# SPATIAL STRUCTURE OF HADRONS AND GPDs

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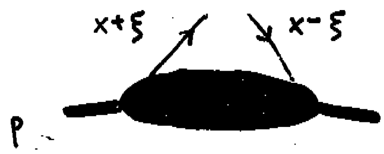
- BRIEF BASICS
- THEORY : SPATIAL STRUCTURE FROM GPDs
- PHENOMENOLOGY : GPDs FROM EXPERIMENT

## WHAT ARE GPDs ?

FOURIER TRANSFORMED HADRONIC MATRIX ELEMENTS

$$z^{\pm} = (z^0 \pm z^3)/\sqrt{2}$$

$$\int dz^- \, e^{i x z^- (p+p')^+} \langle p', s' | \bar{\psi}(-z) \ldots \psi(z) | p, s \rangle_{z^+ = 0}$$

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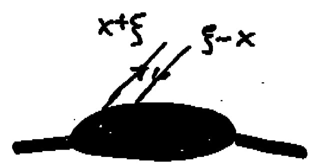



PARAMETERIZE BY FUNCTIONS $H, E, \tilde{H}, \tilde{E}, \ldots$ OF $x, \xi, t$

FOR DIFFERENT HADRON AND PARTON SPINS

$= (p-p')^2$

- $H, \tilde{H}$ CONSERVE HADRON SPIN

  $\rightarrow$ $q$, $\Delta q$ IN FORWARD LIMIT $\xi = 0$, $t = 0$

  $E, \tilde{E}$ CAN FLIP HADRON SPIN

  NOT SEEN IN FORWARD LIMIT $\rightarrow$ VERY UNKNOWN

- MOMENTS IN $x$ $\rightarrow$ FORM FACTORS

$$\int dx \; H(x, \xi, t) = F_1(t)$$

$$\int dx \; E(x, \xi, t) = F_2(t)$$

KNOWN

HIGHER MOMENTS $\int dx \; x^n \, H, \ldots$ UNKNOWN

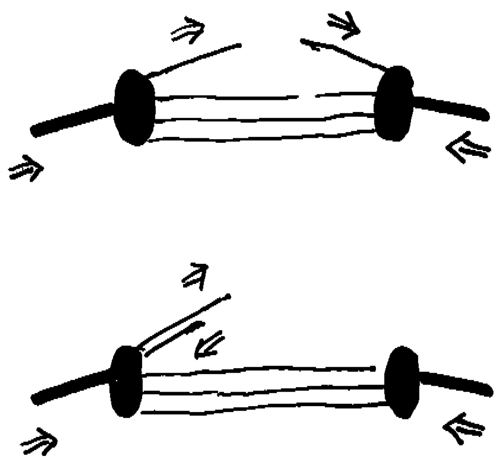
- MOMENTS $\int dx\, x\, H$, $\int dx\, x\, E$

  $\rightarrow$ ENERGY-MOMENTUM TENSOR

$$\frac{1}{2}\left[\int dx\, x\, H^q + \int dx\, x\, E^q\right]_{t=0} = J^q$$

TOTAL ANGULAR MOMENTUM
(HELICITY + ORBITAL)

- CONNECTION $E(x,\xi,t) \leftrightarrow$ O.A.M. IS MORE GENERAL:



$$\propto (\Delta^x + i\Delta^y)\, E(x,\xi,t)$$

MOMENTUM
DEPENDENCE
INTERESTING

HELICITY NOT CONSERVED

$\rightarrow$ O.A.M IN WAVE FUNCTION(S)

# HADRON STRUCTURE IN 2+1 D

$$|p^+, \vec{b}_T\rangle = \int d^2\vec{p}_T \, e^{-i\vec{p}_T \vec{b}_T} \, |p^+, \vec{p}_T\rangle$$

- PROTON STATE LOCALISED IN TRANSVERSE PLANE ("IMPACT PARAMETER")
- CAN CHOOSE $p^+$ LARGE (FAST MOVING PROTON) → CLEAR PARTON INTERPRETATION

