SPECTROSCOPY of PENTAQUARK STATES

talk given by E. Santopinto
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Classification of pentaquark states

. Construction of a complete classification of pentaquark states in terms of $SU_{sf}(6)$ representations

. Only singlets, octects,decuplets,antidecuplets,27-plets and 35-plets are allowed. Antidecuplets,27-plets and 35-plets contains exotics.

. These classification is general and model independent. It is useful both for model builders and experimentalists.

Gürsey Radicati mass formula

- The mass spectrum is obtained from a Gürsey Radicati type mass formula, whose coefficients have been determined previously by a study of qqq baryons.
- The ground state pentaquark which is identified with the recently observed Θ(1540), is predicted isosinglet anti-decuplet state.

Parity of Θ(1540)

- Its parity depends on the interplay between the spin-flavour and orbital contributions to the mass operator and we discuss different cases.
- Independent from the parity of the Θ(1540)

we predict excited exotic baryons at 1660 and 1775 MeV.

Classification of pentaquark states using symmetry principles

We have to consider: spatial degrees of freedom and internal degrees of freedom: color,flavor and spin

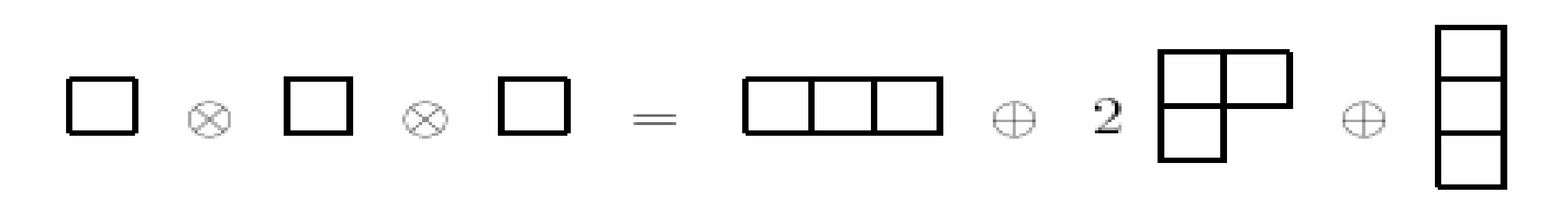
In the construction of the classification scheme we are guided by two conditions: the pentaquark w.ff. should be a colour singlet and should be antisymmetric under any permutation of the four quarks.

SU(6) spin-flavour

	$SU_{sf}(6)$	$\supset$	$SU_f(3)$	$\otimes$	$SU_s(2)$
quark	[1]	$\supset$	[1]	$\otimes$	[1]
	$\square$	$\supset$	$\square$	$\otimes$	$\square$
antiquark	[11111]	$\supset$	[11]	$\otimes$	[1]
	$\square$ $\square$ $\square$ $\square$ $\square$	$\supset$	$\square$ $\square$	$\otimes$	$\square$

qqq system

The allowed $SU_{sf}(6)$ configurations are

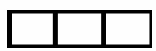
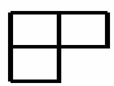
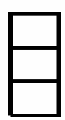


$$[1]_6 \otimes [1]_6 \otimes [1]_6 = [3]_{56} \oplus 2[21]_{70} \oplus [1]_{20}$$

Notation: $[f]_d = [f_1, f_2, \ldots, f_n]_d$

with f_i number of boxes in the i-th row and d dimension of the representation

Allowed spin-flavour, flavour and spin states of qqq baryons and their permutational symmetry($S_3 \sim D_3$)

					Dimension		
D_3	$\sim$	S_3	Young tableau	Multiplicity	$SU(6)$	$SU(3)$	$SU(2)$
A_1	$\sim$	[3]		1	56	10	4
E	$\sim$	[21]		2	70	8	2
A_2	$\sim$	[111]		1	20	1	—

Notation : the spin states are given by the representations [f1,f2] ([30], [21]) or equivalently by their spin S=(f1-f2)/2=3/2, 1/2 , or better by 2S+1= 4, 2.

Decomposition of $SU_{sf}(6)$ into $SU_f(3) \otimes SU_s(2)$

$$
\begin{aligned}
[3]_{56} &= ([21]_8 \otimes [21]_2) \oplus ([3]_{10} \otimes [3]_4) , \\
[21]_{70} &= ([21]_8 \otimes [21]_2) \oplus ([21]_8 \otimes [3]_4) \oplus ([3]_{10} \otimes [21]_2) \oplus ([111]_1 \otimes [21]_2) \\
[111]_{20} &= ([21]_8 \otimes [21]_2) \oplus ([111]_1 \otimes [3]_4) ,
\end{aligned}
$$

or in the usual notation:

$$
\begin{aligned}
[56] &= {}^2 8 \oplus {}^4 10 , \\
[70] &= {}^2 8 \oplus {}^4 8 \oplus {}^2 10 \oplus {}^2 1 , \\
[20] &= {}^2 8 \oplus {}^4 1 .
\end{aligned}
$$

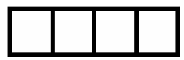
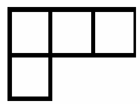
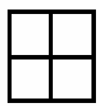
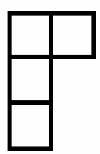
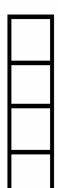
To study pentaquark states it is convenient to first construct qqqq states which should satisfy Pauli statistics and then add the antiquark.

$$[3]_{56} \otimes [1]_6 = [4]_{126} \oplus [31]_{210} \,,$$
$$[21]_{70} \otimes [1]_6 = [31]_{210} \oplus [22]_{105} \oplus [211]_{105} \,,$$
$$[111]_{20} \otimes [1]_6 = [211]_{105} \oplus [1111]_{15} \,.$$

