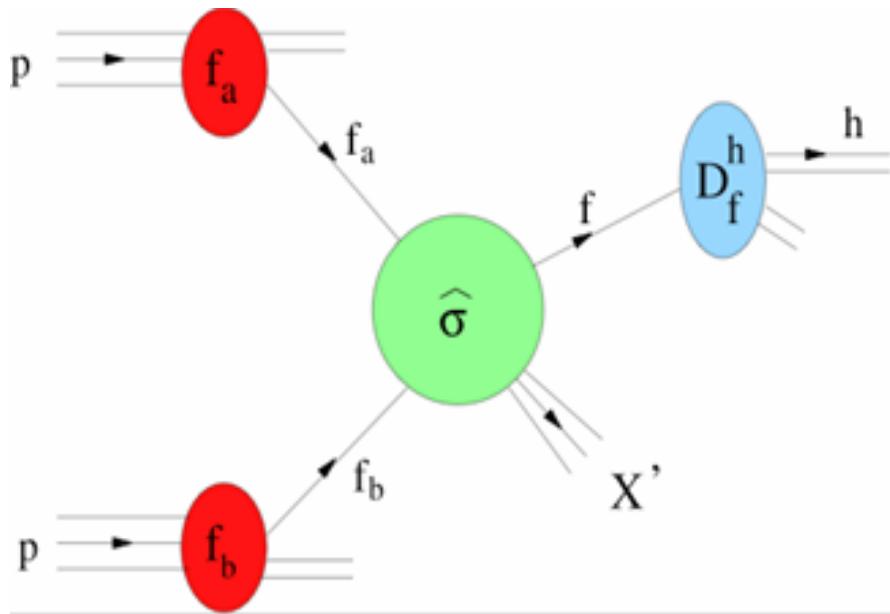


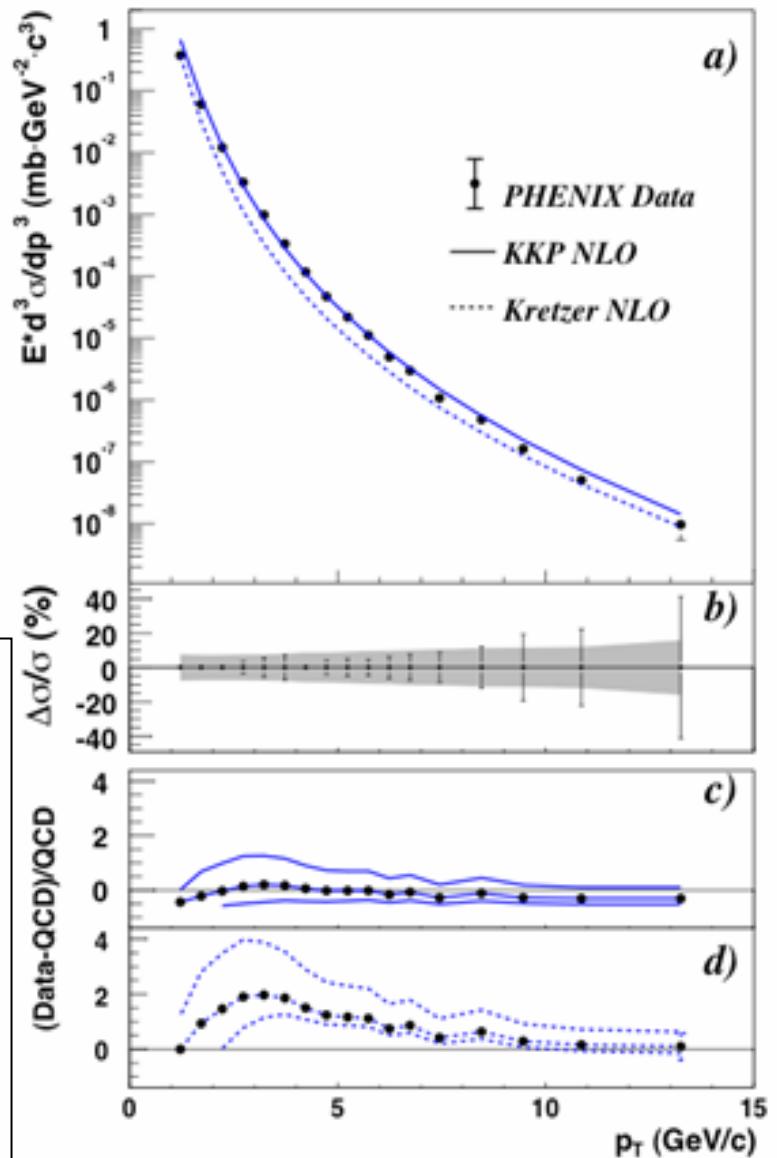
# Universal Aspects of the Dipole Cross Section in eA and pA Collisions