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THIS IS ≠

- LOCALIZATION IN 3D
  - ONLY POSSIBLE WITHIN COMPTON WAVELENGTH

    WELL-KNOWN FROM USUAL SPATIAL INTERPRETATION OF FORM FACTORS
  - PARTON INTERPRETATION NON-TRIVIAL FOR PROTON AT REST

    PROPOSED FOR GPDs BELITSKY, JI, YUAN '03

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$\vec{b}$ IS CENTER OF MOMENTUM OF PROTON

$$\vec{b} = \frac{\sum_i k_i^+ \vec{b}_i}{\sum_i k_i^+}$$

LORENTZ INVARIANCE

ANALOG ↓ NON-RELATIVISTIC

C.O. MASS $\vec{r} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

GALILEAN INVARIANCE

APPLY TO GPDs

BURKHARDT '00
RALSTON, PIRE '01



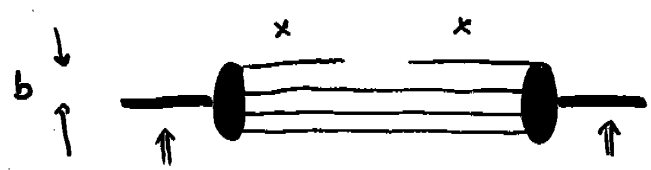
$\boxed{\xi = 0}$

$$\langle p^+, \vec{b} \,|\, \int dz^- \, e^{ix z^- (p+p')^+} \, \bar{\psi}(-z) \ldots \psi(z) \big|_{z^\perp = 0} \,|\, \vec{b}, p^+ \rangle$$

SAME OPERATOR AS BEFORE

$$\propto \int d^2(\vec{p}-\vec{p}') \, e^{i(\vec{p}-\vec{p}')\vec{b}} \, H(x, \xi=0, t=-(\vec{p}-\vec{p}')^2)$$

- JOINT DENSITY OF PARTONS WITH MOM. FRACTION $x$ AND TRANSVERSE POSITION $\vec{b}$




- $E \leftrightarrow$ PROTON HELICITY FLIP

  DENSITY IN TRANSVERSITY BASIS BURKARDT '02

  p SPIN ALONG x - AXIS $\rightarrow$ PARTON DISTRIBUTION SHIFTED IN y - DIRECTION

$\rightarrow$ $t$ DEPENDENCE OF $E$ INTERESTING

APPLY TO GPDs

BURKARDT '00
RALSTON, PIRE '01

6

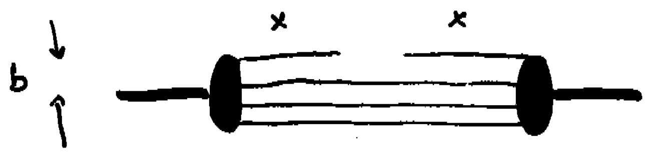
$\boxed{\xi = 0}$

$$\langle p^+, \vec{b} \mid \int dz^- \, e^{ix z^- (p+p')^+} \bar{\psi}(-z) \ldots \psi(z)\Big|_{z^+=0} \mid \vec{b}, p^+ \rangle$$

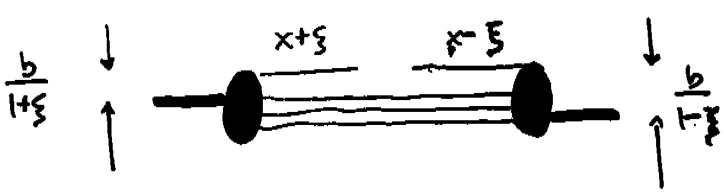
↑
SAME OPERATOR AS BEFORE

$$\propto \int d^2(\vec{p}-\vec{p}') \, e^{i(\vec{p}-\vec{p}')\vec{b}} \, H(x, \xi=0, t=-(\vec{p}-\vec{p}')^2)$$

- JOINT DENSITY OF PARTONS WITH MOM. FRACTION $x$ AND TRANSVERSE POSITION $\vec{b}$

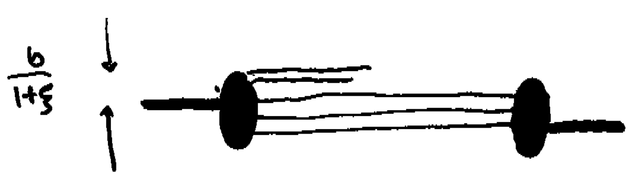



$\boxed{\xi \neq 0}$




PROTON CENTER OF MOMENTUM SHIFTS $\propto \xi \vec{b}$

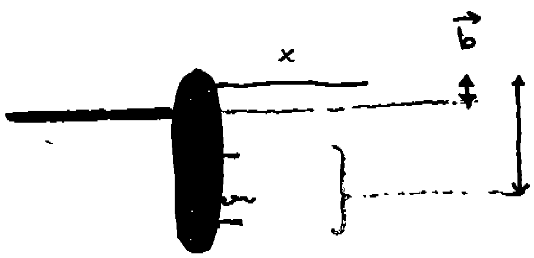
M.D. '02



- AS $x$ BECOMES LARGE :

