HUGS

Introduction to QCD

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Lecture five/six

The plan for my six lectures

- The Goal:

  To understand the strong interaction dynamics, and hadron structure, in terms of Quantum Chromo-dynamics (QCD)

- The Plan (approximately):

  From hadrons to partons, the quarks and gluons in QCD

  Fundamentals of QCD,

  Factorization, Evolution, and

  Elementary hard processes

  Four lectures

  Hadron structures and properties in QCD

  Parton distribution functions (PDFs),

  Transverse momentum dependent PDFs (TMDs),

  Generalized PDFs (GPDs), and

  Multi-parton correlation functions

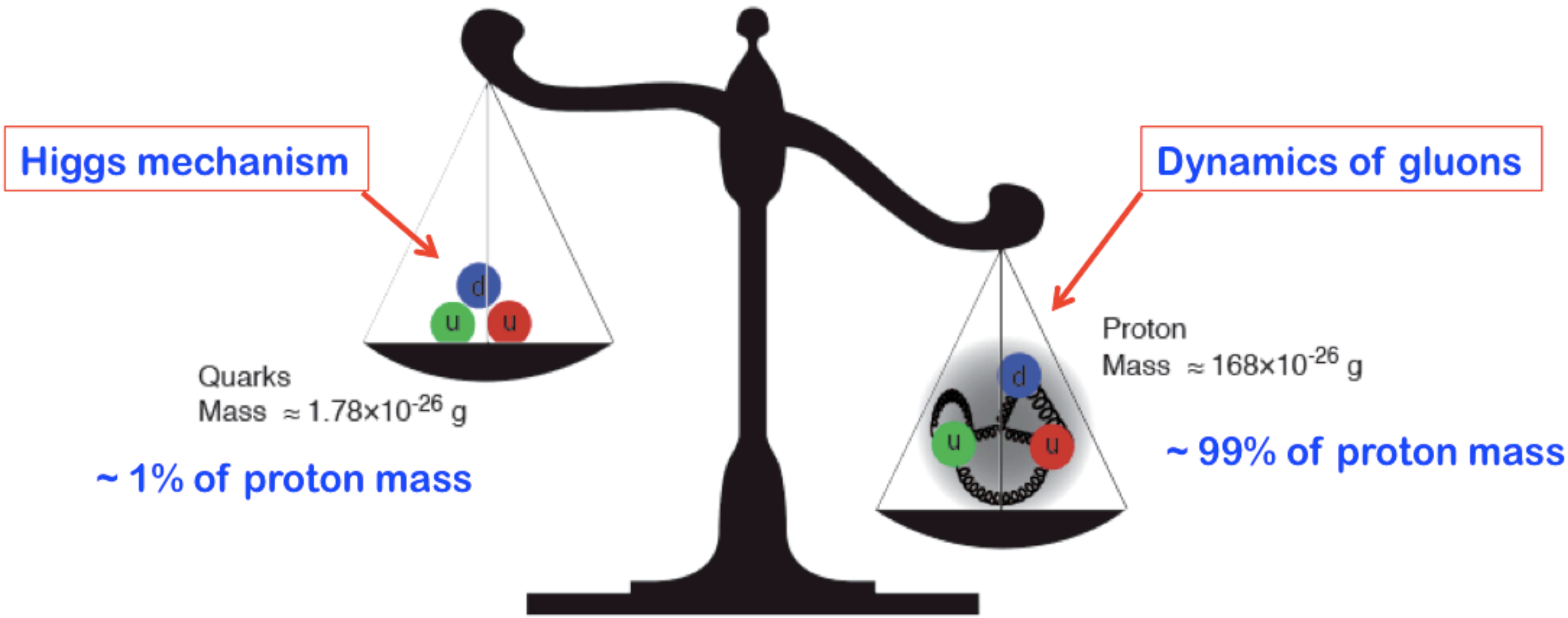
  Two lectures

The proton mass?

- **How does QCD generate the nucleon mass?**

  "… The vast majority of the nucleon's mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. …" *The 2015 Long Range Plan for Nuclear Science*

- **Higgs mechanism is not relevant to hadron mass!**



"Mass without mass!"

Mass vs. Spin

❑ Mass – intrinsic to a particle:

= Energy of the particle when it is at the rest

✧ QCD energy-momentum tensor in terms of quarks and gluons

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\,g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

✧ Proton mass:

X. Ji, PRL (1995)

$$m = \frac{\langle p|\int d^3x\, T^{00}\,|p\rangle}{\langle p|p\rangle} \sim \mathrm{GeV}$$

when proton is at rest!

❑ Spin – intrinsic to a particle:

= Angular momentum of the particle when it is at the rest

✧ QCD angular momentum density in terms of energy-momentum tensor

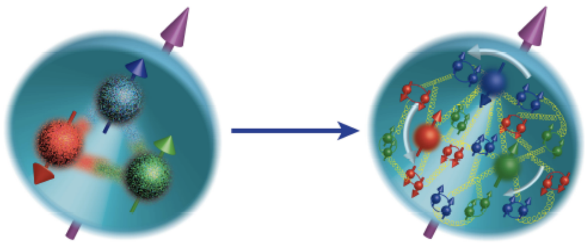
$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

✧ Proton spin:

$$S(\mu) = \sum_f \langle P, S|\hat{J}^z_f(\mu)|P, S\rangle = \frac{1}{2}$$

Hadron Mass

❑ Proton's mass:



- ✧ QCD Lagrangian does not have mass dimension parameters, other than current quark masses
- ✧ Asymptotic freedom ⟷ confinement:

➡ A dynamical scale, $\Lambda_{\rm QCD}$, consistent with $\dfrac{1}{R} \sim 200\ {\rm MeV}$

❑ Bag model:

- ✧ Kinetic energy of three quarks: $K_q \sim 3/R$
- ✧ Bag energy (bag constant B): $T_b = \dfrac{4}{3}\pi R^3\, B$
- ✧ Minimize $K_q + T_b$: $M_p \sim \dfrac{4}{R} \sim \dfrac{4}{0.88fm} \sim 912{\rm MeV}$

❑ Constituent quark model:

- ✧ Spontaneous chiral symmetry breaking:

Massless quarks gain ~300 MeV mass when traveling in vacuum

➡ $M_p \sim 3\, m_q^{\rm eff} \sim 900\ {\rm MeV}$

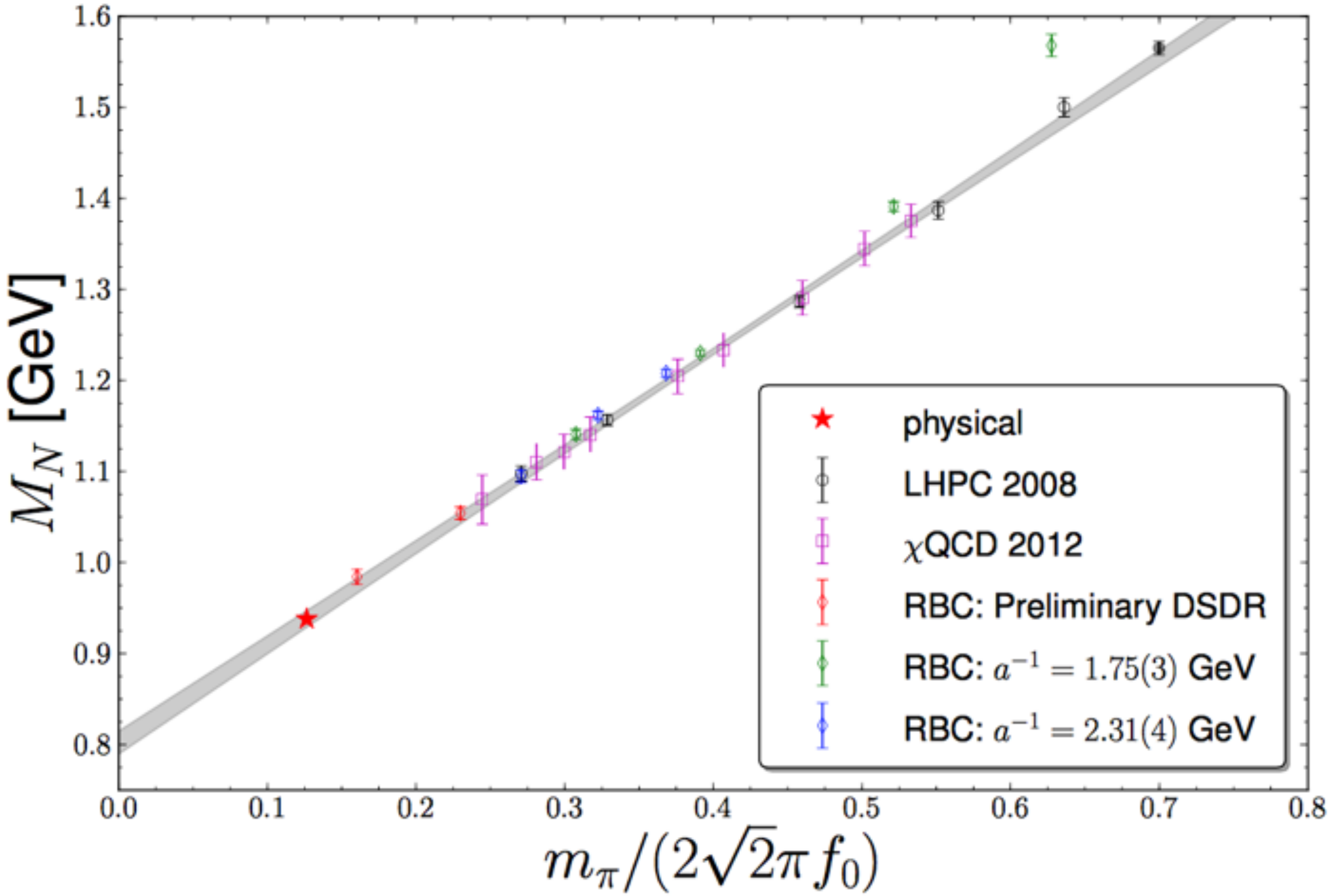
❑ Lattice QCD:

Ratios of hadron masses

Hadron Mass

Martin Savage @ Temple meeting

- Nucleon mass from lattice QCD:

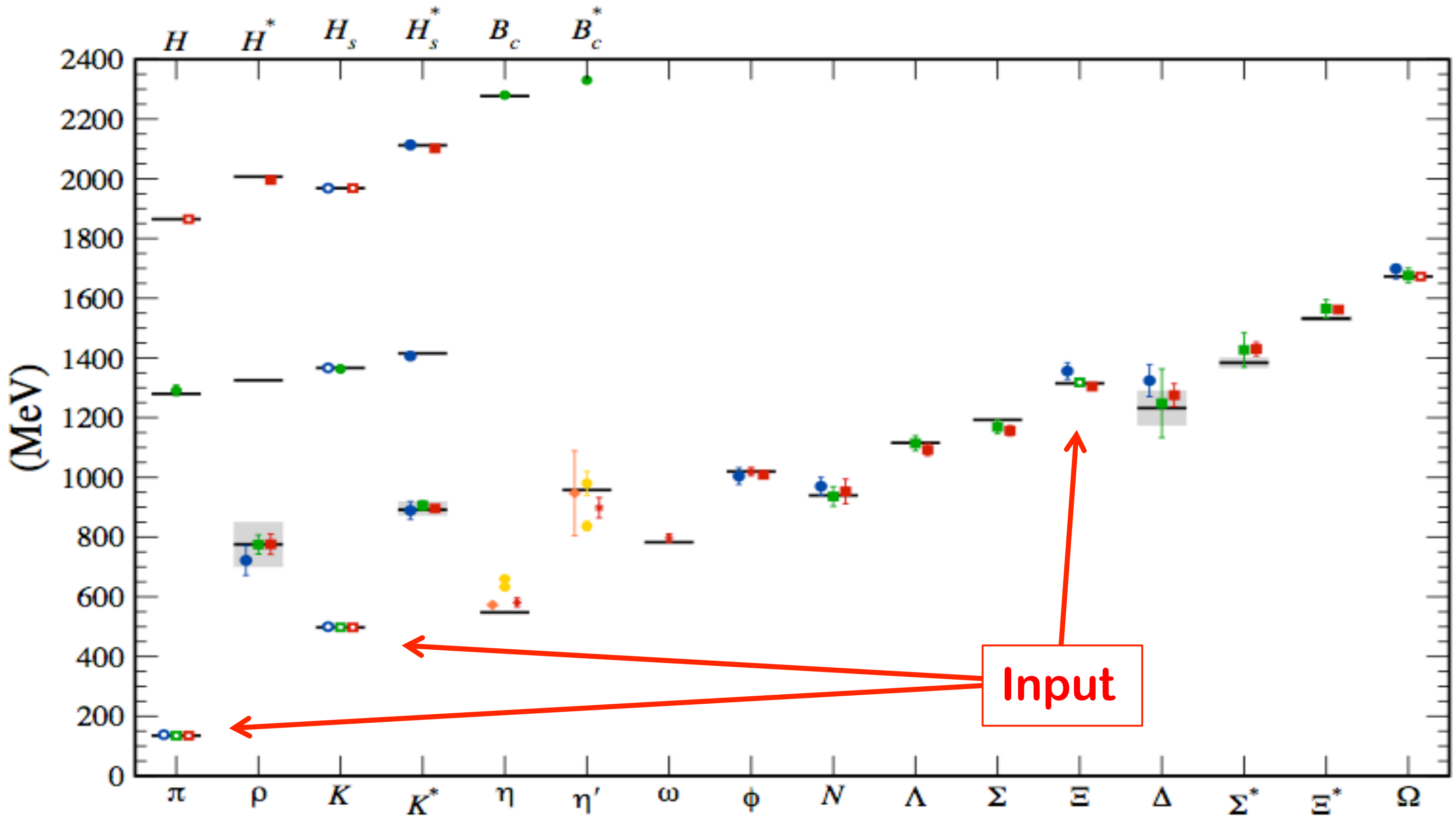


$$M_N = 800 \text{ MeV} + m_\pi$$

Unexpected behavior !!

Hadron Mass

❑ **From Lattice QCD calculation:**



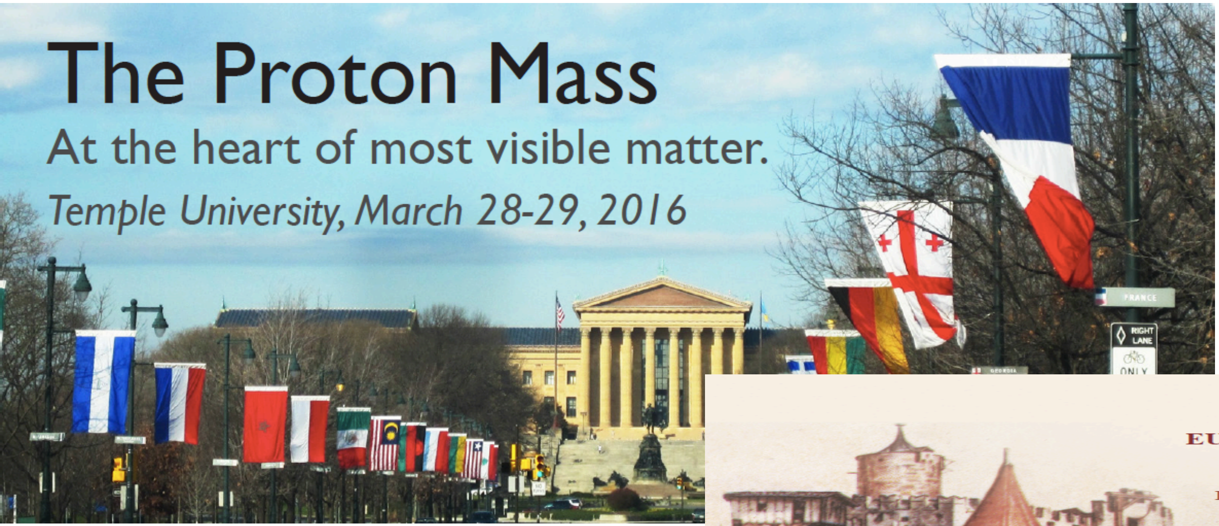
A major success of QCD – is the right theory for the Strong Interaction!

How does QCD generate this? The role of quarks vs that of gluons?

If we do not understand proton mass, we do not understand QCD

New community effort

- Three-pronged approach to explore the origin of hadron mass
  - ✧ Lattice QCD
  - ✧ Mass decomposition – roles of the constituents
  - ✧ Model calculation – approximated analytical approach



https://phys.cst.temple.edu/meziani/proton-mass-workshop-2016/

http://www.ectstar.eu/node/2218

The role of quarks and gluons?

❑ QCD energy-momentum tensor:

$$T^{\mu\nu} = \frac{1}{2}\,\overline{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \frac{1}{4}\, g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}$$

Traceless term: $\overline{T^{\mu\nu}} \equiv T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}$

Trace term: $\widehat{T^{\mu\nu}} \equiv \frac{1}{4} g^{\mu\nu} T^{\alpha}{}_{\alpha}$

Vacuum expectation breaks chiral symmetry

with $T^{\alpha}{}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu,a}F^{a}_{\mu\nu} + \sum_{q=u,d,s} m_q(1+\gamma_m)\overline{\psi}_q\psi_q$

QCD trace anomaly $\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \ldots$

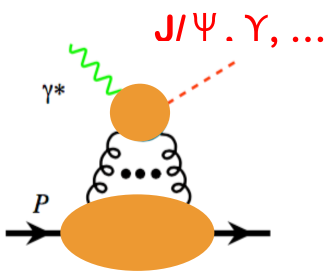
❑ Invariant hadron mass (in any frame):

Pure quantum effect!

$$\langle p|\, T^{\mu\nu}\, |p\rangle \propto p^{\mu}p^{\nu} \;\Rightarrow\; m^2 \propto \langle p| T^{\alpha}{}_{\alpha}\, |p\rangle \;\Rightarrow\; \frac{\beta(g)}{2g}\,\langle p|F^2|p\rangle$$

➡ At the chiral limit, the entire mass is from gluons!

➡ Heavy quarkonium production near the threshold, from JLab12 to EIC



The role of quarks and gluons?

X. Ji, PRL (1995)

❑ Another decomposition:

$$M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = M_q + M_g + M_m + M_a$$

Quark Energy → M_q; Gluon Energy → M_g; Quark Mass → M_m; Trace Anomaly → M_a

✧ **Quark energy contribution:**

$$H_q = \int d^3\vec{x}\; \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi, \qquad M_q = \left.\frac{\langle P|H_q|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}}$$

✧ **Gluon energy contribution:**

$$H_g = \int d^3\vec{x}\; \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2) \qquad M_g = \left.\frac{\langle P|H_g|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}}$$

✧ **Quark mass contribution:**

$$H_m = \int d^3\vec{x}\; \bar{\psi} m \psi \qquad M_m = \left.\frac{\langle P|H_m|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}}$$

✧ **Trace anomaly contribution:**

$$H_a = \int d^3\vec{x}\; \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2) \qquad M_a = \left.\frac{\langle P|H_a|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}}$$

Sum rules are only useful if individual terms can be measured independently

Hadron spin

❑ Spin:

- ✧ Pauli (1924): two-valued quantum degree of freedom of electron
- ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant ħ)
- ✧ Composite particle = Total angular momentum when it is at rest

❑ Spin of a nucleus:

- ✧ Nuclear binding: 8 MeV/nucleon << mass of nucleon
- ✧ Nucleon number is fixed inside a given nucleus
- ✧ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

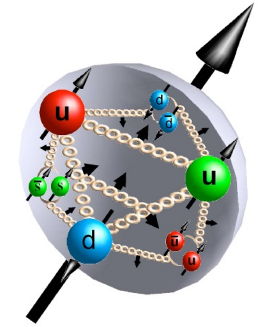
- ✧ If the probing energy << mass of constituent quark
- ✧ Nucleon is made of three constituent (valence) quark
- ✧ Spin of a nucleon = sum of the constituent quark spin

State: $$|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$$

Spin: $$S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \qquad S = \sum_i S_i$$

Carried by valence quarks

Hadron spin



- **Spin of a nucleon – QCD:**
  - ✧ Current quark mass << energy exchange of the collision
  - ✧ Number of quarks and gluons depends on the probing energy
- **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

- **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x \; M^{0jk}_{\rm QCD} \quad \longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

  - ✧ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q \right]$$

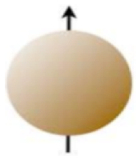
  - ✧ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]$$

Need to have the matrix elements of these partonic operators measured independently

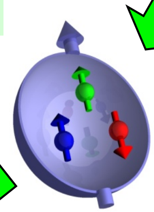
Hadron spin

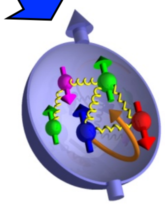
- **How does QCD make up the nucleon's spin?**



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g) = \sum \langle P, S | \hat{J}_f^z(\mu) | P, S \rangle$$

Proton Spin





Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$

$$\sim 30\%$$

Spin "puzzle"

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

$$\sim 20\% (\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

If we do not understand proton spin, we do not understand QCD

Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

- **Cross section:**

  Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$

– Positive definite

- **Asymmetries or difference of cross sections:**

  – Not necessary positive!