As a result the qqqq spin-flavour states

$$[1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [1]_6 = [4]_{126} \oplus \mathbf{3}[31]_{210} \oplus \mathbf{2}[22]_{105} \oplus [1111]_{15}$$

Symmetry properties of four-quark states

$\mathcal{T}_d$	$\sim$	S_4	Young tableau	Multiplicity	Dimension $SU(6)$	Dimension $SU(3)$	Dimension $SU(2)$
A_1	$\sim$	[4]		1	126	15	5
F_2	$\sim$	[31]		3	210	15	3
E	$\sim$	[22]		2	105	6	1
F_1	$\sim$	[211]		3	105	3	–
A_2	$\sim$	[1111]		1	15	–	–

Spin –flavour classification of qqqq states

D_3	T_d	$SU_{sf}(6)$	$\supset$	$SU_f(3)$	$\otimes$	$SU_s(2)$
A_1	A_1	$[4]_{126}$		$[4]_{15}$	$\otimes$	$[4]_5$
				$[31]_{15}$	$\otimes$	$[31]_3$
				$[22]_6$	$\otimes$	$[22]_1$
A_1+E	F_2	$[31]_{210}$		$[4]_{15}$	$\otimes$	$[31]_3$
				$[31]_{15}$	$\otimes$	$[4]_5$
				$[31]_{15}$	$\otimes$	$[31]_3$
				$[31]_{15}$	$\otimes$	$[22]_1$
				$[22]_6$	$\otimes$	$[31]_3$
				$[211]_3$	$\otimes$	$[22]_1$
				$[211]_3$	$\otimes$	$[31]_3$
E	E	$[22]_{105}$		$[4]_{15}$	$\otimes$	$[22]_1$
				$[31]_{15}$	$\otimes$	$[31]_3$
				$[22]_6$	$\otimes$	$[4]_5$
				$[22]_6$	$\otimes$	$[22]_1$
				$[211]_3$	$\otimes$	$[31]_3$
$E+A_2$	F_1	$[211]_{105}$		$[31]_{15}$	$\otimes$	$[31]_3$
				$[31]_{15}$	$\otimes$	$[22]_1$
				$[22]_6$	$\otimes$	$[31]_3$
				$[211]_3$	$\otimes$	$[4]_5$
				$[211]_3$	$\otimes$	$[31]_3$
				$[211]_3$	$\otimes$	$[22]_1$
A_2	A_2	$[1111]_{15}$		$[22]_6$	$\otimes$	$[22]_1$
				$[211]_3$	$\otimes$	$[31]_3$

The allowed $SU_{sf}(6)$ states for qqqq-antiq

$$[4]_{126} \otimes [11111]_6 = [51111]_{700} \oplus [411111]_{56}$$

$$[31]_{210} \otimes [11111]_6 = [42111]_{1134} \oplus [411111]_{56} \oplus [321111]_{70}$$

$$[22]_{105} \otimes [11111]_6 = [33111]_{560} \oplus [321111]_{70}$$

$$[211]_{105} \otimes [11111]_6 = [32211]_{540} \oplus [321111]_{70} \oplus [222111]_{20}$$

$$[1111]_{105} \otimes [11111]_6 = [22221]_{70} \oplus [222111]_{20}$$

As a result, the qqqq-antiq spin-flavour states are

$$[1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [1]_6 \otimes [11111]_6 = [51111]_{700} \oplus 4\,[411111]_{56} \oplus 3\,[42111]_{1134} \oplus 8\,[321111]_{70} \oplus 2\,[33111]_{560} \oplus 3\,[32211]_{540} \oplus [22221]_{70}$$

$\mathcal{T}_d$	$SU_{sf}(6)$	$\supset$	$SU_f(3)$	$\otimes$	$SU_s(2)$
A_1	$[51111]_{700}$		$[51]_{35}$	$\otimes$	$[5]_6$
			$[51]_{35}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[32]_2$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[5]_6$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
$A_1 + F_2$	$[411111]_{56}$		$[411]_{10}$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
F_2	$[42111]_{1134}$		$[51]_{35}$	$\otimes$	$[41]_4$
			$[51]_{35}$	$\otimes$	$[32]_2$
			$[42]_{27}$	$\otimes$	$[5]_6$
			$2([42]_{27}$	$\otimes$	$[41]_4)$
			$2([42]_{27}$	$\otimes$	$[32]_2)$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[5]_6$
			$2([411]_{10}$	$\otimes$	$[41]_4)$
			$2([411]_{10}$	$\otimes$	$[32]_2)$
			$[321]_8$	$\otimes$	$[5]_6$
			$3([321]_8$	$\otimes$	$[41]_4)$
			$3([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[41]_4$
			$[222]_1$	$\otimes$	$[32]_2$
$F_2 + E + F_1$	$[321111]_{70}$		$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[32]_2$

Spin-flavour classification of qqqq antiq and decomposition in terms of their $SU_f(3) \otimes SU_s(2)$ content

$\mathcal{T}_d$	$SU_{\rm sf}(6)$	$\supset$	$SU_{\rm f}(3)$	$\otimes$	$SU_{\rm s}(2)$
E	$[33111]_{560}$		$[51]_{35}$	$\otimes$	$[32]_2$
			$[42]_{27}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[32]_2$
			$[33]_{10}$	$\otimes$	$[5]_6$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[5]_6$
			$2([321]_8$	$\otimes$	$[41]_4)$
			$2([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[41]_4$
F_1	$[32211]_{540}$		$[42]_{27}$	$\otimes$	$[41]_4$
			$2([42]_{27}$	$\otimes$	$[32]_2)$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[5]_6$
			$3([321]_8$	$\otimes$	$[41]_4)$
			$3([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[5]_6$
			$[222]_1$	$\otimes$	$[41]_4$
			$[222]_1$	$\otimes$	$[32]_2$
$F_1 + A_2$	$[222111]_{20}$		$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[41]_4$
A_2	$[22221]_{70}$		$[33]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[32]_2$