EXTRACTED PARTON DOMINATES IN CENTER OF MOMENTUM

$$\vec{0} = \vec{B} = x\vec{b} + \sum_{\text{spect.}} \vec{x}_i \vec{b}_i$$



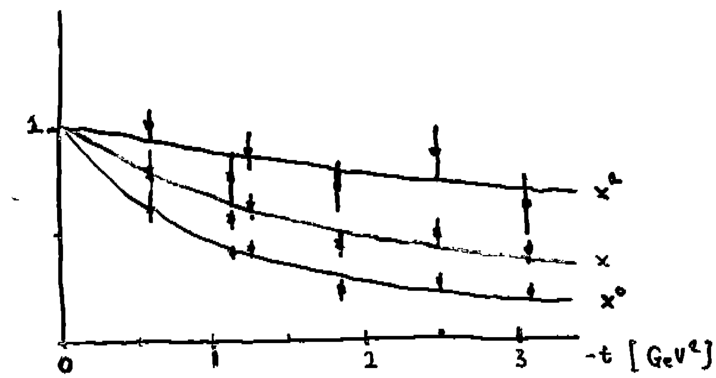

$\frac{\vec{b}}{1-x}$ = DISTANCE EXTRACTED PARTON – SPECTATOR SYSTEM

→ FOR LARGE $x$ DISTRIBUTION IN $\vec{b}$ SHOULD BECOME NARROW

→ DISTRIBUTION IN $t$ FLATTENS

→ LATTICE RESULTS

- HOW FAR CAN A FAST-MOVING QUARK BE SEPARATED FROM SPECTATORS ? (CONFINEMENT !)




J. NEGELE et al.

hep-lat/0309060

$A^{u-d}_{n0}(t) \,/\, A^{u-d}_{n0}(0)$

$n=3$
$n=2$
$n=1$

$$A^{u-d}_{n0}(t) = \int_{-1}^{1} dx\, x^{n-1} \quad H^{u-d}(x, \xi=0, t)$$

$\Rightarrow$ t- DEPENDENCE FLATTENS AS $x$ INCREASES

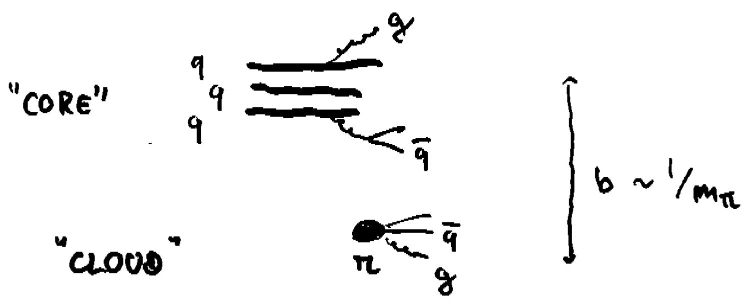
- CORRELATION OF $x$ AND $\vec{b}$ DEPENDENCE

  → DYNAMICS

→ e.g. PARTONS AT LARGE $\vec{b}$ → $\pi$ CLOUD

STRIKMAN, WEISS '03

(ARTIST'S CARTOON:)




EFFECTIVE AT

$x < m_\pi / m_N$

→ e.g. GRIBOV DIFFUSION


$$\langle b^2 \rangle = \langle b^2 \rangle_0 + 4\alpha' \log \frac{x_0}{x}$$

REGGE SLOPE

HERA DATA FOR $J/\psi$ PRODUCTION:

$\alpha' \sim 0.1 \quad GeV^{-2}$

# TWO TYPES OF INFO ON TRANSVERSE STRUCTURE :

GPDs ($\xi = 0$ FOR SIMPLIC.)


$\updownarrow$ FOURIER


E

O.A.M. FROM $\vec{\Delta}$

$k_T$ DEPENDENT PDFs


SIVERS FCT.

