HUGS

Introduction to QCD

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Lecture one/two

Theory Center

Jefferson Lab
EXPLORING THE NATURE OF MATTER

The plan for my six lectures

- **The Goal:**

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- **The Plan (approximately):**

  From hadrons to partons, the quarks and gluons in QCD

  Fundamentals of QCD,

  Factorization, Evolution, and

  Elementary hard processes

  Four lectures

  Hadron structures and properties in QCD

  Parton distribution functions (PDFs),

  Transverse momentum dependent PDFs (TMDs),

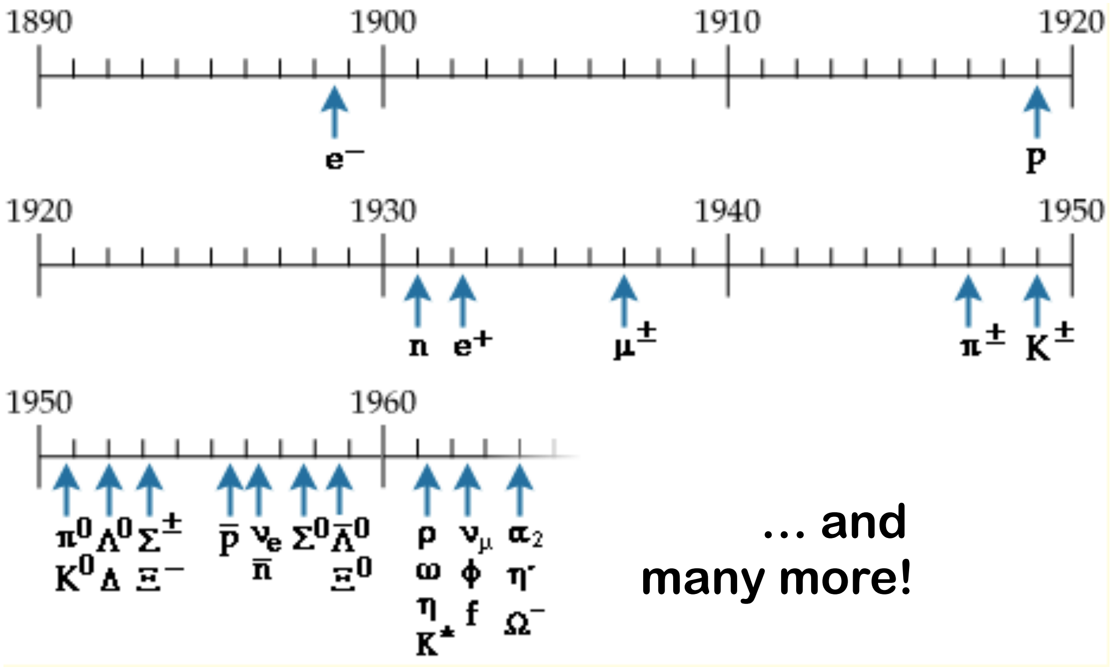
  Generalized PDFs (GPDs), and

  Multi-parton correlation functions

  Two lectures

New particles, new ideas, and new theories

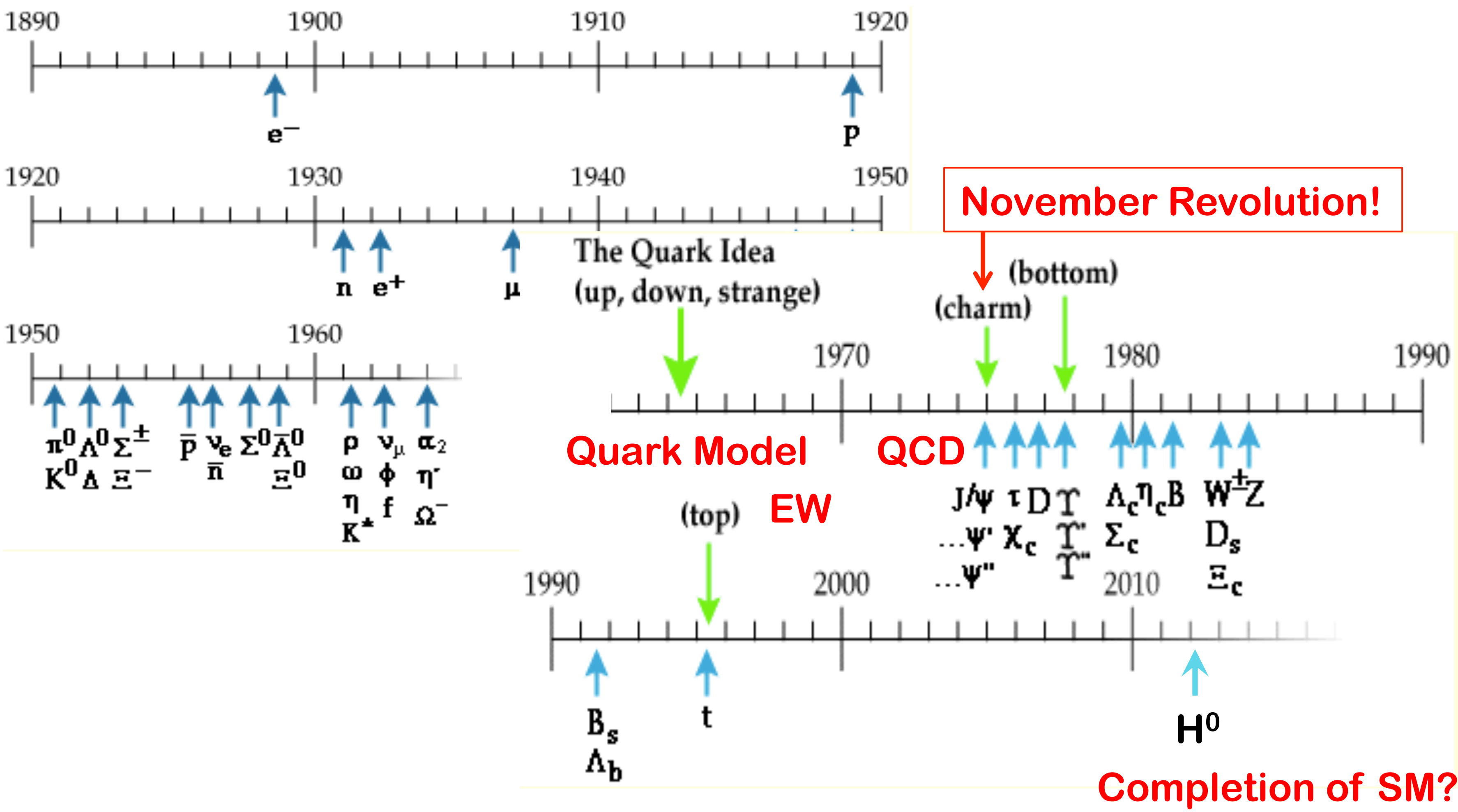
- Early proliferation of new hadrons – "particle explosion":



… and many more!

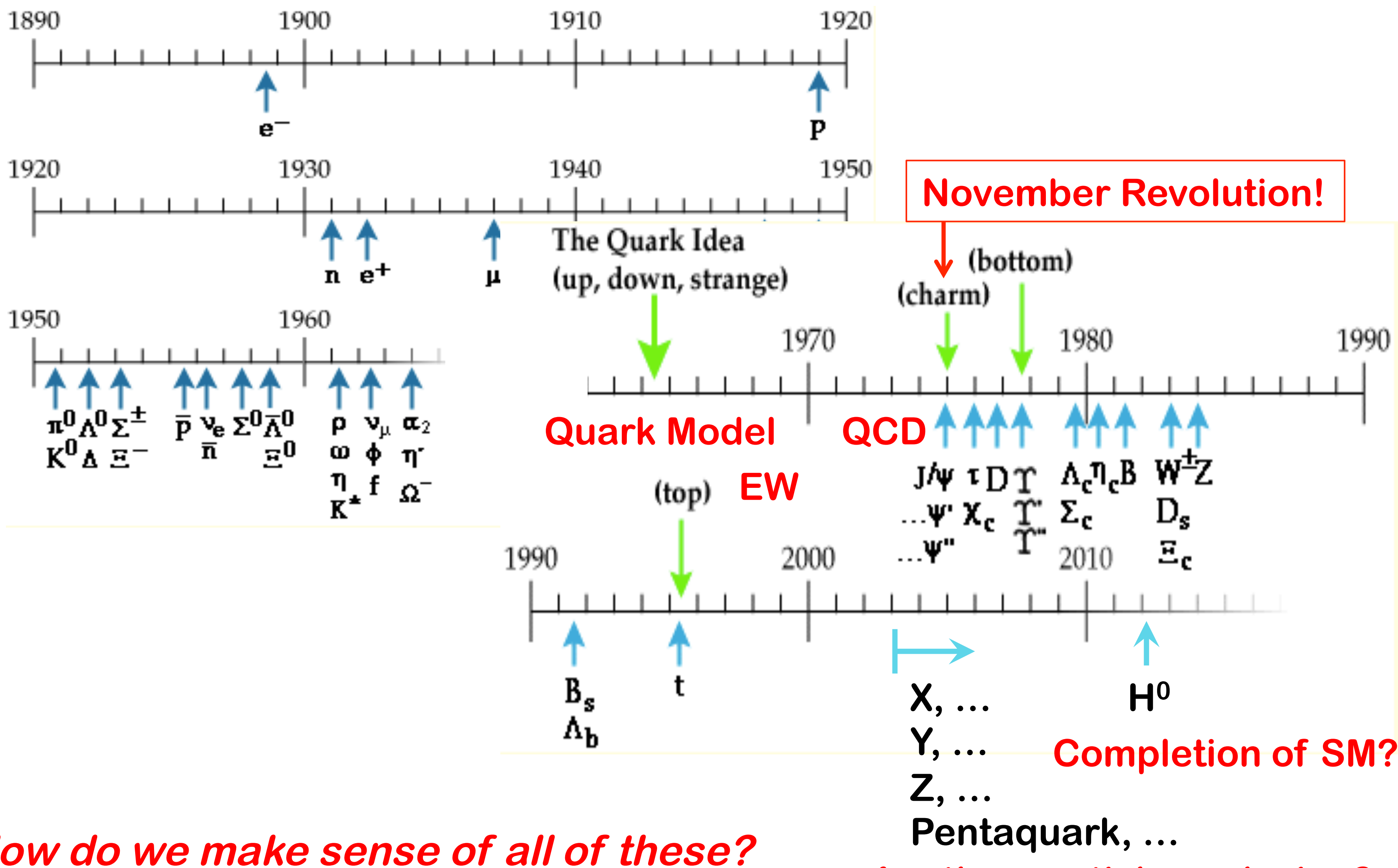
New particles, new ideas, and new theories

❑ Proliferation of new particles – "November Revolution":



New particles, new ideas, and new theories

- Proliferation of new particles – "November Revolution":

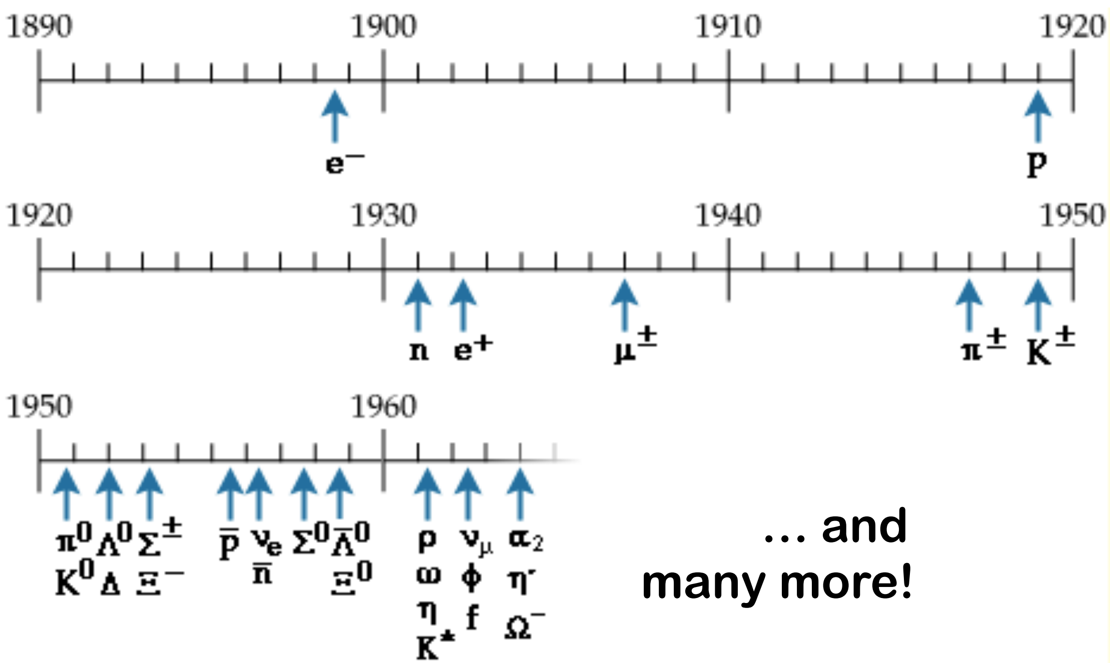


How do we make sense of all of these?

