How do pentaquarks look like

and

why are they narrow

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1. Why chiral fields are important inside baryons

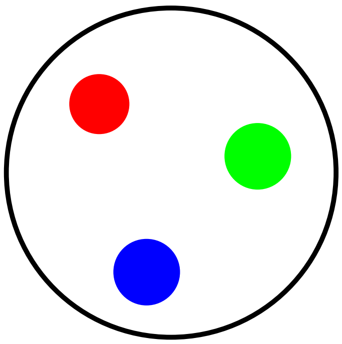
2. Quark structure of “chiral solitons”

3. Why are pentaquarks light

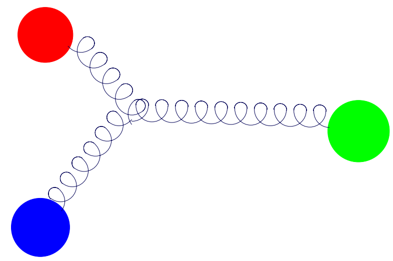
4. Why are pentaquarks narrow

Why chiral fields are important inside baryons

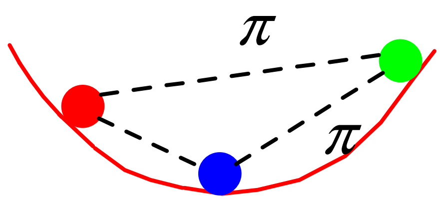
The variety of views on baryons reminds the variety of models of solar system in the beginning of the XVI century



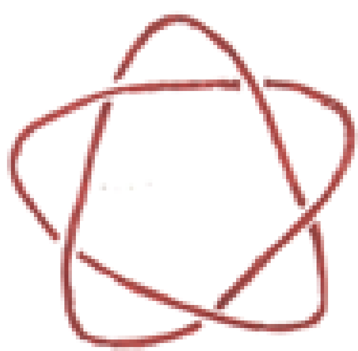
Bag model



String model

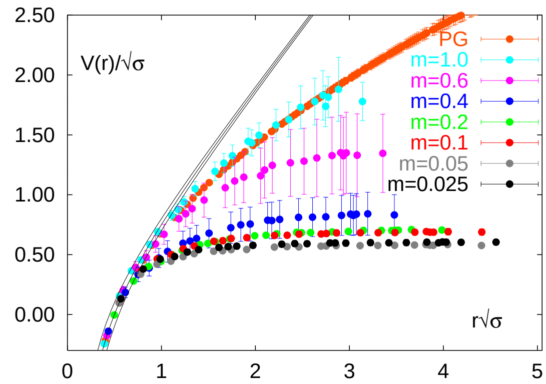


Glozman-Riska model

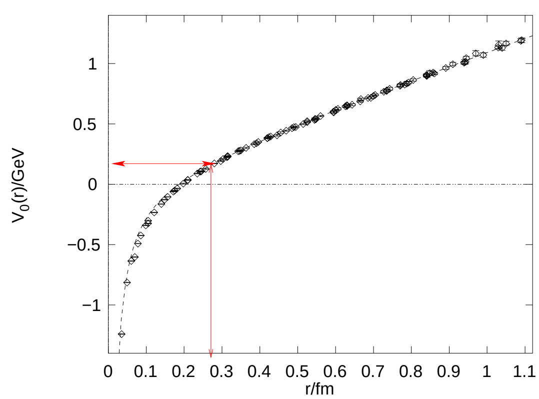


Skyrme-Witten topological chiral soliton model

What do we know about the confining potential?

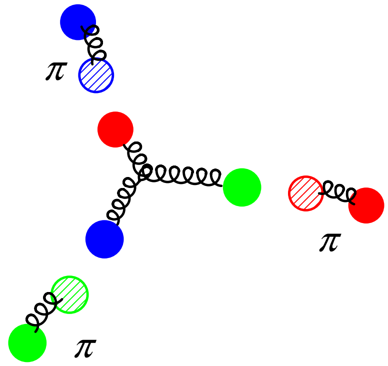


The screening of the linear potential by dynamical quarks is clearly seen in simulations at high temperatures but below the phase transition [Karsch et al. (2001)]. As one lowers the pion mass the string breaks at smaller distances; the scales are $425\,\mathrm{MeV}$ / $0.47\,\mathrm{fm}$.



