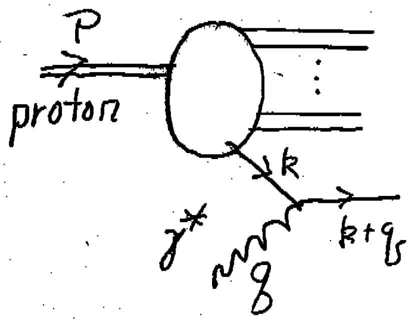
High Density QCD

1. The parton picture

Illustrate in deep inelastic scattering

Partons manifest in Bjorken frame



$$P = \left(p + \frac{m^2}{2p}, 0, 0, p\right)$$

$$q = (q_0, q_\perp, 0)$$

$x = \frac{Q^2}{2P\cdot q}$ and Q^2 are invariants

Virtual photon absorbed by charged parton (quark)

(i) in transverse spatial region $\Delta x_\perp \sim 1/Q$

(ii) over time $\Delta t \simeq \frac{1}{E_k + E_q - E_{k+q}} \sim 1/Q$

So, virtual photon makes localized and instantaneous measurement of quarks.

$$F_2(x, Q^2) = \nu W_2(x, Q^2) = \sum_f e_f^2 \left[x q_f(x, Q^2) + x \bar{q}_f(x, Q^2) \right]$$

Quark and gluon distributions grow rapidly with decreasing x.