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University of Washington

# Perturbative QCD

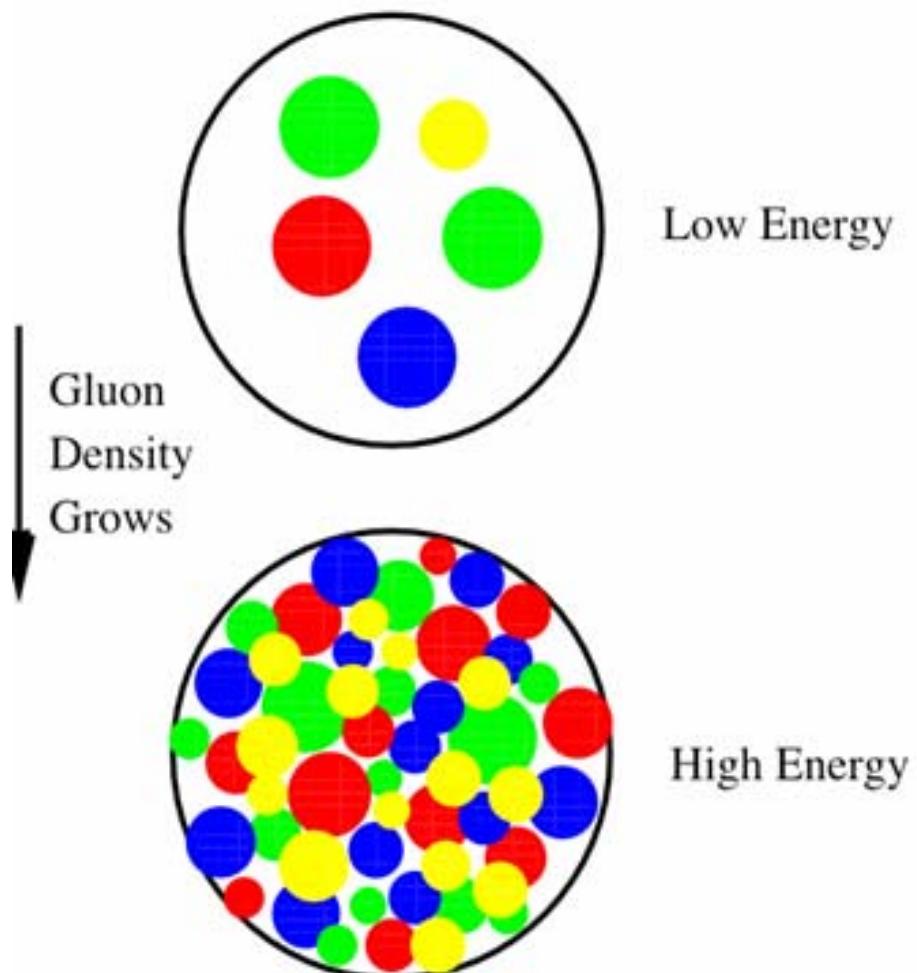


- Quarks, gluons ( $x, Q^2$ )
- Weak coupling ( $\alpha_s \ll 1$ )
- Collinear factorization
- Universality
- *Dilute* systems