  - both beams polarized A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - one beam polarized A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Basics for spin observables

❑ Factorized cross section:

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma}=I,\gamma_5,\gamma^\mu,\gamma_5\gamma^\mu,\sigma^{\mu\nu}$$

❑ Parity and Time-reversal invariance:

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

❑ IF: $\langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

Operators lead to the "+" sign ➡ spin-averaged cross sections

Operators lead to the "-" sign ➡ spin asymmetries

❑ Example:

$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\,\psi(y^-) \;\Rightarrow\; q(x)$$

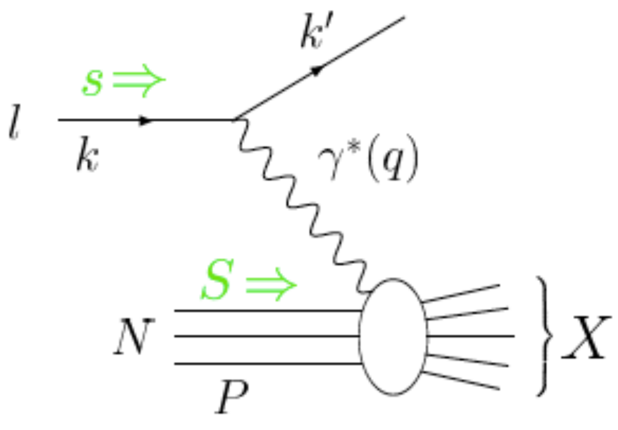
$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \;\Rightarrow\; \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu)=\overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \;\Rightarrow\; \delta q(x)\to h(x)$$

$$\mathcal{O}(\psi,A^\mu)=\frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow \Delta g(x)$$

Polarized deep inelastic scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2\times 10^{-16}\mathrm{m}}{Q/\mathrm{GeV}} \lesssim 10^{-16}\mathrm{m} = 1/10\mathrm{fm}$$

"Inelasticity" – known as Bjorken variable

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ **Recall – from lecture 3:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

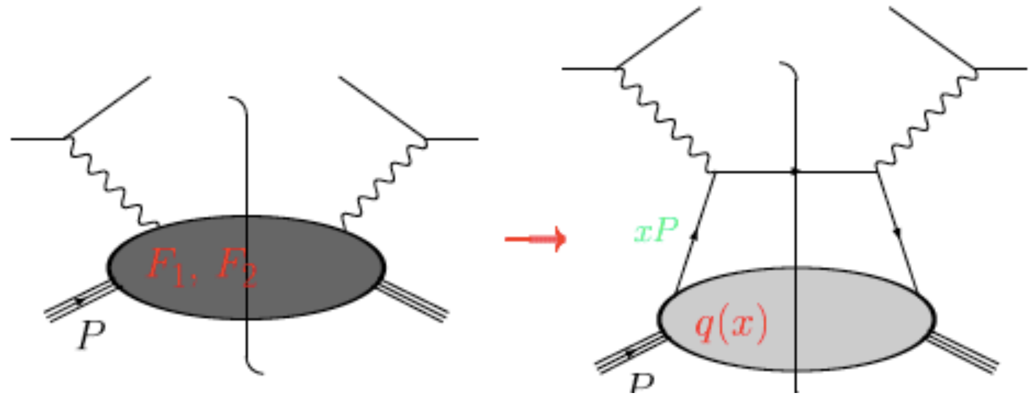
$$+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{\left(p.q\right) S_\sigma - \left(S.q\right) p_\sigma}{\left(p.q\right)^2} g_2\left(x_B, Q^2\right)\right]$$

✧ **Polarized structure functions:**

$$g_1(x_B, Q^2),\ \ g_2(x_B, Q^2)$$

Polarized deep inelastic scattering

❑ Parton model results – LO QCD:



✧ **Structure functions:**

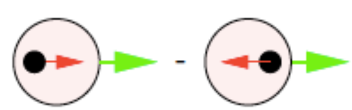
$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \left[q(x) + \bar{q}(x) \right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

$$g_1(x) = \frac{1}{2}\sum e_q^2 \left[\Delta q(x) + \Delta\bar{q}(x) \right] + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}\left(\Delta u + \Delta\bar{u}\right) + \frac{1}{9}\left(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right) \right]$$

✧ **Polarized quark distribution:**

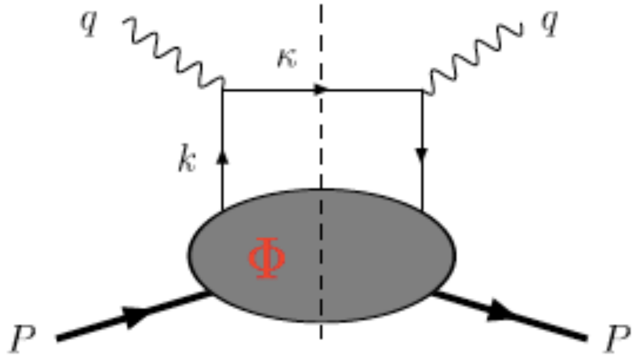
$$\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)$$

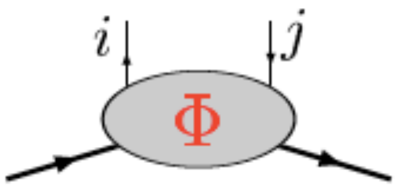


Information on nucleon's spin structure

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:





✧ **Two-quark correlator:**

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X)\, \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\, e^{ik\cdot z}\, \langle\, PS\,|\, \bar{\psi}_j(0)\, \psi_i(z)\,|\, PS\,\rangle$$

✧ **Hadronic tensor (one –flavor):**

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4}\, \delta\left((k+q)^2\right)\, \mathrm{Tr}\left[\, \Phi\, \gamma^\mu (\not{k} + \not{q}) \gamma^\nu \,\right]$$

Polarized deep inelastic scattering

✧ **General expansion of $\phi(x)$:**

must have general expansion in terms of P, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\|}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$

✧ **3-leading power quark parton distribution:**

$$q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

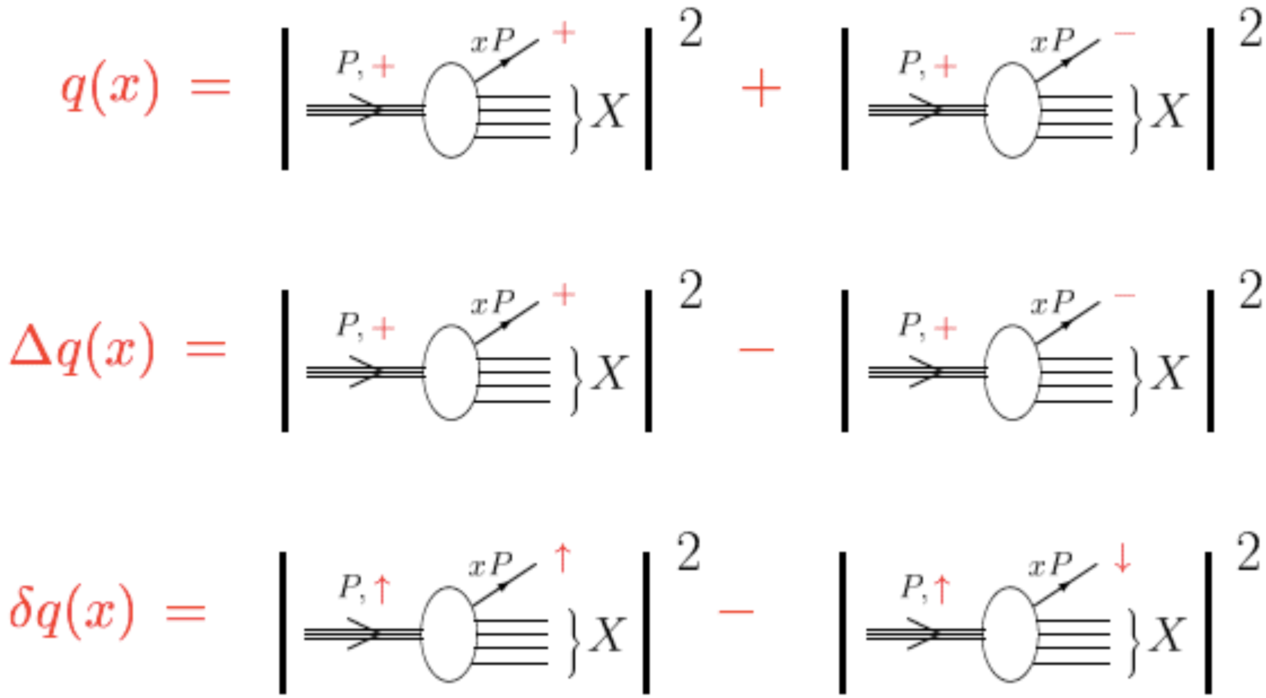
$$\Delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

$$\delta q(x) = \frac{1}{4\pi}\int dz^- \, e^{iz^- xP^+} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\gamma_\perp\gamma_5\,\psi\left(0,z^-,\mathbf{0}_\perp\right)|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Polarized deep inelastic scattering

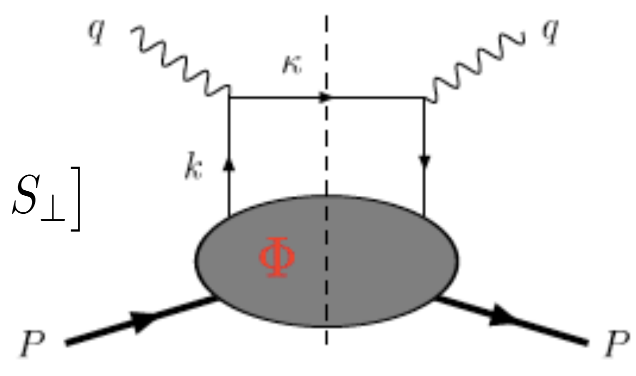
- Pictorially:



- Note:

No transversity contribution to inclusive DIS!

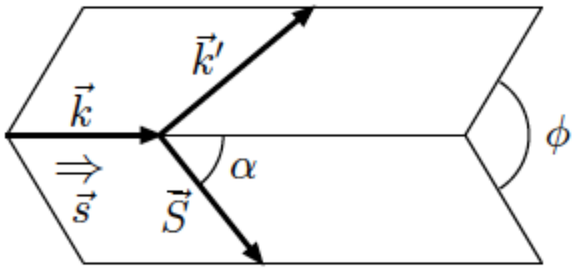
$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\parallel}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$



Polarized deep inelastic scattering

- **Extract the polarized structure functions:**

$$\mathcal{W}^{\mu\nu}(P,q,S) - \mathcal{W}^{\mu\nu}(P,q,-S)$$



- **Define:** $\angle(\hat{k},\hat{S}) = \alpha$, **and lepton helicity** λ

- **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \left\{ \cos\alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2x^2y^2}{Q^2} \right] g_1(x,Q^2) - \frac{2m^2x^2y}{Q^2} g_2(x,Q^2) \right\} \right.$$

$$\left. - \sin\alpha \cos\phi \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2x^2y^2}{Q^2}\right)} \left(\frac{y}{2} g_1(x,Q^2) + g_2(x,Q^2)\right) \right\}$$

- **Spin orientation:**

$$\alpha = 0 : \Rightarrow \quad g_1$$

$$\alpha = \pi/2 : \Rightarrow \quad y\,g_1 + 2\,g_2$$, suppressed m/Q

Polarized deep inelastic scattering

❑ Spin asymmetries – measured experimentally:

✧ **Longitudinal polarization –** $\alpha = 0$

Known function

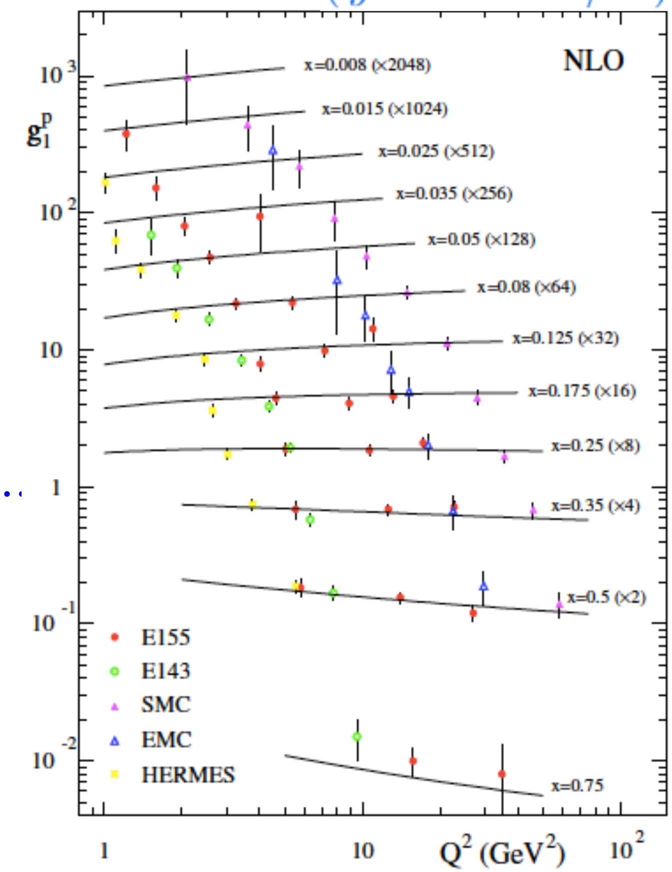
$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

$$(y = 1 - E'/E)$$

✧ **So far only "fixed target" experiments:**

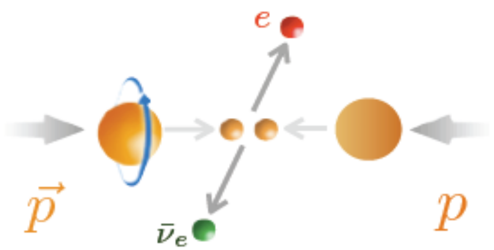
CERN: EMC, SMC, COMPASS
SLAC: E80, E130, E142, E143, E154
DESY: HERMES
JLab: Hall A,B,C, many experiments

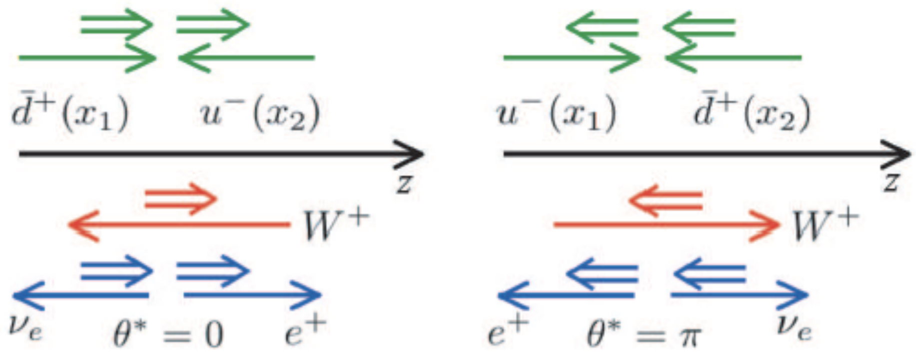
with various polarized targets: $p,\ d,\ {}^3\mathrm{He}, \ldots$



Determination of Δq and $\Delta\bar{q}$

- W's are left-handed:





- Flavor separation:

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}$$

Forward W⁺ (backward e⁺):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

Backward W⁺ (forward e⁺):

$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

- Complications:

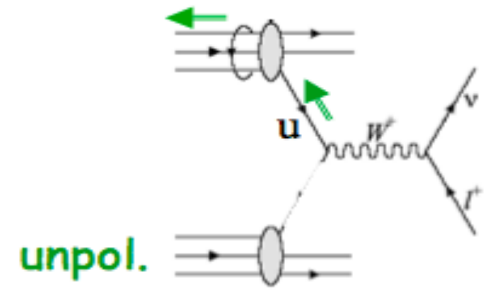
High order, W's p_T-distribution at low p_T

Sea quark polarization – RHIC W program

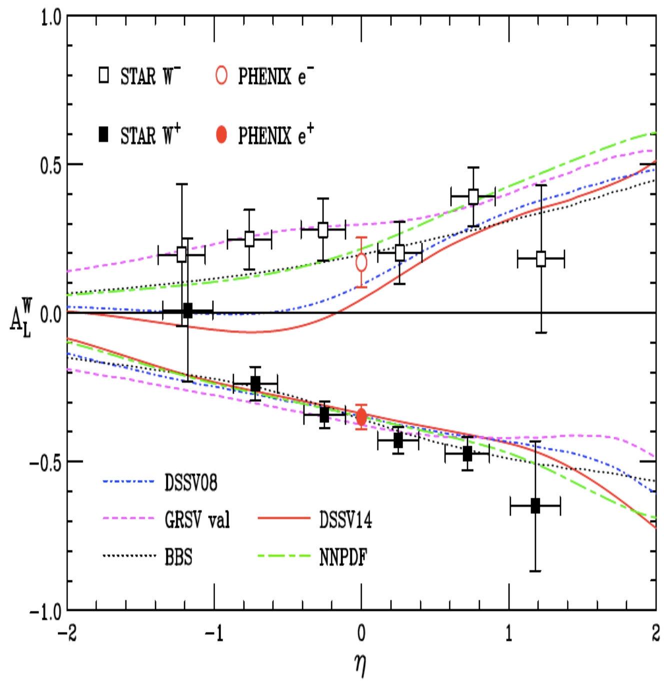
- Single longitudinal spin asymmetries:

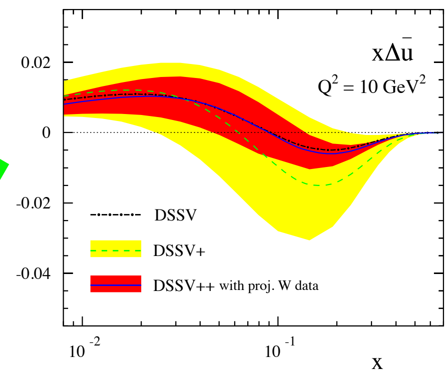
$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s)$$

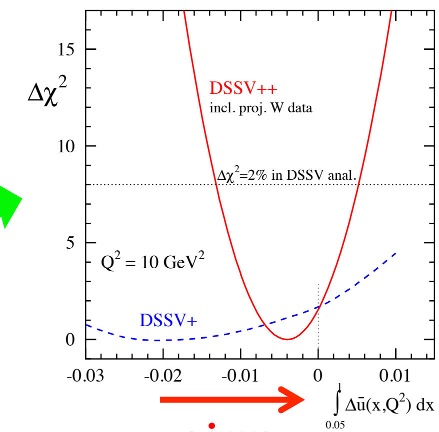
Parity violating weak interaction



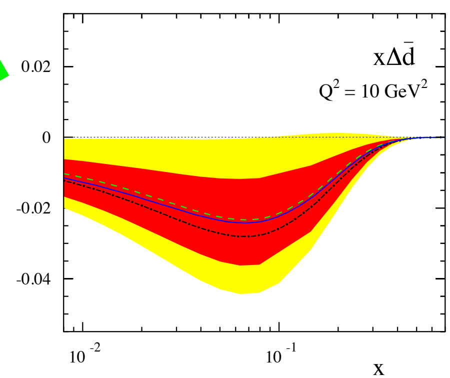
- From 2013 RHIC data:

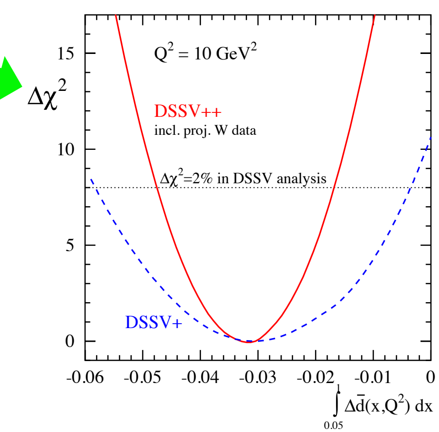




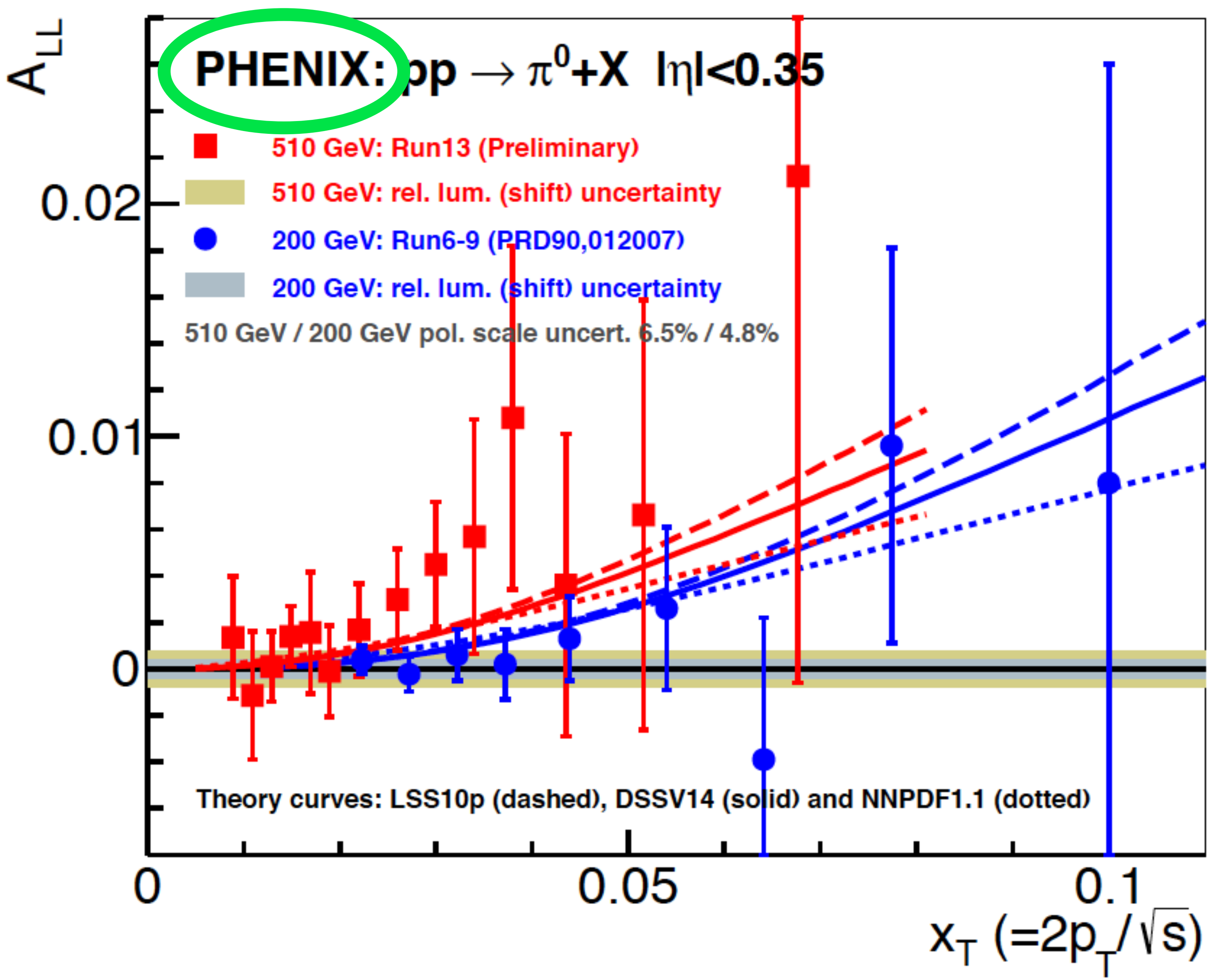


sign

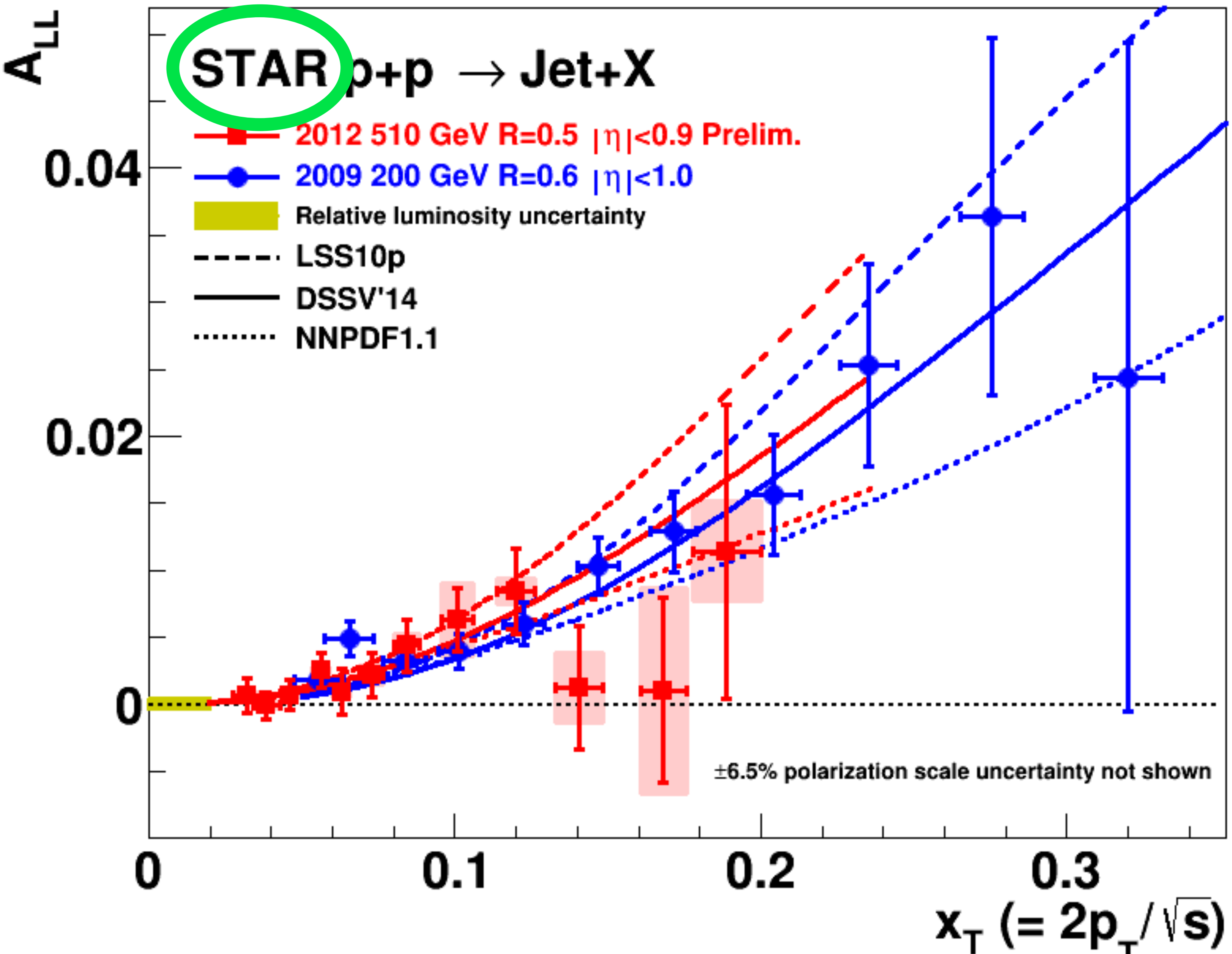




RHIC Measurements on ΔG

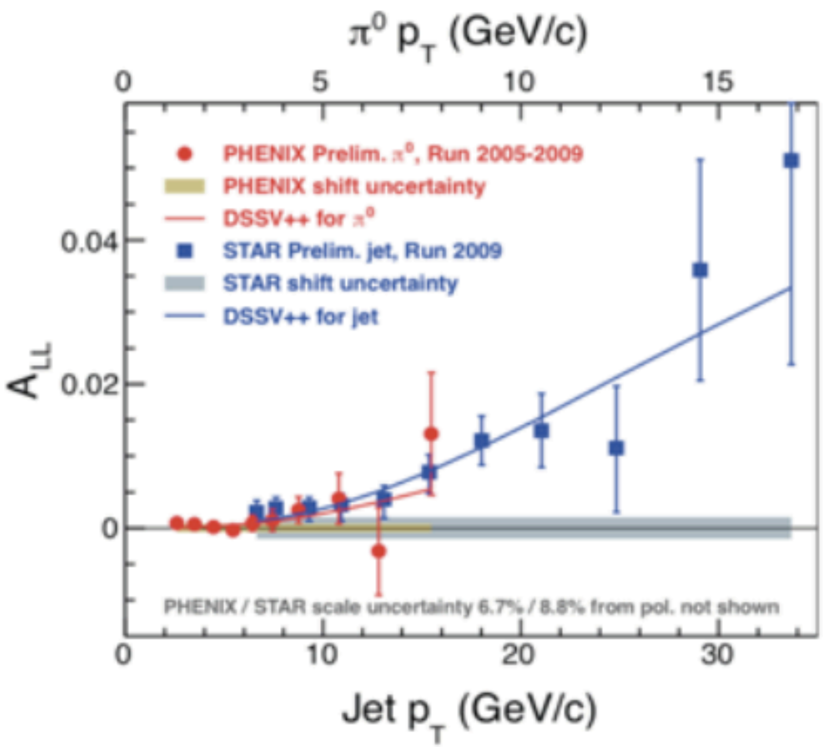


RHIC Measurements on ΔG



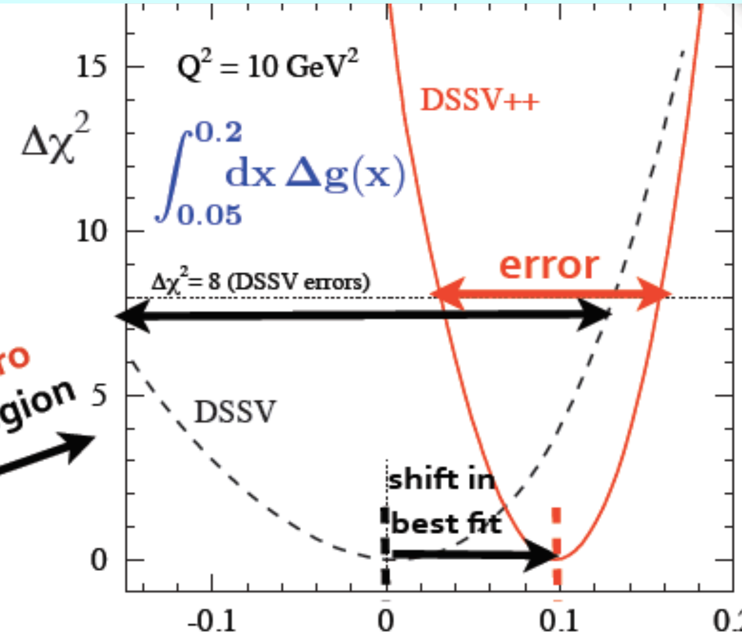
Impact of RHIC measurements

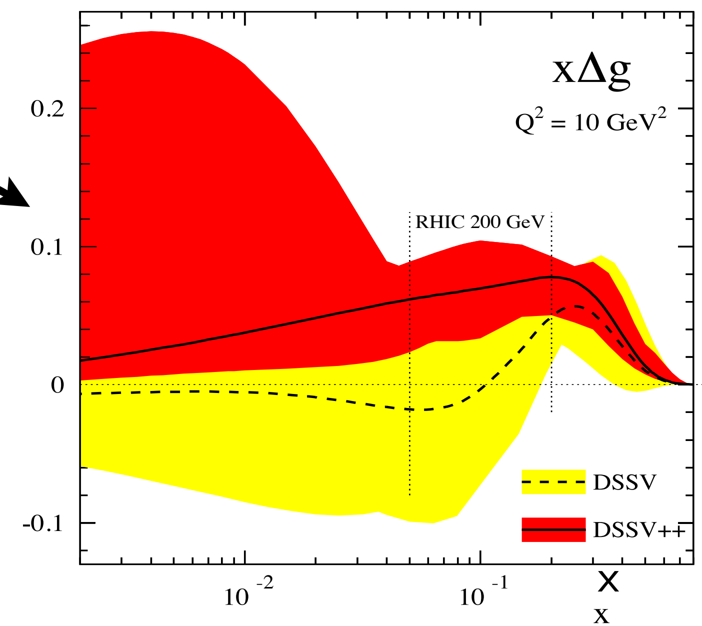
new RHIC data included in DSSV++



lead to non-zero Δg in RHIC x-region

positive Δg in RHIC x-region



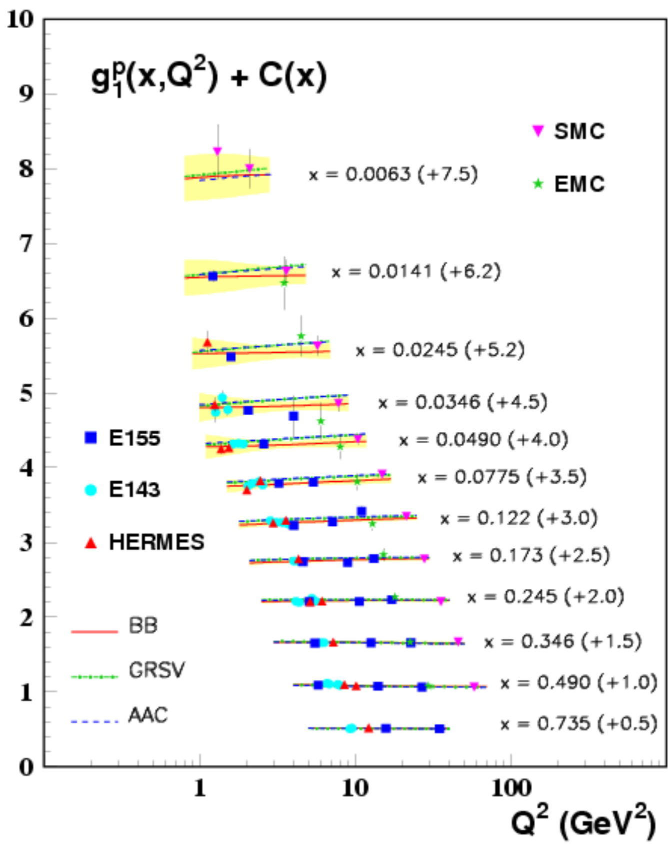


$$\int_{0.05}^{0.2} dx\, \Delta g(x, Q^2 = 10\,\mathrm{GeV}^2) = 0.1^{+0.06}_{-0.07}$$

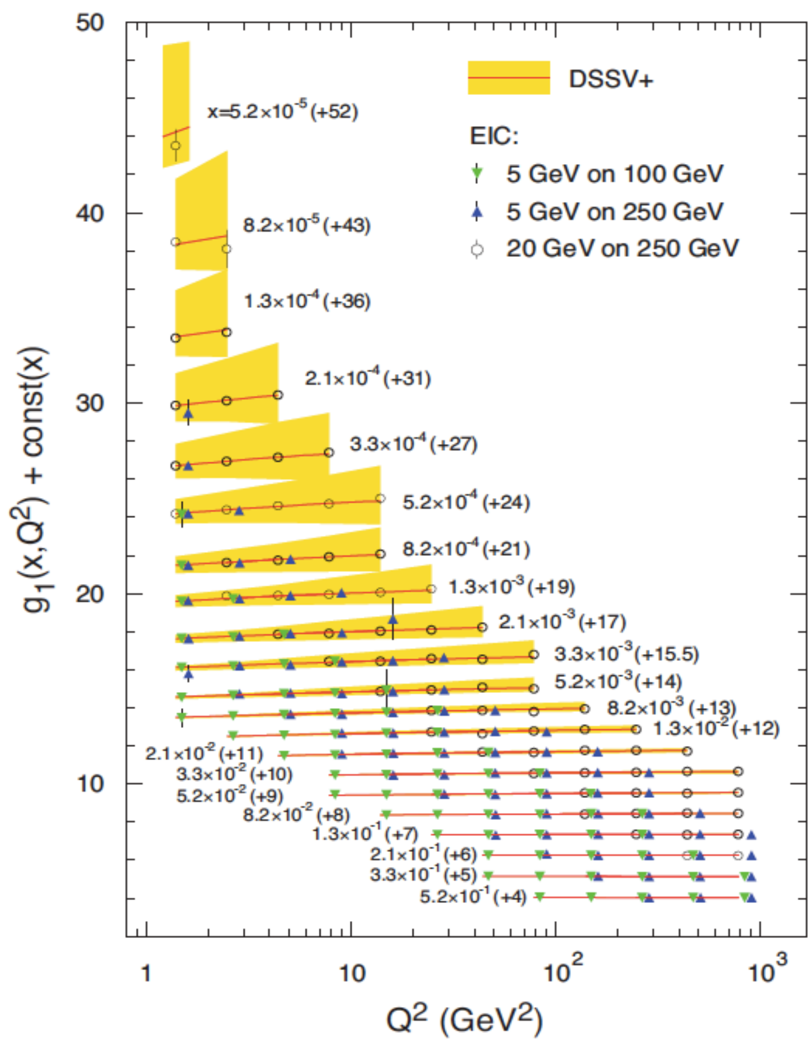
fully compatible with old DSSV error estimate

The Future: Challenges & opportunities

- The power & precision of EIC:



at EIC



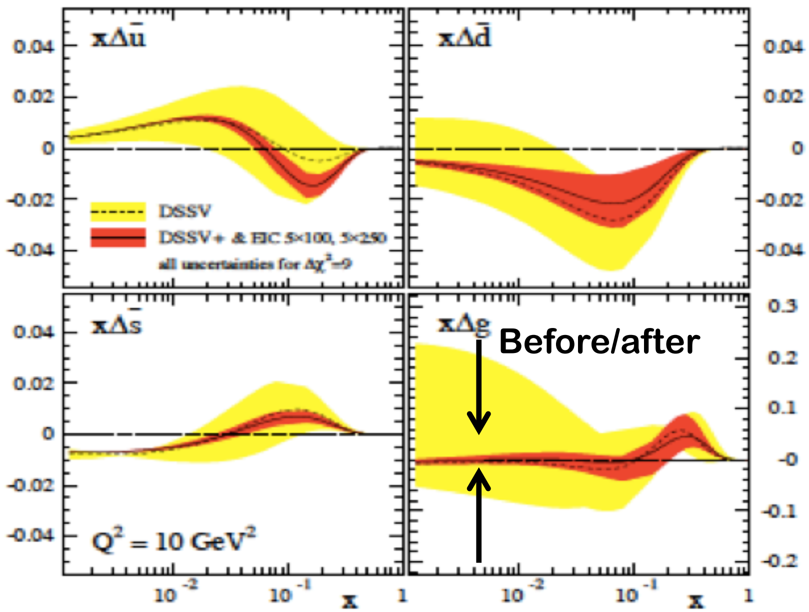
- Reach out the glue:

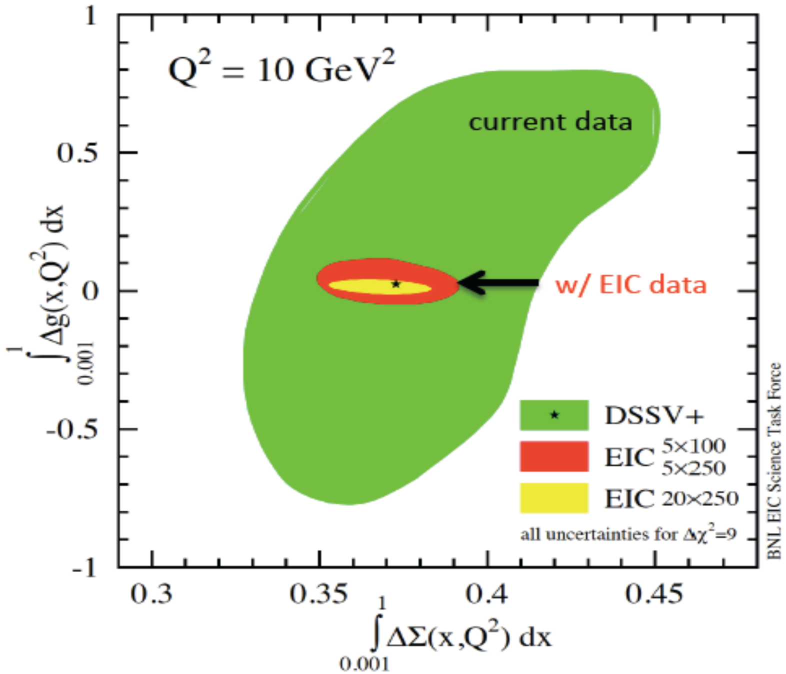
$$\frac{dg_1(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g(x,Q^2) + \cdots$$

The Future: Challenges & opportunities

- One-year of running at EIC:

Wider Q^2 and x range including low x at EIC!





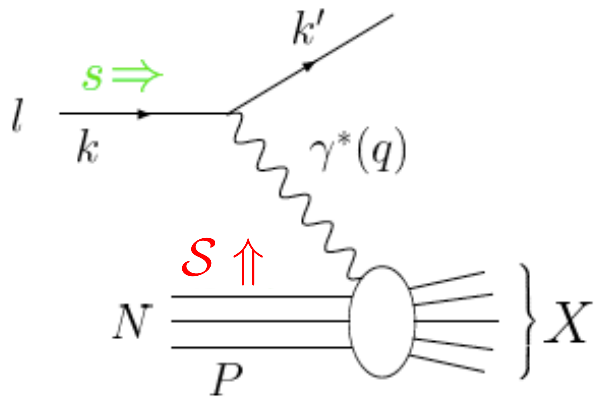
No other machine in the world can achieve this!

