Where are the Cascade Pentaquarks (Ξ_5) and what are their widths?

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Pentaquark ($q^4\bar{q}$)

$SU(3)_F : 3 \otimes 3 \otimes 3 \otimes 3 \otimes \bar{3} \rightarrow$ many possibilities (multiplets)

θ^+ (s = +1) member of 35plet, 27plet or antidecuplet.

Why antidecuplet?

isospin = 0 (searches for $\theta^{++} \rightarrow$ no result)

Other Properties:

- spin = 1/2 ?
- parity ?

All member of multiplet: same mass without $SU(3)_F$ symmetry breaking.

Symmetry Breaking:

1. strange quark mass (m_s)
2. Flavor-Spin interaction

Hidden Strangeness:

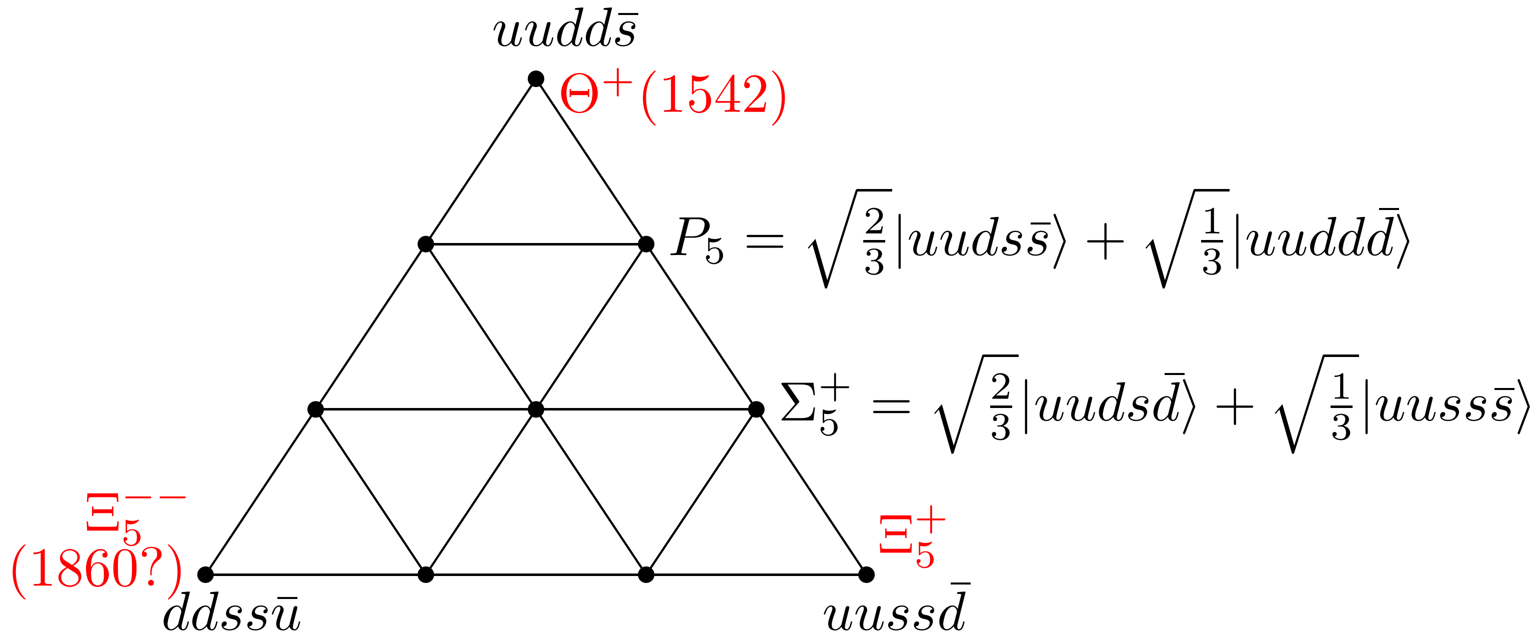

Figure 1: $SU(3)_F$ Antidecuplet for Pentaquark

- naively, $m(\Xi_5^+) - m(\Theta^+) = m_s \approx 150$ MeV.
- Mixing: effect only on N_5 and Σ_5.