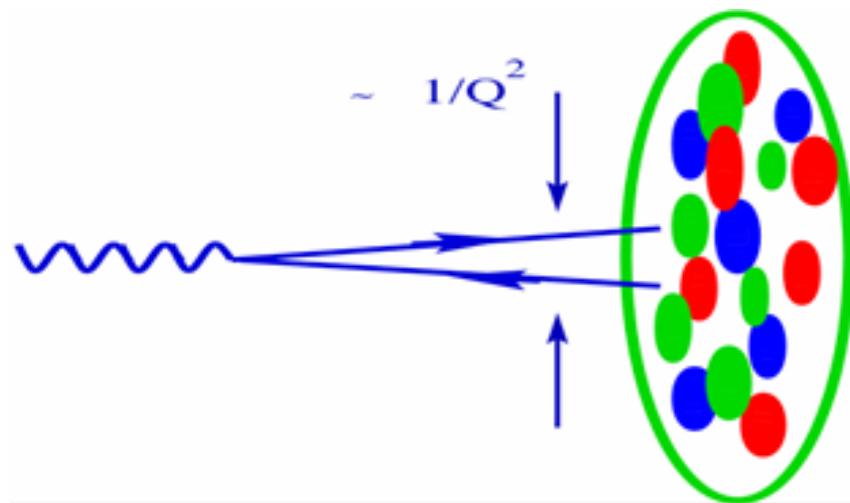
# Gluon Saturation

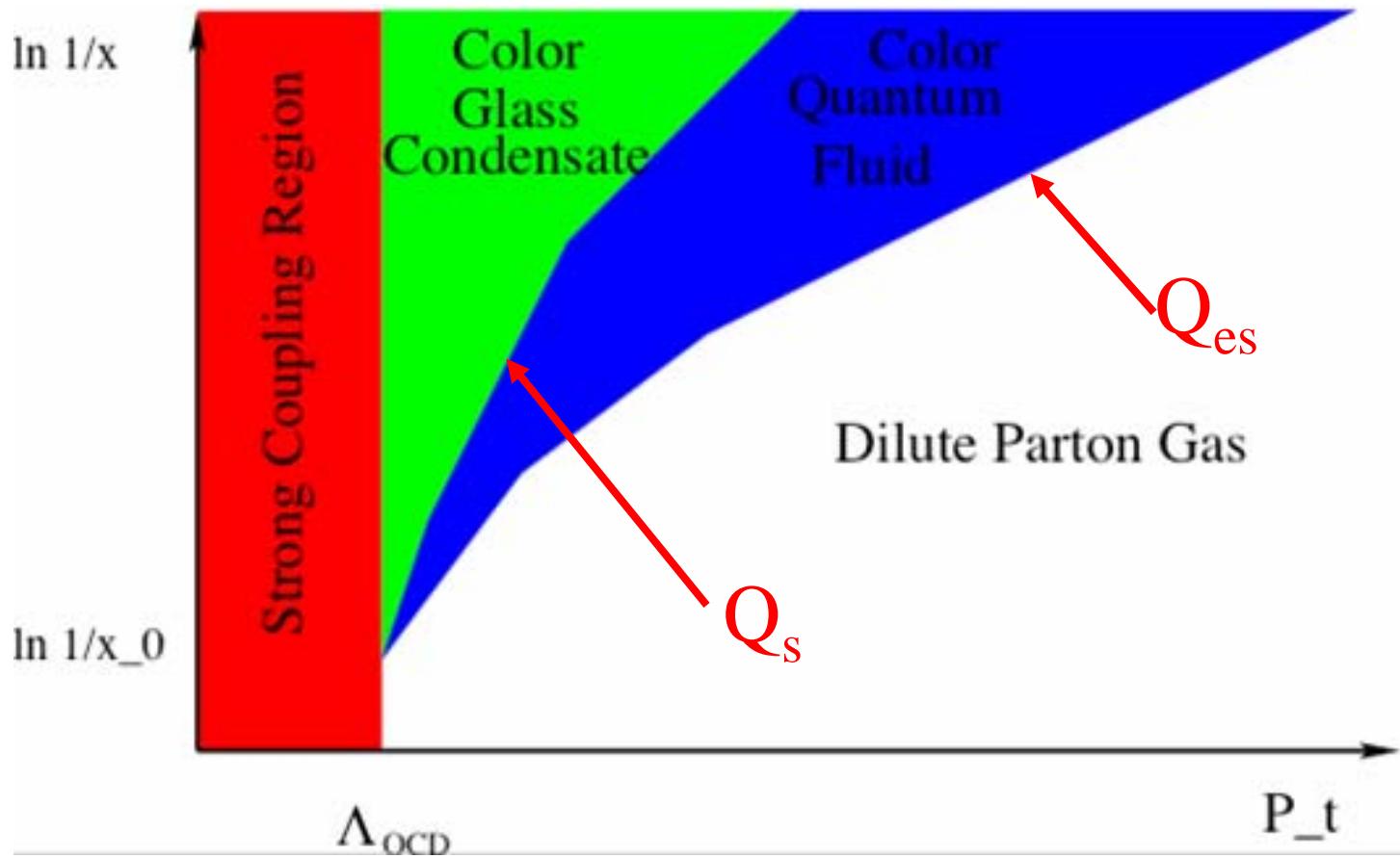
- Small X/Large A
- *Large occupation number*
- *Coherent state*
- *Saturation momentum*  $Q_s(x)$



# Semi-Classical QCD

- Weak coupling ( $\alpha_s \ll 1$ )
- Classical fields + renormalization group
- Coherence
- Wilson lines (dipoles)
- Dense systems





- Color Glass Condensate  $P_t < Q_s(y)$
- Color Quantum Fluid  $Q_s(y) < P_t < Q_{es}(y)$
- Dilute Parton Gas  $P_t > Q_{es}(y)$
- RHIC, LHC, **EIC**

# QCD: Kinematic Regions

- Color Glass Condensate
  - High gluon density
  - Strong classical fields
  - Non-Linear evolution: JIMWLK (BK at large  $N_c$ )
- Color Quantum Fluid
  - Low gluon density
  - Linear evolution: BFKL
  - Anomalous dimension ( $k_t$  factorization)
- Dilute Parton Gas
  - Low gluon density
  - Linear evolution: DGLAP
  - Collinear factorization

