Pentaquark in the SU(3) Skyrme Model and Chiral Quark Soliton Model

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Phys. Lett. **B575** (2003) 234 [hep-ph/0308114]
Talk at the Cracow Workshop on Skyrmions and Anomalies, Mogilany, Poland, Feb 20-24, 1987, World Scientific 1987, p.112.

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Particle Data Group 1986

NOTE ON THE $S = +1$ BARYON SYSTEM

The evidence for strangeness +1 baryon resonances was reviewed in our 1976 edition,[1] and more recently by Kelly[2] and by Oades.[3] Two new partial-wave analyses[4] have appeared since our 1984 edition. Both claim that the P_{13} and perhaps other waves resonate. However, the results permit no definite conclusion — the same story heard for 15 years. The standards of proof must simply be much more severe here than in a channel in which many resonances are already known to exist. The general prejudice against baryons not made of three quarks and the lack of any experimental activity in this area make it likely that it will be another 15 years before the issue is decided.

References

1. Particle Data Group, Rev. Mod. Phys. **48**, S188 (1976).
2. R.L. Kelly, in *Proceedings of the Meeting on Exotic Resonances* (Hiroshima, 1978), ed. I. Endo et al.
3. G.C. Oades, in *Low and Intermediate Energy Kaon-Nucleon Physics* (1981), ed. E. Ferrari and G. Violini.
4. K. Hashimoto, Phys. Rev. **C29**, 1377 (1984); and R.A. Arndt and L.D. Roper, Phys. Rev. **D31**, 2230 (1985).

Chiral Quark Model

constituent quark mass ~ 350 MeV

$$\mathcal{Z} = \int D\pi^A \int D\psi^\dagger D\psi \, \exp \int d^4x \, \psi^\dagger(x) \left[i\partial\!\!\!/ + iMU^{\gamma_5}(x) \right] \psi(x)$$

pions fermions

$$U^{\gamma_5} \equiv \exp i \frac{\pi^A \lambda^A \gamma_5}{F_\pi}$$

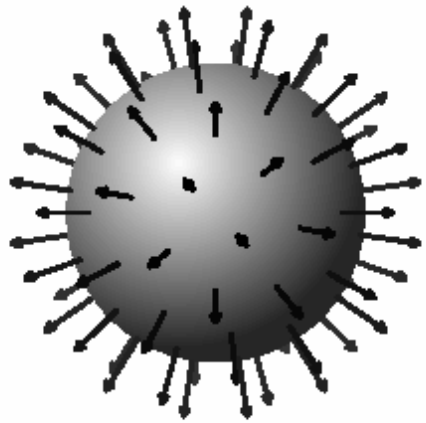
integrate out quarks

"Skyrme" Model

$$\mathcal{L} = \frac{F_\pi^2}{4} \mathrm{Tr} \left(\partial_\mu U^\dagger \partial^\mu U \right) + \frac{1}{32e^2} \mathrm{Tr} \left(\left[\partial_\mu U \, U^\dagger, \partial_\nu U \, U^\dagger \right]^2 \right) + \Gamma_{\mathrm{WZ}} + \ldots.$$

only pion fields, kinetic term + interaction terms

SU(3) soliton: static solution



hedgehog

hedgehog Ansatz:

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}\,P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

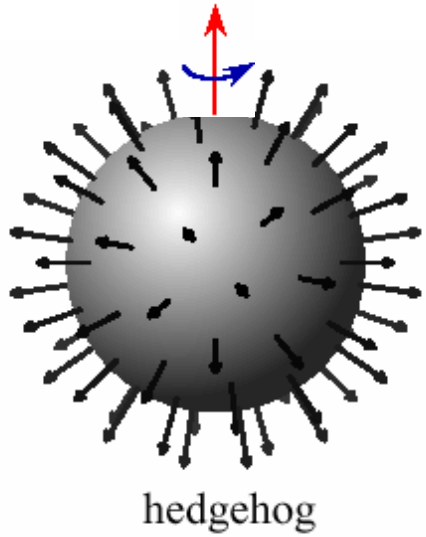
$P(0)=n\pi$ and $P(\infty)=0$ n - winding number

variational approach: $P(r)=P(r/r_0)$, r_0 - soliton size

Energy has minimum as a function of r_0

In the Skyrme model winding number is interpreted as a baryon number.

Quantizing SU(3) Skyrmion



time-dependent rotation

$$U = A(t) U_0 A^\dagger(t)$$

angular velocities:

$$A^\dagger \dot{A} = \frac{i}{2} \Omega_i \lambda_i = \frac{i}{2} \dot{\theta}_k e_{ki}(\theta) \lambda_i$$

