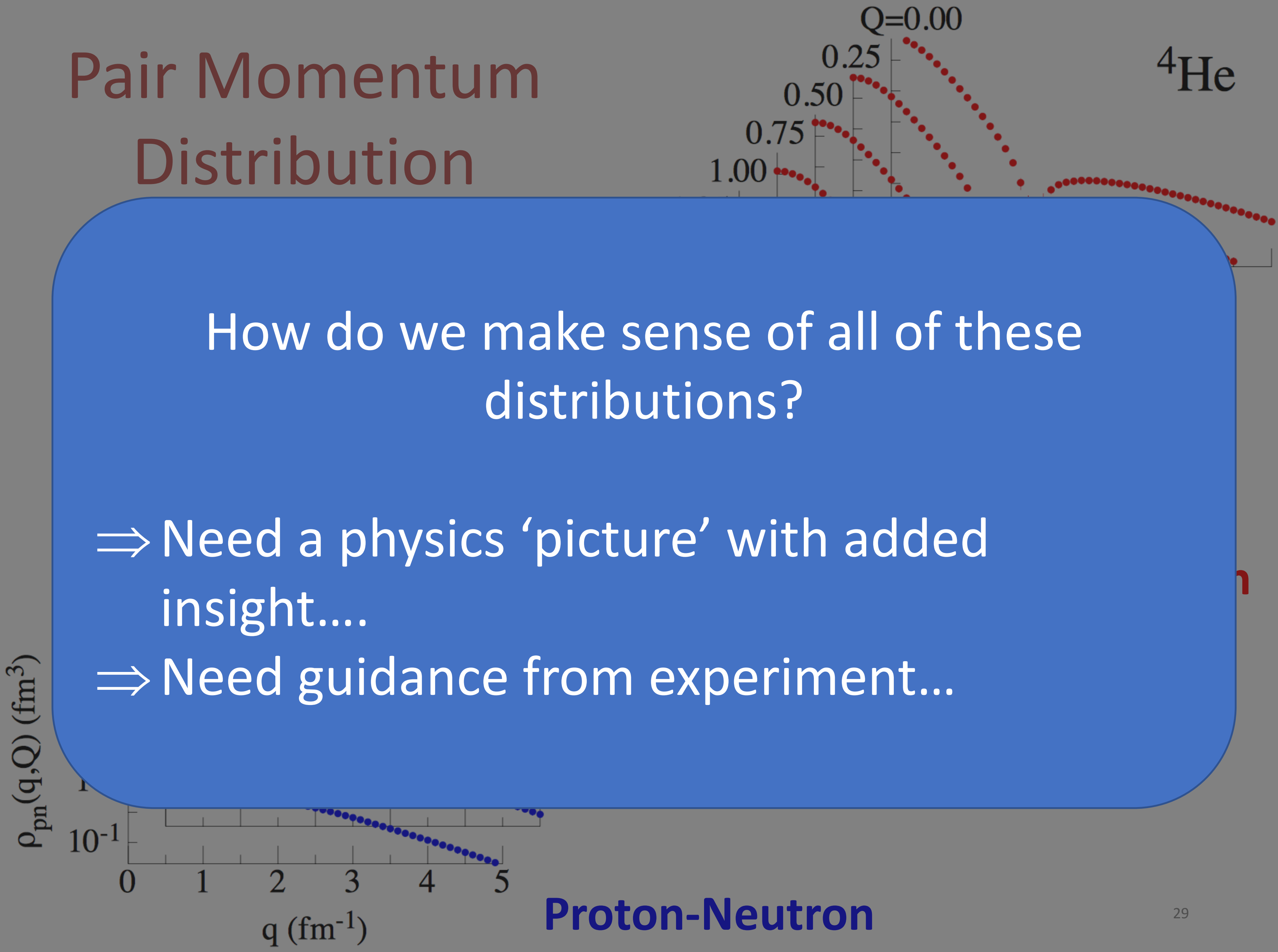


Proton-Proton



Proton-Neutron

Pair Momentum Distribution



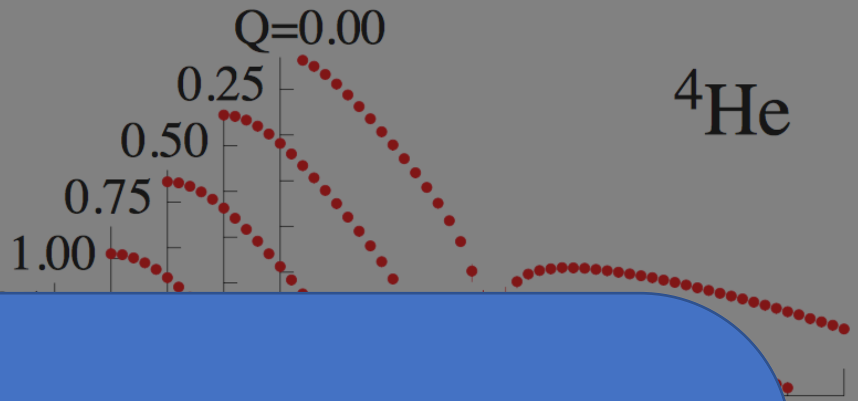
How do we make sense of all of these distributions?

⇒ Need a physics ‘picture’ with added insight....

⇒ Need guidance from experiment...

Proton-Neutron

Pair Momentum Distribution



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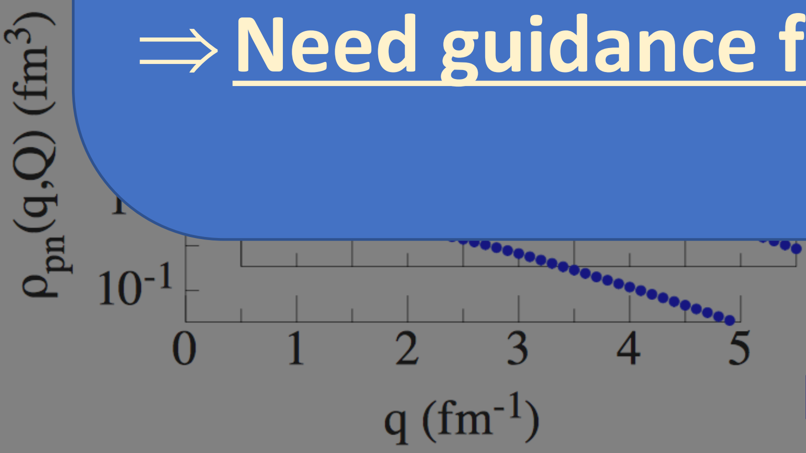
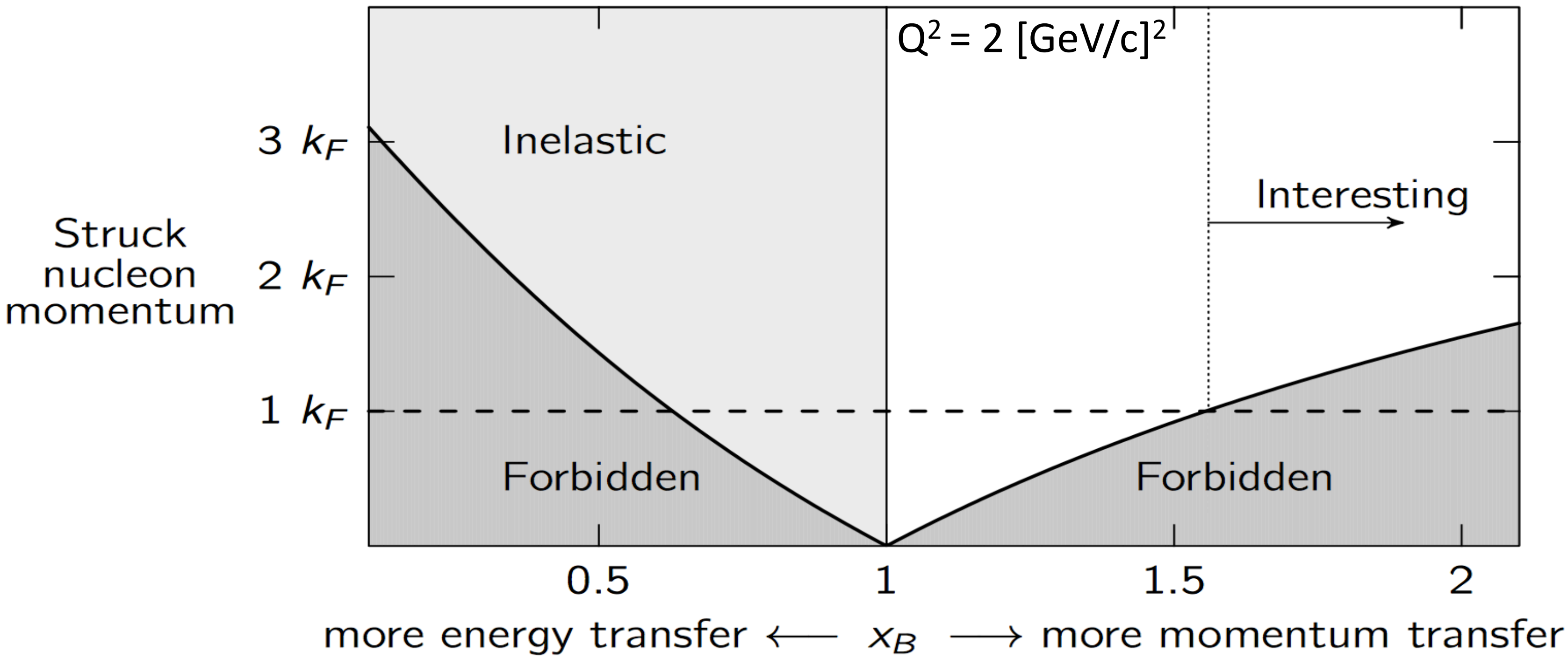


Proton-Neutron

Looking for the missing protons

(e,e') cross section at different kinematics are sensitive to different 'parts' of the nuclear momentum distribution.



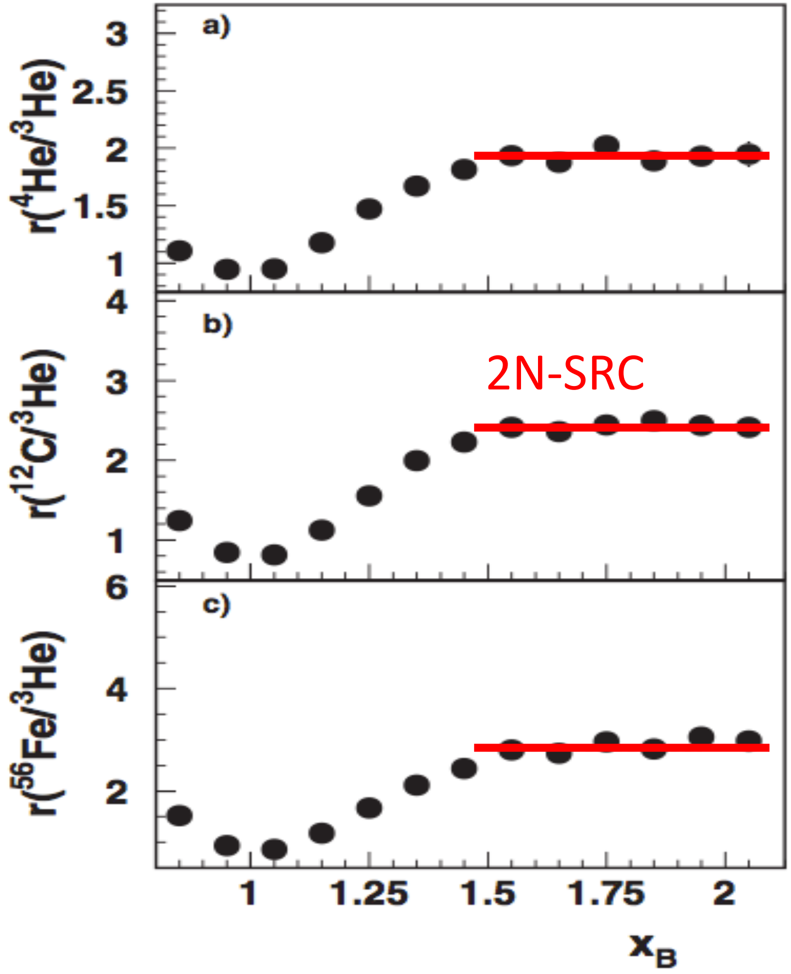
$$(q+p_A-p_{A-1})^2=p_f^2=m_N^2$$

More Scaling ??!?!?

- A/d (e,e') cross section ratios sensitive to $n_A(k)/n_d(k)$

- Observed scaling for $x_B \geq 1.5$.

$$\Rightarrow n_A(k>k_F) = a_2(A) \times n_d(k)$$



K. Egiyan et al., PRL **96**, 082501(2006).

L. Frankfurt et al. , Phys. Rev. C **48**, 2451 (1993).
K. Egiyan et al., Phys. Rev. C **68**, 014313 (2003). N. Fomin et al., Phys. Rev. Lett. **108**, 092502 (2012).

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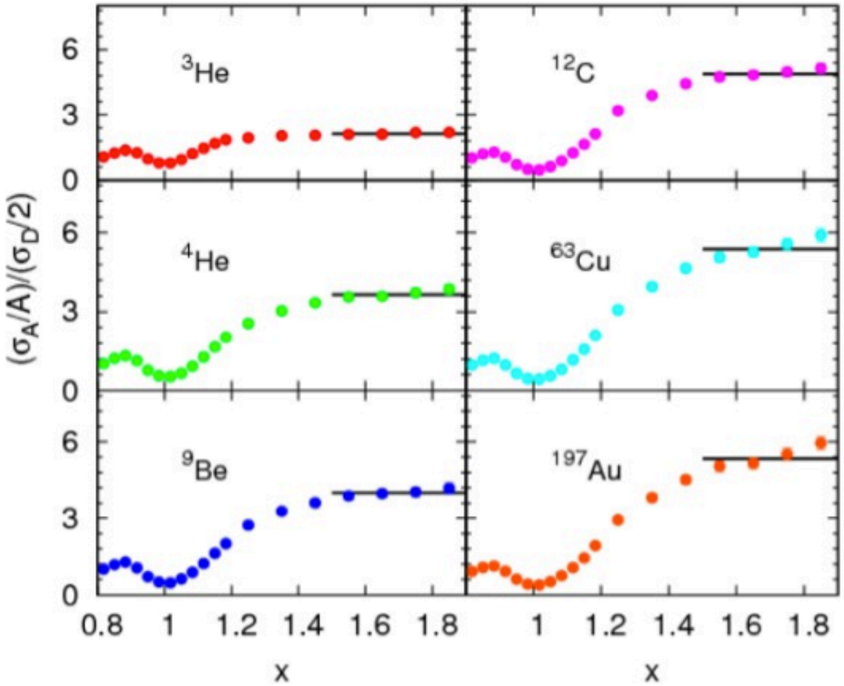

N. Fomin et al., PRL **108**, 092502 (2012)

A	a_2(A/D)	A	a_2(A/D)
^{3}He	**2.1 ± 0.1**	^{12}C	**4.7 ± 0.2**
^{4}He	**3.6 ± 0.1**	^{63}Cu	**5.2 ± 0.2**
^{9}Be	**3.9 ± 0.1**	^{197}Au	**5.1 ± 0.2**

O. Hen et al., PRC **85**, 047301 (2012)

L. Frankfurt et al. , Phys. Rev. C **48**, 2451 (1993).
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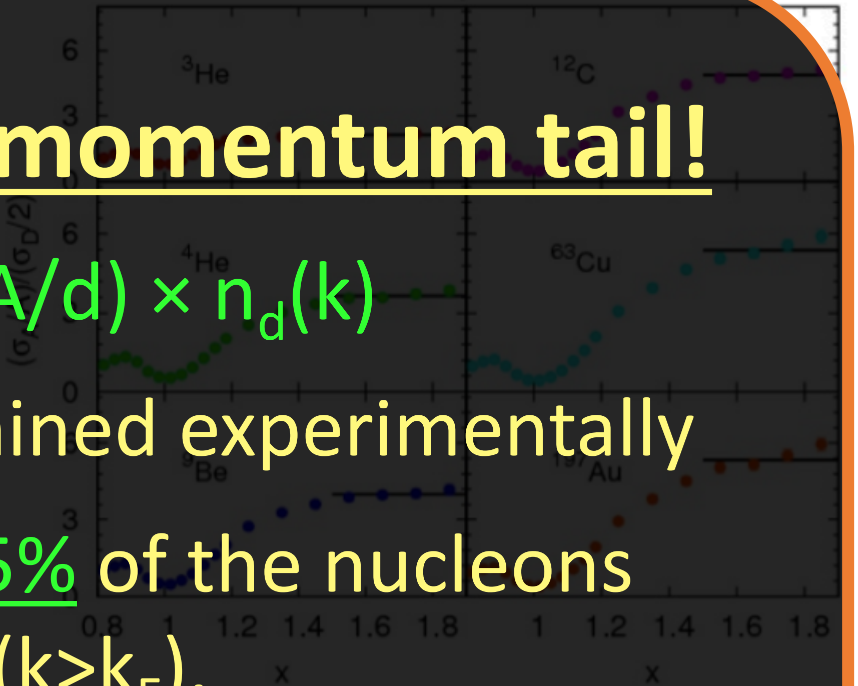
More Scaling ??!?!?

Nuclei have a high-momentum tail!

1. It scales: $n_A(k>k_F) = a_2(A/d) \times n_d(k)$
2. Scale factor, a_2, determined experimentally
3. In A ≥ 12 nuclei, 20 – 25% of the nucleons have high-momentum ($k>k_F$).

- A/d (e,e') cross section ratios sensitive to $n_A(k)/n_d(k)$
- Observed scaling for $x_B > 1.5$

$$=> n_A(k>k_F) = a_2(A) \times n_d(k)$$



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More Scaling ??!?!?

- A/d (e,e') cross section ratios sensitive to $n_A(k)/n_d(k)$
- Observed scaling for $x_B \geq 1.5$:

$$=> n_A(k>k_F) = a_2(A) \times n_d(k)$$

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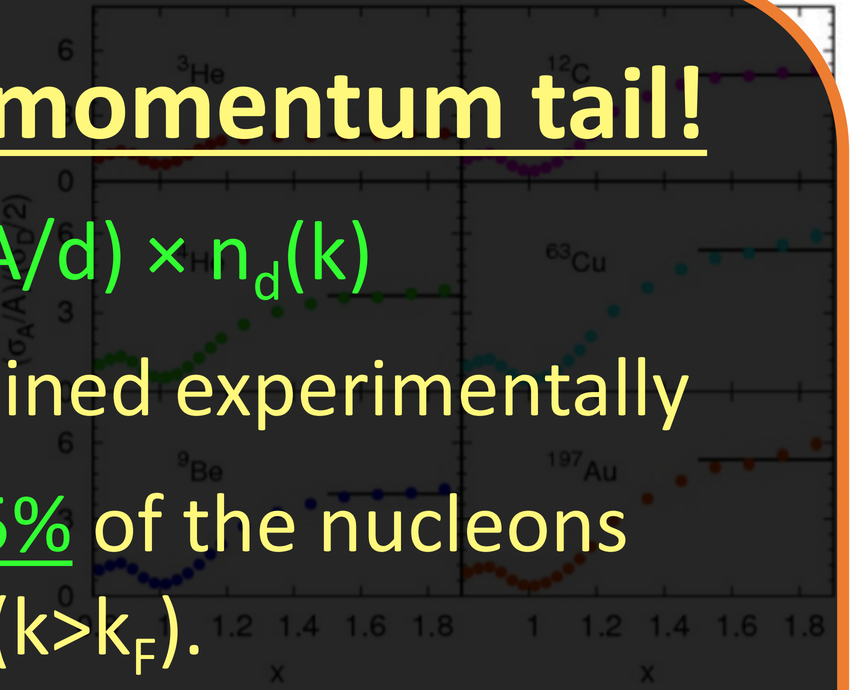
2. Scale factor, a_2, determined experimentally
3. In A ≥ 12 nuclei, 20 – 25% of the nucleons have high-momentum ($k>k_F$).

Do ALL high-momentum nucleons come in pairs? What kind of pairs?



N. Fomin et al., PRL **108**, 092502 (2012)

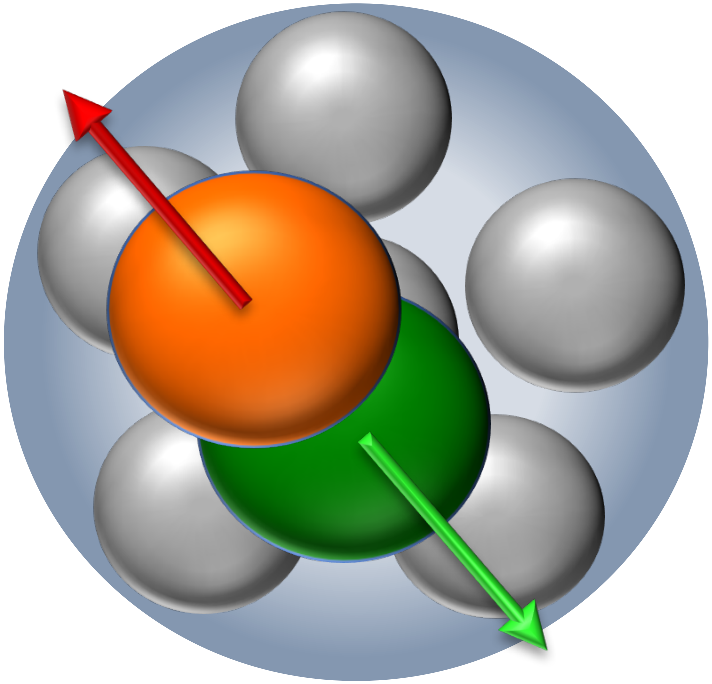
A	a_2(A/D)	A	a_2(A/D)
^{3}He	[illegible]	^{12}C	4.7 ± 0.2
^{4}He	3.6 ± 0.1	^{63}Cu	5.2 ± 0.2
^{9}Be	3.9 ± 0.1	^{197}Au	5.1 ± 0.2

O. Hen et al., PRC **85**, 047301 (2012)

L. Frankfurt et al. , Phys. Rev. C **48**, 2451 (1993).
K. Egiyan et al., Phys. Rev. C **68**, 014313 (2003).

K. Egiyan et al., PRL **96**, 082501 (2006)

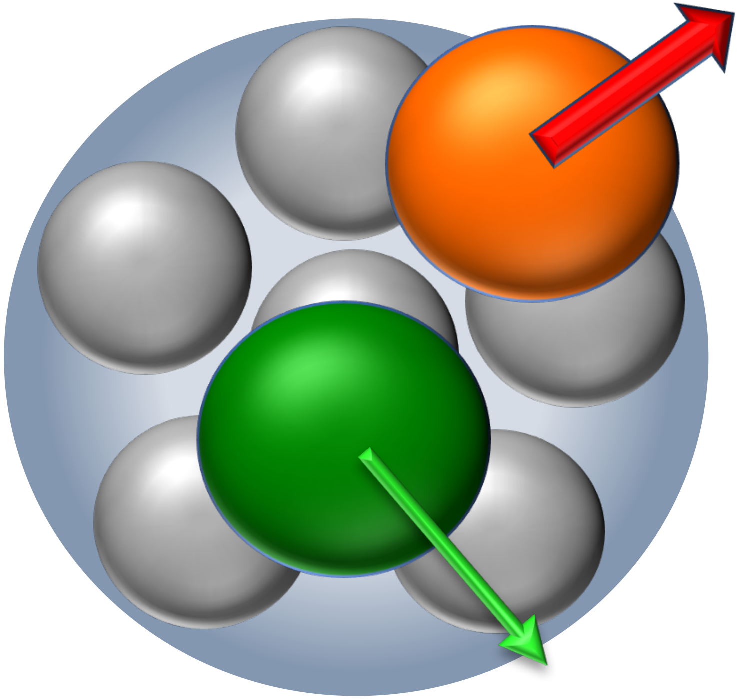
Two-Nucleon Knockout



Two-Nucleon Knockout

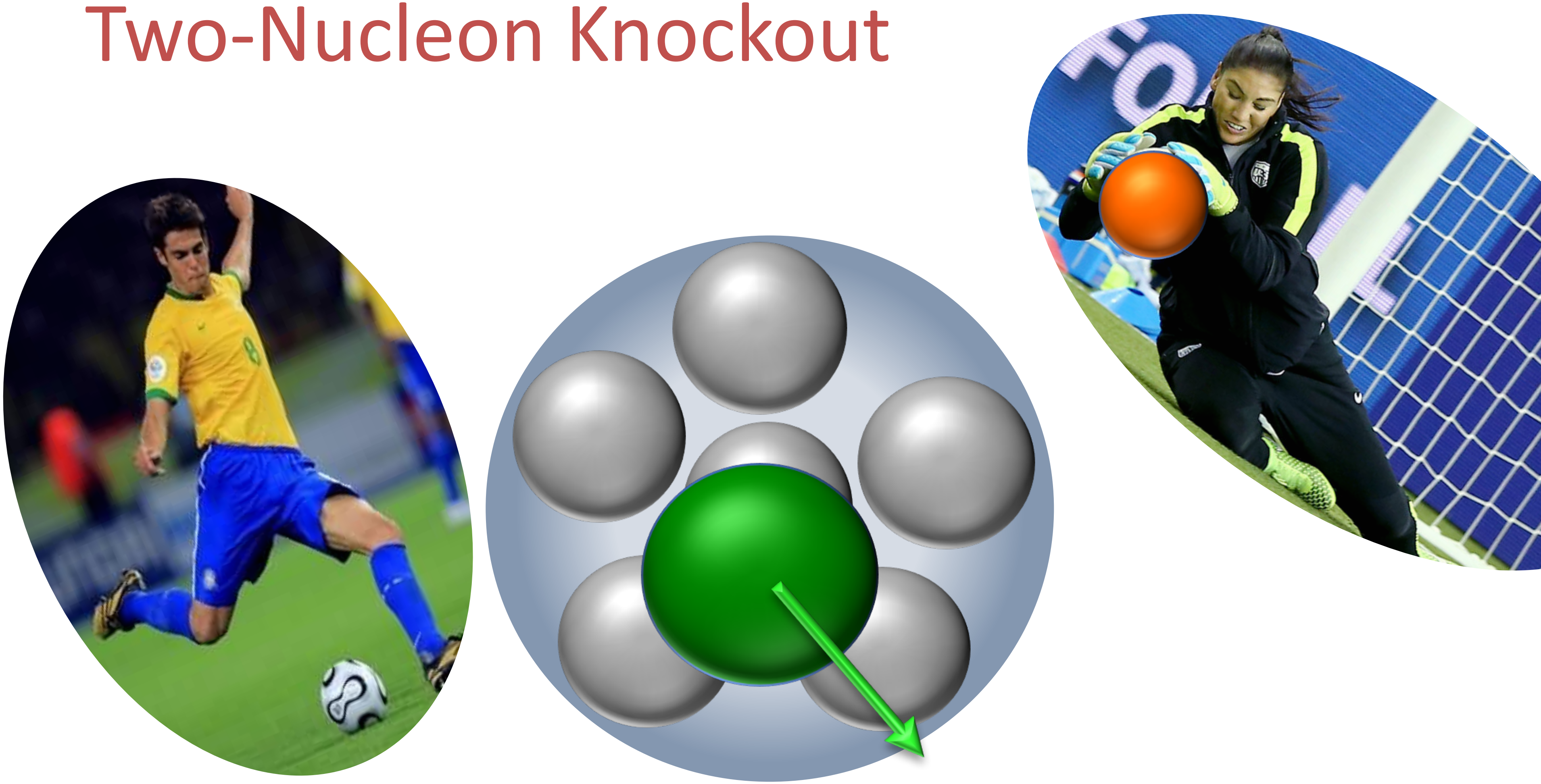
FIFA WORLD CUP
Brasil

Two-Nucleon Knockout



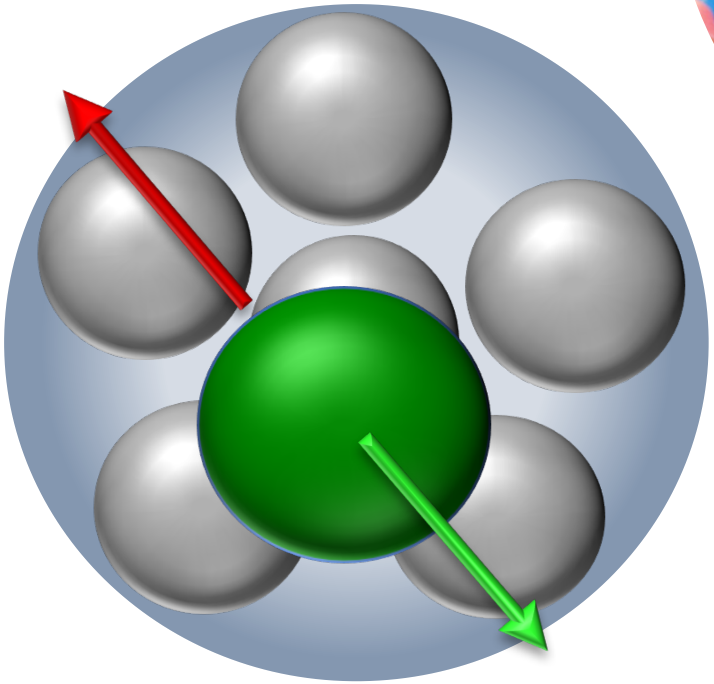
FIFA WORLD CUP
Brasil

Two-Nucleon Knockout



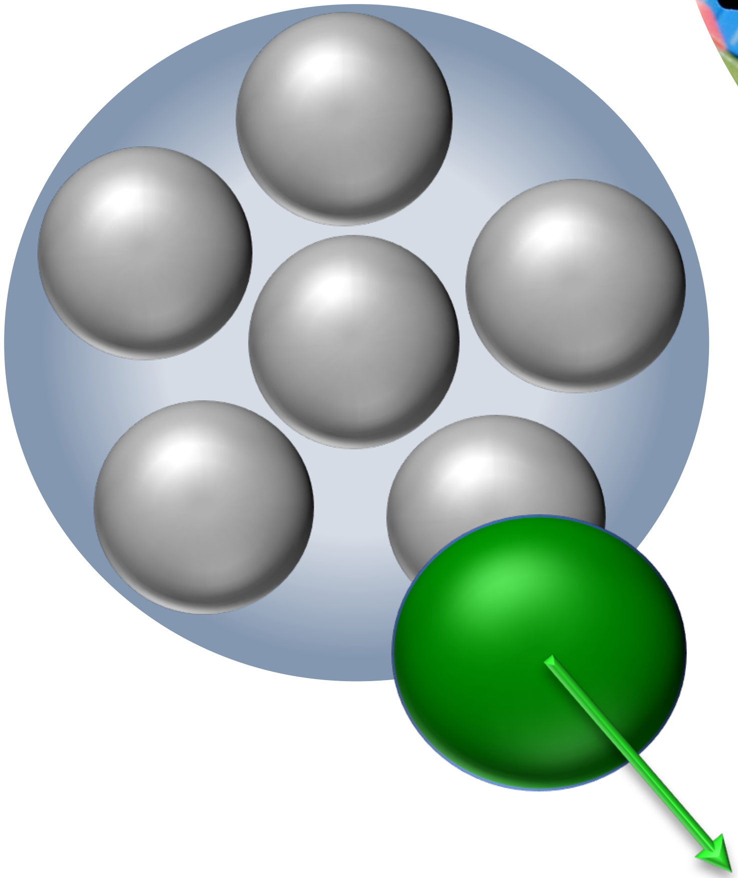
FIFA WORLD CUP
Brasil 2014

Two-Nucleon Knockout



FIFA WORLD CUP
Brasil

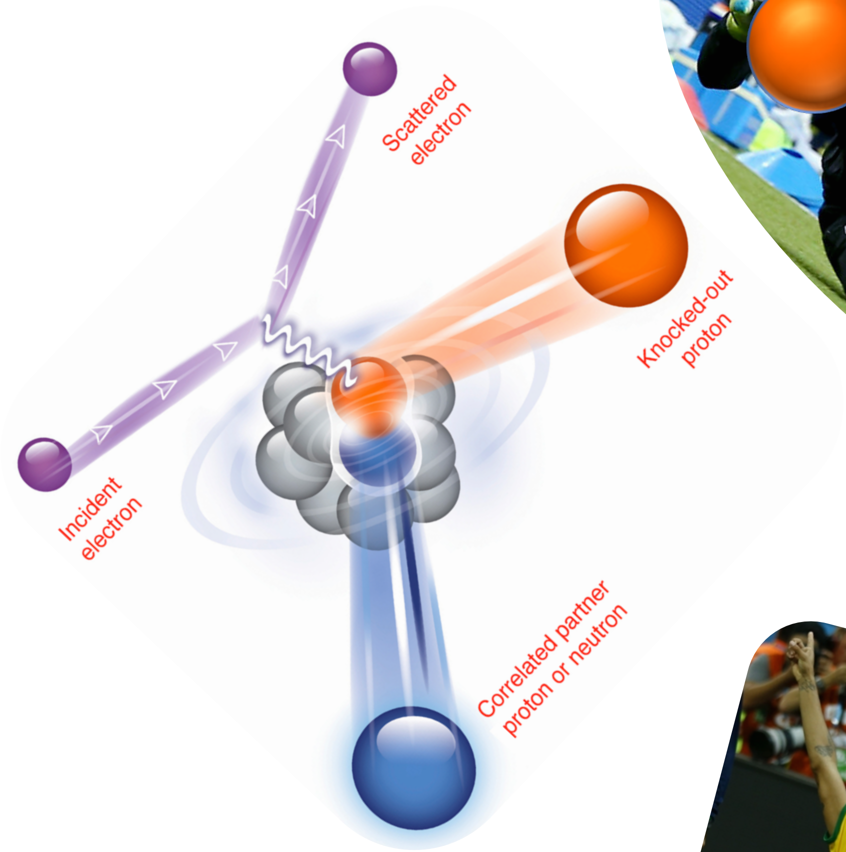
Two-Nucleon Knockout



FIFA WORLD CUP
Brasil
2014

Two-Nucleon Knockout

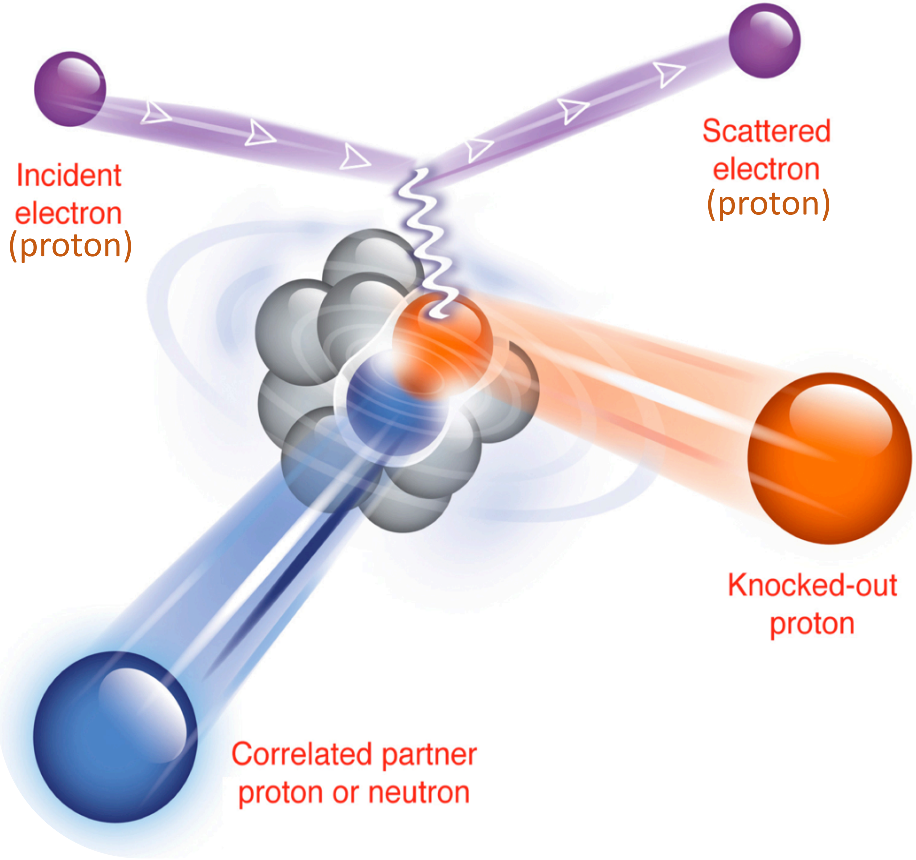
Two-Nucleon Knockout



2014
FIFA WORLD CUP
Brasil

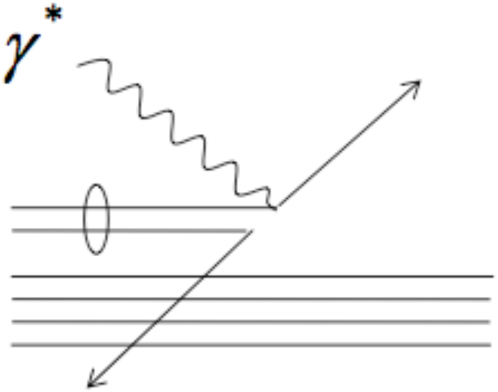
Two-Nucleon Knockout

Breakup the pair =>
Detect both nucleons =>
Reconstruct 'initial' state



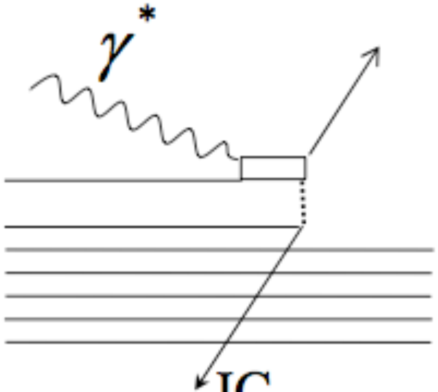
Interlude: Reaction Mechanisms

What we want:

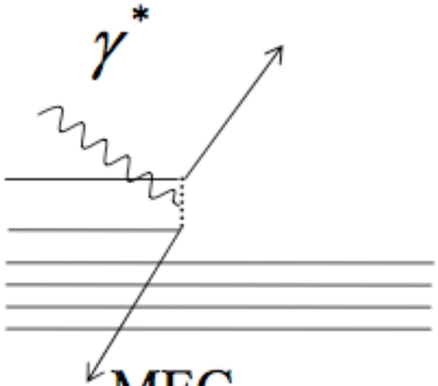


SRC

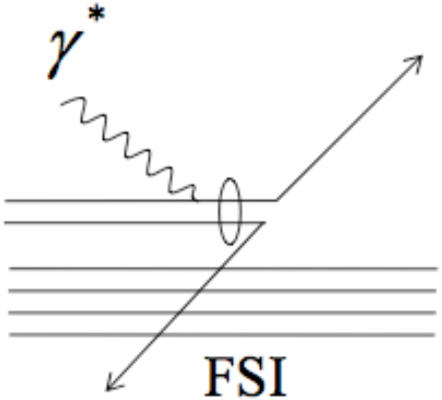
What we (might) get:



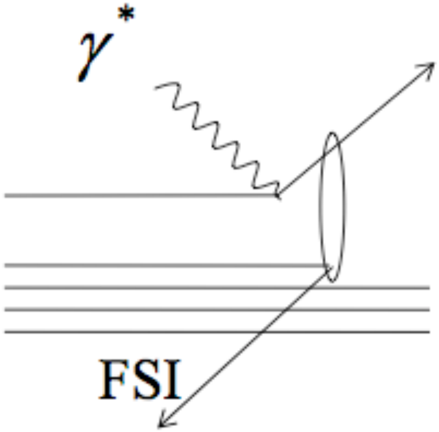
IC



MEC



FSI

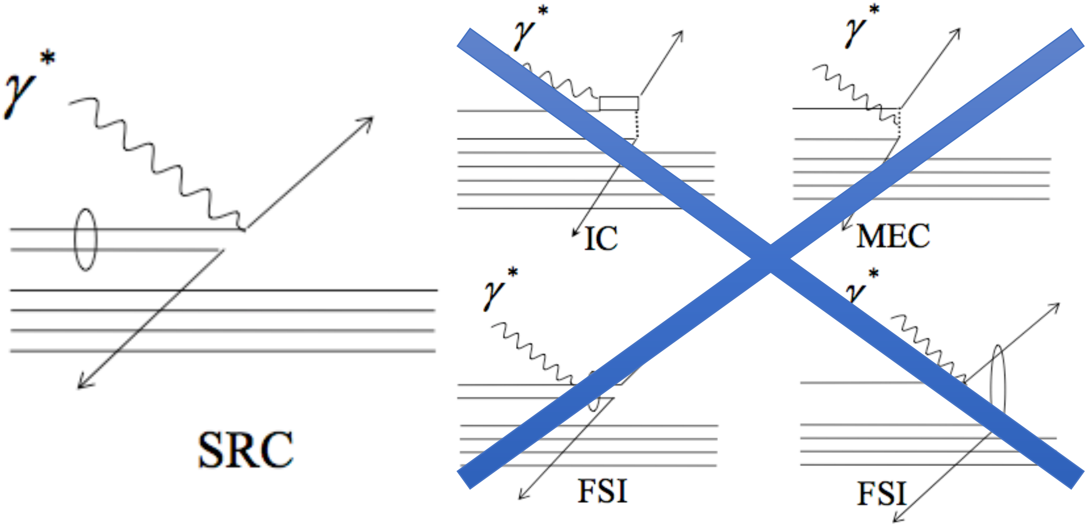


FSI

Interlude: Reaction Mechanisms

Trick: choose ‘good’ kinematics!

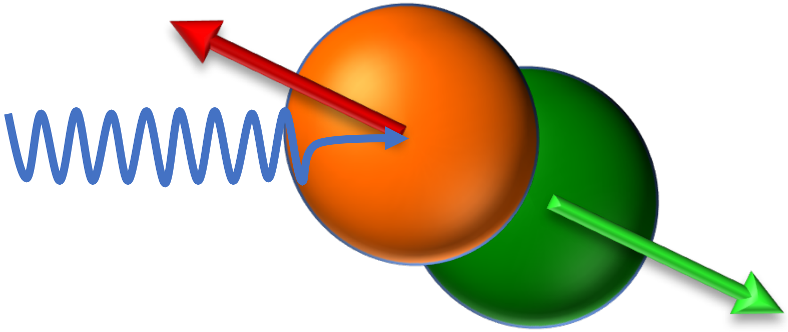
- **$x_B > 1.2$**
- **$Q^2 \sim 2$ (GeV/c^2)**
- **Anti-Parallel Kinematics**



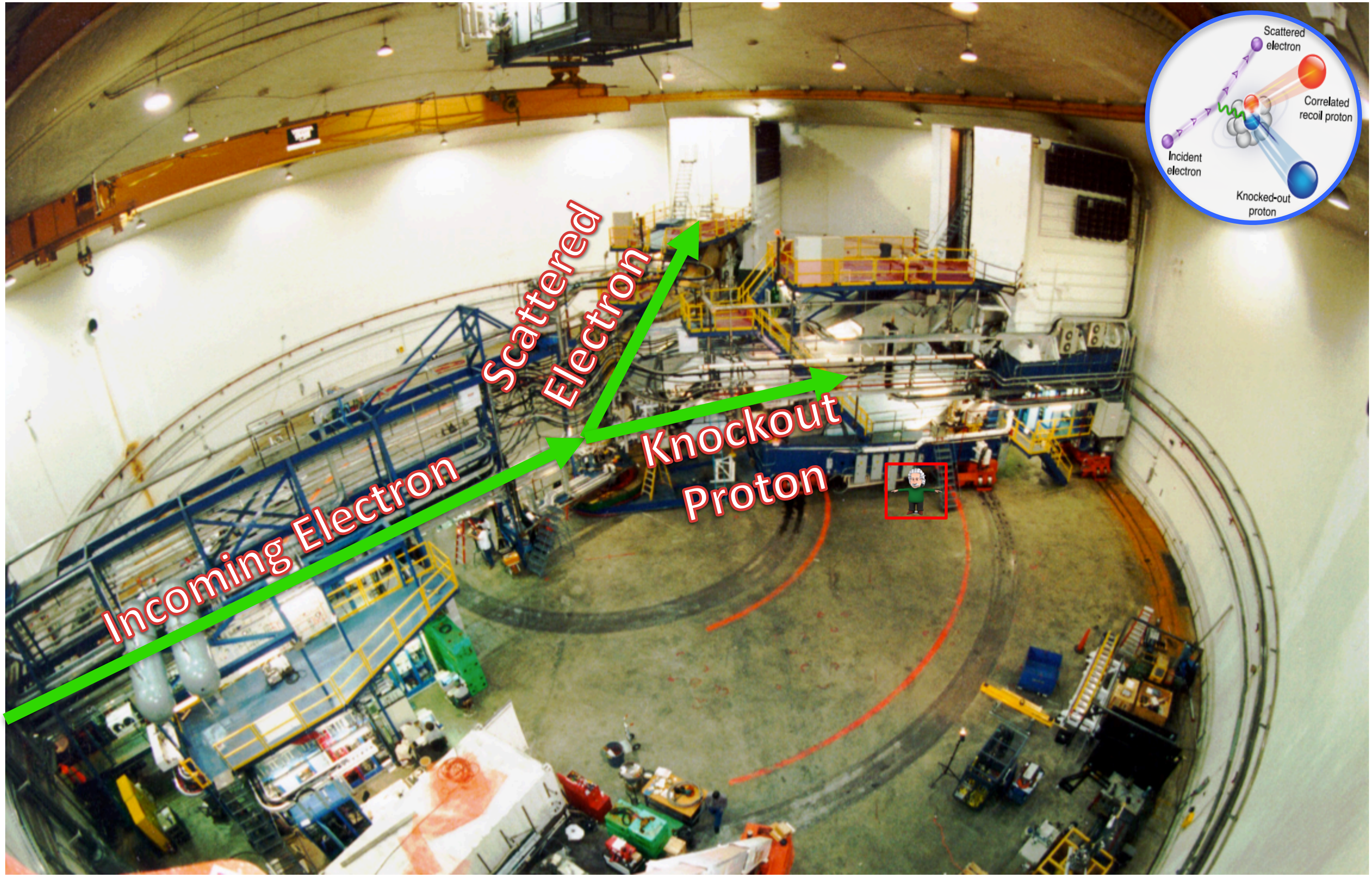
A word on FSI:

- Large-Q^2 (or |t,u|) allows using Eikonal approximation for FSI.
- Combined with x_B>1 ensures FSI largely confined to between the nucleons of the pair.

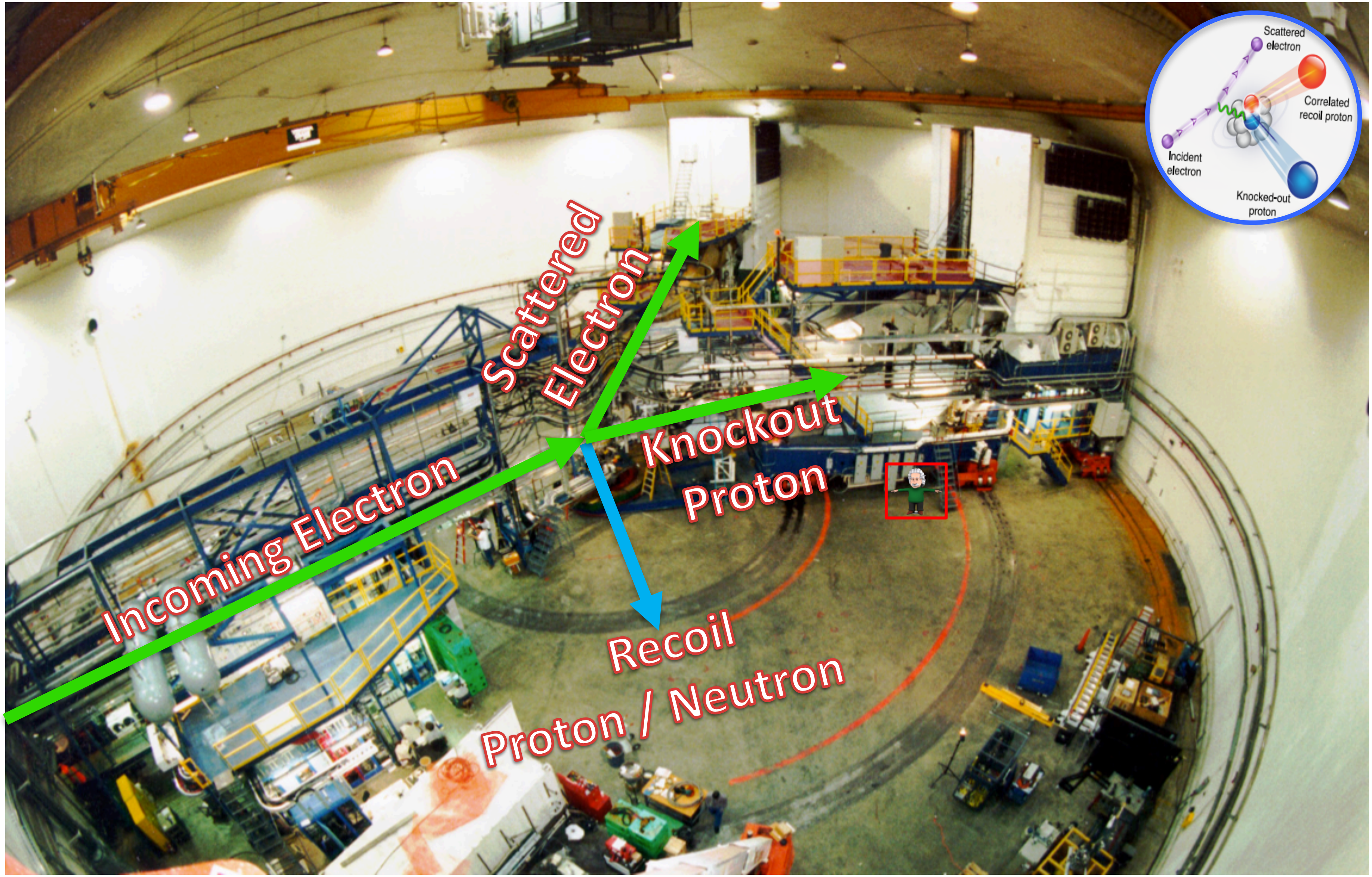
=> Large cancellation in ratios.



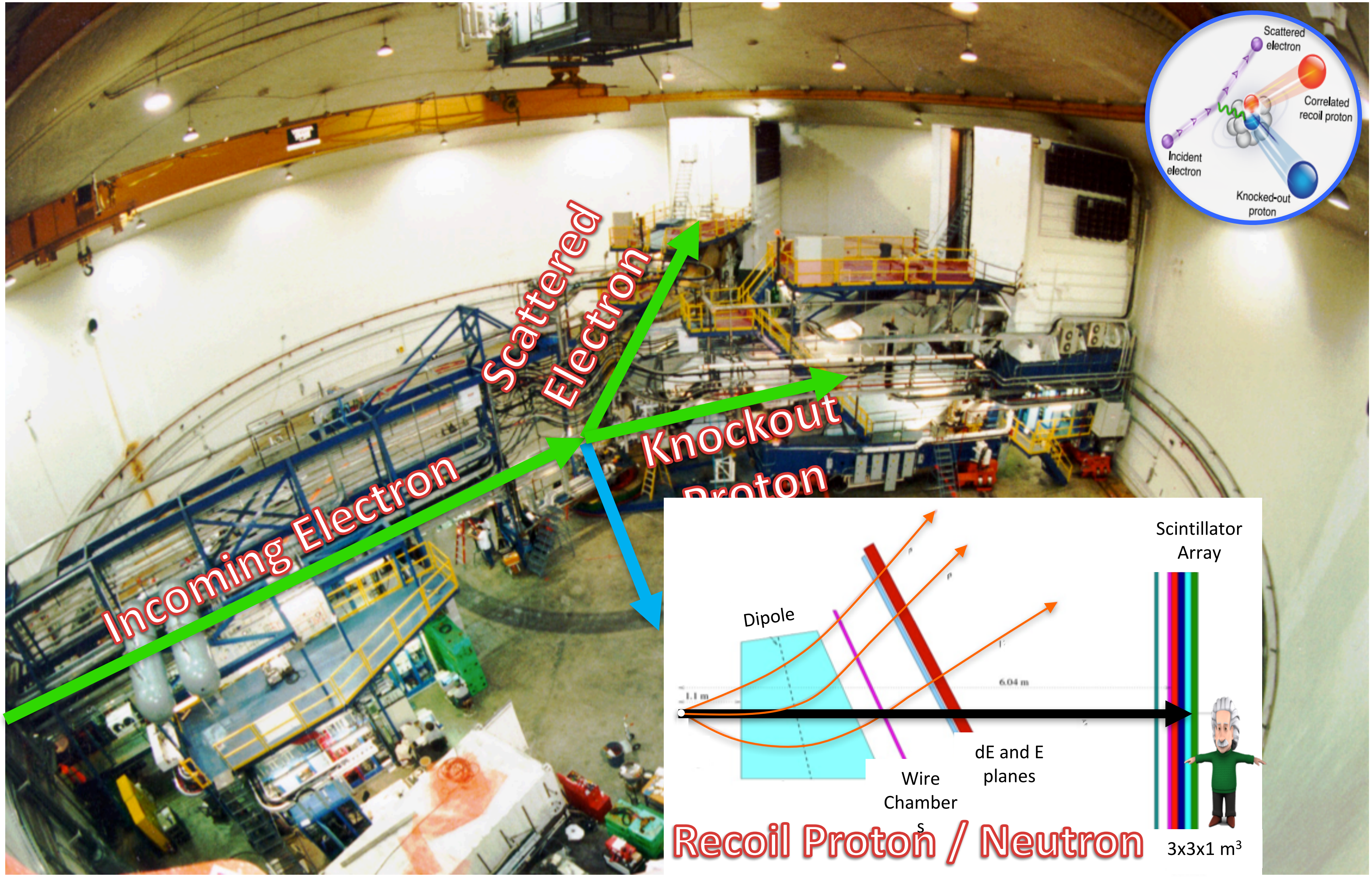
Hall-A: High-Resolution Spectrometers



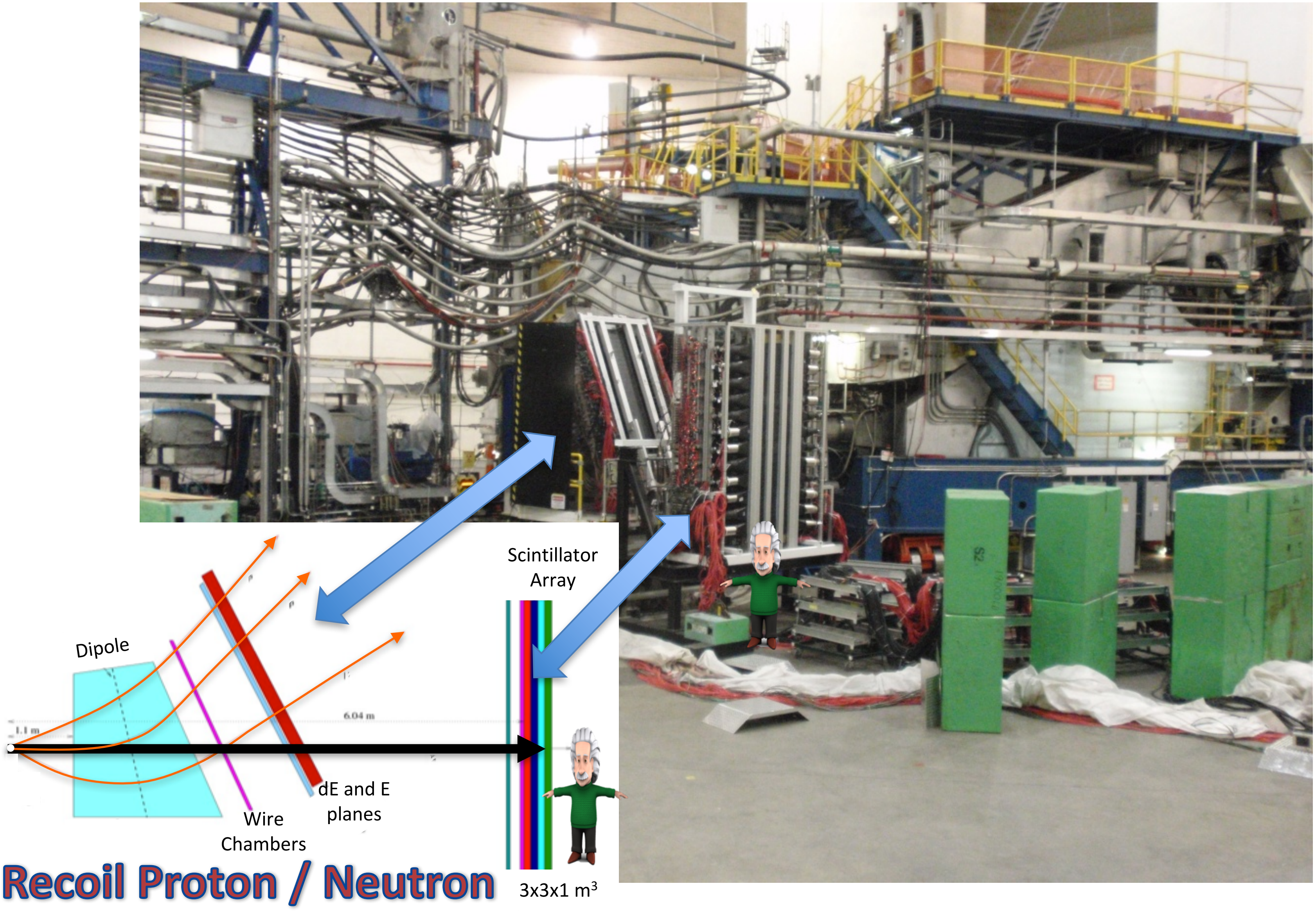
Hall-A: High-Resolution Spectrometers



Hall-A: High-Resolution Spectrometers



Building BigBite and HAND



Detector Inspectors - Or Chen (left) and Moshe Zilka (middle), both of Tel Aviv University, prepare to assemble a neutron detector, while Dominik Wernus (right), a student from Virginia Military Institute, tests a component for the detector. The detector will be used in an upcoming experiment in Hall A. Wernus is spending the summer at JLab in the DOE's Science Undergraduate Laboratory Internships program. (Photo: Jefferson Lab)

Award Winners - Members of Jefferson
World Leader - Jefferson Lab's Free-Ele
Breakthrough Research - Jefferson Lab
Groundbreaking - More than 400 people
Stimulus Dollars - The U.S. Department
Great Job - Jefferson Science Associate
12 GeV Contract - A Virginia Beach com
Second Chance - This accelerator cavity
Neutron Duality - There are two ways to

LAB EVENTS

DOE ACTS
July 7-31, 2009

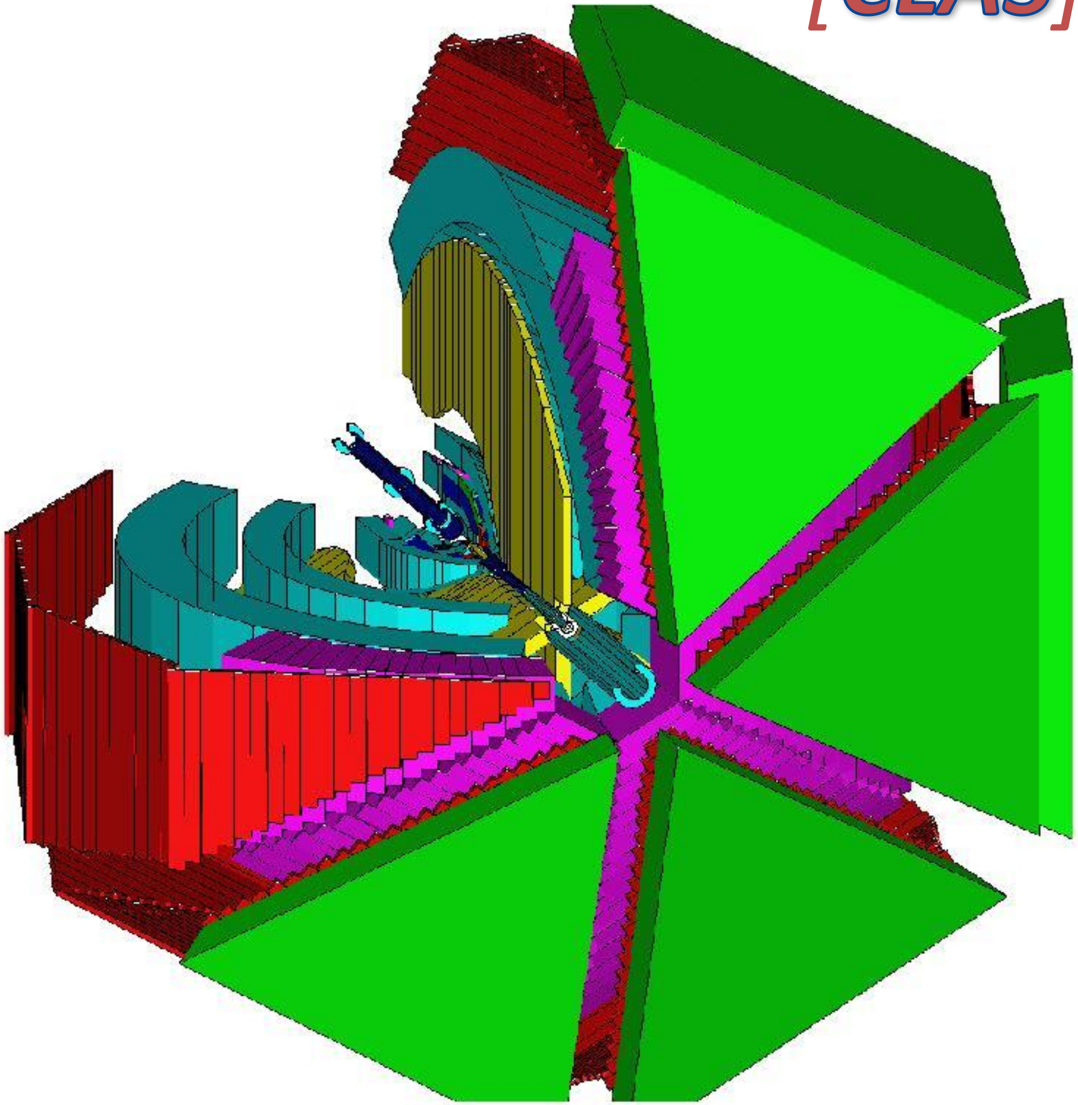
DOE Science Undergrad Lab Internship
May 26-July 31, 2009

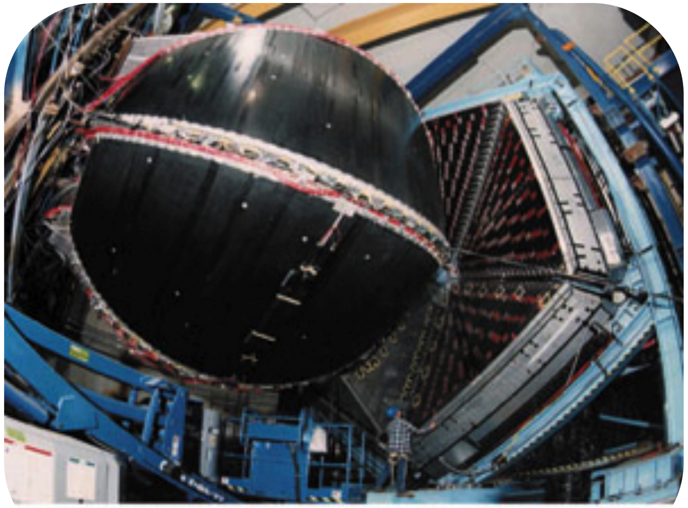
HS Summer Honors Program
June 16-July 31, 2009


100%

CEBAF Large Acceptance Spectrometer

[CLAS]



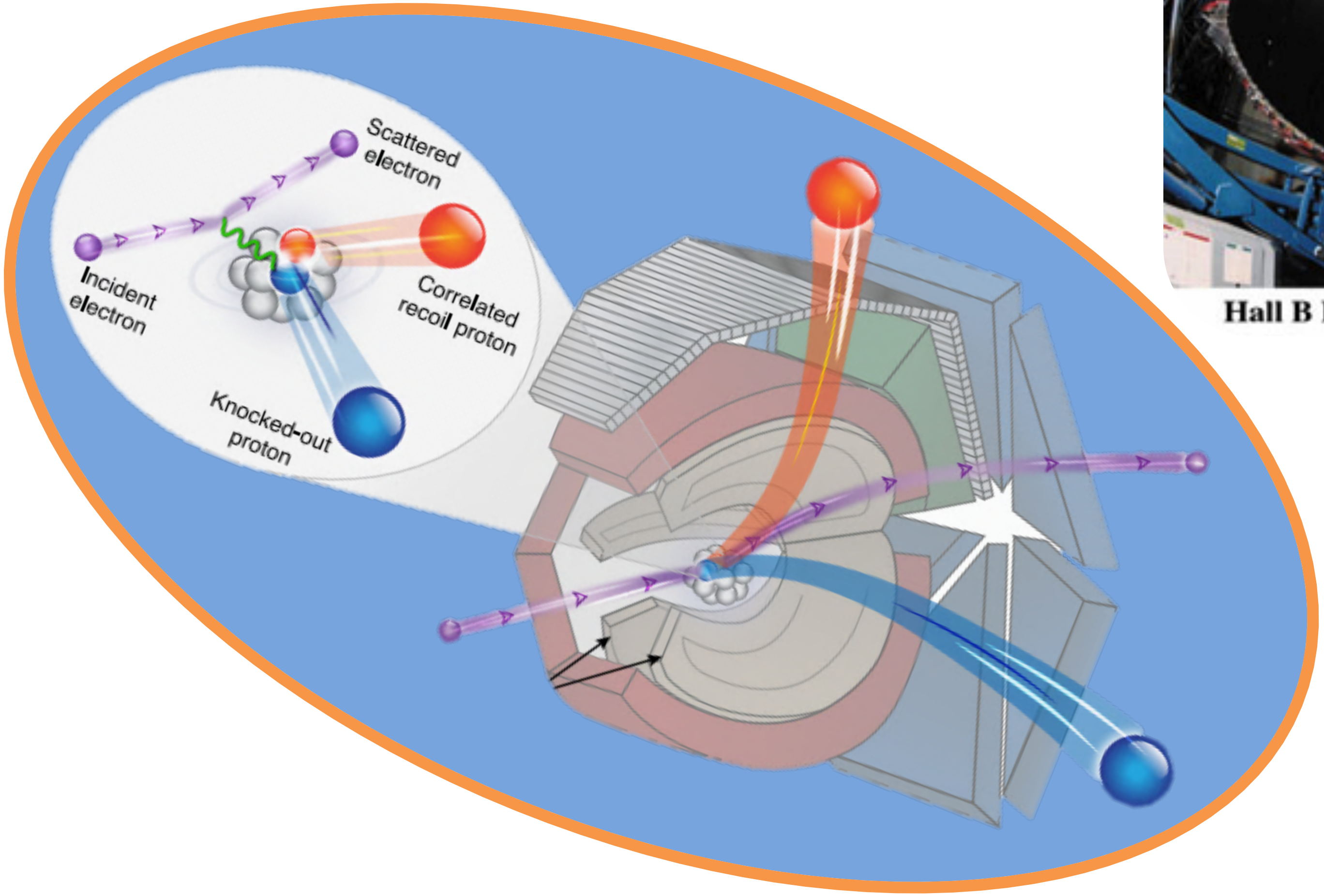


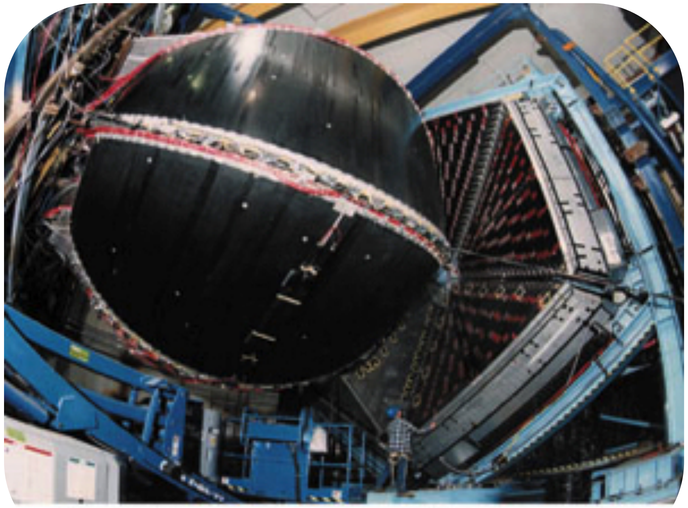
Hall B Large Acceptance Spectrometer

Open (e,e’) trigger, Large-Acceptance, Low luminosity (~10^{34} cm^{-2} sec^{-1})