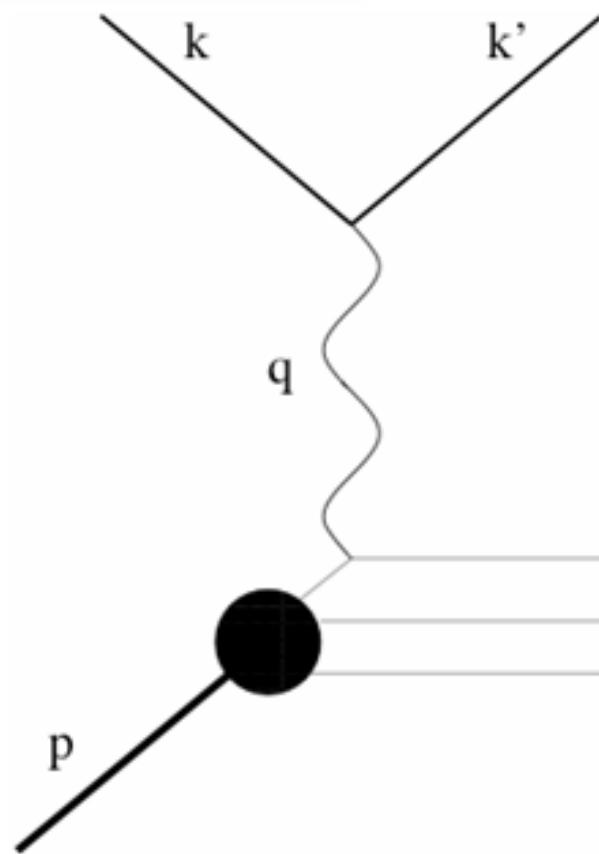
$$\text{DIS: } ep(A) \rightarrow e'X$$

$$\frac{d\sigma^{ep \rightarrow eX}}{dx dQ^2} = \frac{2\pi\alpha_{em}^2}{x Q^4} [Y_+ F_2 + y^2 F_L]$$

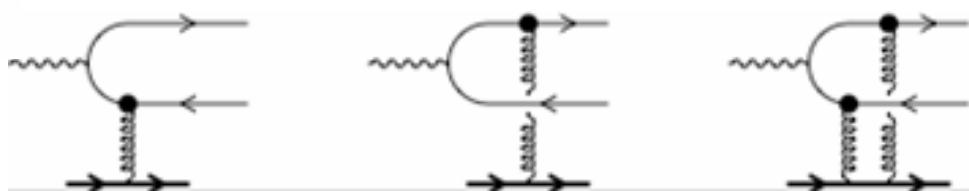
$$Q^2 = -q^2 \quad x = \frac{Q^2}{2 p \cdot q}$$

$$y = \frac{p \cdot q}{p \cdot k}$$

$$Y_+ = 1 + (1 - y)^2$$



$$\mathcal{A}^{\gamma^\star(k)P\rightarrow q(p)\bar{q}(q)X}$$



$$\begin{aligned}
 \mathcal{A}^\mu = & -i \int d^2x_t e^{i(q_t+p_t-k_t)\cdot x_t} (U^\dagger(x_t) - 1) \\
 & \overline{u}(q) \frac{\gamma^\mu(\not{q} - \not{k} + m)\gamma^-}{(q - k)^2 - m^2 + i\epsilon} v(p) + \\
 & i \int d^2y_t e^{i(q_t+p_t-k_t)\cdot y_t} (U(y_t) - 1) \\
 & \overline{u}(q) \frac{\gamma^-(\not{k} - \not{p} + m)\gamma^\mu}{(p - k)^2 - m^2 + i\epsilon} v(p) + \\
 & i \int \frac{d^2l_t}{(2\pi)^2} \int d^2x_t d^2y_t \\
 & e^{il_t\cdot x_t} e^{i(p_t+q_t-k_t-l_t)\cdot y_t} (U(x_t) - 1) (U^\dagger(y_t) - 1) \\
 & \overline{u}(q) \frac{\gamma^-(\not{q} - \not{l} + m)\gamma^\mu(\not{q} - \not{k} - \not{l} + m)\gamma^-}{2p^-[(q_t - l_t)^2 + m^2 - 2q^-k^+] + 2q^-[(q_t - k_t - l_t)^2 + m^2]} v(p)
 \end{aligned}$$

# DIS Structure Functions

$$F_2 = \frac{Q^2}{4\pi^2 \alpha_{em}} \sigma^{\gamma^* p}$$

$$\sigma^{\gamma^* p} = \int_0^1 dz \int d^2 r_t d^2 b_t |\Psi(z, r_t, Q^2)|^2 \sigma_{dipole}(x, r_t, b_t)$$

$$\sigma_{dipole}(x, r_t, b_t) \equiv \frac{2}{N_c} Tr < 1 - U(x_t) U^\dagger(y_t) >_x$$