At $Q^2 \simeq 10\ GeV^2$

$xG(x,Q^2)$ and $x[q(x,Q^2)+\bar{q}(x,Q^2)] \sim x^{-0.3}$

while at $Q^2 \simeq 10\ GeV^2$ and $x \simeq 10^{-4}$

$$xG(x,Q^2) \simeq 20 \qquad x[q(x,Q^2)+\bar{q}(x,Q^2)] \simeq 6$$

2. What causes growth of parton number densities?

Dokshitzer Altarelli Parisi Kuraev

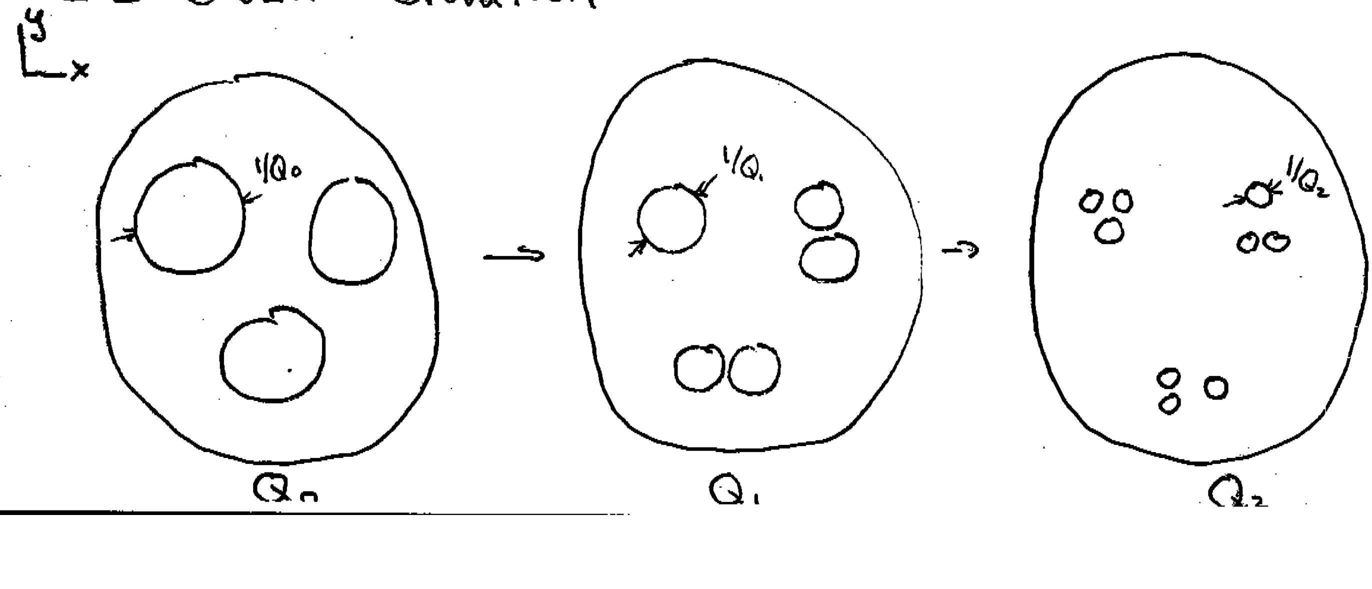
DGLAP and BFKL evolution

Gribov Lipatov Lipatov

DGLAP ↕ Q^2-evolution dominated

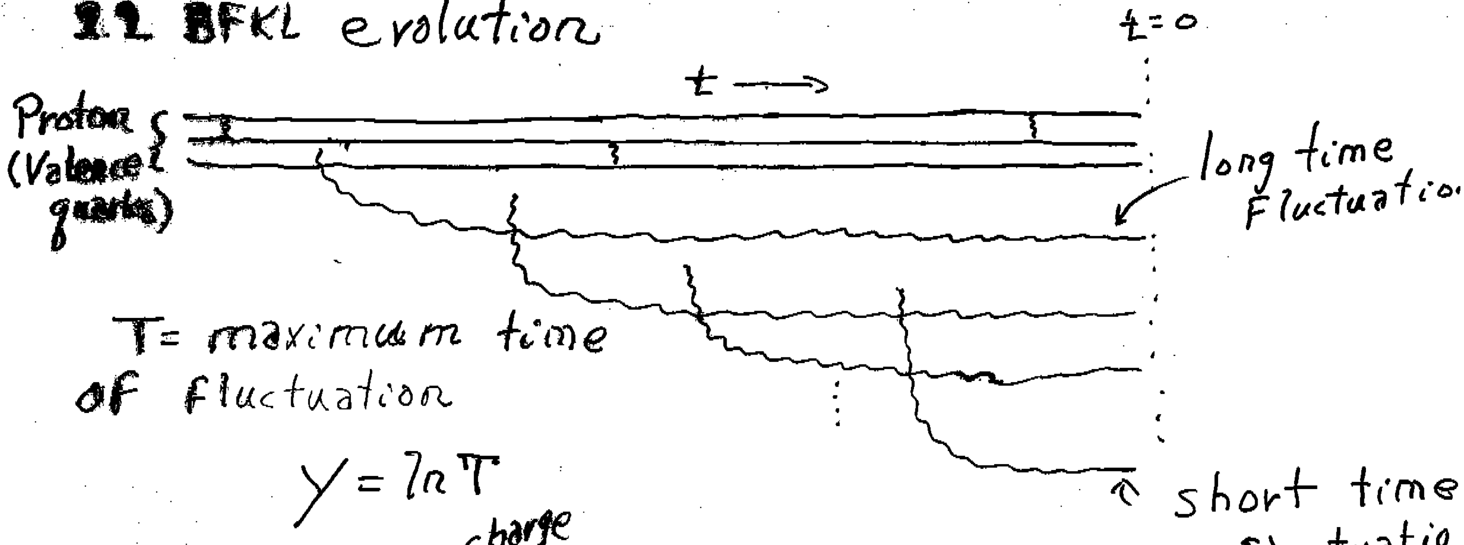
BFKL ↕ x-evolution dominated

2.1 DGLAP evolution



DGLAP evolution increases parton number densities but not phase space densities (occupati numbers). Difficult to free the partons, also.