The lattice-simulated potential between static quarks **in pure glue theory** [Bali et al. (1995)] exceeds m_π at the separation of $0.26\,\mathrm{fm}$, less than the quark size!

The average separation of quarks inside baryons is $\simeq 0.7\,\mathrm{fm}$: At such distances the would-be static potential of the academic pure glue world seems to be perfectly screened.



Because pions are so light, the would-be strings break, and there are plenty of pions inside nucleons, or better to say, the pion field.

The spontaneous chiral symmetry breaking in QCD is responsible for 93-95% of the nucleon mass, since

$$\Sigma = \frac{m_u + m_d}{2} < N|\bar{u}u + \bar{d}d|N >= 45 - 70\,\mathrm{MeV}$$

<u>Three main features of the SCSB:</u>

- Chiral condensate $< \bar{q}q > \simeq -(250\,\mathrm{MeV})^3 \neq 0$

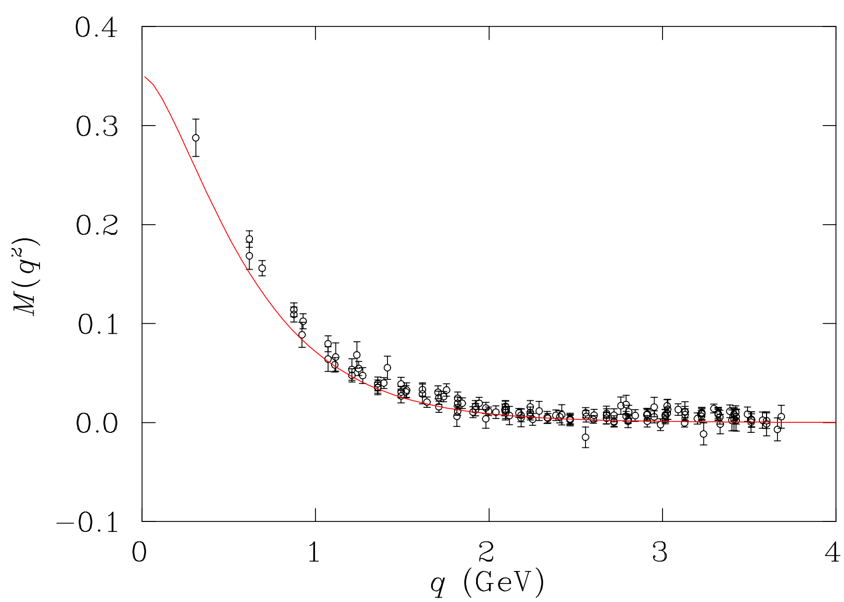
$$< \bar{q}q >= -3 \int \frac{d^4 p}{(2\pi)^4} \mathrm{Tr} \frac{M(p) + \not{p}}{M^2(p) + p^2}$$

- Quarks get dynamical masses: from the 'current' or 'bare' masses

$$\begin{cases} m_u \simeq 4\,\mathrm{MeV} \\ m_d \simeq 7\,\mathrm{MeV} \\ m_s \simeq 150\,\mathrm{MeV} \end{cases} \Rightarrow M_{u,d,s}(p), \qquad M_{u,d}(0) \simeq 350\,\mathrm{MeV}.$$

- The octet of pseudoscalar mesons are anomalously light (pseudo) Goldstone bosons π, K, η

If one has a microscopic theory of the SCSB at hand, one can calculate the momentum-dependent dynamical, or "constituent" quark mass $M(p)$



Extrapolation from the lattice calculation of $M(p)$, courtesy R. Bowler (2002). Solid curve: D.D. and V. Petrov (1986) from instantons.

