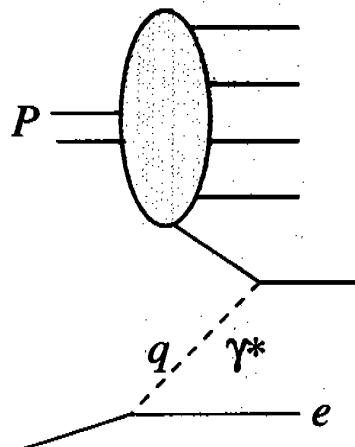


Inclusive signatures of the color glass condensate in ep and eA collisions

Kirill Tuchin (BNL)

Deep Inelastic Scattering in the Breit frame



$$P = (p + \frac{q^2}{2p}, 0, 0, p)$$

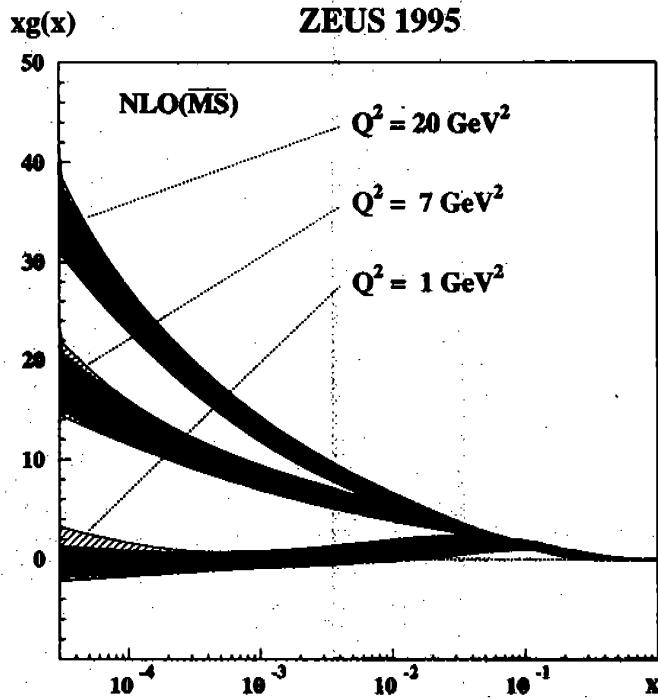
$$q = (0, 0, 0, -Q)$$

$$\alpha = -\frac{q^2}{2(pq)} = \frac{Q}{p} \ll 1$$

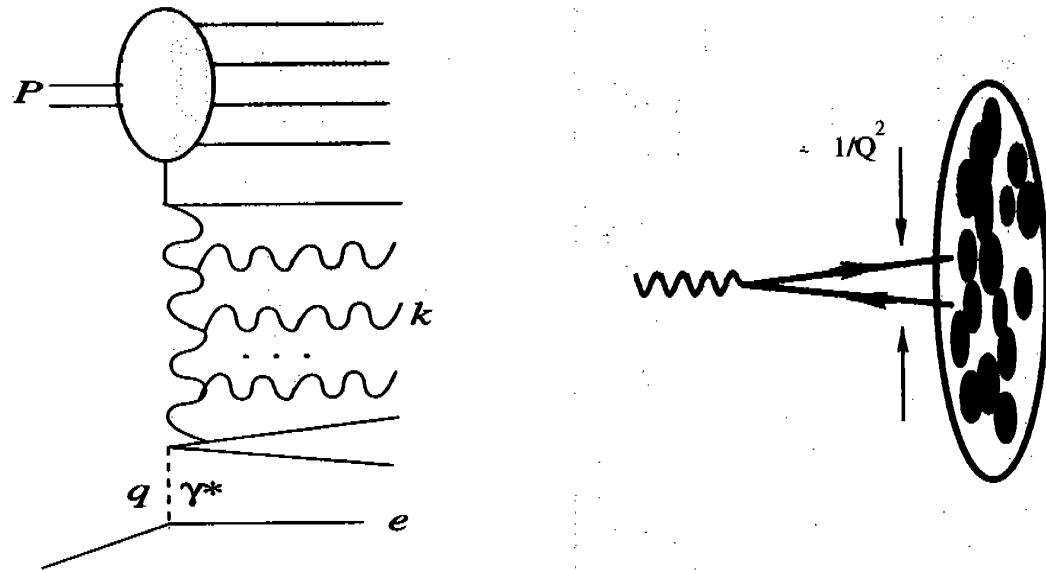
The typical time of interaction is $\tau_{\text{int}} \sim 1/q_z = 1/Q$.

Lifetime of a parton $\tau_{\text{part}} \sim k_+/m_{\perp}^2$. Since $k_z = x p_z \sim q_z$ we have $\tau_{\text{part}} \sim Q/m_{\perp}^2$.

Thus $\tau_{\text{part}} \gg \tau_{\text{int}}$ — photon absorbed by a quark over short period of time \Rightarrow use photon as a microscope.



