SELF-EXCITED LOOP

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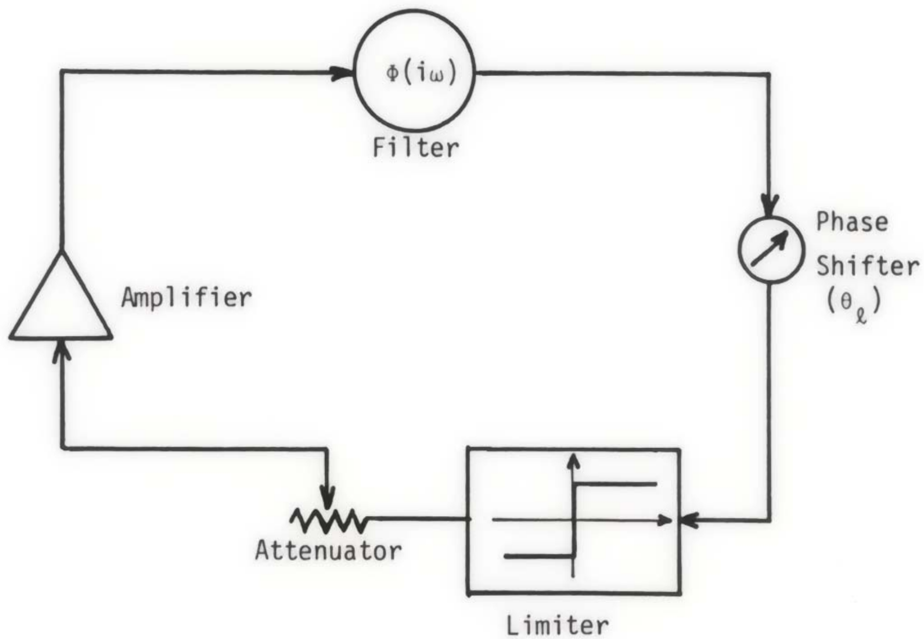
Outline

- Concept of a self-excited loop
- Operation in the unlocked state
- Principle of phase stabilization
- Mechanical modes and ponderomotive effects
- Comparison with generator-driven resonators
- Phase stabilization of a self-excited loop
    - Equations
    - Performance
    - Damping of microphonics and frequency feedback
- Summary and conclusions

Self-Excited Loop - Concept

- A Self-excited loop is:
  - A high-gain, positive feedback loop that is unstable and operates at a limit cycle determined by a non-linear element
  - An oscillator
- Basic elements:
  - Band pass filter
  - Phase shifter
  - Limiter



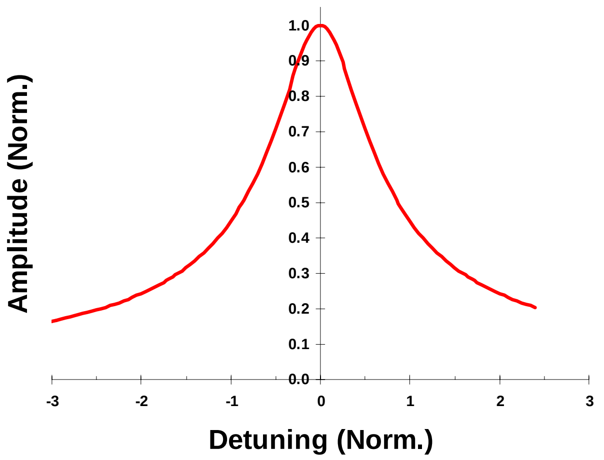
Unlocked Self-Excited Loop-Amplitude Stability

Loop oscillates at a frequency ω given by

$$\theta_c(\omega) + \theta_l = 0 \pmod{2\pi}$$

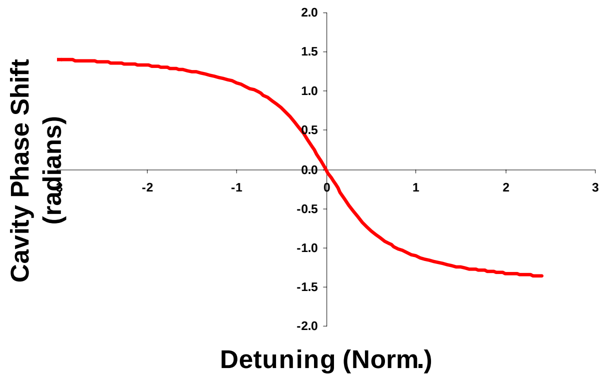
where $\theta_c(\omega) = \text{Arctan}\left[2Q\,\frac{\omega_c - \omega}{\omega_c}\right]$

or $\omega = \omega_c + \frac{\omega_c}{2Q}\tan\theta_l$



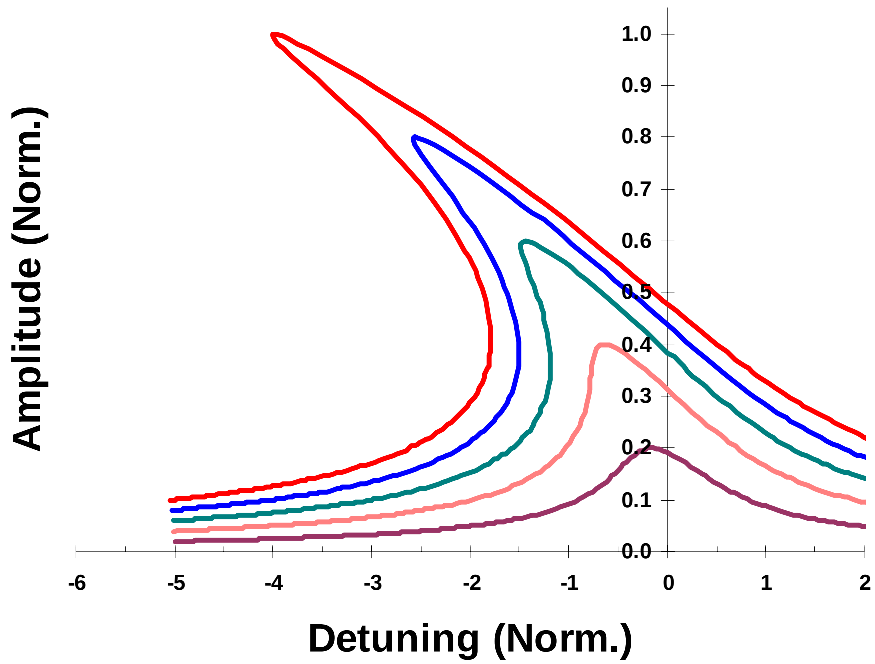
The loop will operate on a fixed point on the resonance curve set by the external phase shifter, independently of the cavity resonant frequency.