Another particle explosion?

New particles, new ideas, and new theories

- Early proliferation of new hadrons – "particle explosion":



… and
many more!

- Nucleons has internal structure!

1933: Proton's magnetic moment

Otto Stern

Nobel Prize 1943

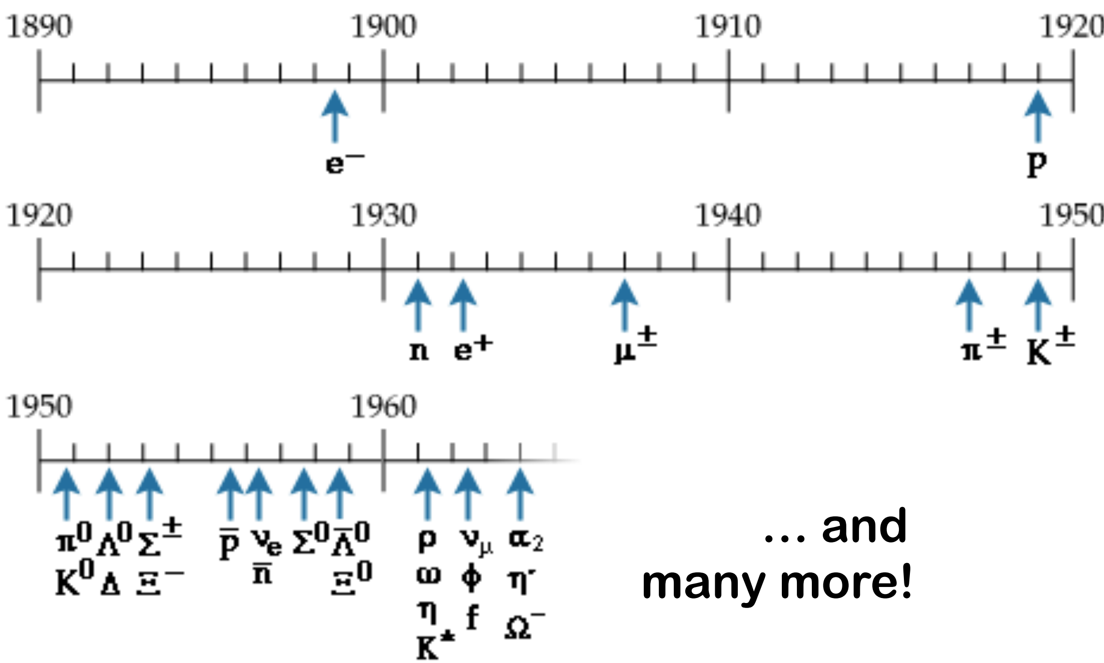
$$\mu_p = g_p \left(\frac{e\hbar}{2m_p} \right)$$

$$g_p = 2.792847356(23) \neq 2!$$

$$\mu_n = -1.913 \left(\frac{e\hbar}{2m_p} \right) \neq 0!$$

New particles, new ideas, and new theories

- **Early proliferation of new hadrons – "particle explosion":**



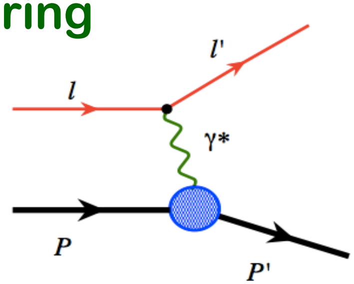
… and
many more!

- **Nucleons has internal structure!**

1960: Elastic e-p scattering

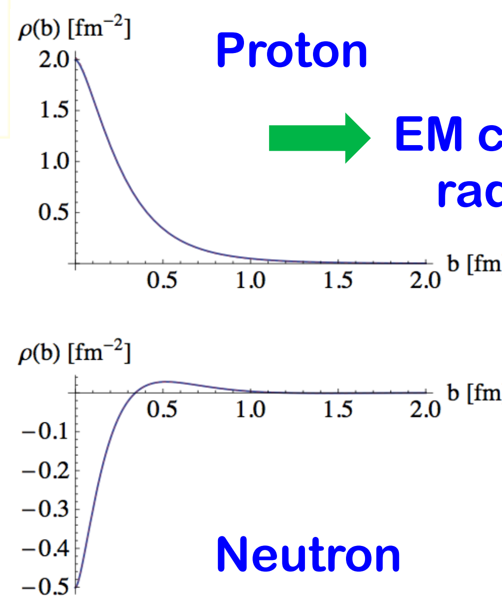
Robert Hofstadter

Nobel Prize 1961



Form factors

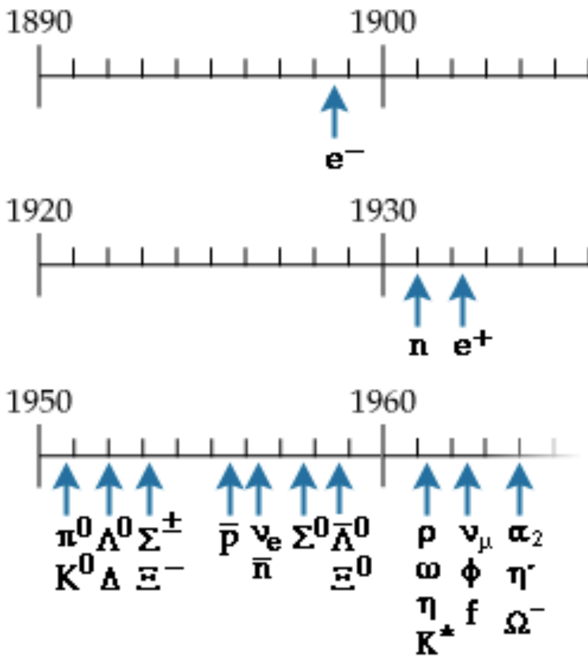
Electric charge distribution



EM charge radius!

New particles, new ideas, and new theories

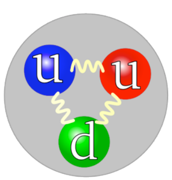
- Early proliferation of new particles – “particle explosion”:



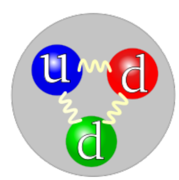
… and
many more!

- Nucleons are made of quarks!

Proton



Neutron



→ **Quark Model**

Murray Gell-Mann

Nobel Prize, 1969

The naïve Quark Model

❑ Flavor SU(3) – assumption:

Physical states for u, d, s, neglecting any mass difference, are represented by 3-eigenstates of the fund'l rep'n of flavor SU(3)

❑ Generators for the fund'l rep'n of SU(3) – 3x3 matrices:

$$J_i = \frac{\lambda_i}{2}$$ with $\lambda_i, i = 1, 2, ..., 8$ Gell-Mann matrices

❑ Good quantum numbers to label the states:

$$J_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad J_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

simultaneously diagonalized

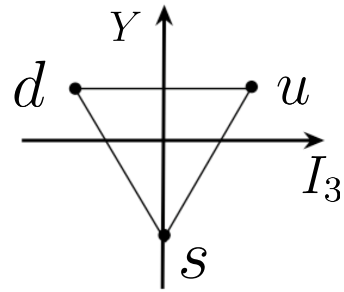
Isospin: $\hat{I}_3 \equiv J_3$, Hypercharge: $\hat{Y} \equiv \frac{2}{\sqrt{3}} J_8$

❑ Basis vectors – Eigenstates: $|I_3, Y\rangle$

$$v^1 \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Longrightarrow u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^2 \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Longrightarrow d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \qquad v^3 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Longrightarrow s = |0,\text{-}\tfrac{2}{3}\rangle$$

The naïve Quark Model

❑ Quark states:



$$u = \left|\tfrac{1}{2}, \tfrac{1}{3}\right\rangle \quad d = \left|\text{-}\tfrac{1}{2}, \tfrac{1}{3}\right\rangle \quad s = \left|0, \text{-}\tfrac{2}{3}\right\rangle$$

Spin: ½

Baryon #: ***B* = ⅓**

Strangeness: ***S* = *Y* – *B*** **Electric charge:** $Q \equiv I_3 + \frac{Y}{2}$

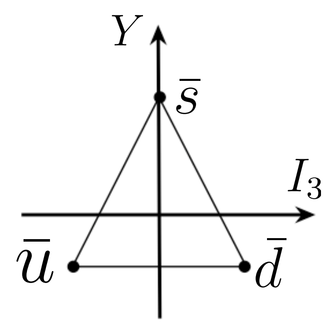
$$u \begin{cases} Q = 2/3\, e \\ s = 1/2 \\ I_3 = 1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad d \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = -1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad s \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = 0 \\ Y = -2/3 \\ B = 1/3 \\ S = -1 \end{cases}$$

❑ Antiquark states: $v_i \equiv \epsilon_{ijk}\, v^j v^k$

$$\hat{I}_3 v_1 = \epsilon_{123}[(\hat{I}_3 v^2)v^3 + v^2(\hat{I}_3 v^3)] + \epsilon_{132}[(\hat{I}_3 v^3)v^2 + v^3(\hat{I}_3 v^2)] = -\frac{1}{2} v_1$$

$$\hat{Y} v_1 = \epsilon_{123}[(\hat{Y} v^2)v^3 + v^2(\hat{Y} v^3)] + \epsilon_{132}[(\hat{Y} v^3)v^2 + v^3(\hat{Y} v^2)] = -\frac{1}{3} v_1$$

$$u \longrightarrow \bar{u} = \left|\text{-}\tfrac{1}{2}, \text{-}\tfrac{1}{3}\right\rangle$$



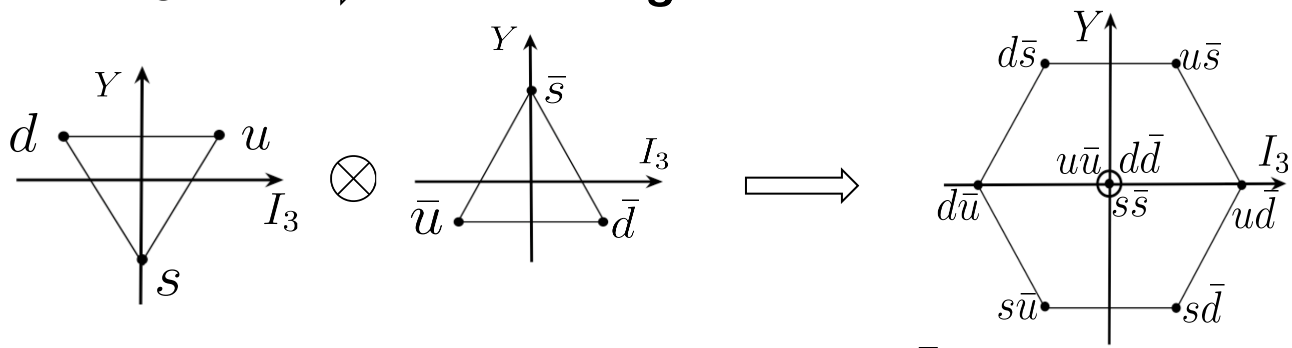
Mesons

Quark-antiquark $q\bar{q}$ flavor states: $B=0$

❑ Group theory says:

$$q(u,d,s)=\mathbf{3},\quad \bar{q}(\bar{u},\bar{d},\bar{s})=\bar{\mathbf{3}},\quad \text{of flavor SU(3)}$$

$\mathbf{3}\otimes\bar{\mathbf{3}}=\mathbf{8}\oplus\mathbf{1}$ $\Longrightarrow$ **1 flavor singlet + 8 flavor octet states**



There are three states with $I_3=0, Y=0$: $u\bar{u}, d\bar{d}, s\bar{s}$

❑ Physical meson states (L=0, S=0):