How to write down the low-momentum lagrangian for constituent quarks?

$$\bar{q}\,(i\partial\!\!\!/ - M)\,q$$

is wrong as it is **not invariant** under chiral rotation

$$q \to \exp(i\gamma_5\alpha^A\lambda^A)\,q, \quad \bar{q} \to \bar{q}\,\exp(i\gamma_5\alpha^A\lambda^A), \quad A = 1..8$$

However

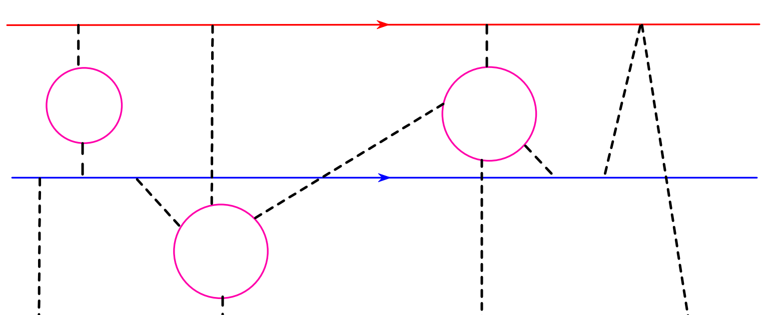
$$\mathcal{L}_{\rm eff} = \bar{q}\,\left[i\partial\!\!\!/ - M\exp(i\,\gamma_5\,\pi^A\lambda^A/F_\pi)\right]\,q$$

is invariant since chiral rotation can be absorbed into the re-definition of the pseudoscalar field $\pi^A = \{\pi, K, \eta\}$.

Quarks that gained a dynamical mass interact with π, K, η very strongly:

$$g_{\pi qq}(0) = \frac{M(0)}{F_\pi} \simeq 4 \quad (!)$$

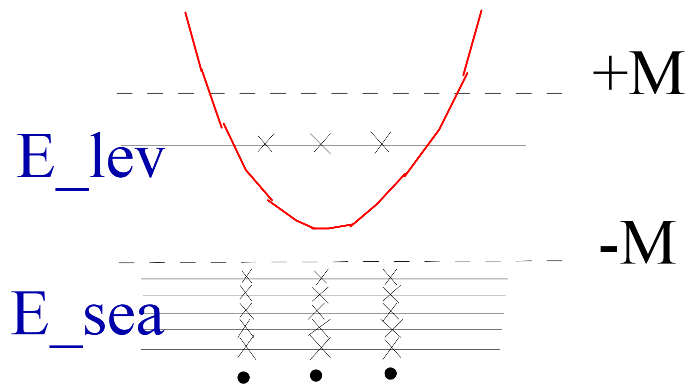
Multiple pion exchanges inside nucleons are important:



Correct, but horrible full-scale relativistic quantum field theory.

Neglect more massive d.o.f.'s (momenta are low); neglect one-gluon exchange (α_s is small); neglect fluctuations of the mean chiral field (suppressed as $\frac{1}{N_c} = \frac{1}{3}$, actually $\times\frac{1}{2\pi}$)

$\Rightarrow$ Chiral Quark Soliton Model [D.D. and V. Petrov (1986)]:



Quarks in the mean chiral field

$\Rightarrow$ Baryon mass $= N_c\,(E_{\rm lev}[\pi(x)] + E_{\rm sea}[\pi(x)])$

There is nothing queer in calling baryons solitons – they are no more solitons than multi-electron Thomas–Fermi atoms, which one may like to call "solitons of the self-consistent electrostatic field".

If one integrates out the quarks one gets the **effective chiral lagrangian**, automatically with the Wess–Zumino–Witten term and correctly describing the low-energy pion scattering.