Quantizing SU(3) Skyrmion

every quantity is a power series in Ω

$$L_0 = -M_{\rm cl} + \frac{I_1}{2}\sum_{a=1}^{3}\Omega_a^2 + \frac{I_2}{2}\sum_{a=4}^{7}\Omega_a^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

no kinetic term for Ω_8

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}\,P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

commutes with λ_8

$$U = A(t)\,hU_0h^\dagger A^\dagger(t) \qquad h = e^{i\varphi_8\lambda_8}$$

Quantizing SU(3) Skyrmion

add small moment of inertia ε

$$L_0 = -M_{\rm cl} + \frac{I_1}{2}\sum_{a=1}^{3}\Omega_a^2 + \frac{I_2}{2}\sum_{a=4}^{7}\Omega_a^2 + \frac{\varepsilon}{2}\Omega_8^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

generalized "momenta":

$$J_a = I_1\Omega_a \text{ for } a = 1,2,3$$

$$J_a = I_2\Omega_a \text{ for } a = 4,5,6,7$$

$$J_8 = \varepsilon\Omega_8 + \frac{N_c}{2\sqrt{3}}$$

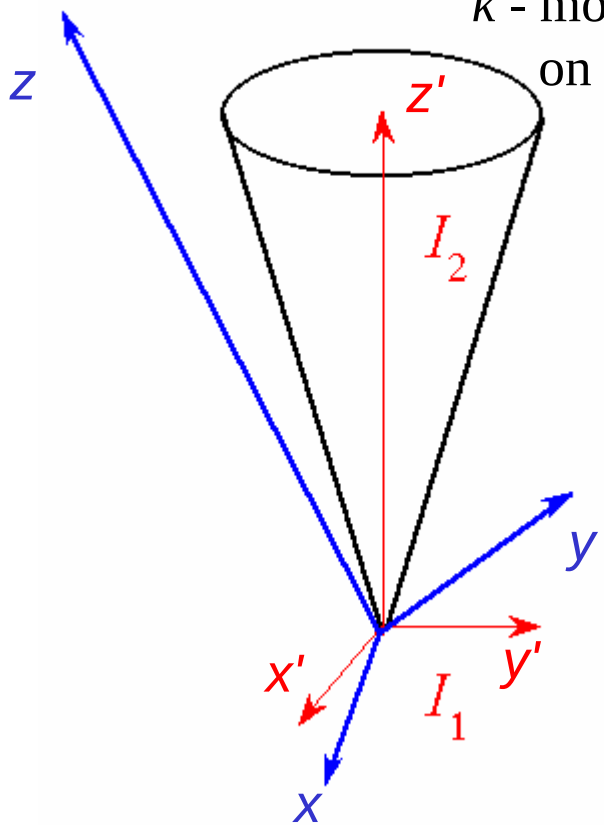
Hamiltonian:

$$H_0 = M_{\rm cl} + \frac{1}{2I_1}\sum_{a=1}^{3}J_a^2 + \frac{1}{2I_2}\sum_{a=4}^{7}J_a^2 + \frac{1}{2\varepsilon}\underbrace{\left(J_8 - \frac{N_c}{2\sqrt{3}}\right)^2}_{\text{constraint for } \varepsilon \to 0}$$