✧ **Octet states:**

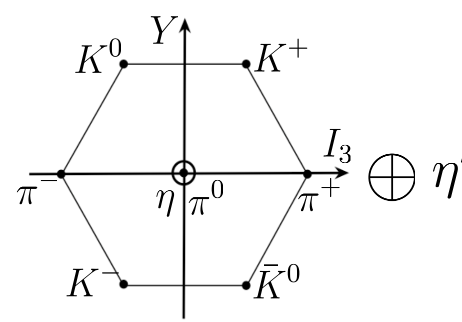
$$A=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) \Longrightarrow \pi^0$$

$$B=\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s}) \Longrightarrow \eta_8$$

✧ **Singlet states:**

$$C=\frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s}) \Longrightarrow \eta_1$$

η_8, η_1 } η, η' $\Longrightarrow$

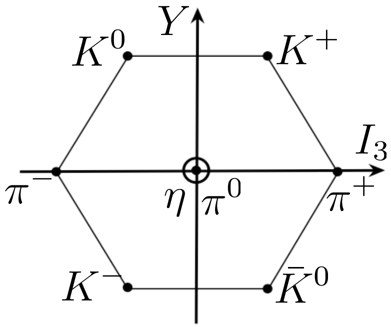


Quantum Numbers

- **Meson states:** J^{PC}
  - **Spin of $q\bar{q}$ pair:** $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \rightarrow S = 0, 1$
  - **Spin of mesons:** $J = S + L$
  - **Parity:** $P = -(-1)^L$
  - **Charge conjugation:** $C = (-1)^{L+S}$

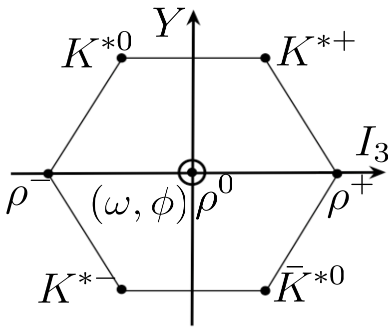
- **L=0 states:**

$J^{PC} = 0^{-+}$: ***(Y=S)***



$\oplus\ \eta'$

$J^{PC} = 1^{--}$: ***(Y=S)***



$\oplus\ \mathrm{mix}(\omega, \phi)$

Flavor singlet, spin octet

Flavor octet, spin octet

- **Color:**

No color was introduced!

Baryons

3 quark qqq states: $B = 1$

❑ Group theory says:

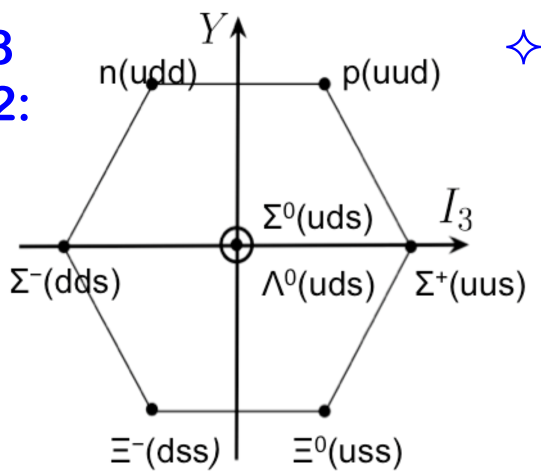
✧ **Flavor:** $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

S: symmetric in all 3 q, M_S : symmetric in 1 and 2,
M_A : antisymmetric in 1 and 2, A : antisymmetric in all 3

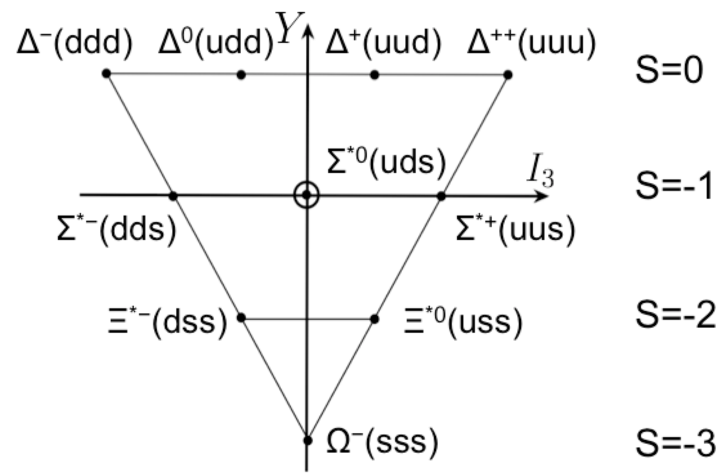
✧ **Spin:** $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4}_S \oplus \mathbf{2}_{M_s} \oplus \mathbf{2}_{M_A} \implies S = \frac{3}{2}, \frac{1}{2}, \frac{1}{2}$

❑ Physical baryon states:

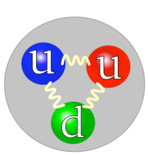
✧ **Flavor-8 Spin-1/2:**



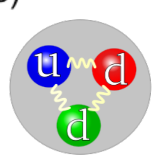
✧ **Flavor-10 Spin-3/2:**



Proton



Neutron



Δ^{++}(uuu), …

Violation of Pauli exclusive principle

➡ ***Need another quantum number - color!***

Color

❑ Minimum requirements:

✧ Quark needs to carry at least 3 different colors

✧ Color part of the 3-quarks' wave function needs to antisymmetric

❑ SU(3) color:

Recall: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

Antisymmetric color singlet state:

$$\Longrightarrow c(\text{Red, Green, Blue})$$

$$\psi_{\text{Color}}(c_1, c_2, c_3) = \frac{1}{\sqrt{6}}[\text{RGB-GRB+RBG-BRG+GBR-BGR}]$$

❑ Baryon wave function:

$$\Psi(q_1, q_2, q_3) = \psi_{\text{Space}}(x_1, x_2, x_3) \otimes \psi_{\text{Flavor}}(f_1, f_2, f_3) \otimes \psi_{\text{Spin}}(s_1, s_2, s_3) \otimes \psi_{\text{Color}}(c_1, c_2, c_3)$$

Antisymmetric | Symmetric | Symmetric | Symmetric | Antisymmetric

A complete example: Proton

- **Wave function – the state:**

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow -2\uparrow\uparrow\downarrow) + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow -2\uparrow\downarrow\uparrow) + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow -2\downarrow\uparrow\uparrow)\right]$$

- **Normalization:**

$$\langle p\uparrow|p\uparrow\rangle = \frac{1}{18}[(1+1+(-2)^2)+(1+1+(-2)^2)+(1+1+(-2)^2)] = 1$$

- **Charge:**

$$\hat{Q} = \sum_{i=1}^{3}\hat{Q}_i$$

$$\langle p\uparrow|\hat{Q}|p\uparrow\rangle = \frac{1}{18}[(\tfrac{2}{3}+\tfrac{2}{3}-\tfrac{1}{3})(1+1+(-2)^2)+(\tfrac{2}{3}-\tfrac{1}{3}+\tfrac{2}{3})(1+1+(-2)^2) +(-\tfrac{1}{3}+\tfrac{2}{3}+\tfrac{2}{3})(1+1+(-2)^2)] = 1$$

- **Spin:**

$$\hat{S} = \sum_{i=1}^{3}\hat{s}_i$$

$$\langle p\uparrow|\hat{S}|p\uparrow\rangle = \frac{1}{18}\{[(\tfrac{1}{2}-\tfrac{1}{2}+\tfrac{1}{2})+(-\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2})+4(\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2})] +[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]+[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]\} = \frac{1}{2}$$

- **Magnetic moment:**

$$\mu_p = \langle p\uparrow|\sum_{i=1}^{3}\hat{\mu}_i(\hat{\sigma}_3)_i|p\uparrow\rangle = \frac{1}{3}[4\mu_u-\mu_d]$$

$$\mu_n = \frac{1}{3}[4\mu_d-\mu_u] \qquad \frac{\mu_u}{\mu_d} \approx \frac{2/3}{-1/3} = -2$$

$\Rightarrow$

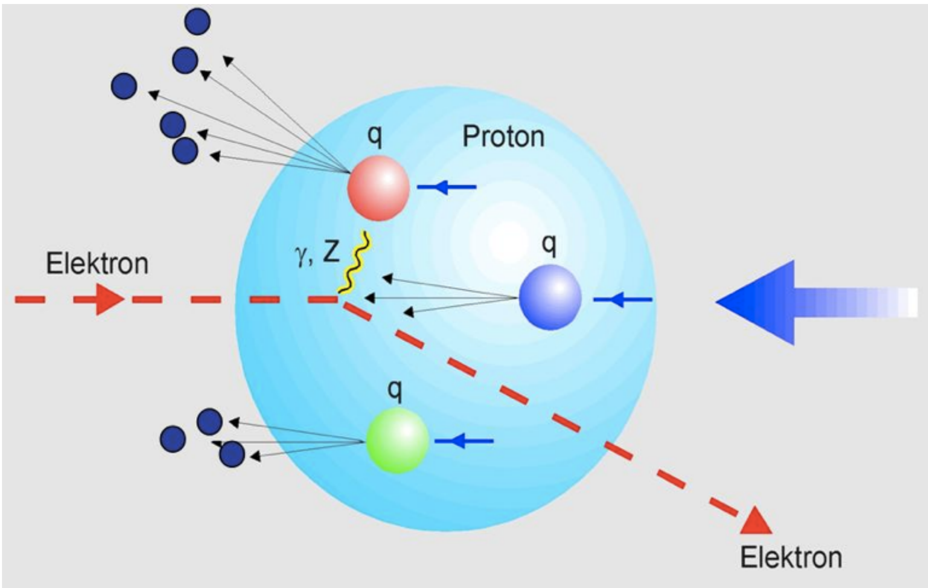
$$\left(\frac{\mu_n}{\mu_p}\right)_{\rm QM} = -\frac{2}{3}$$

$$\left(\frac{\mu_n}{\mu_p}\right)_{\rm Exp} = -0.68497945(58)$$

How to "see" substructure of a nucleon?

❑ Modern Rutherford experiment – Deep Inelastic Scattering:

SLAC 1968: $e(p) + h(P) \to e'(p') + X$



✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \text{fm}$$

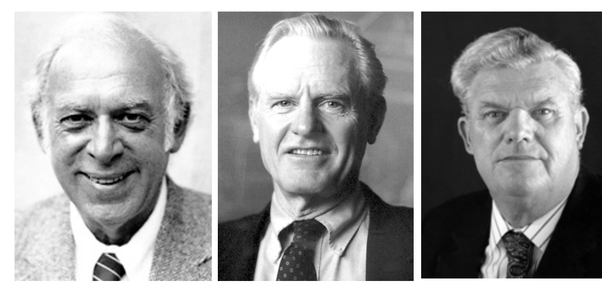
✧ **Two variables:**

$$Q^2 = 4EE' \sin^2(\theta/2)$$

$$x_B = \frac{Q^2}{2m_N \nu}$$

$$\nu = E - E'$$

→ **Discovery of spin ½ quarks, and partonic structure!**



Nobel Prize, 1990

What holds the quarks together?