Saturation at low x



Consider $\alpha_s \ln(1/x) \sim 1$. Radiation of one gluon:

$$\underbrace{\alpha_s}_{\text{strength}} \underbrace{\int \frac{dz_i}{z_i} \int dk_{\perp i}^2}_{\text{phase space}} \underbrace{\frac{k_{\perp i}^2}{k_{\perp i}^4}}_{\text{dynamics}} \sim \alpha_s \ln(1/x) \ln Q^2$$

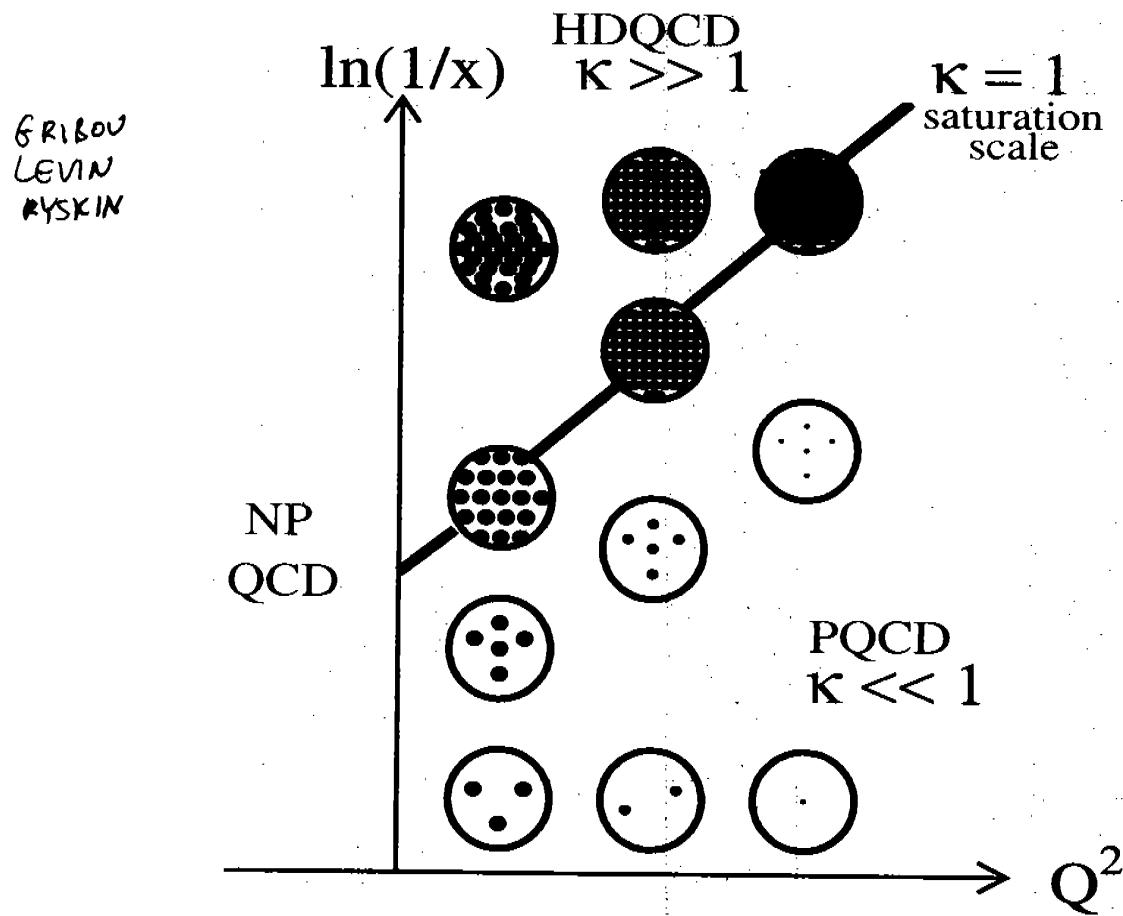
Sum over all radiated gluons:

$$xG(x, Q^2) \sim \sum_n \frac{1}{(n!)^2} (\alpha_s \ln(1/x) \ln Q^2)^n \sim e^{2\sqrt{\alpha_s \ln(1/x) \ln Q^2}}$$

The number of gluons per unit area $xG(x, Q^2)/\pi R^2$ rapidly increases.

The map of QCD

The packing factor : $\kappa = \frac{\pi A \alpha G_N(x, r_\perp^2) \alpha_s \pi r_\perp^2}{2 N_c \pi R_A^2}$



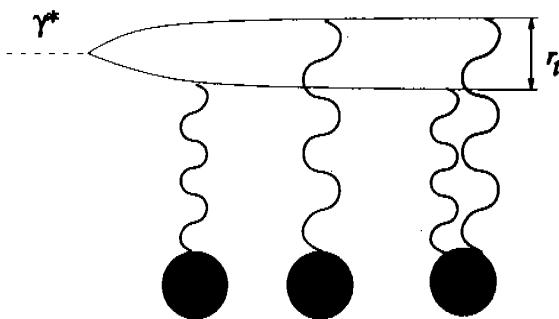
High Density QCD = Color Glass Condensate

$$N(x, Q^2) = \kappa(x, Q^2) \text{ when } N, \kappa \ll 1$$

At HERA: $Q_s^2(x) \simeq (3 \cdot 10^{-4}/x)^{0.288} \text{ GeV}^2$. Golec-Biernat
Wüsthoff

At EIC: $Q_s^2(\text{EIC}) = A^{1/3} Q_s^2(\text{HERA})$

DIS in the target rest frame



$$\mathbf{q} = (\sqrt{\mathbf{q}^2 - Q^2}, 0, 0, -\mathbf{q})$$

$$\mathbf{p} = (M, 0, 0, 0)$$

$$\mathbf{q} \gg \mathbf{Q}$$

Quasi-classical regime:

$$x \ll 1, \alpha_s \ll 1$$

$$\alpha_s \ln(1/x) < 1$$

The life-time of the $q\bar{q}$ state is $\tau_{q\bar{q}} \sim \frac{1}{E_{q\bar{q}} - E_{\gamma^*}} \sim \frac{1}{Mx} \gg \tau_{\text{int}}$

In the rest frame of the nucleus the total cross section for the DIS is

$$\sigma_{\text{tot}}(x, Q^2) = \int \frac{d^2 r_\perp}{2\pi} dz \Phi^{\gamma^*}(r_\perp, z, Q^2) \hat{\sigma}(x, r_\perp)$$