CEBAF Large Acceptance Spectrometer

[CLAS]

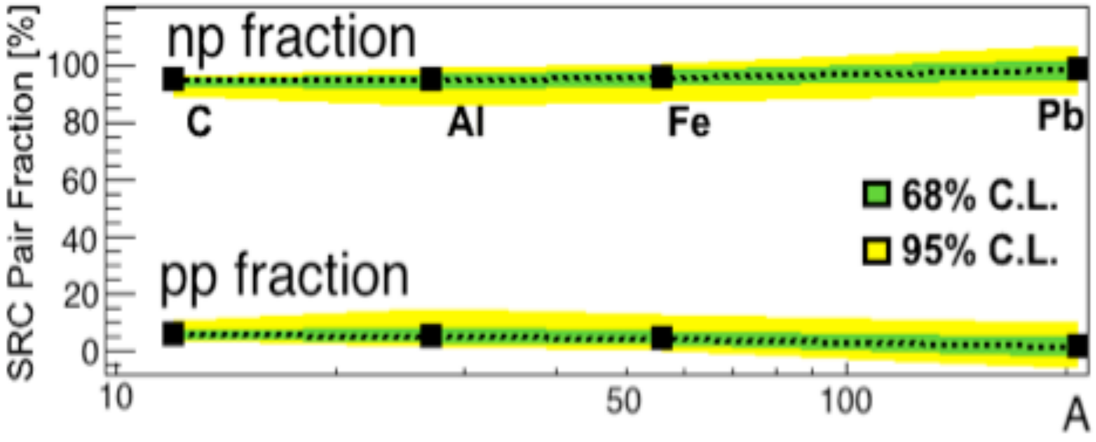




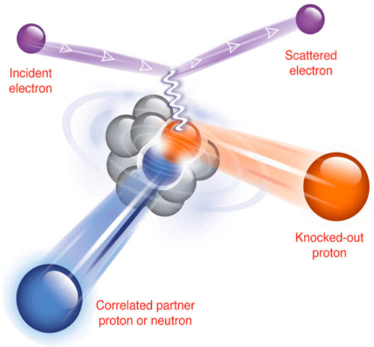
Hall B Large Acceptance Spectrometer

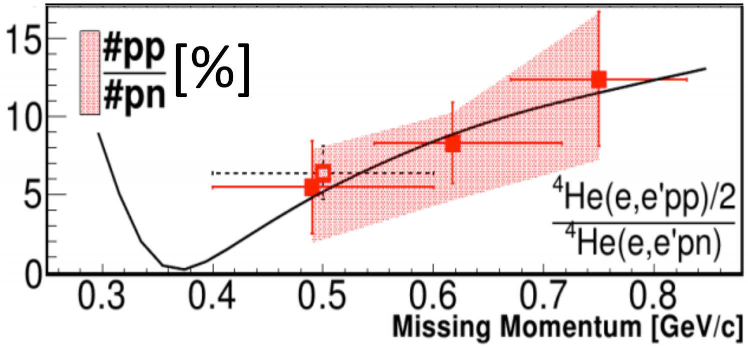
Open (e,e’) trigger, Large-Acceptance, Low luminosity (~10^{34} cm^{-2} sec^{-1})

np dominance results



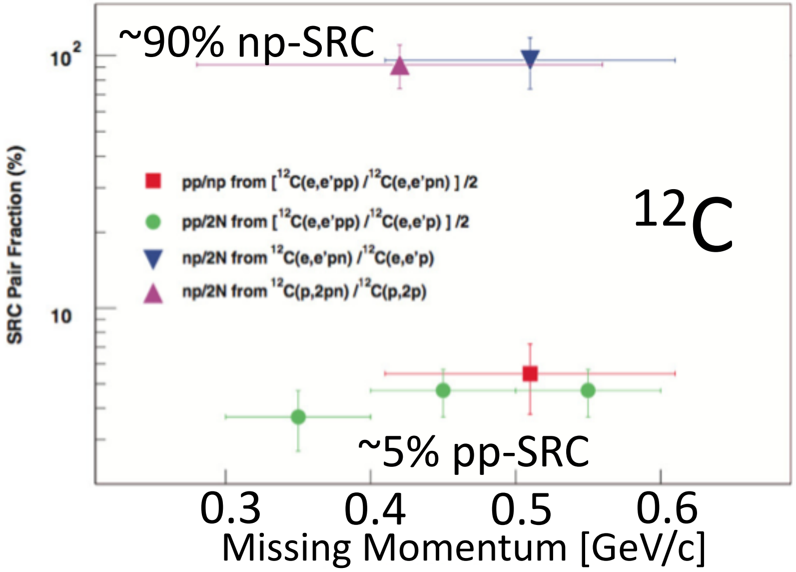
O. Hen et al., Science 364 (2014) 614

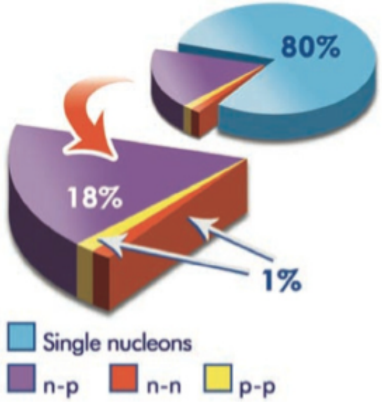


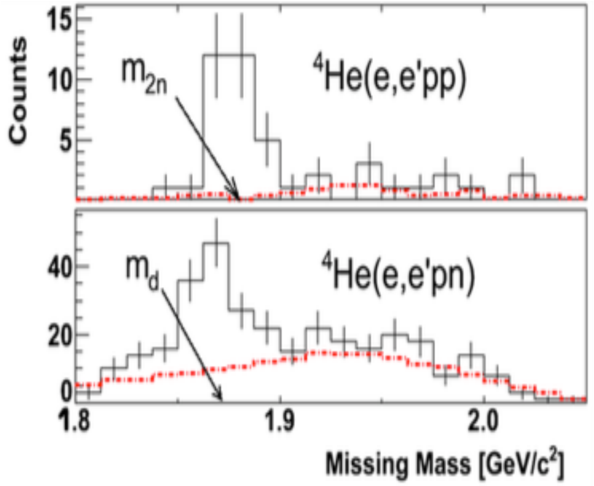


R. Subedi et al., Science 320 (2008) 1476

I. Korover et al., PRL 113 (2014) 022501

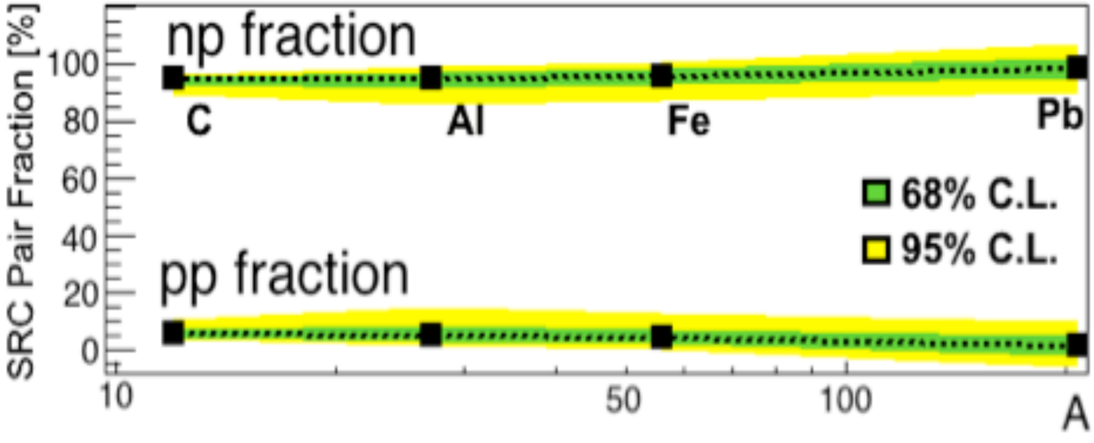




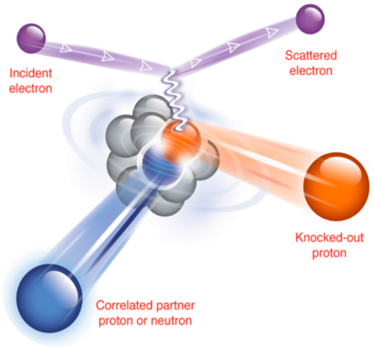


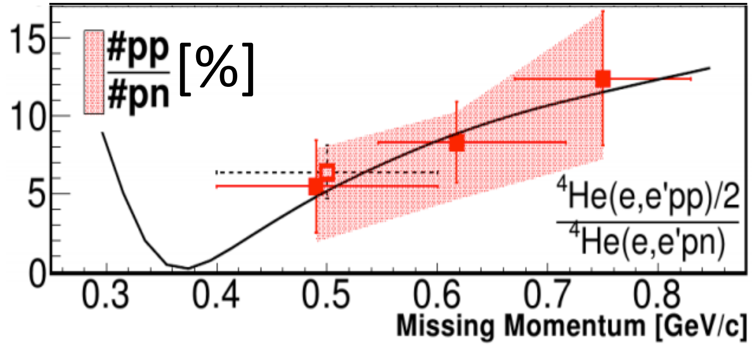
A. Tang et al., PRL (2003); E. Piasetzky et al., PRL (2006); R. Shneor et al., PRL (2007)

np dominance results



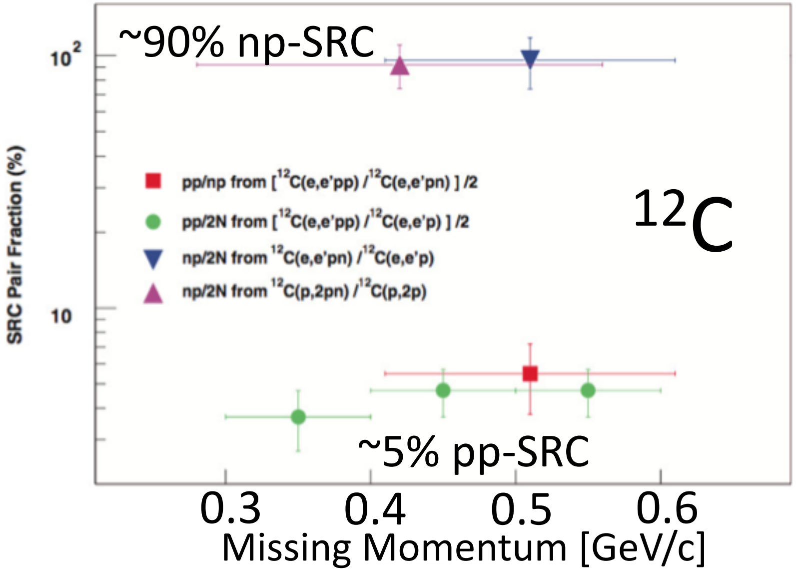
O. Hen et al., Science 364 (2014) 614

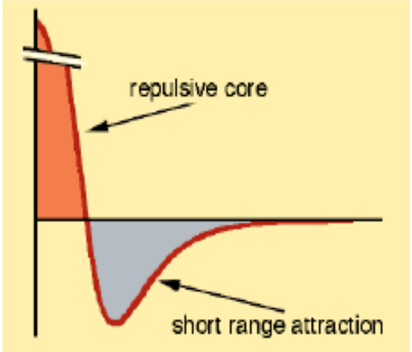


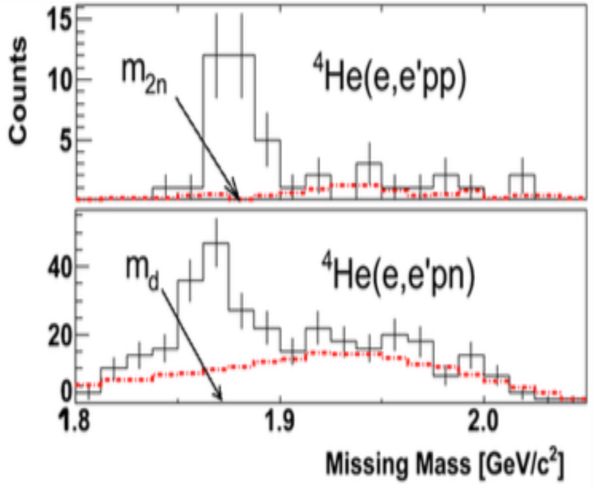


I. Korover et al., PRL 113 (2014) 022501

R. Subedi et al., Science 320 (2008) 1476



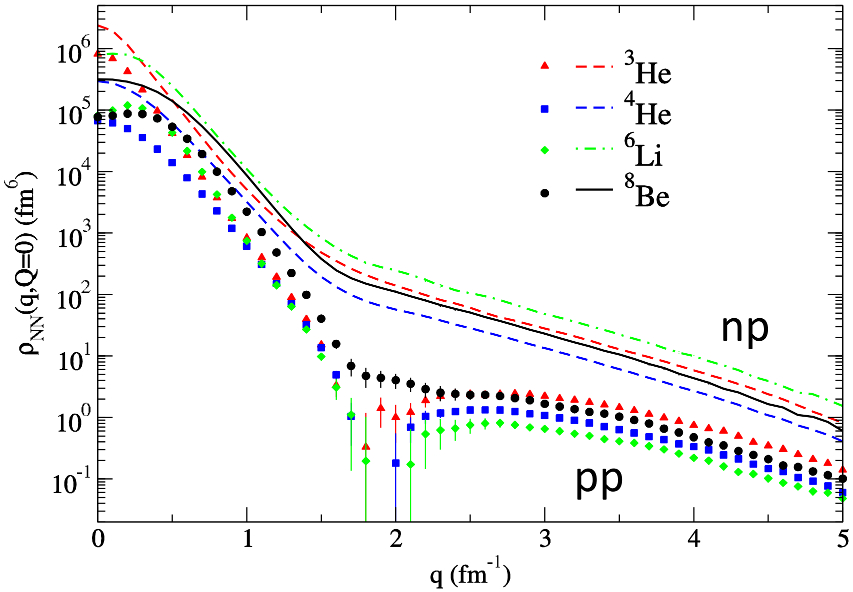




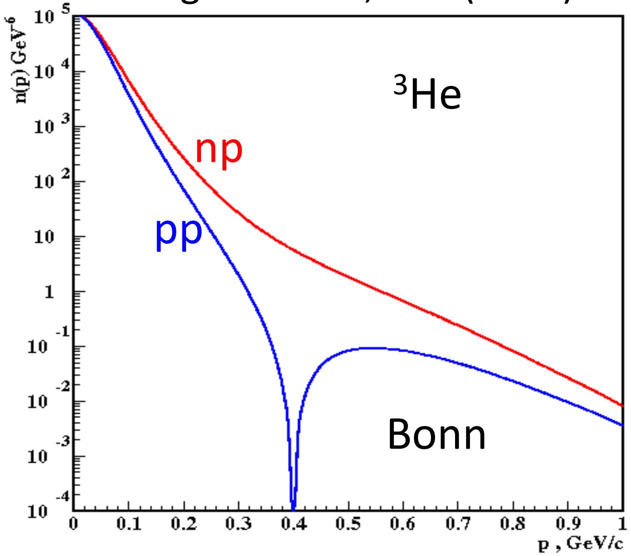
A. Tang et al., PRL (2003); E. Piasetzky et al., PRL (2006); R. Shneor et al., PRL (2007)

Tensor Force Dominance

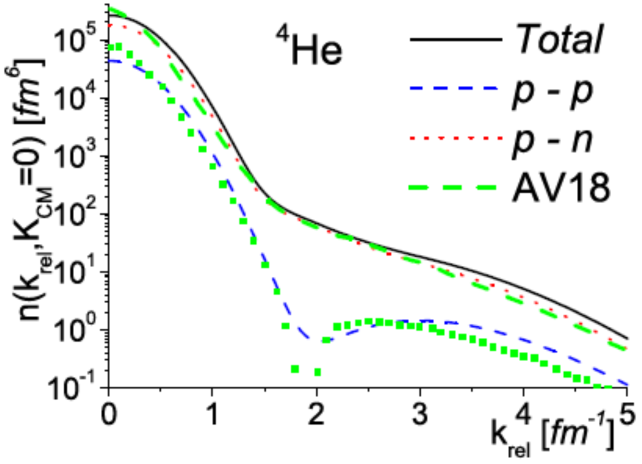
Schiavilla et al., PRL (2007)



Sargsian et al., PRC (2005)

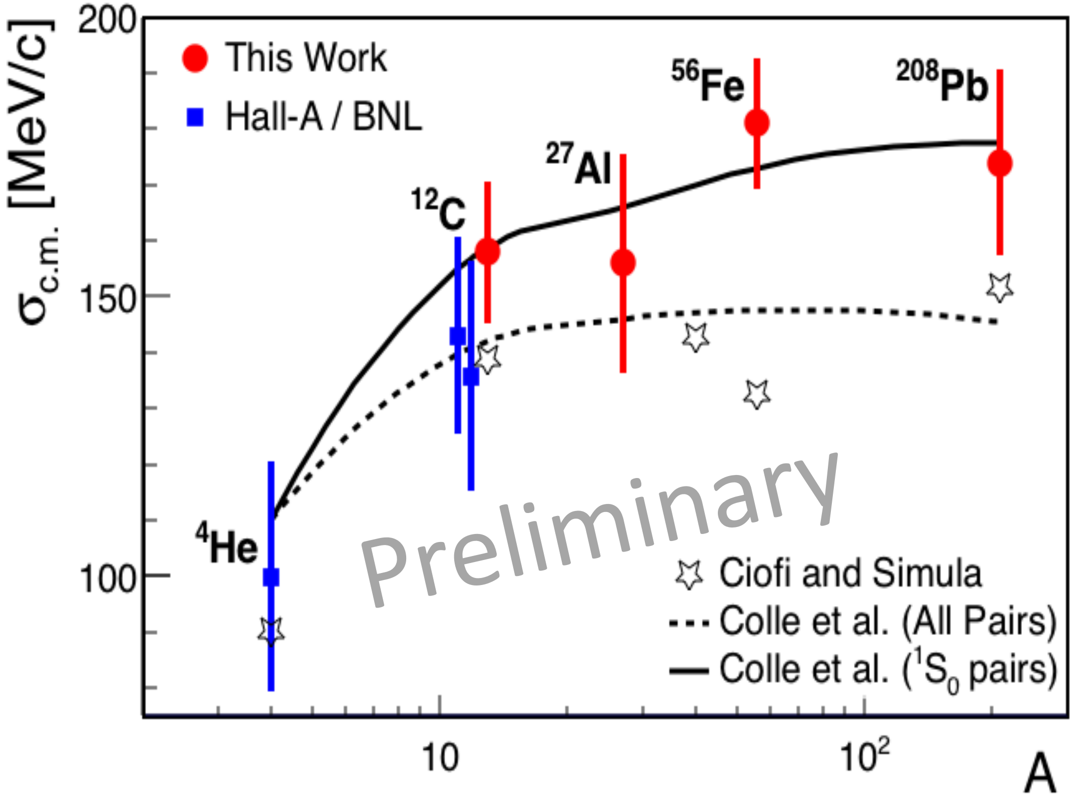


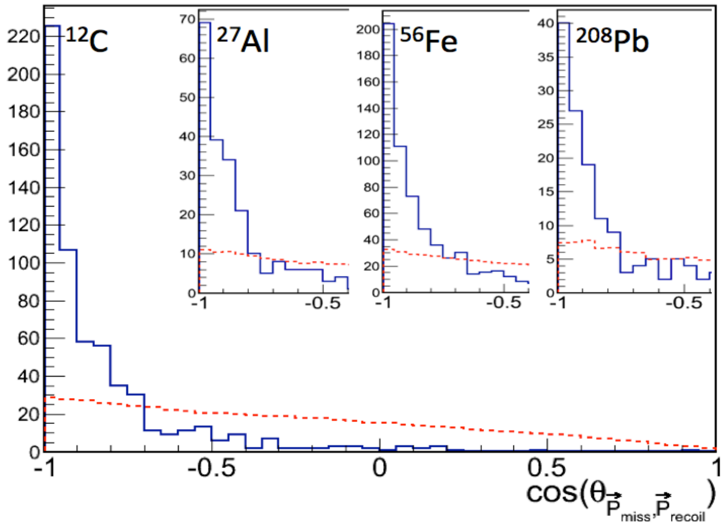
Ciofi and Alvioli PRL (2008)



C.M. Motion and Pairing Mechanisms

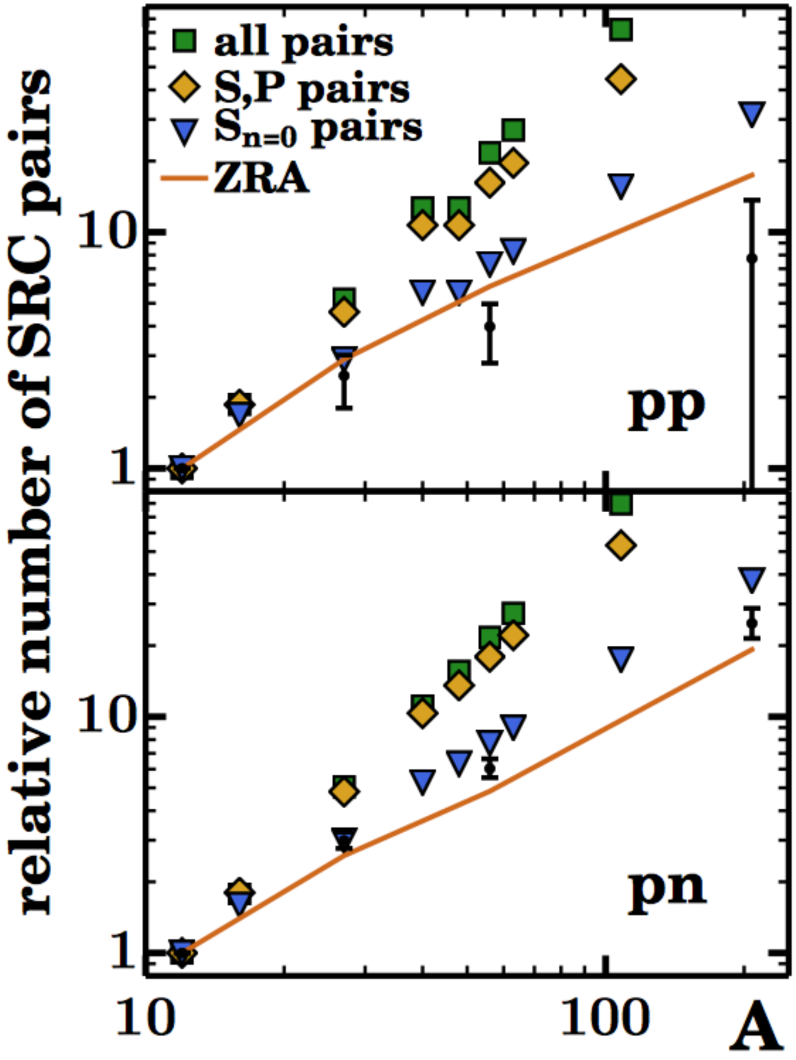
“… *high relative momentum* and ***low c.m. momentum*** compared to the Fermi momentum (k_F)”



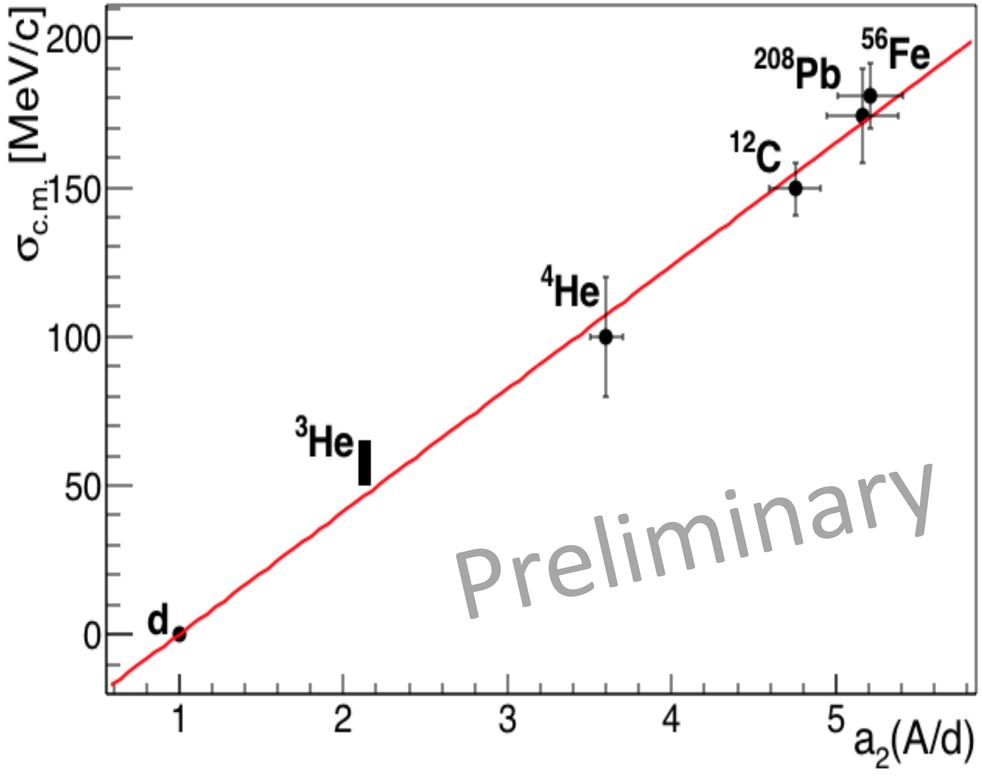


E. Cohen et al. (CLAS Collaboration), In-Preparation (2017)

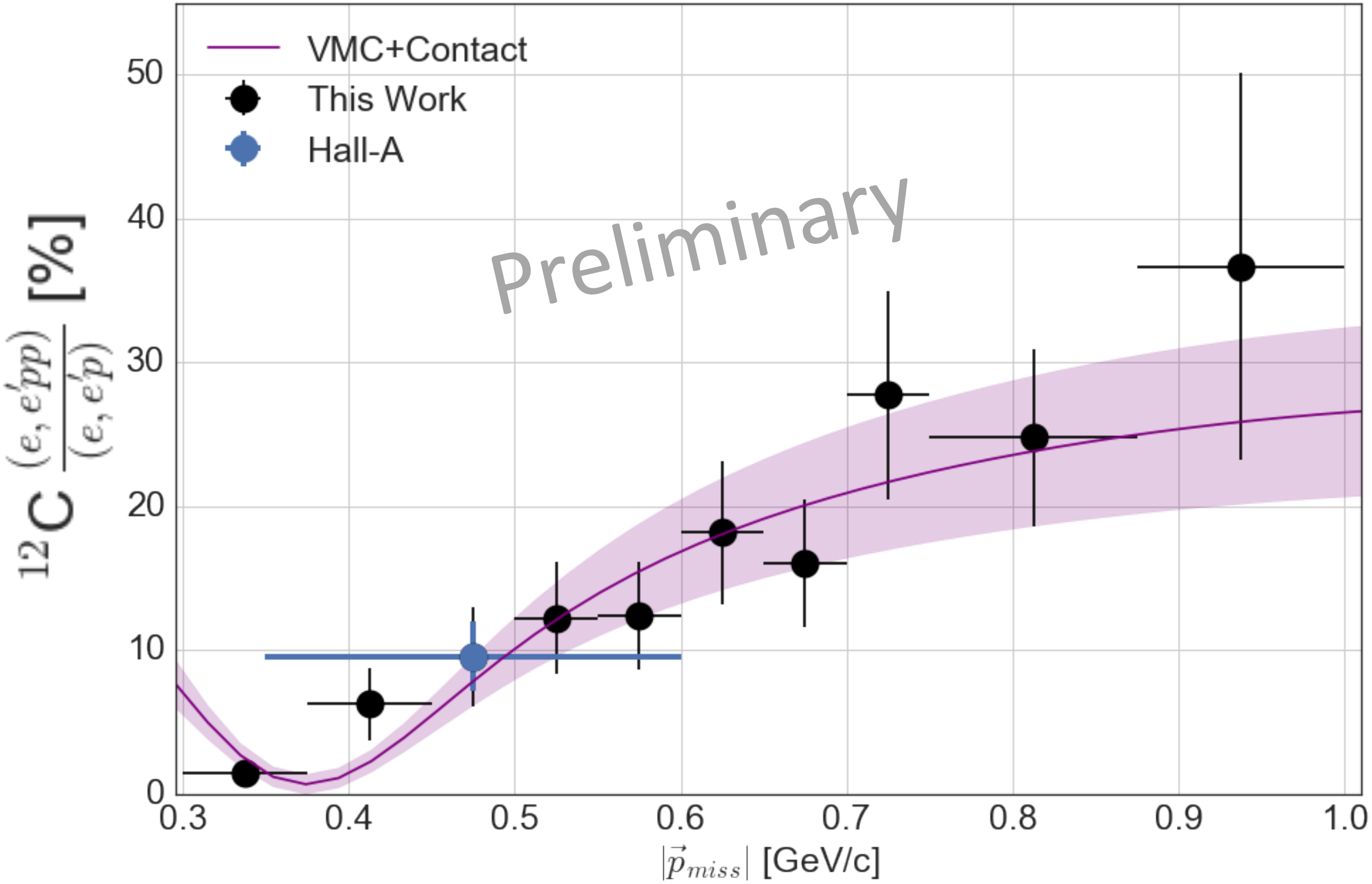
Pairs Counting and Pairing Mechanisms



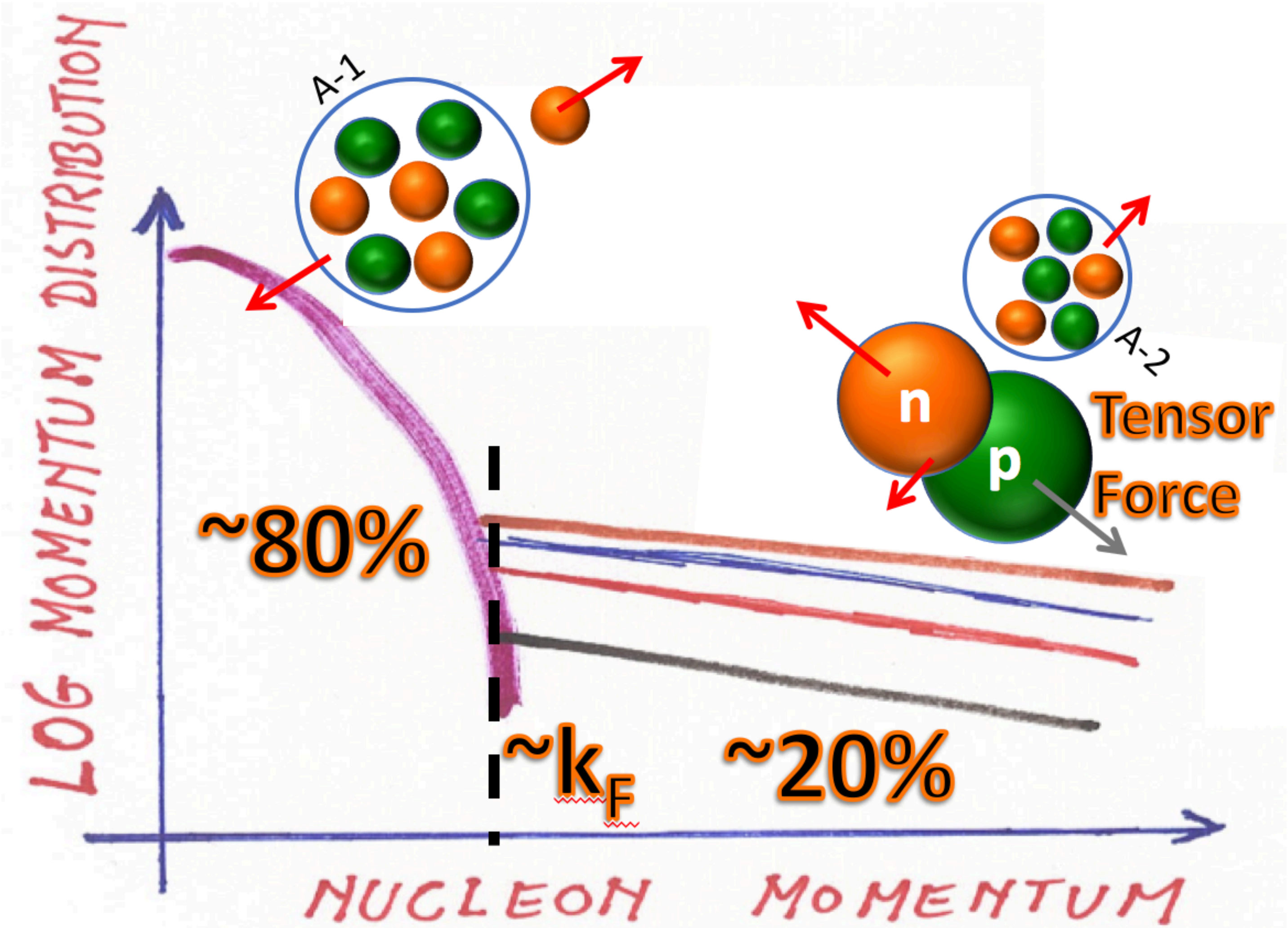
C. Colle et al., PRC (2015)