Wave functions
analogy with a symmetric top

m - momentum projection on *z*

k - momentum projection on *z'*



QM textbook

Landau, Lipschitz: QM sect. 103

$$\psi \sim N\, D^{(J)*}_{mk}(\varphi', \theta, \varphi)$$

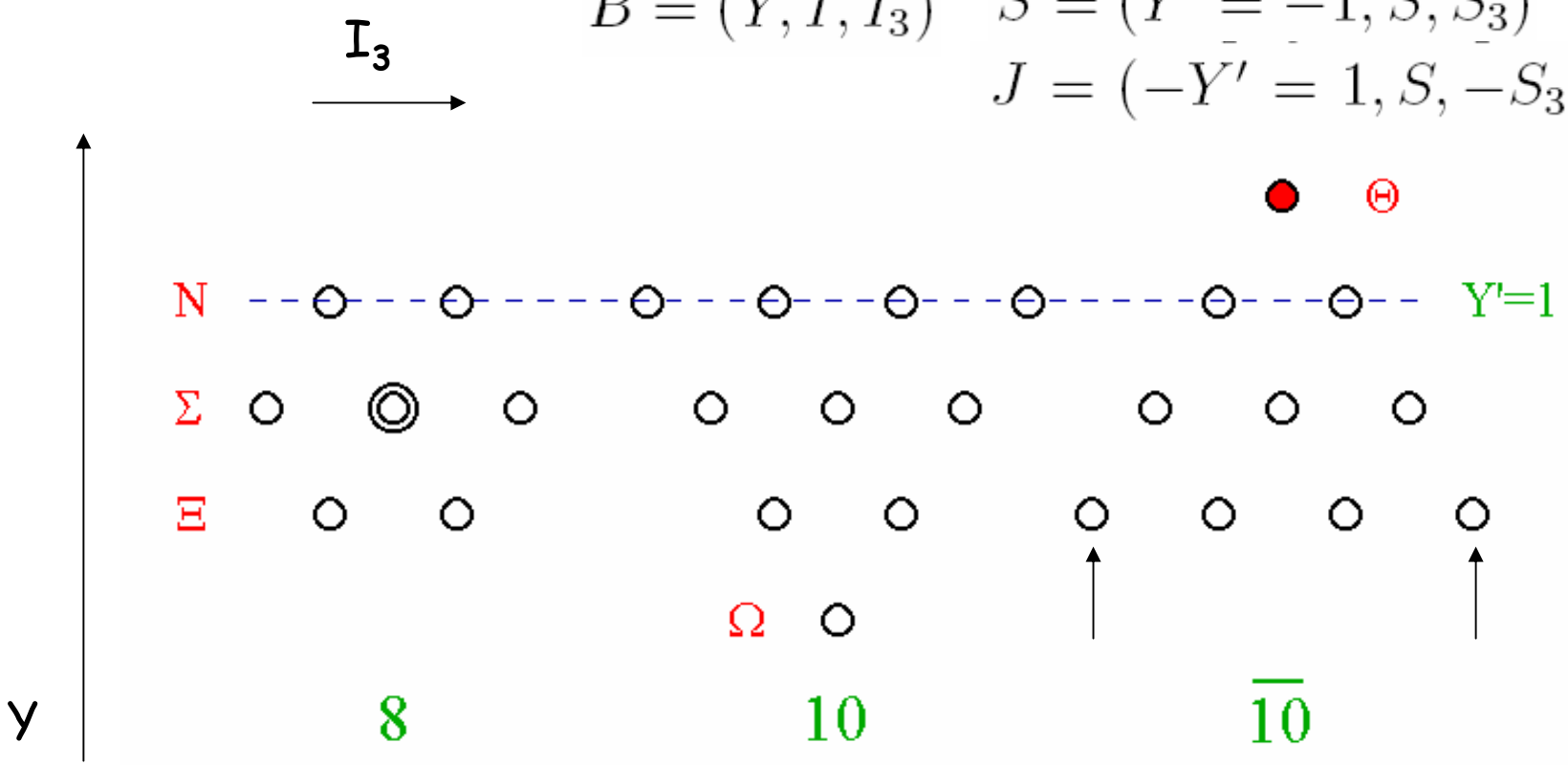
m,k - angular momentum projections

In SU(3) $J \rightarrow R = (p,q)$, $m,k \rightarrow (Y, I, I_3)$
however, because of the constraint
not all k's are allowed but only
those which have $k = (Y=1, I, I_3)$

Wave functions and allowed states

$$\psi_{BS}^{(\mathcal{R})} = \psi_{(\mathcal{R},B)(\mathcal{R}^*,S)} = (-)^{Q_S}\sqrt{\dim(\mathcal{R})}D_{BJ}^{(\mathcal{R})*}$$

$$B = (Y, I, I_3) \quad S = (Y' = -1, S, S_3)$$
$$J = (-Y' = 1, S, -S_3)$$

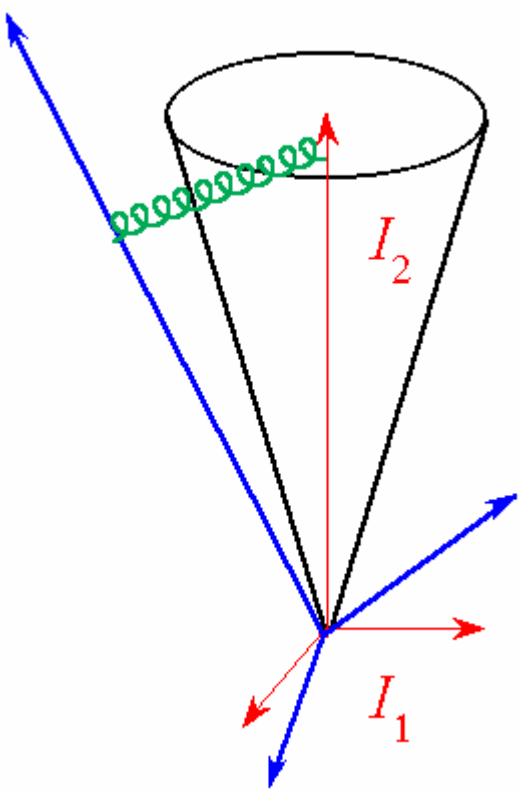


Mass formula

$$H_0 = M_{\rm cl} + \frac{1}{2I_1} S(S+1) + \frac{1}{2I_2}\left(C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12}\right)$$

octet-decuplet splitting

exotic-nonexotic splittings



first order perturbation in the strange quark mass:

$$\sigma = \frac{1}{3}\frac{\Sigma_{\pi N}}{\overline{m}}$$

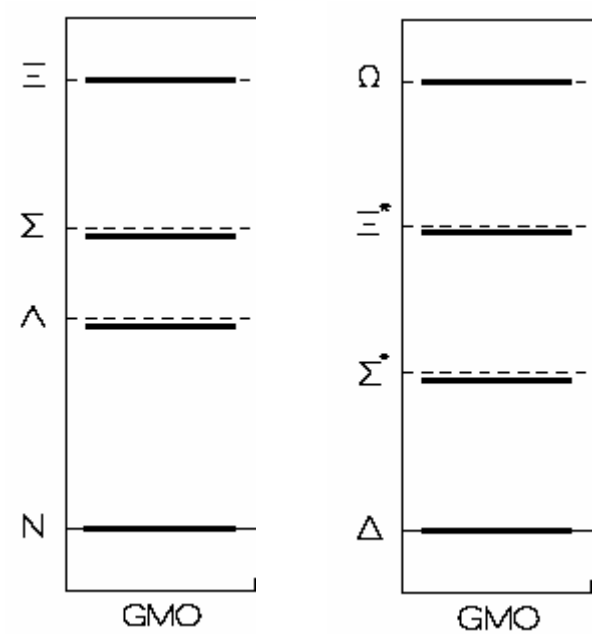
$$H' = m_{\rm cl} - \alpha D_{88}^{(8)}(A) = m_s\sigma - m_s\sigma D_{88}^{(8)}(A)$$