$\Rightarrow$ The amplitude will be stable and unaffected by the microphonics



Unlocked Self-Excited Loop-Amplitude Stability

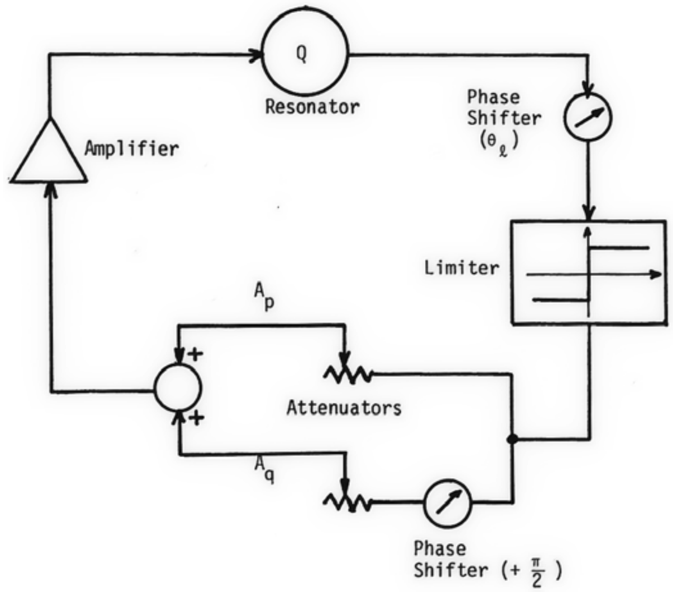
During transient operation (rise time and decay time) the loop frequency automatically tracts the resonator frequency. Lorentz detuning has no effect and is automatically compensated



Self-Excited Loop-Principle of Stabilization

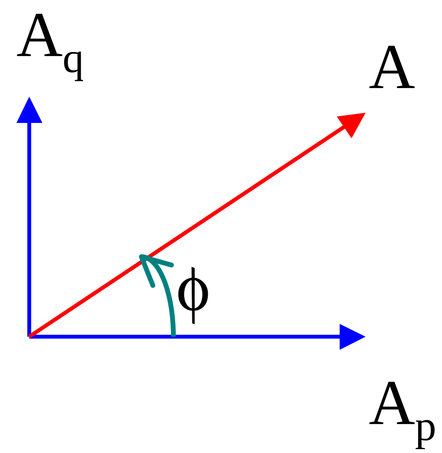
Controlling the external phase shift θ_l can compensate for the fluctuations in the cavity frequency ω_c so the loop is phase locked to an external frequency reference ω_r.

$$\omega = \omega_c + \frac{\omega_c}{2Q}\tan\theta_l$$



Instead of introducing an additional external controllable phase shifter, this is usually done by adding a signal in quadrature

$\Rightarrow$ The cavity field amplitude is unaffected by the phase stabilization even in the absence of amplitude feedback.



Ponderomotive Effects – Mechanical Modes

- In addition to electromagnetic modes, a cavity is also a system of mechanical modes of oscillations of frequency Ω_μ and decay time τ_μ .
- The amplitude of a mechanical mode will be represented by its contribution to the frequency shift $\Delta\omega_\mu$ of the electromagnetic mode.

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\Delta\dot{\omega}_\mu + \Omega_\mu^{\ 2}\Delta\omega_\mu = -\Omega_\mu^2 k_\mu V_o^{\ 2} + n(t)$$

k_μ : Lorentz coefficient

$n(t)$: external noise

Steady state: $\Delta\omega_{\mu o} = -k_\mu V_o^{\ 2}$

Ponderomotive Effects – Mechanical Modes (cont.)

Fluctuations around steady state:

$$\Delta\omega_\mu = \Delta\omega_{\mu o} + \delta\omega_\mu$$

$$V = V_0(1+\delta v)$$

Linearized equation of motion for mechanical mode:

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu}\delta\dot{\omega}_\mu + \Omega_\mu^2\delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o{}^2\delta v$$

The mechanical mode is driven by fluctuations in the electromagnetic mode amplitude.

Variations in the mechanical mode amplitude causes a variation of the electromagnetic mode frequency, which can cause a variation of its amplitude.

⇒ Closed feedback system between electromagnetic and mechanical modes, that can lead to instabilities.

Generator-Driven Resonator

- In a generator-driven resonator the coupling between the electromagnetic and mechanical modes can lead to two ponderomotive instabilities

- Monotonic instability

  Jump phenomenon where the amplitudes of the electromagnetic and mechanical modes increase or decrease exponentially until limited by non-linear effects

- Oscillatory instability

  The amplitudes of both modes oscillate and increase at an exponential rate until limited by non-linear effects

Generator-Driven Resonator

Approximate stability criteria:

- Monotonic $$-y\,k_\mu V_o^2 < \frac{1}{2\tau}$$

- Oscillatory $$y\,k_\mu V_o^2 < \frac{1}{2\tau_\mu}\frac{\left(1+\tau\,\Omega_\mu\right)^2}{\tau^2\Omega_\mu{}^2}$$

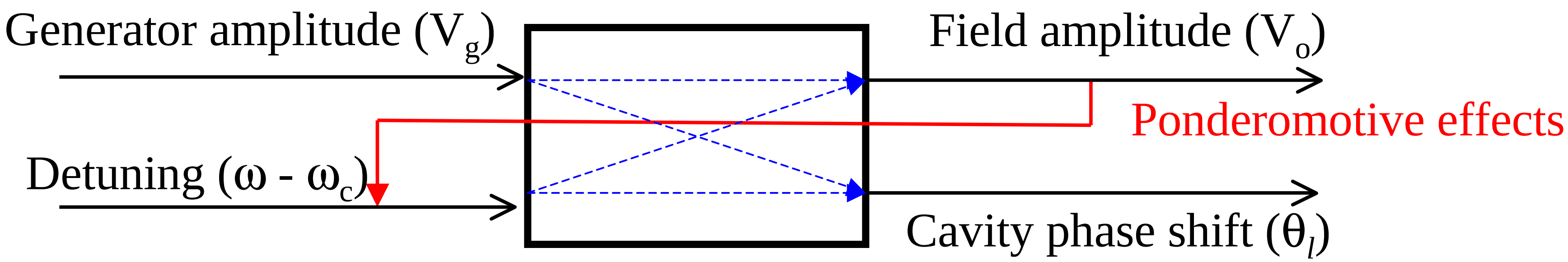
where $y = \tau\left(\omega_g - \omega_{co}\right)$: normalized detuning

The monotonic instability can occur on the low frequency side when the Lorentz detuning is of the order of an electromagnetic bandwidth.
The oscillatory instability can occur on the high frequency side when the Lorentz detuning is of the order of a mechanical bandwidth.