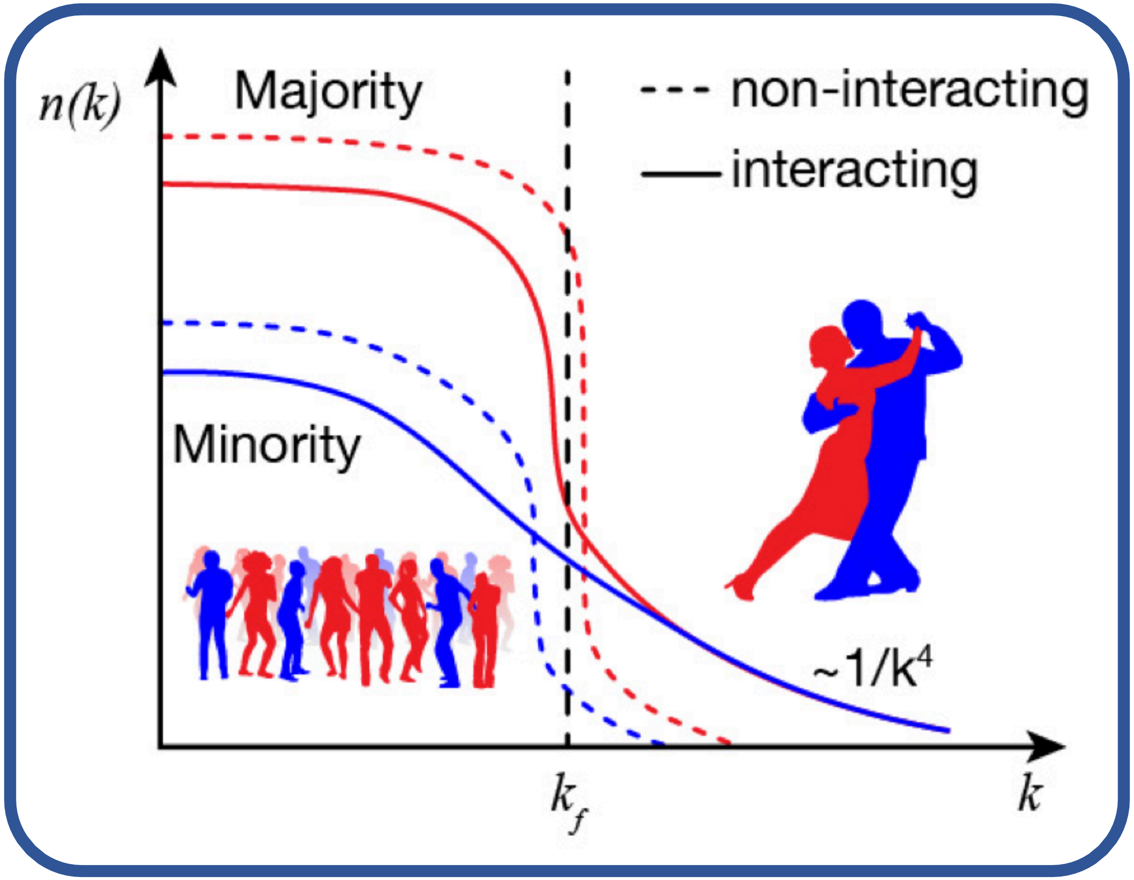
NN interaction at Short Distances

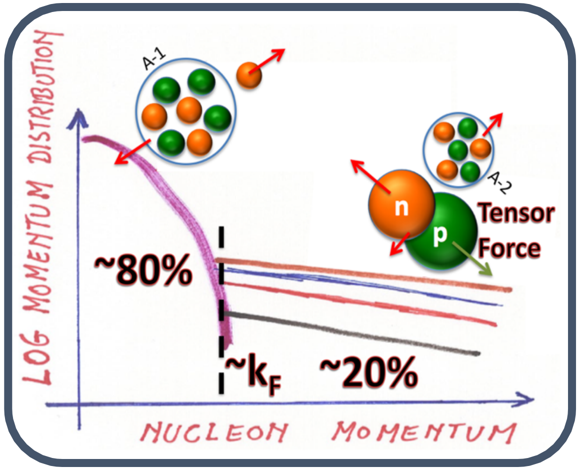


O. Hen et al. (CLAS Collaboration), In-Preparation (2017)

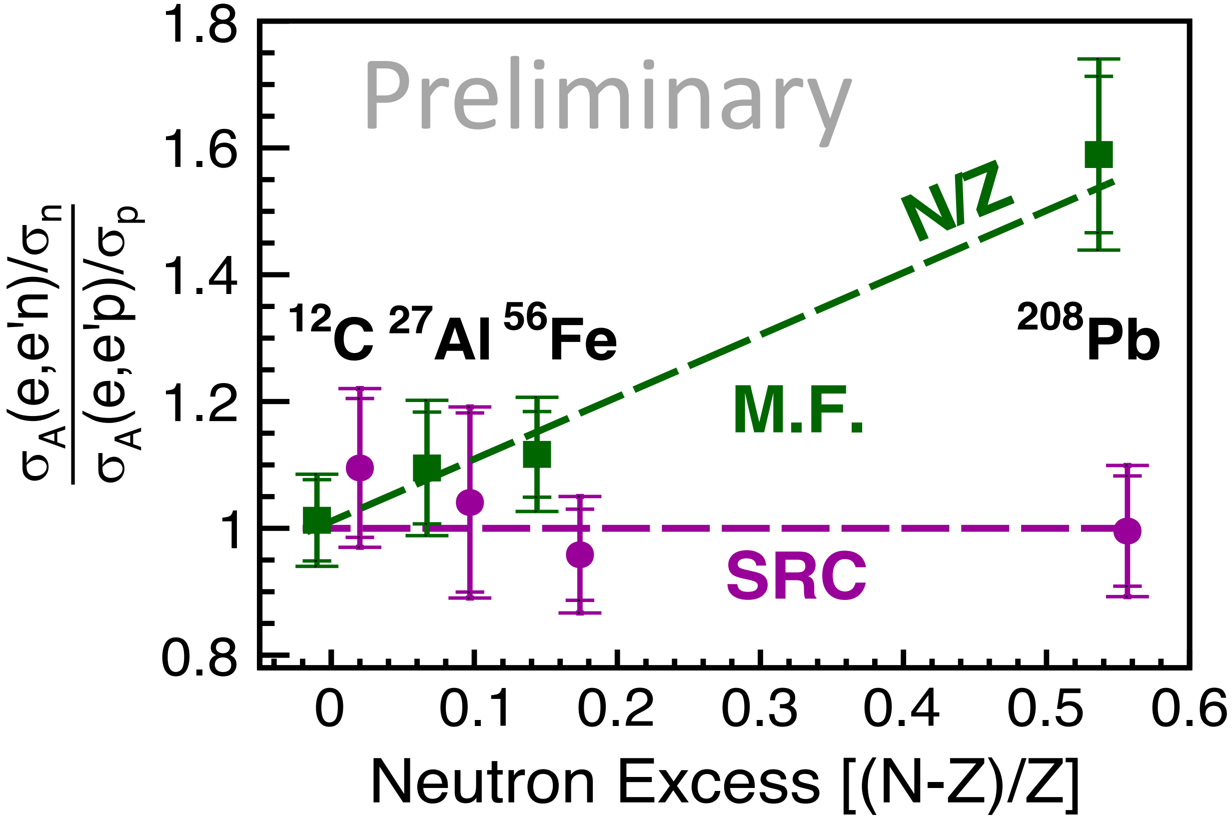

LOG MOMENTUM DISTRIBUTION
A-1
A-2
n
p
Tensor
Force
~80%
~k_F
~20%
NUCLEON MOMENTUM

Nuclear Asymmetry Dependence





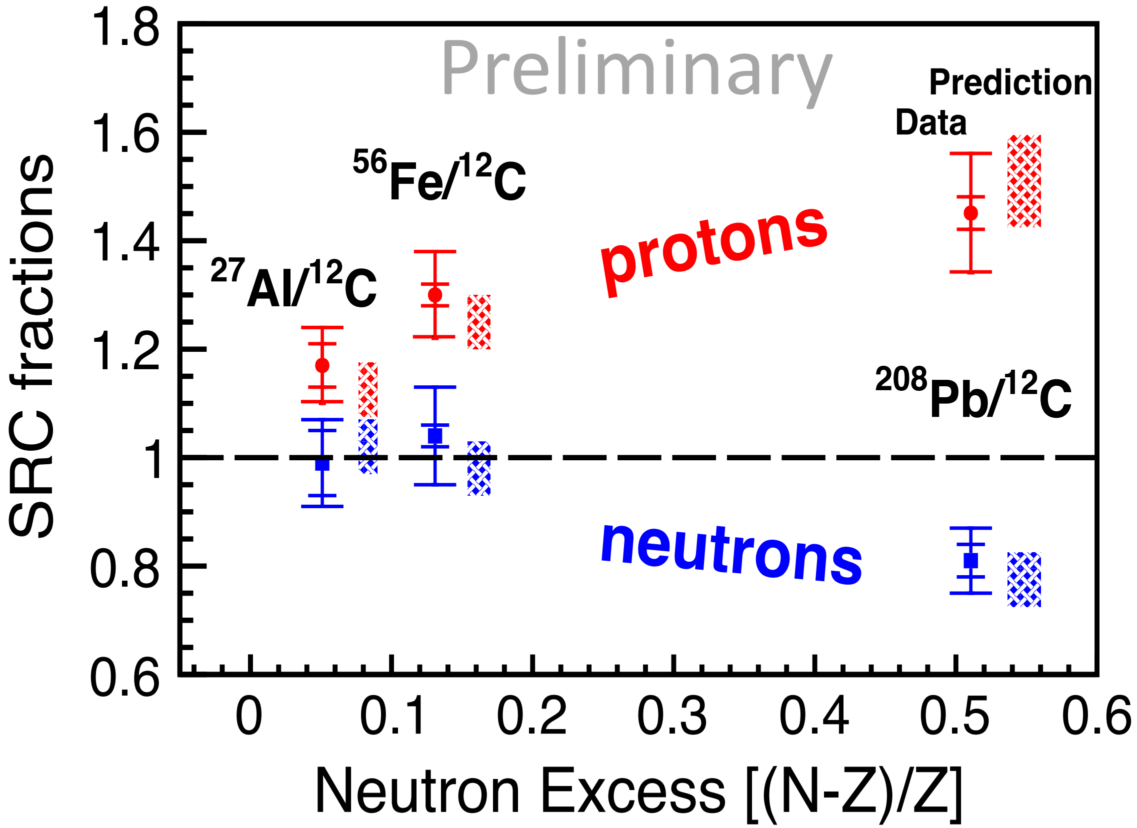
Nuclear Asymmetry Dependence



=> *Same* number of high-P protons and neutrons!

M. Duer et al. (CLAS Collaboration), In-Preparation (2017)

Nuclear Asymmetry Dependence



=> Protons more correlated in neutrons rich nuclei!

M. Duer et al. (CLAS Collaboration), In-Preparation (2017)

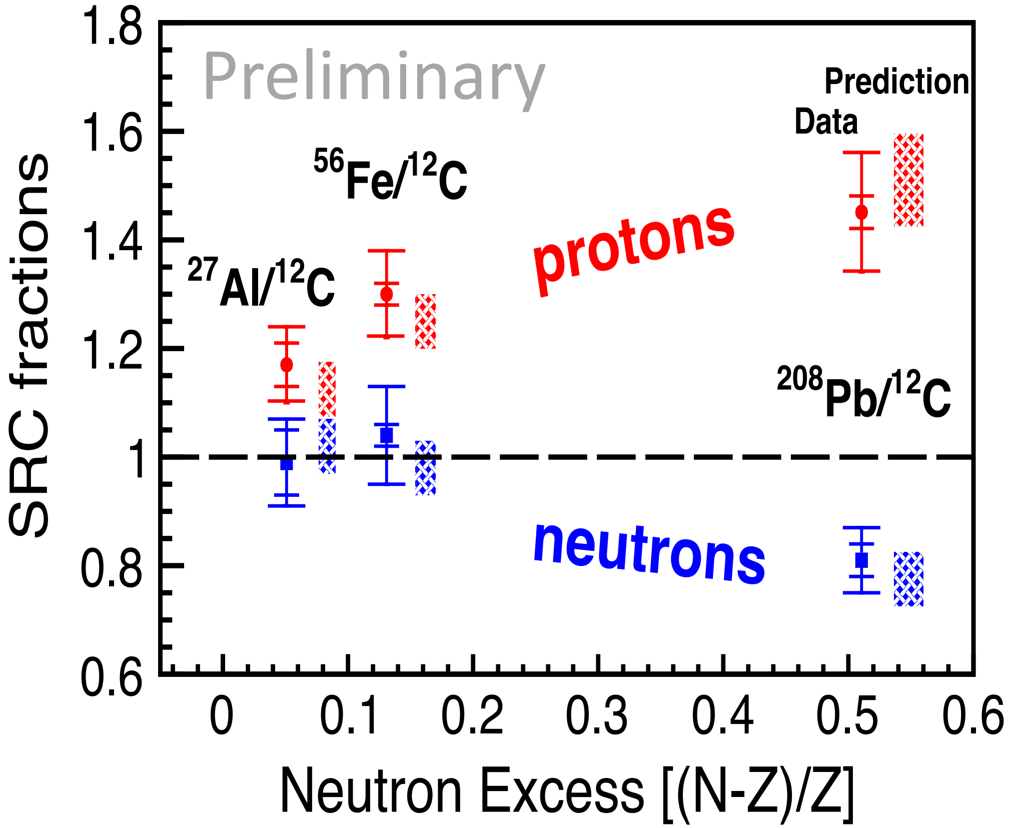
Nuclear Asymmetry Dependence

Theory model:
depleted mean-field +
scaled deuteron tail.

Simplistic, but works.

Need to test real
calculations!

[more to come
on how to do it]



M. Duer et al. (CLAS Collaboration), In-Preparation (2017)

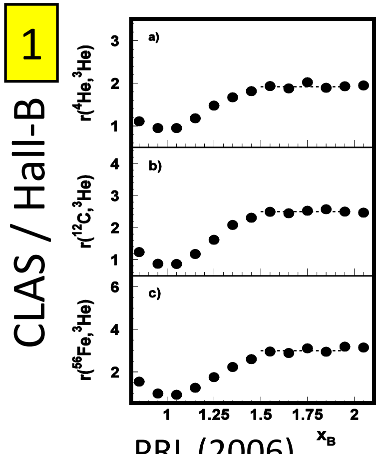
What do we know about SRC

1 Account for ~ 25% of nucleons in nuclei.

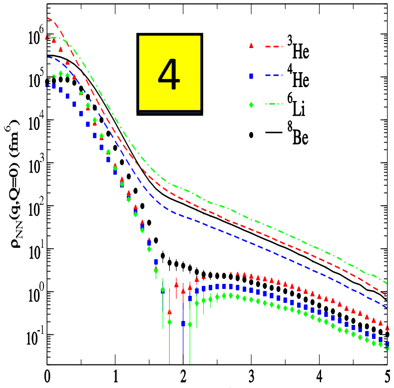
2 Dominate the momentum distribution for k ≥ 300 MeV/c.

3 Probability for np-SRC is ~18 times larger than pp-SRC. Also true for heavy asymmetric nuclei.

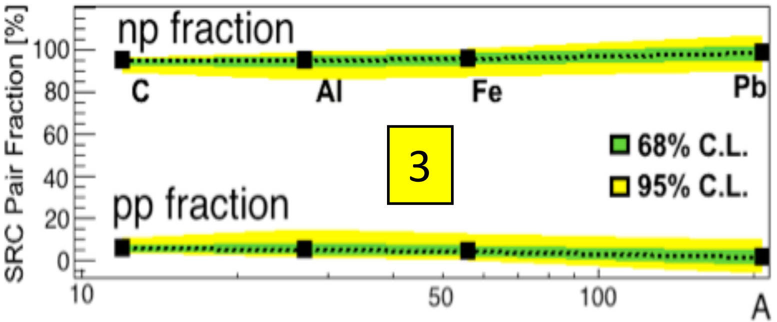
4 Dominant NN force in the 2N-SRC is tensor force.
High momentum tail (300-600 MeV/c)
dominated by L=0,2 S=1 np-SRC pairs.



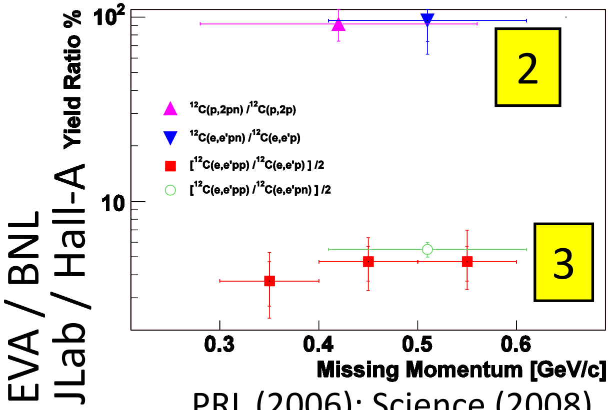
PRL (2006)



PRL (2007)

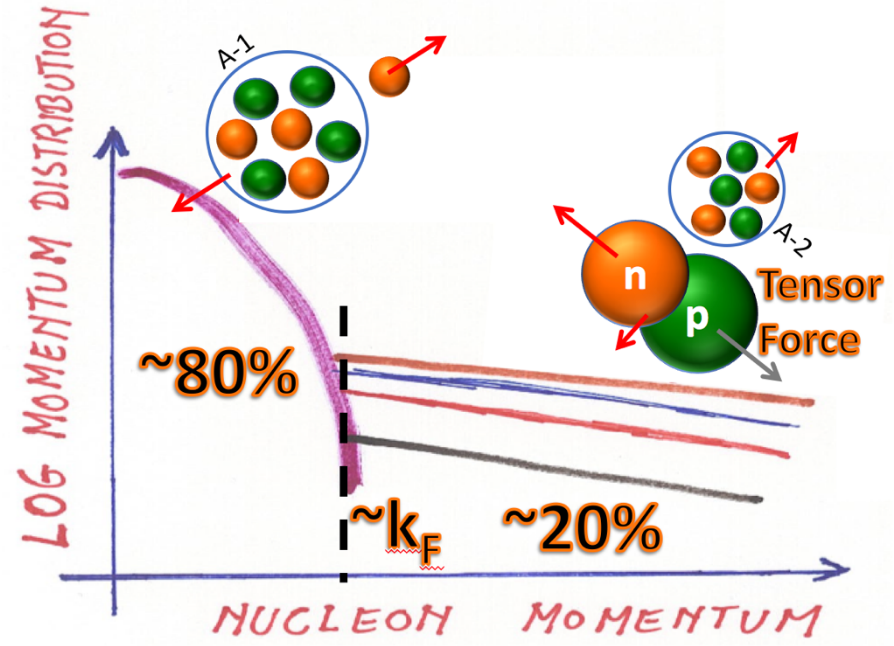


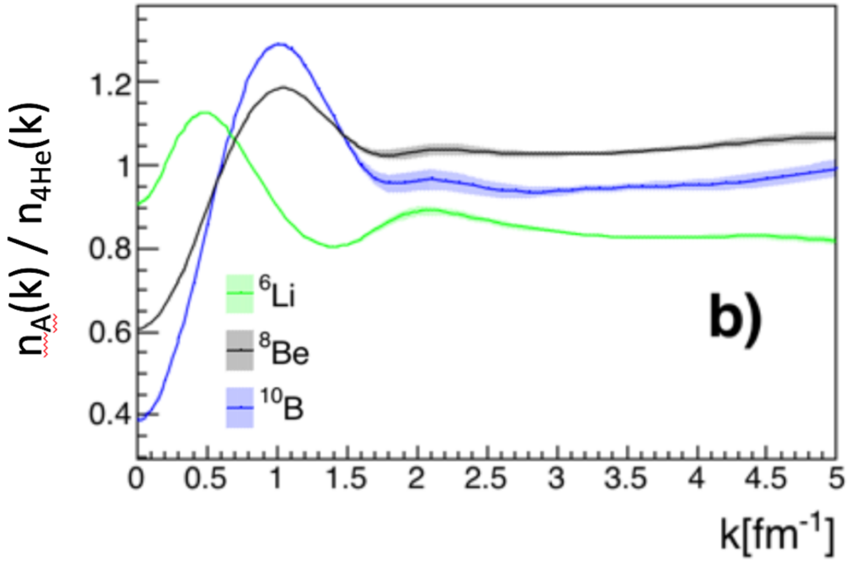
Science 346, 614 (2014)

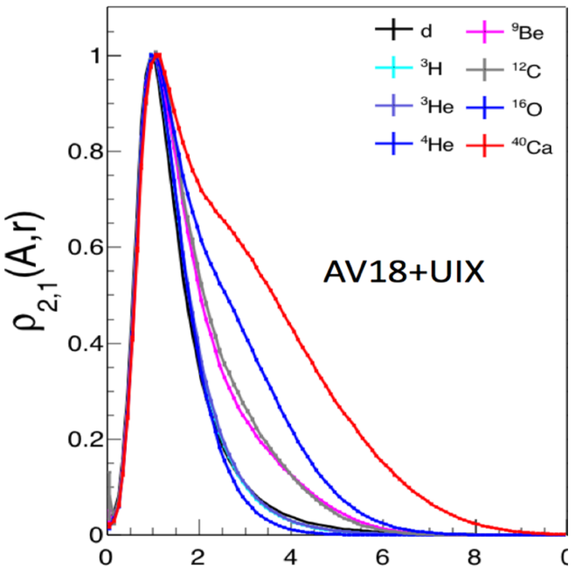


PRL (2006); Science (2008).

NOW…. Can we ‘nail’ it all to one consistent picture?







Two-component interacting Fermi systems

Lets start with a simpler system – Atomic gas

Please forget about nuclear physics for a moment

The Contact and Universal Relations

Concept developed for dilute two-component Fermi systems with a short-range (δ-like) interaction.

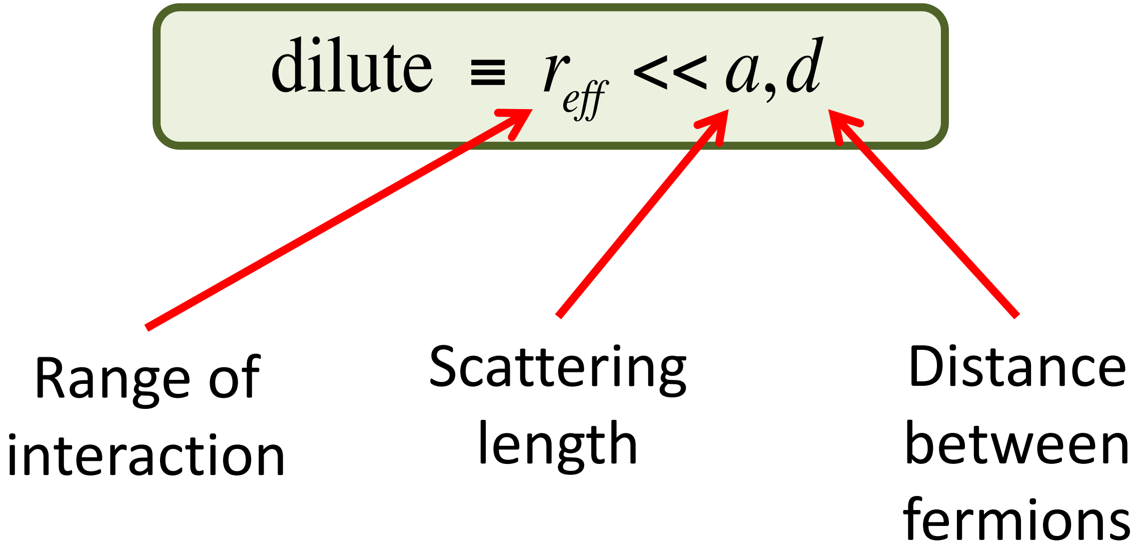
$$\text{dilute} \equiv r_{eff} << a, d$$

Dilute System

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

The Contact and Universal Relations

Concept developed for dilute two-component Fermi systems with a short-range (δ-like) interaction.



$$\text{dilute} \equiv r_{eff} << a, d$$

Range of interaction

Scattering length

Distance between fermions

Range of interaction much smaller than the other relevant length scales in the problem

Dilute System

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

The Contact and Universal Relations

Contact interaction is represented through a boundary condition

Imposing this B.C. on the Schrödinger equation yields an asymptotic wave function when two fermions get very close

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a)A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

Dilute System → **Short Distance Factorization**

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

The Contact and Universal Relations

Contact interaction is represented through a boundary condition

Imposing this B.C. on the Schrödinger equation yields an asymptotic wave function when two fermions get very close

$$\Psi \xrightarrow[r_{ij}\to 0]{} \underbrace{(1/r_{ij} - 1/a)}_{\text{Two Body}}\underbrace{A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})}_{\text{A-2}}$$

Dilute System → **Short Distance Factorization**

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

The Contact and Universal Relations

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a)A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j})$$

$$n(k) = C/k^4 \quad \text{for} \quad k > k_F$$

Dilute System → Short Distance Factorization → **High Momentum Tail**

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$$\Rightarrow \boxed{n(k) = C/k^4 \quad \text{for} \quad k > k_F}$$

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Tan’s Contact term:

1. Measures the number of SRC different fermion pairs.
2. Determines the thermodynamics through a series of universal relations.

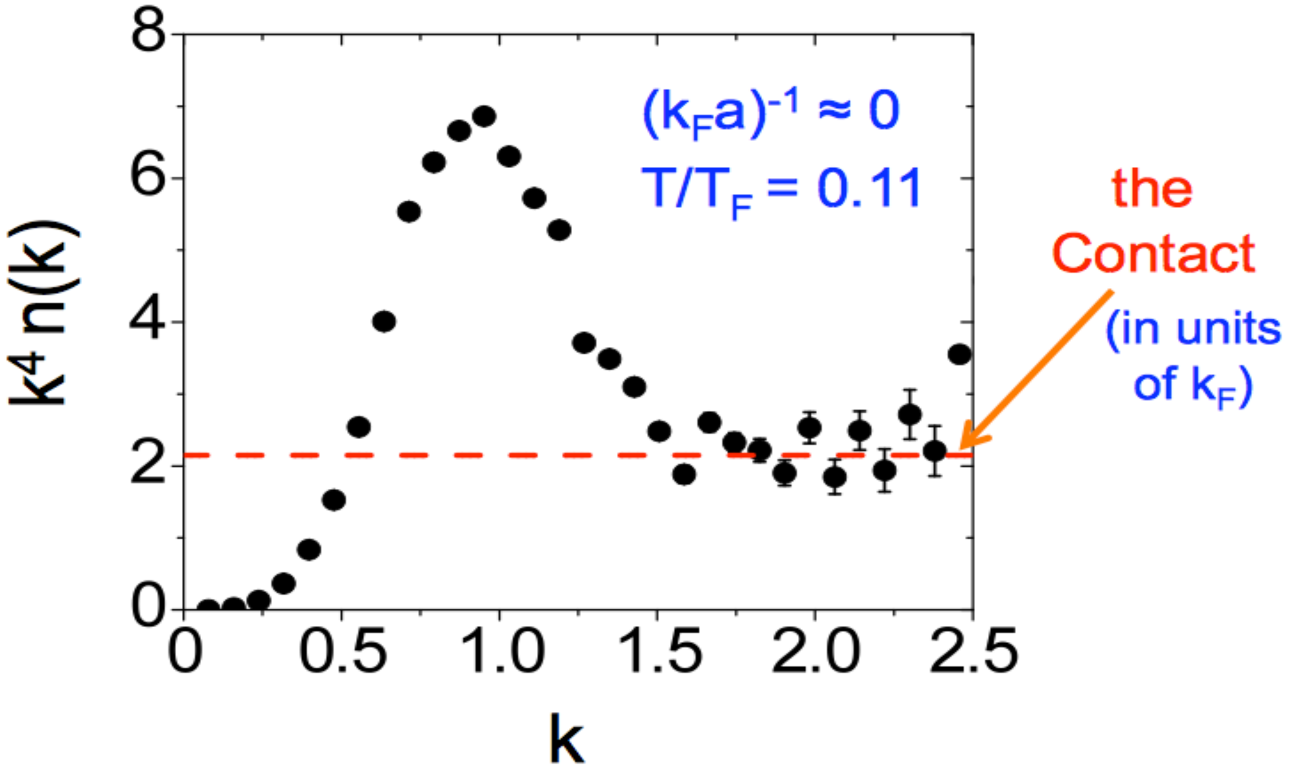
Dilute System → Short Distance Factorization → **High Momentum Tail**

S. Tan Annals of Physics 323 (2008) 2952, ibid 2971, ibid 2987

Experimental Validation

Two spin-state mixtures of ultra-cold ^{40}K and ^{6}Li atomic gas systems.

=> extracted the contact and verified the universal relations



Stewart et al. PRL **104**, 235301 (2010)

What About a *Nuclear* Contact ?

A Nuclear Contact?

Concept developed for:

dilute two-component Fermi systems with a short-range interaction.