Allowed $SU_f(3)$ multiplets

$[51]_{35}$	(35-plet)
$[42]_{27}$	(27-plet)
$[411]_{10}$	(10-plet)
$[33]_{\overline{10}}$	(anti-decuplet)
$[321]_{8}$	(octet)
$[222]_{1}$	(singlet)

It is difficult to distinguish the pentaquark flavour singlets, octets and decuplets from standard three-quark states, since they have the same values if Y and I

On the contrary, antidecuplets, 27-plets and 35-plets contain in addition exotic states
(which cannot be obtained from 3q configurations only)

Complete list of all exotic pentaquark states

$SU_f(3)$	Y	I	Q	Flavour States	Notation
$[33]_{10}$	2	0	1	$dduu\bar{s}$	Θ^+
	-1	3/2	-2,1	$ddss\bar{u}$, $uuss\bar{d}$	$\Xi_{3/2}$
$[42]_{27}$	2	1	0,1,2	$dddu\bar{s}$, $dduu\bar{s}$, $duuu\bar{s}$	Θ_1
	0	2	-2,2	$ddds\bar{u}$, $uuus\bar{d}$	Γ
	-1	3/2	-2,1	$ddss\bar{u}$, $uuss\bar{d}$	Π
	-2	1	-2,0	$dsss\bar{u}$, $usss\bar{d}$	Ω_1
$[51]_{35}$	2	2	-1,0,1,2,3	$dddd\bar{s}$, $dddu\bar{s}$, $dduu\bar{s}$, $duuu\bar{s}$, $uuuu\bar{s}$	Θ_2
	1	5/2	-2, 3	$dddd\bar{u}$, $uuuu\bar{d}$	Φ
	0	2	-2, 2	$ddds\bar{u}$, $uuus\bar{d}$	Γ
	-1	3/2	-2, 1	$ddss\bar{u}$, $uuss\bar{d}$	Π
	-2	1	-2, 0	$dsss\bar{u}$, $usss\bar{d}$	Ω_1
	-3	1/2	-2,-1	$ssss\bar{u}$, $ssss\bar{d}$	Ψ

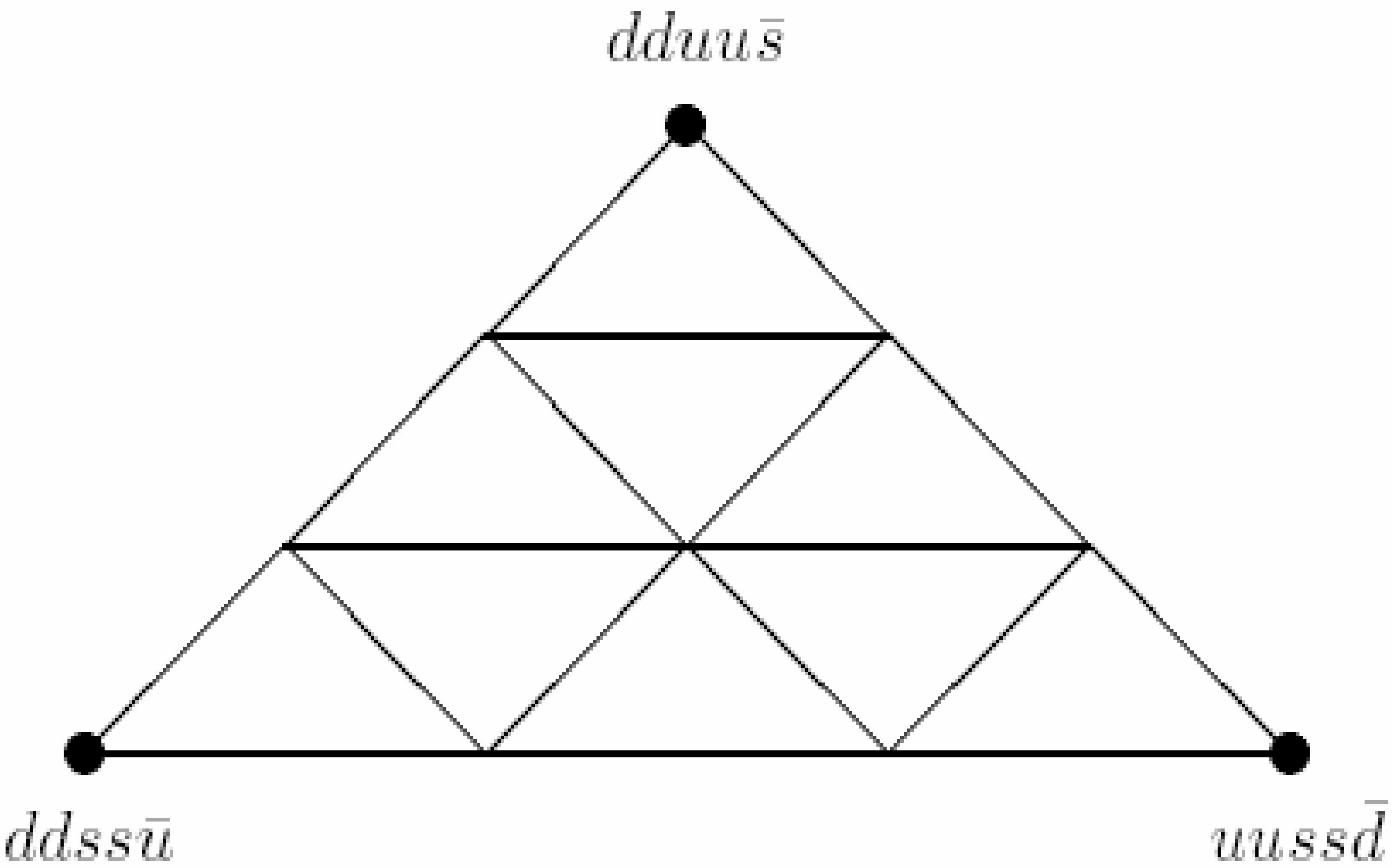
$Q=I_3+Y/2$

Whithin each isospin multiplet we have identified the states whose combination of Y and Q cannot be obtained for 3q configurations only.