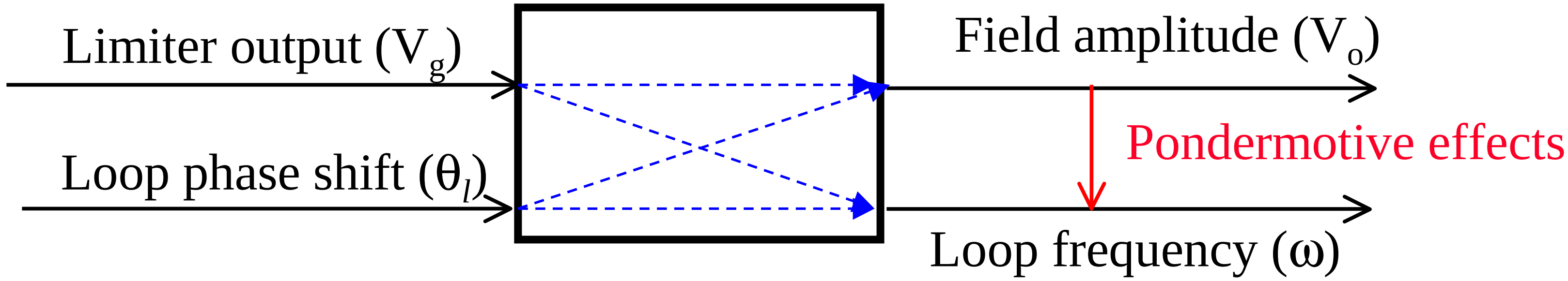
Amplitude feedback can stabilize system with respect to ponderomotive instabilities