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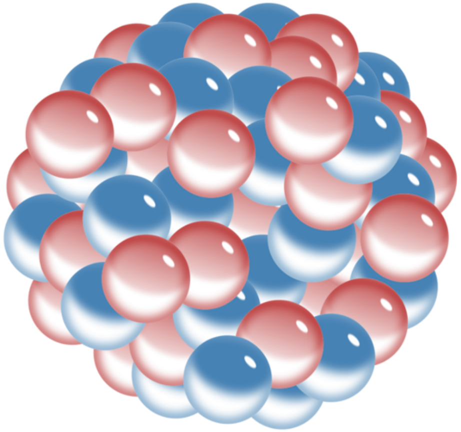

protons and neutrons

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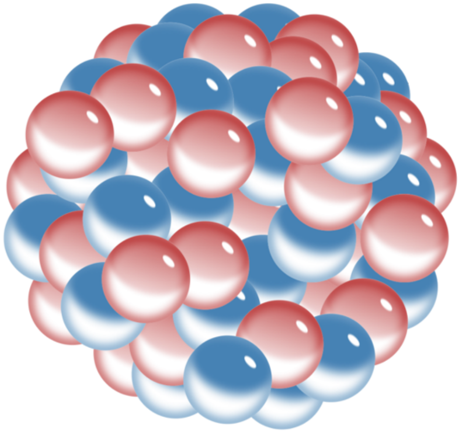

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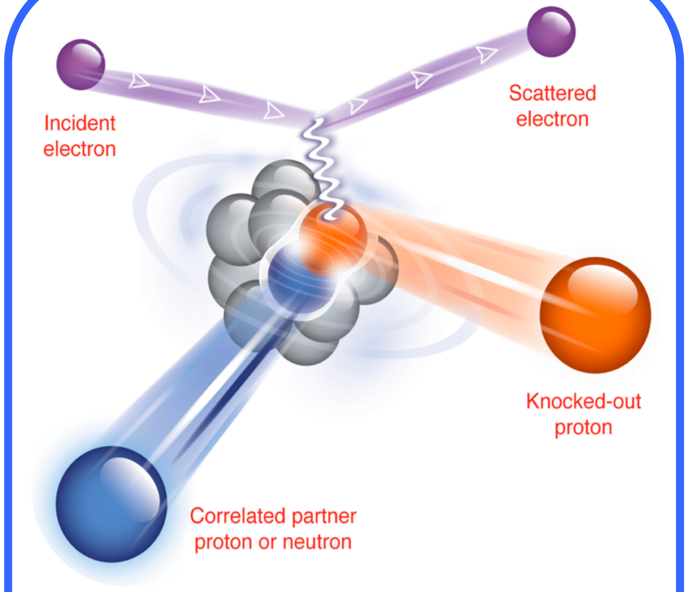

2N-SRC

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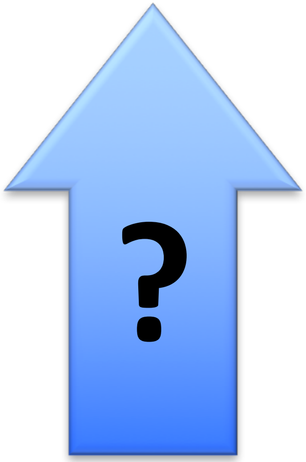

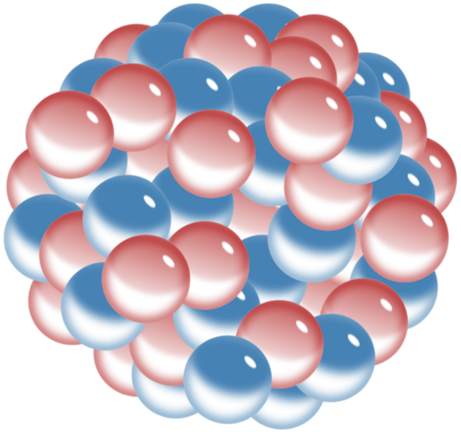

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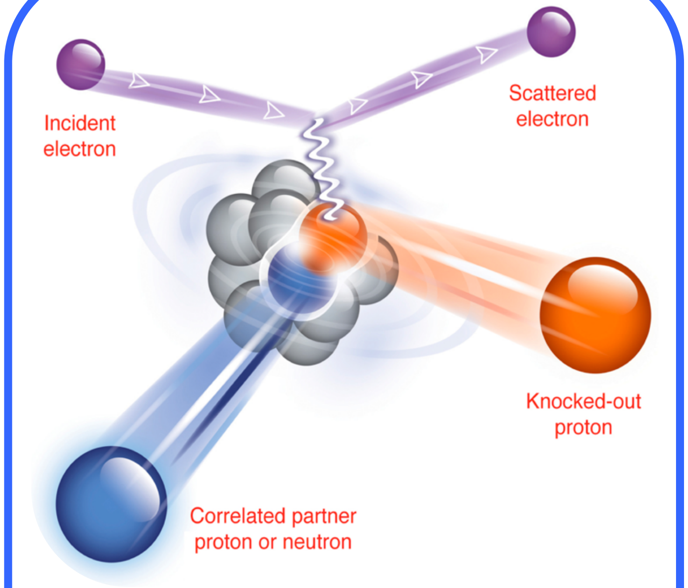

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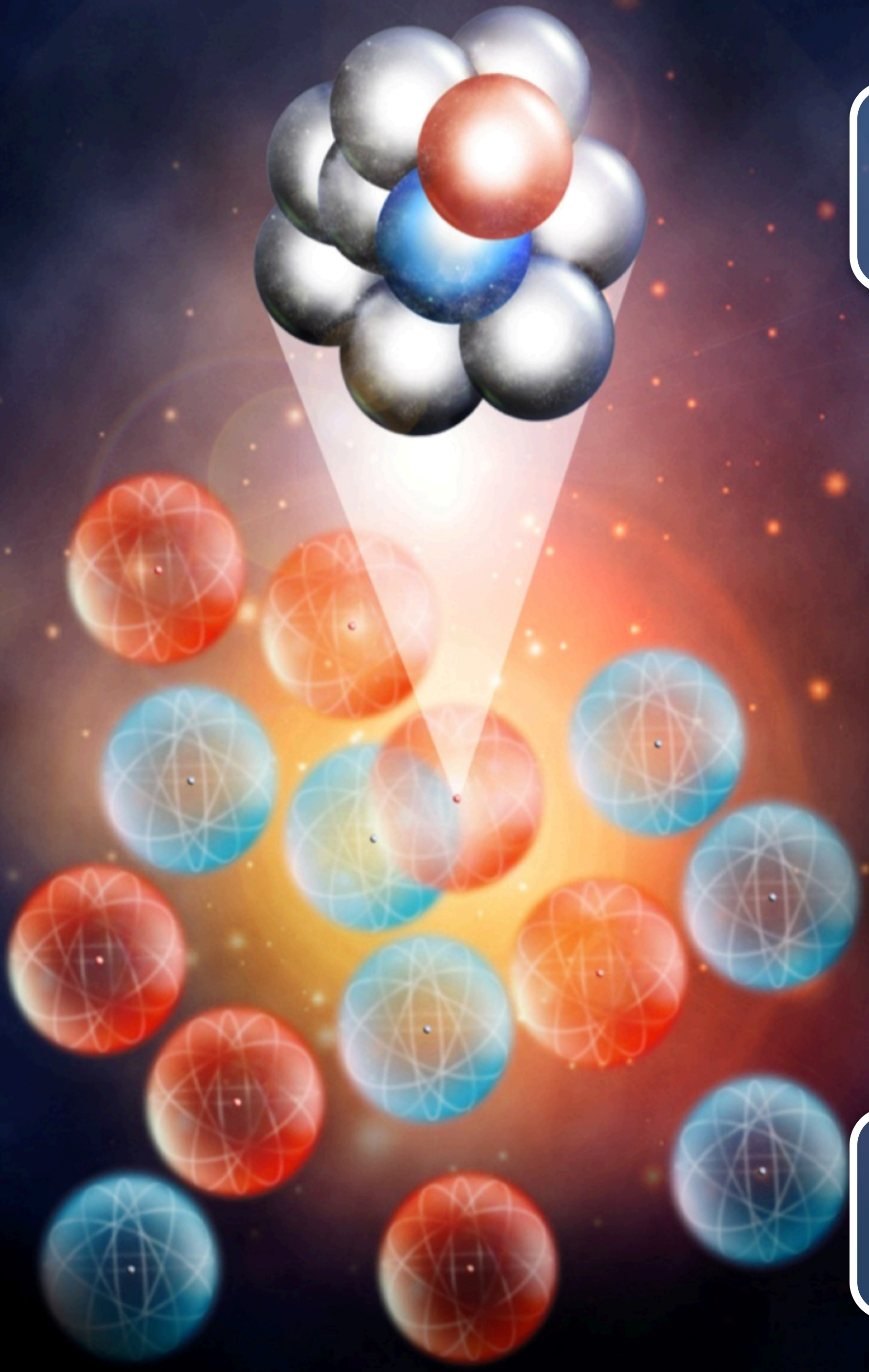

nucleons in nuclei

$\rho = 10^{44}\ m^{-3}$

Ultra-cold atoms in a trap

$\rho = 10^{21}\ m^{-3}$

$$\sigma_1 \approx 1 \ \text{person/m}^2$$

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$$\sigma_2 \approx 1 \ \text{person/km}^2$$

$$\sigma_1/\sigma_2 \approx 10^6$$

Theory Says: not so much

Are nuclei dilute? (i.e. $r_{eff} << a,d$)

$$d = \left(\rho/2\right)^{-1/3} \approx 2.3 \text{ fm}$$

$$r_{eff} \approx \frac{\hbar}{2 \cdot m_\pi \cdot c} \approx 0.7 \text{ fm}$$

[Tensor force]

$$a\left({}^3S_1\right) = 5.42 \text{ fm}$$

$$r_{eff}(0.7 \text{ fm}) < d(2.3 \text{ fm}), a(5.4 \text{ fm})$$

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$$r_{eff} \approx \frac{}{2}$$

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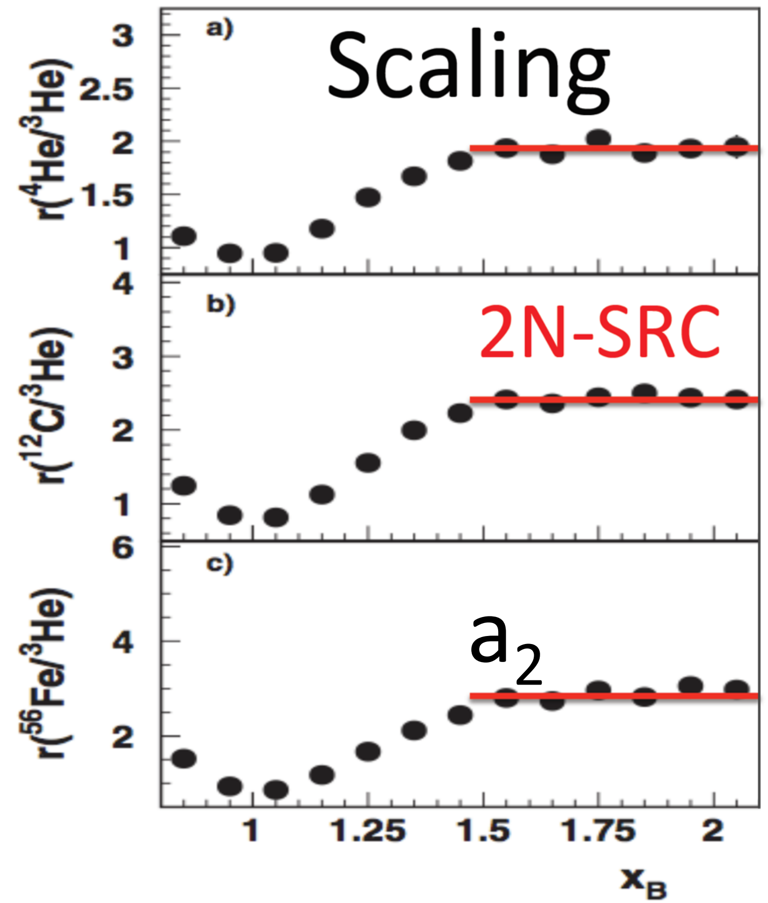
What can we learn from experiment ?

Is there $1/k^4$ scaling regardless?

$$1.5k_F < k < 3k_F$$

$$n_A(k) = a_2(A/d) \cdot n_d(k)$$



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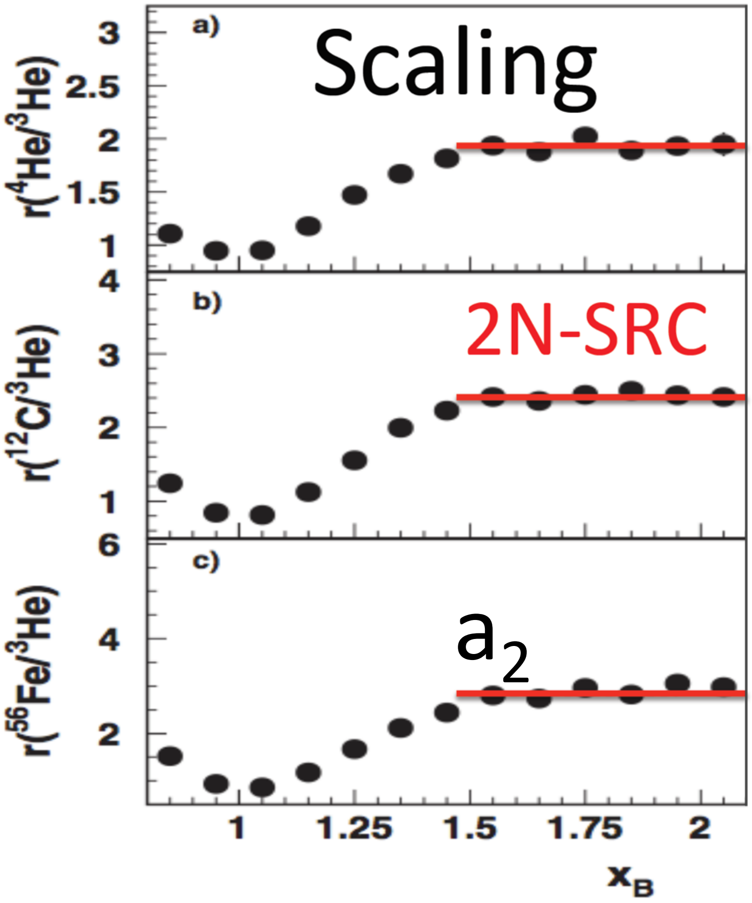
nucleus A momentum distribution

experimental constant

deuteron momentum distribution



The momentum distribution of nucleons in medium to heavy nuclei is proportional to that of deuteron at high momenta.

But Experiment Says....   **Yes!**

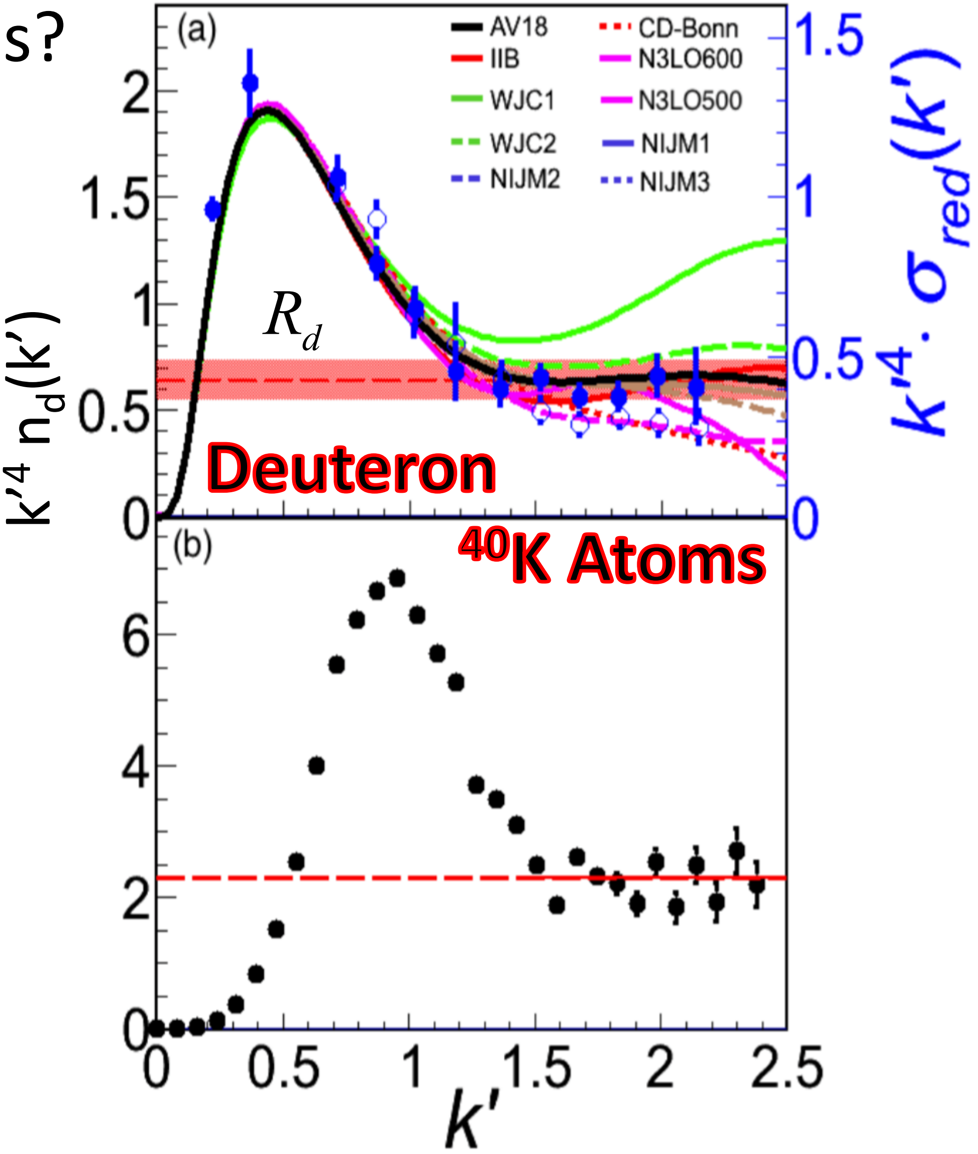
Is there $1/k^4$ scaling regardless?

$$1.5k_F < k < 3k_F$$

$$n_A(k) = a_2(A/d) \cdot n_d(k)$$

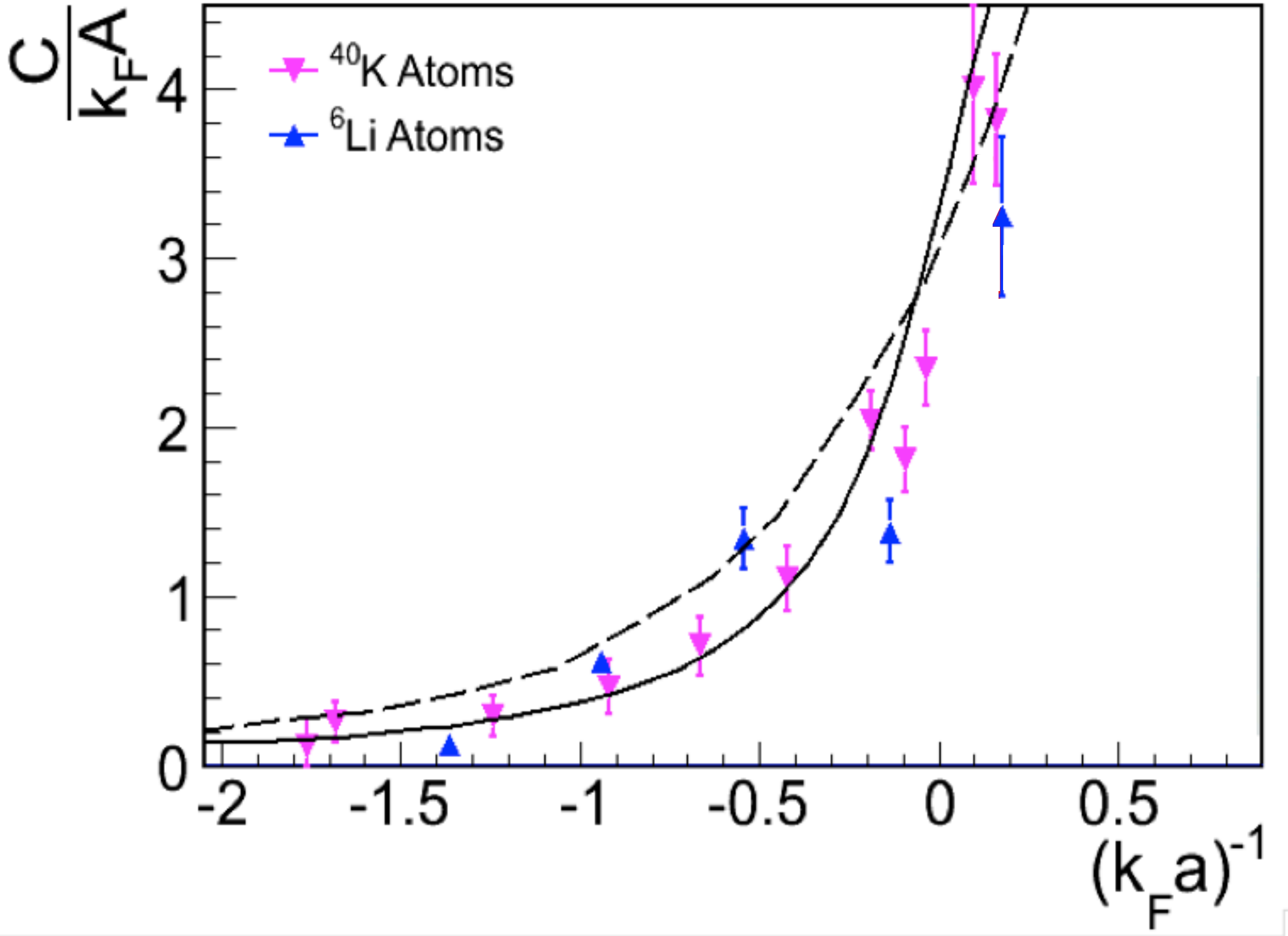
$$\frac{C}{k_F \cdot A} = a_2(A) \cdot R_d$$

Nucleus	$a_2(A)$	$\frac{C}{k_F A}$
^{12}C	4.75 ± 0.16	3.04 ± 0.49
^{56}Fe	5.21 ± 0.20	3.33 ± 0.54
^{197}Au	5.16 ± 0.22	3.30 ± 0.53



O. Hen et al. Phys. Rev. C **92**, 045205 (2015)

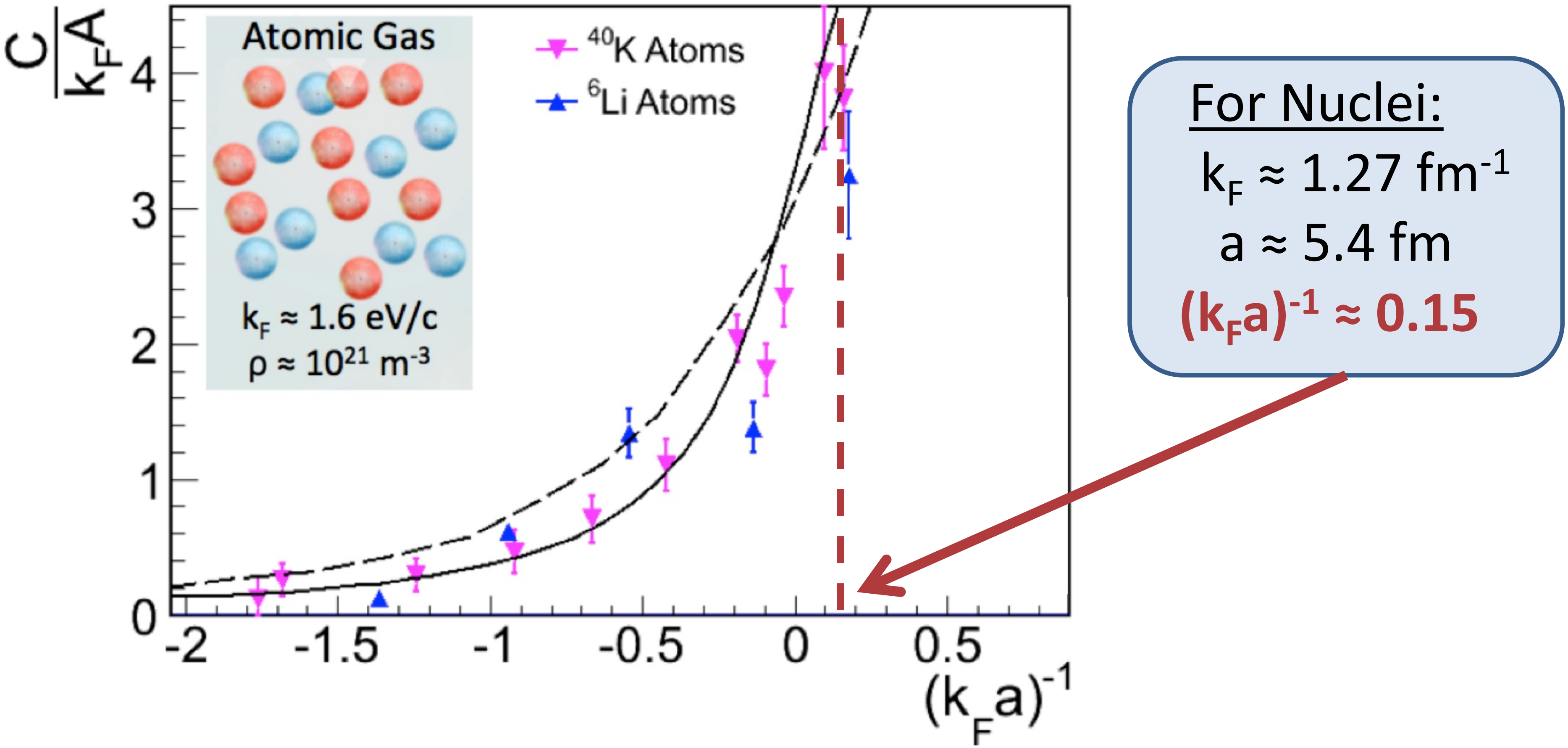
Comparing with atomic systems



Stewart et al. Phys. Rev. Lett. **104**, 235301 (2010)
Kuhnle et al. Phys. Rev. Lett. **105**, 070402 (2010)

Comparing with atomic systems

Finding the same *dimensionless* interaction strength



For Nuclei:

$k_F \approx 1.27$ fm^{-1}

$a \approx 5.4$ fm

$(k_F a)^{-1} \approx 0.15$

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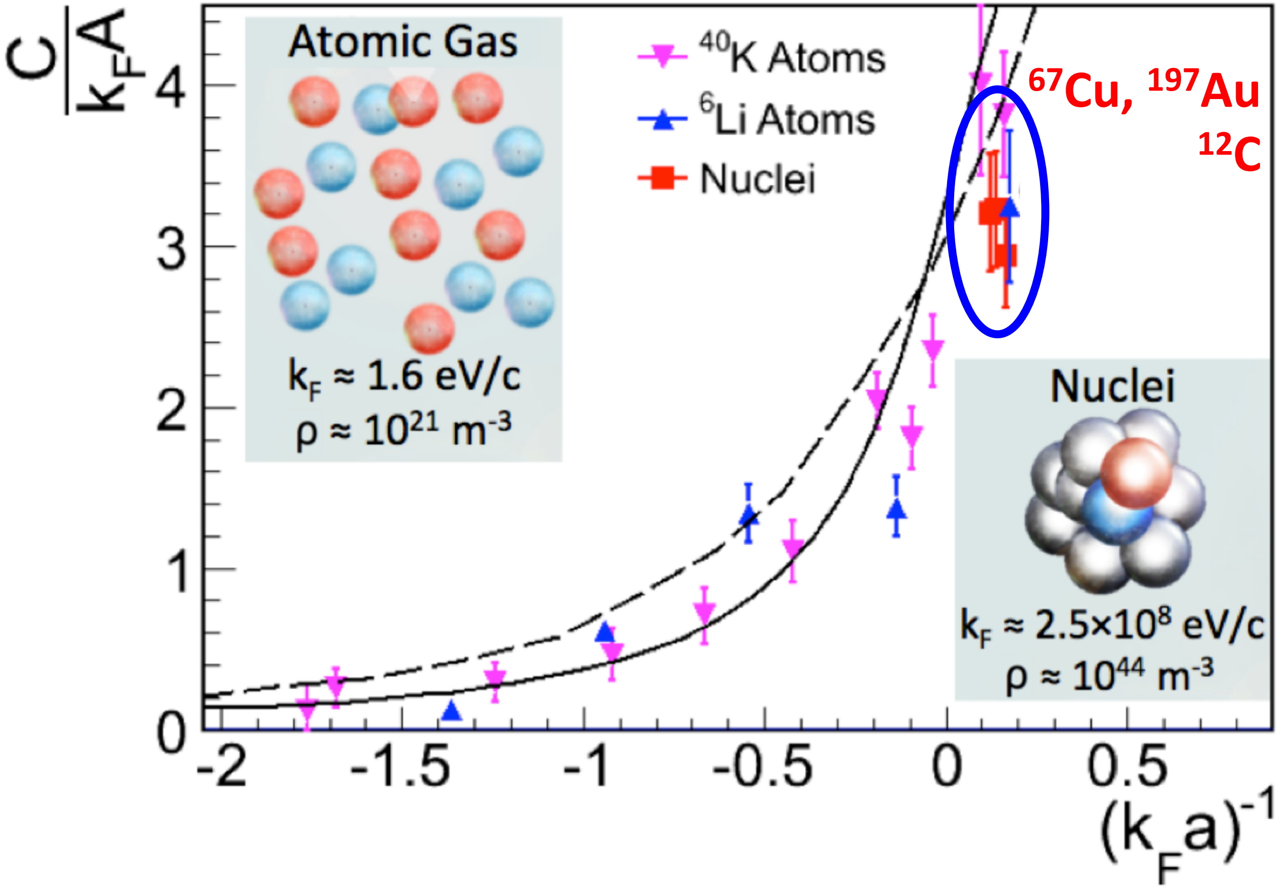
Comparing with atomic systems

Equal contacts for equal interactions strength!



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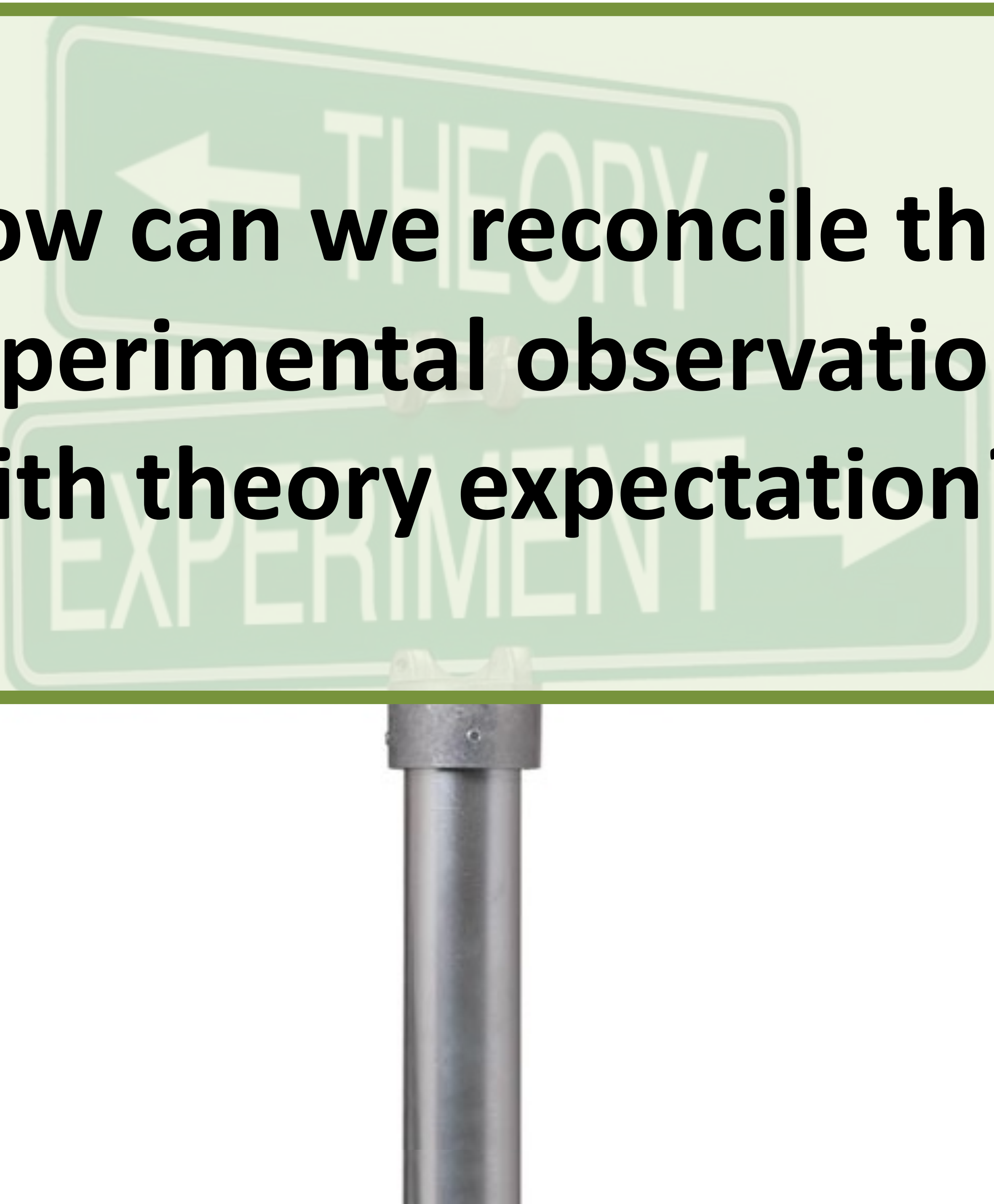
THEORY
EXPERIMENT

How can we reconcile the experimental observation with theory expectation?



Going Back to the Theory…

1. Generalize the contact formalism to nuclear systems.
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The scale separation does not necessarily work in nuclear systems.

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=>We need to assume a more general form for the wavefunction.