⇒ **The birth of QCD (1973)**

– Quark Model + Yang-Mill gauge theory

Quantum Chromo-dynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:**

$\psi_i^f(x)$ **Quark fields: spin-½ Dirac fermion (like electron)**

Color triplet: $i = 1, 2, 3 = N_c$

Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$ **Gluon fields: spin-1 vector field (like photon)**

Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - g A_{\mu,a} (t_a)_{ij}) \gamma^\mu - m_f \delta_{ij} \right] \psi_j^f$$
$$- \frac{1}{4} \left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - g C_{abc} A_{\mu,b} A_{\nu,c} \right]^2$$
$$+ \text{ gauge fixing} + \text{ghost terms}$$

- **QED – force to hold atoms together:**

$$\mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}^f \left[(i\partial_\mu - e A_\mu)\gamma^\mu - m_f \right] \psi^f - \frac{1}{4} \left[\partial_\mu A_\nu - \partial_\nu A_\mu \right]^2$$

QCD is much richer in dynamics than QED

Gluons are dark, but, interact with themselves, NO free quarks and gluons

Gauge property of QCD

- **Gauge Invariance:**

$$\psi_i(x) \to \psi'_j(x) = U(x)_{ji}\,\psi_i(x)$$

$$A_\mu(x) \to A'_\mu(x) = U(x)\,A_\mu(x)\,U^{-1}(x) + \frac{i}{g}\,[\partial_\mu U(x)]\,U^{-1}(x)$$

where $A_\mu(x)_{ij} \equiv A_{\mu,a}(x)(t_a)_{ij}$

$$U(x)_{ij} = \left[e^{i\,\alpha_a(x)\,t_a}\right]_{ij} \quad \text{Unitary} \quad [\,\text{det=1, SU(3)}]$$

- **Color matrices:**

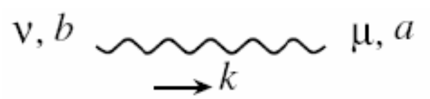
$$[\,t_a, t_b\,] = i\,C_{abc}\,t_c$$

Generators for the fundamental representation of SU3 color

- **Gauge Fixing:**

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}(\partial_\mu A_a^\mu)(\partial_\nu A_a^\nu)$$

Allow us to define the gauge field propagator:



$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

with $\lambda = 1$ **the Feynman gauge**

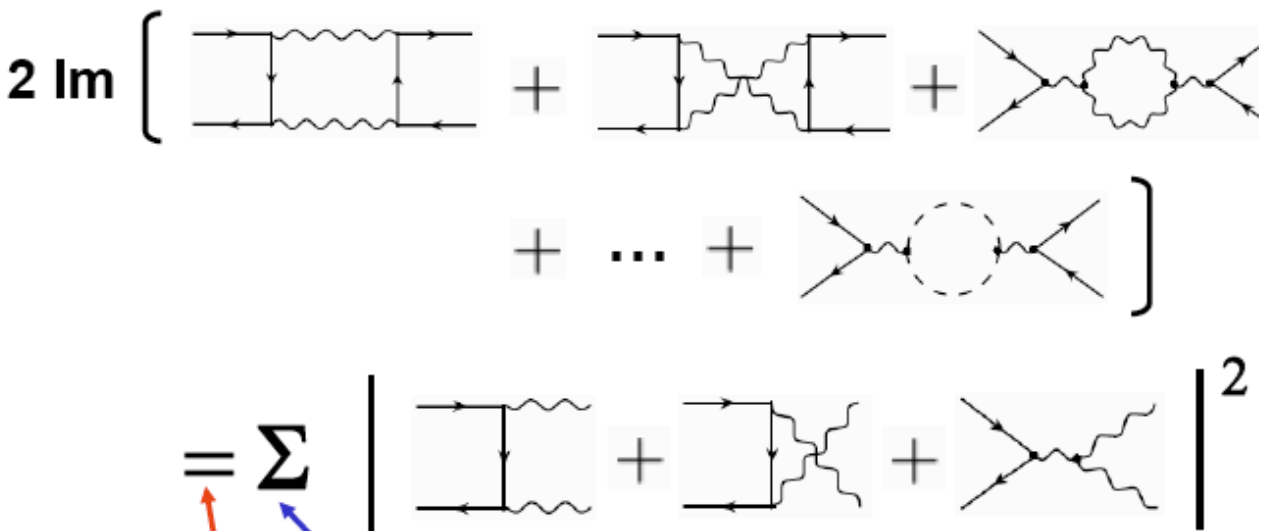
Ghost in QCD

- **Ghost:**

Ghost

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a(x))(\partial^\mu \eta_a(x) - g\, C_{abc}\, A_b^\mu(x)\, \eta_c(x))$$

so that the optical theorem (hence the unitarity) can be respected



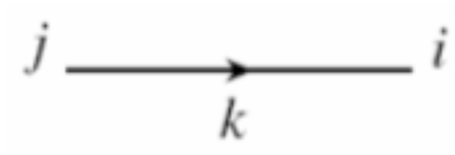
Sum over all physical polarizations

Fail without the ghost loop

Feynman rules in QCD

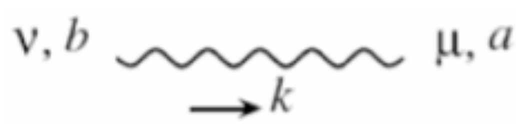
❑ Propagators:

Quark:



$$\frac{i}{\gamma \cdot k - m}\delta_{ij}$$

Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

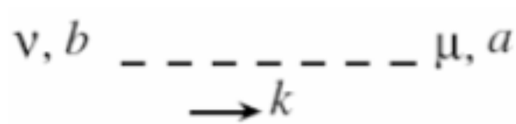
for a covariant gauge

$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n}\right]$$

for a light-cone gauge

$$n \cdot A(x) = 0 \quad \text{with} \quad n^2 = 0$$

Ghost::

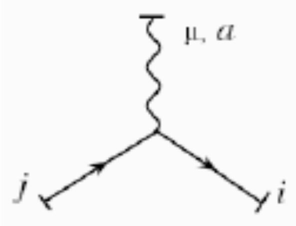


$$\frac{i\delta_{ab}}{k^2}$$

Feynman rules in QCD

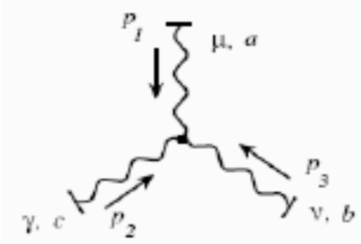
❑ Interactions:

$$-g\bar{\psi}\gamma^{\mu}A_{\mu,a}t_a\psi$$



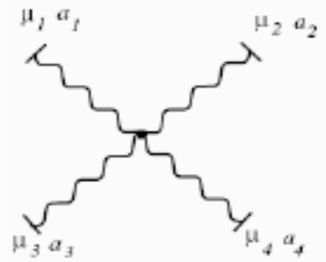
$$-ig(t_a)_{ij}\gamma_{\mu}$$

$$\frac{1}{2}gC_{abc}(\partial_{\mu}A_{\nu,a} - \partial_{\nu}A_{\mu,a})A^{\mu}_{b}A^{\nu}_{c}$$



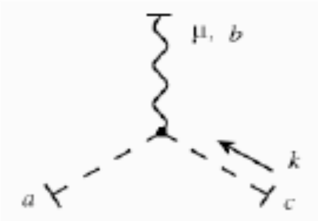
$$-gC_{abc}\left[g_{\mu\nu}(p_1-p_2)_{\gamma} + g_{\nu\gamma}(p_2-p_3)_{\mu} + g_{\gamma\mu}(p_3-p_1)_{\nu}\right]$$

$$-\frac{g^2}{4}C_{abc}C_{ab'c'} * A^{\mu}_{b}A^{\nu}_{c}A_{\mu,b'}A_{\nu,c'}$$



$$-ig^2\left[C_{ea_1a_2}C_{ea_3a_4} * (g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) + \ldots\right]$$

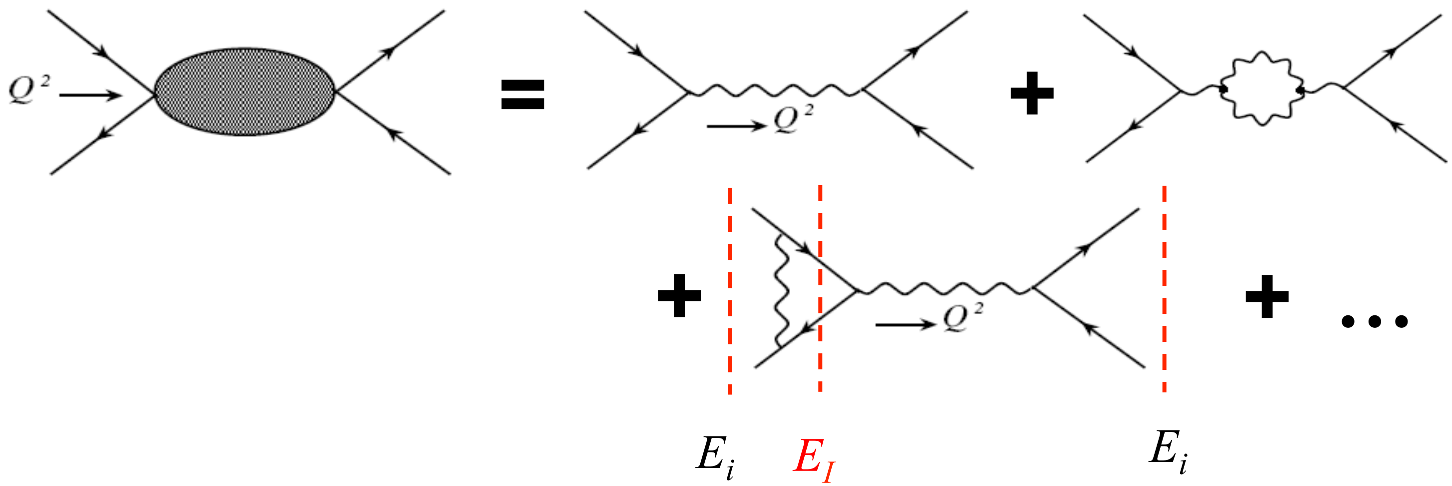
$$\partial_{\mu}\bar{\eta}_a\,(gC_{abc}A^{\mu}_{b})\,\eta_c$$



$$gC_{abc}k_{\mu}$$

Renormalization, why need?

- Scattering amplitude:



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \ldots \right) + \ldots \Rightarrow \infty$$

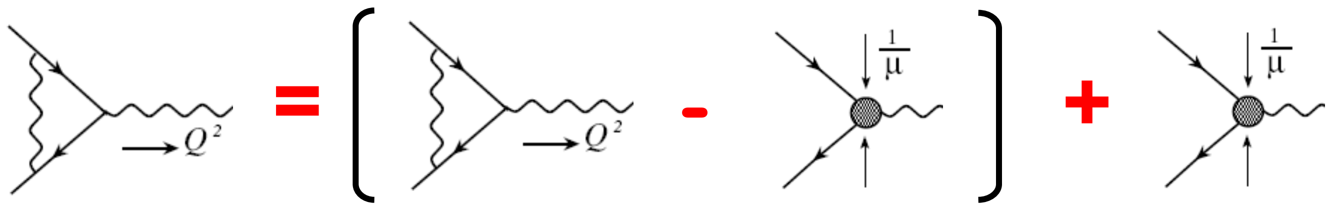
UV divergence: result of a "sum" over states of high masses

Uncertainty principle: High mass states = "Local" interactions

No experiment has an infinite resolution!

Physics of renormalization

- UV divergence due to "high mass" states, not observed

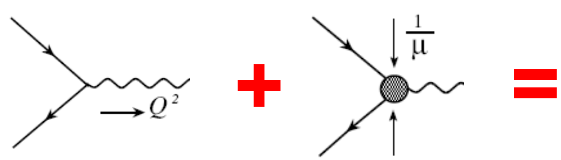


"Low mass" state

"High mass" states

- Combine the "high mass" states with LO

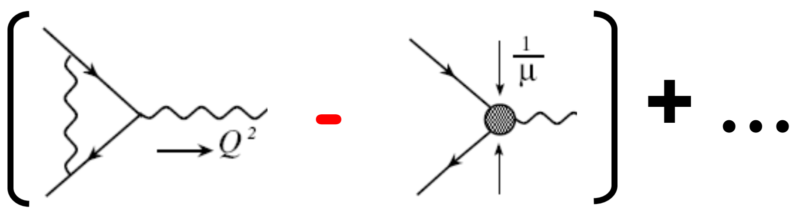
LO:



$g(\mu)$

Renormalized coupling

NLO:



No UV divergence!

- Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantity should not depend on renormalization scale μ ➡ renormalization group equation:

$$\mu^2 \frac{d}{d\mu^2} \sigma_{\text{Phy}} \left(\frac{Q^2}{\mu^2}, g(\mu), \mu \right) = 0 \quad \Longrightarrow \quad \sigma_{\text{Phy}}(Q^2) = \sum_n \hat{\sigma}^{(n)}(Q^2, \mu^2) \left(\frac{\alpha_s(\mu)}{2\pi} \right)^n$$

- Running coupling constant:

$$\mu \frac{\partial g(\mu)}{\partial \mu} = \beta(g) \qquad \alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- QCD β function:

$$\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu} = +g^3 \frac{\beta_1}{16\pi^2} + \mathcal{O}(g^5) \qquad \beta_1 = -\frac{11}{3} N_c + \frac{4}{3} \frac{n_f}{2} < 0 \quad \text{for } n_f \leq 6$$

- QCD running coupling constant:

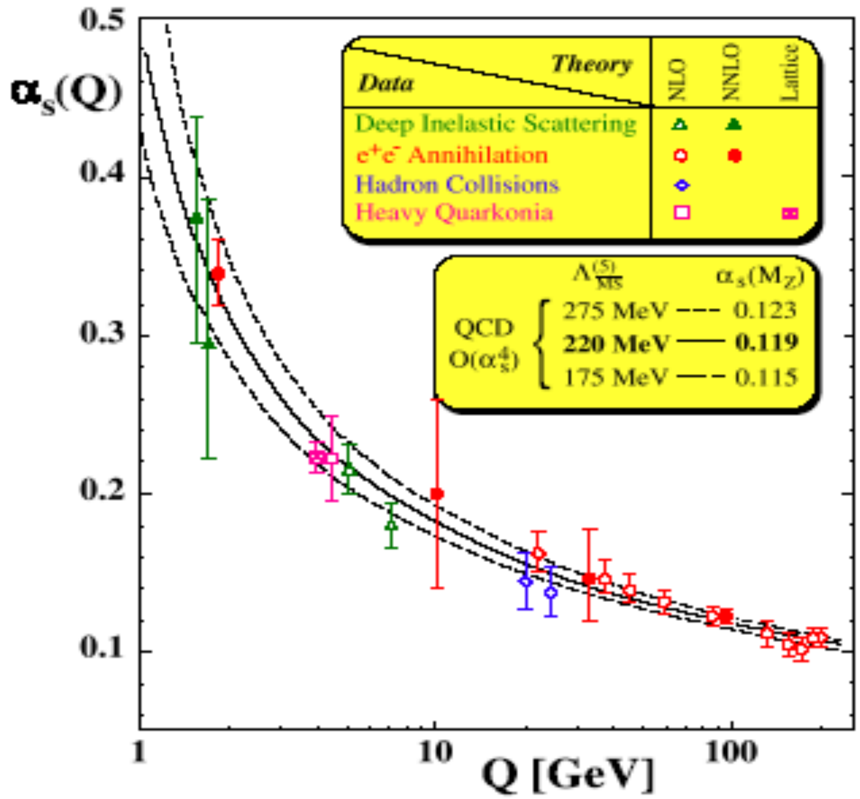
$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi} \alpha_s(\mu_1) \ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \quad \text{as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Asymptotic freedom!