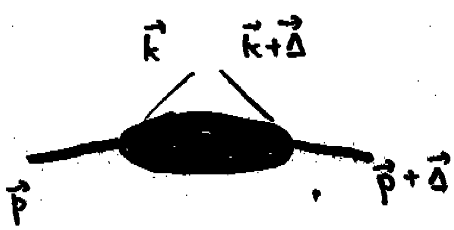
O.A.M. FROM $\vec{k}$

- RELATION FOUND IN SIMPLE MODEL

BURKARDT, HWANG '03

TASK FOR THEORY : UNDERSTAND RELATIONSHIP

$k_T$ DEPENDENT GPDs




$$f(x, \vec{\Delta}, \vec{k})$$

$\downarrow$ FOURIER

$$\tilde{f}(x, \vec{b}, \vec{k})$$

WIGNER FUNCTIONS

BELITSKY, JI, YUAN '03

# STUDYING GPDs IN LATTICE QCD

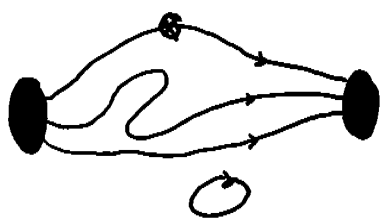
x - MOMENTS OF GPDs

→ MATRIX ELEMENTS OF LOCAL TWIST-2 OPERATORS

$$\langle p', s' | \, \bar{q}(0) \, \overleftrightarrow{D}^{\mu} \ldots \, \gamma^{\nu} \, q(0) \, | p, s \rangle$$

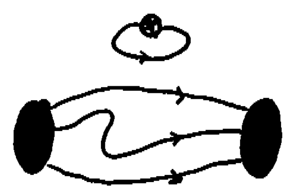
DEDICATED STUDIES LPHC, SESAM '03
QCDSF '03

SO FAR
CONNECTED GRAPHS



→ EUCLID. TIME $\tau$

NOT YET
DISCONNECTED



CANCELS IN $u - d$

MOMENTS :

$$\int_{-1}^{1} dx \; H^q = F_1^q(t) \quad \text{DIRAC}$$

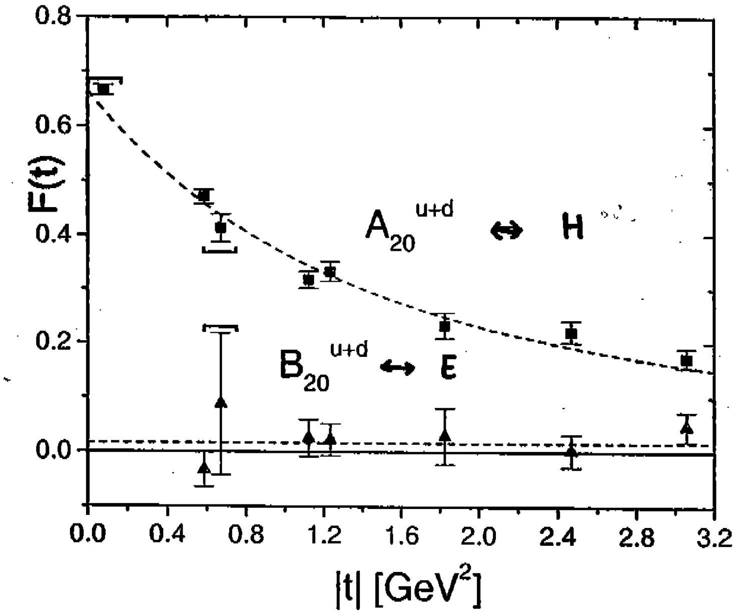
$$\int_{-1}^{1} dx \; E^q = F_2^q(t) \quad \text{PAULI}$$