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(known) Solution for the two-body problem

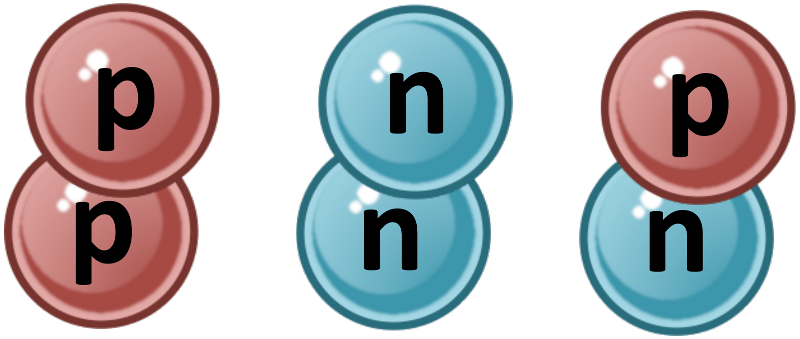
Factorization in Nuclei

Consider the factorized wave function:

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi_{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

In nuclear physics we have 3 possible types of pairs:

ij = {pp, nn, pn}



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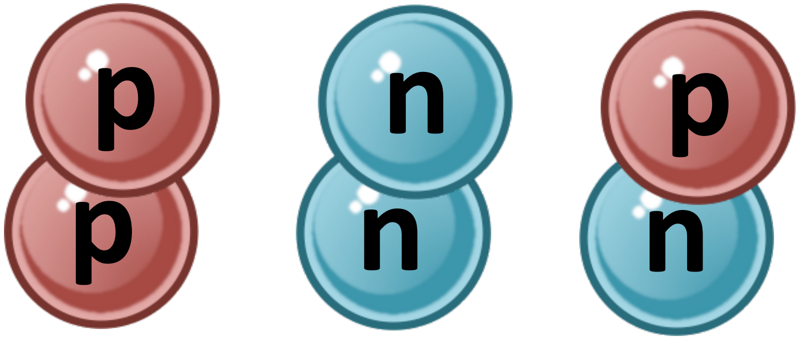
In nuclear physics we have 3 possible types of pairs:

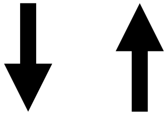
ij = {pp, nn, pn}

+

For each pair we have different channels

α = (s,l)jm





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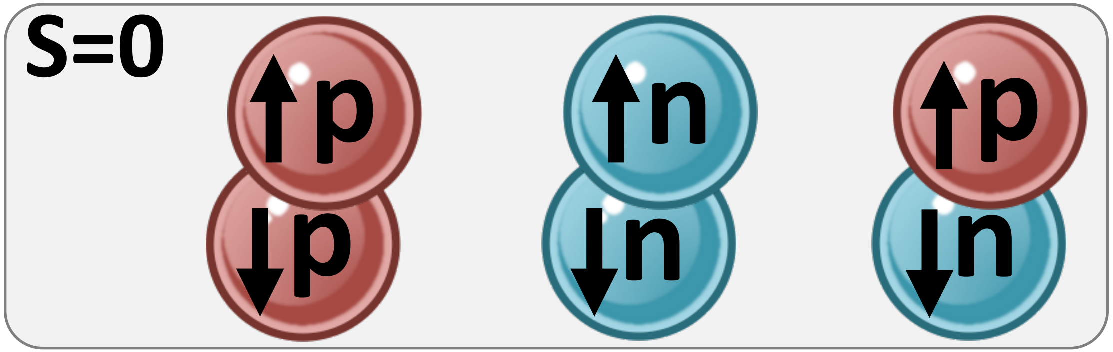
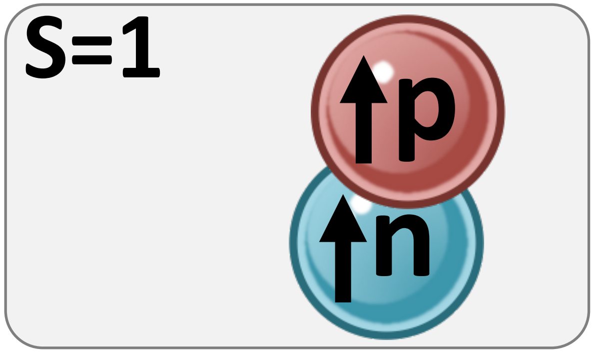
4 wave functions and contacts for L=0





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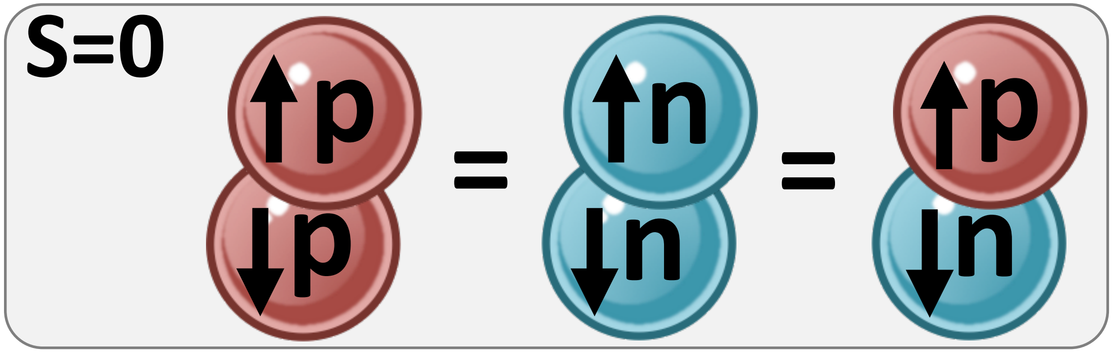
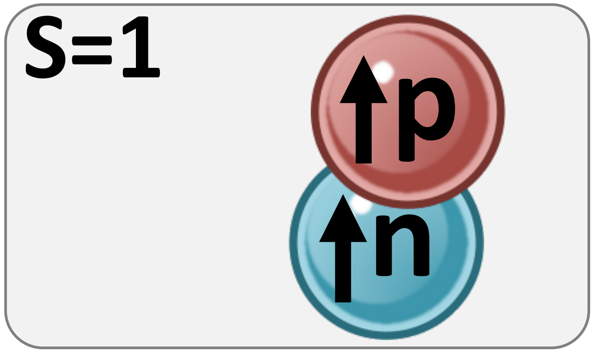
Reduced to 2 contacts from symmetry considerations





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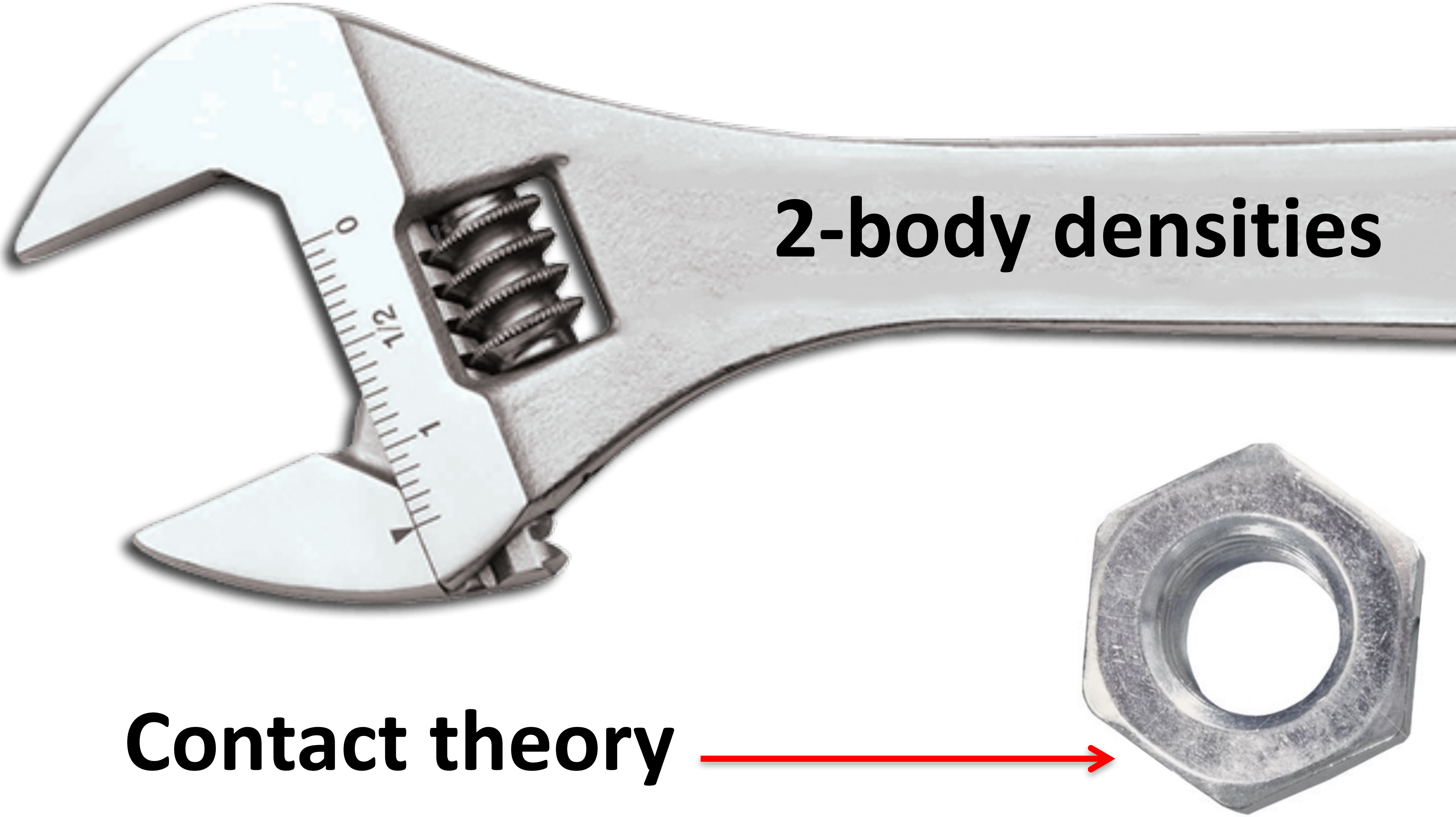
Finding the right tool for the job!



2-body densities

Contact theory →

2-Body momentum distributions

- One Body momentum distribution [$n_N(k)$]: Probability to find a nucleon, N, in the nucleus with momentum k.

- Two Body momentum distribution [$n_{NN}(q,Q)$]: Probability to find a NN pair in the nucleus with relative (c.m.) momentum q (Q).

$n_{NN}(q,Q)$ – computational Frontier!

Relating to Momentum Space

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi_{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

One Body:

$$\boldsymbol{n_p(k)} = \sum_{\alpha} \left|\widetilde{\boldsymbol{\varphi}}_{\boldsymbol{pp}}^{\boldsymbol{\alpha}}(\boldsymbol{k})\right|^2 2\boldsymbol{C}_{\boldsymbol{pp}}^{\boldsymbol{\alpha}} + \sum_{\alpha} \left|\widetilde{\boldsymbol{\varphi}}_{\boldsymbol{pn}}^{\boldsymbol{\alpha}}(\boldsymbol{k})\right|^2 \boldsymbol{C}_{\boldsymbol{pn}}^{\boldsymbol{\alpha}}$$

Two body:

$$\boldsymbol{F_{ij}(k)} = \sum_{\alpha} \left|\widetilde{\boldsymbol{\varphi}}_{\boldsymbol{ij}}^{\boldsymbol{\alpha}}(\boldsymbol{k})\right|^2 \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha}}$$

Momentum Space Factorization

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_{\alpha} \varphi_{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

One Body:

$$n_p(k) = \sum_{\alpha} \left|\tilde{\varphi}_{pp}^{\alpha}(k)\right|^2 2C_{pp}^{\alpha} + \sum_{\alpha} \left|\tilde{\varphi}_{pn}^{\alpha}(k)\right|^2 C_{pn}^{\alpha}$$

Two body:

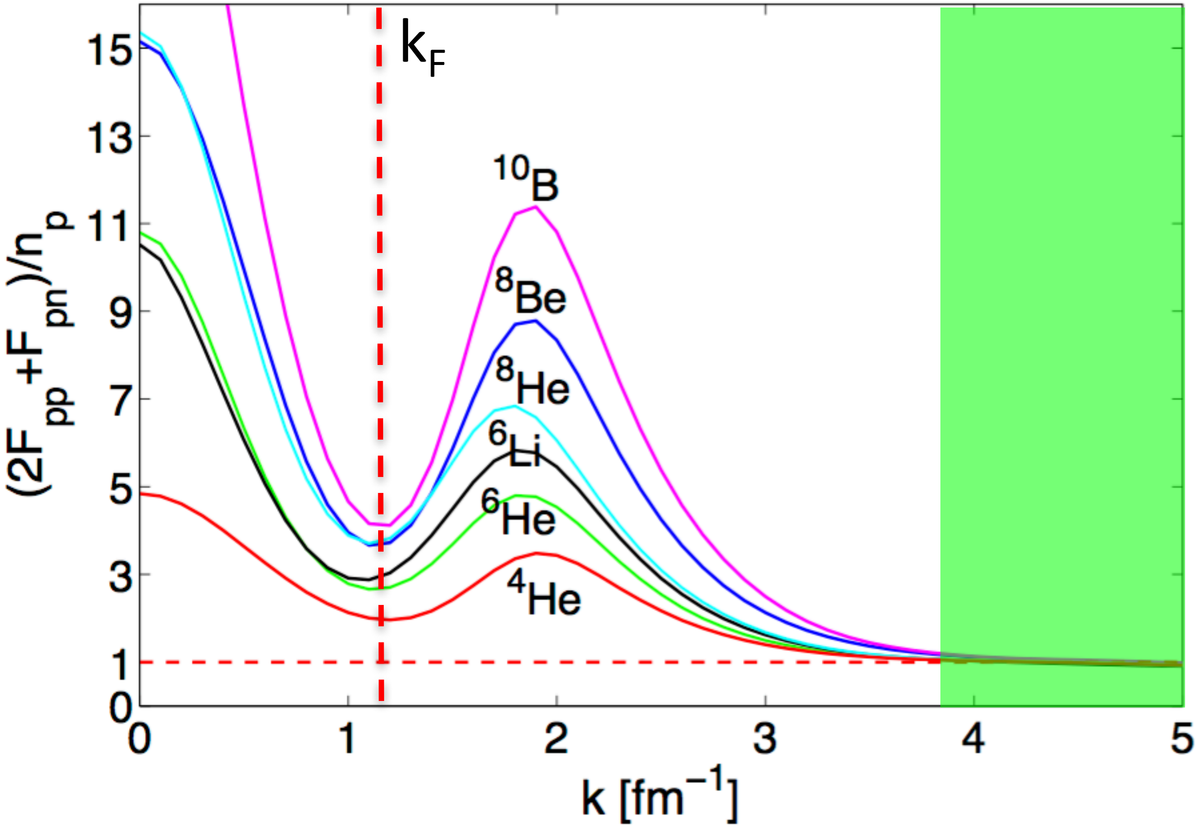
$$F_{ij}(k) = \sum_{\alpha} \left|\tilde{\varphi}_{ij}^{\alpha}(k)\right|^2 C_{ij}^{\alpha}$$

Clearly:

$$n_p(k) \xrightarrow[k\to\infty]{} 2F_{pp}(k) + F_{pn}(k)$$

Two-Body Scaling

- Weiss and Barnea (PRC 2015): contact interactions dominate when $n_{pn}(q)+2n_{pp}(q) = n_p(k)$



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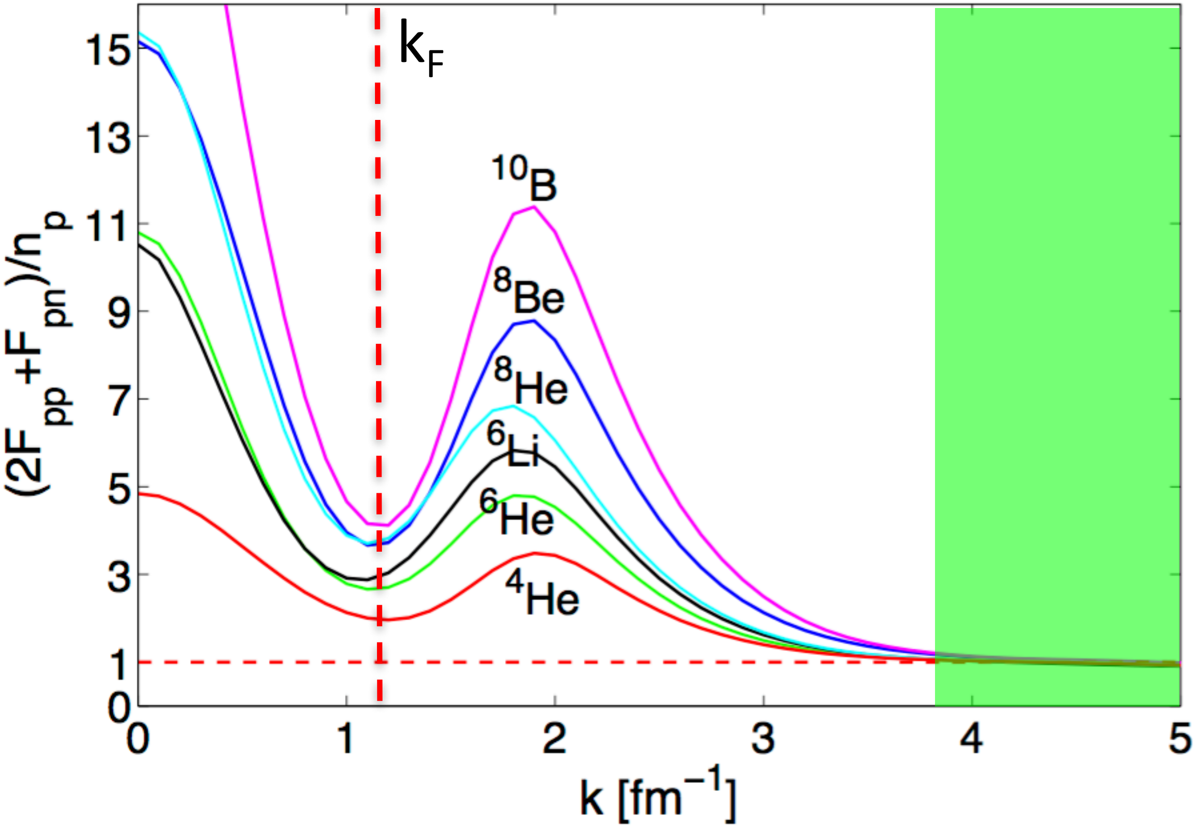
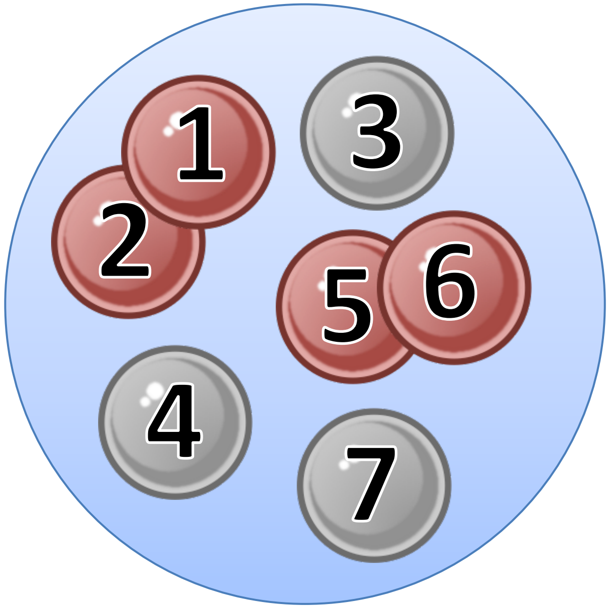


But… experiment observe SRC pair dominance starting at k_F and here we see it at ~3xk_F

Two-Body Momentum Distributions

- $n_{NN}(q,Q)$ – Mathematical object that counts all possible NN pairs, regardless of their state:



Consider all NN pairs:

1-2
1-3
1-4
1-5
1-6
1-7

2-3
2-4
2-5
2-6
2-7

3-4
3-5
3-6
3-7

4-5
4-6
4-7

5-6
5-7

6-7

$n_{NN}(q,Q)$

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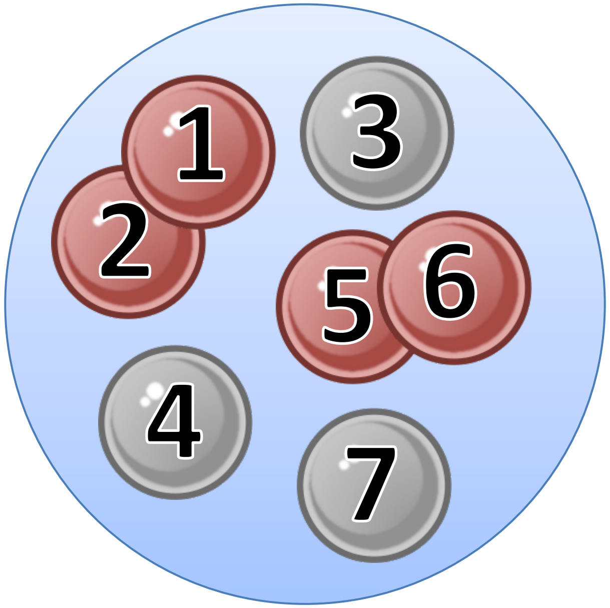

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2-7

3-4
3-5
3-6
3-7

4-5
4-6
4-7

5-6
5-7

6-7

$n_{NN}(q,Q)$

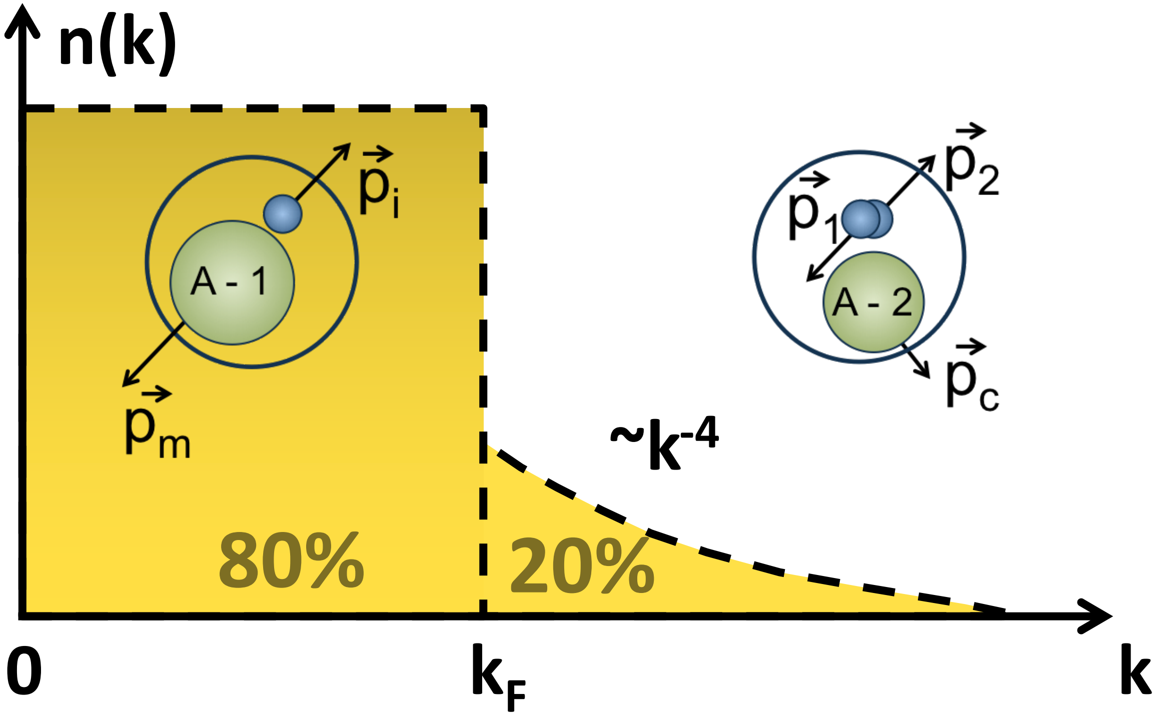
Toy model to the rescue

Build an ideal “contact interaction” system:

Free Fermi Gas for $k<k_F$

2N Correlations with $1/k^4$ for $k>k_F$

Correlated Fermi Gas Model



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Use Monte Carlo to raffle nucleons (pairs) and construct the one body and two body momentum distributions

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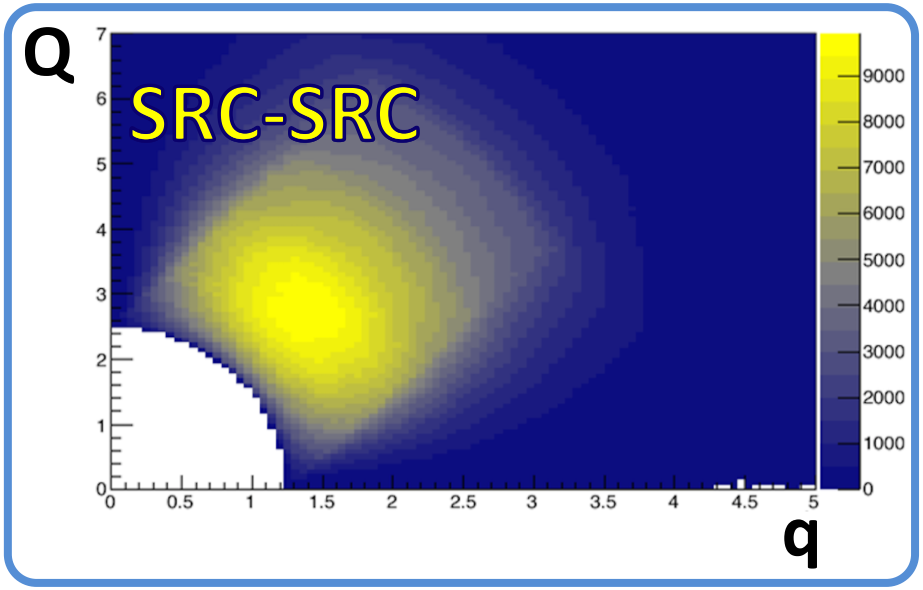
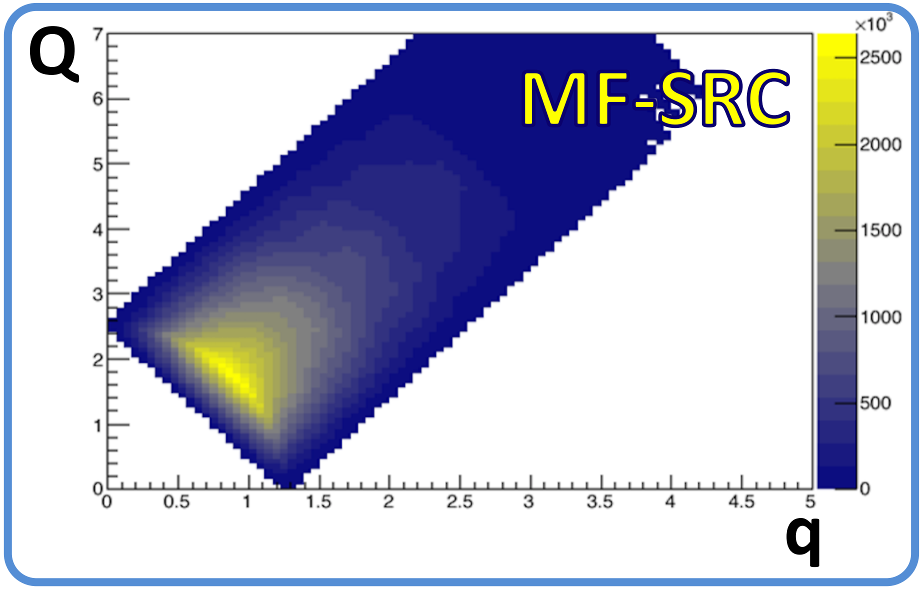
2N Correlations with $1/k^4$ for $k>k_F$

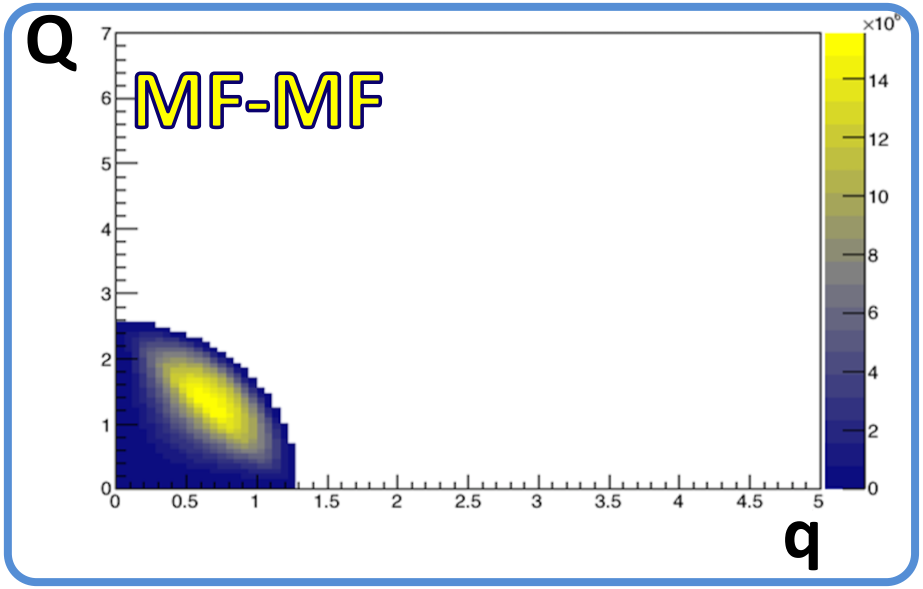
Use Monte Carlo to raffle nucleons (pairs) and construct the one body and two body momentum distributions

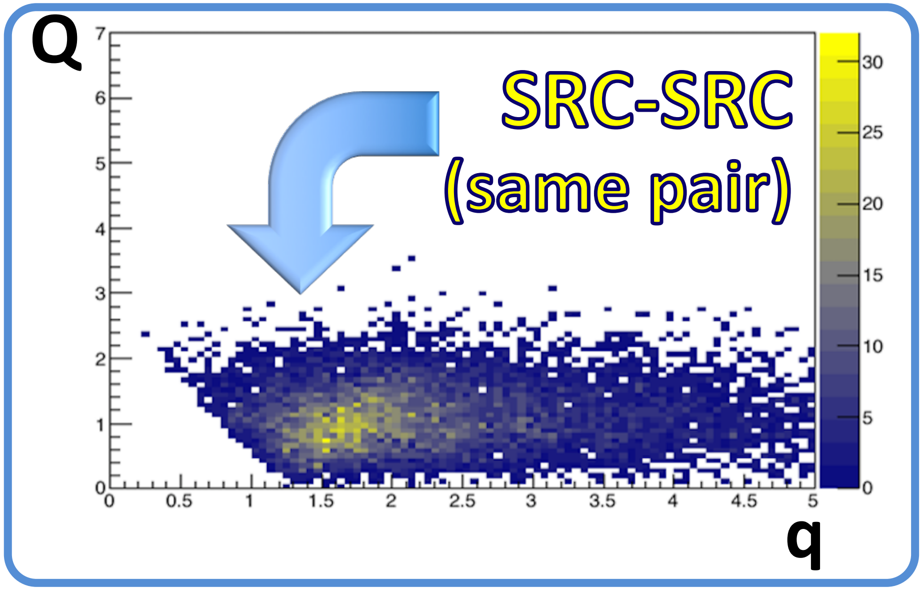
Separate different pairs based on their ‘origin’ (mean-field vs. SRC)

Toy model to the rescue









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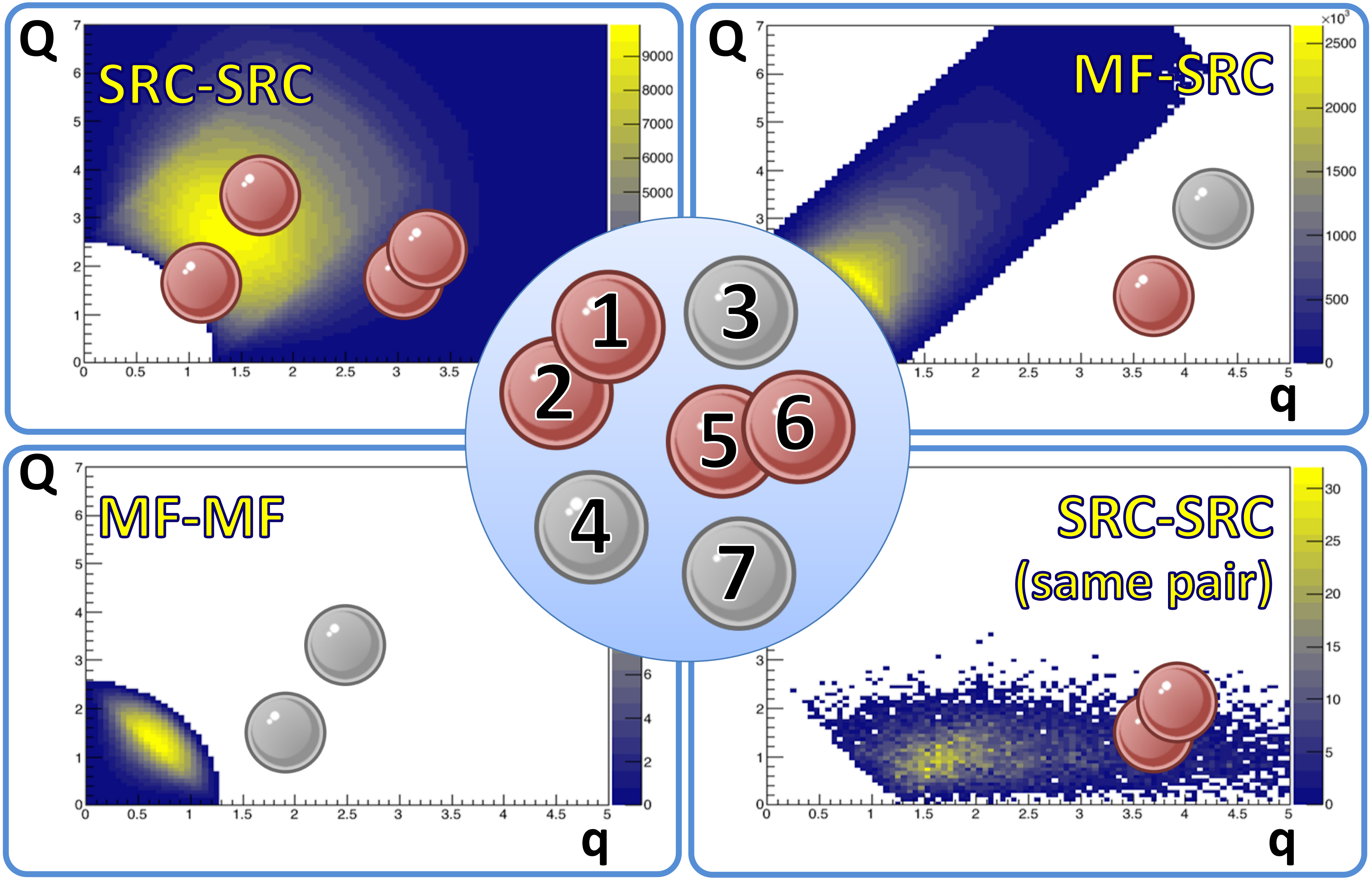
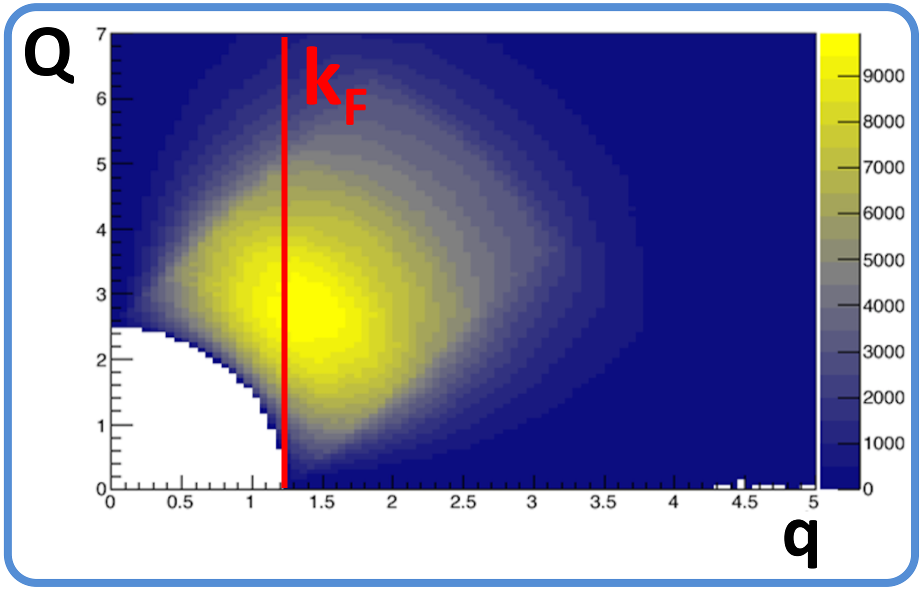
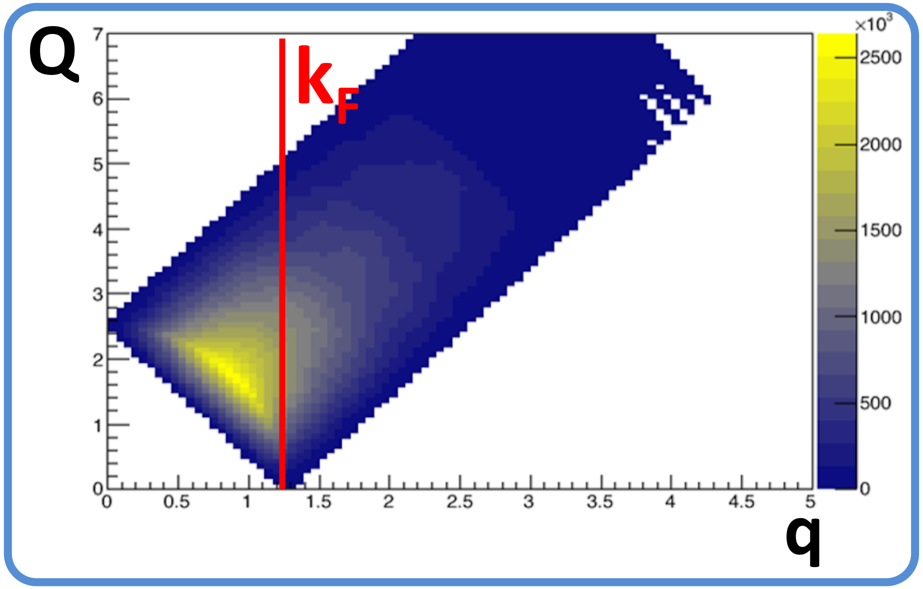
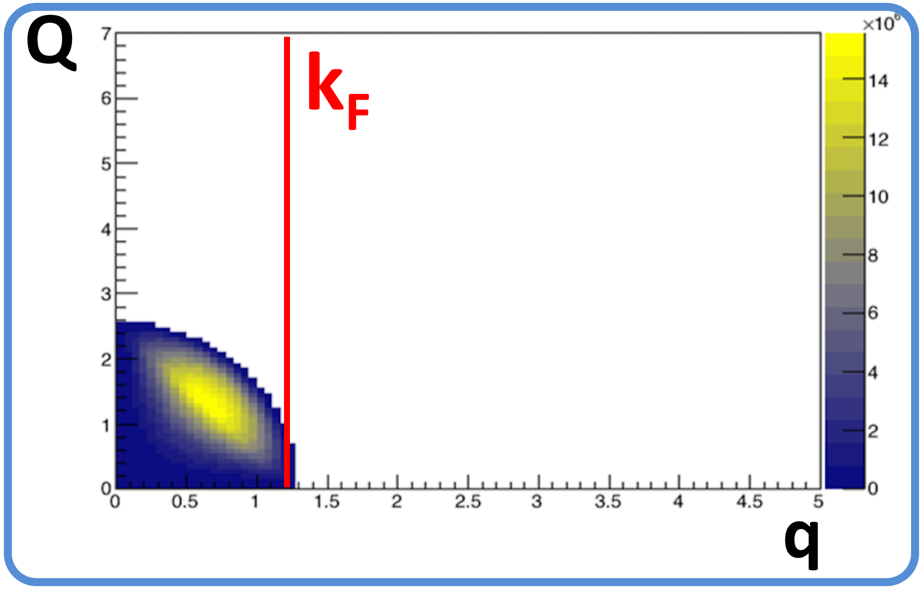
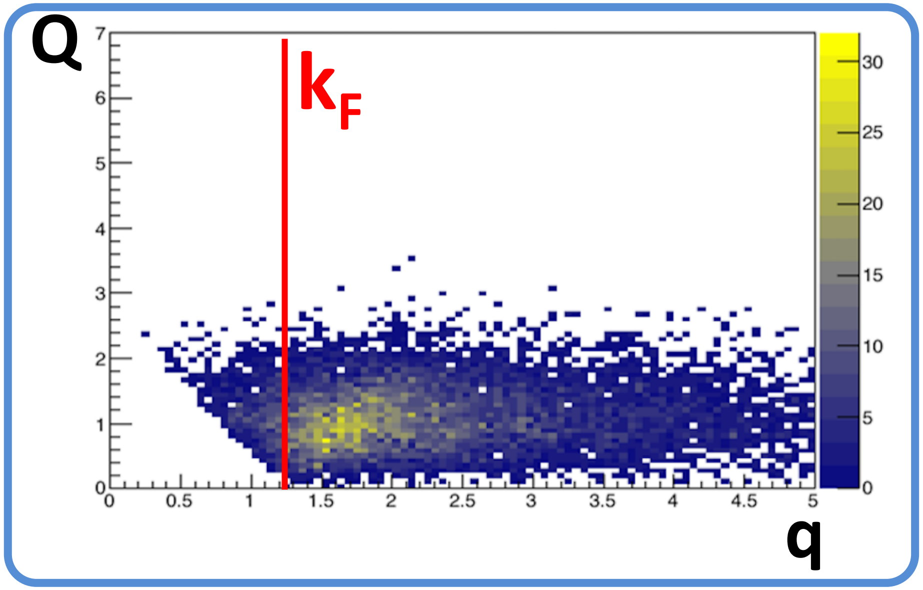