Gell-Man Okubo mass formulae

$$\Delta M_B^{(8)} = F \begin{pmatrix} 8 & 8 & 8_- \\ 000 & B & B \end{pmatrix} + D \begin{pmatrix} 8 & 8 & 8_+ \\ 000 & B & B \end{pmatrix}$$

$$\Delta M_B^{(10)} = C \begin{pmatrix} 8 & 10 & 10 \\ 000 & B & B \end{pmatrix}$$

three independent constants:
F, C and D

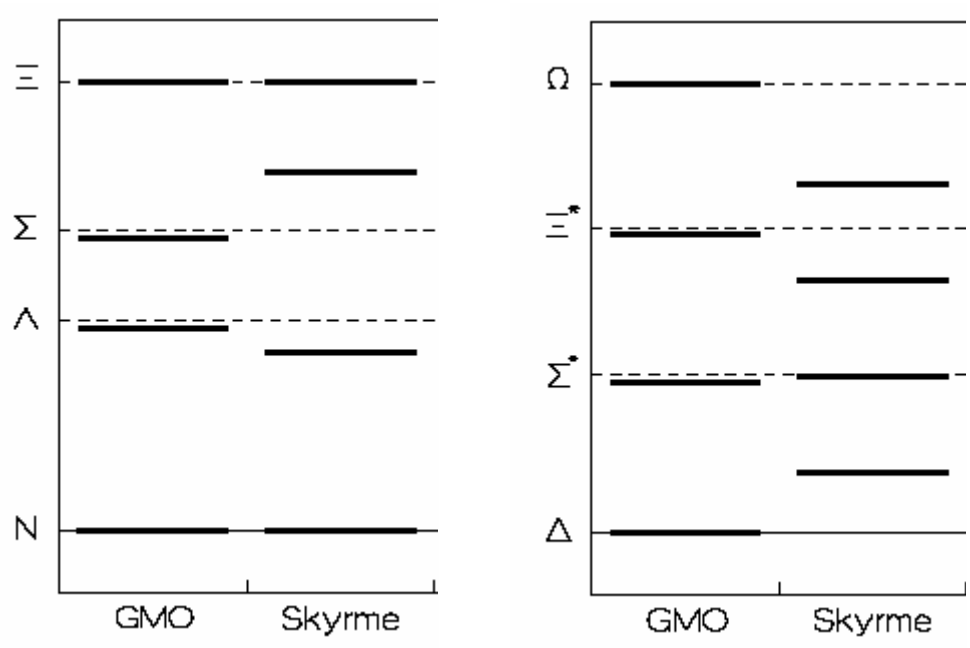


$$F = 379, \quad D = 79 \pm 17 \quad \text{and} \quad C = 415 \pm 15 \quad \text{MeV}$$

Chiral model relations

$$F = -\frac{1}{2}\,\alpha, \qquad D = -\frac{1}{2\sqrt{5}}\,\alpha, \qquad C = -\frac{1}{2\sqrt{2}}\,\alpha$$

$$\frac{F}{D} = \sqrt{5} = 2.24 \quad (\text{exp.} \quad 4.40), \qquad \frac{F}{C} = \sqrt{2} = 1.41 \quad (\text{exp.} \quad 0.91)$$



symmetry breaking Hamiltonian is too primitive: richer structure is needed

Yabu-Ando: higher orders in m_s

$$H'_{\rm SM} = -\alpha D^{(8)}_{88}(A)$$

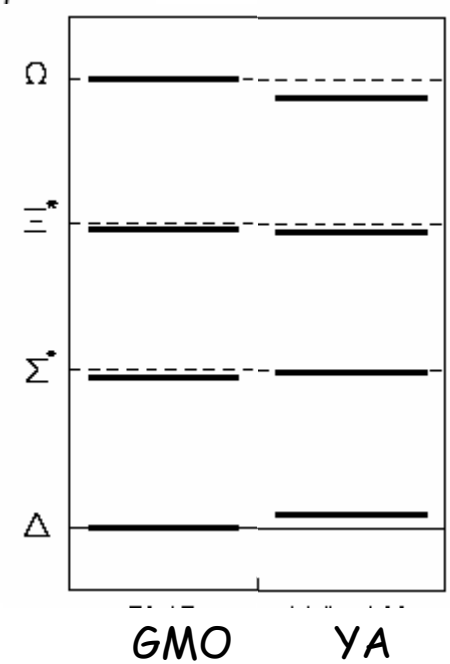
H. Yabu, K. Ando, Nucl.Phys.B301 (1988) 601

second order:

$$M^{(2)}_B(R) = -\alpha^2 \sum_{R'} \frac{\left|\left\langle R', B, S \mid D^{(8)}_{88} \mid R, B, S \right\rangle\right|^2}{M^{(0)}(R') - M^{(0)}(R)}$$