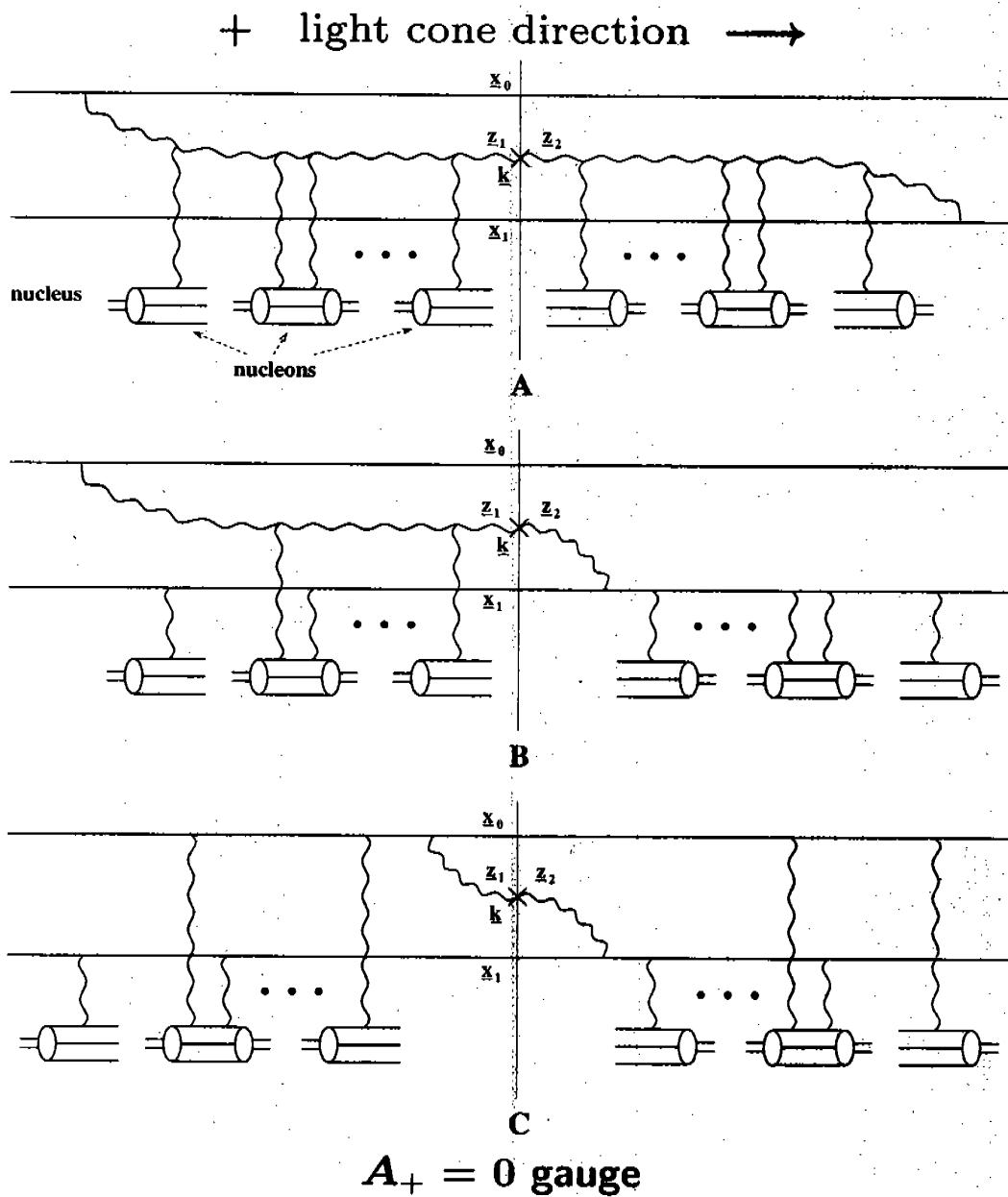
where Φ stands for the wave function of the $q\bar{q}$. Interaction of the color dipole with target is given by

$$\hat{\sigma}(x, r_\perp) = 2 \int d^2 b_\perp \text{Im} N(x, r_\perp, b_\perp)$$

$N(x, r_\perp, b_\perp)$ is the forward scattering amplitude of $q\bar{q}$ off the target at given impact parameter b_\perp .

Gluon production in quasi-classics (Mueller,Kovchegov, 1998, 2001)

Quasi-classical approximation: each rescattering happens via one or two gluon exchange with nucleon.



Gluon production in quasi-classics

$$\frac{d\sigma^{\gamma^* A}}{d^2 k \, dy} = \int \frac{d^2 \underline{x}_{01}}{2\pi^2} d\alpha \, \Phi^{\gamma^*}(\underline{x}_{01}, \alpha, Q^2) \, \frac{d\hat{\sigma}(\underline{x}_{01})}{d^2 k \, dy}$$

Scattering of $q\bar{q}$ (\underline{x}_{01}) + emitted G (\underline{z}_1 or \underline{z}_2) off the target:

$$\begin{aligned} \frac{d\hat{\sigma}(\underline{x}_{01})}{d^2 k \, dy} &= \frac{\alpha_s C_F}{\pi^2} \frac{1}{(2\pi)^2} \int d^2 b \, d^2 z_1 \, d^2 z_2 \, e^{-ik \cdot (z_1 - z_2)} \\ &\times \sum_{i,j=0}^1 (-1)^{i+j} \frac{\underline{z}_1 - \underline{x}_i}{|\underline{z}_1 - \underline{x}_i|^2} \cdot \frac{\underline{z}_2 - \underline{x}_j}{|\underline{z}_2 - \underline{x}_j|^2} \left(e^{-\frac{1}{4}(\underline{x}_i - \underline{x}_j)^2 Q_{0g}^2} \right. \\ &\quad \left. - e^{-\frac{1}{4}(\underline{z}_1 - \underline{x}_j)^2 Q_{0g}^2} - e^{-\frac{1}{4}(\underline{z}_2 - \underline{x}_i)^2 Q_{0g}^2} + e^{-\frac{1}{4}(\underline{z}_1 - \underline{z}_2)^2 Q_{0g}^2} \right) \end{aligned}$$