Toy model to the rescue









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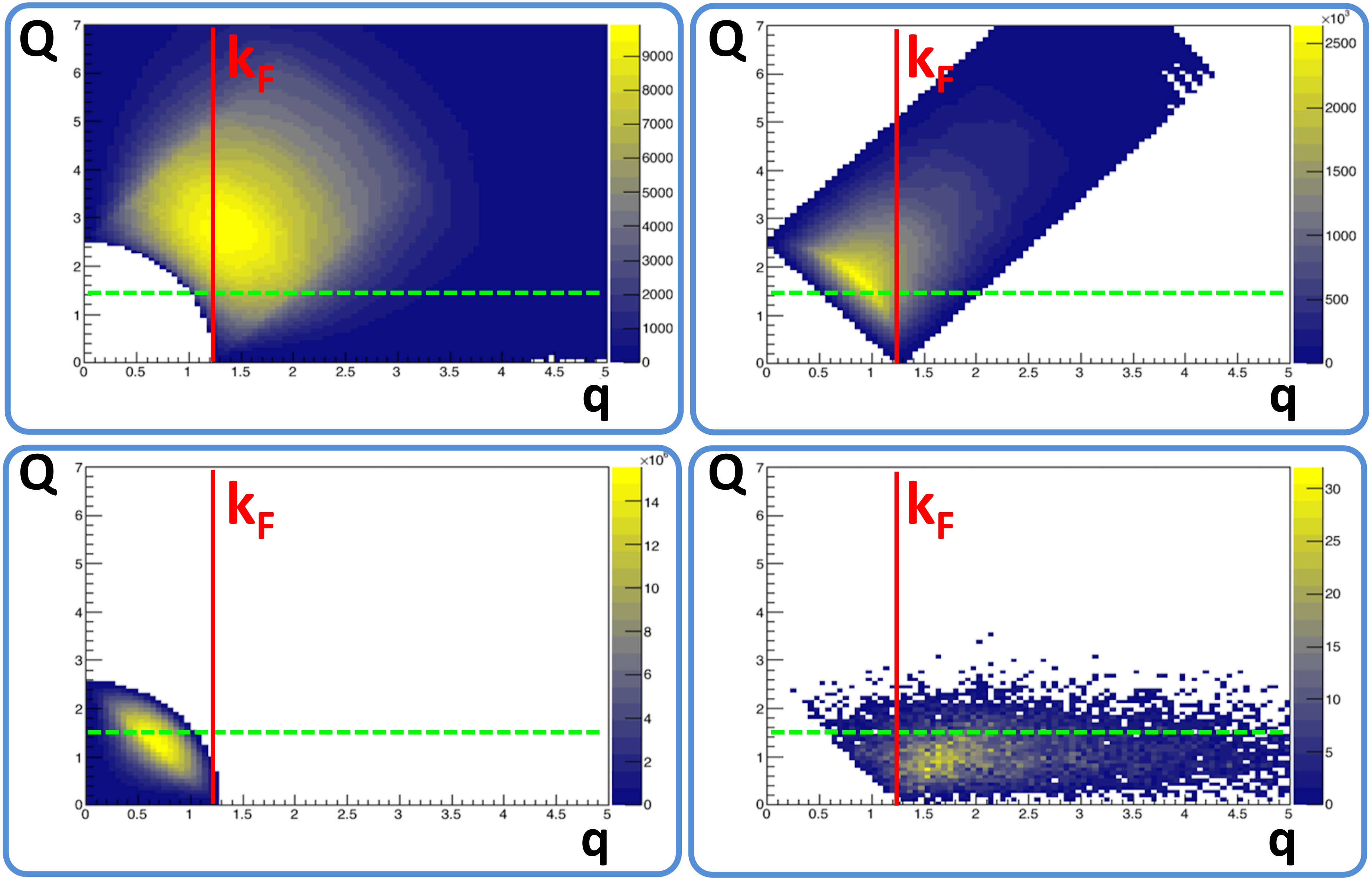
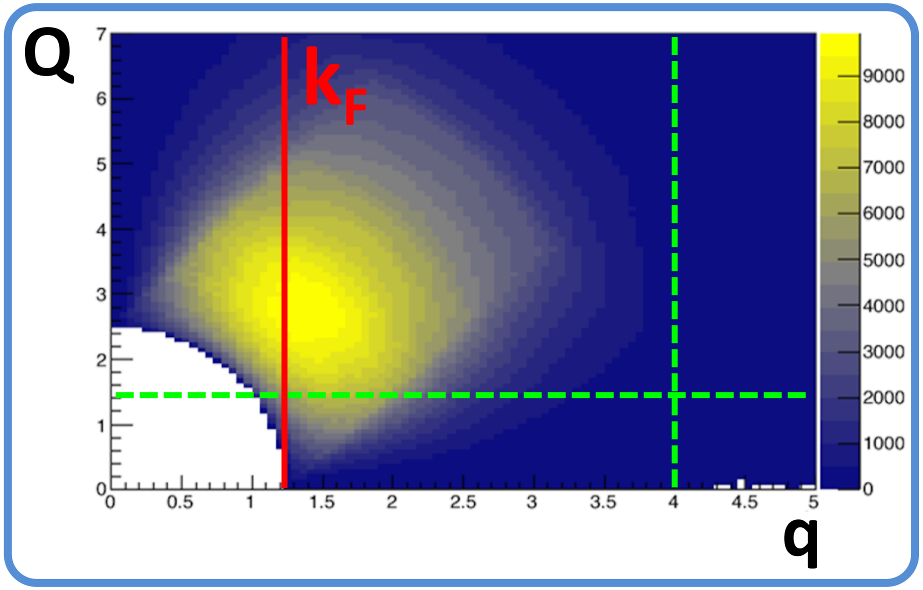
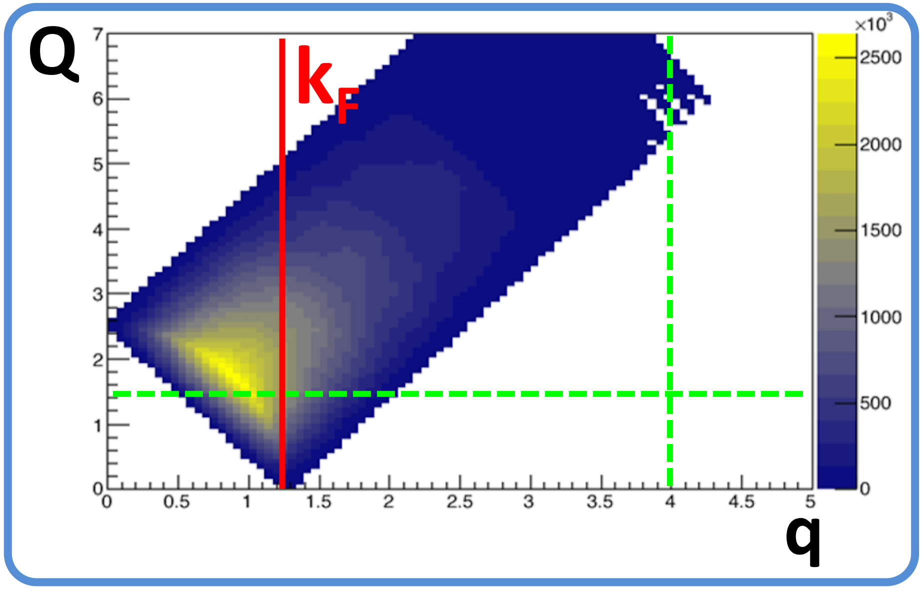
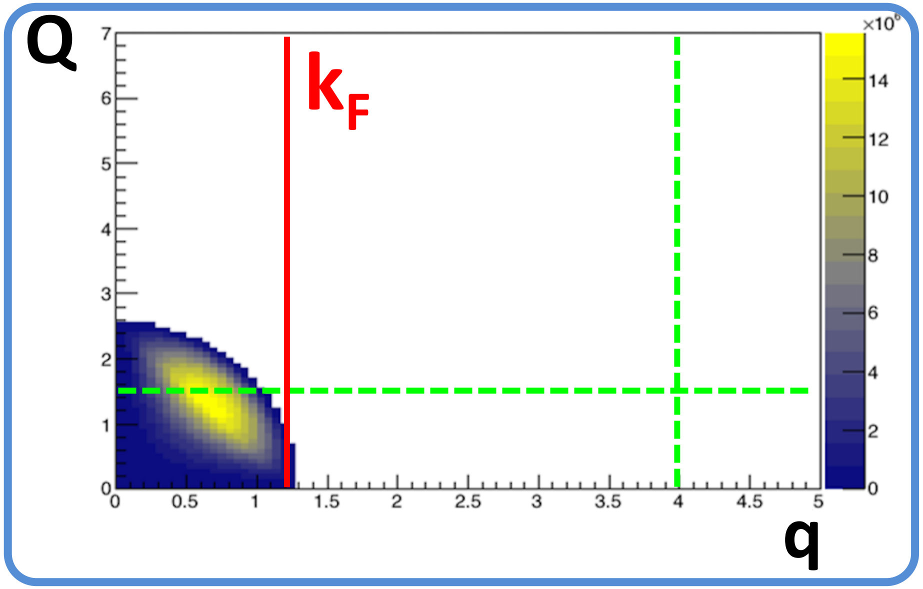
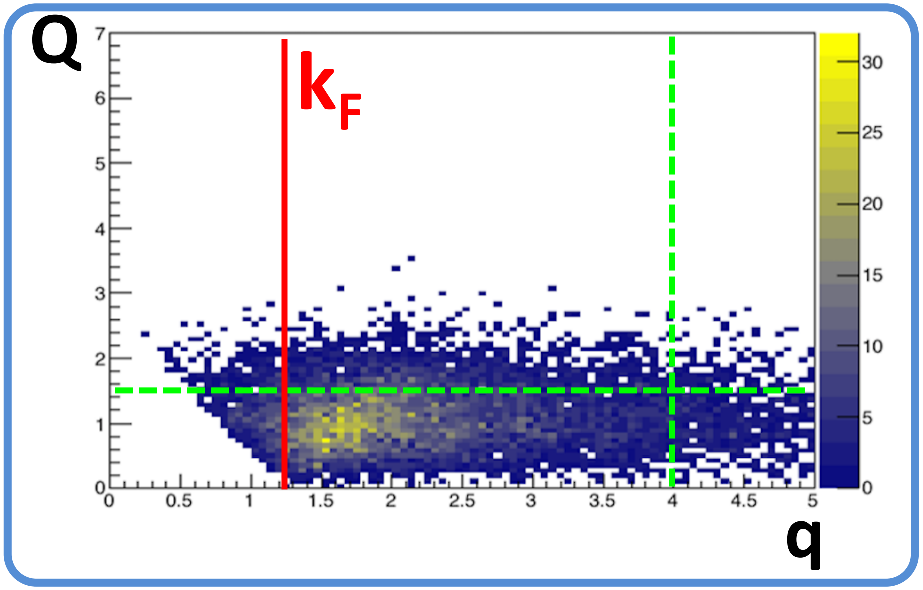


Toy model to the rescue

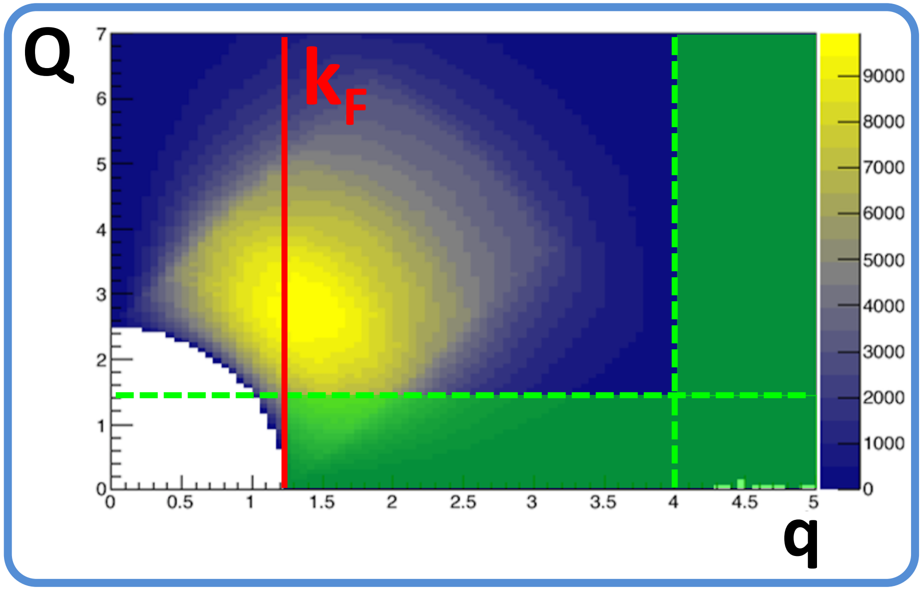


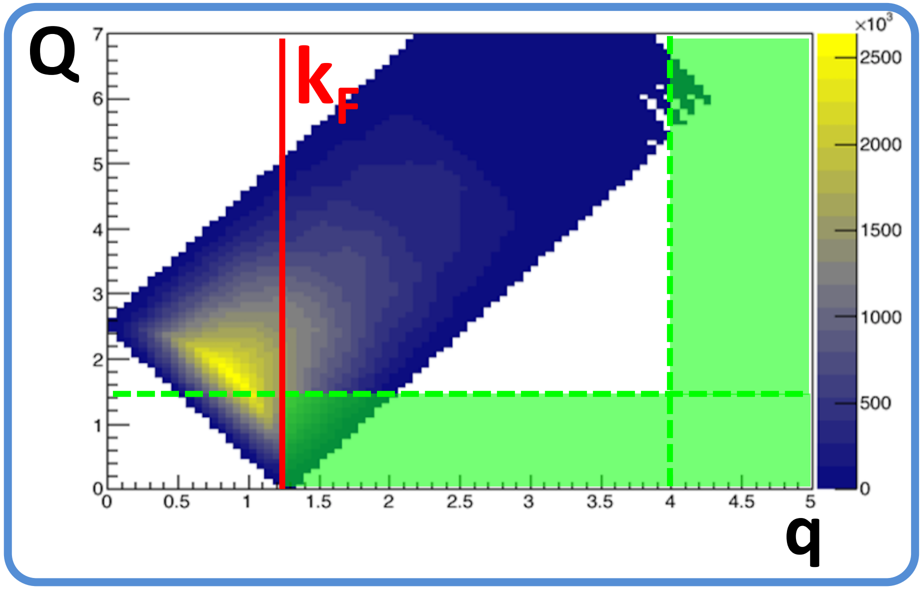


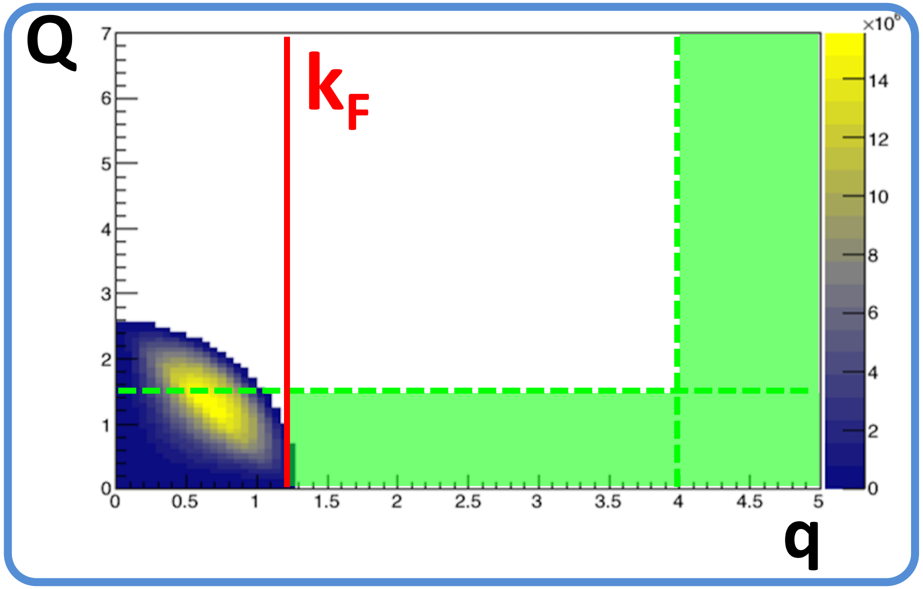


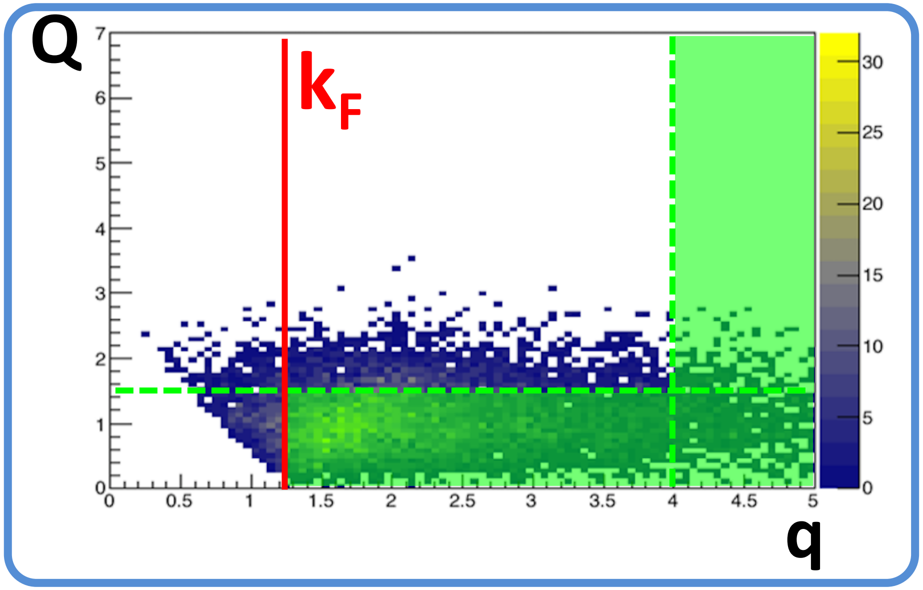


Toy model to the rescue



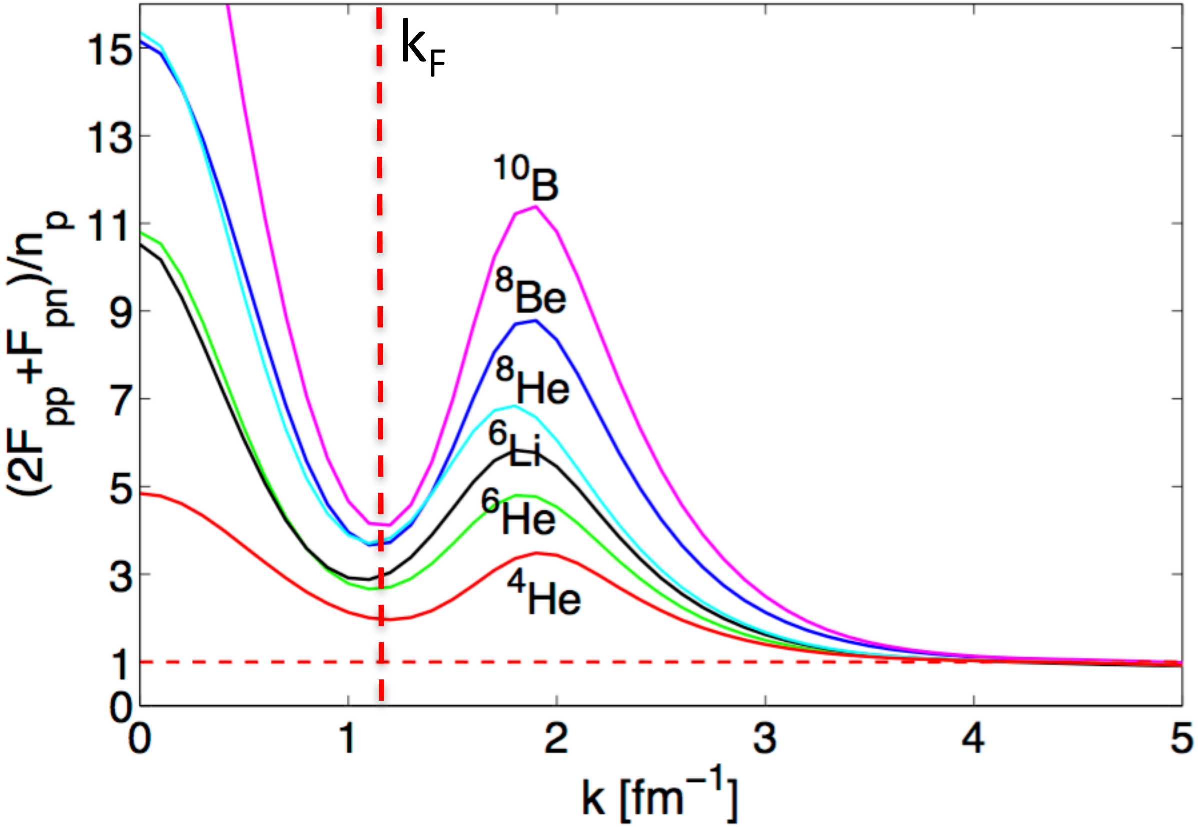




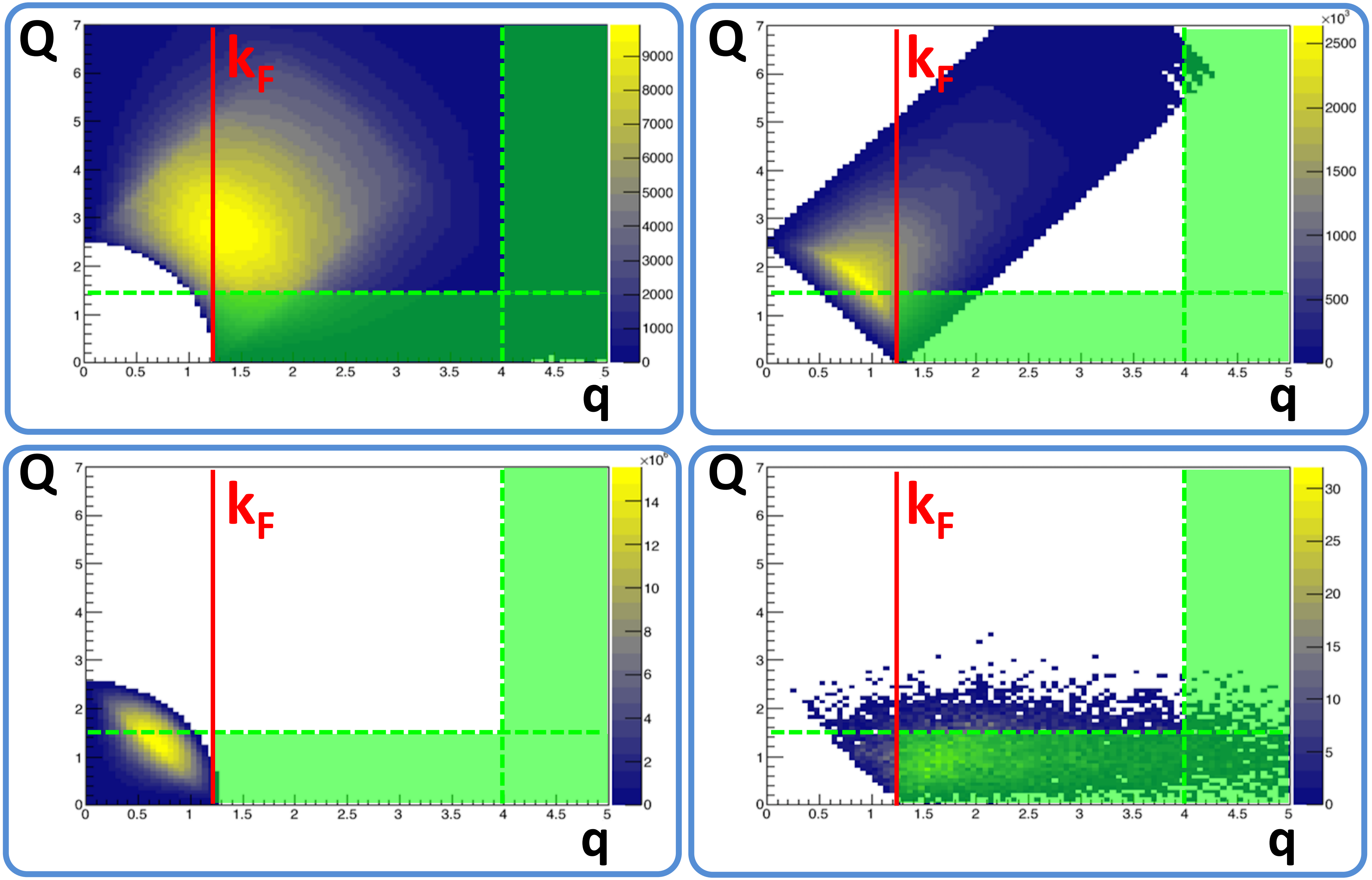


Two-Body Scaling for **High q**

- Weiss and Barnea (PRC 2015): contact interactions dominate when $n_{pn}(q)+2n_{pp}(q) = n_p(k)$

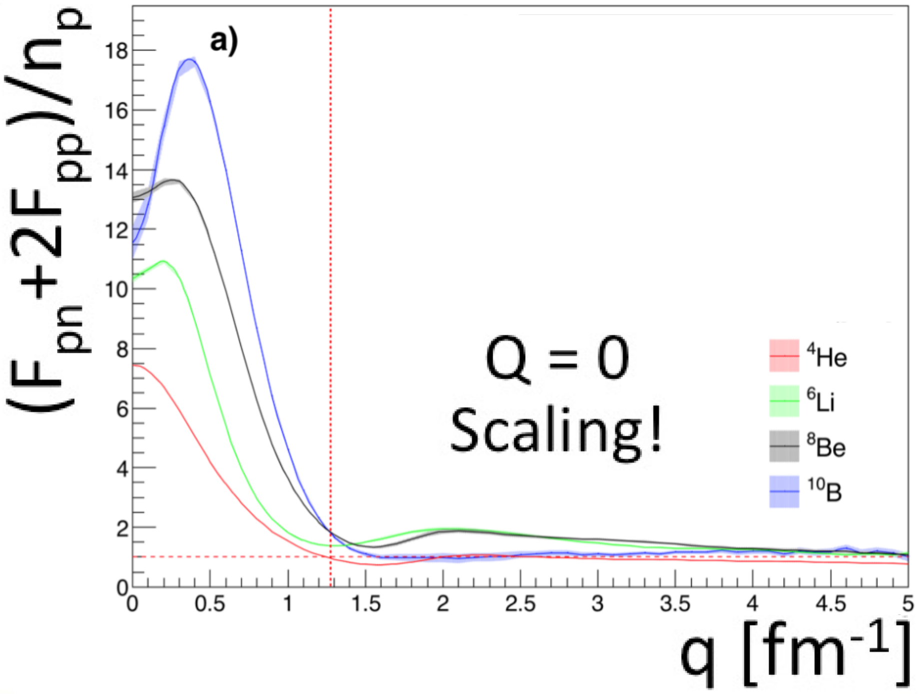


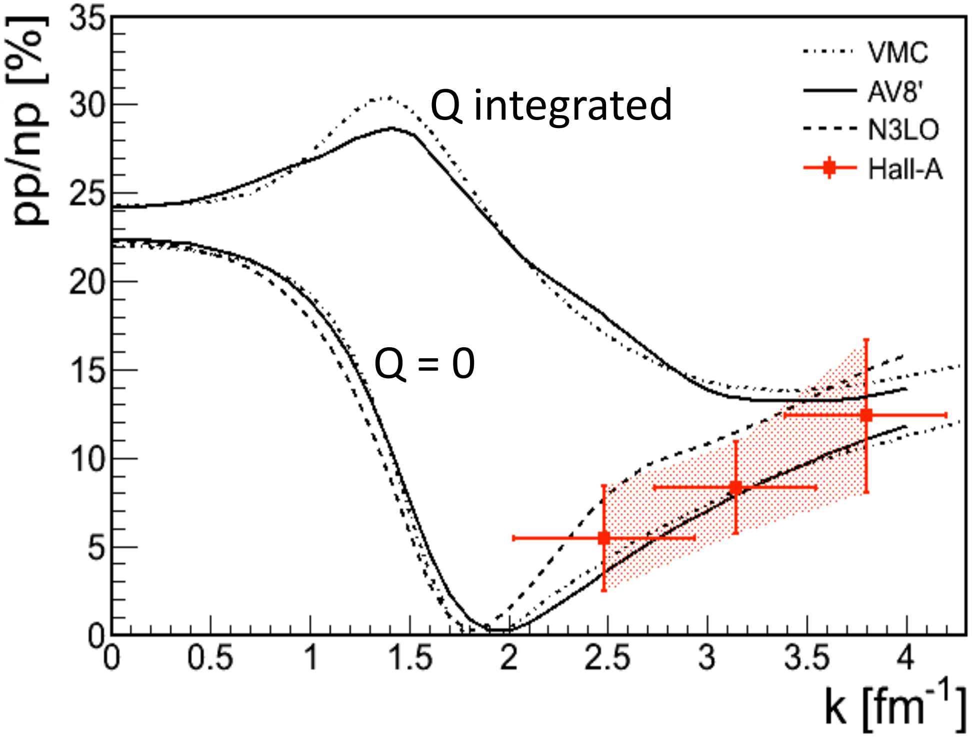
Toy model to the rescue



Two-Body Scaling for **Low Q**

- Restricting Q=0 restores scaling starting from k>k_F AND gives consistent results with experimental data!