Rotational states of $SU(3)$ baryons

To describe baryons with strangeness, one uses the *Ansatz* for π, K, η fields [Witten(83), Guadagnini(84)]:

$$\begin{aligned} \pi^a(\mathbf{x}) &= \frac{n^a}{r}P(r), \qquad \mathbf{n} = \frac{\mathbf{x}}{r} \\ K(\mathbf{x}), \eta(\mathbf{x}) &= 0. \end{aligned}$$

The rotation in the flavor $SU(3)$ space is not equivalent to ordinary space rotations, and there are **two** moments of inertia. The rotation along the 8^{th} axis does not affect the *Ansatz*:

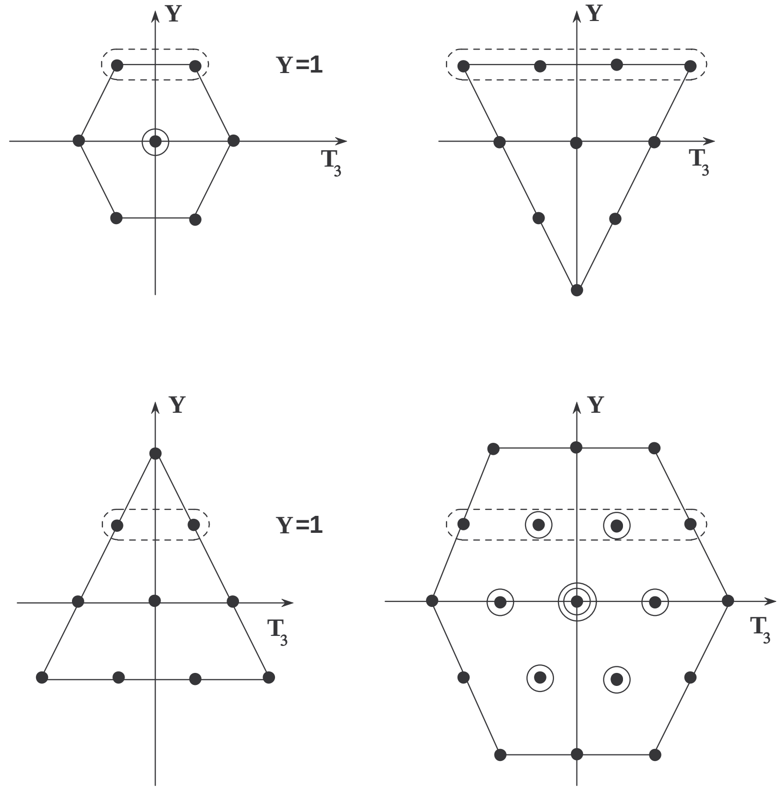
$$R_8 = e^{i\alpha\lambda_8}, \qquad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The rotational Lagrangian is

$$\mathcal{L}_{\text{rot}} = \frac{I_1}{2}\sum_{a=1}^{3}\Omega_a^2 + \frac{I_2}{2}\sum_{A=4}^{7}\Omega_A^2 + \frac{\sqrt{3}}{2}\Omega_8.$$

Hence, only rotational states with $J_8 = \frac{\sqrt{3}}{2}$ have finite energy. Very important, not all spin and $SU(3)$ multiplets are allowed but only those which contain $Y=1$; the number of particles with $Y=1$ gives the spin multiplicity $2J+1$, i.e. the spin is also known.

Examples of allowed baryons: [D.D. and Petrov (84), Mazur, Nowak and Praszalowicz (84), Chemtob (84), Jain and Wadia (85)]



The first two are exactly the lowest multiplets in nature; the antidecuplet with spin and parity $\frac{1}{2}^+$ must be also observed!