Because of the uniqueness of their quantum numbers, these states are more easily identified experimentally

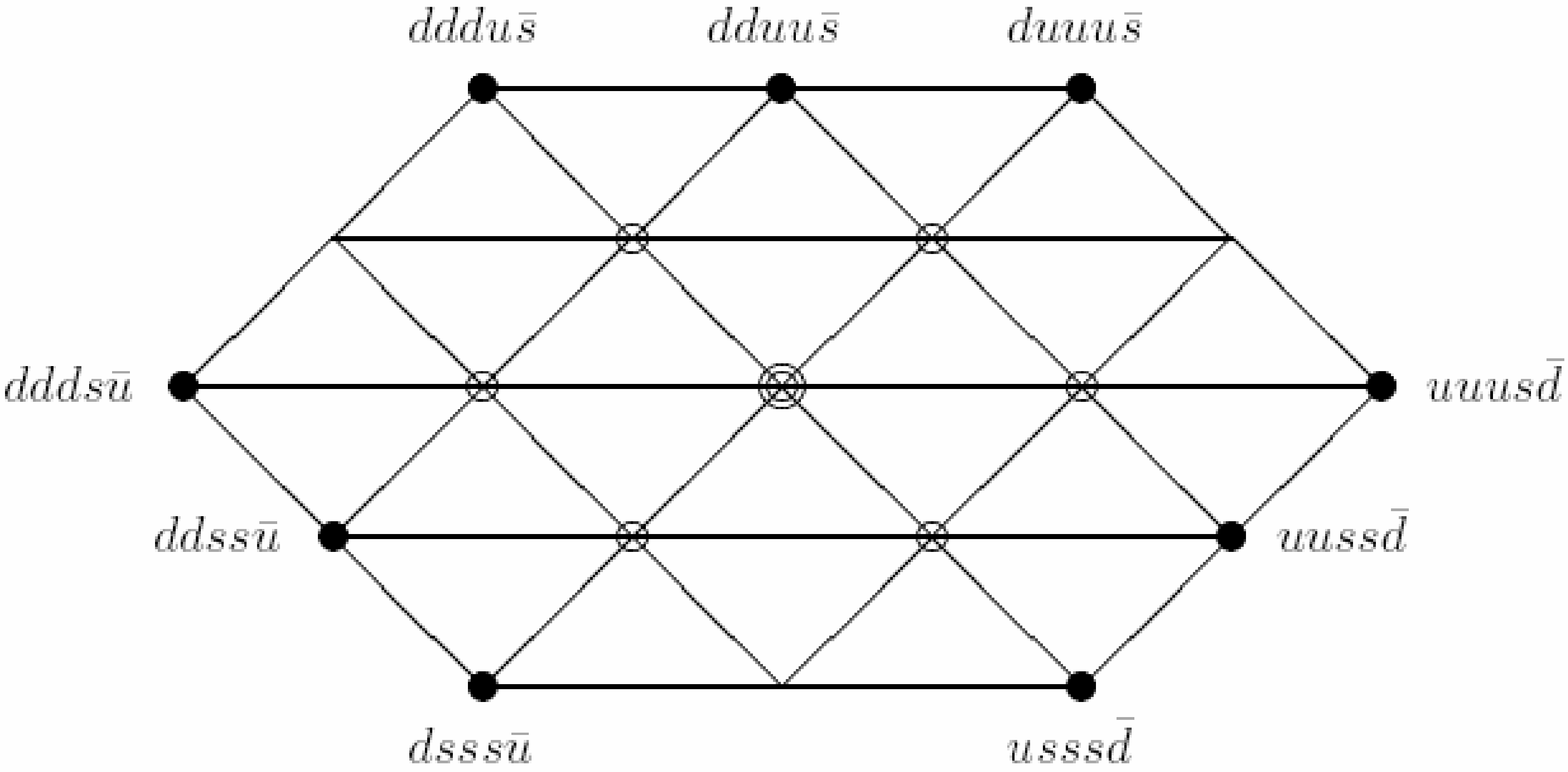
(in next fig. ·)

SU(3)flavour $[33]_{10}$ (anti-decuplet) with E sym.