$$\int_{-1}^{1} dx \; x \, H^q = A^q(t) + 4\xi^2 \, C^q(t)$$

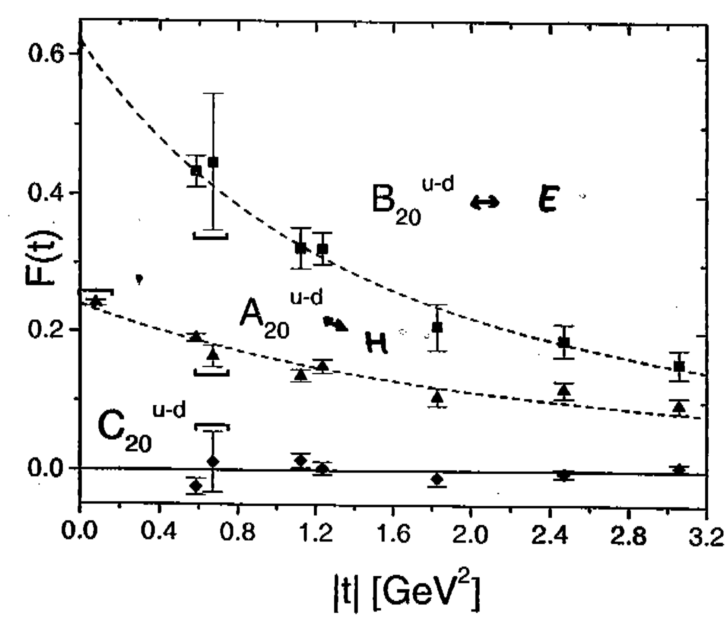
$$\int_{-1}^{1} dx \; x \, E^q = B^q(t) - 4\xi^2 \, C^q(t)$$

FORM FACTORS OF $T^{\mu\nu}_q$
(ENERGY - MOMENTUM)

LPC, SESAM Collab hep-lat/0304018




u+d




u-d

LARGE - $N_c$ COUNTING :

$H^{u+d} \sim N_c^2$ $H^{u-d} \sim N_c$

$E^{u+d} \sim N_c^2$ $E^{u-d} \sim N_c^3$

# UNDERSTANDING THE SHAPE AND SIZE OF GPDs

THEORET. CONSIDERATIONS

- SYMMETRIES

e.g. DOUBLE DISTRIBUT'S

or WAVE FUNCTION REPRESENTATION

LATTICE QCD

MOMENTS IN x

- SPIN-FLAVOR STRUCTURE
- t - DEPENDENCE

- LIMITS

e.g. CHIRAL DYNAMICS

or LARGE $N_c$

MODELS

GPDS

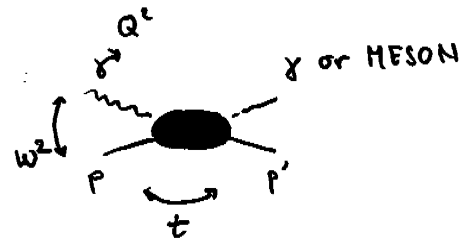
LONGITUDINAL : REGION $|x| > \xi$ $|x| = \xi$ $|x| < \xi$

HELP FROM LIMIT $\xi = 0$ > > LESS AND LESS KNOWN

TRANSVERSE $t$ DEPEND'CE AND INTERPLAY WITH $x, \xi$

DATA

# KEY PROCESSES + OBSERVABLES




## 1) VARIABLES

FACTORIZATION THEOREMS REQUIRE

LARGE $W^2$, $Q^2$ $\gg$ $-t$ , $\Lambda^2_{QCD}$

✗ NEED SOME RANGE IF WANT FOURIER TRANSFORM

- $x_B$ AND $Q^2$ TIED TOGETHER BY EVOLUTION

  ✗ NEED LEVER ARM

  TEST SCALING $\left\{ \begin{array}{l} \text{BEHAVIOR} \\ \text{BREAKING} \end{array} \right.$

- $x_B$ AND $t$ CORRELATE LONG. ⊗ TRANSV. STRUCTURE

## 2) GPDs → PROCESS AMPLITUDES

- AT BORN LEVEL ( DVCS , MESONS )

$$\mathcal{A} \propto \underbrace{\int_{-1}^{1} dx \frac{1}{x-\xi} f(x,\xi,t)}_{\text{ALL } x} - i\pi \underbrace{f(\xi,\xi,t)}_{x=\xi}$$

$$\downarrow$$

$$|x| \geq \xi$$

- AT $\mathcal{O}(\alpha_s)$

EVOLUTION LINKS $x$ AND $Q^2$ DEPENDENCE

(AS IN INCLUSIVE DIS)

$$\xi = \frac{x_B}{2-x_B}$$

IF CANNOT RECONSTRUCT $x$ - DEPENDENCE

→ TYPICAL $x \sim \xi$

"MIX" REGIONS  AND 

UNLESS HAVE SEPARATE $\mathrm{Im}\,\mathcal{A}$

→ $t$ DEPENDENCE → TRANSVERSE STRUCTURE

FOR "TYPICAL $x \sim \xi$"

3) DVCS BEST THEORY CONTROL

LEADING TWIST (2) : LO, NLO KNOWN

TWIST 3 : LO, NLO IN W.W. APPROX.