Splitting inside multiplets

$$\begin{aligned}
\Delta_{\overline{10}} &= \frac{35}{16}x + \frac{5}{4}y - \frac{1}{6}\frac{m_s}{m_u+m_d}\Sigma \\
&= 170\,\mathrm{MeV}, \quad \Sigma = 45\,\mathrm{MeV}\ [1] \\
&= 110\,\mathrm{MeV}, \quad \Sigma = 70\,\mathrm{MeV}\ [2]
\end{aligned}$$

$\Xi_{\frac{3}{2}} - \Theta^+ = 1862 - 1539 = 323\,\mathrm{MeV} = 3 * 108\,\mathrm{MeV}.$

[1] Gasser, Leutwyler and Sanio (91)

[2] Pavan, Arndt, Strakovsky, Workman (01)

The splitting between multiplet centers

$$\mathcal{M}_{\mathbf{10},\frac{3}{2}} - \mathcal{M}_{\mathbf{8},\frac{1}{2}} = \frac{3}{2I_1}$$
$$\mathcal{M}_{\overline{\mathbf{10}},\frac{1}{2}} - \mathcal{M}_{\mathbf{8},\frac{1}{2}} = \frac{3}{2I_2}.$$

Both moments of inertia $I_{1,2}\to\infty$ for large-size baryons.

Hadron masses $(M \approx 350\,\mathrm{MeV})$

Pseudoscalar mesons: $m_P^2 = (M + M - 2M)^2 + \frac{m_q <\bar{q}q>}{F_\pi}$

Vector mesons: $m_V \approx 2M$

3-quark baryons: $\mathcal{M} \approx 3M$ (+ strangeness)

5-quark baryons, naively: $\mathcal{M} \approx 5M$ (+ strangeness)
$\approx 1900\,\mathrm{MeV}$ for Θ^+ ???

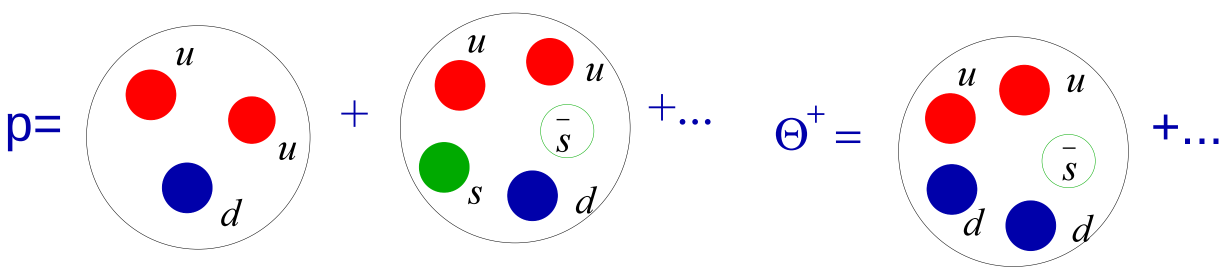
5-quark baryons, correct:

$$\mathcal{M} \approx 3M + \frac{1}{\text{baryon size}} + (\text{splitting from strangeness})$$

In pentaquarks, the additional quark-antiquark pair is added in the form of the excitation of the (nearly massless) chiral field. Energy penalty would be zero, had not the chiral field been restricted to the baryon volume.

There is a wealth of evidence that normal nucleons are not just three 'valence' quarks:

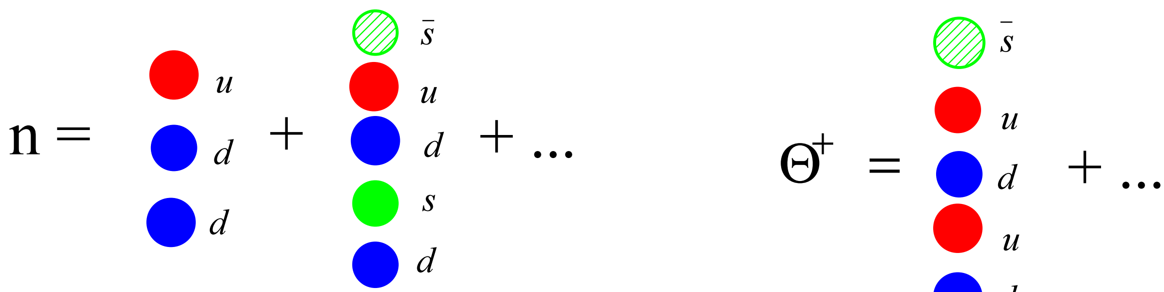
- Drell–Yan pair production and $\bar{\nu}$ DIS has shown in the 70's that there are more antiquarks at low virtuality than following from perturbative bremsstrahlung
- 'spin crisis' of the 80's has shown that nucleon's spin is partly carried by the angular momentum between 'valence' and 'sea' quarks
- nucleon Σ-term is 3-5 times larger than from naive quark counting: the quark-antiquark 'sea' makes it what it is
- ...