Effective Flavor-Spin Interaction,

isospin conserving, breaking $SU(3)_F$

Mass correction:

$$
\begin{aligned}
\Delta M = & - C_{SI} \sum_{\alpha<\beta} (\tau\sigma)_\alpha \cdot (\tau\sigma)_\beta - C_{47} \sum_{\alpha<\beta, i=4}^{7} (\lambda^i \sigma)_\alpha \cdot (\lambda^i \sigma)_\beta \\
& - C_8 \sum_{\alpha<\beta} (\lambda^8 \sigma)_\alpha \cdot (\lambda^8 \sigma)_\beta
\end{aligned}
$$

C_{SI}, C_{47} and C_8 fit to baryon octet and decuplet (q^3)

Mass formula for (q^3)baryon

$$M = M_0^{(3)} + x_1 C_{SI} + x_2 C_{47} + x_3 C_8 + n_s \Delta m_s$$

Result from fitting:

$$
\begin{aligned}
M_0^{(3)} &= 1340.5 \pm 5.3 \text{ MeV}, \quad \Delta m_s = 136.3 \pm 2.5 \text{ MeV} \\
C_{SI} &= 28.2 \pm 0.5 \text{ MeV}, \quad C_{47} = 20.7 \pm 0.5 \text{ MeV}, \\
C_8 &= 19.7 \pm 1.2 \text{ MeV}
\end{aligned}
$$

Negative Parity Pentaquark

- all $q's$ and $\bar{q}$ are in orbital ground state
- totally antisymmetric Color, Flavor, Spin (CFS) wavefunction (q^4)

The CFS wavefunction are:

$(\mathbf{qq})(\mathbf{qq})$

$$
\begin{aligned}
|(\mathbf{3},\bar{\mathbf{6}},1)\rangle &= \frac{1}{\sqrt{3}}|(\bar{\mathbf{3}},\mathbf{6},1)(\bar{\mathbf{3}},\mathbf{6},1)\rangle + \frac{1}{\sqrt{12}}\left[|(\mathbf{6},\mathbf{6},0)(\bar{\mathbf{3}},\mathbf{6},1)\rangle + |(\bar{\mathbf{3}},\mathbf{6},1)(\mathbf{6},\mathbf{6},0)\rangle\right] \\
&- \frac{1}{2}\left[|(\mathbf{6},\bar{\mathbf{3}},1)(\bar{\mathbf{3}},\bar{\mathbf{3}},0)\rangle + |(\bar{\mathbf{3}},\bar{\mathbf{3}},0)(\mathbf{6},\bar{\mathbf{3}},1)\rangle\right]
\end{aligned}
$$

combined with $|(\bar{\mathbf{3}},\bar{\mathbf{3}},1/2)\rangle$ to form $|(\mathbf{1},\overline{\mathbf{10}},1/2)\rangle$

$(\mathbf{qqq})(\mathbf{q\bar{q}})$

$$
\begin{aligned}
|(\mathbf{1},\overline{\mathbf{10}},1/2)\rangle &= \frac{1}{2}|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},0)\rangle + \frac{1}{\sqrt{12}}|(\mathbf{1},\mathbf{8},1/2)(\mathbf{1},\mathbf{8},1)\rangle - \frac{1}{\sqrt{3}}|(\mathbf{8},\mathbf{8},3/2)(\mathbf{8},\mathbf{8},1)\rangle \\
&+ \frac{1}{2}|(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},0)\rangle + \frac{1}{\sqrt{12}}|(\mathbf{8},\mathbf{8},1/2)(\mathbf{8},\mathbf{8},1)\rangle
\end{aligned}
$$

Mass formula for Pentaquark:

$$M = M_0^{(5)} + x_1 C_{SI} + x_2 C_{47} + x_3 C_8 + n_s^{eff} \Delta m_s$$

Note:

1. M_0^5: no reliable theoretical prediction, largest effect for $q^3 \to q^4 \bar{q}$
2. C_{SI}, C_{47} and C_8: assumed constant

State	x_1	x_2	x_3	n_s^{eff}	M (MeV)	
Θ^+	-10	0	10/3	1	1542	
N_5	$-20/3$	2	-2	4/3	1618	w/o
Σ_5	$-25/9$	$-2/9$	$-11/3$	5/3	1694	mixing
Ξ_5	5/3	$-20/3$	$-5/3$	2	1771	

Table 1: Prediction for Negative Parity.