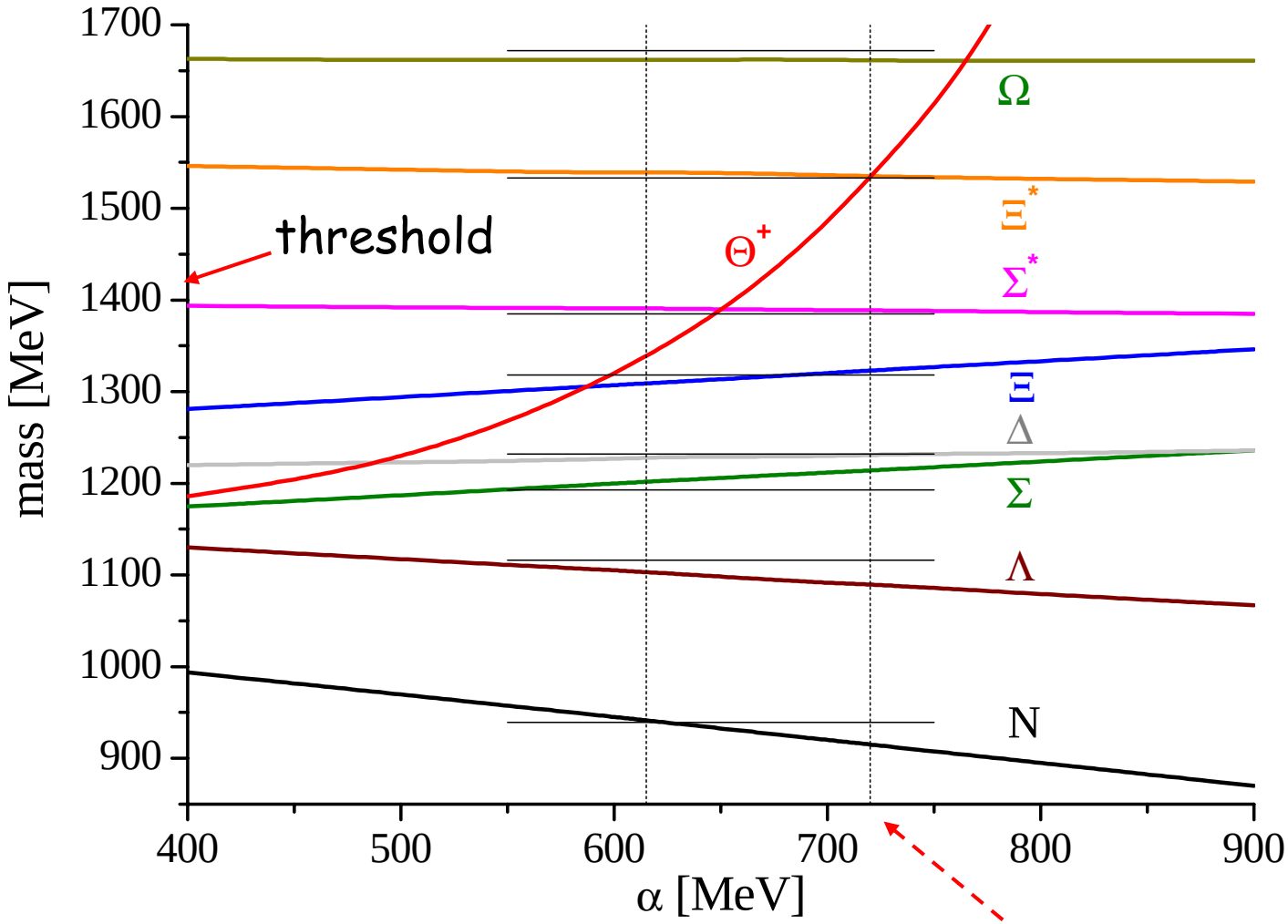
$$M^{(0)}(R') - M^{(0)}(R) = \frac{\Delta_{R'R}}{2I_2}$$