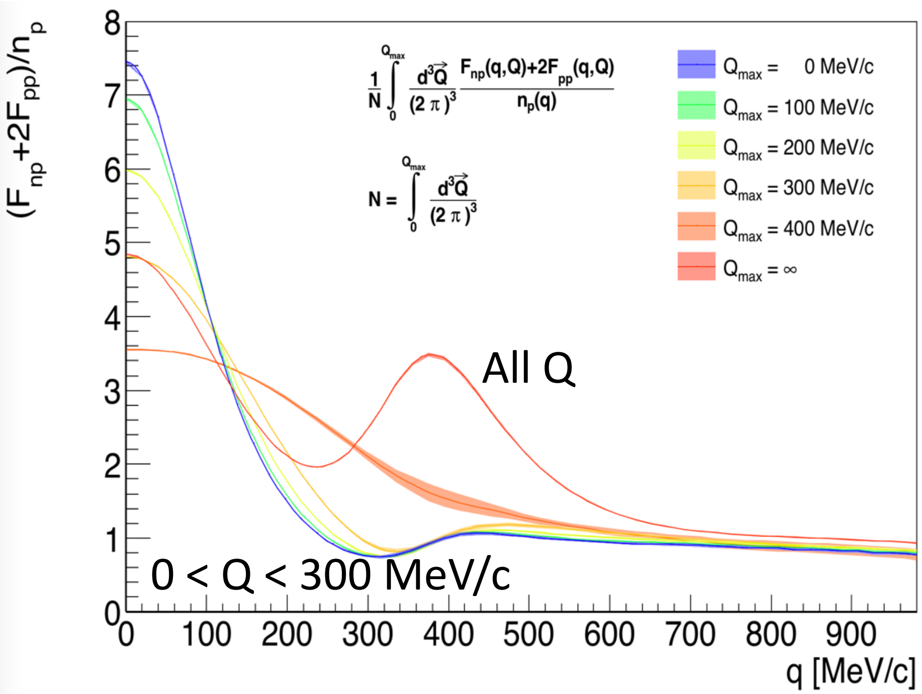


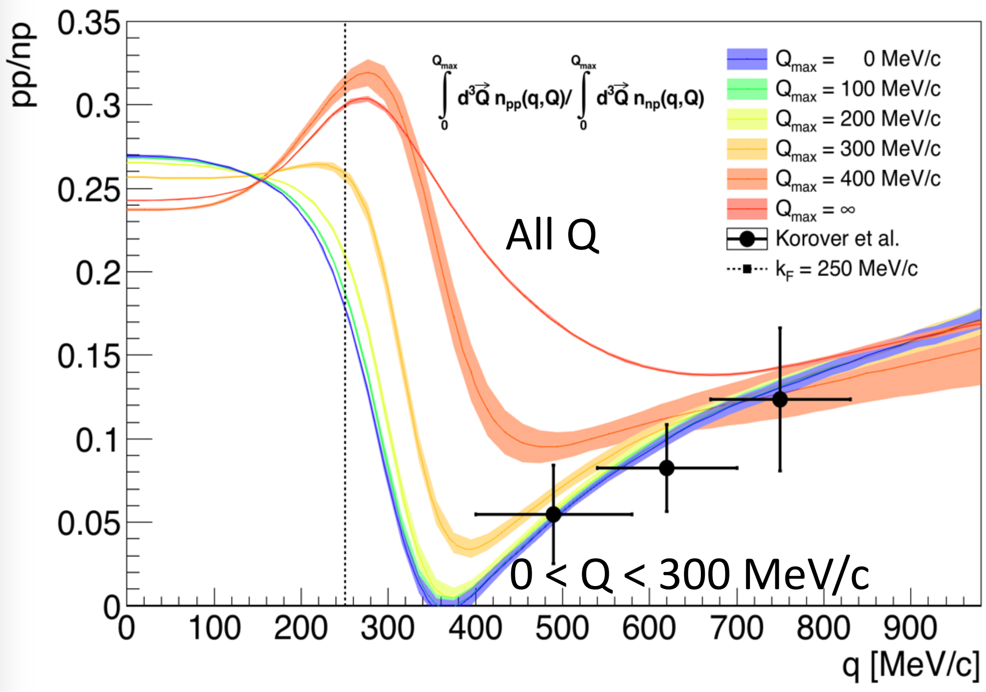
SRC pairs are consistent with Q = 0 *back-to-back* pairs

R. Weiss, **R. Cruz-Torres** et al., In-Preperation (2016)
R. Wiringa et al., Phys. Rev. C 89, 024305 (2014).
T. Neff, H. Feldmeier and W. Horiuchi, Phys. Rev. C 92, 024003 (2015).
I. Korover, N. Muangma, and O. Hen et al., Phys. Rev. Lett 113, 022501 (2014).

Two-Body Scaling for **Low Q**

- Restricting Q=0 restores scaling starting from $k>k_F$ AND gives consistent results with experimental data!





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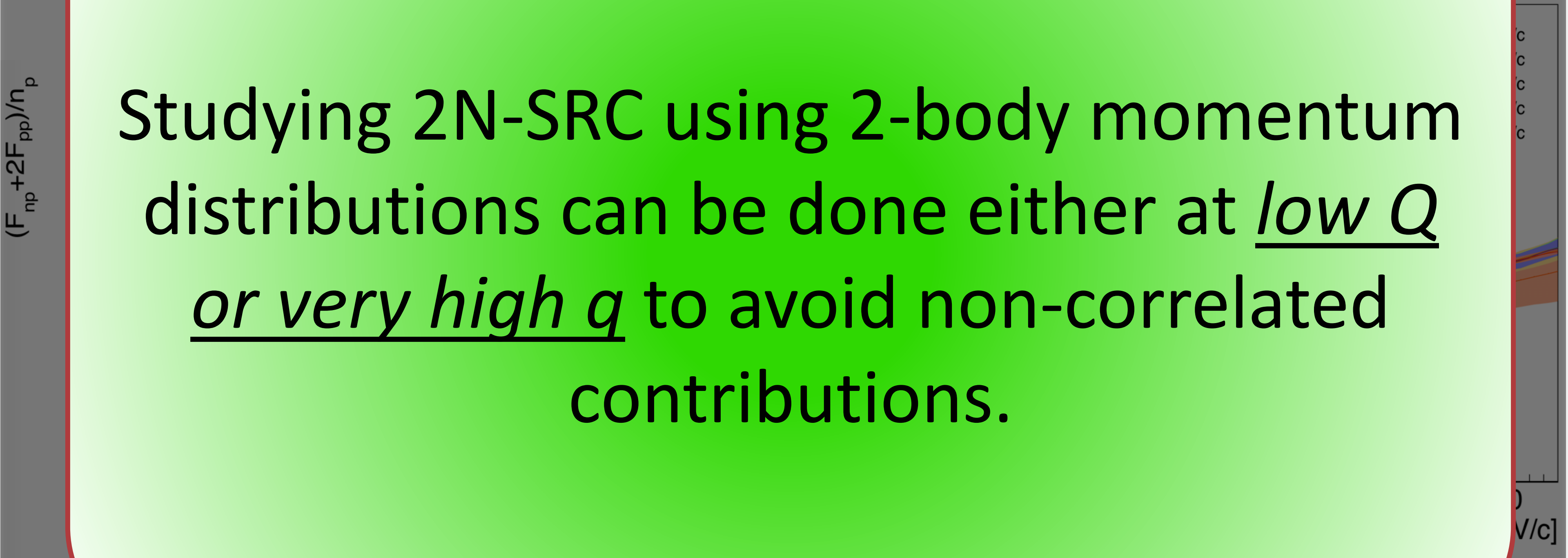
Two-Body Scaling for **Low Q**

- Re



Studying 2N-SRC using 2-body momentum distributions can be done either at *low Q* *or very high q* to avoid non-correlated contributions.

with
$Q \leq k_F$ *back-to-back* pairs

R. Weiss, **R. Cruz-Torres** et al., In-Preperation (2016)
R. Wiringa et al., Phys. Rev. C 89, 024305 (2014).
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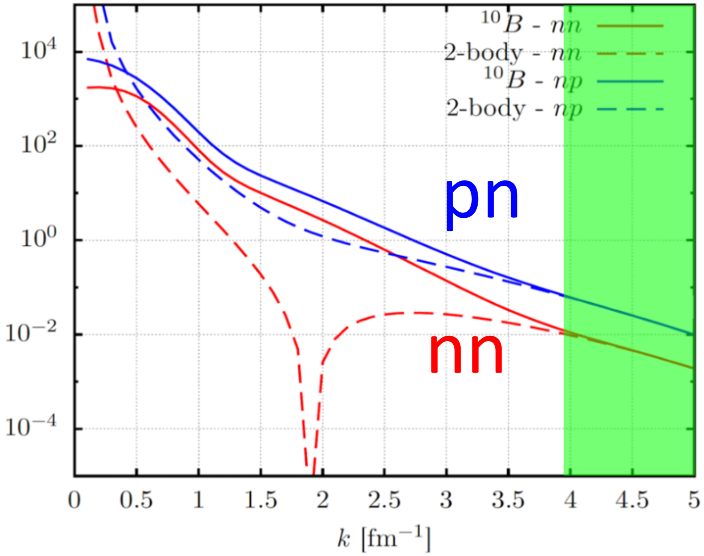
Extracting the nuclear contact(s)

Extracting the Contacts



2-Body momentum distributions

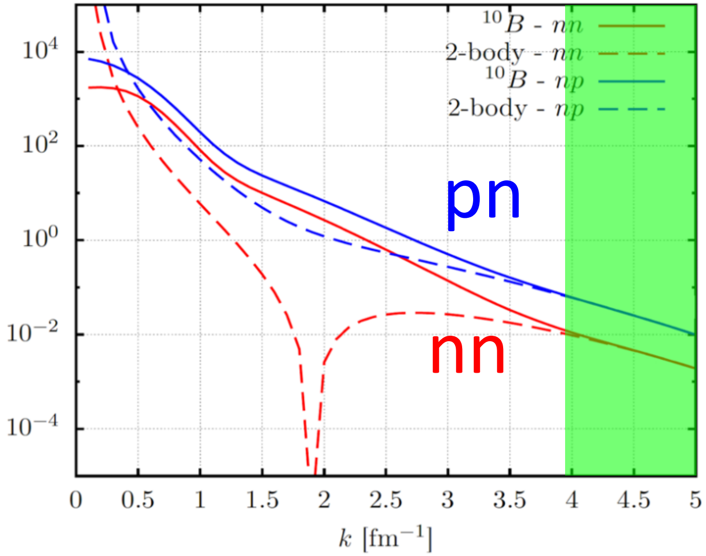
$$F_{pn}(k) \xrightarrow[k\to\infty]{} |\varphi^0_{pn}(k)|^2 C^0_{pn} + |\varphi^d_{pn}(k)|^2 C^d_{pn}$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} |\varphi^0_{nn}(k)|^2 C^0_{nn}$$

Fitting range ~ 4-5 fm^{-1}

R. B. Wiringa *et al.*, PRC 89, no. 2, 024305 (2014).

Extracting the Contacts



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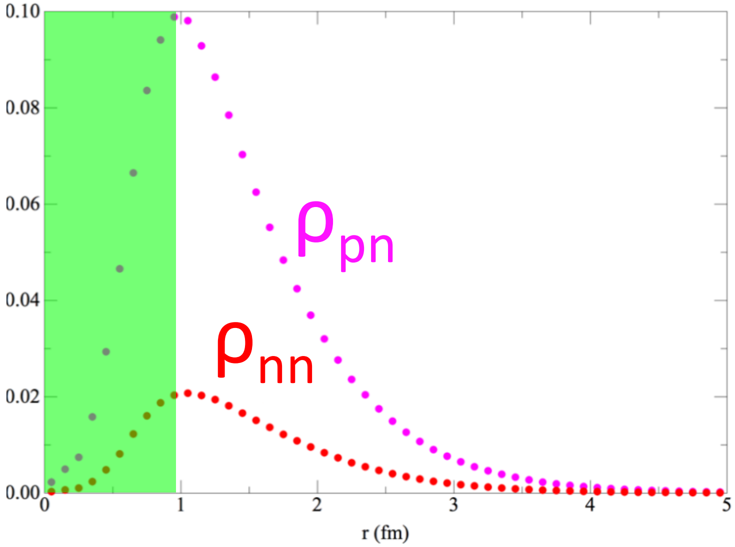


2-Body density distributions

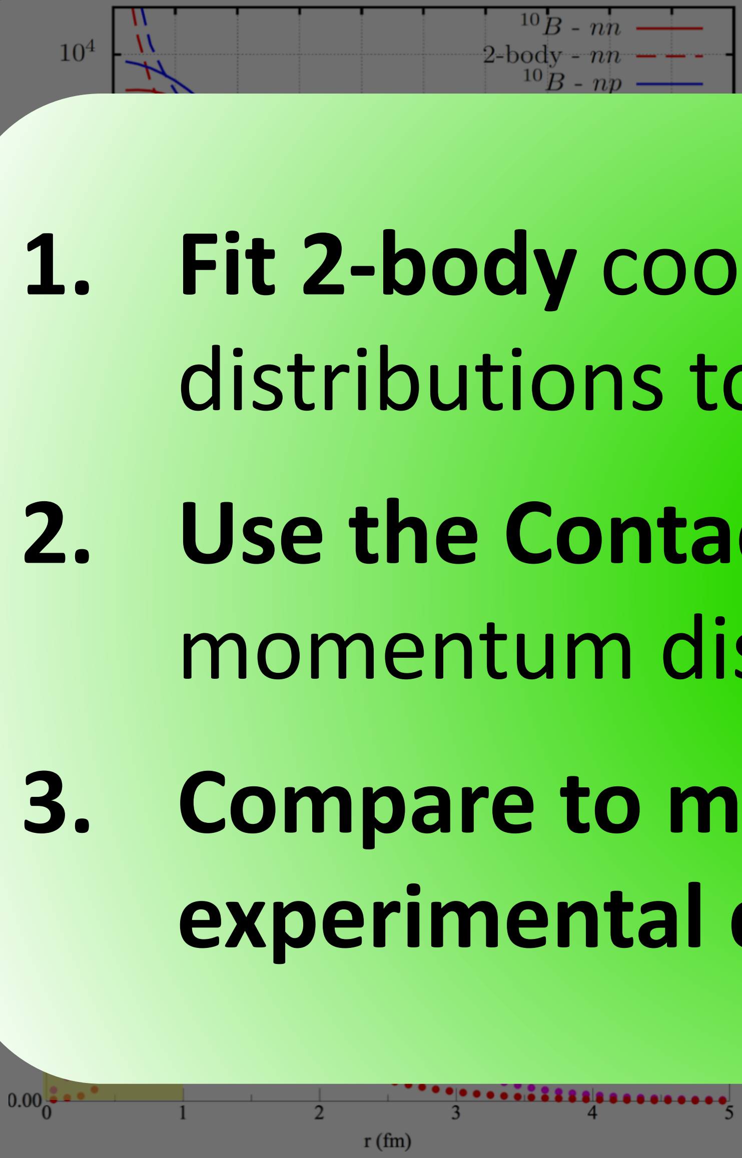
$$\rho_{pn}(r) \xrightarrow[r\to 0]{} \left|\varphi^0_{pn}(r)\right|^2 C^0_{pn} + \left|\varphi^d_{pn}(r)\right|^2 C^d_{pn}$$

$$\rho_{nn}(r) \xrightarrow[r\to 0]{} \left|\varphi^0_{nn}(r)\right|^2 C^0_{nn}$$

Fitting range ~ 0.25-1 fm

R. B. Wiringa *et al.*, PRC 89, no. 2, 024305 (2014).

Extracting the Contacts



2-Body momentum distributions

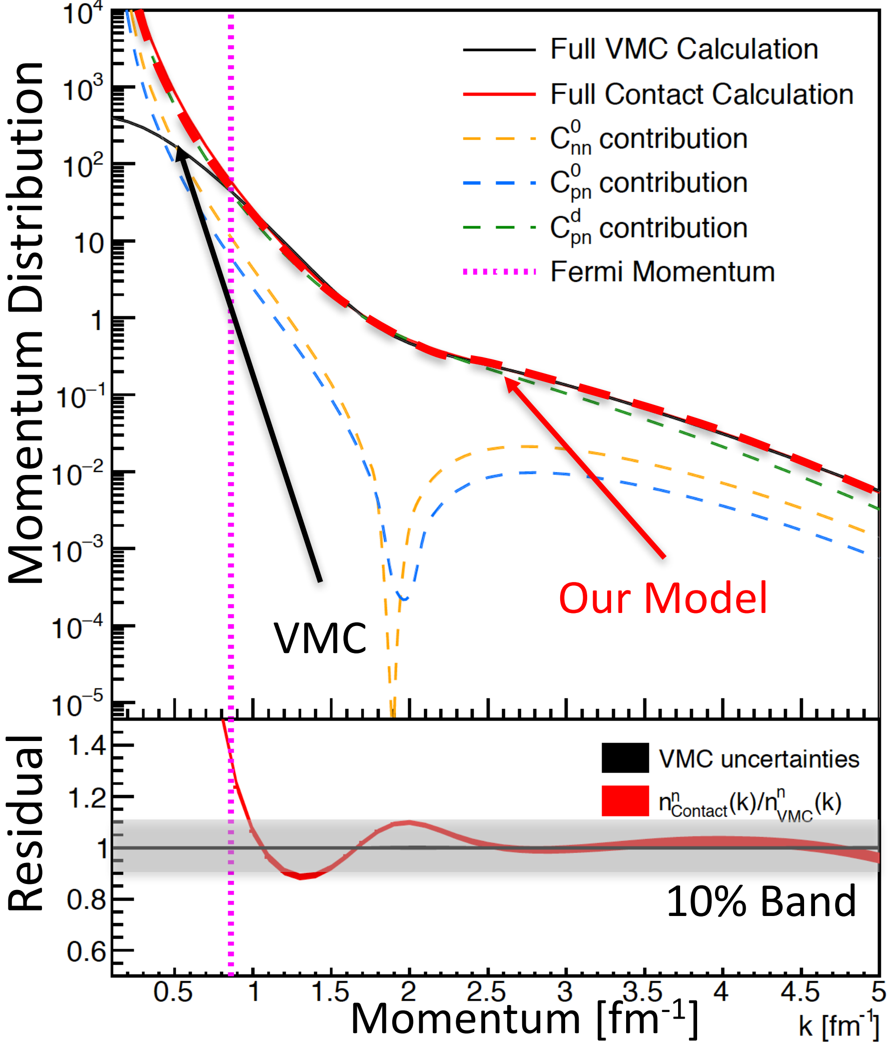
1. **Fit 2-body** coordinate and/or momentum distributions to **determine the Contacts**.
2. **Use the Contacts** to **calculate 1-body** momentum distributions.
3. **Compare to many-body calculations / experimental data**

Fitting range ~ 0.25-1 fm

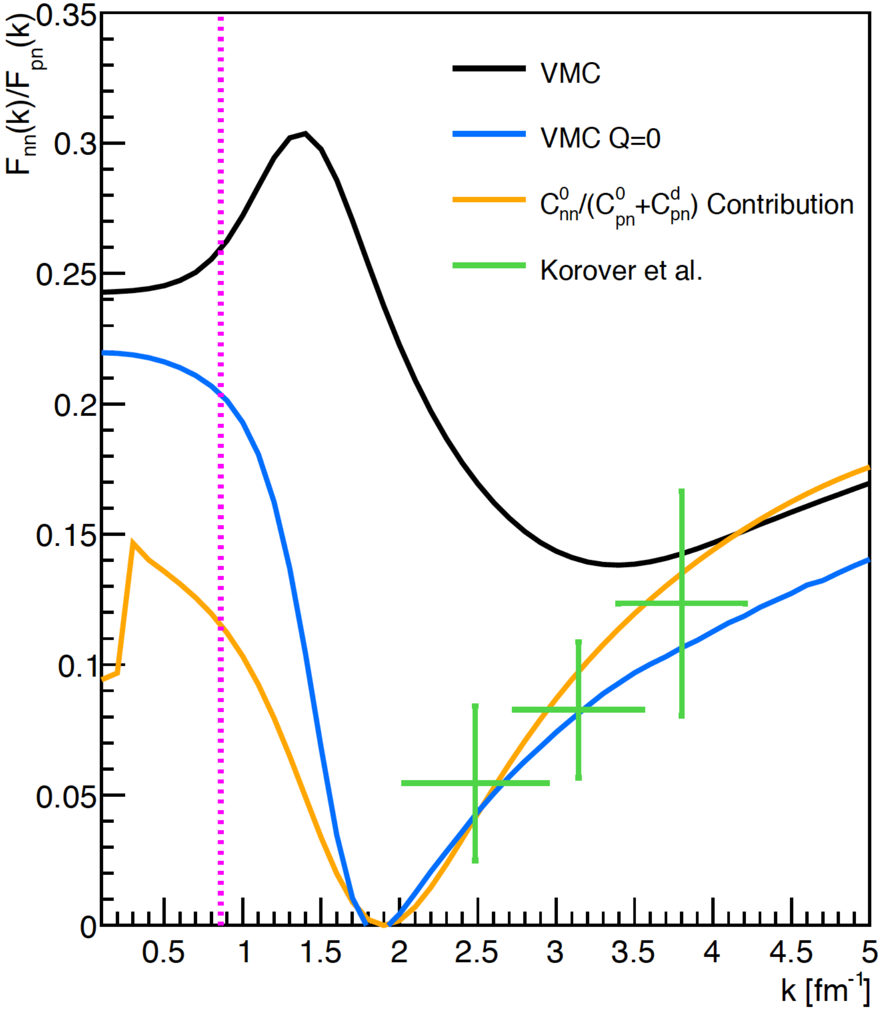
R. B. Wiringa *et al.*, PRC 89, no. 2, 024305 (2014).

^{4}He Results

1-body Momentum dist.

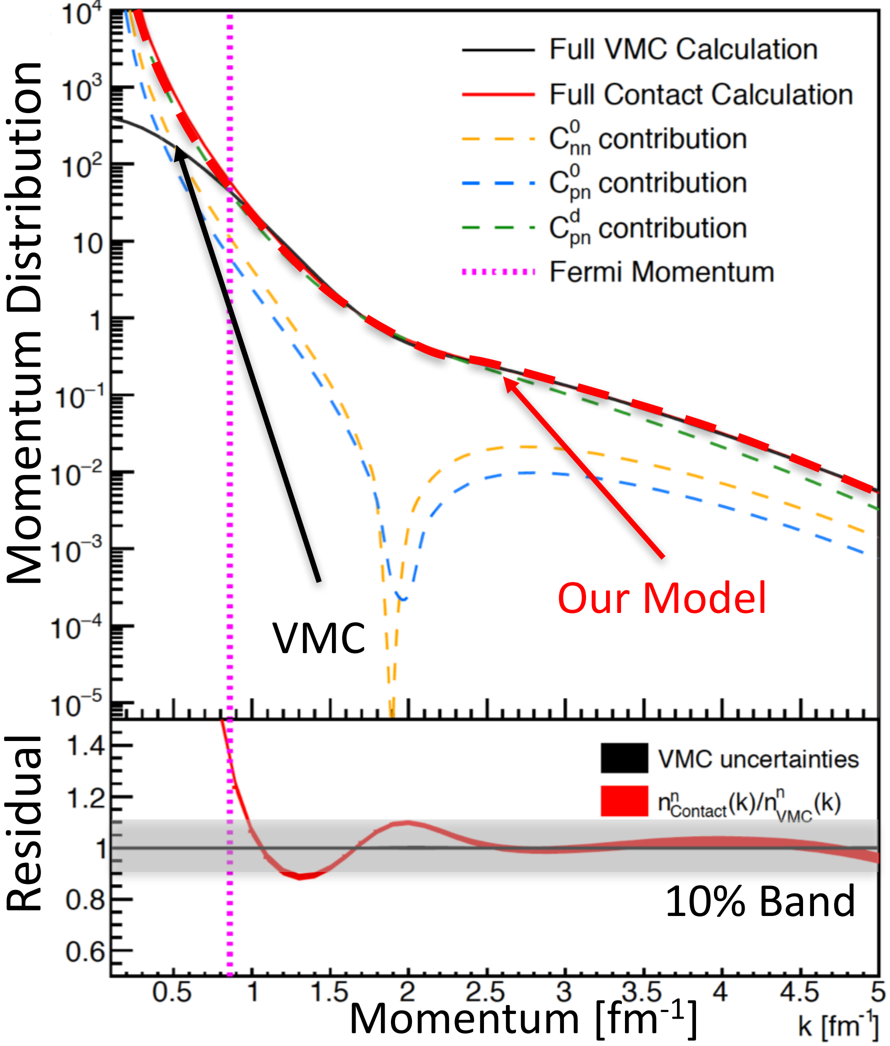


pp / np ratio

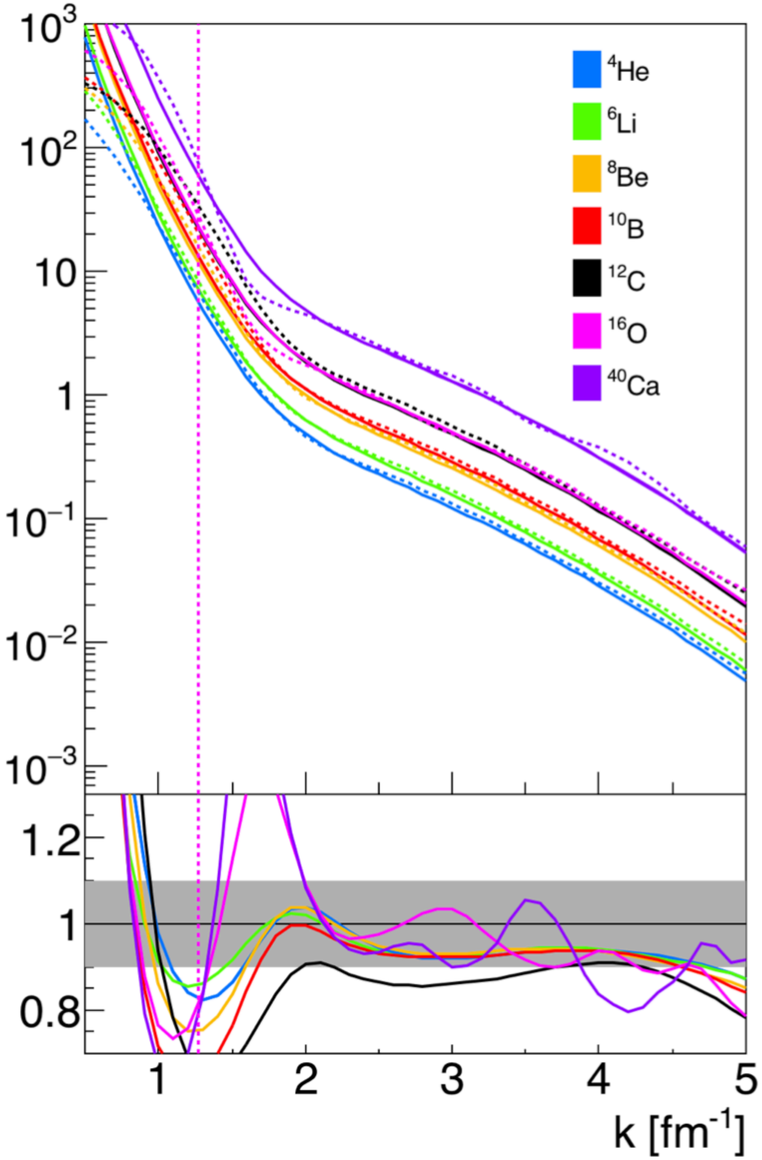


Works For ALL Nuclei!

1-body Momentum dist.



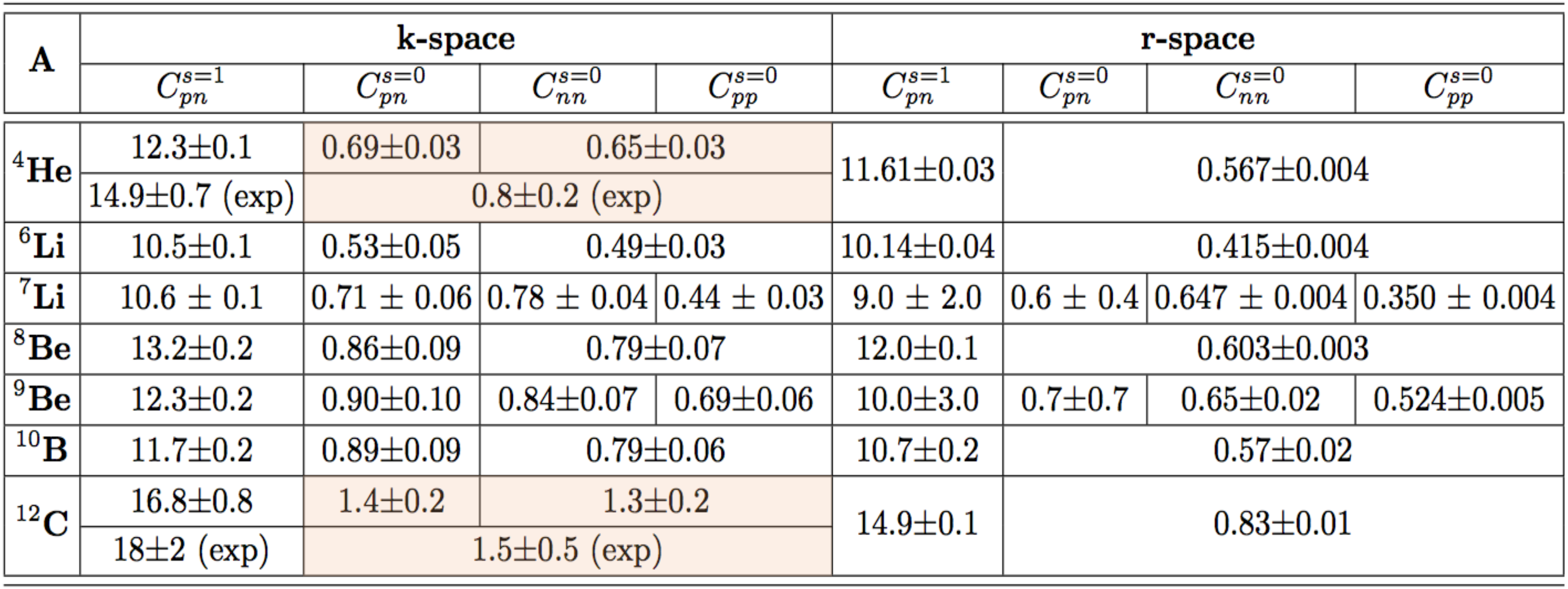
Works for all nuclei!



Universal Nuclear Structure!

$$n_p(k) = \sum_{\alpha} \left|\tilde{\varphi}^{\alpha}_{pp}(k)\right|^2 2C^{\alpha}_{pp} + \sum_{\alpha} \left|\tilde{\varphi}^{\alpha}_{pn}(k)\right|^2 C^{\alpha}_{pn}$$

Nuclear contacts extracted from many-body densities in k- and r-space and from experiment



A	k-space				r-space			
	$C^{s=1}_{pn}$	$C^{s=0}_{pn}$	$C^{s=0}_{nn}$	$C^{s=0}_{pp}$	$C^{s=1}_{pn}$	$C^{s=0}_{pn}$	$C^{s=0}_{nn}$	$C^{s=0}_{pp}$
^{4}He	12.3±0.1	0.69±0.03	0.65±0.03		11.61±0.03	0.567±0.004		
	14.9±0.7 (exp)	0.8±0.2 (exp)						
^{6}Li	10.5±0.1	0.53±0.05	0.49±0.03		10.14±0.04	0.415±0.004		
^{7}Li	10.6 ± 0.1	0.71 ± 0.06	0.78 ± 0.04	0.44 ± 0.03	9.0 ± 2.0	0.6 ± 0.4	0.647 ± 0.004	0.350 ± 0.004
^{8}Be	13.2±0.2	0.86±0.09	0.79±0.07		12.0±0.1	0.603±0.003		
^{9}Be	12.3±0.2	0.90±0.10	0.84±0.07	0.69±0.06	10.0±3.0	0.7±0.7	0.65±0.02	0.524±0.005
^{10}B	11.7±0.2	0.89±0.09	0.79±0.06		10.7±0.2	0.57±0.02		
^{12}C	16.8±0.8	1.4±0.2	1.3±0.2		14.9±0.1	0.83±0.01		
	18±2 (exp)	1.5±0.5 (exp)						

Weiss, Cruz-Torres, Barnea, Piasetzky and Hen, arXiv 1612.00923 (2017)

=> Pair counting constrains EMC calculations, even in EFT approaches! (More Tomorrow)

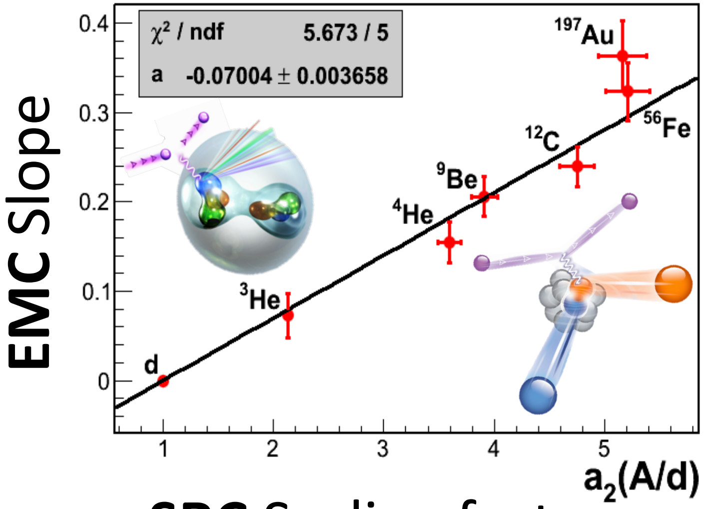
EFT description of bound nucleon structure:

$$F_2^A(x,Q^2)/A = F_2^N(x,Q^2) + g_2(A,\Lambda) f_2(x,Q^2,\Lambda).$$

$$g_2(A,\Lambda) = \underbrace{\frac{1}{A}\langle A|\left(N^\dagger N\right)^2|A\rangle_\Lambda}_{\text{SRC contact}}$$

$$a_2(A, x>1) = \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)}$$

[SRC Scaling Factor]



SRC Scaling factors

arXiv: 1607.03065 (2016)