Pentaquarks are not qualitatively different from the 5-quark Fock component of the nucleon.

Treating antiquarks in nucleons (not to mention pentaquarks) requires a fully relativistic approach. Caution: Non-relativistic wave-function description with a fixed number of quarks makes no sense [Landau and Peierls (1931)]. "Measuring" quark position with an accuracy higher than the pion Compton wave length (1 fm!) produces a new pion, i.e. a new $Q\bar{Q}$ pair.

What makes sense, is describing baryons in the infinite momentum frame. In the IMF there is no production and annihilation, and the baryon wave function falls into separate sectors of the Fock space: 3 quarks, 5 quarks,...



Neutron and Theta in the infinite momentum frame

Wave functions depend on quark momenta $(z, \mathbf{p}_\perp)$, colors, (α) flavors (f) and polarizations (λ), and also on the polarization of the neutron, $\lambda = 1, 2$.

3-quark component of the neutron wave function (in the non-relativistic limit):

$$\begin{aligned} & (|n>_\lambda)^{f_1 f_2 f_3, \lambda_1 \lambda_2 \lambda_3} = \epsilon^{f_1 f_2}\, \epsilon^{\lambda_1 \lambda_2}\, \delta_2^{f_3}\, \delta_\lambda^{\lambda_3} \\ \cdot \quad & h(z_1, p_{1\perp}) h(z_2, p_{2\perp}) h(z_3, p_{3\perp}) \\ + \quad & \text{permutations of } 1, 2, 3 \end{aligned}$$

coincides with the well-known $SU(6)$ wave function!

5-quark component:

$$(|n>_\lambda)^{f_1 f_2 f_3 f_4, \lambda_1 \lambda_2 \lambda_3 \lambda_4}_{f_5, \lambda_5} = \text{lengthy} \ldots$$

Θ's 5-quark component: $(\Theta^+ = uudd\bar{s} + ...)$

$$
\begin{aligned}
&(|\Theta>_\lambda)^{f_1 f_2 f_3 f_4, \lambda_1 \lambda_2 \lambda_3 \lambda_4}_{f_5, \lambda_5} \\
= \quad & \epsilon^{f_1 f_2}\, \epsilon^{f_3 f_4}\, \delta^3_{f_5}\, \epsilon^{\lambda_1 \lambda_2}\, W^{\lambda_3 \lambda_4}_{\lambda\, \lambda_5}(z_4, z_5)\, h(z_1) h(z_2) h(z_3) \\
+ \quad & \text{permutations of } 1, 2, 3
\end{aligned}
$$

In the CQSM all functions of $z_1 ... z_5$ are known! The color structure is, of course $\epsilon^{\alpha_1 \alpha_2 \alpha_3}\, \delta^{\alpha_4}_{\alpha_5}$.