$$\Delta_{R'R} = C_2^{R'}(\mathrm{SU}(3)) - C_2^{R}(\mathrm{SU}(3))$$



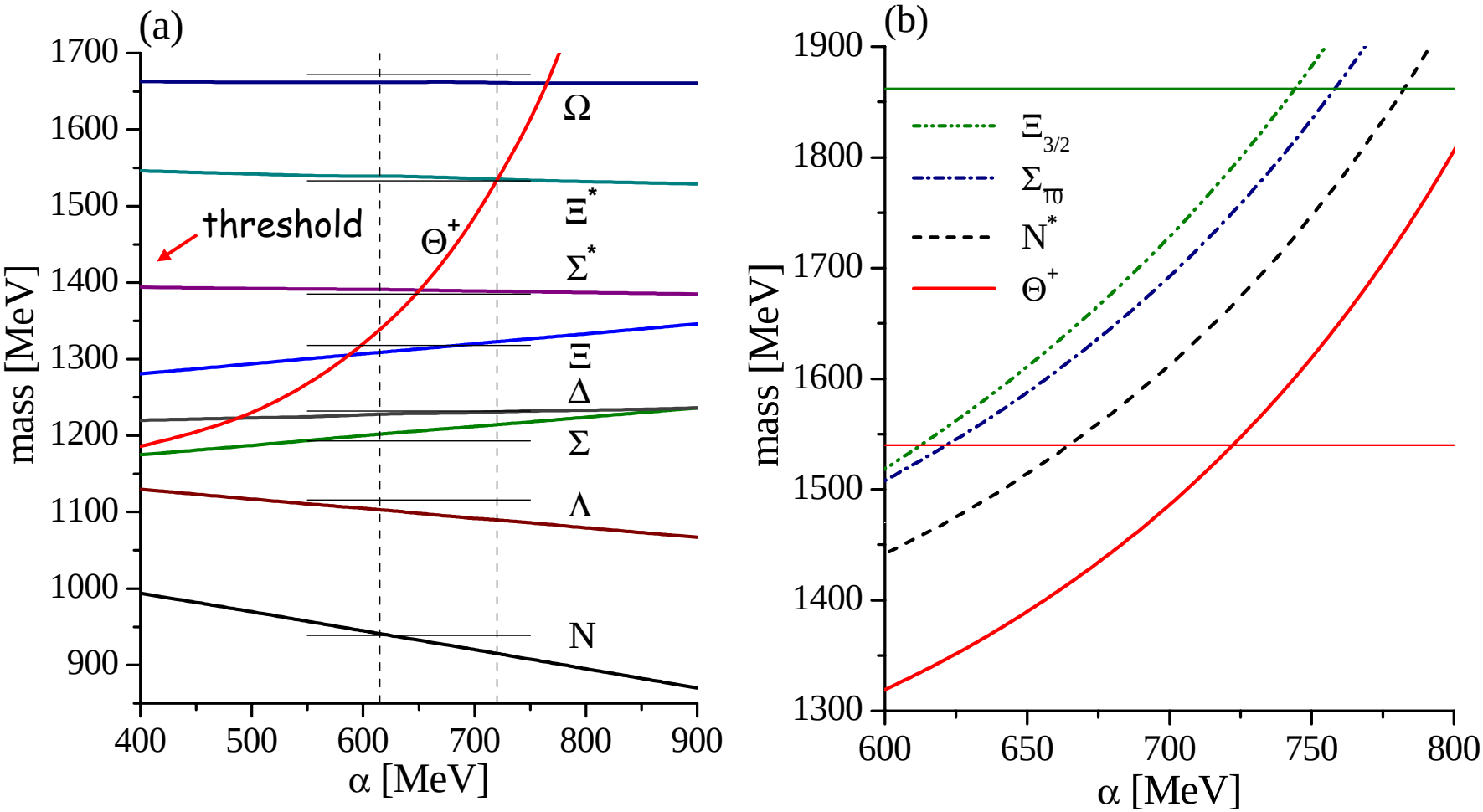
4 free parameters: M_{sol}, I_1, I_2 and α,
but now I_2 contributes to nonexotic splittings

fix α and then minimize χ^2 with respect to the remaining parameters



$$\alpha = \frac{m_s}{m_u + m_d}\frac{2}{3}\Sigma_{\pi N}$$

$$1987 \rightarrow \Sigma_{\pi N} \sim 60 \text{ MeV}, \quad m_s \sim 200 \text{ MeV} \rightarrow \alpha = 720 \text{ MeV}$$



the simplest Skyrme model predicts light Θ^+

prediction for Ξ matches NA49 data within model accuracy

Numerical values for all masses

	exp.	$\alpha = 720$	push
N	938	915	-23
Λ	1116	1090	-26
Σ	1193	1214	$+21$
Ξ	1318	1323	$+5$
Δ	1322	1321	-1
Σ^*	1385	1389	$+4$
Ξ^*	1533	1535	$+2$
Ω^-	1672	1662	-10
Θ^+	1539	1534	-5
N^*		1667	
$\Sigma_{\overline{10}}$		1751	
$\Xi_{3/2}$	1862	1784	-78

χQM breaking hamiltonian

calculate next-to-leading contributions to H'

E. Guadagnini
Nucl.Phys.**B236** (1984) 35

$$H'_{\chi QM} = -\alpha' D^{(8)}_{88} + \beta Y + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D^{(8)}_{8i} S_i$$

$O(N_c)+O(1)$ $O(1)$ $O(1)$ all $O(m_s)$

equivalent to Guadagnini mass formula:

$$H_G = \alpha\, D^{(8)}_{8,8}(A) + \beta\, \mathbf{Y}$$

in the Skyrme model
$\beta, \gamma = 0$

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

richer H':

* no handle on I_2
* only 2 linear combinations of parameters

  α', β and γ enter nonexotic splittings

  splittings in $10 \neq 1\overline{0}$

Width

Diakonov, Petrov, Polyakov, Z.Phys A359 (97) 305

Weigel, hep-ph/0006619

$$\hat{O} = -i\frac{3}{2M_B} \times \left[G_0 D^{(8)}_{mi}(A) - G_1 d_{abi} D^{(8)}_{ma} \hat{S}_b - G_2 \frac{1}{\sqrt{3}} D^{(8)}_{m8} \hat{S}_i \right] \times p_i$$

$\boxtimes \overline{N_c} \diamond$ $\left[O(N_c) \qquad O(1) \qquad O(1) \right]$

one can extract constants $G_{0,1,2}$ from known decuplet decays

$G_2 \sim 0$, then

$$G_{10} = G_0 + G_1/2 \sim 19 \quad \text{and} \quad G_{\overline{10}} = G_0 - G_1 \sim 0.5$$

note that $\Gamma_R \sim G_R^2$; that explains smallness of the Θ width

exact result is: $G_{\overline{10}} = G_0 - G_1 - G_2/2$

hence: $G_{\overline{10}} = 0$!