2.2 BFKL evolution



T = maximum time of fluctuation

$$y = \ln T$$

$$\frac{d}{dy} N_g(y) = \underbrace{Q^2(y)}_{g_s^2 N_g(y)}$$

(charge) ... random charge in fixed $(\Delta x_\perp)^2$ where evolution occurs

$$N_g(y) \sim \frac{e^{(\alpha_P - 1)y}}{\sqrt{\alpha_s y}} \qquad \alpha_P - 1 = \frac{4\alpha_s N_c}{\pi} \ln 2$$

Now $\frac{dN_g}{dy} \sim$ phase space density.

A collision can free all the gluons.

Should be maximum occupation number.

Yang-Mills is stable theory ⇒ repulsion when occupation number large.

3. The saturated QCD wavefunction. (Color Glass Condensate)

[illegible]
McLerran, Venugopalan
Iancu, Leonidov, Jalilian-Marian
Kovner, Weigert, ...

3.1 Gluons

Q_s = Saturation scale $\qquad Q_s \simeq 1\,\mathrm{GeV}$ at HERA and for nuclei at RHIC

$k_\perp \lesssim Q_s$

$$f_g \simeq \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dxG}{d^2b_\perp d^2k_\perp} \simeq \frac{c_1}{\alpha_s N_c}\left(\ln Q_s^2/k_\perp^2 + c_2\right)$$

c_1, c_2 not yet ~~under~~ control

Q_s increases with energy.

Total number of gluons increase (Fairly rapidly) with energy but occupation numbers reach limit.

3.2 Quarks

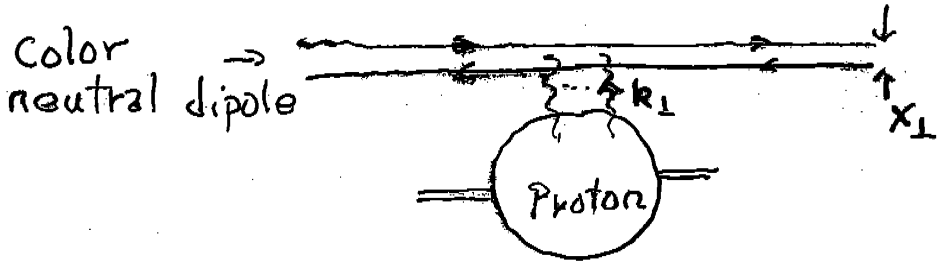
$\bar{Q}_s$ = Saturation scale $\qquad \bar{Q}_s^2 = \frac{C_F}{N_c} Q_s^2$

$k_\perp \lesssim Q_s$

$$f_q \simeq \frac{(2\pi)^3}{2N_c \cdot N_f \cdot 2} \frac{d\,x[q+\bar{q}]}{d^2b_\perp d^2k_\perp} \simeq 1/\pi$$

3.3 Saturation ~ Unitarity limit

Scatter quark dipole on hadron



$k_\perp \sim 2/x_\perp$ to resolve q and $\bar{q}$ parts of dipol

So long as $k_\perp \lesssim Q_s$ coupling is strong

$A(k_\perp) \sim 1/g$

$\Rightarrow S \simeq 0$ for $x_\perp \lesssim 2/Q_s$

4. Phenomenology

Vigorous attempts to check Saturation-CGC picture at HERA and at RHIC.

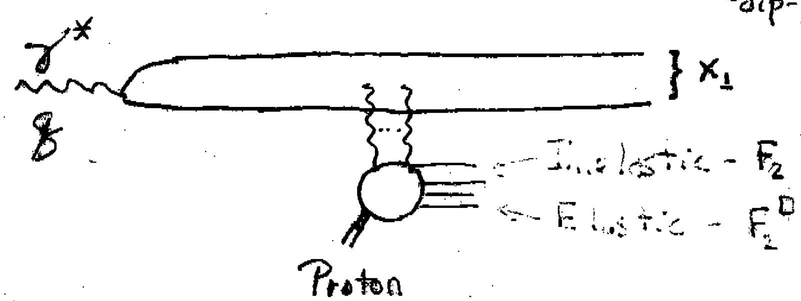
HERA probes the small-x proton wavefunction, but does not produce (free) the partons