- Ultimate solution to the proton spin puzzle:
  - *Precision measurement of $\Delta g(x)$ – extend to smaller x regime*
  - *Orbital angular momentum contribution – measurement of GPDs!*

Transverse single-spin asymmetry (TSSA)

- Over 50 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- **DIS cross section:** $\sigma(\vec{s}_\perp) \propto L^{\mu\nu} W_{\mu\nu}(\vec{s}_\perp)$
- **Leptionic tensor is symmetric:**

$$L^{\mu\nu} = L^{\nu\mu}$$

- **Hadronic tensor:**

$$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle$$

- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right]$$

- **Vanishing single spin asymmetry:**

$$A_N = 0 \quad \Leftrightarrow \quad \langle P, \vec{s}_\perp | j^\dagger_\mu(0)\, j_\nu(y) | P, \vec{s}_\perp \rangle \overset{?}{=} \langle P, -\vec{s}_\perp | j^\dagger_\nu(0)\, j_\mu(y) | P, -\vec{s}_\perp \rangle$$

A_N for inclusive DIS

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P, \vec{s}_\perp | \, j^\dagger_\mu(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P, \vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T\,|P, \vec{s}_\perp\rangle = |-P, -\vec{s}_\perp\rangle$$

$$\begin{aligned} |\beta_T\rangle &= V_T\,\left[j^\dagger_\mu(0)\, j_\nu(y)\right]^\dagger |P, \vec{s}_\perp\rangle \\ &= \left(V_T j^\dagger_\nu(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle \end{aligned}$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\, V_T^\dagger\, V_T\, |\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\Longrightarrow \langle -P, -\vec{s}_\perp|\,\left(V_T j^\dagger_\nu(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right) |-P, -\vec{s}_\perp\rangle$$

$$= \langle P, \vec{s}_\perp|\, j^\dagger_\mu(0)\, j_\nu(y)\, |P, \vec{s}_\perp\rangle$$

A_N for inclusive DIS

- **Parity invariance:**

$$1 = U_P^{-1} U_P = U_P^{\dagger} U_P$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp\rangle$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp\rangle$$

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(-y)\, j_\mu(0) | P, -\vec{s}_\perp\rangle$$

Translation invariance:

$$\langle P, -\vec{s}_\perp | j_\nu^\dagger(0)\, j_\mu(y) | P, -\vec{s}_\perp\rangle = \langle P, \vec{s}_\perp | j_\mu^\dagger(0)\, j_\nu(y) | P, \vec{s}_\perp\rangle$$

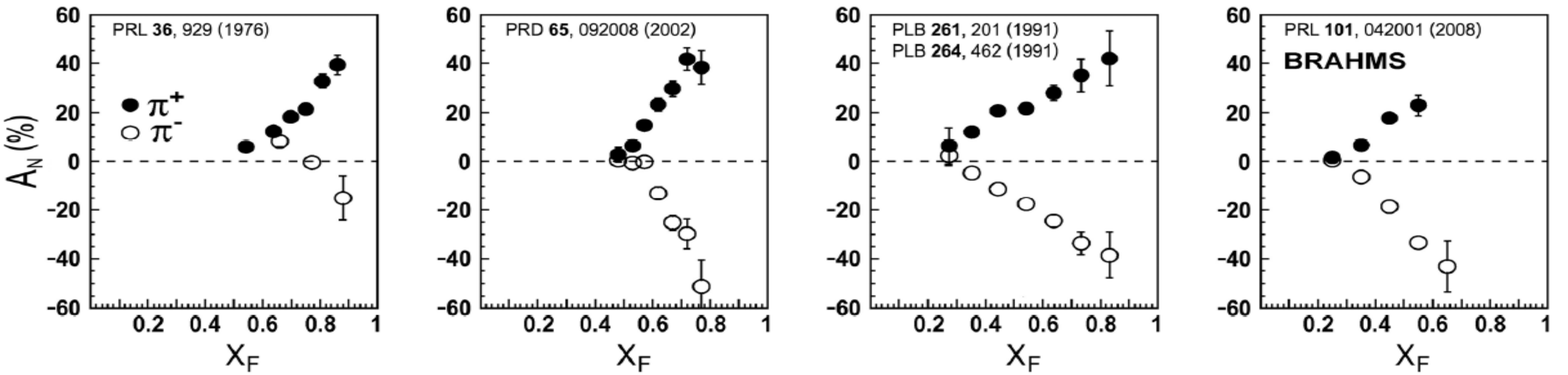
- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right] = L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp)\right] = 0$$

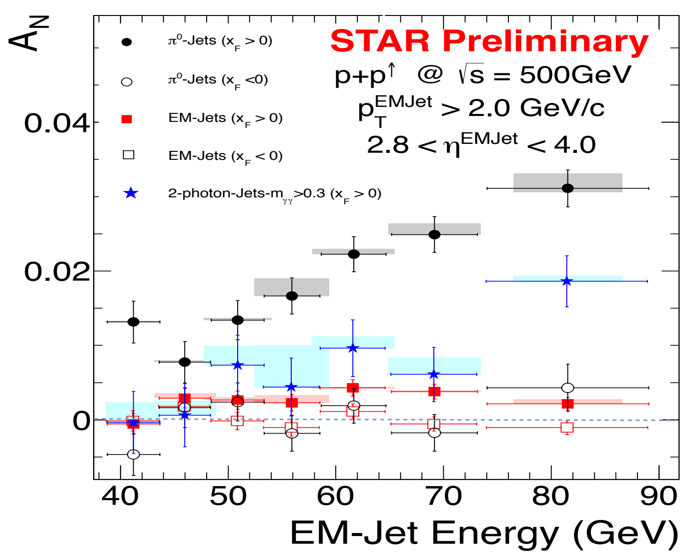
A_N in hadronic collisions

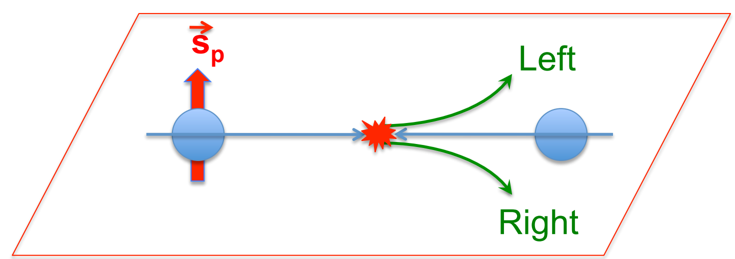
- A_N - consistently observed for over 35 years!

ANL – 4.9 GeV **BNL – 6.6 GeV** **FNAL – 20 GeV** **BNL – 62.4 GeV**



- Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

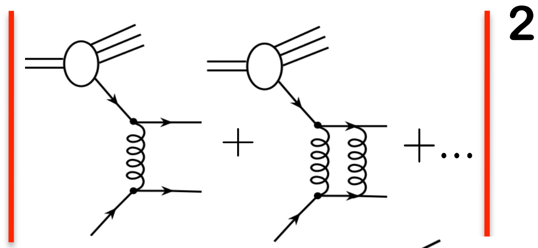
Do we understand this?

Do we understand it?

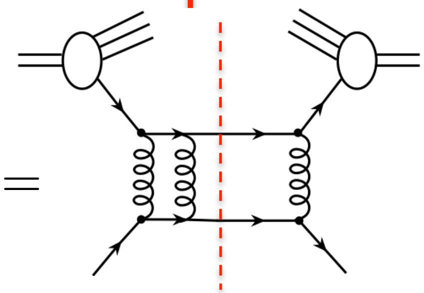
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$



$\propto \alpha_s \frac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

❑ Vanish without parton's transverse motion:

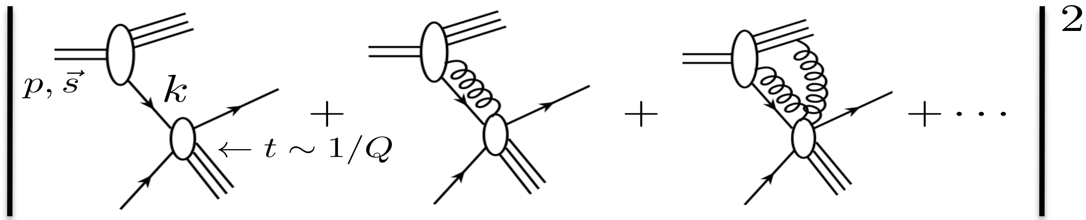
➡ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

How collinear factorization generates TSSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q, \vec{s}) \propto$



$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete! (↑ f_2)

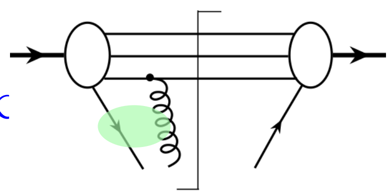
Three-parton correlation (↑ f_3)

❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

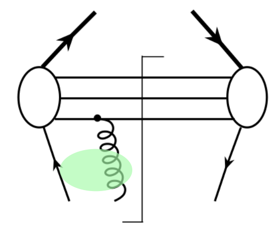
$$\Delta\sigma(s_T) \propto T^{(3)}(x, x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \ldots$$

$T^{(3)}(x, x) \propto$



Qiu, Sterman, 1991, …

$D^{(3)}(z, z) \propto$



Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: Integration of "dx" using unpinched poles

Twist-3 distributions relevant to A_N

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

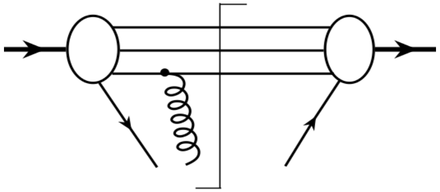
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P,S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P,S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P,S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{T}_{q,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{T}^{(f,d)}_{G,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int\frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

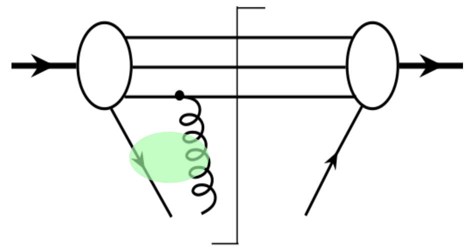
❑ Twist-3 fragmentation functions:

See Kang, Yuan, Zhou, 2010, Kang 2010

"Interpretation" of twist-3 correlation functions

❑ Measurement of direct QCD quantum interference:

Qiu, Sterman, 1991, …

$T^{(3)}(x, x, S_\perp) \;\propto$ 

Interference between a single active parton state and an active two-parton composite state

❑ "Expectation value" of QCD operators:

$$\langle P,s|\overline{\psi}(0)\gamma^+\psi(y^-)|P,s\rangle \;\rightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[\epsilon_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

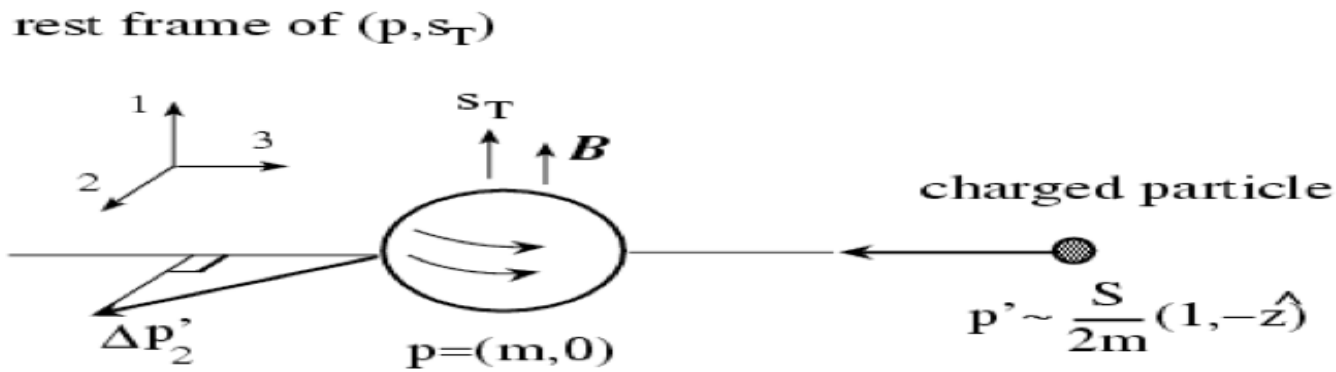
$$\langle P,s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P,s\rangle \;\rightarrow\; \langle P,s|\overline{\psi}(0)\gamma^+\left[i\,g_\perp^{\alpha\beta}s_{T\alpha}\int dy_2^- F_\beta{}^+(y_2^-)\right]\psi(y^-)|P,s\rangle$$

How to interpret the "expectation value" of the operators in **RED**?

A simple example

❑ The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



❑ Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

❑ In the c.m. frame:

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}$$

❑ The total change:

$$\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Test QCD at twist-3 level

Kang, Qiu, 2009

❑ Scaling violation – "DGLAP" evolution:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix} = \begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix} \otimes \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

$(x, x+x_2, \mu, s_T)$ $\qquad$ $(\xi, \xi+\xi_2; x, x+x_2, \alpha_s)$ $\qquad$ $\int d\xi \int d\xi_2$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

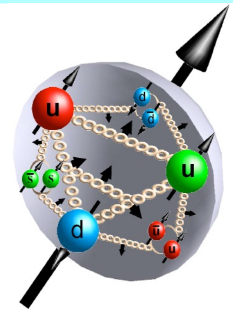
Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

Introduction to QCD

Jianwei Qiu
Theory Center, Jefferson Lab
May 31 – June 2, 2017

Lecture five/six

3D structure of hadrons



❑ What do we need to know for the structure?

✧ **In theory:** $\langle P,S|\mathcal{O}(\overline{\psi},\psi,A^\mu)|P,S\rangle$ – Hadronic matrix elements

with all possible operators: $\mathcal{O}(\overline{\psi},\psi,A^\mu)$

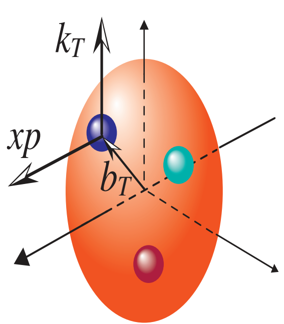
✧ **In fact:** *None of these matrix elements is a direct physical observable in QCD – color confinement!*

✧ **In practice:** Accessible hadron structure
= hadron matrix elements of quarks and gluons, which

1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
2) can be calculated in lattice QCD

❑ Single-parton structure "seen" by a short-distance probe:

✧ **5D structure:**

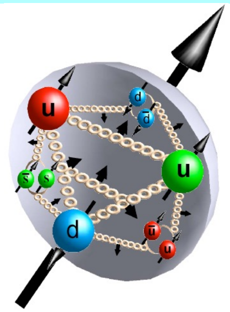


1) $\int d^2b_T$ ➡ $f(x,k_T,\mu)$ – **TMDs:** *2D confined motion!*

2) $\int d^2k_T$ ➡ $F(x,b_T,\mu)$ – **GPDs:** *2D spatial imaging!*

3) $\int d^2k_T d^2b_T$ ➡ $f(x,\mu)$ – **PDFs:** *Number density!*

3D structure of hadrons



❑ What do we need to know for the structure?

✧ **In theory:** $\langle P, S | \mathcal{O}(\overline{\psi}, \psi, A^\mu) | P, S \rangle$ – Hadronic matrix elements

with all possible operators: $\mathcal{O}(\overline{\psi}, \psi, A^\mu)$

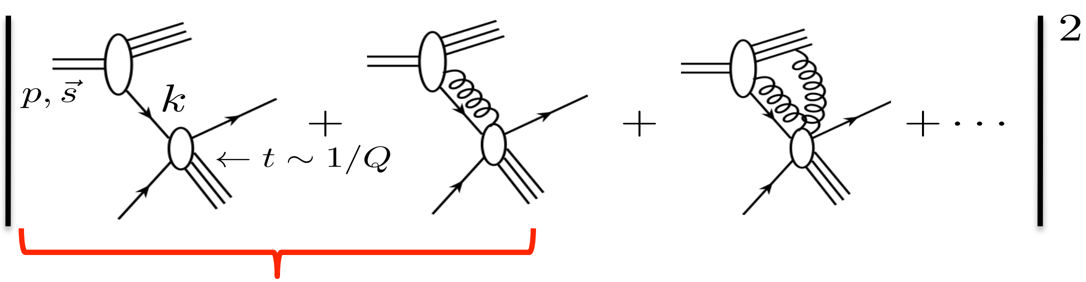
✧ **In fact:** *None of these matrix elements is a direct physical observable in QCD – color confinement!*

✧ **In practice:** Accessible hadron structure
= hadron matrix elements of quarks and gluons, which

1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
2) can be calculated in lattice QCD

❑ Multi-parton correlations:

$\sigma(Q, \vec{s}) \propto$



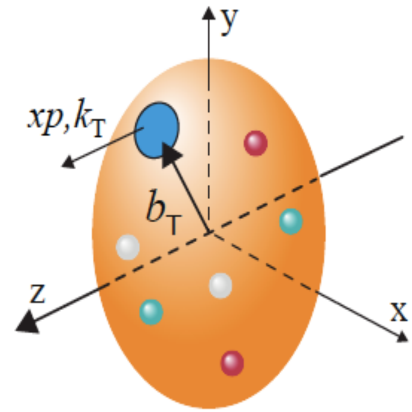
$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

Quantum interference → 3-parton matrix element – not a probability!

Two-momentum-scale observables

❑ Cross sections with two-momentum scales observed:

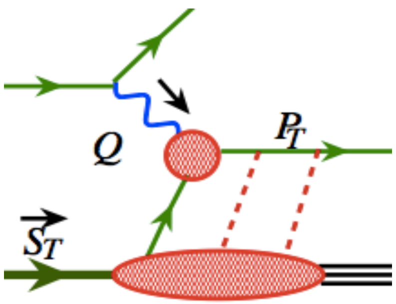
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$



✧ **Hard scale:** Q_1 localizes the probe to see the quark or gluon d.o.f.

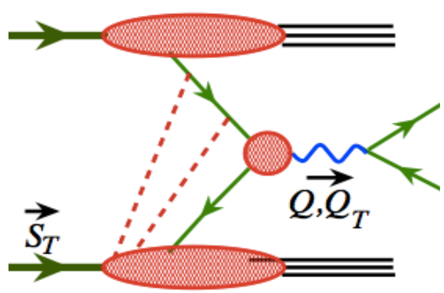
✧ **"Soft" scale:** Q_2 could be more sensitive to hadron structure, e.g., confined motion

❑ Two-scale observables with the hadron broken:



SIDIS: $Q \gg P_T$

\+



DY: $Q \gg P_T$

\+ Two-jet momentum imbalance in SIDIS, …

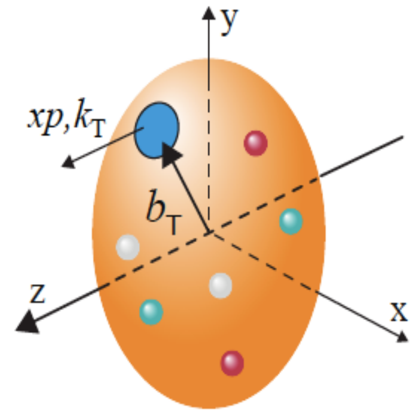
✧ Natural observables with TWO very different scales

✧ TMD factorization: partons' confined motion is encoded into TMDs

Two-momentum-scale observables

❑ Cross sections with two-momentum scales observed:

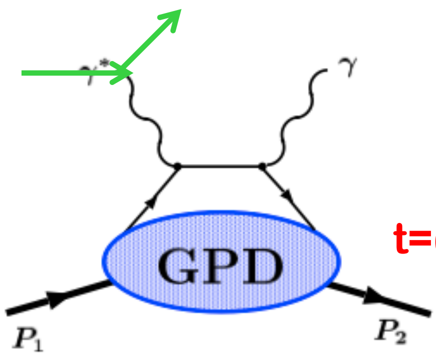
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\mathrm{QCD}}$$



✧ **Hard scale:** Q_1 **localizes the probe to see the quark or gluon d.o.f.**

✧ **"Soft" scale:** Q_2 **could be more sensitive to hadron structure, e.g., confined motion**

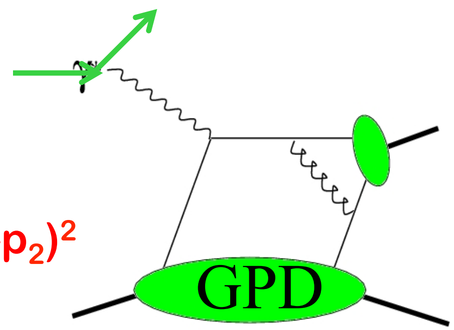
❑ Two-scale observables with the hadron unbroken:



DVCS: $Q^2 >> |t|$

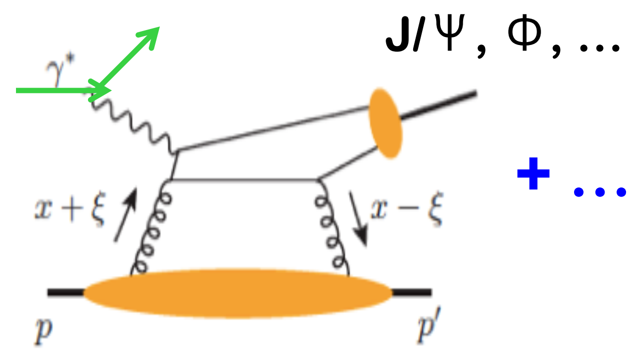
+

$t=(p_1-p_2)^2$



DVEM: $Q^2 >> |t|$

+



EHMP: $Q^2 >> |t|$

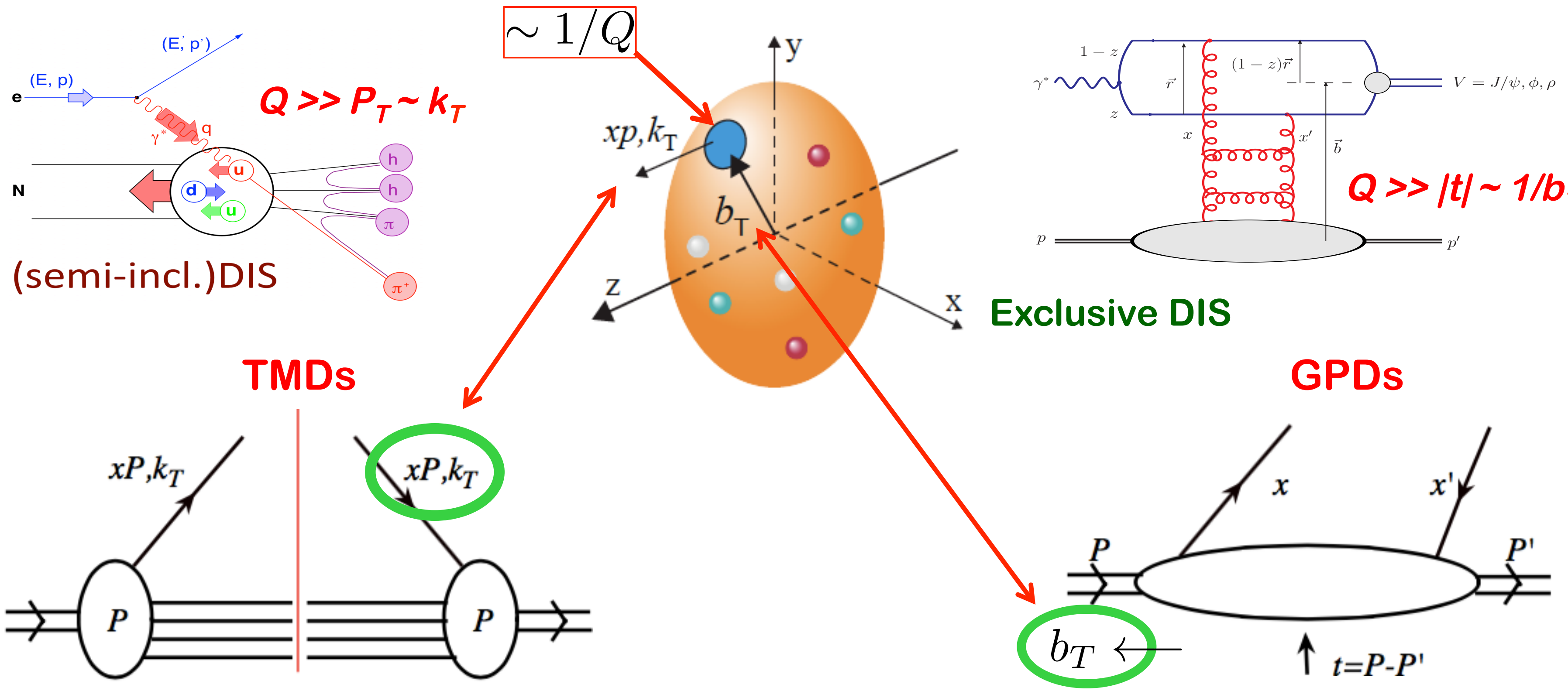
+ …

✧ Natural observables with TWO very different scales

✧ GPDs: Fourier Transform of t-dependence gives spatial b_T-dependence

How to quantify the hadron structure?