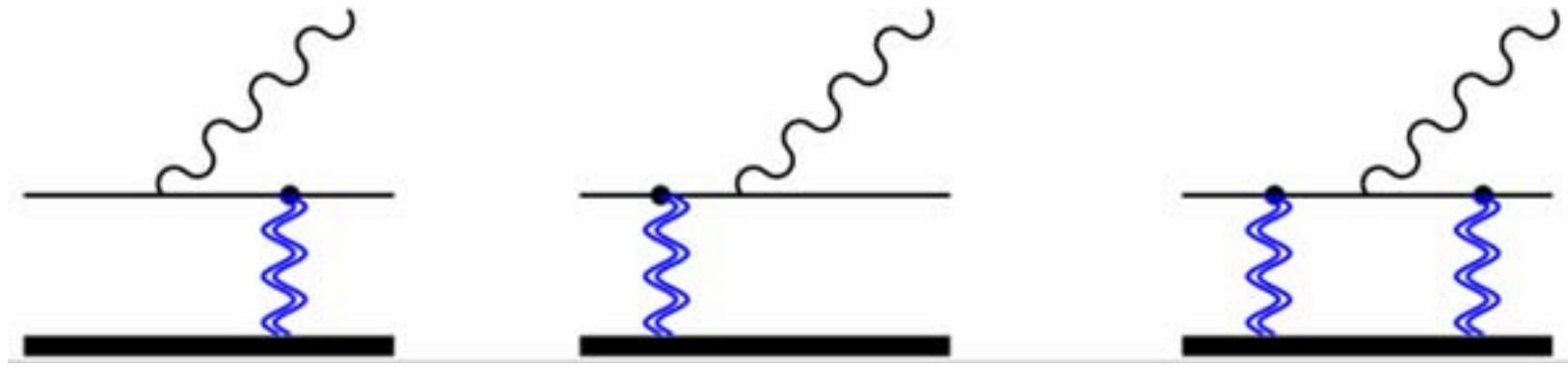
$$b_t = (x_t + y_t)/2 \quad \quad r_t = x_t - y_t$$

$$U(x_t) = \hat{P} e^{ig \int_{-\infty}^{\infty} dx^- A_a^+(x^-, x_t) t_a}$$

# Virtual Photon Production

(F.G. + J.J-M. PRD67, 2003)

$$q(p)P \rightarrow q(q)\gamma^\star(k)X$$



$$\begin{aligned} \mathcal{A}^\mu = & i \int d^2x_t e^{i(q_t + k_t - p_t) \cdot x_t} (U(x_t) - 1) \\ & \times \bar{u}(q) \left[ \frac{\gamma^\mu (q + k + m) \gamma^-}{(q + k)^2 - m^2 + i\epsilon} + \frac{\gamma^- (p - k + m) \gamma^\mu}{(p - k)^2 - m^2 + i\epsilon} \right] u(p) \end{aligned}$$

# Virtual Photon Production

$$\sigma = \int_0^1 dz \int d^2 r_t d^2 b_t |\phi(p^-, p_t | k^+, z, r_t)|^2 \times \sigma_{\text{dipole}}(r_t, b_t, x)$$

$$|\phi|^2 \equiv \frac{1}{64\pi p^{-2} z(1-z)} \int \frac{d^2 q_t}{(2\pi)^2} \frac{d^2 k_t}{(2\pi)^2} e^{i(q_t + k_t - p_t) \cdot r_t} \left[ M(p_t - k_t, p_t - k_t) + M(q_t, q_t) - M(q_t, p_t - k_t) - M(p_t - k_t, q_t) \right]$$

# Dilepton Production in pA

$$\frac{d\sigma^{d(p) A \rightarrow l^+ l^- X}}{d^2 b_t \, dM^2 \, dx_F} = \frac{\alpha_{em}^2}{6\pi^2} \frac{1}{x_q + x_g} \int_{x_q}^1 dz \int dr_t^2 \frac{1-z}{z^2}$$

$$F_2^{d(p)}(x_q/z) \sigma_{dipole}(x_g, b_t, r_t)$$

$$\left[ [1 + (1 - z)^2] K_1^2 \left[ \frac{\sqrt{1-z}}{z} M r_t \right] + \right.$$

$$\left. 2(1-z) K_0^2 \left[ \frac{\sqrt{1-z}}{z} M r_t \right] \right] \quad \text{with}$$

$$x_q = \frac{1}{2} \left[ \sqrt{x_F^2 + 4 \frac{M^2}{s}} + x_F \right]$$

$$x_g = \frac{1}{2} \left[ \sqrt{x_F^2 + 4 \frac{M^2}{s}} - x_F \right]$$

# Models of Dipole Cross Section

Golec-Biernat+Wusthoff

$$\hat{\sigma}(x, r_t) = \sigma_0 \left[ 1 - e^{-r_t^2 Q_s^2 / 4} \right]$$

Bartels+Golec-Biernat+Kowalski

$$\hat{\sigma}(x, r_t) = \sigma_0 \left[ 1 - e^{-\frac{\pi^2 \alpha_s r_t^2 x G(x, \mu^2)}{3\sigma_0}} \right]$$