where

$$Q_{0\text{gluon}}^2 = 2 Q_{0\text{quark}}^2$$

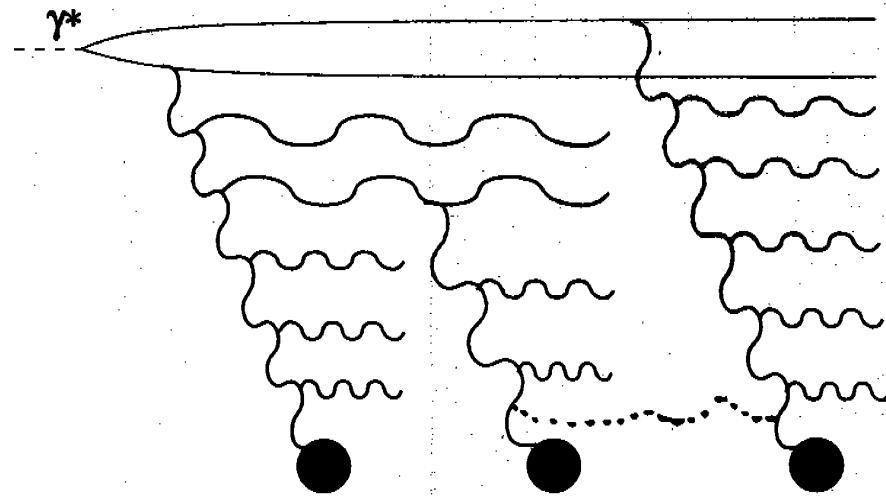
Origin of factor 2: $Q_{0q}^2 \sim 1/N_c$ while $Q_{0g}^2 \sim 2N_c/(N_c^2 - 1)$.

Alternatively, from the evolution equation:

- i. take initial condition for $q\bar{q}$ $N^{(0)} = 1 - e^{-\Omega_q}$ as a zeroth iteration of nonlinear equation,
- ii. calculate $N^{(1)} = \mathcal{K} \circ (2N^{(0)} - N^{(0)2}) = 1 - e^{-2\Omega_q}$

Unitarity constraint in QCD

The very small x regime: $\alpha_s \ln(1/x) \sim 1$



Unitarity constraint:

$$2 \operatorname{Im} N(x, r_\perp, b_\perp) = |N|^2 + G^{\text{in}}$$

At high energy N is almost purely imaginary \Rightarrow

$$N = i \left(1 - e^{-\frac{\Omega}{2}}\right) \quad G^{\text{in}} = \left(1 - e^{-\Omega}\right)$$

This implies: $|N(x, r_\perp, b_\perp)| < 1$. Black disk regime is $|N(x, r_\perp, b_\perp)| = 1$ in which case $\hat{\sigma} = 2\pi R^2$.