- SEPARATION OF $\mathrm{Re}\,\mathcal{A}$, $\mathrm{Im}\,\mathcal{A}$
- MIGHT USE $Q^2$ DEPENDENCE TO CONSTRAIN $x$

COMPETITION $\sigma_{\text{BETHE-HEITLER}} \sim \frac{1}{t}$

$\sigma_{\text{COMPTON}} \sim \frac{1}{Q^2 y^2}$ $\qquad y = \frac{Q^2}{x_B S_{ep}}$

↳ COMPTON X SECTION, TARGET SPIN ASYMM'S

↳ INTERFERENCE FROM BEAM OR TARGET SSA'S

$\propto \mathrm{Im}\,\mathcal{A}_{DVCS}$

↳ INTERFERENCE FROM $e^+$ VS $e^-$ BEAM

$\propto \mathrm{Re}\,\mathcal{A}_{DVCS}$

X NEED TO MEASURE

$\phi$ ($\measuredangle$ LEPTON vs HADRON PLANE)

# HOW SENSITIVE TO $E$ ?

IN INTERFERENCE

UNPOL. TARGET $\approx \dfrac{\sqrt{t_0 - t}}{2m_p} \left[ F_1 H + \xi (F_1 + F_2) \tilde{H} - \dfrac{t}{4m_p^2} F_2 E \right]$

LONG. POL. TARGET $\approx \dfrac{\sqrt{t_0 - t}}{2m_p} \left[ F_1 \tilde{H} + \xi (F_1 + F_2) H + \xi^2 (F_1 + F_2) E + \ldots \tilde{E} \right]$

X TRANSV. POL. TARGET $\approx \dfrac{t}{4m_p^2} \left[ F_2 H - F_1 E + \ldots \right]$

FOR PROTON : INDISPENSABLE TO GET $E$

SAME FOR DVCS X SECTION

FOR NEUTRON : UNPOL. TARGET HAS SENSITIVITY

DVCS → DOUBLE DVCS

$$\int dx \, \frac{H(x,\xi,t)}{x-\xi+i\epsilon} = \ldots - i\pi \, H(\xi,\xi,t)$$

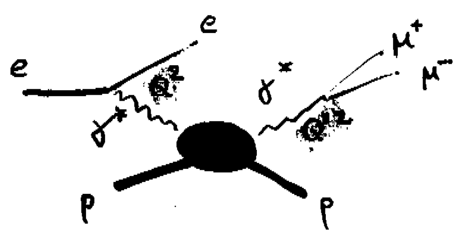
$$\int dx \, \frac{H(x,\xi,t)}{x-\rho+i\epsilon} = \ldots - i\pi \, H(\rho,\xi,t)$$

NEED EITHER $Q^2$ OR $Q'^2$ LARGE

$$\xi = \frac{Q^2+Q'^2}{2\ell.q} \qquad \rho = \frac{Q^2-Q'^2}{2\ell.q}$$

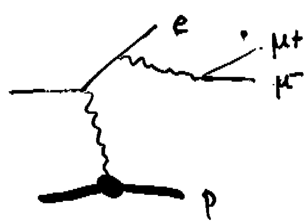
→ SCAN GPDs IN REGION

$\sim \frac{1}{Q'^2} \frac{1}{Q^2 y^2}$

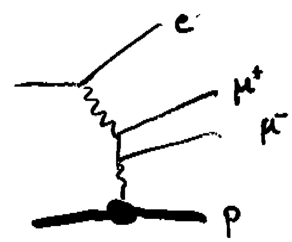



GUIDAL, VANDERHAEGHEN '02
BELITSKY, MÜLLER '02
+ KIRCHNER '03

- 2 TYPES OF BETHE - HEITLER




$\sim \frac{1}{Q'^2} \frac{1}{t}$




$\sim \frac{1}{Q^2 y^2} \frac{1}{t}$

ALWAYS BIG IF $Q'^2$ LARGE

$\mu^+\mu^-$ ANGULAR DISTRIBUTION
→ INTERFERENCE TERM

BERGER, M.D., PIRE '01

# 4) MESON PRODUCTION


- LEADING TWIST : $\sigma_L$ AND TRANSVERSE TARGET ASY (*) } BOTH $\gamma^*$ AND MESON LONGITUDINAL

  NLO CORR'S CALCULATED JUST RECENTLY

  D. IVANOV et al '03, '04

  CAN BE LARGE DEPENDING ON KINEMATICS AND OBSERVABLE

  TASK FOR THEORY :