Iancu+Itakura+Munier

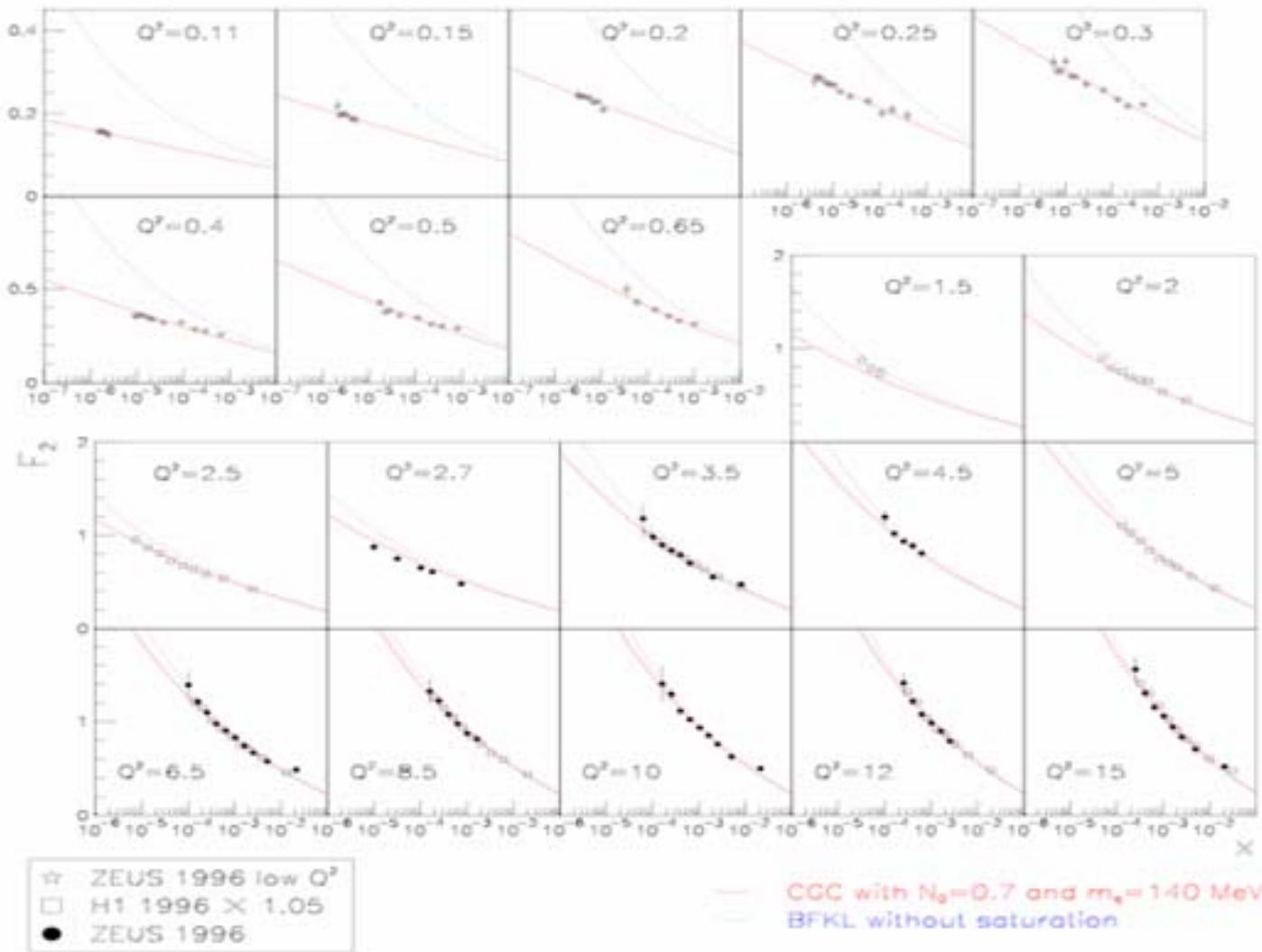
$$\hat{\sigma}(x, r_t) = 2\pi R^2 \mathcal{N}(r_t Q_s, x)$$

$$\mathcal{N}(r_t Q_s, x) = \mathcal{N}_0 \left( \frac{r_t Q_s}{2} \right)^{2\gamma_s + \frac{2 \log(2/r_t Q_s)}{\kappa \lambda \log 1/x}} \quad r_t Q_s \leq 2$$