Input-Output Variables – Self-Excited Loop vs. Generator-Driven Cavity

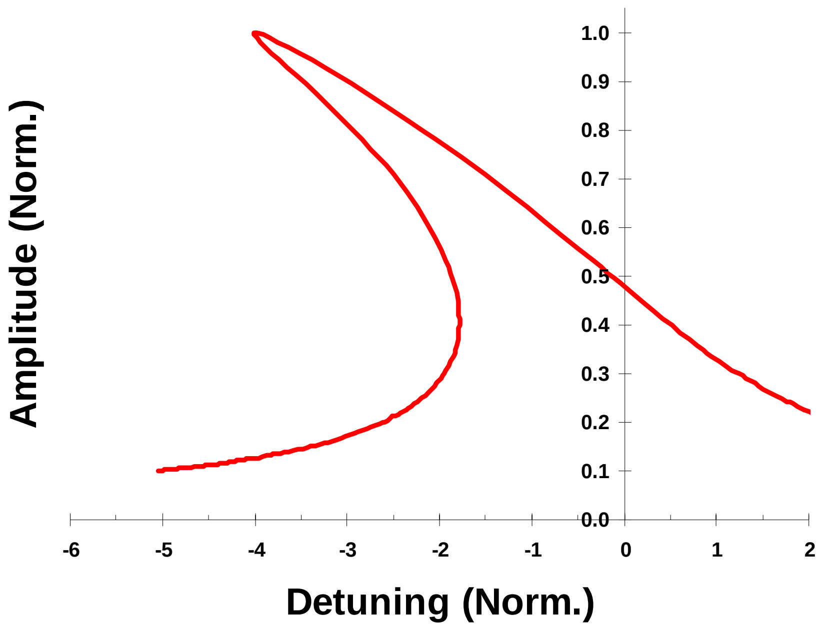
- Generator - driven cavity

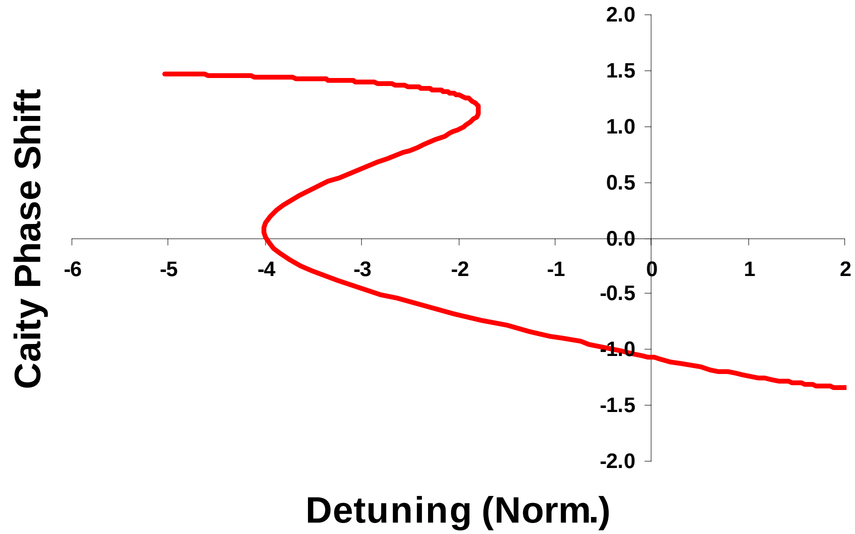


- Cavity in a self-excited loop

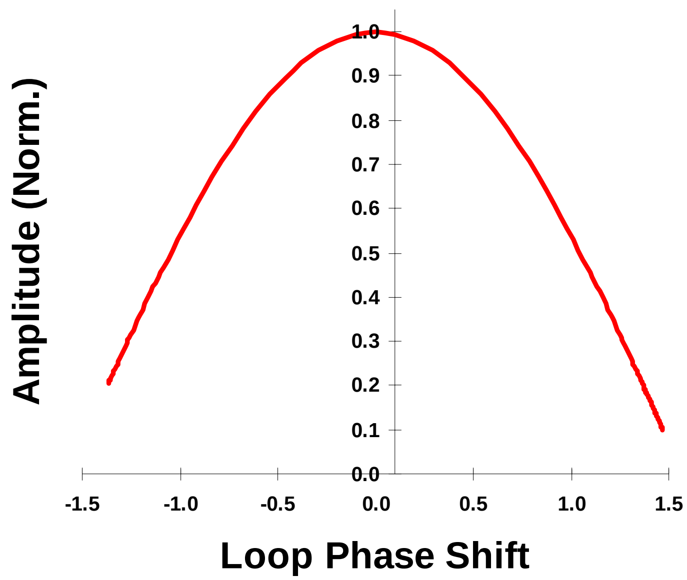


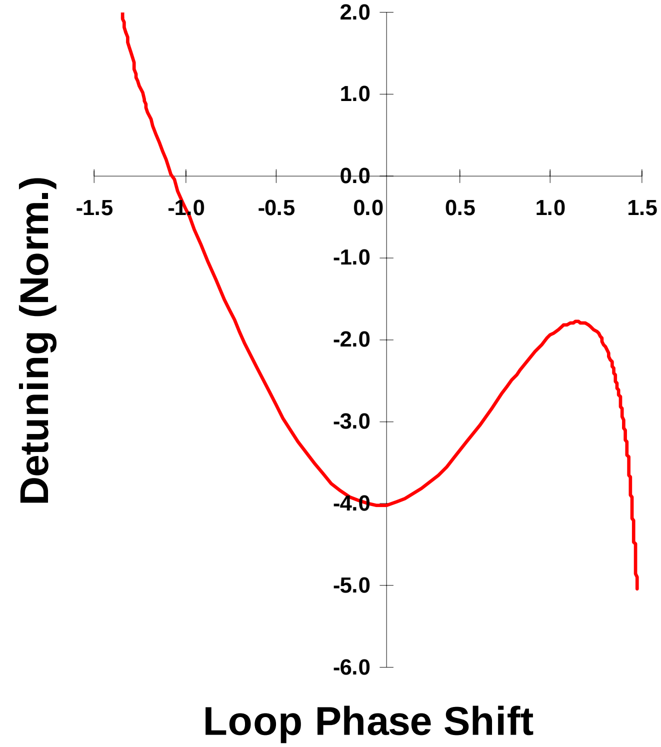
Input-Output Variables – Generator-Driven Resonator



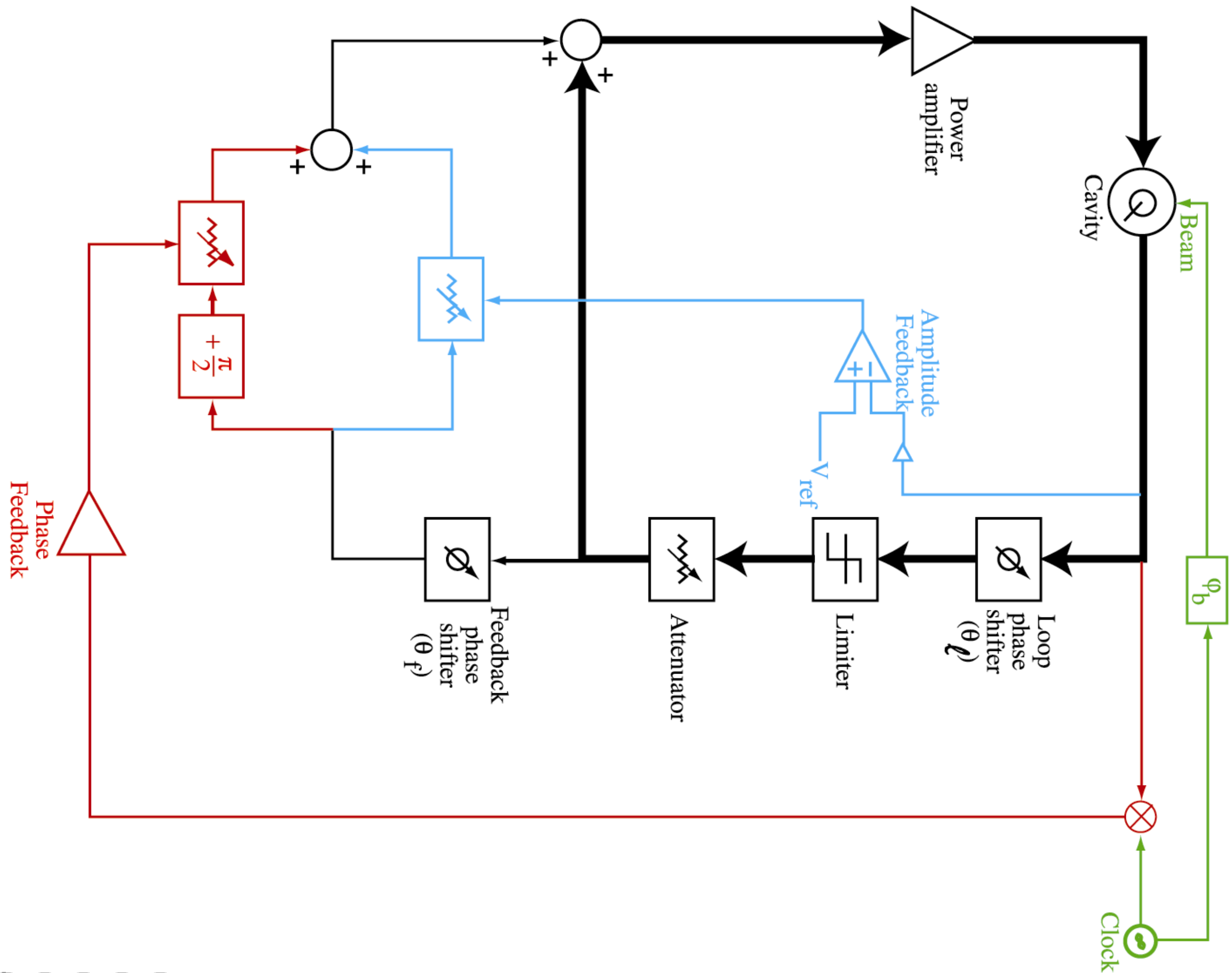


Input-Output Variables – Self-Excited Loop





Self-Excited Loop - Block Diagram



Self-Excited Loop – Field Equations

$$\ddot{v} + 2\frac{1+\beta}{\tau_0}\dot{v} + \omega_c^2 v = \frac{2}{\tau_0}\dot{v}_g - \frac{2}{\tau}\dot{v}_b$$

$$v = Ve^{i\alpha} \qquad\qquad v_b = \frac{i_b R_{sh}}{2} e^{i(\omega_r t + \varphi_b)}$$