Positive Parity Pentaquark

- one of the $q's$ is in P-state and $\bar{q}$ is in S-state $\rightarrow$ higher mass due to excitation energy (in HO $= \hbar\omega$),

- Flavor-Spin wavefunction: totally symmetric(q^4) $\rightarrow$ maximal Flavor-Spin interaction $\rightarrow$ lower total mass.
  W/O $SU(3)_F$ symmetry breaking:

$$\Delta M_\chi = \begin{cases} -20/3C_\chi & S^4 \quad \text{(negative)} \\ -28C_\chi & S^3P \quad \text{(positive)} \end{cases}$$

  Assuming $\hbar\omega \approx 250$ MeV and $C_\chi \approx 25$ MeV,

$$M(S^3P) - M(S^4) = \hbar\omega - 64/3C_\chi \approx -280 \text{ MeV}$$

- Color-Orbital wavefunction: totally antisymmetric(q^4).

Flavor-Spin wavefunction

$$\left|(\overline{\mathbf{10}}, 1/2)\right\rangle = \{\frac{1}{\sqrt{2}}\,|(\bar{\mathbf{3}}, 0)(\bar{\mathbf{3}}, 0)\rangle_{\bar{\mathbf{6}},0} + \frac{1}{\sqrt{2}}\,|(\mathbf{6}, 1)(\mathbf{6}, 1)\rangle_{\bar{\mathbf{6}},0}\} \otimes |(\bar{3}, 1/2)\rangle$$

State	x_1	x_2	x_3	n_s^{eff}	M (MeV)	
Θ^+	-30	0	2	1	1542	
N_5	-20	-8	0	$4/3$	1665	w/o
Σ_5	$-31/3$	$-44/3$	-3	$5/3$	1786	mixing
Ξ_5	-1	-20	-7	2	1906	

Table 2: Prediction for Positive Parity.

Note:

$q\bar{q}$ does not contribute to the matrix element $(\langle\, q\bar{q}|\,(\lambda^F\sigma)_\alpha(\lambda^F\sigma)_\beta\,|q\bar{q}\,\rangle = 0)$

Width Prediction

$$\Gamma = \frac{M}{32\pi}\sqrt{\left(1-\left(\frac{m+\mu}{M}\right)^2\right)\left(1-\left(\frac{m-\mu}{M}\right)^2\right)}$$
$$\times \left[\left(1\pm\frac{m}{M}\right)^2-\left(\frac{\mu}{M}\right)^2\right][A]^2\left|\langle nK^+|\Theta^+\rangle\right|^2$$

with M: pentaquark mass, m: q^3 baryon mass and μ: meson mass
$+$: S-wave decay, negative parity
$-$: P-wave decay, positive parity
$[A]^2$ = A number from group theory

Decay	$\lvert A/A_0\rvert^2$	Γ/Γ_0(+ parity)	Γ/Γ_0(- parity)
$\Theta^+ \to pK^0$	1	0.97	0.99
$\Xi_5^+ \to \Xi^0\pi^+$	1	3.23	1.69
$\Xi_5^+ \to \Sigma^+\bar{K}^0$	1	2.22	0.99

SU(3) decay predictions for the highest isospin members of antidecuplet.
Γ_0 is for $\Theta^+ \to nK^+$

Conclusion

- mass splitting in multiplet: from m_s and Flavor-Spin interaction
- overall Positive parity mass is less than Negative parity
- Positive parity has wider split in mass spectrum than negative parity

Numerical results:

Negative Parity

$$M(\Xi_5) = 1771 \quad \text{MeV} \qquad \frac{\Gamma(\Xi_5)}{\Gamma(\Theta^+)} = 1.35$$

Positive Parity

$$M(\Xi_5) = 1906 \quad \text{MeV} \qquad \frac{\Gamma(\Xi_5)}{\Gamma(\Theta^+)} = 2.76$$

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