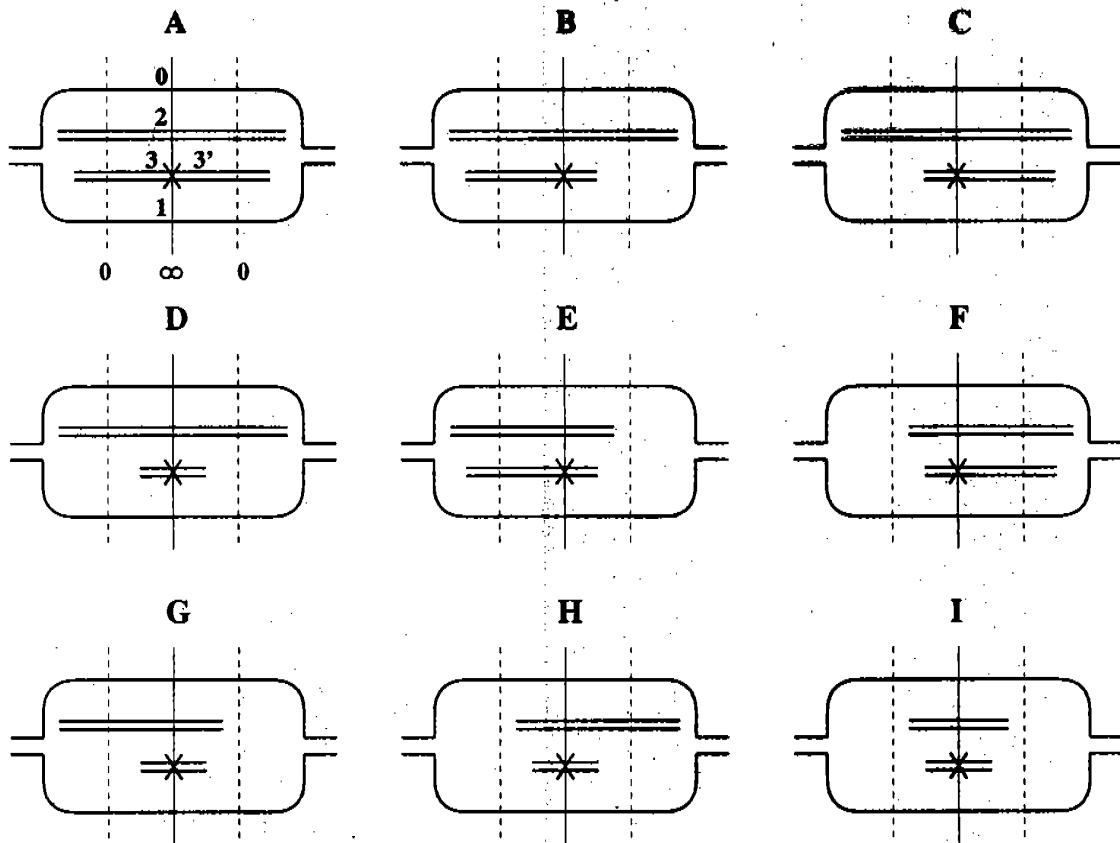
Nonlinear evolution can arise from

interaction of two cascades with the target $\sim \alpha_s^2 A^{1/3} \sim 1$

interactions of partons in the cascade $\sim \alpha_s^2$

Emission of faster gluons

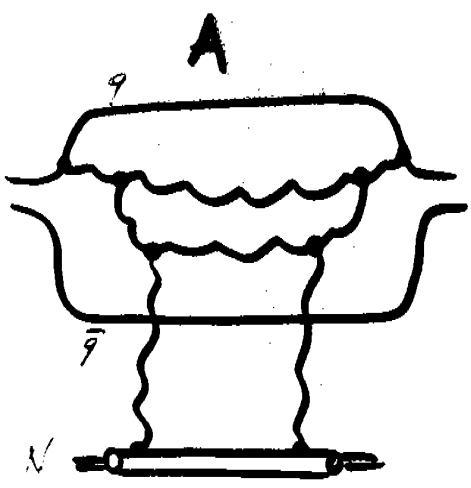
Evolution preceding emission of an inclusive gluon consists of gluons emitted before interaction both in amplitude and in CC amplitude



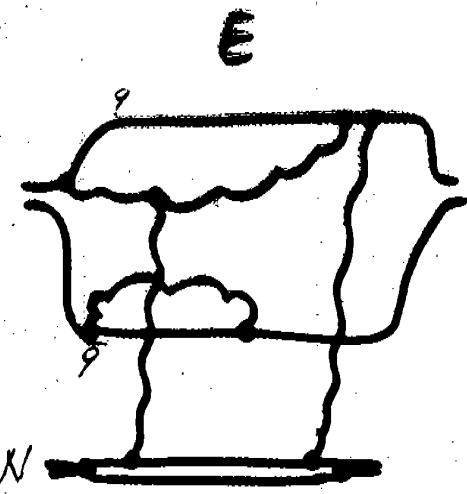
Graphs E-I do not have $\ln s$ enhancement, they are suppressed by α_s . Graphs A-D contribute to the real part of the evolution kernel.

Evolution is linear:

- simultaneous evolution happens in different dipoles isolated in large N_c
- all interactions in an isolated dipole cancel out (real-virtual cancellations): probability conservation