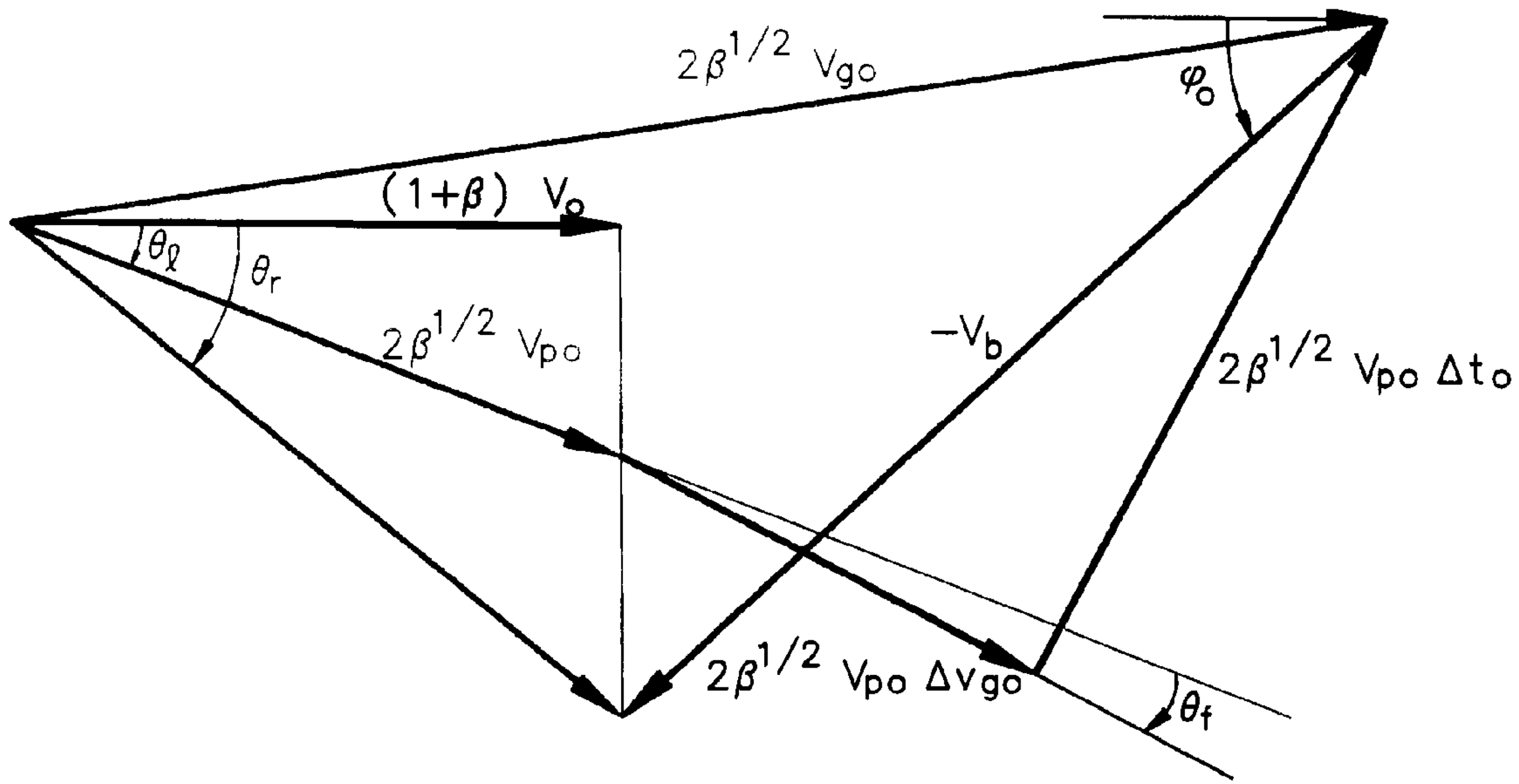
$$v_g = V_{po} 2\beta^{1/2} e^{i\alpha} e^{i\theta_l} \left[1 + e^{i\theta_f} \left(\Delta v_g + i\Delta t\right)\right]$$

Assume variations are slow compared to electromagnetic time-scale

Separate real and imaginary parts

$$\begin{cases} \tau_o \dot{V} + (1+\beta)V = 2\beta^{1/2} V_{po} \left[\cos\theta_l + \Delta v_g \cos(\theta_l + \theta_f) - \Delta t \sin(\theta_l + \theta_f)\right] - V_b \cos\varphi_s \\ V\tau_o(\omega - \omega_c) = 2\beta^{1/2} V_{po} \left[\sin\theta_l + \Delta v_g \sin(\theta_l + \theta_f) + \Delta t \cos(\theta_l + \theta_f)\right] - V_b \sin\varphi_s \end{cases}$$

Self-Excited Loop – Steady State



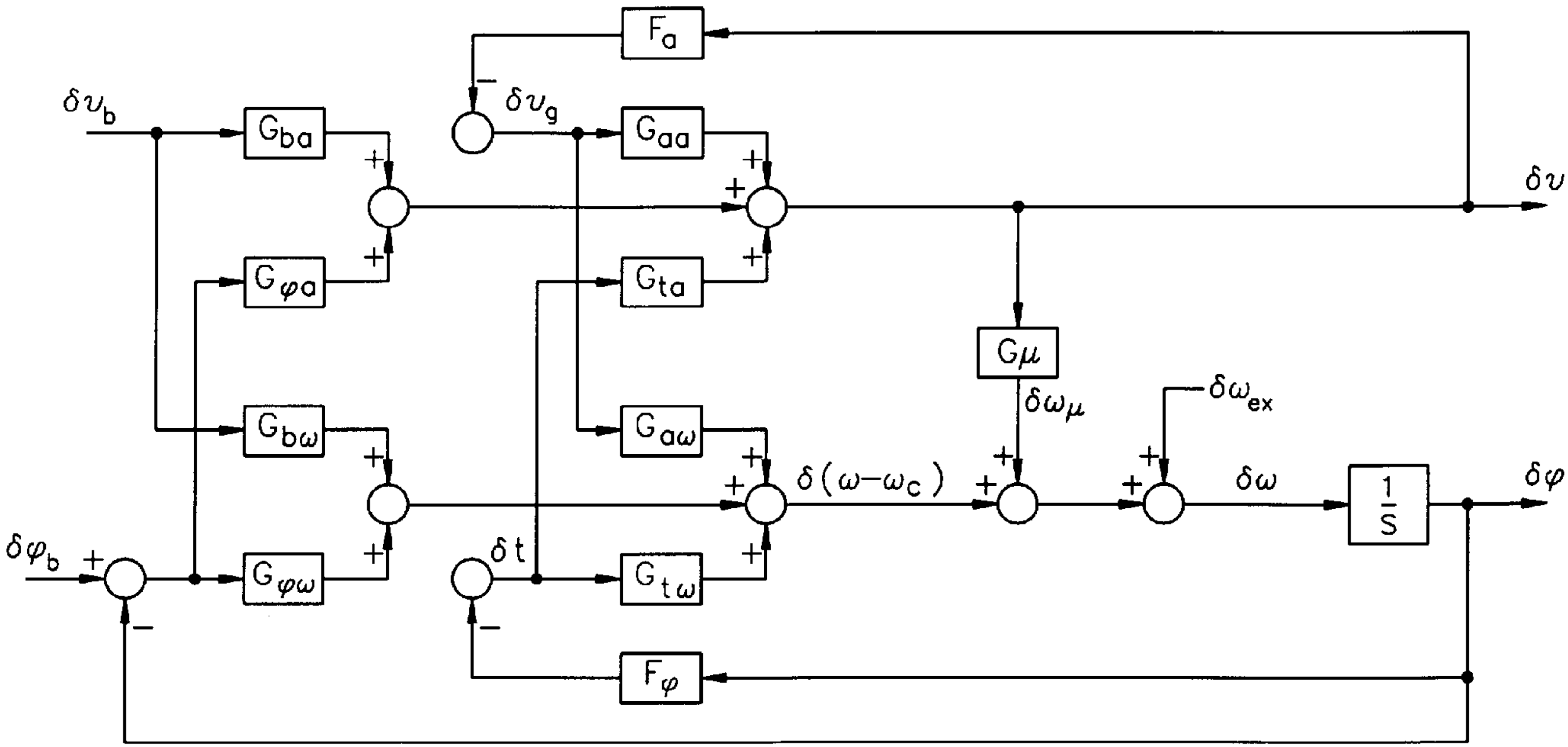
Self Excited Loop – Fluctuations from Steady State