QCD Asymptotic Freedom

❑ **Interaction strength:** $\alpha_s(\mu_2) = \dfrac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \dfrac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{\rm QCD}^2}\right)}$

μ_2 and μ_1 not independent



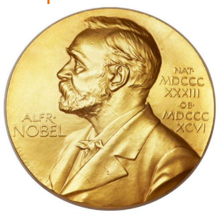
Asymptotic Freedom ⇔ antiscreening

$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)



Nobel Prize, 2004

→ **Discovery of QCD Asymptotic Freedom**

➡ **Collider phenomenology**
– *Controllable perturbative QCD calculations*

Effective Quark Mass

- **Ru2nning quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since } \gamma_m(g(\lambda)) > 0$$

- **Choice of renormalization scale:**

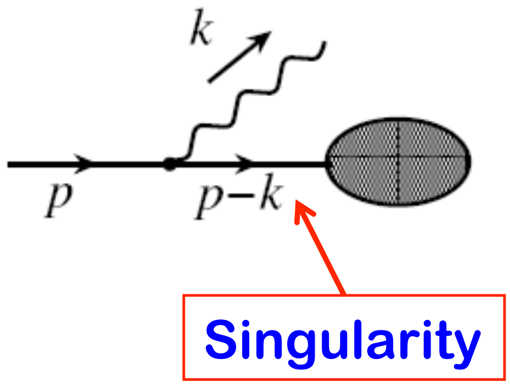
$\mu \sim Q$ **for small logarithms in the perturbative coefficients**

- **Light quark mass:** $m_f(\mu) \ll \Lambda_{\rm QCD}$ for $f = u, d,$ even s

QCD perturbation theory (Q>> $\Lambda_{\rm QCD}$)
is effectively a massless theory

Infrared and collinear divergences

- **Consider a general diagram:**



$p^2 = 0, \quad k^2 = 0$ **for a massless theory**

✧ $k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$

$\Longrightarrow$ **Infrared (IR) divergence**

✧ $k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1$

$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2 \, p^2 = 0$

$\Longrightarrow$ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

Infrared Safety

- Infrared safety:

$$\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) + \mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = $\kappa > 0$

Asymptotic freedom is useful only for quantities that are infrared safe

Foundation of perturbative QCD

- ❑ Renormalization
  - – QCD is renormalizable

Nobel Prize, 1999
't Hooft, Veltman

- ❑ Asymptotic freedom
  - – weaker interaction at a shorter distance

Nobel Prize, 2004
Gross, Politzer, Welczek

- ❑ Infrared safety and factorization
  - – calculable short distance dynamics
  - – pQCD factorization – connect the partons to physical cross sections

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe and factorizable observables!

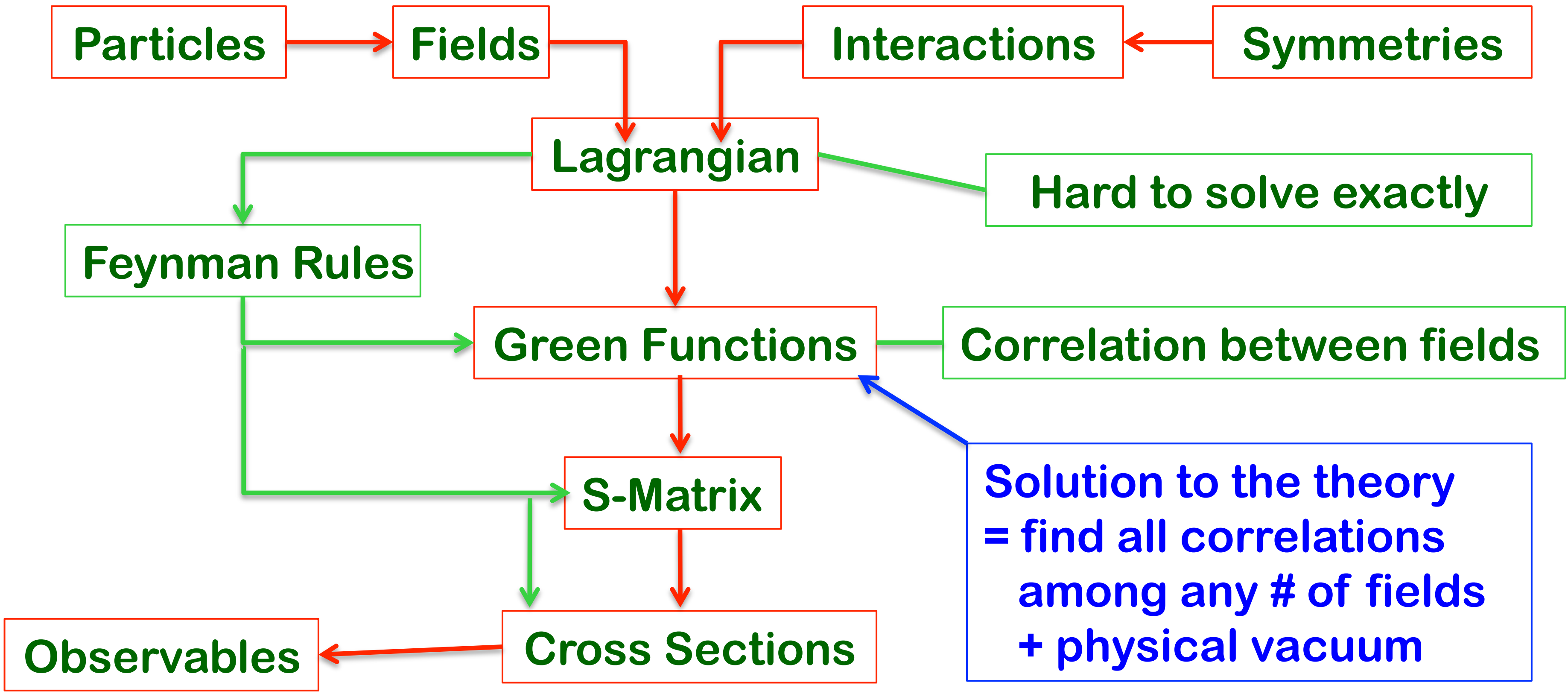
Introduction to QCD

Jianwei Qiu
Theory Center, Jefferson Lab
May 31 – June 2, 2017

Lecture one/two

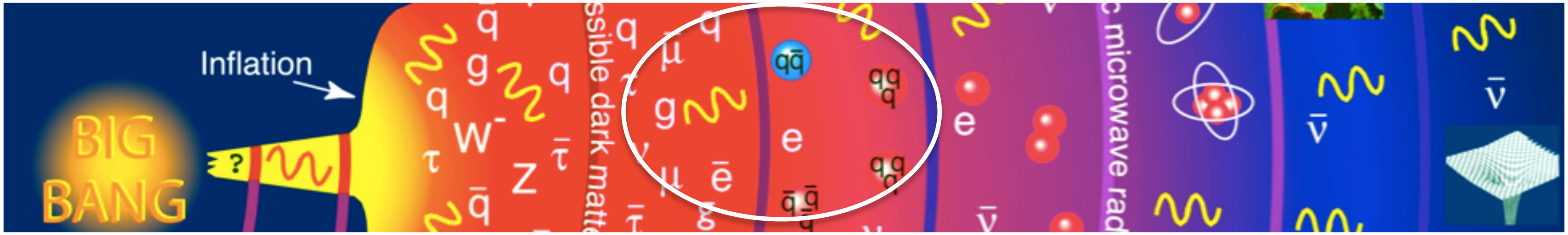
From Lagrangian to Physical Observables

- Theorists: Lagrangian = "complete" theory
- Experimentalists: Cross Section ➡ Observables
- A road map – from Lagrangian to Cross Section:



QCD is everywhere in our universe

- ❑ What is the role of QCD in the evolution of the universe?



- ❑ How hadrons are emerged from quarks and gluons?

- ❑ How does QCD make up the properties of hadrons?

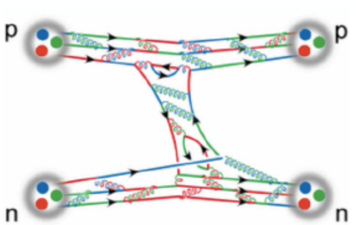
  Their mass, spin, magnetic moment, …

- ❑ What is the QCD landscape of nucleon and nuclei?

Color Confinement — Asymptotic freedom

Q (GeV)

200 MeV (1 fm) — 2 GeV (1/10 fm) — Probing momentum



- ❑ How do the nuclear force arise from QCD?

- ❑ …

Unprecedented Intellectual Challenge!

- **Facts:**

  No modern detector has been able to see quarks and gluons in isolation!

  Gluons are dark!

- **The challenge:**

  How to probe the quark-gluon dynamics, quantify the hadron structure, study the emergence of hadrons, …, if we cannot see quarks and gluons?

- **Answer to the challenge:**

  Theory advances:

  QCD factorization – matching the quarks/gluons to hadrons with controllable approximations!

  Experimental breakthroughs:

  Jets – *Footprints of energetic quarks and gluons*

  Quarks – *Need an EM probe to "see" their existence, …*

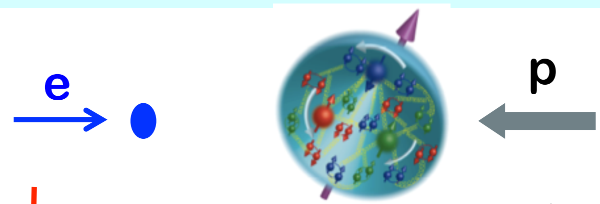
  Gluons – *Varying the probe's resolution to "see" their effect, …*

  Energy, luminosity and measurement – Unprecedented resolution, event rates, and precision probes, especially EM probes, like one at Jlab, …

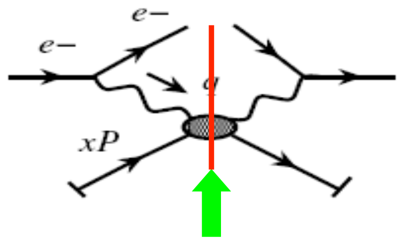
Theoretical approaches – approximations

❑ Perturbative QCD Factorization:

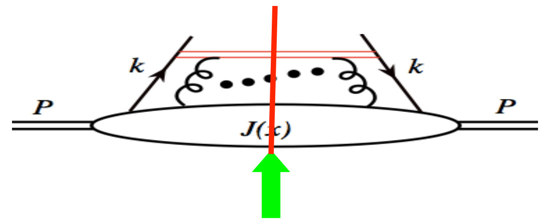
– Approximation at Feynman diagram level



$\sigma^{\rm DIS}_{\rm tot}$:



$\otimes$



$+ \, O\left(\frac{1}{QR}\right)$

Probe Hard-part	Structure Parton-distribution	Approximation Power corrections

❑ Effective field theory (EFT):

– Approximation at the Lagrangian level

Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

❑ Other approximation or model approaches:

Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

❑ Lattice QCD:

– Approximation mainly due to computer power

Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

Physical Observables

Cross sections with identified hadrons are non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not a perturbative scale

Purely infrared safe quantities

Observables without identified hadron(s)

Fully infrared safe observables – I

Fully inclusive, without any identified hadron!