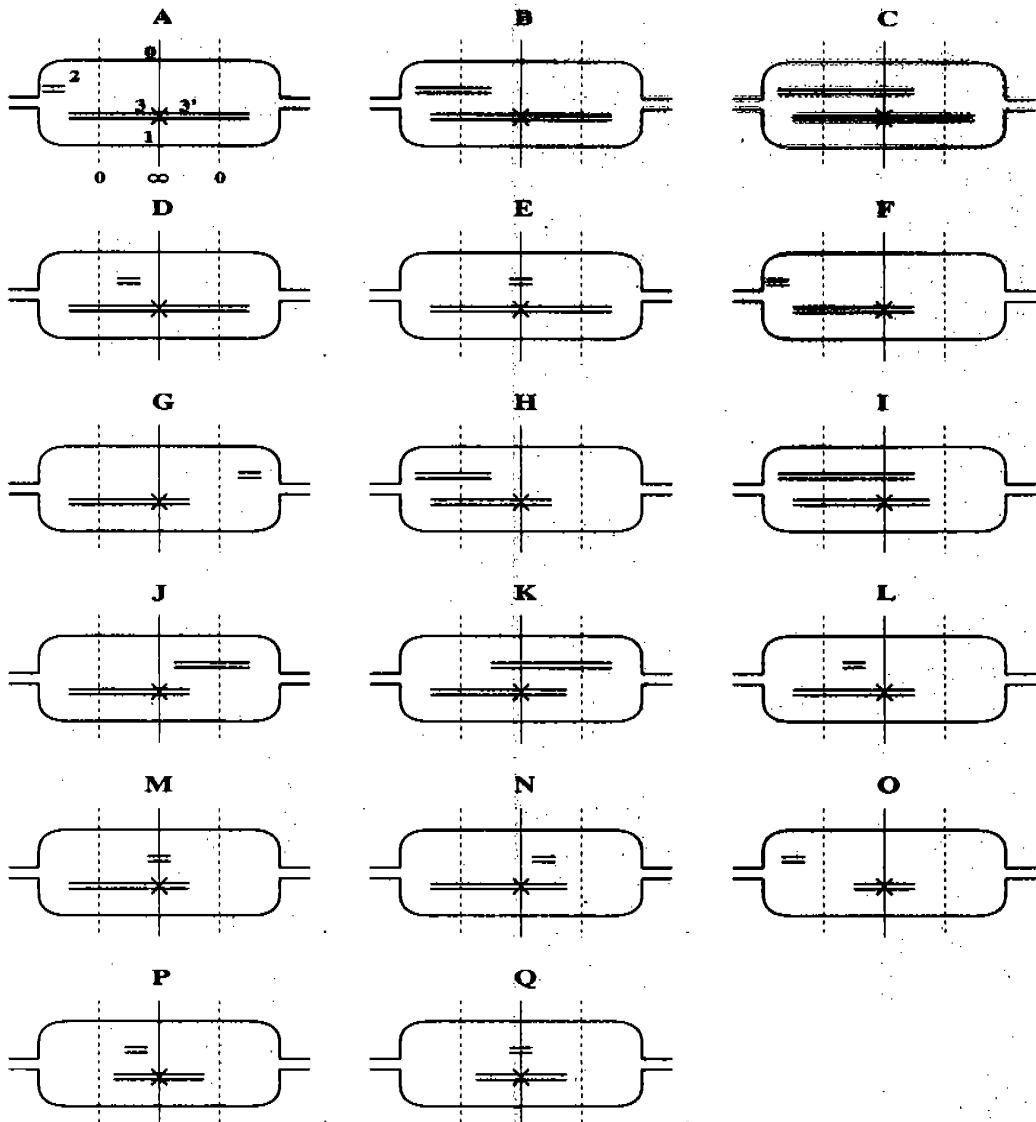


ENHANCED



SUB LEADING

Emission of faster gluons

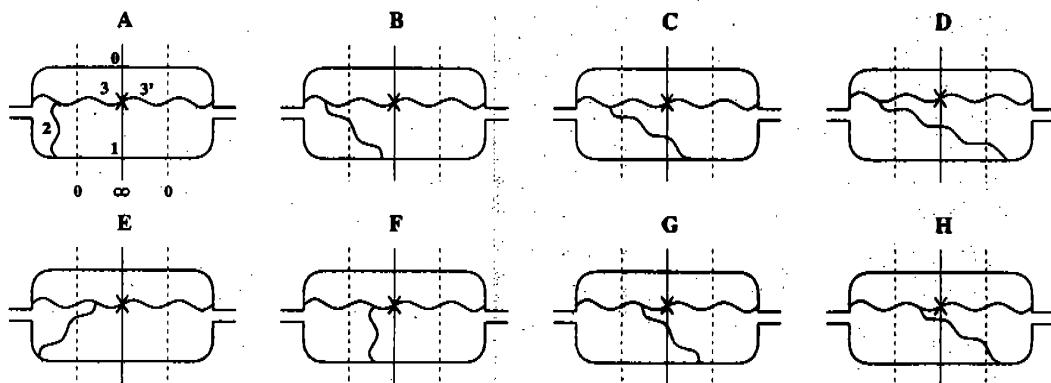


Graphs A,F,G,O + CC contribute to the virtual part of the evolution kernel. Other graphs + CC cancel: cancellation of final state interactions.

Emission of slower gluons

Production of inclusive gluon at \underline{z}_1 in the amplitude and \underline{z}_2 in the CC \Rightarrow effective scattering of 4 dipoles $\underline{z}_1 - \underline{z}_2$, $\underline{z}_1 - \underline{x}_1$, $\underline{z}_2 - \underline{x}_0$ and $\underline{x}_1 - \underline{x}_0$. Evolution in each one is independent on evolution in others.

Consider $\underline{z}_1 - \underline{z}_2 = 33'$



$$A + D = B + C = F + G = E + H = 0 \Rightarrow$$

$$1 - e^{-\frac{1}{4}x^2 Q_{0g}^2(b)} \rightarrow N_G(\underline{x}, \underline{b}, y)$$

Evolution equation for the forward gluon dipole scattering amplitude N_G can be found from unitarity constraint

$$N_G(\underline{x}, \underline{b}, y) = 2 N(\underline{x}, \underline{b}, y) - N^2(\underline{x}, \underline{b}, y)$$

The main result (Yu. Kovchegov and K.T.)

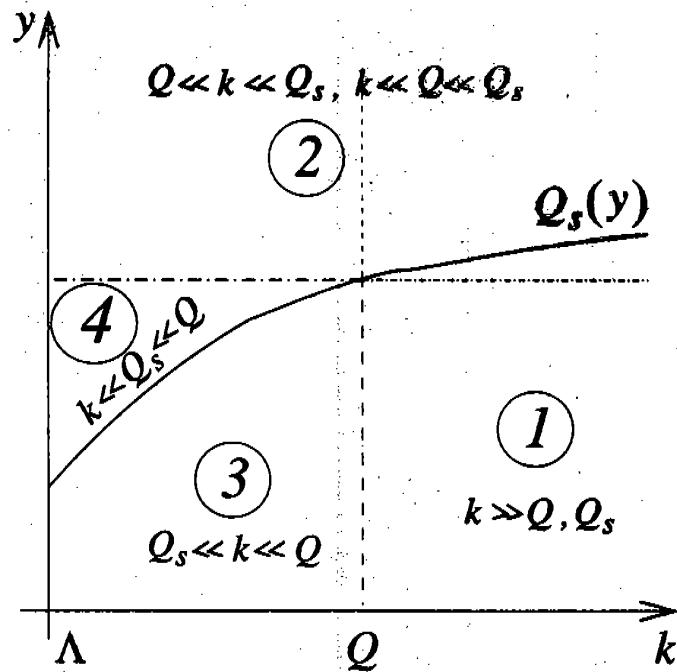
$$\frac{d\sigma^{\gamma^* A \rightarrow q\bar{q}G X}}{d^2 k \, dy} = \frac{1}{2\pi^2} \int d^2 r \, d\alpha \, \Phi^{\gamma^*}(r, \alpha) \frac{2\alpha_s}{C_F k^2} \int d^2 q \, \varphi_0(\underline{q}, \underline{r}, Y - y) \, \varphi_A(\underline{k} - \underline{q}, y)$$

where the unintegrated gluon distributions are defined as

$$\varphi(\underline{k}, y) = \frac{C_F}{\alpha_s(2\pi)^3} \int d^2 b \, d^2 r \, e^{-i\underline{k}\cdot\underline{r}} \nabla_r^2 N_G(\underline{r}, \underline{b}, y)$$

here N_G is the forward scattering amplitude of gluon dipole of size \underline{r} satisfying the nonlinear evolution equation (Balitsky, Kovchegov)