Linearize around steady state and apply Laplace Transform

$$\delta v = \delta v_g \frac{\cos(\theta_l + \theta_f)}{\cos\theta_l} \frac{1+\beta}{1+\beta+\tau_o s} - \delta t \frac{\sin(\theta_l + \theta_f)}{\cos\theta_l} \frac{1+\beta}{1+\beta+\tau_o s}$$
$$-v_b \frac{b}{1+\beta+\tau_o s} + (\delta\varphi_b - \delta\varphi) \frac{b\tan\phi_o}{1+\beta+\tau_o s}$$

$$\tau_o(\delta\omega - \delta\omega_c) = \delta v_g (1+\beta) \frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[\tan(\theta_l+\theta_f) - \tan\theta_r \frac{1+\beta}{1+\beta+\tau_o s}\right]$$
$$+\delta t (1+\beta) \frac{\cos(\theta_l+\theta_f)}{\cos\theta_l}\left[1 + \tan\theta_r \frac{1+\beta}{1+\beta+\tau_o s}\right]$$
$$+\delta v_b\, b\left[-\tan\phi_o + \tan\theta_r \frac{1+\beta}{1+\beta+\tau_o s}\right]$$
$$+(\delta\varphi_b - \delta\varphi) b\left[-1 - \tan\phi_r + \tan\phi_o \frac{1+\beta}{1+\beta+\tau_o s}\right]$$

Transfer Function Representation



Monotonic Stability

The system will be stable if: $-\left(y - \dfrac{m\, y_o}{K_\varphi} \right) k_\mu\, V_0^2 < \dfrac{1}{\tau}\, B_{mo}\,(K_a, K_\varphi)$, with:

$$B_{mo}(K_a, K_\varphi) = \frac{1}{2}\left[K_a(1+y^2) + 1 + yy_r - \frac{m}{K_\varphi}\left[K_a(1+yy_r) + 1 + y_0 y_r \right] \right]$$

If: $\theta_l = 0$ $\theta_f \ll 1$ no beam loading

$$k_a, k_\varphi \gg 1,\ \tau\,\Omega_\mu \qquad \frac{\tau}{\tau_\mu} \ll 1$$

$$-y\, k_\mu V_0^2 < \frac{k_a + 1}{2\tau}$$

The stability boundary can be pushed arbitrarily far with amplitude feedback

Oscillatory Stability

The system will be stable if: $\left(y - \frac{m\,y_o}{K_\varphi}\right) k_\mu V_0^2 < \frac{1}{\tau_\mu} B_{os}(K_a, K_\varphi)$ with

$$B_{os}(K_a,K_\varphi) = \frac{K_a + K_\varphi + 1 - m}{\tau^2\ \Omega_\mu^2\ K_\varphi} \times \left\{ \tau^2\Omega_\mu^2 + \left[\frac{(K_a+1)(K_\varphi - m) + (K_\varphi y - m y_0)(y_r + K_a y) - \tau^2\Omega_\mu^2}{1 + \frac{2\tau}{\tau_\mu} + K_a + K_\varphi - m} \right] \right.$$

$$\left. \times \left[\frac{(K_a+1)(K_\varphi - m + (K_\varphi y - m y_0)(y_r + K_a y) - \tau^2\Omega_\mu^2}{1 + \frac{2\tau}{\tau_\mu} + K_a + K_\varphi - m} + \frac{2\tau}{\tau_\mu} \right] \right\},$$

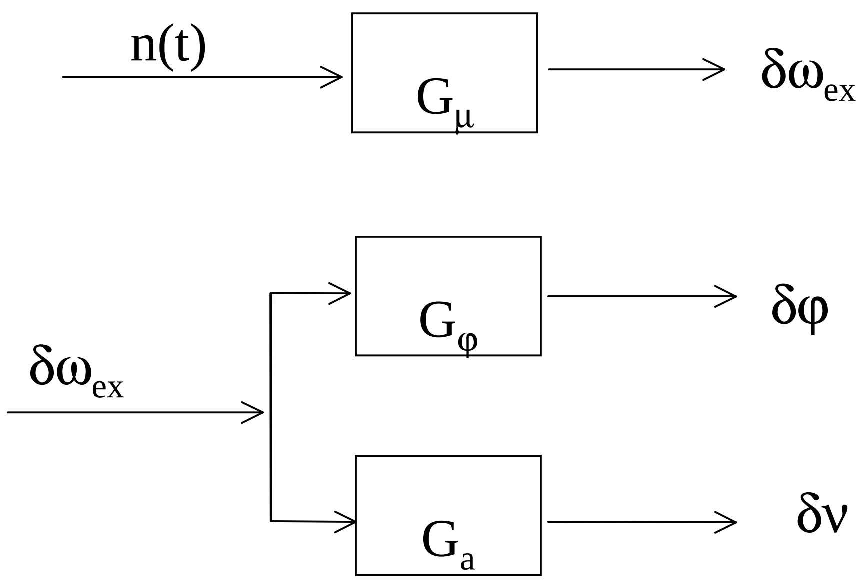
If: $\theta_l = 0$ $\theta_f \ll 1$ no beam loading $k_a, k_\varphi \gg 1,\ \tau\,\Omega_\mu$ $\frac{\tau}{\tau_\mu} \ll 1$

$$y\,k_\mu V_0^2 < \frac{1}{2\tau_\mu} \frac{(k_a+1)^2 k_\varphi}{\tau^2\ \Omega_\mu^2 (k_\varphi + k_a + 1)}$$

The stability boundary can be pushed arbitrarily far with amplitude feedback

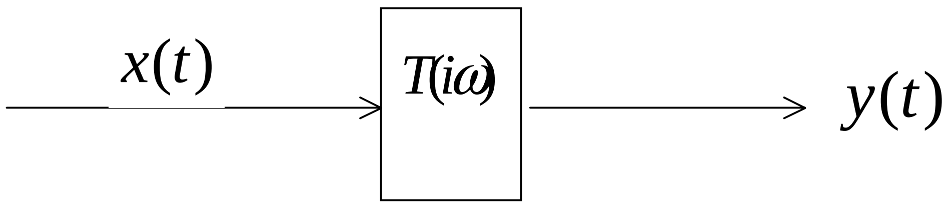
Performance of a Self-Excited Loop

- Residual phase and amplitude errors caused by microphonics
- Can also be done for beam current amplitude and phase fluctuations
- Assume a single mechanical oscillator of frequency Ω_μ and decay time τ_μ excited by white noise of spectral density A^2



Performance of a Self-Excited Loop (cont.)