❑ Quantifying the "structure":



$$\propto \langle P|\overline{\psi}\ [\mathrm{Spin}]\ \psi|P\rangle$$

$$\propto \langle P'|\overline{\psi}\ [\mathrm{Spin}]\ \psi|P\rangle$$

Cross sections

Two-scales observables

Amplitudes

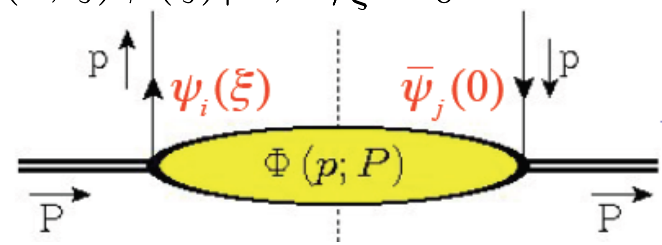
- ✧ ***Confined transverse motion***
- ✧ ***Confined spatial distribution – imagining***

Definition of TMDs

❑ Non-perturbative definition:

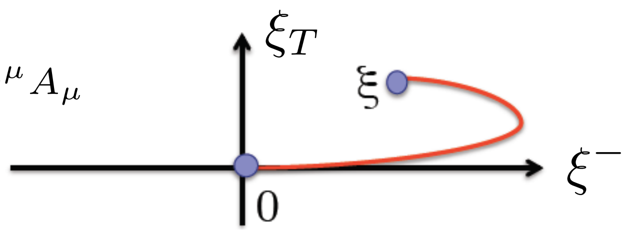
✧ **In terms of matrix elements of parton correlators:**

$$\Phi^{[U]}(x,p_T;n) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,p\cdot\xi}\, \langle P,S|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P,S\rangle_{\xi^+=0}$$



✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$



✧ **Decomposes into a list of TMDs:**

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + ih_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

❑ Gives "unique" TMDs, IF we knew proton wave function!

But, we do NOT know proton wave function (calculate it on lattice?)

TMDs are NOT direct physical observables!

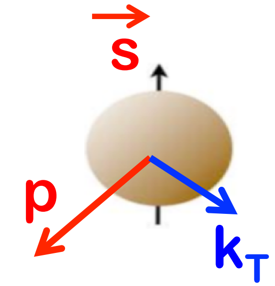
TMDs: confined motion, its spin correlation

- ❑ Power of spin – many more correlations:

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 =		$h_1^{\perp}$ = − (Boer-Mulders)
L		g_{1L} = − (Helicity)	$h_{1L}^{\perp}$ = −
T	$f_{1T}^{\perp}$ = − (Sivers)	$g_{1T}^{\perp}$ = −	h_1 = − (Transversity); $h_{1T}^{\perp}$ = −

Nucleon Spin · Quark Spin

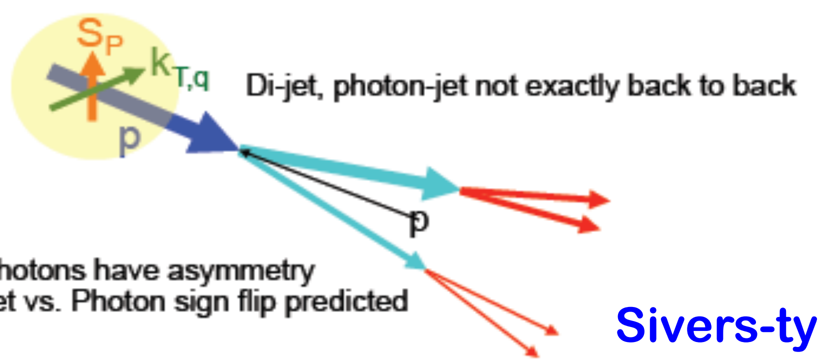
Similar for gluons



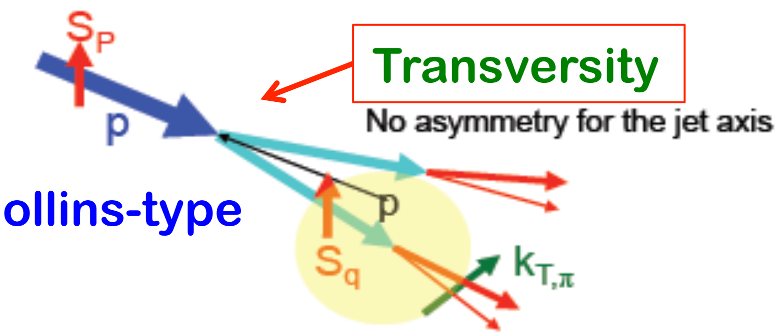
Require two Physical scales

More than one TMD contribute to the same observable!

- ❑ A_N – single hadron production:



Sivers-type

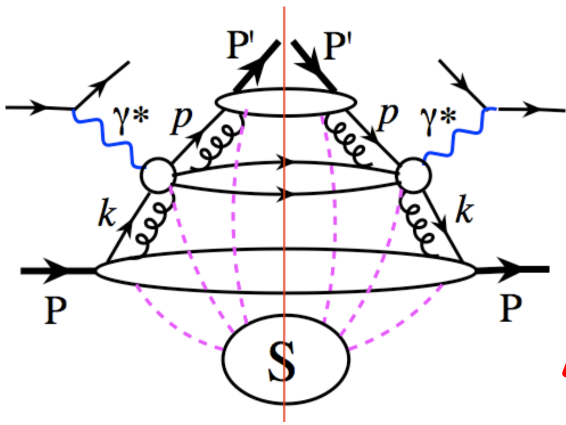


Collins-type

TMDs extracted from data

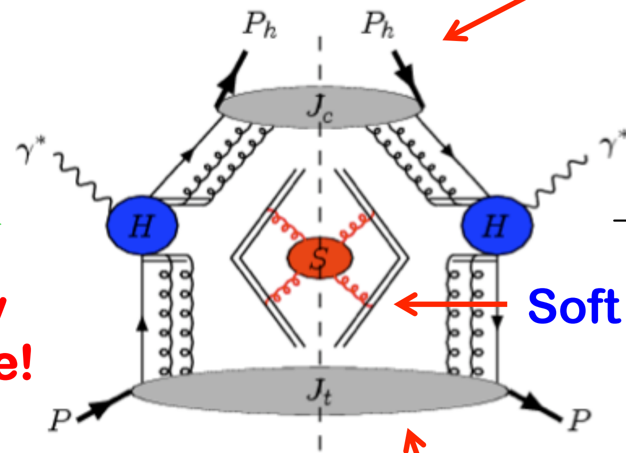
❑ **Perturbative definition – in terms of TMD factorization:**

SIDIS as an example:



Theory Advance!

TMD fragmentation



$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ **Extraction of TMDs:**

$$\sigma_{\text{SIDIS}}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

TMDs are extracted by fitting DATA using the factorization formula

✧ **Depending on the perturbatively calculated** $\hat{H}(Q;\mu)$
perturbative orders, renormalization, factorization schemes, …

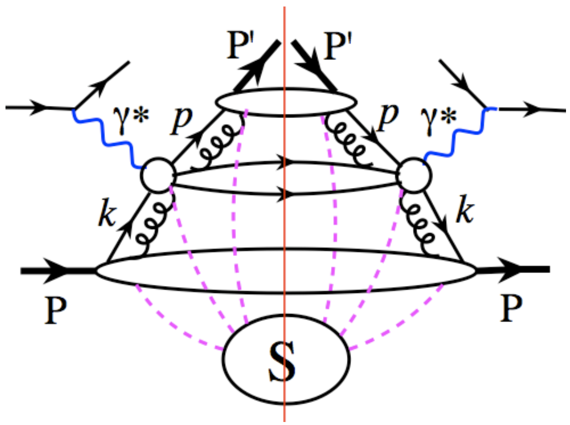
✧ **Depending on the approximation of neglecting the power corrections, ..**

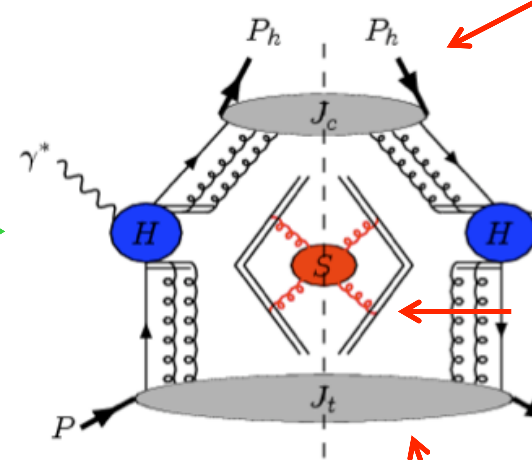
➡ ***Importance of lattice QCD calculations, …***

TMDs extracted from data

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





TMD fragmentation

$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle p^2\rangle}{Q^2}\right)$$

Soft factors

TMD parton distribution

❑ Low P_{hT} – TMD factorization:

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

❑ High P_{hT} – Collinear factorization:

$$\sigma_{\rm SIDIS}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

❑ P_{hT} Integrated - Collinear factorization:

$$\sigma_{\rm SIDIS}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

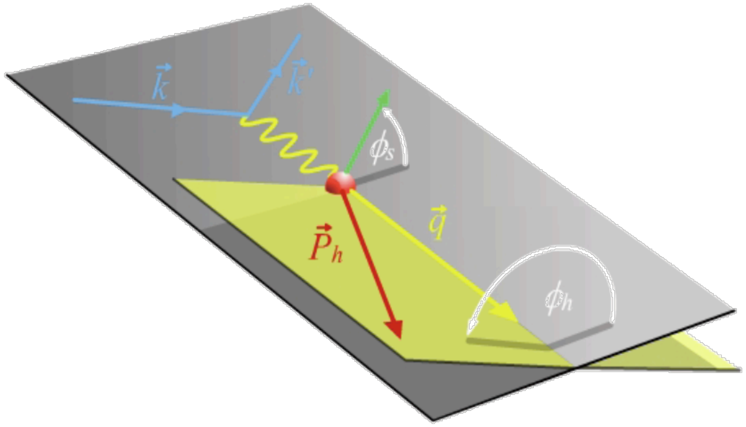
SIDIS is the best for probing TMDs

- **Naturally, two scales & two planes:**

$$A_{UT}(\varphi_h^l,\varphi_S^l) = \frac{1}{P}\frac{N^{\uparrow}-N^{\downarrow}}{N^{\uparrow}+N^{\downarrow}}$$

$$= A_{UT}^{Collins}\sin(\phi_h+\phi_S) + A_{UT}^{Sivers}\sin(\phi_h-\phi_S)$$

$$+ A_{UT}^{Pretzelosity}\sin(3\phi_h-\phi_S)$$



- **Separation of TMDs:**

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h+\phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h-\phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h-\phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

Collins frag. Func. from e^+e^- collisions

Hard, if not impossible, to separate TMDs in hadronic collisions

Using a combination of different observables (not the same observable): jet, identified hadron, photon, …

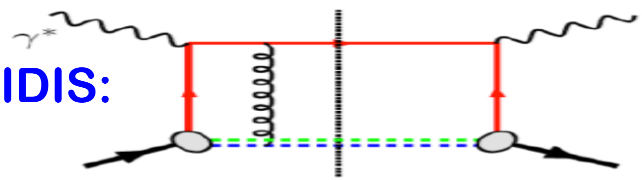
Modified universality for TMDs

❑ Definition:

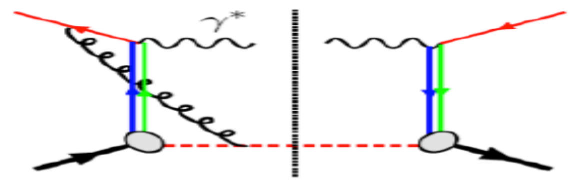
$$f_{q/h^\uparrow}(x, \mathbf{k}_\perp, \vec{S}) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\, \mathbf{k}_\perp \cdot \mathbf{y}_\perp} \langle p, \vec{S} | \overline{\psi}(0^-, \mathbf{0}_\perp) \boxed{\text{Gauge link}} \frac{\gamma^+}{2} \psi(y^-, \mathbf{y}_\perp) | p, \vec{S} \rangle$$

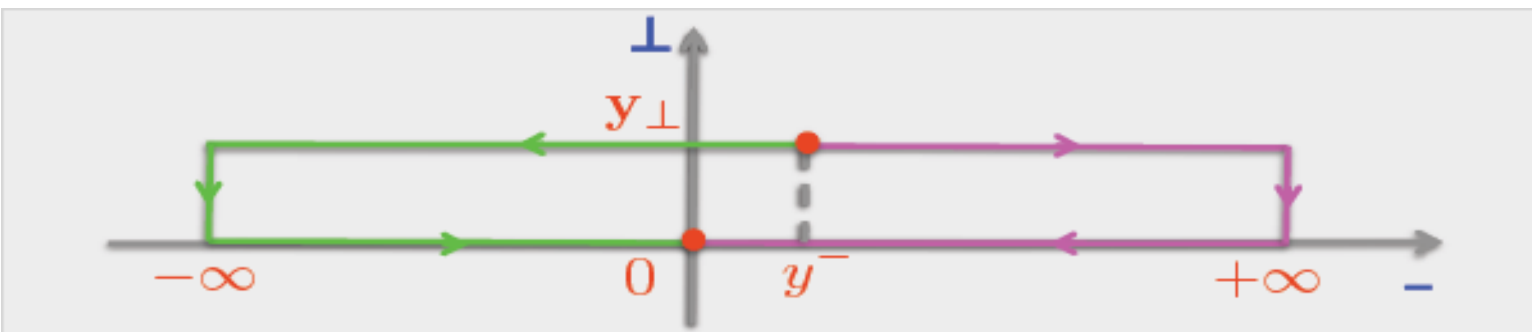
❑ Gauge links:

SIDIS:



DY:





❑ Process dependence:

$$f_{q/h^\uparrow}^{\text{SIDIS}}(x, \mathbf{k}_\perp, \vec{S}) \neq f_{q/h^\uparrow}^{\text{DY}}(x, \mathbf{k}_\perp, \vec{S})$$

Collinear factorized PDFs are process independent

Critical test of TMD factorization

- **Parity – Time reversal invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- **Definition of Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- **Modified universality:**

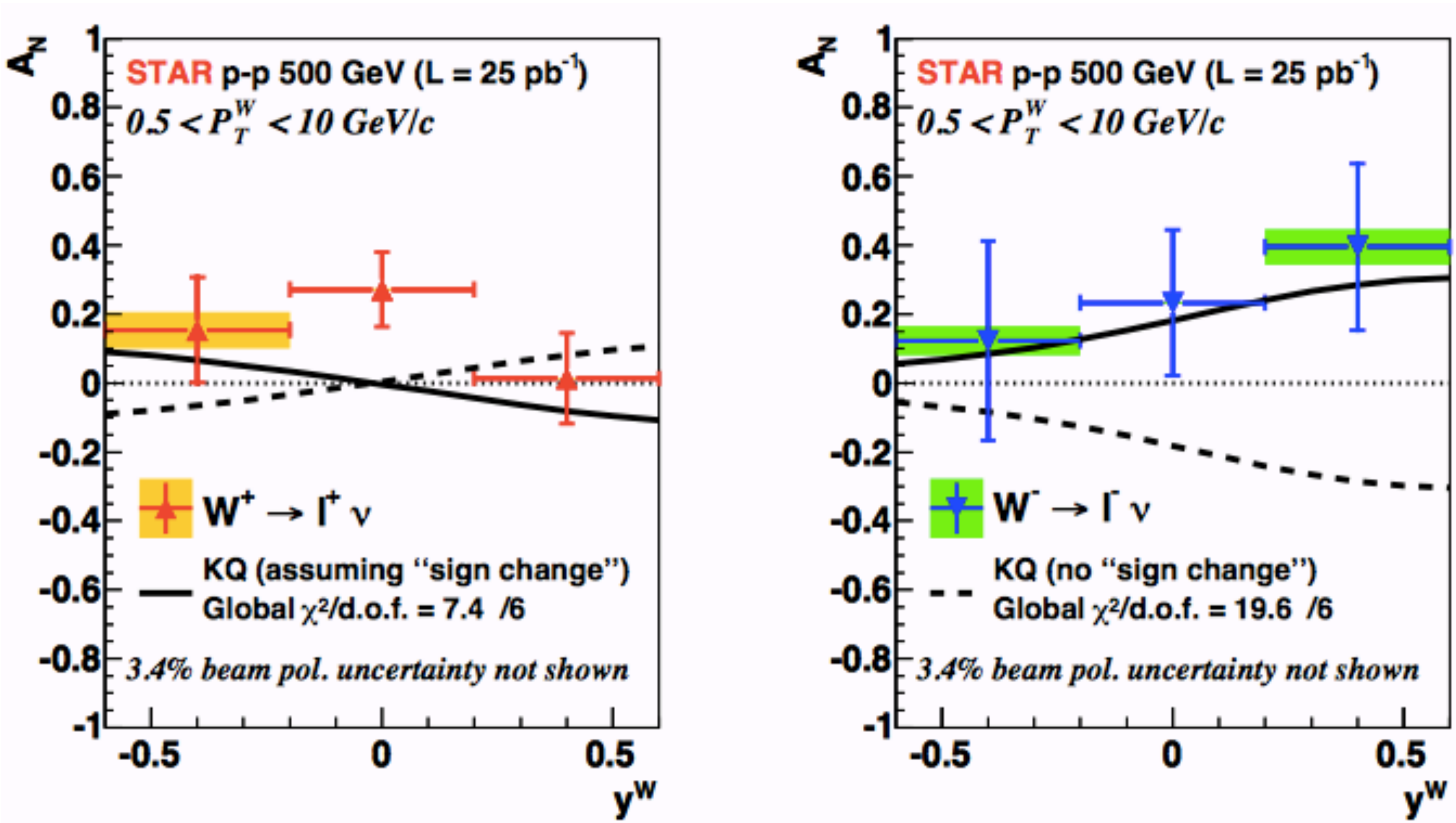
$$\Delta^N f_{q/h^\uparrow}^{\rm SIDIS}(x,k_\perp) = -\Delta^N f_{q/h^\uparrow}^{\rm DY}(x,k_\perp)$$

The spin-averaged part of this TMD is process independent,

but, spin-averaged Boer-Mulder's TMD requires the sign change!

Same PT symmetry examination needs for TMD gluon distributions!