RHIC frees the (mainly) gluons and creates, at time of production, an extremely dense non equilibrium system

4.1 HERA low Q^2 phenomenology

Golec-Biernat and Wüsthoff model (Many refinements)

$$F_2 = \frac{Q^2}{8\pi^2 \alpha_{em}} \int d^2x_\perp \int_0^1 dz \sum_f e_f^2 |\psi_{Tf}(x_\perp, z, Q)|^2 \underbrace{\sigma_0 2(1 - e^{-x_\perp^2 \bar{Q}_s^2/4})}_{\sigma_{dip\text{-}prot}}$$



$$F_2^D = \frac{Q^2}{8\pi^2 \alpha_{em}} \int d^2x_\perp \int dz \sum_f e_f^2 |\psi_{Tf}(x_\perp, z, Q)|^2 \underbrace{\sigma_0 (1 - e^{-x_\perp^2 \bar{Q}_s^2/4})^2}_{\sigma^{el}_{dip\text{-}prot}} + q\bar{q}g \text{ part}$$

Impact parameter averaged picture

F_2^D quadratically divergent in perturbation theory.
Divergences eliminated by $e^{-x_\perp^2 \bar{Q}_s^2/4} \xrightarrow[x_\perp \to \infty]{} 0$.

3-parameters

$\underline{\sigma_0} = 23\,mb$ $\quad \bar{Q}_s^2 = \left(\frac{x_0}{x}\right)^\lambda$ in GeV^2 $\quad x_0 = 3 \times 10^{-4}$, $\lambda = 0.3$

$$Q_s^2 = 9/4\, \bar{Q}_s^2 \simeq 2\text{-}4\ GeV^2 \text{ in HERA regime}$$

(A bit too big; see refinements)

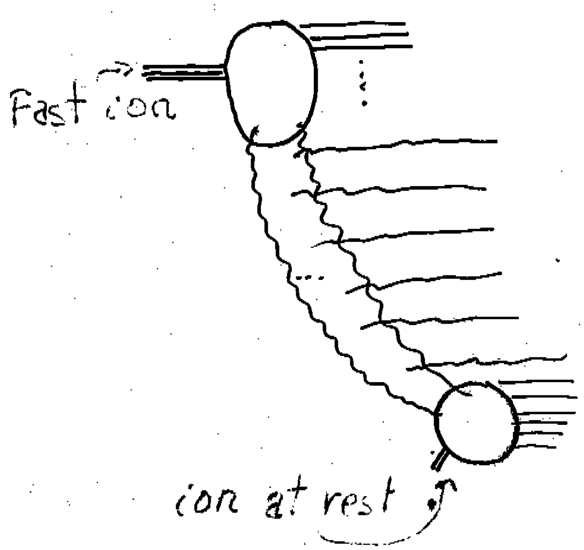
G-B W model gives impressive fits to HERA data in small x and moderate Q^2 region.

5. Heavy Ion Collisions

Because of Lorentz contraction, gluon and sea quark densities and occupation numbers are proportional to $A^{1/3}$. (Recall, there is only one layer of partons, in the longitudinal direction at small x.)

5.1 Gluon production

View in rest frame of one ion



Light cone gauge picture suggests produced gluons are, roughly, gluons in the light cone wavefunction which are "freed" during the collision.

Since occupation numbers in the wavefunction are large classical Yang-Mills equations should be valid.

So far no complete analytic calculation has been. Should be possible?

Numerical simulations of classical Y-M theory by Krasnitz, Nara, Venugopalan and by Lappi.