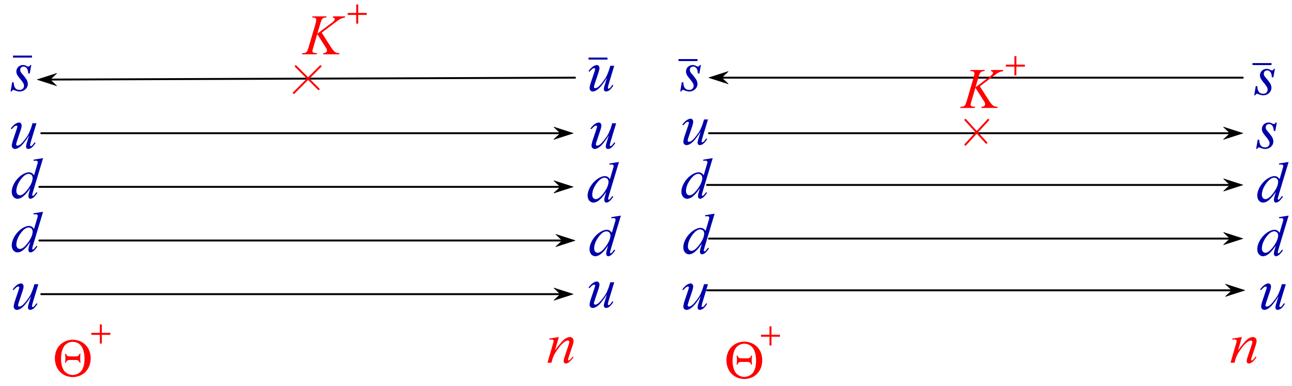
From this wave function, one can compute Θ's parton distributions, formfactors, quark longitudinal and transverse distributions, etc. Similarly for $\Xi_{\frac{3}{2}}$.

Θ, $\Xi_{3/2}$ decay widths

The decay amplitude $\Theta \to KN$ is proportional, thanks to Goldberger–Treiman, to the transition axial charge,

$$
g_{\Theta NK} = \frac{g_A(\Theta \to NK)(\mathcal{M}_\Theta + \mathcal{M}_N)}{F_K} \sim < N | \bar{u}\, \gamma_0 \gamma_5\, s | \Theta >
$$

In the IMF the axial charge does not create or annihilate quarks but measures the transition between existing quarks. It means that it is $\neq 0$ only in transition between Θ and the 5-quark component of the n wave function.



Two diagrams contributing to Theta's decay

It is suppressed to the extent the 5-quark component of the neutron is less than its 3-quark component. Additional suppression comes from the peculiar **flavor** structure of the neutron's 5-quark component where the $\bar{Q}$ is in the flavor-singlet combination with one of the four Q.

Why name it Θ^+?

- The tradition is to name baryons by capital Greek characters: $\Delta, \Lambda, \Sigma, \Xi, \Omega$
- The name must be distinct from anything used before and carry no associations with mesons
- The character must exist in LaTeX: $\cancel{\Gamma}, \cancel{\Delta}, \Theta, \cancel{\Lambda}, \cancel{\Xi}, \cancel{\Pi}, \cancel{\Sigma}, \cancel{\Upsilon}, \cancel{\Phi}, \cancel{\Psi}, \cancel{\Omega}$.

Notice: no free upper-case Greek characters left! $\Xi(1862)(?)$ must be named either $\Xi_{3/2}$ or $\Xi_{\overline{10}}$.

Summary

1. 95% of the nucleons' mass is due to the Spontaneous Chiral Symmetry Breaking. It implies that quarks get a large dynamically-generated mass, which inevitably leads to their strong coupling to chiral fields (π, K, η).

2. Assuming that chiral forces are essential in binding quarks together in baryons, one gets the lowest multiplets $(\mathbf{8}, \frac{1}{2})$, $(\mathbf{10}, \frac{3}{2})$ and $(\overline{\mathbf{10}}, \frac{1}{2})$ whose properties are related by symmetry.

3. The predicted $\Theta^+ \simeq uudd\bar{s}$ is light because it is **not** a sum of constituent quark masses but rather a collective excitation of the mean chiral field binding baryons. It is narrow for the same reasons.

4. The non-relativistic wave-function description of an atom is valid at distances $10^{-8}\,\mathrm{cm}$, but fails at $1/m_e c = 10^{-11}\,\mathrm{cm}$. For baryons, "$10^{-11}\,\mathrm{cm}$" is $1\,\mathrm{fm}$. Relativistic field-theoretic description is a must.

5. The discovery of Θ^+ has demonstrated how poorly we understand QCD. Hopefully, the lesson will be learned and we'll know what the ordinary proton and neutron are "made of" and "how do they work".