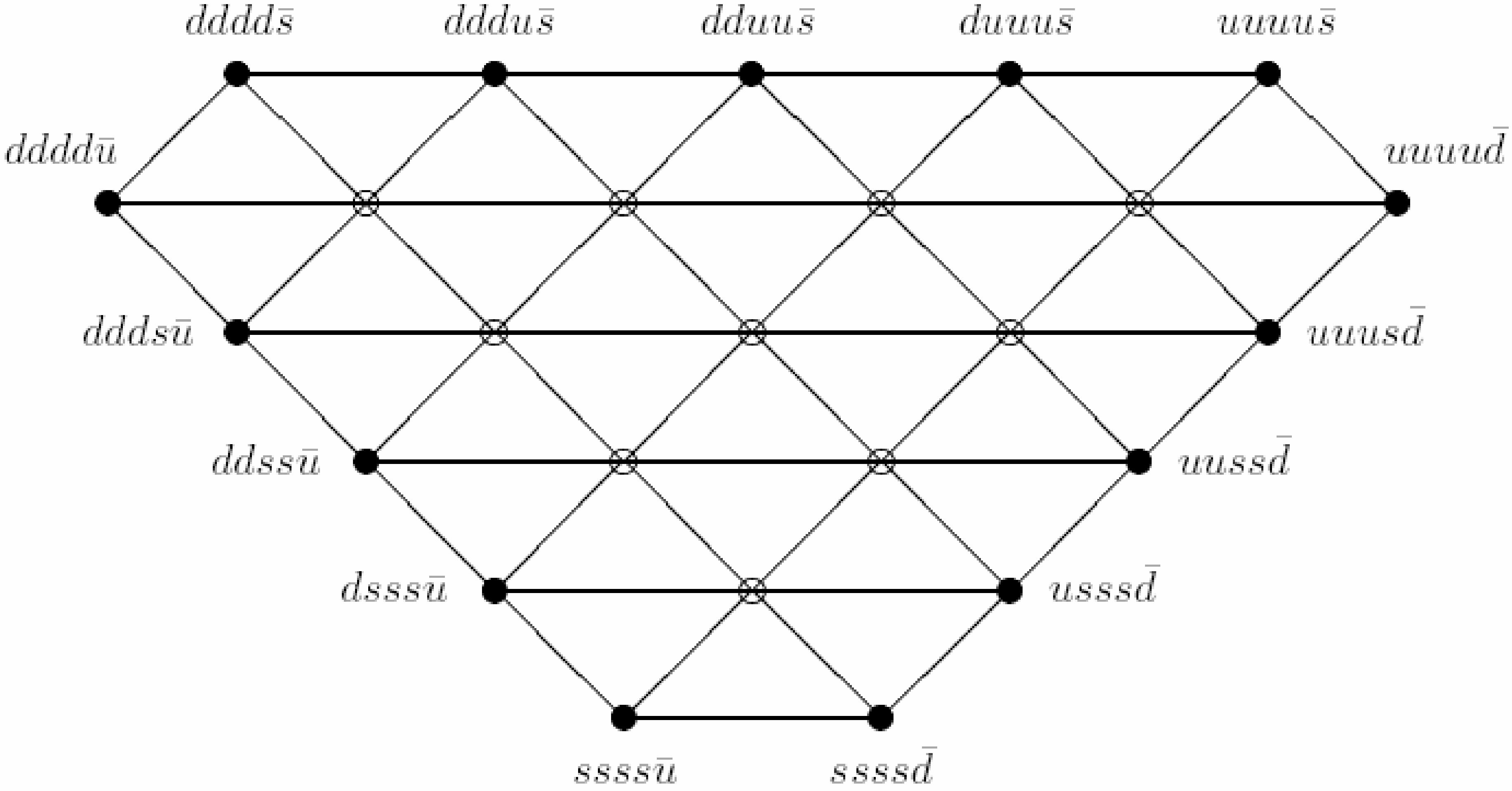
The isospin-hypercharge multiplets are (I,Y) = (0,2), (1/2,1), (1,0) and (3/2,-1).   In fig. · indicates the exotic states.

SU(3)flavour $[42]_{27}$ (**27-plet**) with F2 symmetry



The isospin-hypercharge multiplets are (I,Y) = (1,2), (3/2,1), (1/2,1), (2,0), (1,0), (0,0) (3/2,-1), (1/2,-1) and (1,-2)

SU(3)flavour [51]35 (**35-plet**) with A1 sym.



The isospin-hypercharge multiplets are (I,Y) = (2,2), (5/2,1), (3/2,1), (2,0), (1,0), (3/2,-1), (1/2,-1), (1,-2),(0,-2) and (1/2,-3).

We have to combine the $SU_{sf}(6)$ part with a given S_4 symmetry with the color part and the orbital part, in such a way that the total pentaq state is a $[222]_1$ color-singlet state and the 4q obey the Pauli principle.

Since the color part of the pentaquark is $[222]_1$ singlet and the antiquark is a $[11]_3$ anti-triplet, the color w.f. of the 4q is a $[211]_3$ triplet with F_1 symmetry. The total w. f. of 4q is antisymmetric (A_2), so the orbital-spin-flavour part is a [31] with F_2 symmetry.

$\psi_c(q^4)$	$[211]$	F_1
$\psi_{osf}(q^4)$	$[31]$	F_2

Symmetry properties of the pentaquark orbital w.f.

We know that the orb.-spin-flavour. part is [31] with F_2 symmetry, so we can derive the symmetry of the orbital part.

If the 4q are in a S-wave ground state with A_1 the only allowed $SU_{sf}(6)$ representation is [31] with F_2.In the table with the SU(6) pentaquark states, the only F_2 configuration containing exotics is the $[42111]_{1134}$

If the 4q are in a P-wave state with F_2 symmetry, there are several allowed $SU_{sf}(6)$ representations

- [4] with A_1,
- [31] with F_2,
- [22] with E and
- [211] with F_1
- The corresponding pentaquark states which contain exotics are: $[51111]_{700}$, $[42111]_{1134}$, $[33111]_{560}$ respectively.

For each symmetry type of the orbital w.f. the symmetry of the associated qqqq SU(6) part and the pentaquark SU(6) multiplet containing exotics

Orbital Symmetry	Spin-Flavour Symmetry	$q^4\bar{q}$ Configuration with Exotic States
A_1	F_2	[42111]
F_2	A_1	[51111]
	F_2	[42111]
	E	[33111]
	F_1	[32211]
E	F_2	[42111]
	F_1	[32211]
F_1	A_2	–
	F_2	[42111]
	E	[33111]
	F_1	[32211]
A_2	F_1	[32211]

We have constructed explicitly the S_4 invariant wave functions which are at disposal under request.

(R.Bijker, M. Giannini and E. Santopinto to be published)

The classification scheme we have derived is general

and it is based only on the symmetries

The precise ordering of the various SU(6) multiplets in the mass spectrum depends on the specific dynamical model (Skyrme, CQM, GBE, Instanton etc.)

Pentaquark spectrum

- In the limit of SU(6) symmetry, all states belonging to a definite spin-flavour representation are degenerate.