  UNDERSTAND PATTERN OF CORRECTIONS

  IDENTIFY "SAFE" OBSERVABLES

- AT LEVEL OF $1/Q$ CORRECTIONS : IN GENERAL FAC~~TORI~~ZATION

(*) AST IS $f\left(\frac{E}{H}\right)$ OR $f\left(\frac{\tilde{E}}{\tilde{H}}\right)$

REVIEW GOEKE et al '01

# MESON CHANNELS

→ HELP SEPARATE SPIN-FLAVOR STRUCTURE

N.B. FORM FACTORS CONSTRAIN (INTEGRATED OVER $x$)

$F_1(t)$, $F_2(t)$ $\quad$ $q-\bar{q}$ $\quad$ FOR $u, d$

$g_A(t)$ $\quad$ $\Delta q + \Delta\bar{q}$ $\quad$ $u-d$

+ INFO FROM PARITY VIOLATING ELASTIC SCATT.

- VECTOR MESONS $\rho^0, \omega, \phi$ ; $J/\psi$

  $H, E$ IN COMBINATIONS $q+\bar{q}$ AND $g$

  SAME AS IN COMPTON SCATT.

  e.g. $\rho^0$ : $\frac{2}{3}(u+\bar{u}) + \frac{1}{3}(d+\bar{d}) + \frac{3}{4} g$

- $u/d$ SEPARATION : DVCS ON $p$, $n^{(*)}$ X

  MESON CHANNELS

- ALSO $K^*$ PRODUCTION, IF USE FLAVOR SU(3) RELATIONS

- $f_2$, $\pi^+\pi^-$ IN CONTINUUM : $q-\bar{q}$

- $\pi^0$, $K^0$ $\quad$ $\Delta q - \Delta\bar{q}$ $\quad$ $\pi^+, K^+$ ALSO $\Delta q + \Delta\bar{q}$

  $\eta, \eta'$

X (*) MUST KNOW IF $d \to d$ OR $d \to n + p$

# SUMMARY + OUTLOOK

GPDs

- CORRELATE $x$ AND $\vec{b}$ $\rightsquigarrow$ PARTON DYNAMICS

- $E(x,\xi,t)$ $\rightarrow$ ORBITAL ANGULAR MOM.