Inclusive spectrum in asymptotic regions



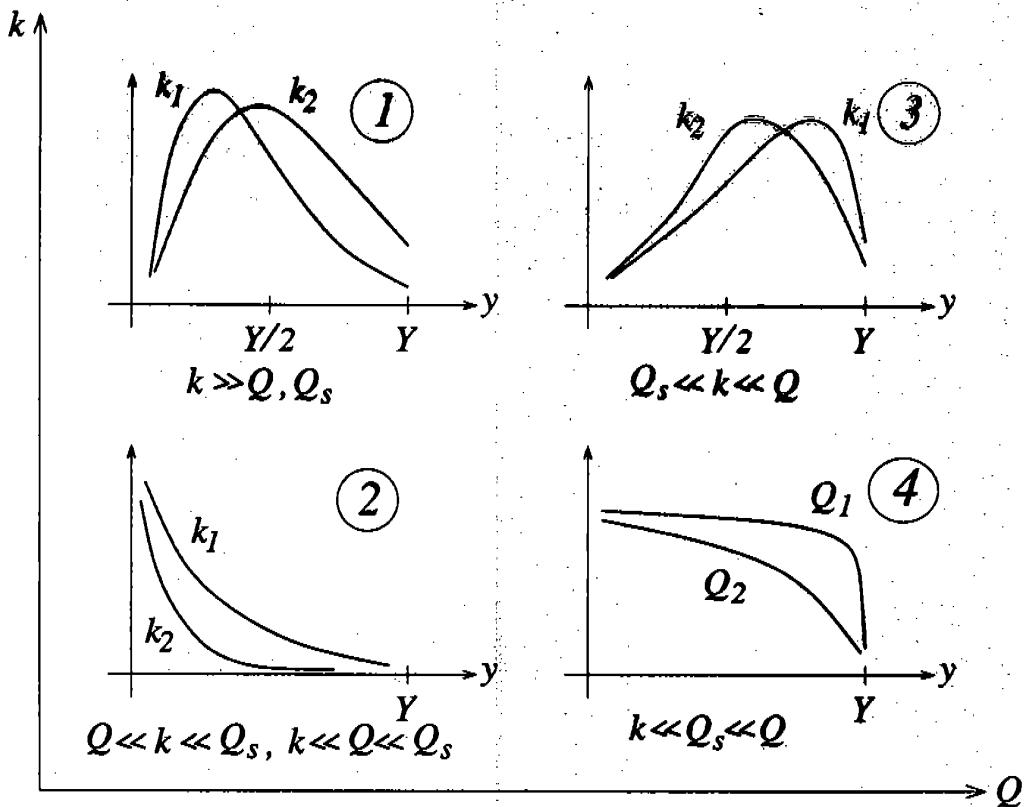
$$\frac{d\hat{\sigma}}{d^2k dy} \Big|_1 \propto \frac{\bar{\alpha}_s Q_{0g}^2 R^2}{\underline{k}^4} e^{2\sqrt{\bar{\alpha}_s(Y-y)\ln(k^2 r^2)} + 2\sqrt{\bar{\alpha}_s y \ln(k^2/Q_{0g}^2)}}$$

$$\frac{d\hat{\sigma}}{d^2k dy} \Big|_3 \propto \frac{\bar{\alpha}_s Q_{0g}^2 R^2}{\underline{k}^2} r^2 e^{2\sqrt{\bar{\alpha}_s(Y-y)\ln(1/k^2 r^2)} + 2\sqrt{\bar{\alpha}_s y \ln(k^2/Q_{0g}^2)}}$$

$$\frac{d\hat{\sigma}}{d^2k dy} \Big|_2 \propto \frac{\bar{\alpha}_s R^2}{\underline{k}^2} \ln \frac{Q_g^2(y)}{\underline{k}^2} e^{2\sqrt{\bar{\alpha}_s(Y-y)\ln(k^2 r^2)}}$$

$$\frac{d\hat{\sigma}}{d^2k dy} \Big|_4 \propto \frac{\bar{\alpha}_s Q_s^2(y) R^2}{\underline{k}^2} r^2 e^{2\sqrt{\bar{\alpha}_s(Y-y)\ln(1/Q_s^2(y)r^2)}}$$

Inclusive spectrum in asymptotic regions. Energy dependence.



In the perturbative regions 1 and 3 where gluon momentum k is the largest momentum scale in the problem, inclusive spectrum exhibits maximum as a function of rapidity. Position of maximum is

$$y_0 = Y \frac{1}{1 \pm \frac{\ln^2(kr)}{\ln^2(k/Q_{0g})}} \xrightarrow{k \gg Q, Q_{0g}} \frac{1}{2}Y,$$

If the largest momentum scale in DIS is Q_s (region 2), the spectrum is decreasing function of Y .

The spectrum can take the step-like form if $k \ll Q_s \lesssim Q$

"Extended" Geometric scaling

(Levin, K.T.; Iancu, Itakura, McLerran).