Wave function just before collision.

$$\frac{dN_g}{dy\, d^2k_\perp d^2b_\perp} = \frac{N_c^2-1}{4\pi^3}\frac{1}{\alpha N_c}\int_1^\infty \frac{dt}{t^2} e^{-t k_\perp^2/Q_s^2}$$

$\langle k_\perp \rangle \simeq 0.6\, Q_s$

At time gluons are freed, $\tau \simeq 3/Q_s$

$$\frac{dN_g}{dy\, d^2k_\perp d^2b_\perp} = \frac{N_c^2-1}{4\pi^3}\frac{1}{\alpha N_c}\frac{a_1}{e^{k_\perp/T_{eff}}-1}$$

$T_{eff} \simeq 0.52\, Q_s$

$\langle k_\perp \rangle \simeq 0.75\, Q_s$

Freeing fraction $\simeq \frac{1}{2}$ (Light cone wave function picture ok)

5.2 Toward equilibration

Accurate numerical calculations from first principles are very difficult. ($Q_s \simeq 1\,GeV$)

For large Q_s dynamics of approach to equilibrium (Quark Gluon Plasma) understood, parametrically (Baier, Schiff, ...)

Interesting question of instability in the early-time pre-equilibrium gluon system. (Arnold, Lenaghan, Moore)

Great laboratory for non-equilibrium QCD!

5.3 High-$p_\perp$ hadrons at RHIC

At $y=0$

$$R_{AA} = \frac{p_\perp^2 \frac{dN_{AA\to h}}{dp_\perp^2}}{N_{col}\, p_\perp^2 \frac{dN_{pp\to h}}{dp_\perp^2}} \simeq 0.2$$

while $\qquad p_\perp \simeq$ A Few GeV up to $\simeq 10$ GeV

$$R_{dA} = \frac{p_\perp^2 \frac{dN_{dA\to h}}{dp_\perp^2}}{N_{col}\, p_\perp^2 \frac{dN_{pp\to h}}{dp_\perp^2}} \simeq 1$$

Attributed to jet energy loss arguing for high density matter produced in A-A collision.

BRAHMS

But at $y \simeq 3$ R_{dA} is much less than 1.

Suggests gluon shadowing in the $x \simeq 10^{-3}$ regime.

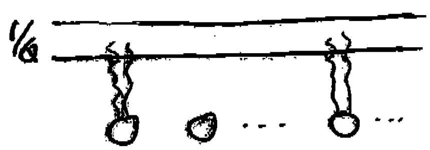
Evidence for Color Glass Condensate - Saturation?

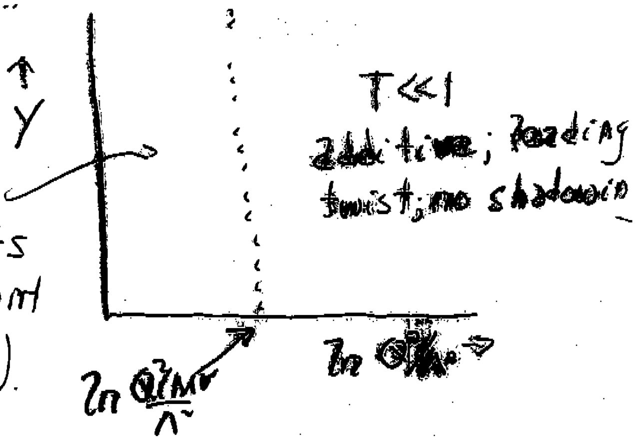
Unfortunately no e-A data in this regime!

5.4 QCD evolution and leading twist shadowing (Relation to QCD evolved saturation)

5.4.1 McLerran-Venugopalan model

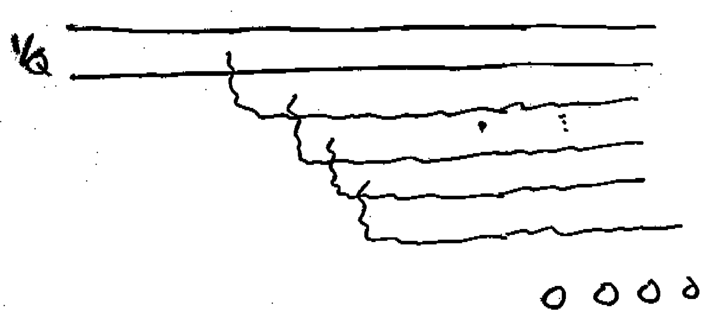
no leading twist shadowing $\Leftrightarrow$ no evolution