  $\int dx\, x\,[E+H]\Big|_{t=0}$ IS ONE IMPORTANT NUMBER

- UNIFYING FORMALISM

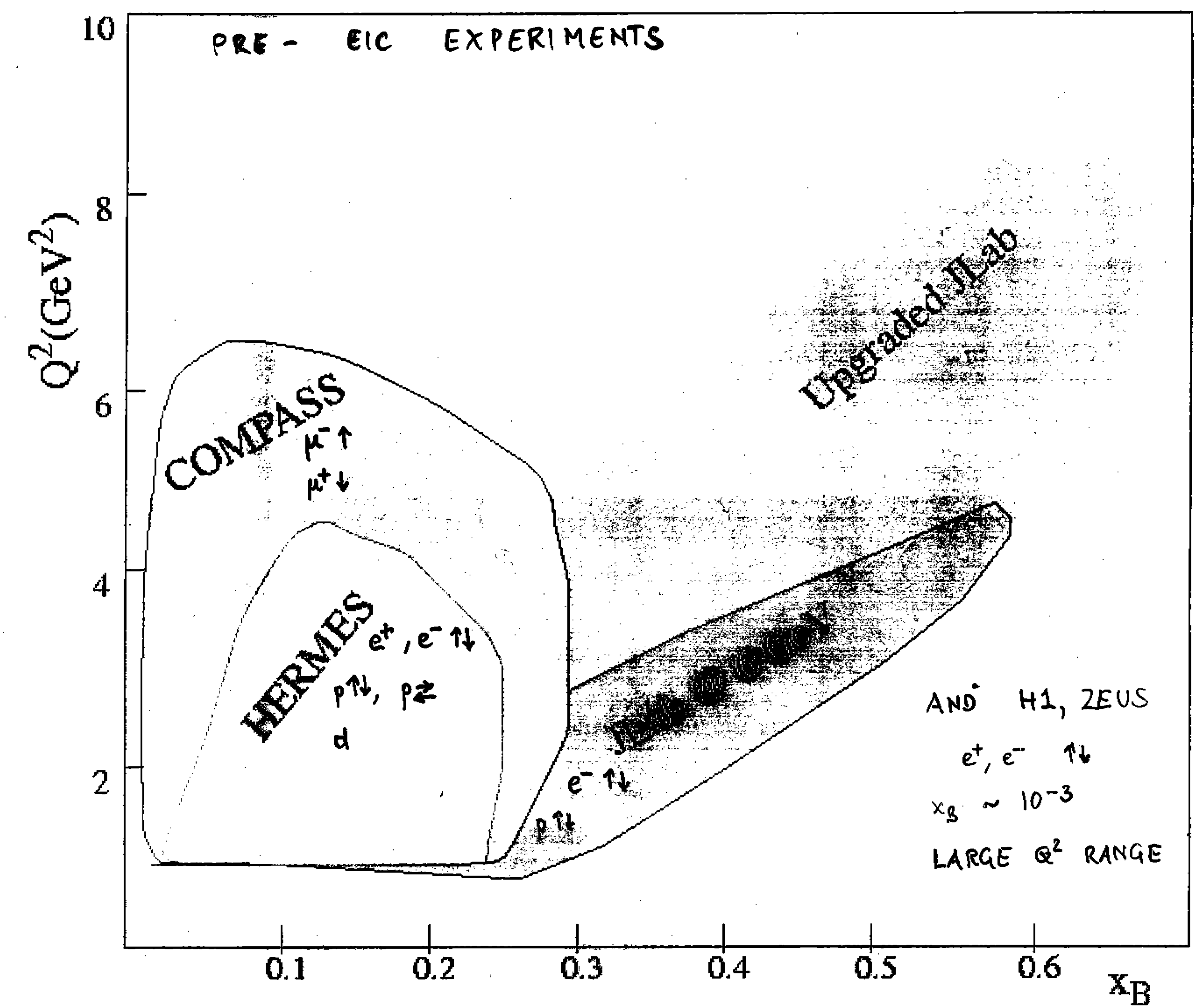
  ALLOWS THEORISTS TO QUANTIFY HADRON STRUCTURE IN NOVEL WAYS

  + TO CONNECT PIECES OF THE PUZZLE

  e.g. TRANSVERSE SPIN, MOMENTUM, POSITION

  CONTACT WITH EXPERIMENT

EXPECT FURTHER PROGRESS FROM THEORY, INCLUDING LATTICE QCD


PRE - EIC EXPERIMENTS
COMPASS
$\mu^- \uparrow$
$\mu^+ \downarrow$
HERMES
$e^+, e^- \uparrow\downarrow$
$p \uparrow\downarrow$, $p \rightleftarrows$
d
Upgraded JLab
$e^- \uparrow\downarrow$
$p \uparrow\downarrow$
AND H1, ZEUS
$e^+, e^- \uparrow\downarrow$
$x_B \sim 10^{-3}$
LARGE $Q^2$ RANGE
$Q^2 (GeV^2)$
$x_B$
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
0.6

- FROM RUNNING EXPERIMENTS EXPECT TO
  - EXPLORE VALIDITY OF LARGE $Q^2$ LIMIT
  - —//— FEASIBILITY TO ANALYSE VARIOUS CHANNES
  - UNCOVER BASIC QUALITATIVE + QUANTITATIVE FEATURES OF GPDS

MOSTLY @ MODERATE $Q^2$ (EXCEPT FOR HERA COLLIDER)

- JLAB @ 12 GeV EXPECT IN-DEPTH STUDIES AT MODERATE TO LARGE $x_B$
- TO OPTIMIZE PHYSICS HARVEST

→ GOOD LEVER ARM IN $Q^2$ AND $t$ AT GIVEN $x_B$

→ TARGET POL. TO DISENTANGLE SPIN STRCT.

→ POLARIZED $e^-$, $e^+$ BEAMS FOR COMPTON SCATT.

→ CAPABILITY TO IDENTIFY FINAL STATE (+ GOOD KINEMATIC RESOLUTION)

→ POSSIBLY $n$ TARGETS