Hint of the sign change: A_N of W production



Data from STAR collaboration on A_N for W-production are consistent with a sign change between SIDIS and DY

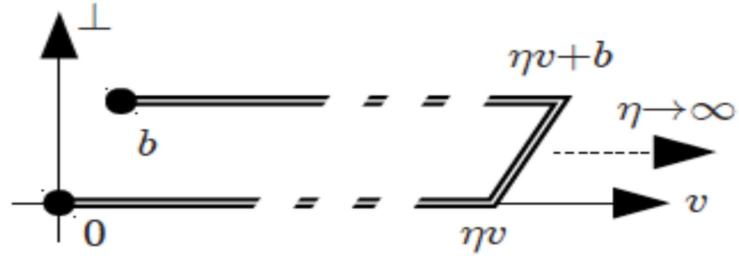
STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)

Hint of the sign change from lattice QCD

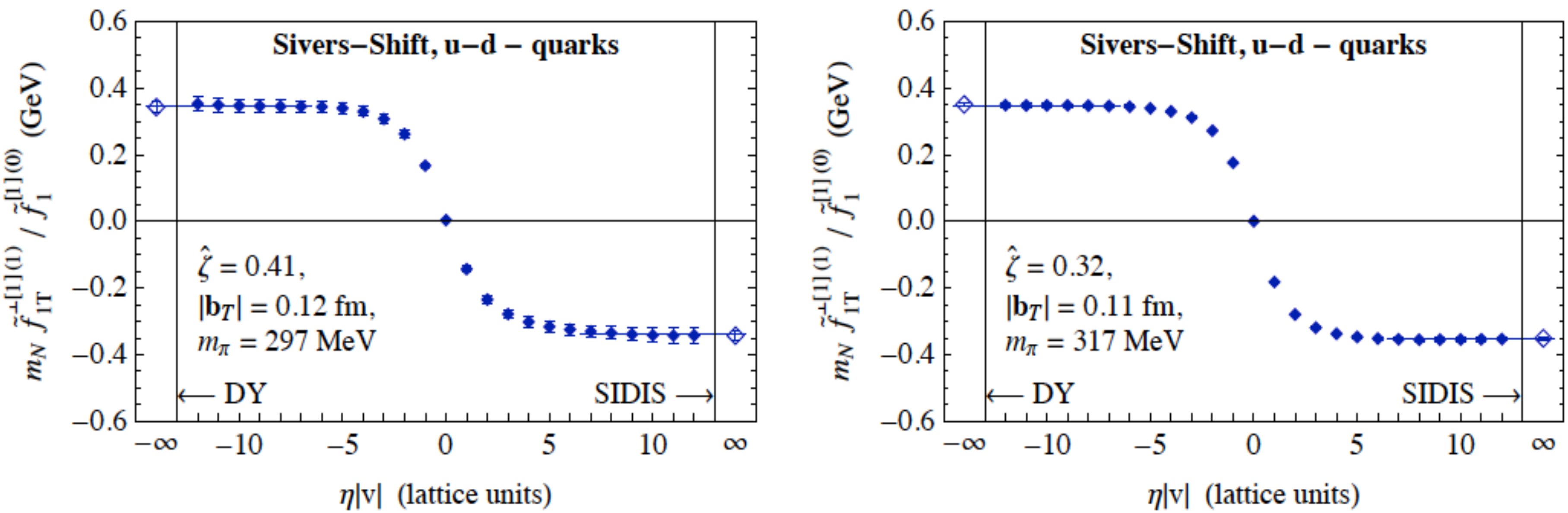
Engelhardt@TMD Collaboration meeting

❑ Gauge link for lattice calculation:

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



❑ Normalized moment of Sivers function – at given b_T:



Hint of the sign change from lattice QCD

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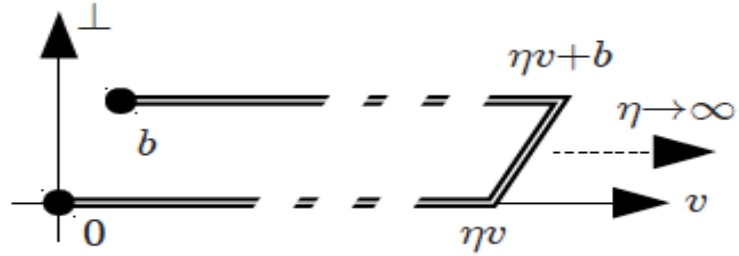
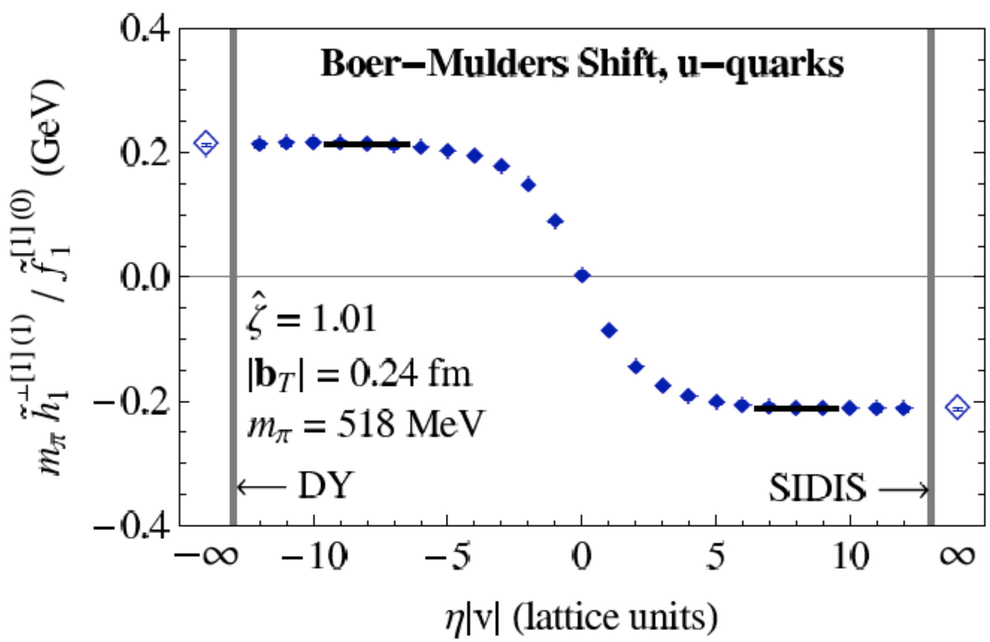


- Normalized moment of Boer-Mulders function – at given b_T:

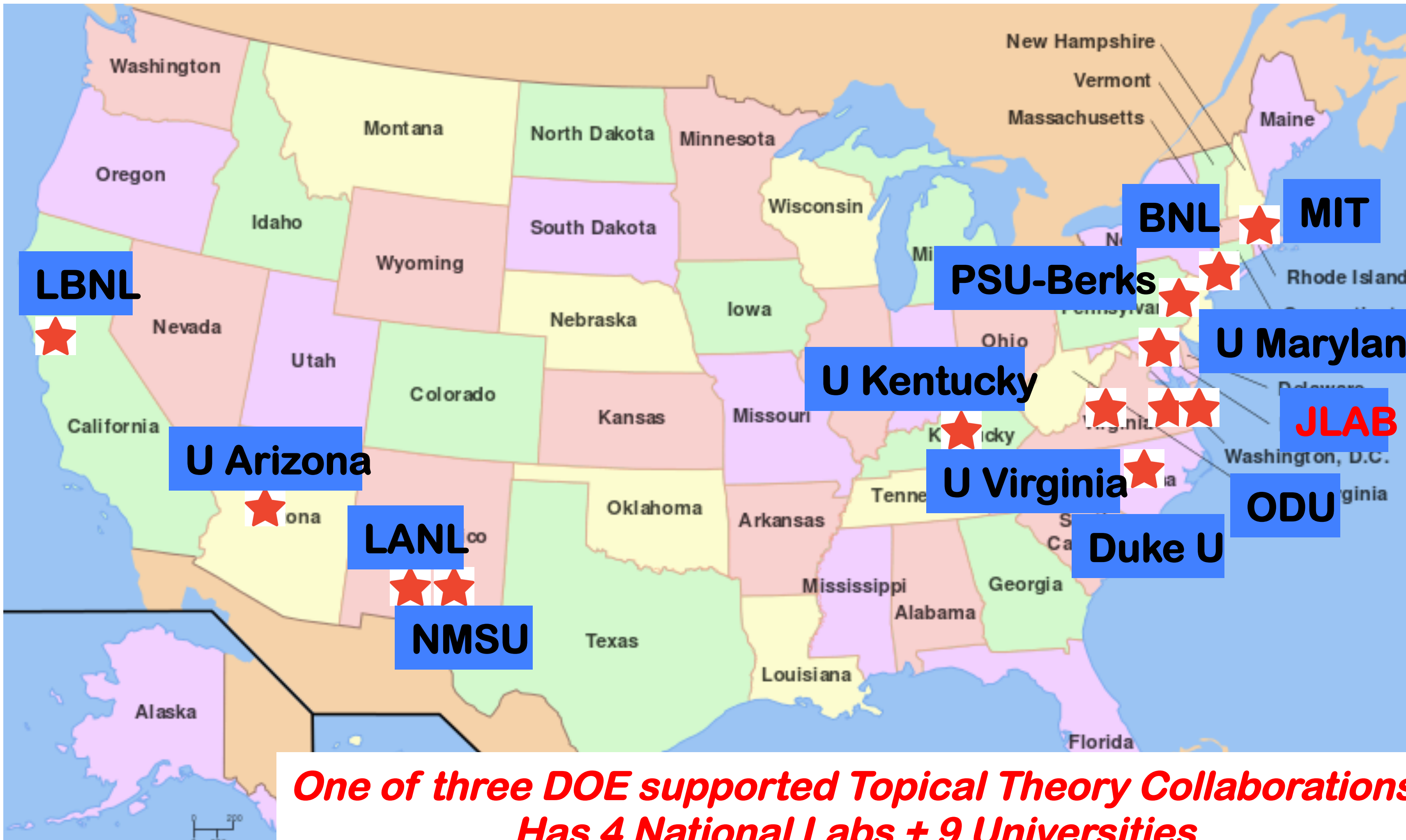


TMD Topical Theory Collaboration

Coordinated Theoretical Approach to Transverse Momentum Dependent Hadron Structure in QCD (TMD Collaboration)

Co-spokespersons: W. Detmold, J.W. Qiu

TMD Collaboration



One of three DOE supported Topical Theory Collaborations
Has 4 National Labs + 9 Universities

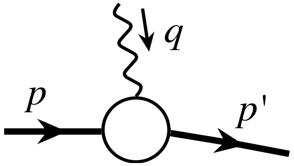
Proton's radius in color distribution?

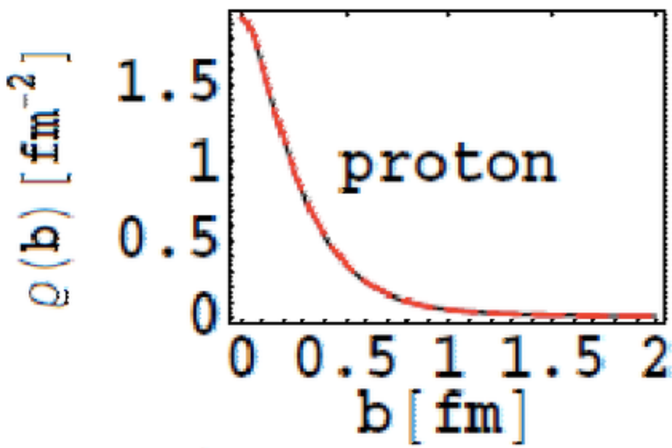
- ❑ **The "big" question:**

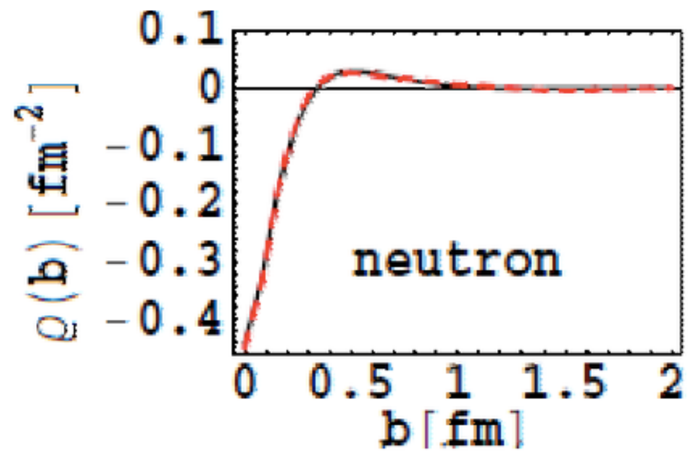
  How color is distributed inside a hadron? (clue for color confinement?)

- ❑ **Electric charge distribution:**

  Elastic electric form factor ➡ Charge distributions

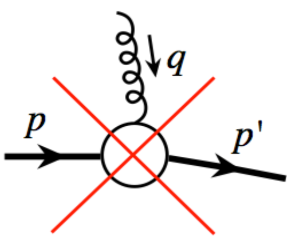






- ❑ **But, NO color elastic nucleon form factor!**

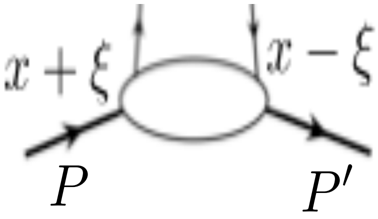
  Hadron is colorless and gluon carries color



➡ *Parton density's spatial distributions – a function of x as well (more "proton"-like than "neutron"-like?) – GPDs*

Definition of GPDs

❑ **Quark "form factor":**



$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' | \bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2) | P\rangle$$

$$\equiv H_q(x,\xi,t,\mu^2)\,[\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P)]\,\frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M}\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2 \;\; \text{and} \;\; t=(P'-P)^2 \;\Rightarrow\; -\Delta_\perp^2 \;\; \text{if} \;\; \xi\to 0$

$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

❑ **Total quark's orbital contribution to proton's spin:** Ji, PRL78, 1997

$$J_q = \frac{1}{2}\lim_{t\to 0}\int dx\, x\,[H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

❑ **Connection to normal quark distribution:**

$$H_q(x,0,0,\mu^2) = q(x,\mu^2)$$ **The limit when** $\xi\to 0$

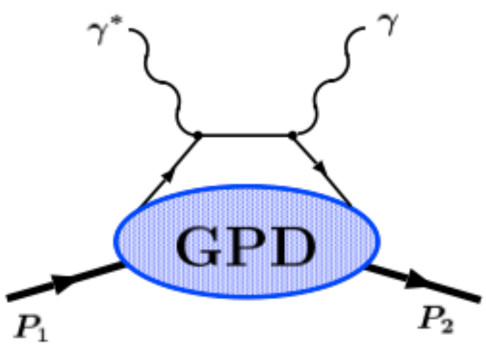
Exclusive DIS: Hunting for GPDs

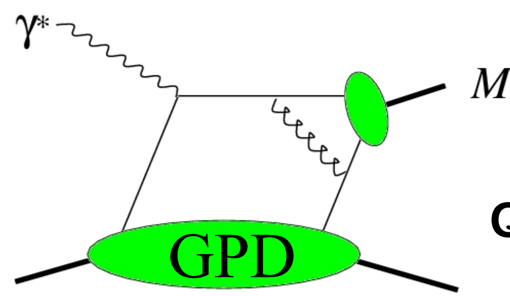
- **Experimental access to GPDs:**

Mueller et al., 94;
Ji, 96;
Radyushkin, 96

  - Diffractive exclusive processes – high luminosity:

    DVCS: Deeply virtual Compton Scattering
    DVEM: Deeply virtual exclusive meson production





Require

$Q^2 >> (-t), \Lambda^2_{QCD}, M^2$

  - No factorization for hadronic diffractive processes – EIC is ideal

- **Much more complicated – (x, ξ, t) variables:**

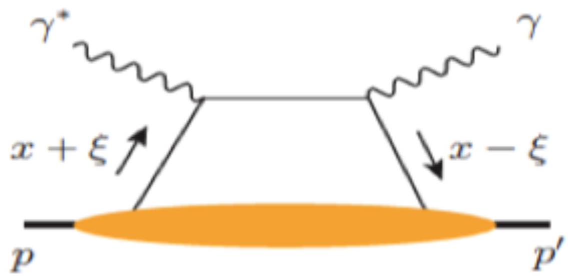
  Challenge to derive GPDs from data

- **Great experimental effort:**

  HERA, HERMES, COMPASS, JLab → JLab12, COMPASS-II, EIC

Deep virtual Compton scattering

- **The LO diagram:**



$$\xi \;=\; Q^2/(2\bar{P}\cdot q)$$
$$P' = P + \Delta$$

- **Scattering amplitude:**

$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int dx \left(\frac{1}{x-\xi/2+i\epsilon} + \frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times \left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P) + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int dx \left(\frac{1}{x-\xi/2+i\epsilon} - \frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times \left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5 U(P)\right]$$

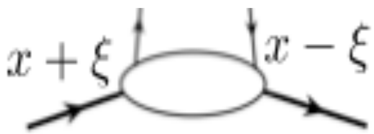
- **GPDs:**

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P)$$
$$+E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + \ldots$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu\gamma_5\psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu\gamma_5 U(P)$$
$$+\tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P) + \ldots$$

What can GPDs tell us?

- **GPDs of quarks and gluons:**

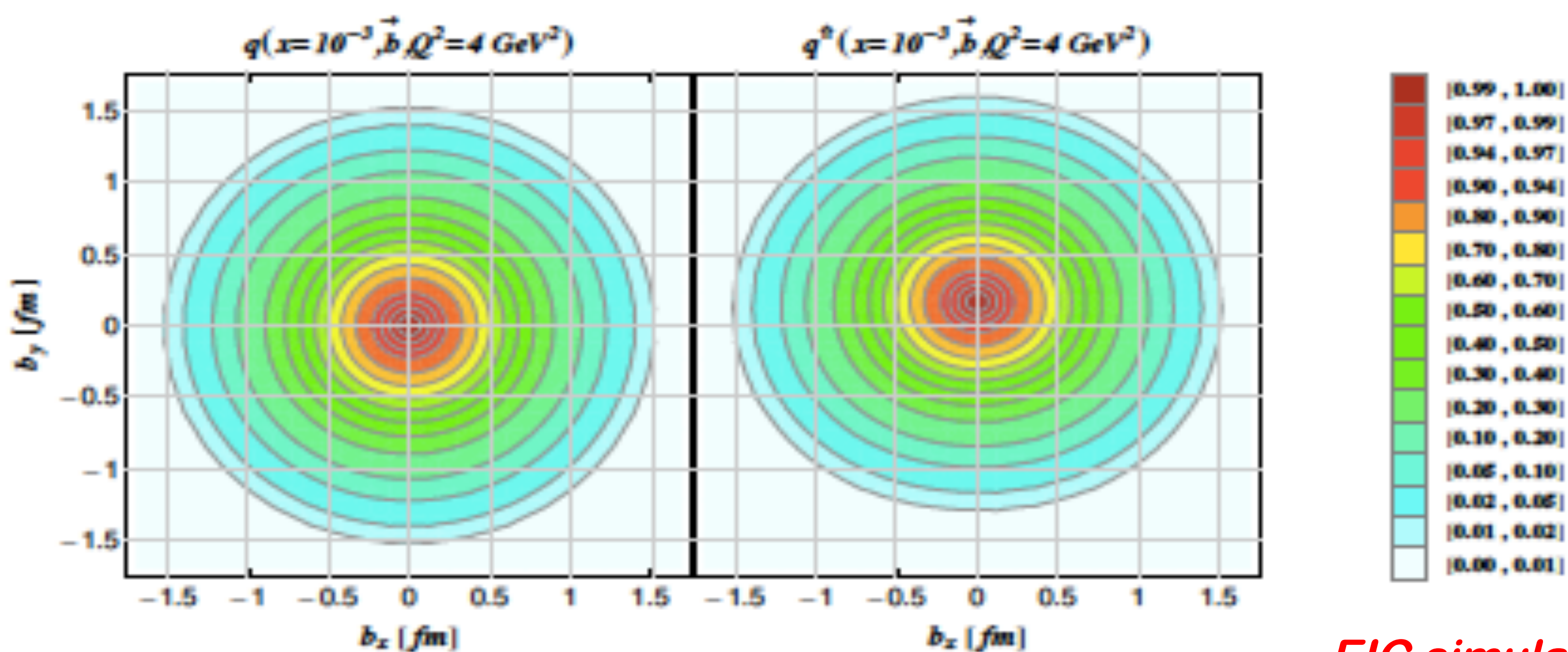


$$H_q(x,\xi,t,Q), \quad E_q(x,\xi,t,Q),$$
$$\tilde{H}_q(x,\xi,t,Q), \quad \tilde{E}_q(x,\xi,t,Q)$$

Evolution in Q
– gluon GPDs

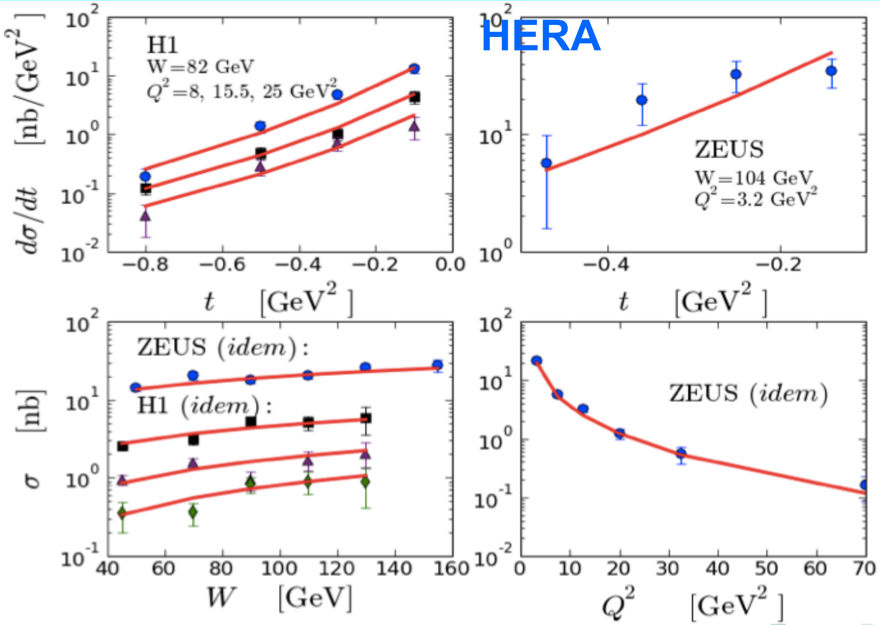
- **Imaging ($\xi \to 0$):** $q(x,b_\perp,Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x,\xi=0,t=-\Delta_\perp^2,Q)$