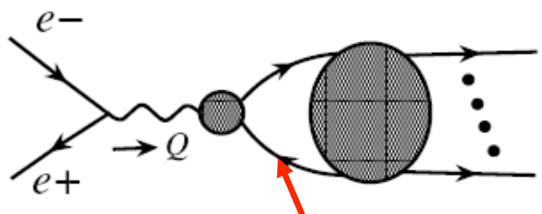
$$\sigma^{\rm total}_{e^+e^-\to{\rm hadrons}} \;=\; \sigma^{\rm total}_{e^+e^-\to{\rm partons}}$$

The simplest observable in QCD

$e^+e^- \rightarrow$ hadrons inclsusive cross sections

- $e^+e^- \rightarrow$ hadron **total** cross section – not a specific hadron!

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \left| \quad \right|^2$$



If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \sum_n {\rm P}_{e^+e^-\to n} = \sum_n \sum_m {\rm P}_{e^+e^-\to m} {\rm P}_{m\to n} = \sum_m {\rm P}_{e^+e^-\to m} \sum_n {\rm P}_{m\to n} = 1$$

Unitarity

$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}} \propto \sum_m {\rm P}_{e^+e^-\to m}$$

$$\Rightarrow \quad \sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} = \sigma^{\rm tot}_{e^+e^-\to {\rm partons}}$$

Finite in perturbation theory – KLN theorem

- $e^+e^- \rightarrow$ parton total cross section:

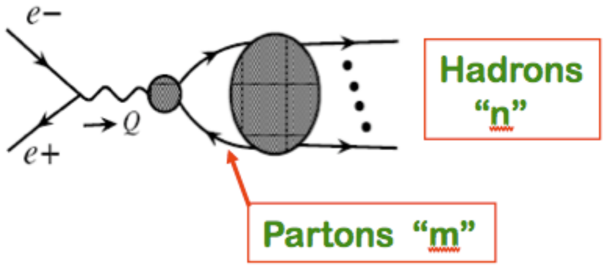
$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2) \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

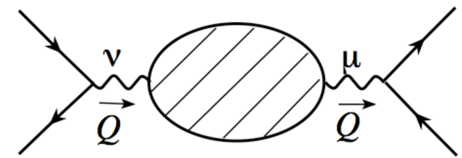
Calculable in pQCD

Infrared Safety of e⁺e⁻ Total Cross Sections

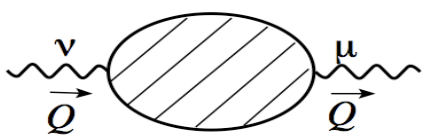
❑ Optical theorem:

$$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\left| \quad \right|^2 \propto {\rm Im}$$





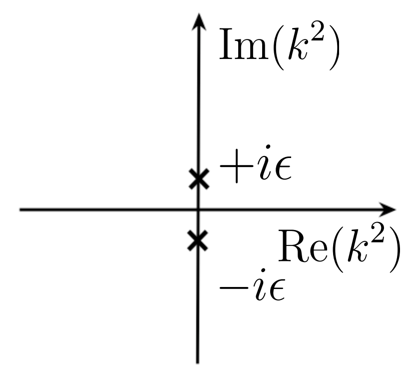
❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

IR safety of $\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

❑ IR safety of $\Pi(Q^2)$



If there were pinched poles in Π(Q²),

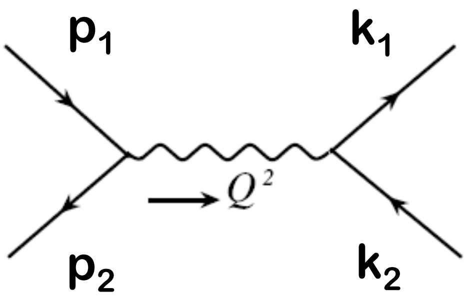
✧ **real partons moving away from each other**

✧ **cannot be back to form the virtual photon again!**

Rest frame of the virtual photon

Lowest order (LO) perturbative calculation

- Lowest order Feynman diagram:



- Invariant amplitude square:

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1 + m_Q\right)\gamma_\mu\left(\gamma\cdot k_2 - m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right]$$

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - k_1)^2$$
$$u = (p_2 - k_1)^2$$

- Lowest order cross section:

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \quad \text{where } s = Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1 + \frac{2m_Q^2}{s}\right]\sqrt{1 - \frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

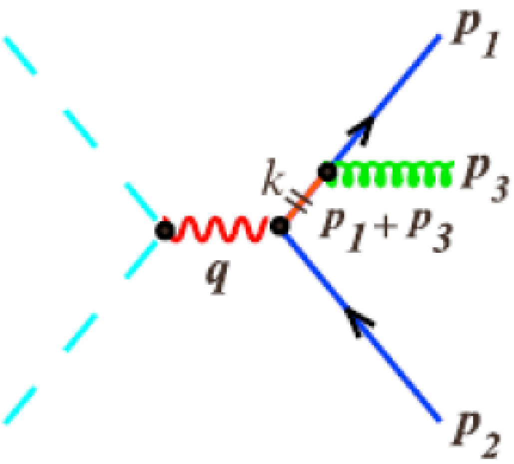
Next-to-leading order (NLO) contribution

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2p_i.q}{s} \quad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

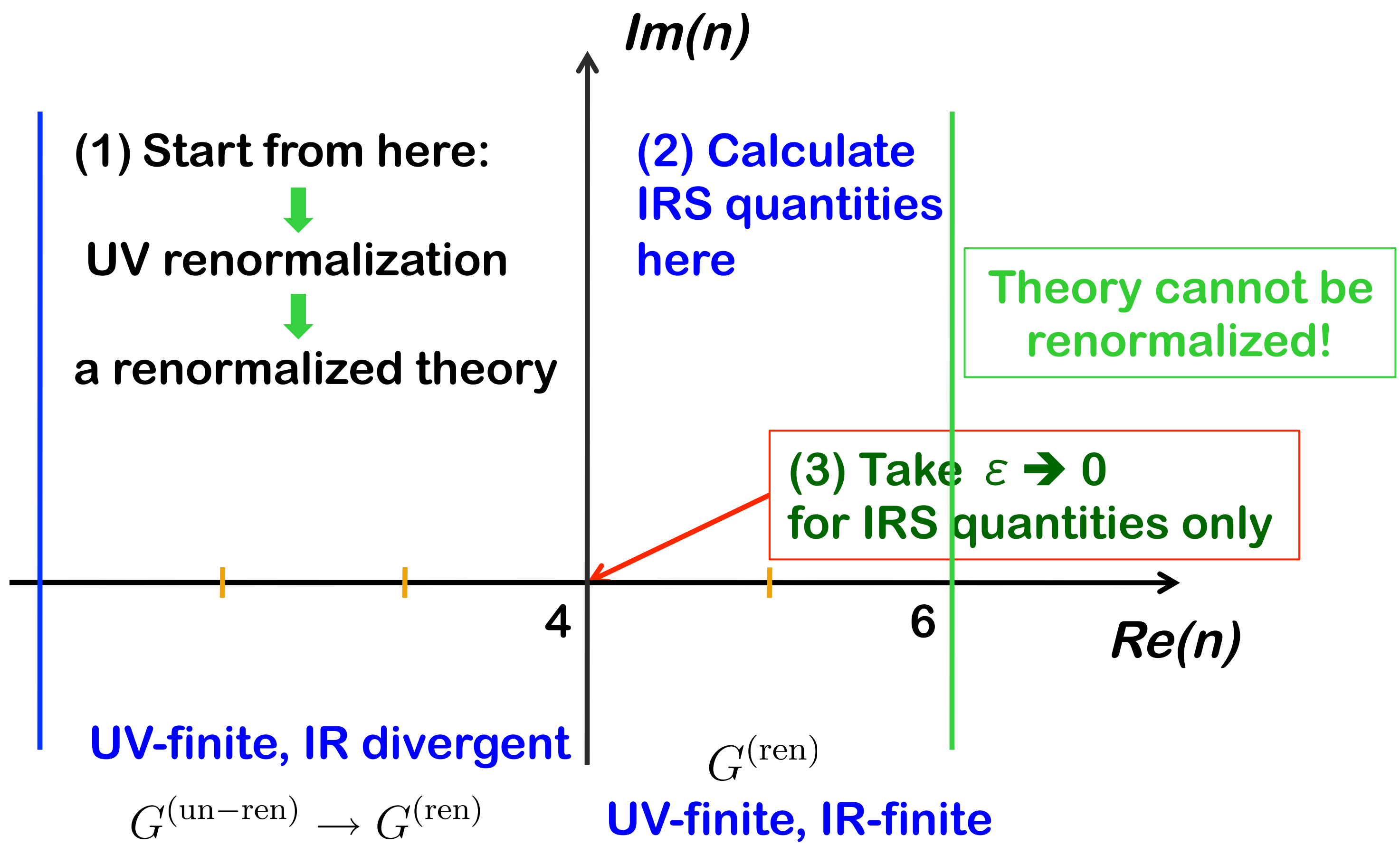
IR as $x3 \to 0$
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

- **Complex *n*-dimensional space:** $\int d^n k\, F(k,Q)$



Dimensional regularization for both IR and CO

❑ NLO with a dimensional regulator:

✧ **Real:**

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

✧ **Virtual:**

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

✧ **NLO:**

$$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_{2}^{(0)}\left[\frac{\alpha_s}{\pi} + O(\varepsilon)\right]$$

No ε dependence!

✧ **Total:**

$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O(\alpha_s^2) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O(\alpha_s^2)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Go beyond the inclusive total cross section?

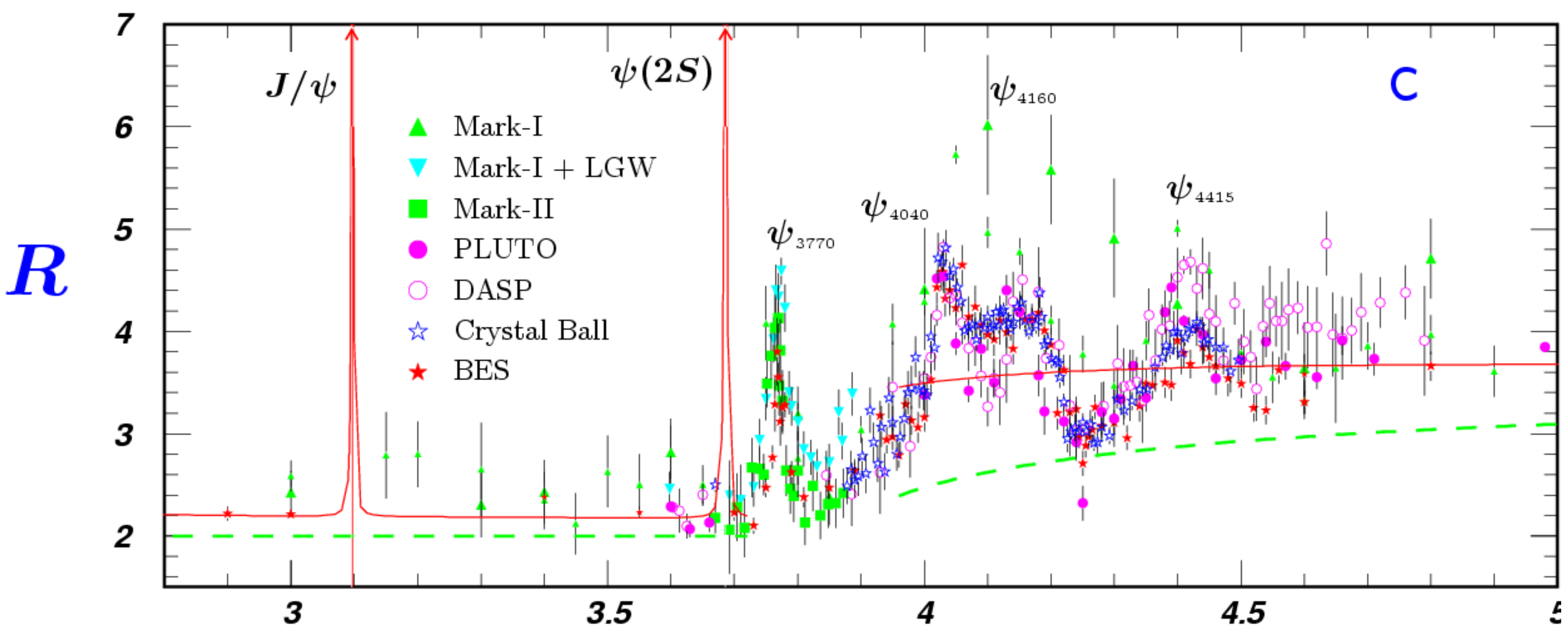
Hadronic cross section in e+e- collision

- **Normalized hadronic cross section:**

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+ N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Fully infrared safe observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\mathrm{Jets}}_{e^+e^-\to\mathrm{hadrons}} = \sigma^{\mathrm{Jets}}_{e^+e^-\to\mathrm{partons}}$$

Jets – “trace” of partons

Thrust distribution in e^+e^- collisions

etc.