- The description of the splittings within the $SU_{sf}(6)$ multiplets can be described by means of a Gürsey-Radicati mass formula

$$M=M_0 + b\, C_2(SU_f(3)) + c\, s\,(s+1) + d\, Y + e\,[\, I\,(I+1)-1/4\, Y^2]$$

- This formula is a general way to describe the SU(6) breaking of the strong interaction and it is valid for any quark system.

Mass for exotic pentaquark states with the GR mass formula

$SU_F(3)$	s	Y	I	Notation	Mass (MeV) [51111] $1^-_{F_2}$	[42111] $0^+_{A_1}, 1^-_{A_1,F_2}$	[33111] $1^-_{F_2}$	[32211] $1^-_{F_2}$
$[33]_{10}$	1/2	2	0	Θ^+	1540	1540	1540	1540
		-1	3/2	$\Xi_{3/2}$	2305	2305	2305	2305
$[33]_{10}$	3/2	2	0	Θ^+		1655	1655	1655
		-1	3/2	$\Xi_{3/2}$		2420	2420	2420
$[33]_{10}$	5/2	2	0	Θ^+			1846	
		-1	3/2	$\Xi_{3/2}$			2612	
$[42]_{27}$	1/2	2	1	Θ_1	1660	1660	1660	1660
		0	2	Γ	2247	2247	2247	2247
		-1	3/2	Π	2348	2348	2348	2348
		-2	1	Ω_1	2449	2449	2449	2449
$[42]_{27}$	3/2	2	1	Θ_1	1775	1775	1775	1775
		0	2	Γ	2362	2362	2362	2362
		-1	3/2	Π	2463	2463	2463	2463
		-2	1	Ω_1	2564	2564	2564	2564
$[42]_{27}$	5/2	2	1	Θ_1		1966		
		0	2	Γ		2553		
		-1	3/2	Π		2654		
		-2	1	Ω_1		2755		

The lowest pentaquark state of each multiplet is normalized to the observed mass of the Θ^+ (1540). The states are labeled by their spin s, hypercharge Y and isospin I

$SU_{f}(3)$	s	Y	I	Notation	Mass (MeV) [51111] $1^-_{F_2}$	[42111] $0^+_{A_1}, 1^-_{A_1,F_2}$	[33111] $1^-_{F_2}$	[32211] $1^-_{F_2}$
$[51]_{35}$	1/2	2	2	Θ_2		1899	1899	
		1	5/2	Φ		2231	2231	
		0	2	Γ		2332	2332	
		-1	3/2	Π		2433	2433	
		-2	1	Ω_1		2534	2534	
		-3	1/2	Ψ		2635	2635	
$[51]_{35}$	3/2	2	2	Θ_2	2014	2014		
		1	5/2	Φ	2346	2346		
		0	2	Γ	2447	2447		
		-1	3/2	Π	2548	2548		
		-2	1	Ω_1	2649	2649		
		-3	1/2	Ψ	2750	2750		
$[51]_{35}$	5/2	2	2	Θ_2	2205			
		1	5/2	Φ	2537			
		0	2	Γ	2638			
		-1	3/2	Π	2739			
		-2	1	Ω_1	2840			
		-3	1/2	Ψ	2941			

Discussion of results

For all configurations we find that the lowest pentaquark state is an isosinglet anti-decuplet [33] state with s=1/2, in agreement with the available experimental data which indicate that the Θ^+ (1540) is an isosinglet (J. Bart et al. hep-ex/0307083).

For all configurations there are other lowlying excited pentaquark states belonging to the 27-plet at 1660 MeV and 1775 MeV

The anti-decuplet state with strangeness S=-2 (Y=-1) and isospin I=3/2 is calculated at an energy of 2305 MeV, to be compared with the recently observed resonance at 1826 MeV (NA49 collaboration) which was suggested as a candidate for the $\Xi_{3/2}$ exotic with charge Q=-2.

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Generalized Gürsey-Radicati mass formula

$$M = M_0 + aC_2[SU_{SF}(6)] + bC_2[SU_F(3)] + cC_2[SU_S(2)] +$$

$$dY + e[C_2(SU_{SF}(6)) - \tfrac{1}{4} Y^2]$$

Results for all the possible exotic states belonging to the allowed SU$_{sf}$(6) representations

$SU_F(3)$	s	Y	I	Notation	Mass (MeV) [51111] $1^-_{F_2}$	[42111] $0^+_{A_1}, 1^-_{A_1,F_2}$	[33111] $1^-_{F_2}$	[32211] $1^-_{F_1}$
$[33]_{10}$	1/2	2	0	Θ^+	1263	1540	1678	1817
		-1	3/2	$\Xi_{3/2}$	2028	2305	2444	2582
$[33]_{10}$	3/2	2	0	Θ^+		1655	1793	1932
		-1	3/2	$\Xi_{3/2}$		2420	2558	2697
$[33]_{10}$	5/2	2	0	Θ^+			1985	
		-1	3/2	$\Xi_{3/2}$			2750	
$[42]_{27}$	1/2	2	1	Θ_1	1383	1660	1798	1936
		0	2	Γ	1970	2247	2385	2524
		-1	3/2	Π	2071	2348	2486	2625
		-2	1	Ω_1	2172	2449	2587	2726
$[42]_{27}$	3/2	2	1	Θ_1	1498	1775	1913	2051

Spin and parity of the Θ^+ (1540) pentaquark

What are the consequences of these calculations for the spin and parity of the Θ^+ (1540) ? They depend in part on the assignment of quantum numbers and in part on the choice of a particular model to describe the orbital motion.

We identify the Θ^+ (1540) with the lowest pentaquark state.

We consider a simple model in which the orbital motion of the pentaquark is limited to excitations up to N=1quantum.