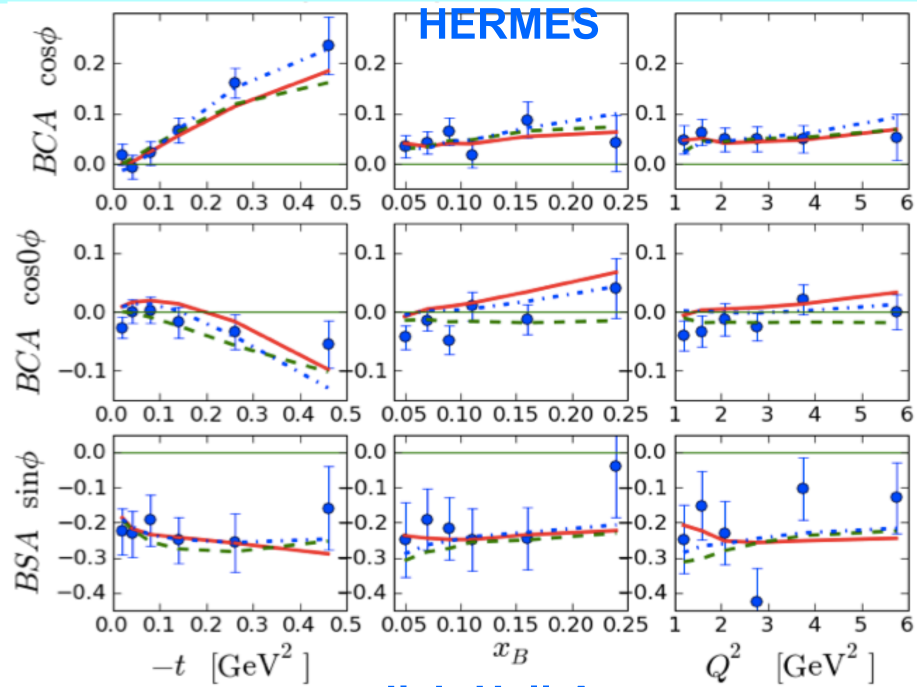
- **Influence of transverse polarization – shift in density:**

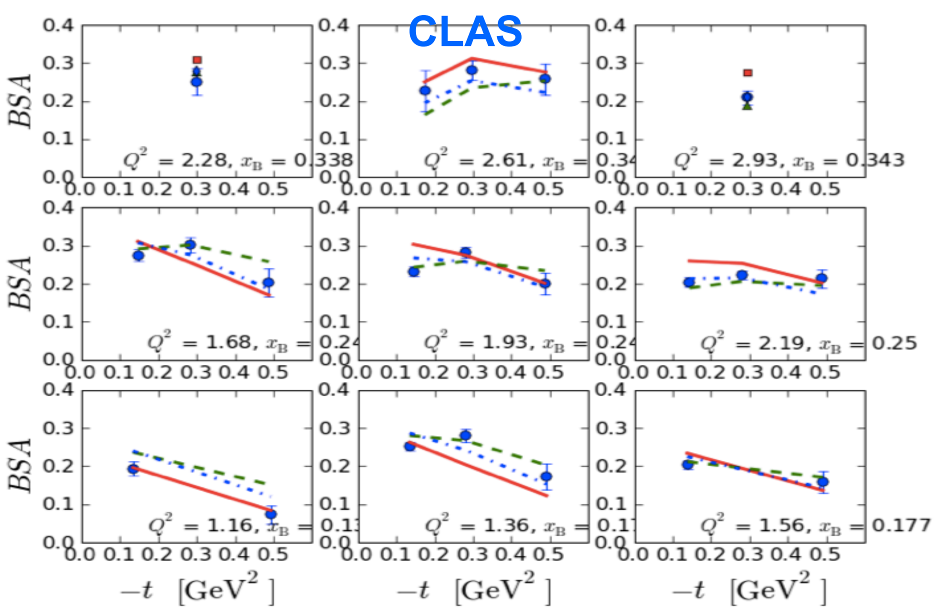


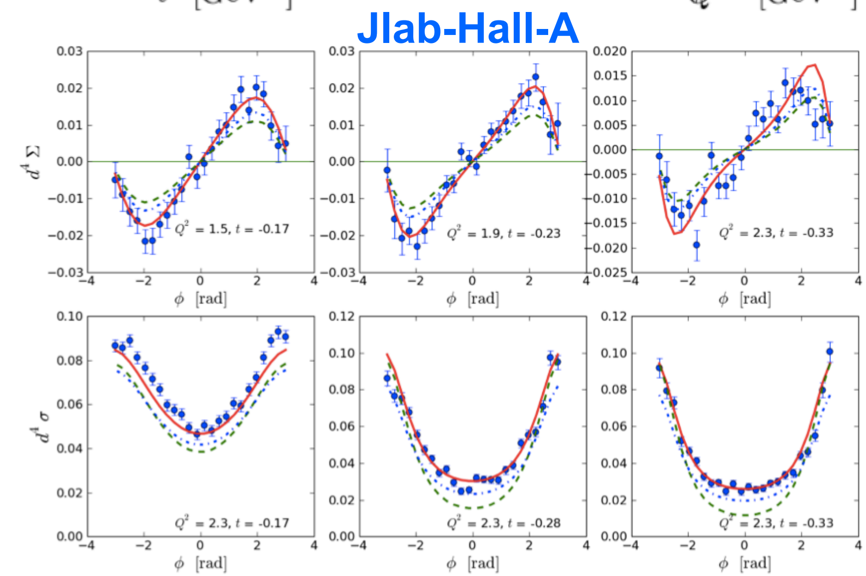
EIC simulation

GPDs: just the beginning



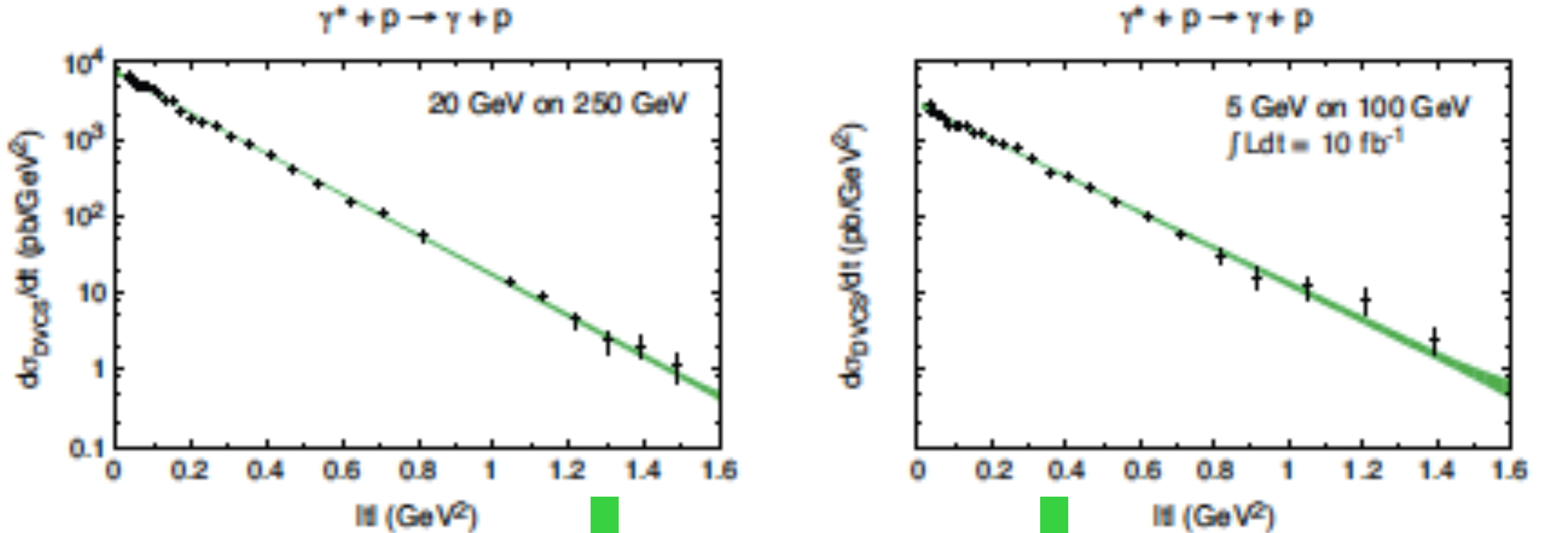




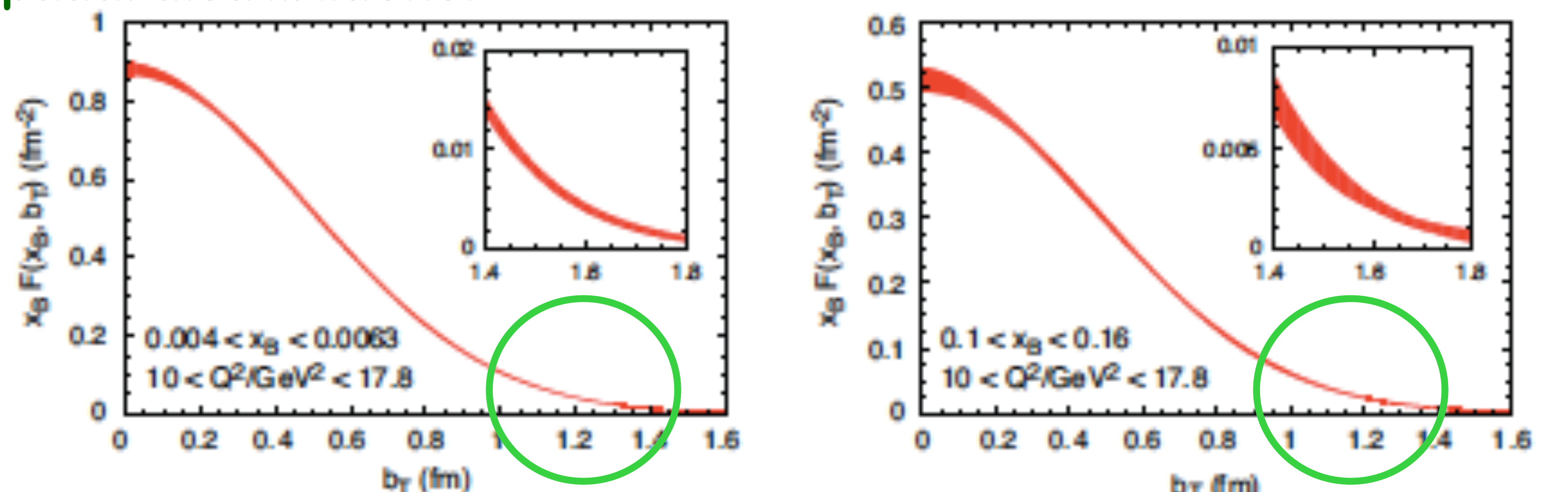


DVCS @ EIC

- ❑ **Cross Sections:**



- ❑ **Spatial distributions:**



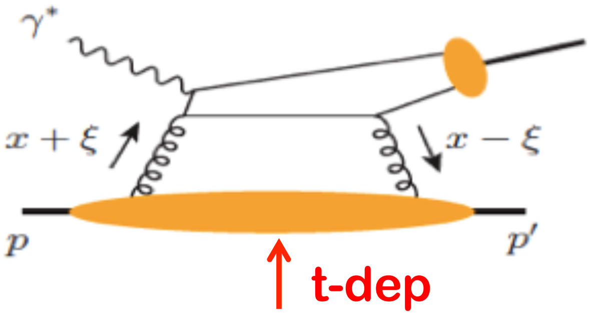
Quark radius (x)!

Spatial distribution of gluons

❑ **Exclusive vector meson production:**

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



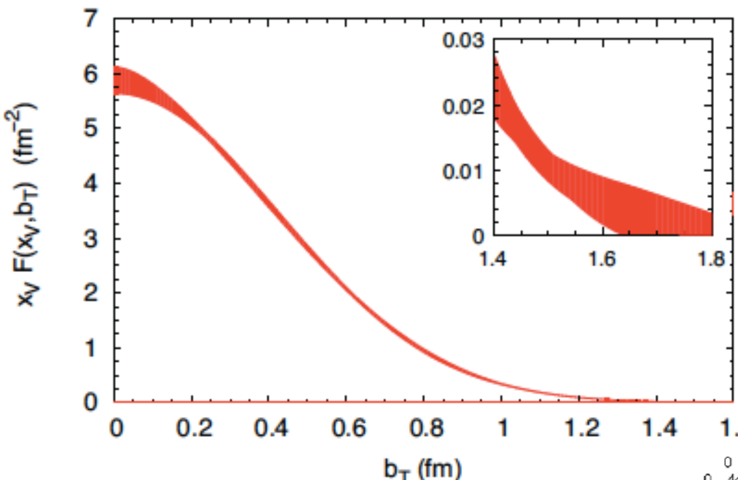
J/Ψ, Φ, …

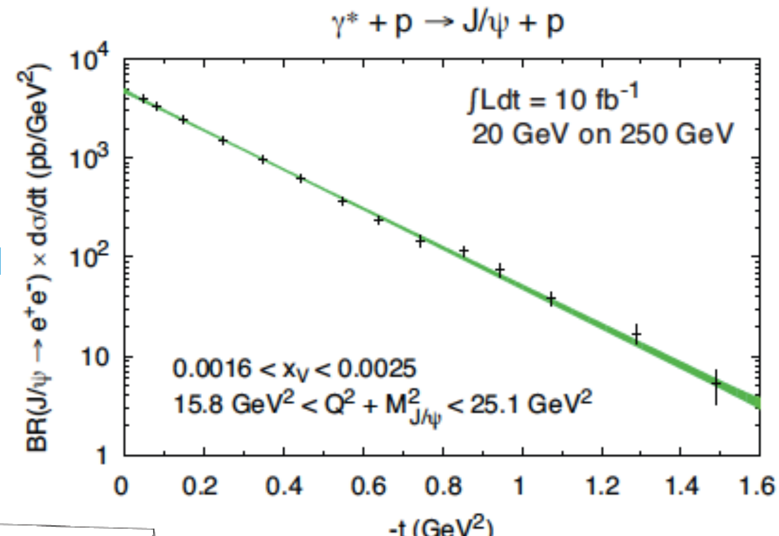
✧ Fourier transform of the t-dep

➡ Spatial imaging of glue density

✧ Resolution ~ 1/Q or $1/M_Q$

❑ **Gluon imaging from simulation:**

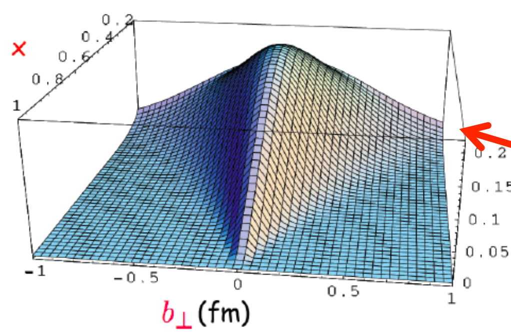




Only possible at the EIC

Gluon radius?

Gluon radius (x)!



How spread at small-x?

Color confinement

Unified view of nucleon structure

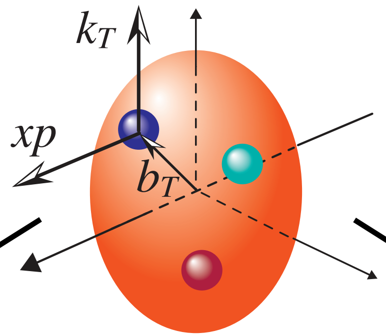
❑ Wigner distribution:

Momentum Space

TMDs

Confined motion

$\int d^2 b_T$



$\int d^2 k_T$

Coordinate Space

GPDs

Spatial distribution

$f(x, k_T)$ Two-scales observables $f(x, b_T)$

❑ Note:

- ✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD
- ✧ But, the TMDs and GPDs that represent them are not unique!
  - – Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

Position r x Momentum p → Orbital Motion of Partons

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ compensated by difference between gluon OAM density

✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q \left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\, \overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\, \psi(y)\, |P\rangle_{y^+=0}$$

JM: "staple" gauge link

Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a "torque" of color Lorentz force

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-) \times \sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right] \Phi(z^-, y)\psi(y)|P\rangle_{y^+=0}$$

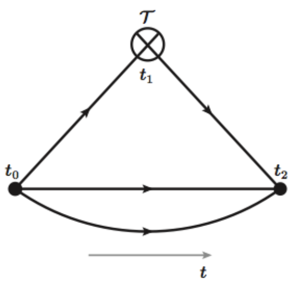
"Chromodynamic torque"

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

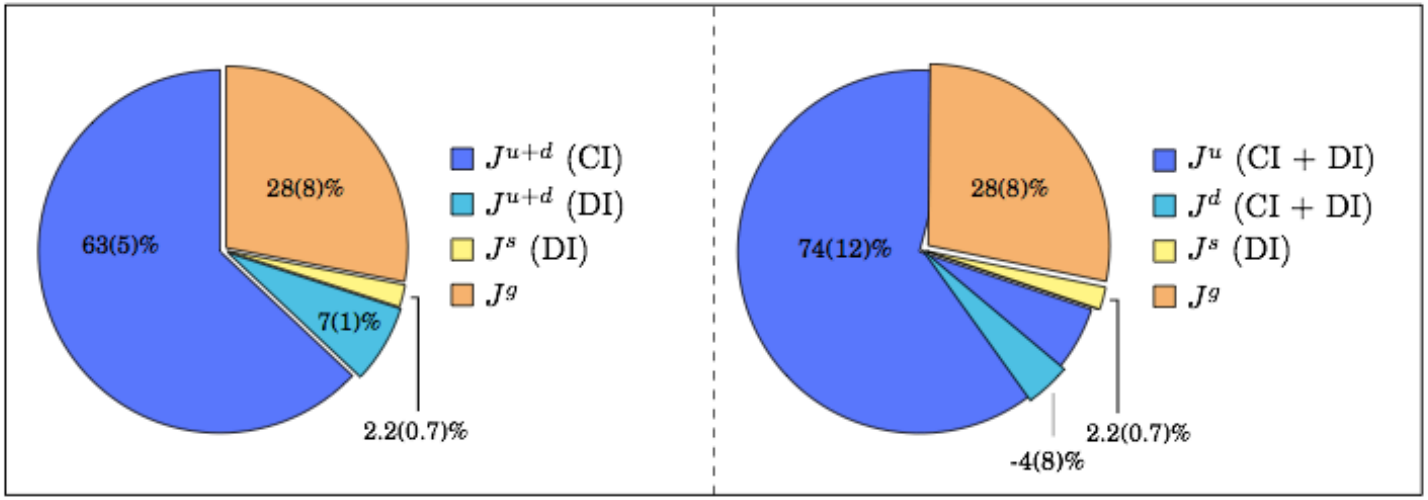
Nucleon spin and OAM from lattice QCD

❑ χQCD Collaboration:

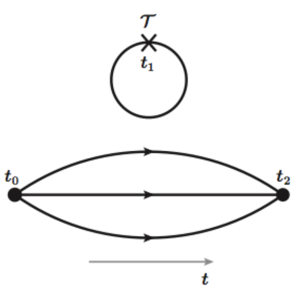
[Deka *et al.* arXiv:1312.4816]



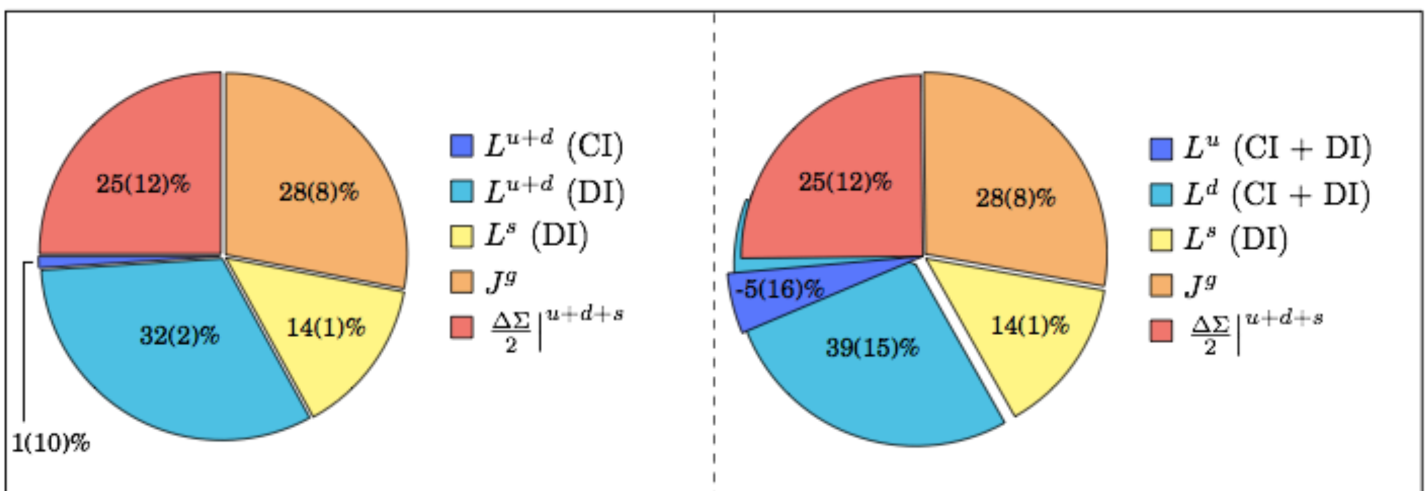
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)

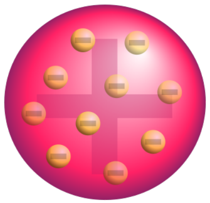


(c)

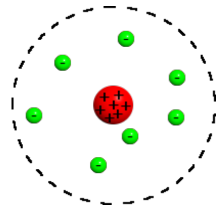
Why 3D hadron structure?