Jets – trace of partons

- ❑ Jets – "total" cross-section with a limited phase-space

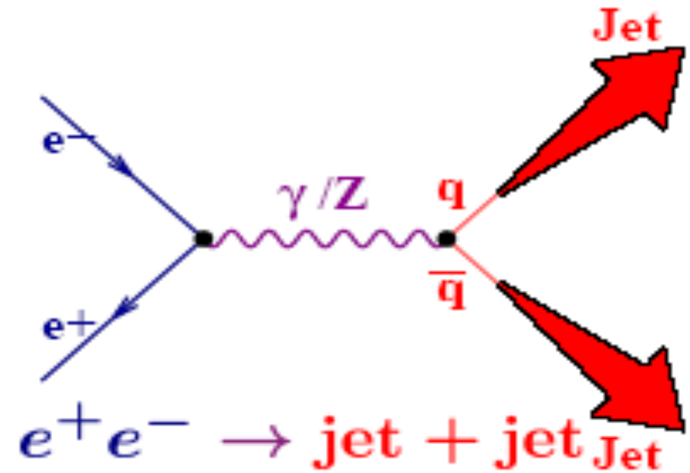
  Not any specific hadron!

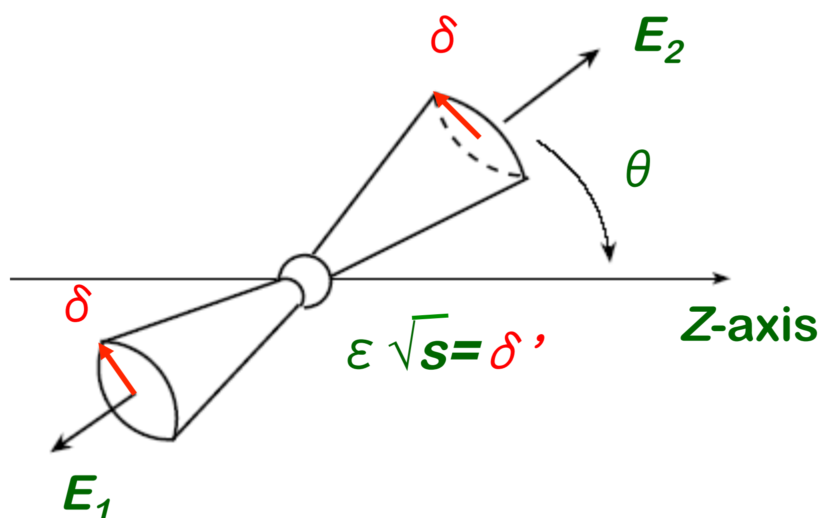
- ❑ Q: will IR cancellation be completed?

  - ✧ Leading partons are moving away from each other

  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton

- ❑ Many Jet algorithms



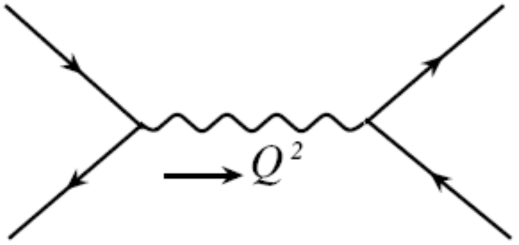


Sterman-Weinberg Jet

Two-jet cross section in e+e- collisions

❑ **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



❑ **Two-jet in pQCD:**

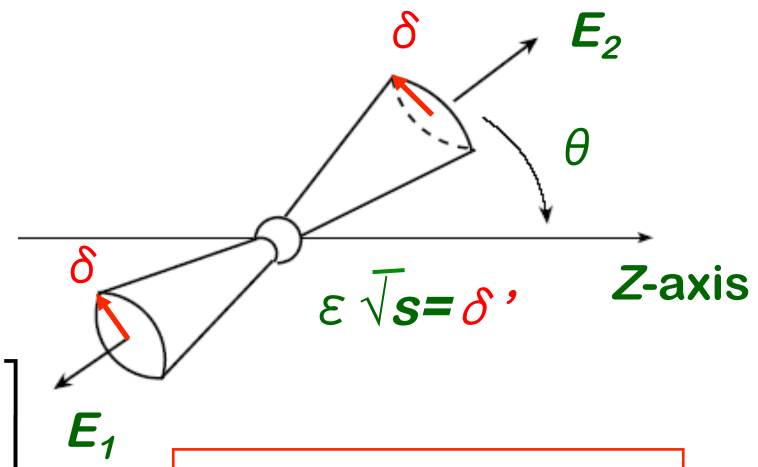
$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n(\delta)$

❑ **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\times\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$

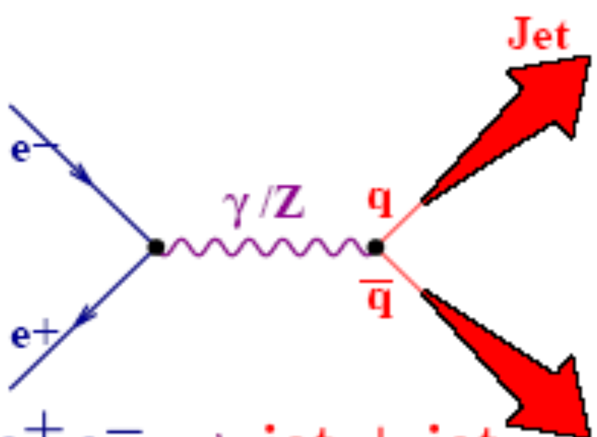


Sterman-Weinberg Jet

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as } Q \to \infty$$

An early clean two-jet event

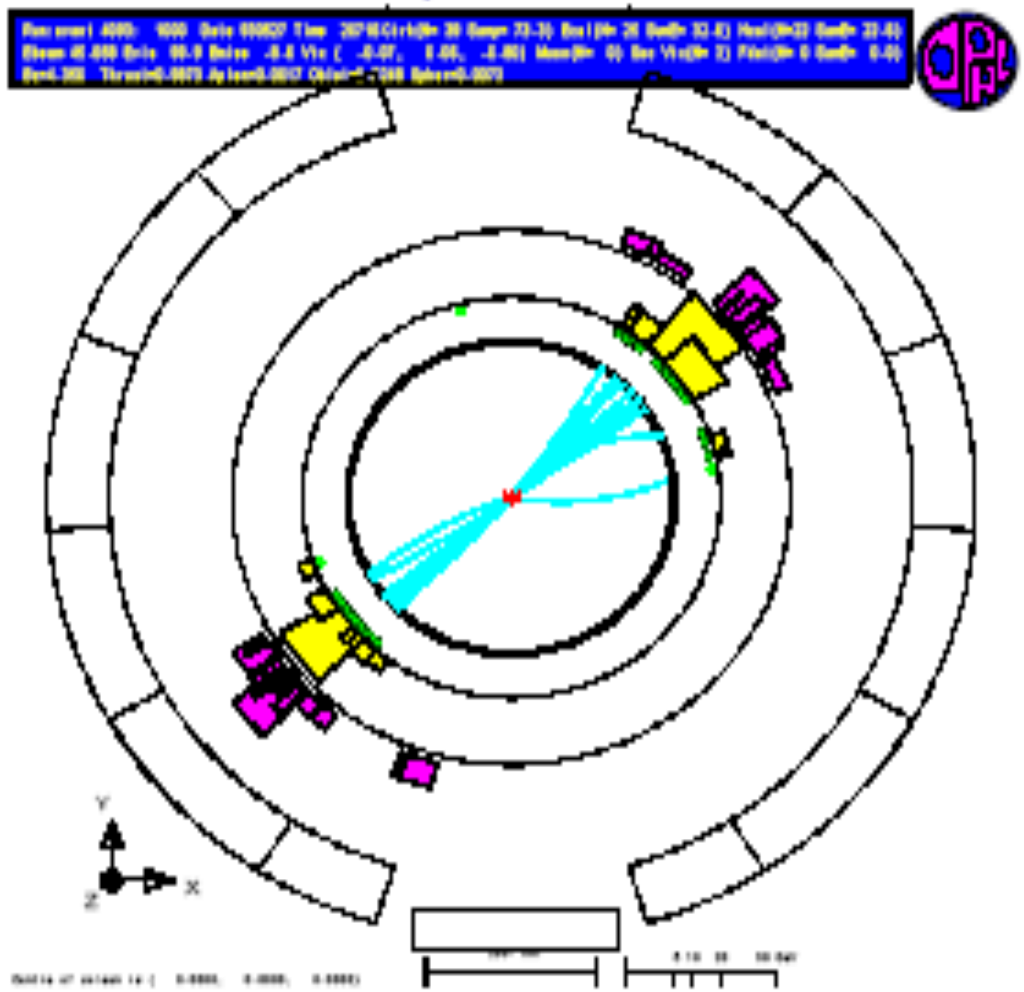
Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \rightarrow \text{jet} + \text{jet}$

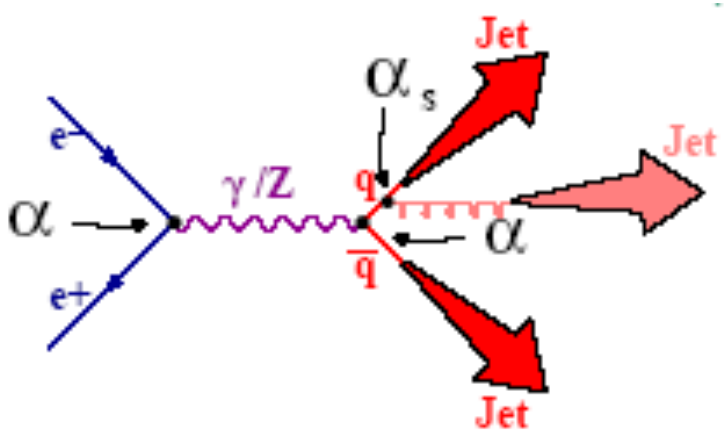
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

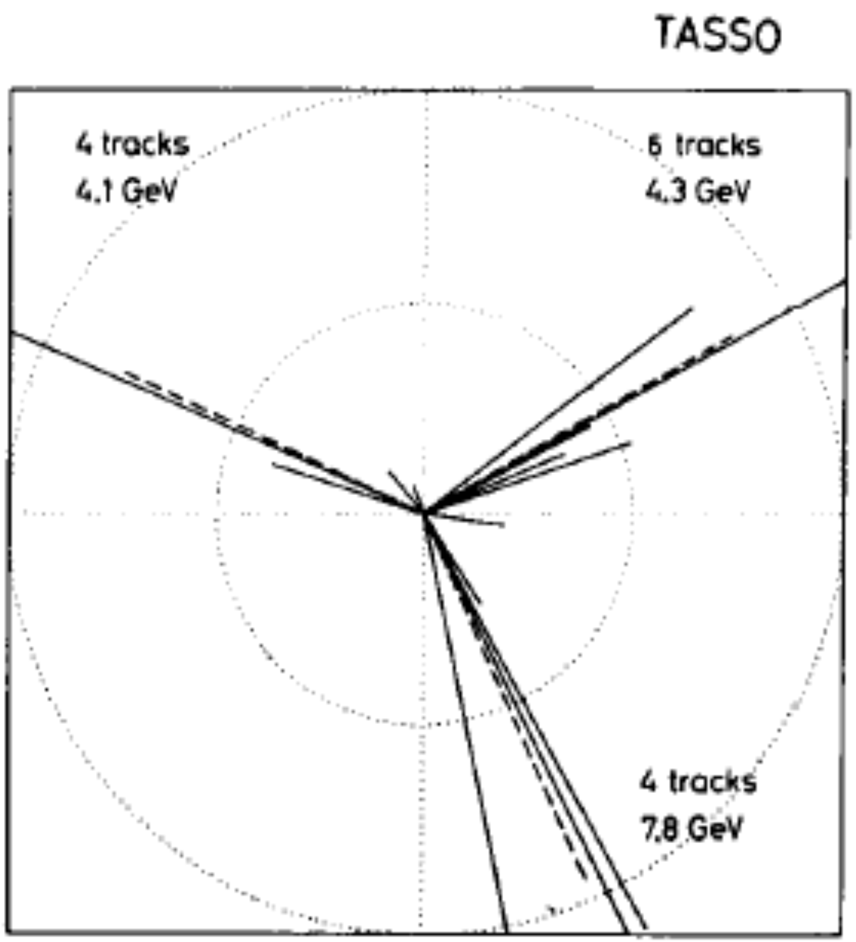
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