$$\mathcal{N}(r_t Q_s, x) = 1 - e^{-a \log^2(b r_t Q_s)} \quad r_t Q_s > 2$$

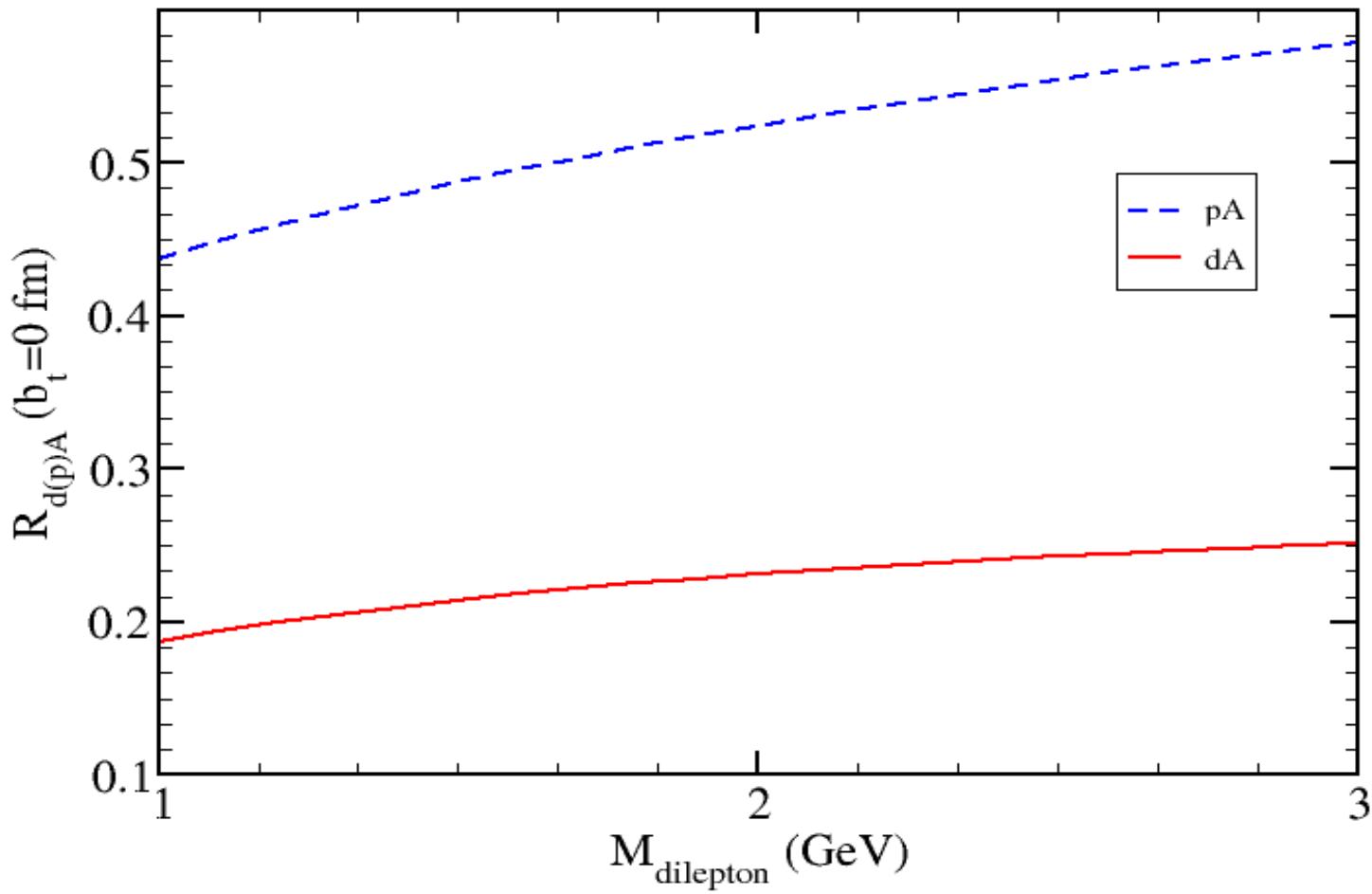
# Models of Dipole Cross Section

Iancu, Itakura and Munier hep-ph/0310338

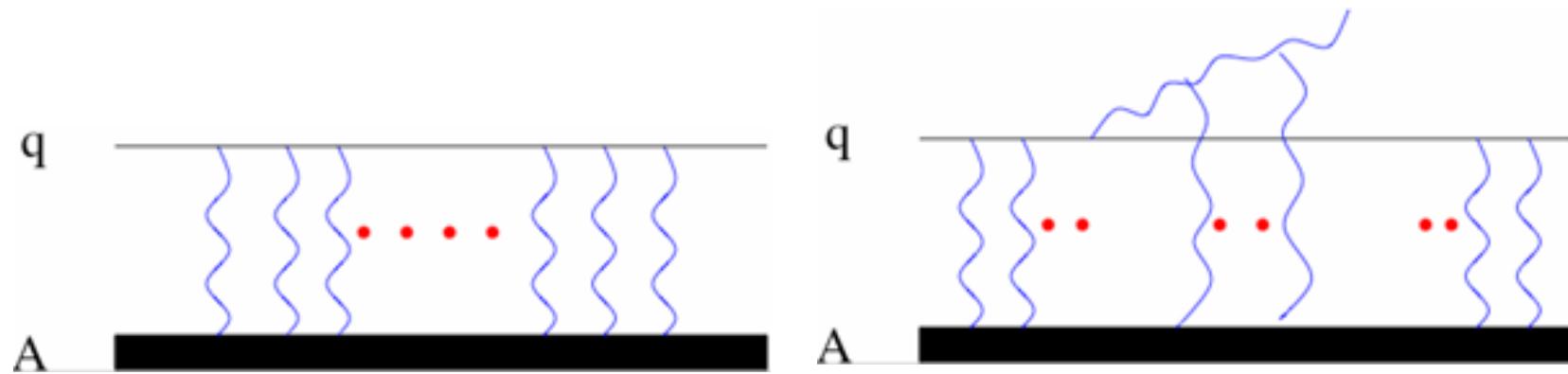


# Dilepton Production in dA

J.J-M nucl-th/0402014



# Dipoles and Hadron Production



- Quark "production"