in QM limit $G_1/G_0 = 4/5$ and $G_2/G_0 = 2/5$

Wave functions and allowed states

$$\psi_{BS}^{(\mathcal{R})} = \psi_{(\mathcal{R},B)(\mathcal{R}^*,S)} = (-)^{Q_S}\sqrt{\dim(\mathcal{R})}D_{BJ}^{(\mathcal{R})*}$$

$$B = (Y, I, I_3) \quad S = (Y' = -1, S, S_3)$$
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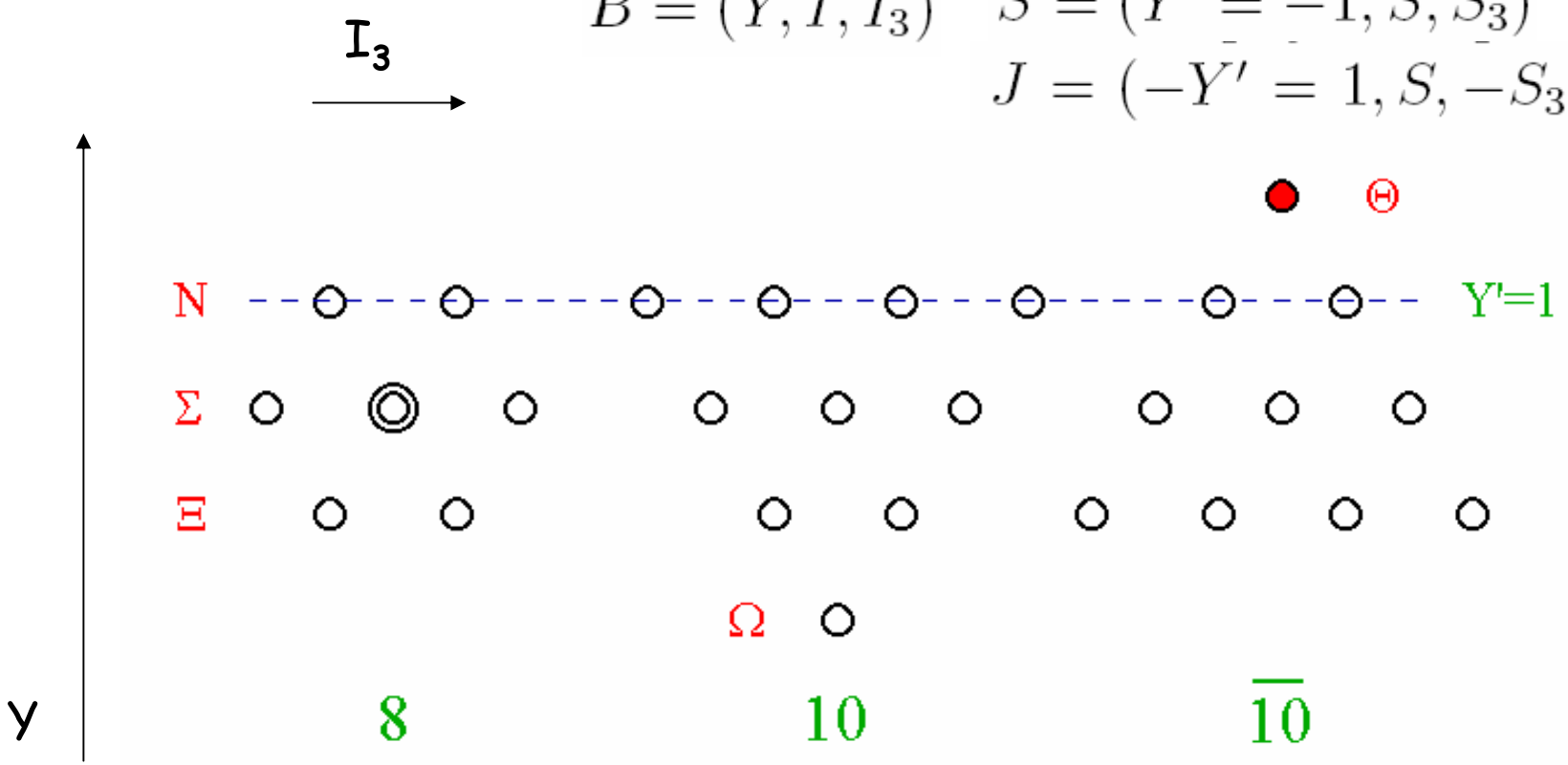