The ground state is in an S-state with $L^P=0_{A1}^+$ and A_1 symmetry for the four quarks. Since the orbital excitations are described by four relative coordinates, there are four excited P-wave states with $L^P= 1^-$. As a consequence of the S_4 symmetry of the 4q, the first 3 excitations form a degenerate triplet with three-fold F_2 symmetry, and the fourth has A_1 symmetry.

$$L_t^P = 0^+_{A_1}, 1^-_{F_2}, 1^-_{A_1}$$

The allowed SUsf(6) representations for each orbital excitations are in the following table

Orbital Symmetry	Spin-Flavour Symmetry	$q^4\bar{q}$ Configuration with Exotic States
A_1	F_2	[42111]
F_2	A_1	[51111]
	F_2	[42111]
	E	[33111]
	F_1	[32211]
E	F_2	[42111]
	F_1	[32211]
F_1	A_2	–
	F_2	[42111]
	E	[33111]
	F_1	[32211]
A_2	F_1	[32211]

$\mathcal{T}_d$	$SU_{sf}(6)$	$\supset$	$SU_f(3)$	$\otimes$	$SU_s(2)$
A_1	$[51111]_{700}$		$[51]_{35}$	$\otimes$	$[5]_6$
			$[51]_{35}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[32]_2$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[5]_6$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
A_1+F_2	$[411111]_{56}$		$[411]_{10}$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
F_2	$[42111]_{1134}$		$[51]_{35}$	$\otimes$	$[41]_4$
			$[51]_{35}$	$\otimes$	$[32]_2$
			$[42]_{27}$	$\otimes$	$[5]_6$
			$2([42]_{27}$	$\otimes$	$[41]_4)$
			$2([42]_{27}$	$\otimes$	$[32]_2)$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[5]_6$
			$2([411]_{10}$	$\otimes$	$[41]_4)$
			$2([411]_{10}$	$\otimes$	$[32]_2)$
			$[321]_8$	$\otimes$	$[5]_6$
			$3([321]_8$	$\otimes$	$[41]_4)$
			$3([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[41]_4$
			$[222]_1$	$\otimes$	$[32]_2$
F_2+E+F_1	$[321111]_{70}$		$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[32]_2$

The exotic states with

$$L_t^P = 0^+_{A_1}, 1^-_{A_1}$$

are associated with $[42111]_{1134}$

$$L_t^P = 1^-_{F_1}$$

$[51111]_{700}$ $[42111]_{1134}$
$[33111]_{560}$ $[32211]_{540}$

$\mathcal{T}_d$	$SU_{\rm sf}(6)$	$\supset$	$SU_{\rm f}(3)$	$\otimes$	$SU_{\rm s}(2)$
E	$[33111]_{560}$		$[51]_{35}$	$\otimes$	$[32]_2$
			$[42]_{27}$	$\otimes$	$[41]_4$
			$[42]_{27}$	$\otimes$	$[32]_2$
			$[33]_{10}$	$\otimes$	$[5]_6$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[5]_6$
			$2([321]_8$	$\otimes$	$[41]_4)$
			$2([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[41]_4$
F_1	$[32211]_{540}$		$[42]_{27}$	$\otimes$	$[41]_4$
			$2([42]_{27}$	$\otimes$	$[32]_2)$
			$[33]_{10}$	$\otimes$	$[41]_4$
			$[33]_{10}$	$\otimes$	$[32]_2$
			$[411]_{10}$	$\otimes$	$[41]_4$
			$[411]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[5]_6$
			$3([321]_8$	$\otimes$	$[41]_4)$
			$3([321]_8$	$\otimes$	$[32]_2)$
			$[222]_1$	$\otimes$	$[5]_6$
			$[222]_1$	$\otimes$	$[41]_4$
			$[222]_1$	$\otimes$	$[32]_2$
$F_1 + A_2$	$[222111]_{20}$		$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[41]_4$
A_2	$[22221]_{70}$		$[33]_{10}$	$\otimes$	$[32]_2$
			$[321]_8$	$\otimes$	$[41]_4$
			$[321]_8$	$\otimes$	$[32]_2$
			$[222]_1$	$\otimes$	$[32]_2$

The mass spectrum of pentaquark states can be obtained combining the spin-flavour contribution and the orbital excitation energies, Δ_1 Δ_2