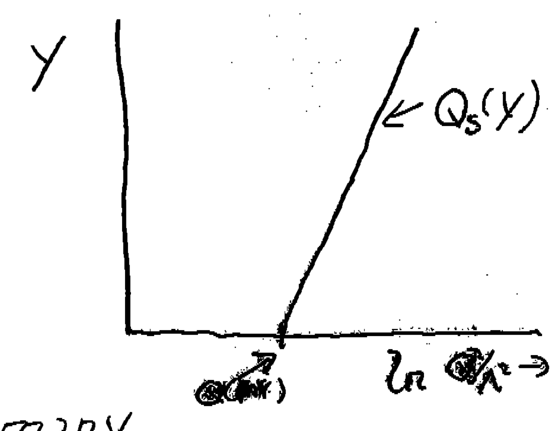
$T_{MV}(b,Q,Y) =$  $\simeq 1 - e^{-Q^2_{MV}/Q^2}$



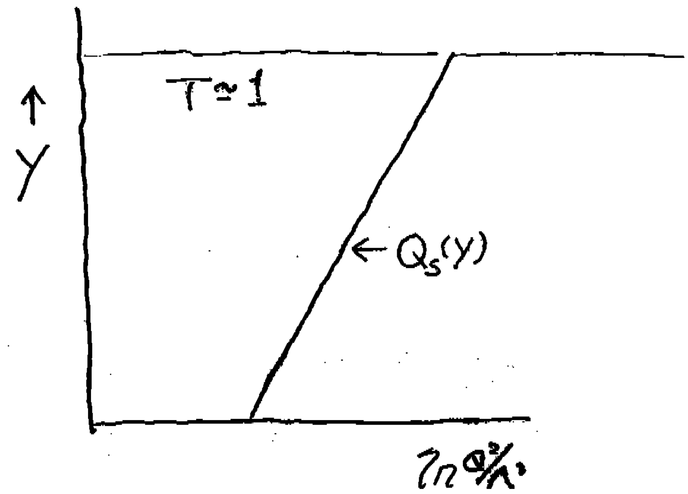
Sharp transition between complete shadowing (all twists) and no shadowing (leading twist).

5.4.2 Evolution and leading twist shadowing





Evolved 1/Q-dipole has many gluons for large Y. Scattering on nucleus clearly larger



When $Q/Q_s > 1$

$$T(b, Q^2, Q_s^2(MV), Y) = \left(\frac{Q_s^2(Y)}{Q^2}\right)^{1-\lambda(Q,Y)}$$

$$Q_s^2(Y) = Q_s^2(MV)\, f(Y)$$

IF Q-dependence of λ is weak

$$-Q^2 \frac{\partial}{\partial Q^2} T = (1-\lambda)\, T$$

$$T \sim (A^{1/3})^{1-\lambda}$$

Q-dependence and A-dependence <u>strongly</u> related

(i) Q/Q_s very large. DGLAP evolution dominates. anomalous dimension, λ, is small. Little or no shadowing.

(ii) $Q/Q_s > 1$ but moderate. BFKL evolution dominates. $\lambda \simeq \lambda_0 = 0.372$ giving significant anomalous dimension. A-dependence is $(A^{1/3})^{1-\lambda_0}$ showing shadowing determined by anomalous dimension.

6. The role of the EIC

EIC studies a physics region similar to HERA and RHIC. (Roughly a trade-off of energy vs nuclear enhancement in comparing HERA and EIC.)

RHIC produces dense matter. Probes are very difficult. EIC will give excellent probes of high density ion wave function.

Theory role crucial in connecting heavy ion and electron scattering physics.

Dense QCD matter is genuinely new domain.

In the next 10 years high density QCD, stimulated by heavy ion physics, should have become much more quantitative. Precise data on F_2, F_L, vector meson production etc. will provide tests and new input for our understanding.