Wave functions and allowed states

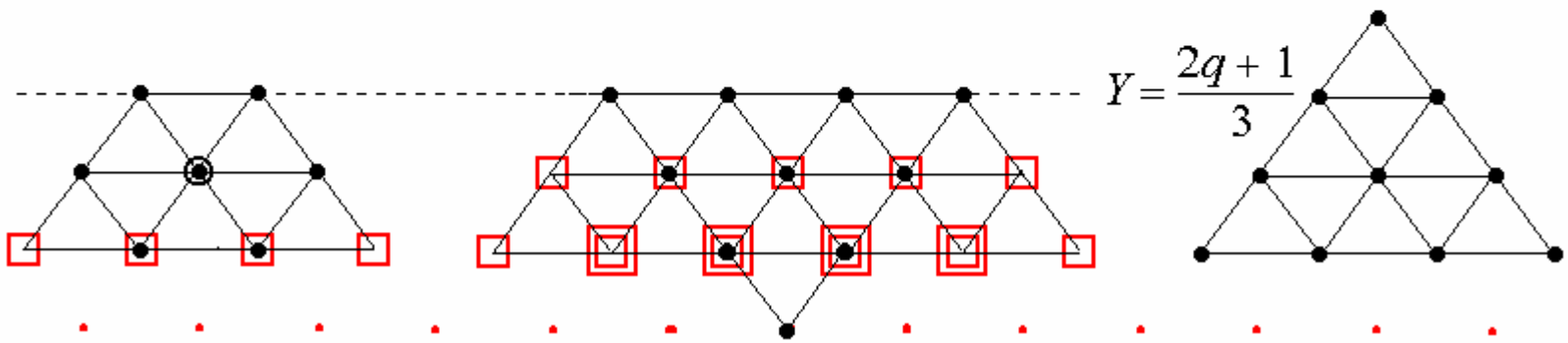
$$\psi_{BS}^{(\mathcal{R})} = \psi_{(\mathcal{R},B)(\mathcal{R}^*,S)} = (-)^{Q_S}\sqrt{\dim(\mathcal{R})}D_{BJ}^{(\mathcal{R})*}$$

G. Karl, J. Patera, S. Perantonis, Phys. Lett **172B** (1986) 49,
J. Bijnens, H. Sonoda, M. Wise, Can. J. Phys. **64** (1986) 1,
Z. Duliński, M. Praszałowicz, Acta Phys.Pol. **B18** (1988) 1157.

$$q = \frac{N_c - 1}{2}$$

$8 = (1,q)$ $\quad$ $10 = (3,q-1)$ $\quad$ $\overline{10} = (0,q+2)$

$$Y = \frac{2q+1}{3}$$



Mass formula

$O(N_c)$ $O(N_c,m_s)$ unknown corrections $O(1)$ $O(1)$ $O(N_c)$

$$M = \underbrace{M_{\rm cl} + m_{\rm cl} - \mathcal{M}}_{M_{\rm sol}} + \frac{1}{2I_1} S(S+1) + \frac{1}{2I_2}\left(C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12}\right) - \alpha d_B^R$$

$O(1/N_c)$ $O(N_c,m_s)$

$$\Delta - N \sim \mathcal{O}(\frac{1}{N_c})$$

$$\Theta - N \sim \mathcal{O}(1)$$

$\Delta \to \pi N$

$$\overline{\mathcal{M}}_\Delta^2 = \frac{3}{(M_N + M_\Delta)^2} \frac{q(q+3)}{2(q+1)(q+4)} \left[G_0 + \frac{1}{2} G_1 \right]^2 p^2$$

$$= \frac{3}{(M_N + M_\Delta)^2} \boxed{\frac{(N_c - 1)(N_c + 5)}{2(N_c + 1)(N_c + 7)}} \left[G_0 + \frac{1}{2} G_1 \right]^2 p^2$$

$O(1/N_c^2)$ $O(1)$ $O(N_c^3)$

$\Theta^+ \to KN$

$$\overline{\mathcal{M}}_\Theta^2 = \frac{3}{(M_N + M_\Theta)^2} \frac{3(q+1)}{2(q+2)(q+4)} \left[G_0 - \frac{q+1}{2} G_1 - \frac{1}{2} G_2 \right]^2 p^2$$

$$= \frac{3}{(M_N + M_\Theta)^2} \boxed{\frac{3(N_c + 1)}{(N_c + 3)(N_c + 7)}} \left[\boxed{G_0 - \frac{N_c + 1}{4} G_1} - \frac{1}{2} G_2 \right]^2 p^2$$

$O(1/N_c^2)$ $O(1/N_c)$ $O(N_c^3)$

$$\Gamma_\Delta \sim \mathcal{O}(N_c) \times p^3$$
$$\Gamma_\Theta \sim \mathcal{O}(1) \times p^3$$

where p for $1 \to 2$ decay reads

$$p = \frac{\sqrt{M_1^2 - (M_2 + m)^2}\sqrt{M_1^2 - (M_2 - m)^2}}{2M_1}$$

chiral limit:

$$\Delta \to \pi N \quad p \sim \mathcal{O}(\frac{1}{N_c}) \qquad \Gamma_\Delta \sim \mathcal{O}(\frac{1}{N_c^2})$$
$$\Theta \to KN \quad p \sim \mathcal{O}(1) \qquad \Gamma_\Theta \sim \mathcal{O}(1)$$

nonzero meson masses:

$$p \sim \mathcal{O}(1) \qquad \Gamma_\Delta \sim \mathcal{O}(N_c)$$
$$\Gamma_\Theta \sim \mathcal{O}(1)$$

Congres International sur le Rayonnement Cosmique, Bagneres de Bigorre, Juillet 1953

"The particles described
in this conference
are not entirely fictitious
and every analogy
with the particles
really existing in nature
is not entirely coincindental."