$SU_f(3)$	s	Y	I	Notation	Mass (MeV) [51111] $1^-_{F_2}$	[42111] $0^+_{A_1}$, $1^-_{A_1,F_2}$	[33111] $1^-_{F_2}$	[32211] $1^-_{F_2}$
$[33]_{10}$	1/2	2	0	Θ^+	1263	1540	1678	1817
		-1	3/2	$\Xi_{3/2}$	2028	2305	2444	2582
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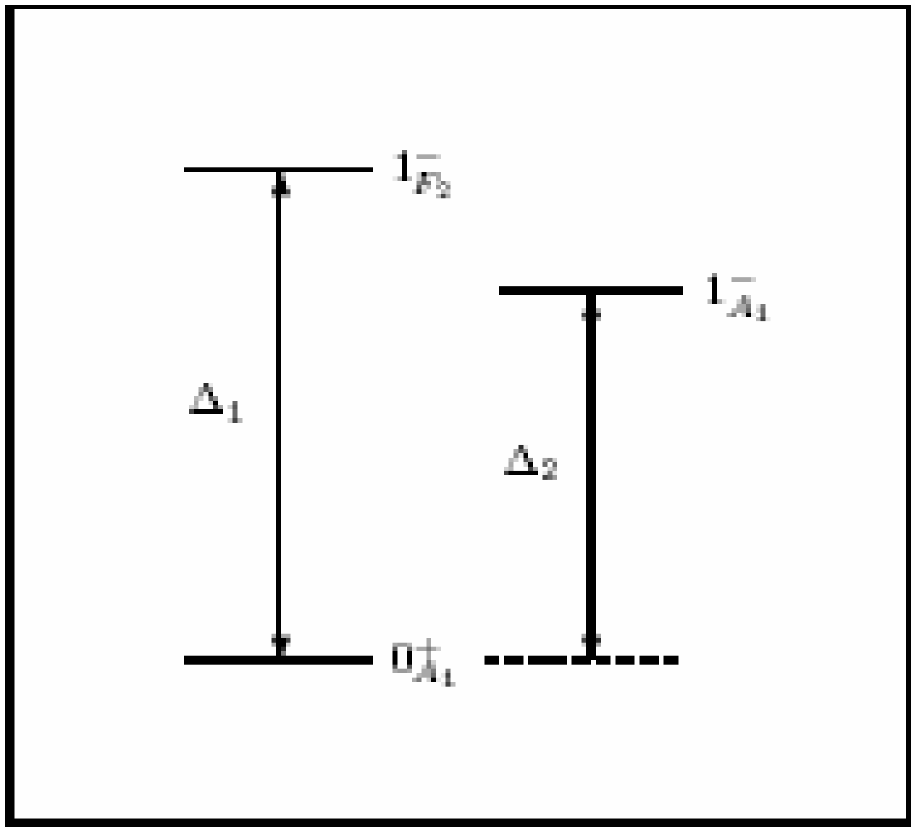
		-1	3/2	$\Xi_{3/2}$		2420	2558	2697
$[33]_{10}$	5/2	2	0	Θ^+			1985	
		-1	3/2	$\Xi_{3/2}$			2750	
$[42]_{27}$	1/2	2	1	Θ_1	1383	1660	1798	1936
		0	2	Γ	1970	2247	2385	2524
		-1	3/2	Π	2071	2348	2486	2625
		-2	1	Ω_1	2172	2449	2587	2726
$[42]_{27}$	3/2	2	1	Θ_1	1498	1775	1913	2051
		0	2	Γ	2085	2362	2500	2638
		-1	3/2	Π	2186	2463	2601	2739
		-2	1	Ω_1	2287	2564	2702	2841
$[42]_{27}$	5/2	2	1	Θ_1		1966		
		0	2	Γ		2553		
		-1	3/2	Π		2654		
		-2	1	Ω_1		2755		

$SU_F(3)$	s	Y	I	Notation	Mass (MeV) [51111] $1^-_{F_2}$	[42111] $0^+_{A_1}$, $1^-_{A_1,F_2}$	[33111] $1^-_{F_2}$	[32211] $1^-_{F_2}$
$[51]_{SS}$	1/2	2	2	Θ_2		1899	2037	
		1	5/2	Φ		2231	2369	
		0	2	Γ		2332	2470	
		-1	3/2	Π		2433	2571	
		-2	1	Ω_1		2534	2672	
		-3	1/2	Ψ		2635	2773	
$[51]_{SS}$	3/2	2	2	Θ_2	1737	2014		
		1	5/2	Φ	2069	2346		
		0	2	Γ	2170	2447		
		-1	3/2	Π	2271	2548		
		-2	1	Ω_1	2372	2649		
		-3	1/2	Ψ	2473	2750		
$[51]_{SS}$	5/2	2	2	Θ_2	1928			
		1	5/2	Φ	2260			
		0	2	Γ	2362			
		-1	3/2	Π	2463			
		-2	1	Ω_1	2564			
		-3	1/2	Ψ	2665			


$1^-_{F_2}$
$1^-_{A_1}$
Δ_1
Δ_2
$0^+_{A_1}$

We assume Δ_1 Δ_2 >0 The total angular momentum is given by

$\vec{J} = \vec{L} + \vec{S}$ whereas the parity is opposite to the orbital excitation due to to the intrinsic parity of the antiquark

For Δ_2 >277 MeV the ground state pentaquark is associated with

and the [42111]1134, so the $J^P = \frac{1}{2}^-$

Another possible identification of the observed Θ+ (1540)

is [42111]1134 anti-decuplet with s=3/2, so that $J^P = \frac{3}{2}^-$

and in this case there would be another pentaquark state with s=1/2

$J^P = \frac{1}{2}^-$ at an energy lower than the observed one.

For $0 < \Delta_1 < 277$ MeV the parity I+ since now the ground state is has $L_t^P = 1_{F_2}^-$

and antidecuplet with spin ½ of the $[51111]_{700}$.

In absence of spin-orbit splitting, we find in this case a ground state doublet with angular momentum and parity

$$J^P = \frac{1}{2}^+, \frac{3}{2}^+$$

The ground state properties depend on the interplay between the orbital and the spin-flavour part:

for small spin –flavour part the parity is negative, whereas for large splitting due to the $SU_{sf}(6)$ term compared to that between the orbital states, it is positive

Conclusions

Construction of pentaquark classification scheme useful for model builders and experimentalists
It is general and complete.

Exotic pentaquark state can be found only in flavour antidecuplet, 27-plets and 35-plets

In order to obtain to total w.f. , the spin-flavour part has to be combined with the color and orbital part, so that the total w.f. is a color singlet and it is antisymm. for any of the four quarks

We have calculated the mass spectrum with a G.R. mass formula which corresponds to the dynamical symmetry

$$\left| \begin{array}{ccccccccccc} SU_{SF}(6) & \supset & SU_F(3) & \otimes & SU_S(2) & \supset & SU_I(2) & \otimes & U_Y(1) & \otimes & SU_S(2) \\ | & & | & & | & & | & & | & & \\ (\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5) & & (\lambda_f,\mu_f) & & S & & I & & Y & & \end{array} \right\rangle$$

which encodes the broken symmetry of the strong interaction

The spectroscopy of exotic baryons will be a testing ground for models of baryons and their structure

The determination of the angular momentum and parity will help to distinguish between different approaches