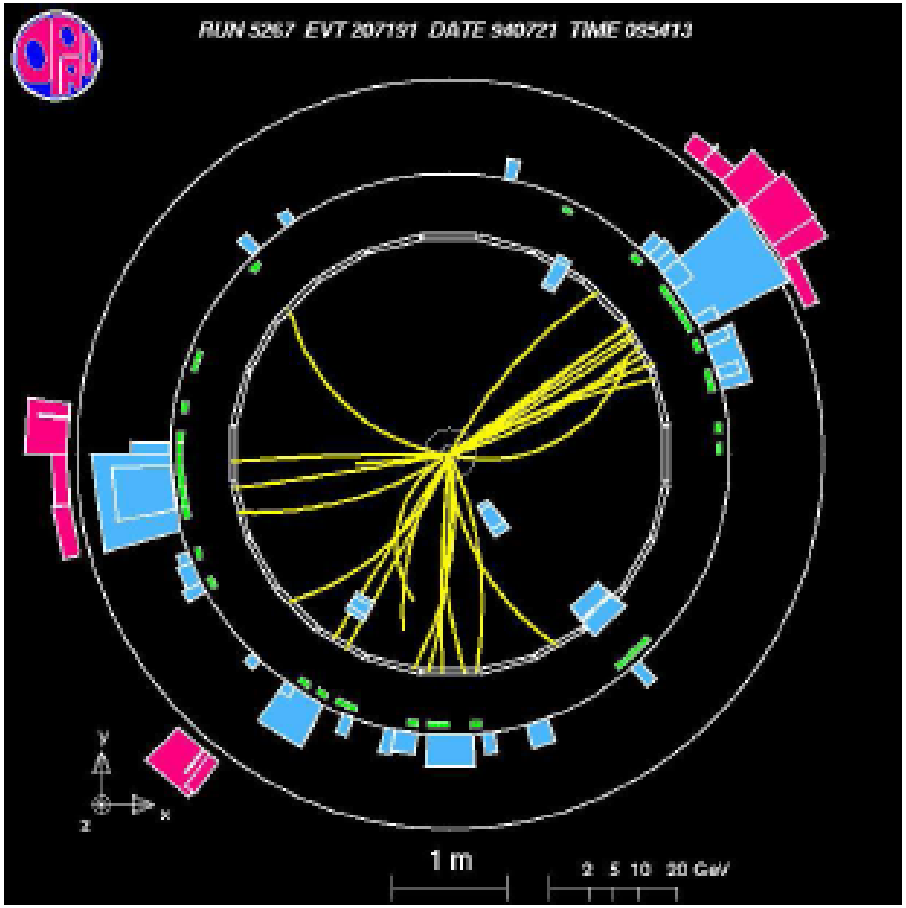
JADE Collab., Phys. Lett. B91 (1980) 142

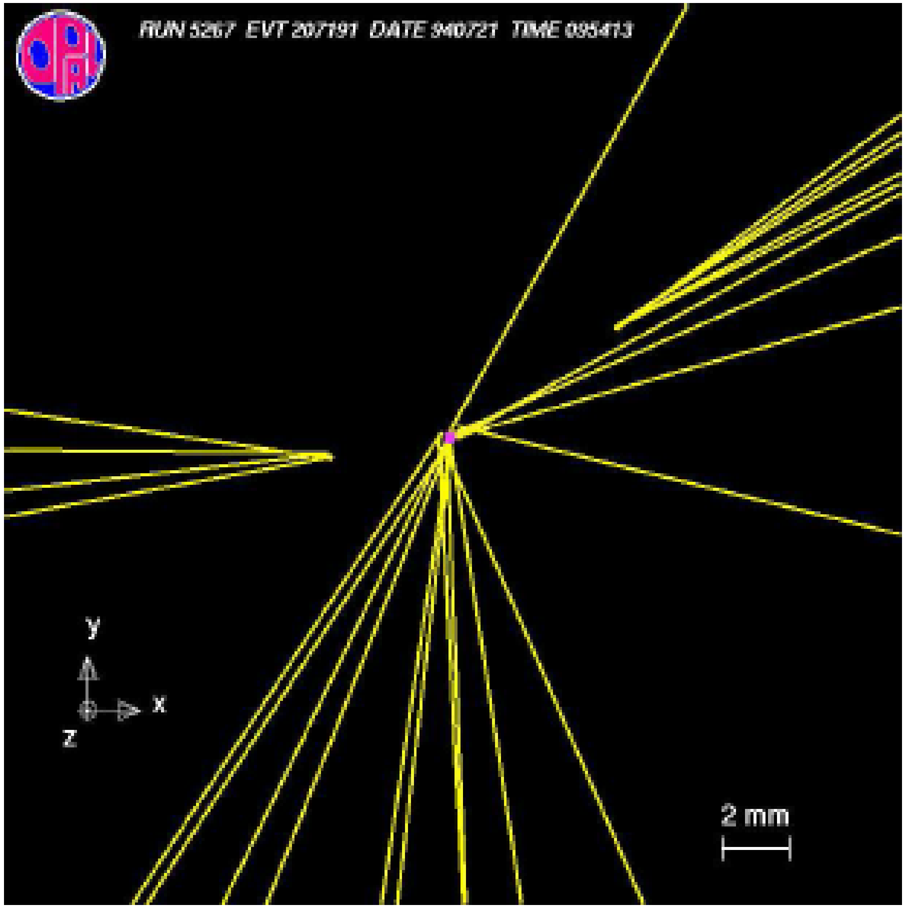
PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Tagged three-jet event from LEP





Gluon Jet

Basics of jet finding algorithms

- **Recombination jet algorithms (almost all e+e- colliders):**

  Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$ $\quad M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

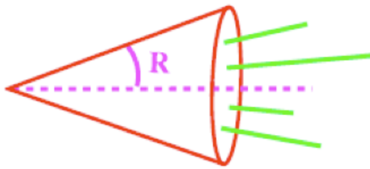
  for Durham k_T

  - different algorithm = different choice of M_{ij}^2 :
  - Combine the particle pair (i,j) with the smallest y_{ij} : $(i,j) \to k$

    $$e.g.\ \mathrm{E\ scheme}: \quad p_k = p_i + p_j$$

  - iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$

- **Cone jet algorithms (CDF, …, colliders):**



  - Cluster all particles into a cone of half angle R to form a jet:
  - Require a minimum visible jet energy: $E_{jet} > \epsilon$

  Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$

  with $\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$

  - Classical choices: ***p=1*** – "k_T algorithm", ***p= -1*** – "anti-k_T", …

Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \quad 0 \le \lambda \le 1$$

Physical meaning:

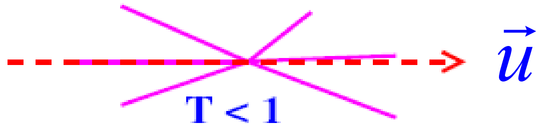
Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without the parton

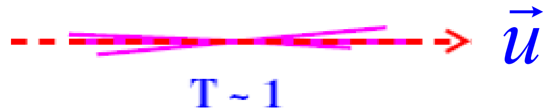
Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1 \text{ for all } n \quad \Rightarrow \quad \sigma^{(\mathrm{tot})}$

Thrust distribution

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}}\left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|}\right)$$





- **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to\text{hadrons}}}{dT} \quad \text{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

  - Contribution from p=0 particles drops out the sum

  - Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

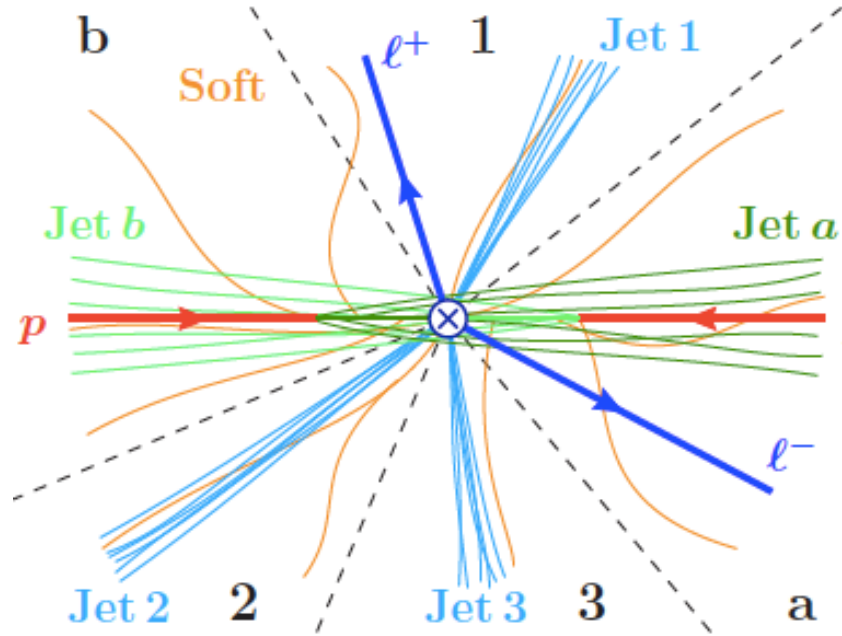
and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

N-Jettiness

- Event structure:

  $pp \to$ leptons plus jets



- N-Jettiness:

  (Stewart, Tackmann, Waalewijin, 2010)

$$\tau_N = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

The sum include all final-state hadrons *excluding* more than N jets

Allows for an event-shape based analysis of multi-jets events (a generalization of Thrust)

- N-infinitely narrow jets (jet veto):

  As a limit of N-Jettiness: $\tau_N \to 0$

Generalization of the thrust distribution in e^+e^- initial-state identified hadron!

The harder question

- **Question:**

  How to test QCD in a reaction with identified hadron(s)?
  – to probe the quark-gluon structure of the hadron

- **Facts:**

  Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

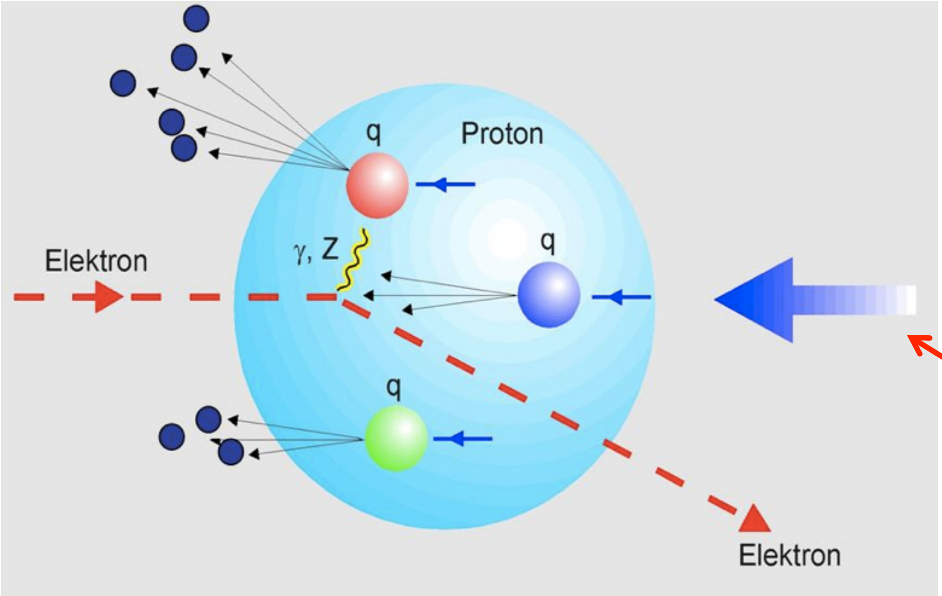
  Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

- **Solution – Factorization:**

  ✧ Isolate the calculable dynamics of quarks and gluons

  ✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
  – provide information on the partonic structure of the hadron

Observables with ONE identified hadron



$\sigma^{\text{DIS}}_{\ell p \to \ell' X(\text{everything})}$

Identified initial-state hadron-proton!

Cross section is infrared divergent, and nonperturbative!

QCD factorization (approximation!)

Cross Section $=$ Infrared-Safe $\otimes$ Nonperturbative-distribution

↑ **Measured** ↑ **Hard-probe** ↑ **Universal-hadron structure**

Backup slides