❑ Rutherford's experiment – atomic structure (100 years ago):

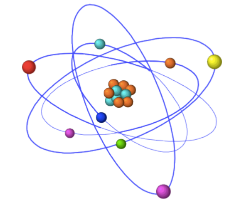
Atom:



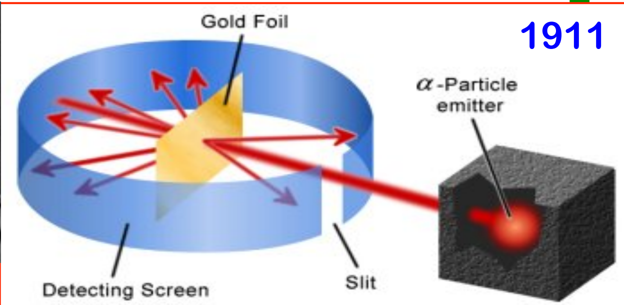
J.J. Thomson's
plum-pudding model



Rutherford's
planetary model



Modern model
Quantum orbitals



Discovery of nucleus
A localized
charge/force center
A vast
"open" space

Discovery of
Quantum Mechanics,
and
the Quantum World!

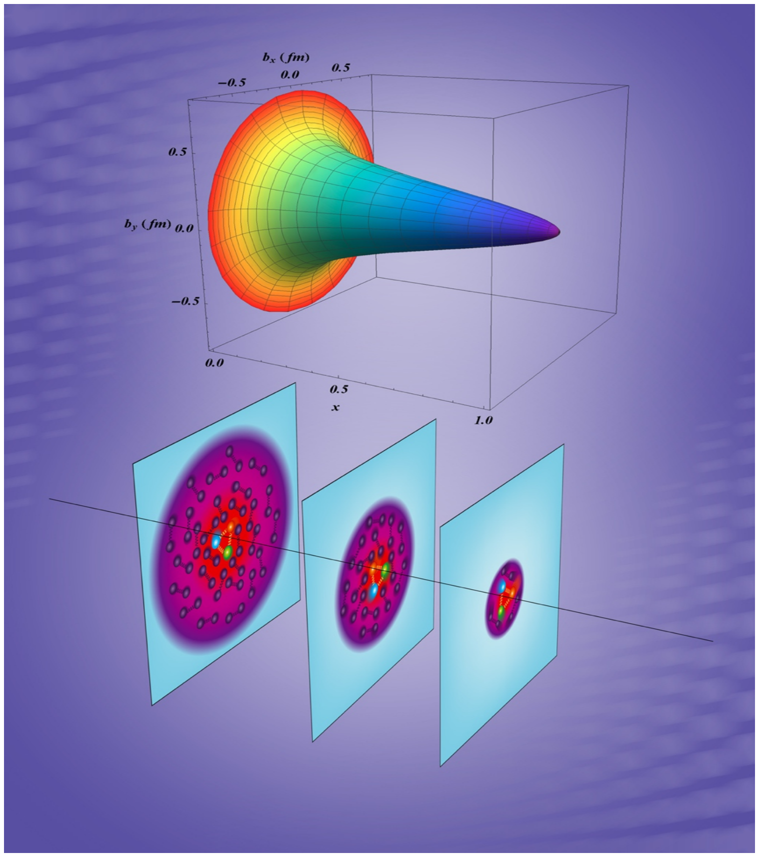
❑ Completely changed our "view" of the visible world:

- ✧ Mass by "tiny" nuclei – *less than 1 trillionth in volume of an atom*
- ✧ Motion by quantum probability – *the quantum world!*
- ✧ Provided infinite opportunities to improve things around us, …

What would we learn from the hadron structure in QCD, …?

Paradigm shift: 3D imaging of the "Proton"

❑ This is transformational!

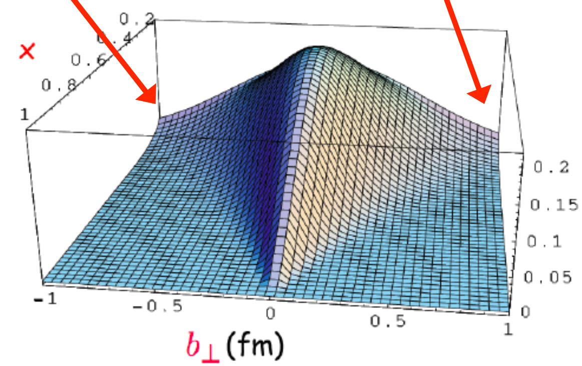


JLab12 – valence quarks,
EIC – sea quarks and gluons

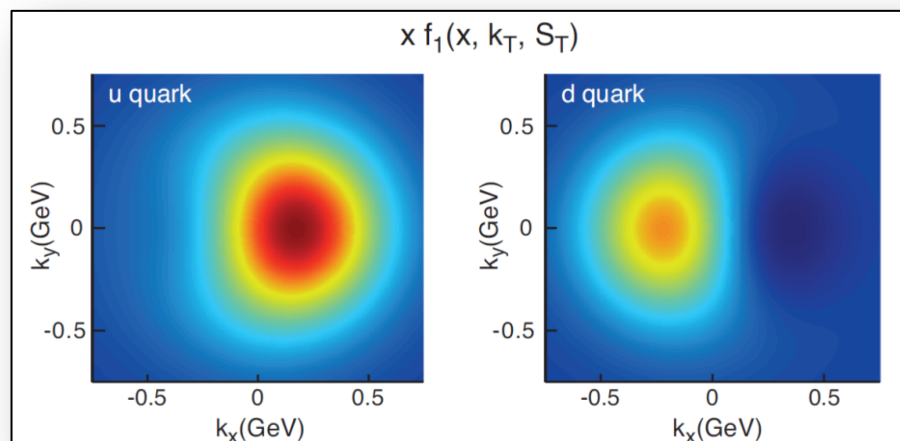
✧ How color is confined?

How far does glue density spread?

How fast does glue density fall?



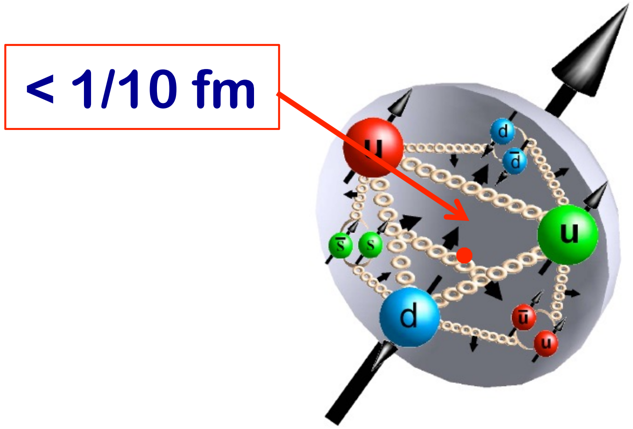
✧ Why there is preference in motion?



Summary

- ❑ QCD has been extremely successful in interpreting and predicting high energy experimental data!

- ❑ But, we still do not know much about hadron structure – work just started!



- ❑ Cross sections with large momentum transfer(s) and identified hadron(s) are the source of structure information

- ❑ QCD factorization is necessary for any controllable "probe" for hadron's quark-gluon structure!

- ❑ But, EIC is a ultimate QCD machine, and will provide answers to many of our questions on hadron structure, in particular, the confined transverse motions (TMDs), spatial distributions (GPDs), and multi-parton correlations, …

Thank you!

Backup slides

Transversity distributions

- **Transversity:** $\delta q(x)$ or $h_1(x)$ Jaffe and Ji, 1991

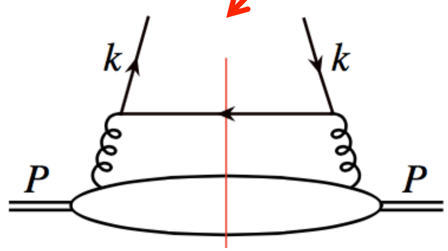
$$h_1(x) = \frac{1}{\sqrt{2}p^+}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\perp|\psi_+^\dagger(0)\gamma_\perp\gamma_5\psi_+(\lambda n)|PS_\perp\rangle$$ **+ UVCT**

with $\psi_\pm = P_\pm\psi$ and $P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$

- **Unique for the quarks:**

No mixing with gluons!

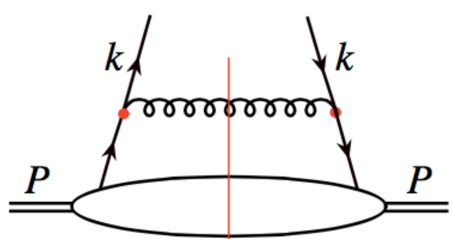
$\gamma\cdot n\gamma_\perp\gamma_5$ ← Even # of γ 's



= 0

No mixing with PDFs, helicity distributions

- **Perturbatively UV and CO divergent:**



+ wave function renormalization

$$\Delta_T P_{qq}^{(0)}(x) = C_F\left[\frac{2x}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

→ "DGLAP" evolution kernels

NLO - Vogelsang, 1998

Soffer's inequality

❑ **Relation between quark distributions:**

$$h_1(x) \leq \frac{1}{2}\left[q(x) + \Delta q(x)\right] = q^+(x)$$

Derived by using the positivity constraint of
quark + nucleon -> quark + nucleon
forward scattering helicity amplitudes

Cautions:

- ✧ Quark field of the Transversity distribution is NOT on-shell
- ✧ Quark + nucleon -> quark + nucleon
forward scattering amplitude is perturbatively divergent

❑ **Testing vs using as a constraint:**

It is important to test this inequality, rather than using it
as a constraint for fitting the transversity

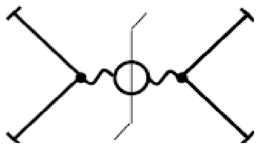
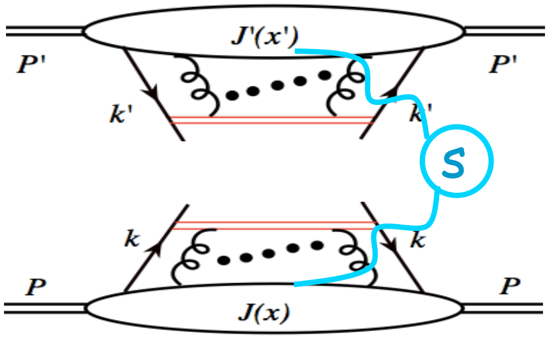
Perturbatively calculated evolution kernels seem to be consistent
with the inequality – the scale dependence

Connection to physical observables

- **Need two-chiral odd distributions – two hadrons:**

Soper, Ralston, 1978
Jaffe, Ji, 1991, 1992

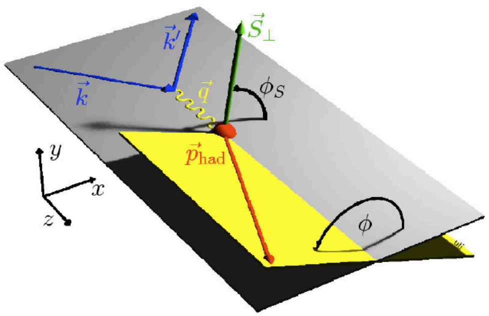
– **Drell-Yan:**

$$\sigma^{\rm DY}_{\rm tot} \sim$$  $$\otimes$$ 

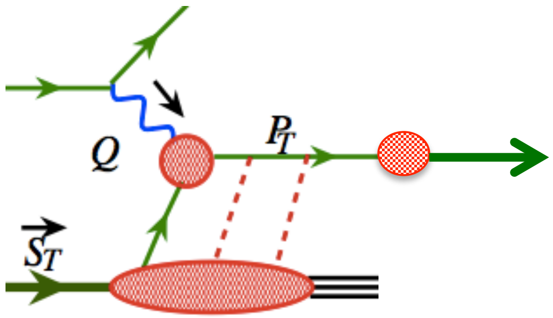
$$+\, O\left(\frac{1}{QR}\right)$$

$$\sim\ h_1(x)\otimes h_1(x')$$

Predictive power: Universal Transversity



– **SIDIS:**



$$\sim\ h_1(x)\otimes D_{\rm Collins}(z)$$

$$\sim\ A_T^{\sin(\phi+\phi_s)}$$

- **Caution:**

Transversity extracted depends on the "scheme" or UVCT

Cross section is always positive!

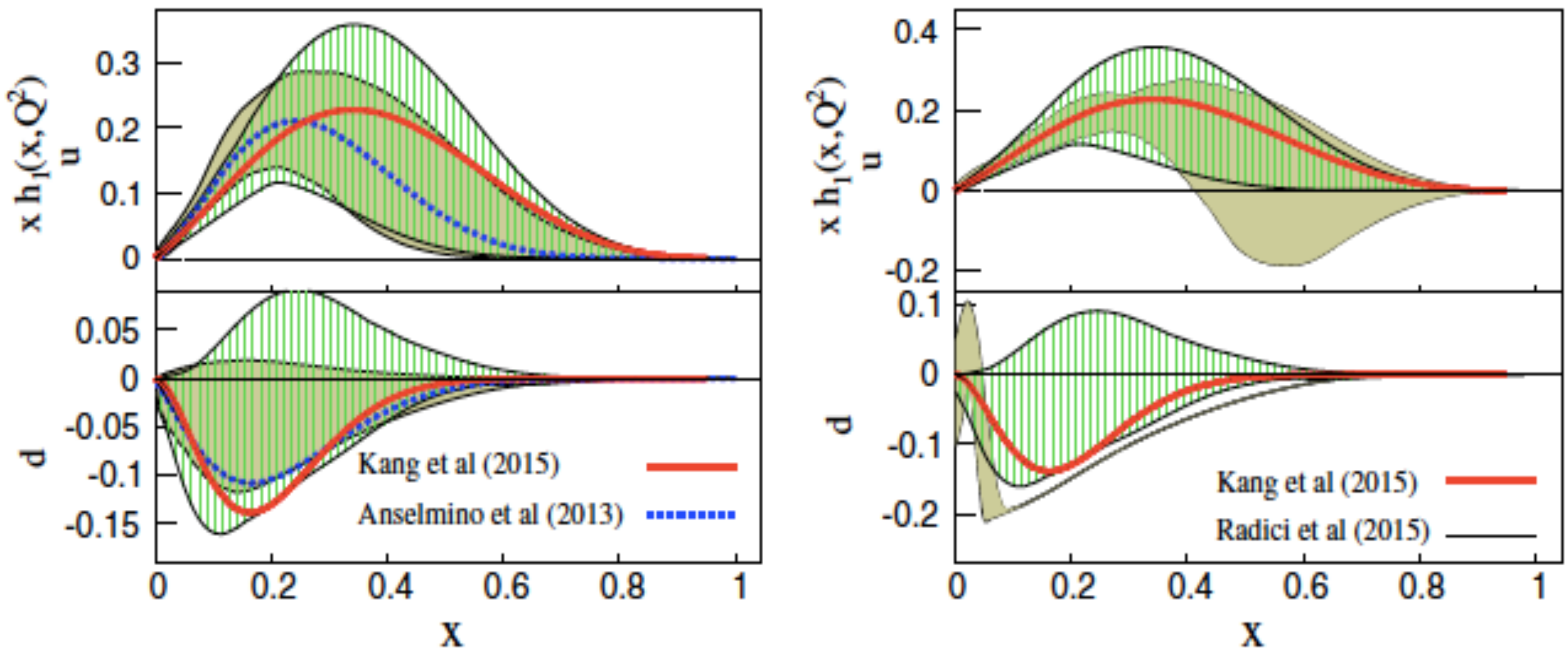
Like PDFs (helicity distributions), transversity does not have to be positive

Extraction of Transversity

- **Transversity comparison:**

Anselmino et al., PRD 87, 094019 (2013)

Kang et al, PRD, 2016



✧ Consistent in overall shape and sign, but, different in details

✧ Large uncertainties!

- **Future:**

JLab12, Compass, EIC; Transverse polarized Drell-Yan?

Tensor charge

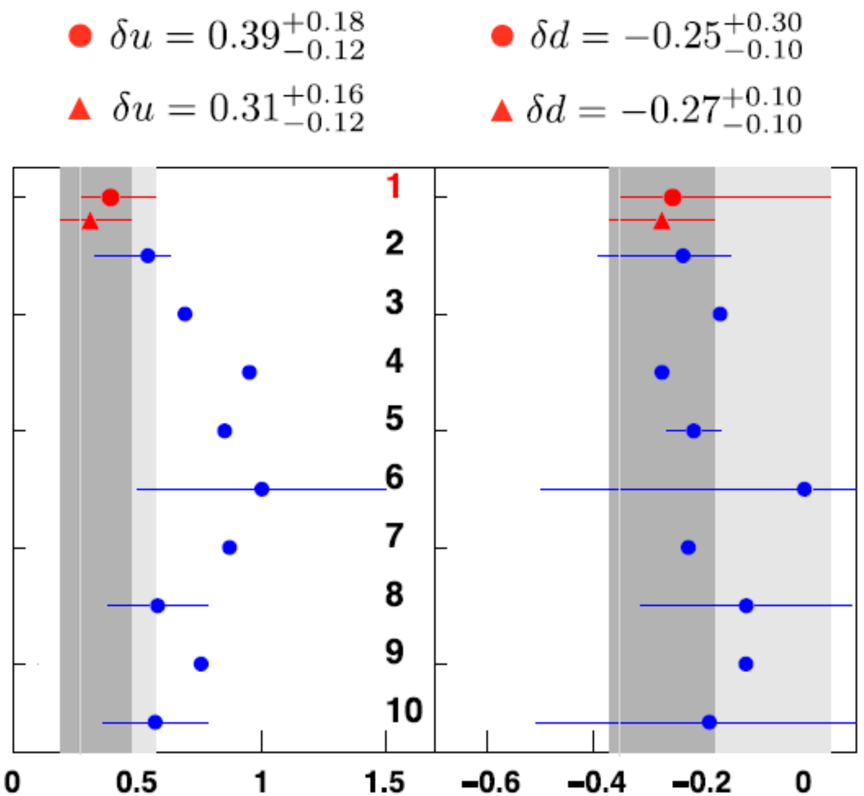
❑ **Definition:**

$$\delta q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]dx$$

Moment – matrix elements of local operators
– fundamental QCD quantity – calculable on lattice or using models

❑ **Extraction:**

Anselmino et al., PRD 87, 094019 (2013)



✧ **Extracted from global fits**

by using two different parameterizations for Collins FF)

✧ **Predictions from various models (including LQCD)**

✧ **Tensor charges are expected to be smaller than axial charge**

$$\Delta u = 0.787 \quad \Delta d = -0.319$$

Tensor charge

- **Definition:**

$$\delta q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]dx$$

Moment – matrix elements of local operators
– fundamental QCD quantity – calculable on lattice or using models

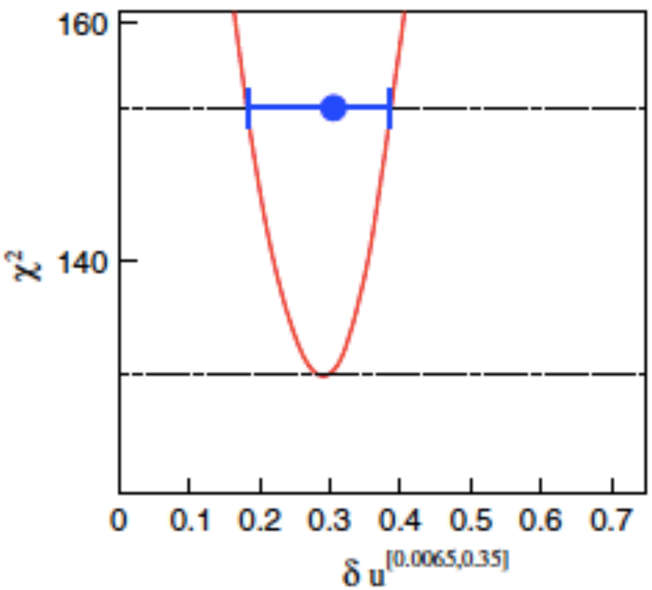
- **Extraction from global fits :**

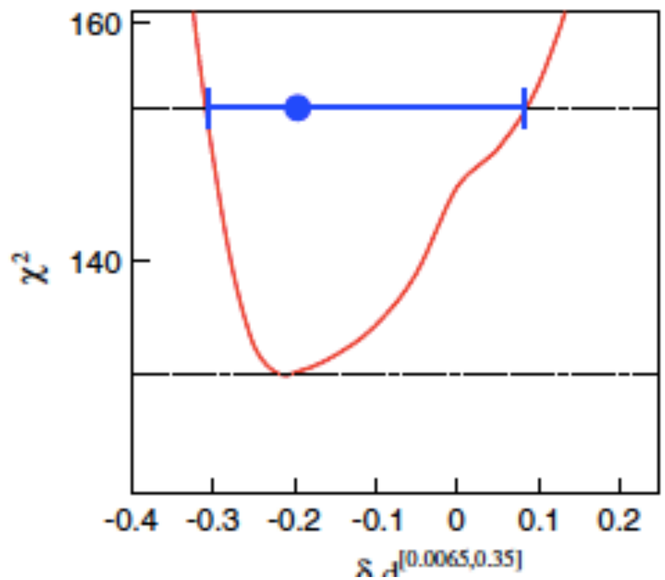
Kang et al, PRD, 2016

$$\delta q^{[x_{\min}, x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} dx h_1^q(x, Q^2)$$

$$\delta u^{[0.0065,0.35]} = +0.30^{+0.08}_{-0.12}$$

$$\delta d^{[0.0065,0.35]} = -0.20^{+0.28}_{-0.11}$$





$Q^2 = 10\ \mathrm{GeV}^2$

90% C.L.

$\Delta u = 0.787 \quad \Delta d = -0.319$