- For a stationary random process driving a linear system



$$< y^2 >= R_y(0) = \int_{-\infty}^{+\infty} S_y(\omega)$$

$R_y(\tau)$: auto correlation function of $y(t)$

$S_y(\omega)$: spectral density of $y(t)$

$$\boxed{S_y(\omega) = S_x(\omega)\,|T(i\omega)|^2}$$

Performance of a Self-Excited Loop (cont.)

$$< \delta\omega_{ex}^2 >= A^2 \int_{-\infty}^{+\infty} \frac{d\omega}{\left| -\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2 \right|^2} = A^2 \frac{\pi\tau_\mu}{2\Omega_\mu^2}$$

$$< \delta v^2 >= A^2 \int_{-\infty}^{+\infty} \left| \frac{G_a(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2} \right|^2 d\omega = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{+\infty} \left| \frac{G_a(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2} \right|^2 d\omega$$

$$< \delta\varphi^2 >= A^2 \int_{-\infty}^{+\infty} \left| \frac{G_\varphi(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2} \right|^2 d\omega = < \delta\omega_{ex}^2 > \frac{2\Omega_\mu^2}{\pi\tau_\mu} \int_{-\infty}^{+\infty} \left| \frac{G_\varphi(i\omega)}{-\omega^2 + \frac{2}{\tau_\mu} i\omega + \Omega_\mu^2} \right|^2 d\omega$$

Electronic Damping

- In the limits:

$$\theta_l = 0 \qquad \theta_f \ll 1 \qquad \text{no beam loading}$$

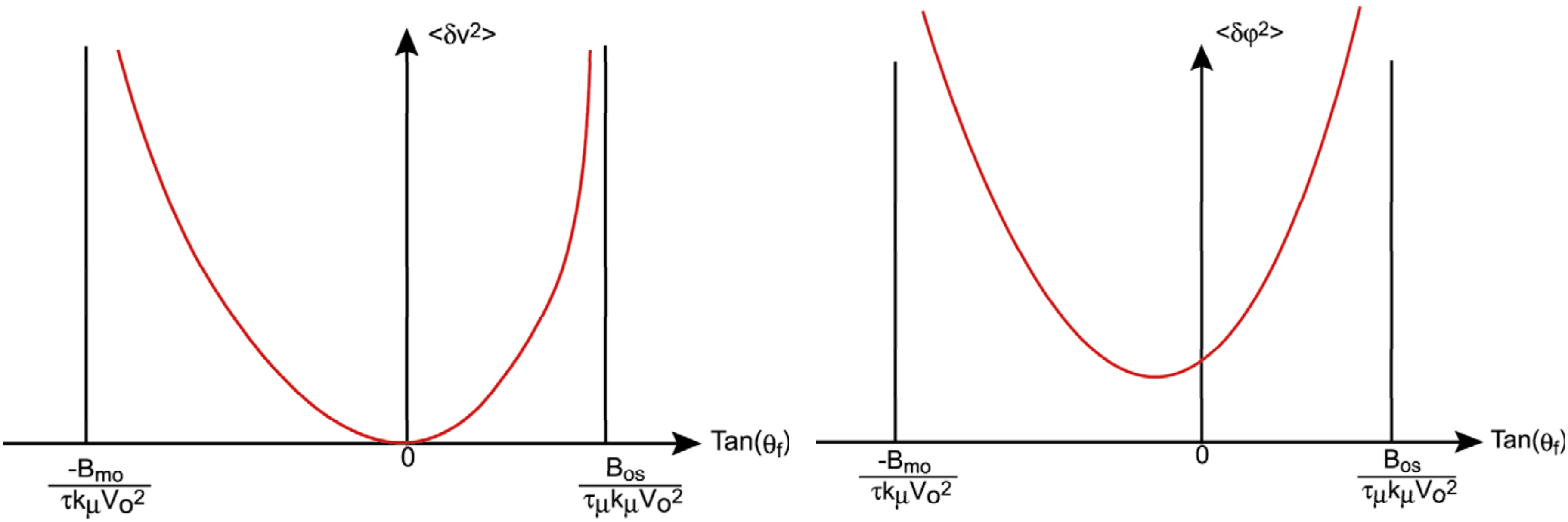
$$k_a, k_\varphi \gg 1,\ \tau\Omega_\mu \qquad \frac{\tau}{\tau_\mu} \ll 1$$

$$< \delta v^2 >= \frac{\tau^2 < \delta\omega_{ex}^2 >}{(k_a + 1)^2}\left[\theta_f\right]^2$$

$$< \delta\varphi^2 >= \frac{\tau^2 < \delta\omega_{ex}^2 >}{k_\varphi^2}\left[1 + \theta_f k_\mu V_0^2 \frac{2\tau}{k_a + 1}\left(1 - \frac{\tau_\mu}{2\tau}\tau^2\Omega_\mu^2 \frac{k_\varphi + k_a + 1}{k_\varphi (k_a + 1)}\right)\right]$$

[ ]: electronic damping or excitation of microphonics by introduction of a feedback phase (coupling between phase and amplitude feedback)

Residual Amplitude and Phase Errors



Frequency Feedback

- Frequency feedback can be added to phase and amplitude feedback to reduce microphonics