Recall the solution to the DGLAP equation at low x ($\underline{k}^2 > Q_s^2$):

$$N(\underline{k}, x) \simeq \frac{Q_0^2}{\underline{k}^2} \exp \sqrt{4 \frac{\alpha_s N_c}{\pi} \ln(1/x) \ln \underline{k}^2/Q_0^2}$$

We can write identically

$$\ln \underline{k}^2/Q_0^2 \equiv \ln \underline{k}^2/Q_s^2 + \ln Q_s^2/Q_0^2$$

Consider the kinematic region

$$Q_s^2 \ll \underline{k}^2 \ll k_{\text{geom}}^2 = \frac{Q_s^4}{Q_0^2}$$

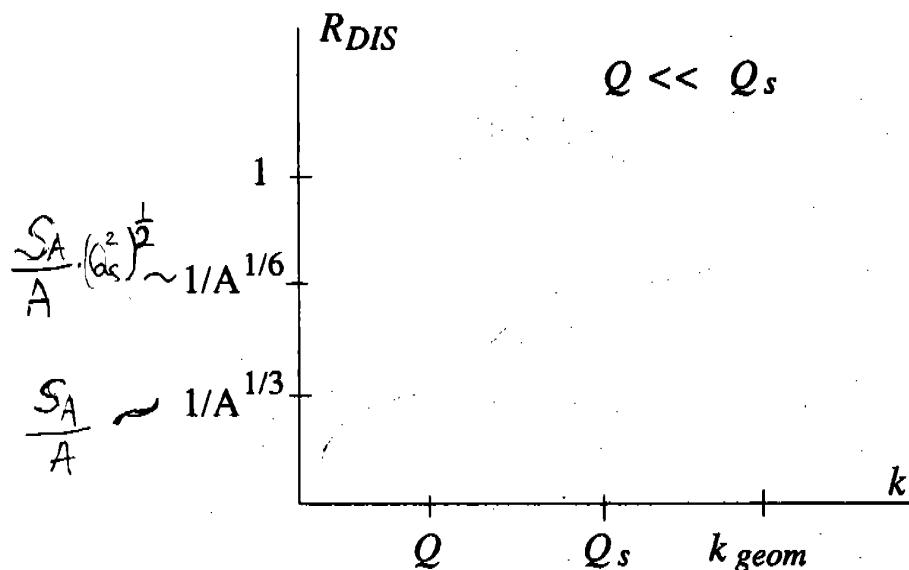
We can expand the solution as follows:

$$N(\underline{k}_\perp, x) \approx \left(\frac{Q_s^2(x)}{\underline{k}^2} \right)^{1/2} + \mathcal{O} \left(\frac{\underline{k}^2}{\tilde{Q}^2(x)} \right)$$

$N(x, \underline{k})$ is a function of only one variable $Q_s^2(y)/\underline{k}^2$!

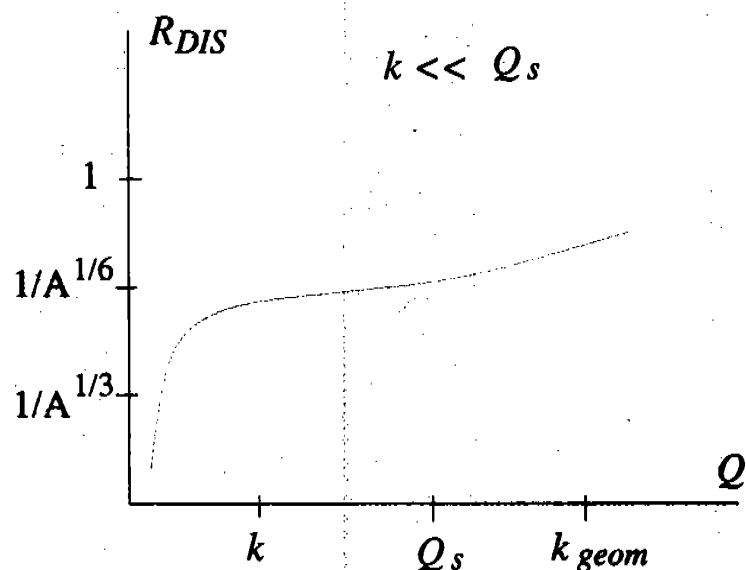
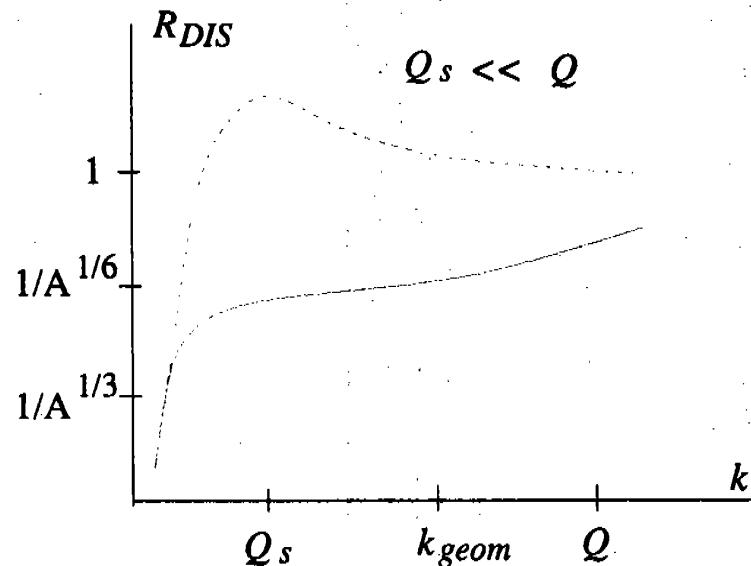
Nuclear Modification Factor in DIS

$$R_{\text{DIS}}(x, Q^2, \underline{k}^2) = \frac{\frac{d\sigma}{d^2 k d y}(\gamma^* A)}{A \frac{d\sigma}{d^2 k d y}(\gamma^* p)}$$



This picture assumes that $\log(k_{\text{geom}}^2/Q_s^2) \gg 1$ and $\log(Q_s^2/Q^2) \gg 1$. At EIC these strong inequalities will hardly hold. Therefore, the suppression curve will be much smoother.

Nuclear Modification Factor in DIS



Conclusion

DIS experiments at EIC will be able to provide much cleaner signatures of the Color Glass Condensate than pA or AA collisions