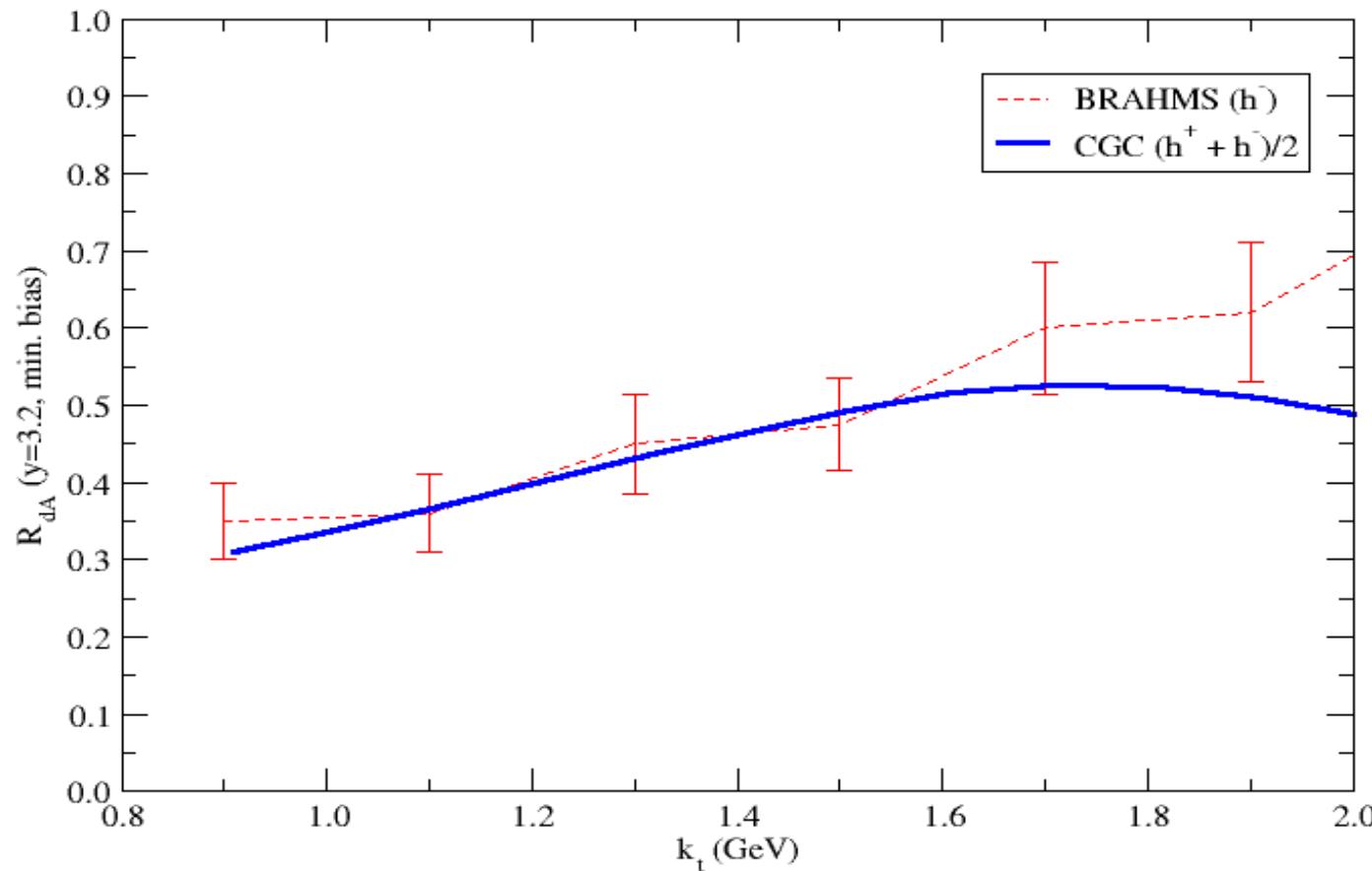
$$\frac{d\sigma^{pA}}{dy d^2b_t d^2k_t} \sim \int d^2r_t e^{ik_t \cdot r_t} \sigma_{dipole}(x, r_t, b_t)$$

- Gluon production

$$\frac{d\sigma^{pA}}{dy d^2b_t d^2k_t} \sim \int \frac{d^2r_t}{r_t^2} e^{ik_t \cdot r_t} \sigma_{dipole}(x, r_t, b_t)$$

# Forward Rapidity Hadron Production

J.J-M. nucl-th/0402080



# From RHIC To EIC

- RHIC experimental coverage (Ex.:  $p_t = 2 \text{ GeV}$ )
  - $y=0 \rightarrow 4$
  - $x_1 \sim 10^{-2} \rightarrow 5 \times 10^{-1}$
  - $x_2 \sim 10^{-2} \rightarrow 10^{-4}$
- Limited  $p_t$  coverage at large  $y$
- Convolution
- EIC: the ultimate CGC machine
  - $x, Q^2$  coverage
  - Luminosity
  - $A$  dependence
  - $F_L, \dots$