- Signal driving the resonator (Assume $\theta_f = 0$, no current)

$$V_g = V_{go}\left[1+\delta v_g + i\,\delta t\right]$$

where $\delta v_g = -\,k_a \delta v = -k_a\left(\dfrac{V-E}{E}\right)$

E : reference amplitude

Modulate reference amplitude

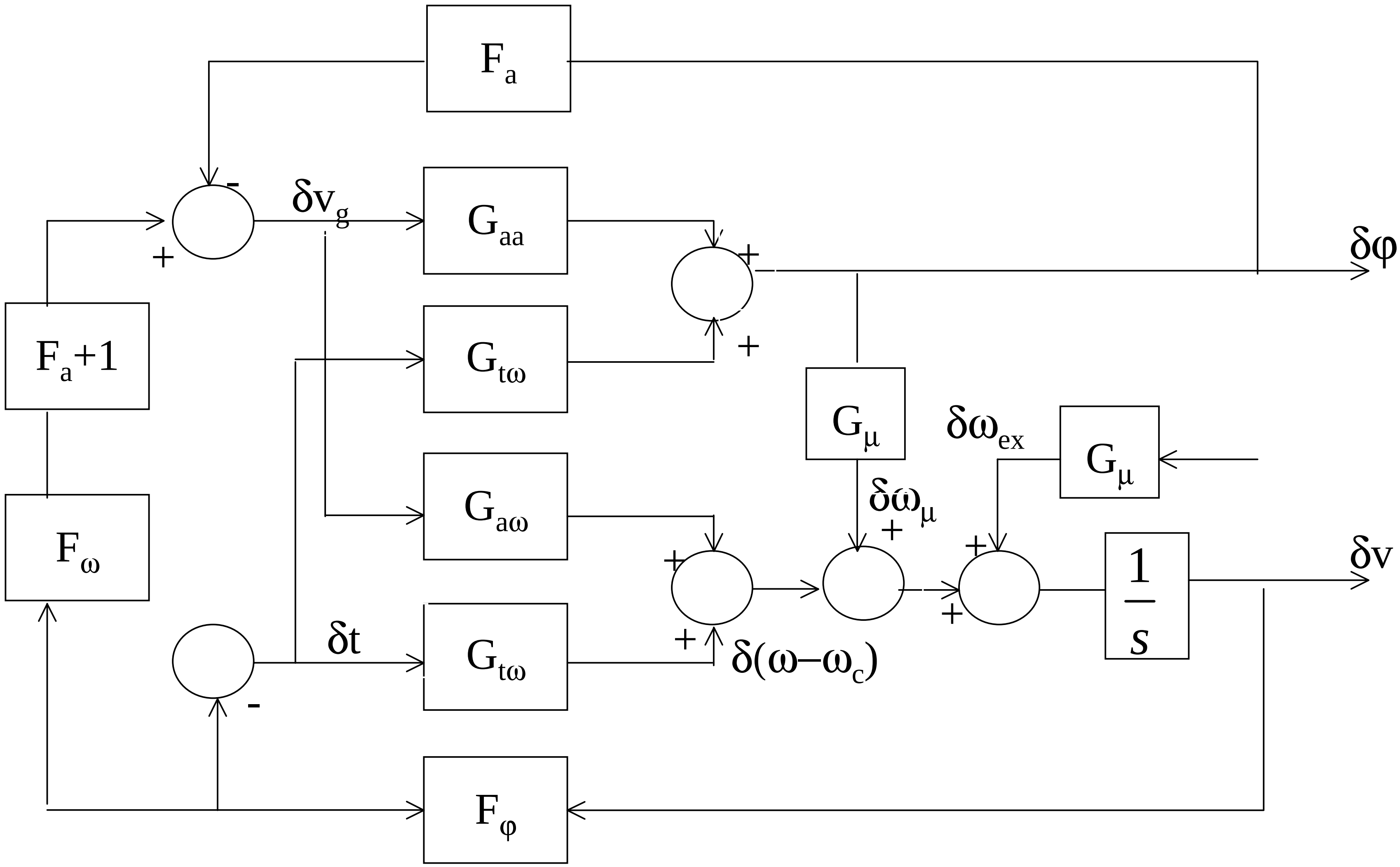
$$E = E_0\left(1+\delta e\right)$$

δe : frequency feedback

$$\delta v_g\left(s\right) = -F_a\,\delta v\left(s\right) + \delta e\left(s\right)\left(1+F_a\right)$$

$$\delta e\left(s\right) = -\,F_\omega\,\delta t\left(s\right) = \delta\varphi\left(s\right)F_\varphi\,F_\omega$$

Frequency Feedback (cont.)



Frequency Feedback (cont.)

Assume $F_a = k_a$ $\quad F_\varphi = k_\varphi$ Proportional feedbacks

$F_\omega = k_\omega \dfrac{s}{\Omega_\mu}$ Derivative feedback

$$< \delta\omega_c^2 >= \frac{< \delta\omega_{ex}^2 >}{1 + k_\omega \tau \tau_\mu \Omega_\mu k_\mu V_o^2}$$

$$< \delta\varphi^2 >= \frac{\tau^2 < \delta\omega_{ex}^2 >}{k_\varphi{}^2} \frac{1}{1 + k_\omega \tau \tau_\mu \Omega_\mu k_\mu V_o^2}$$

$$< \delta\nu^2 >= \tau^2 < \delta\omega_{ex}^2 > \frac{k_\omega{}^2}{1 + k_\omega \tau \tau_\mu \Omega_\mu k_\mu V_o^2}$$

Microphonics and residual phase error can be reduced at the expense of an increase in residual amplitude error

Frequency Feedback (cont.)

Amount of modulation required to reduce frequency excursions by a given amount:

$$\frac{<\delta\omega_{ex}^2>}{<\delta\omega_c^2>} = K^2 \qquad K : \text{reduction factor}$$

$$<\delta\nu^2> = \frac{<\delta\omega_{ex}^2>}{4Q_\mu^2\left(k_\mu V_o^2\right)^2}\;\frac{\left[K^2-1\right]^2}{K^2}$$

If $\quad <\delta\omega_{ex}^2> = \left(2\pi\times 5\right)^2 = 1000$

$$\left(k_\mu V_o^2\right)^2 = \left(2\pi\times 300\right)^2 = 4\ \ 10^6$$

$$\left(2Q_\mu\right)^2 = \left(2\times 2\pi\times 100\times 0.25\right)^2 = 10^5$$

for $\quad K = 2 \qquad <\delta\nu^2> \simeq 0.5\ \ 10^{-8}$

Self Excited Loop – Positive Features

- Able to operate cavities without external frequency source
  - Large number of cavities can be operated simultaneously and independently
- In the unlocked state
  - Free of ponderomotive instabilities
  - No need for frequency tracking during turn-on or amplitude ramping
  - No need for amplitude stabilization (inherent in process)
  - Ponderomotive instabilities can appear in the locked state but are easily eliminated by amplitude feedback

Self-Excited Loop – Negative Features

- Loop can oscillate in several cavity modes
  - Control loop phase shift
  - Band-pass filter in the SEL
- Randomness in the turn-on process
- Need for high gain at zero amplitude (instability that grows from thermal noise)
  - Inject signal from master oscillator

Conclusions

- SEL are ideally suited for high gradient, high-loaded Q cavities operated cw.
- Less suited for low-Q cavities operated for short pulse length.