Emittance Compensation

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Emittance minimization in the RF photoinjector

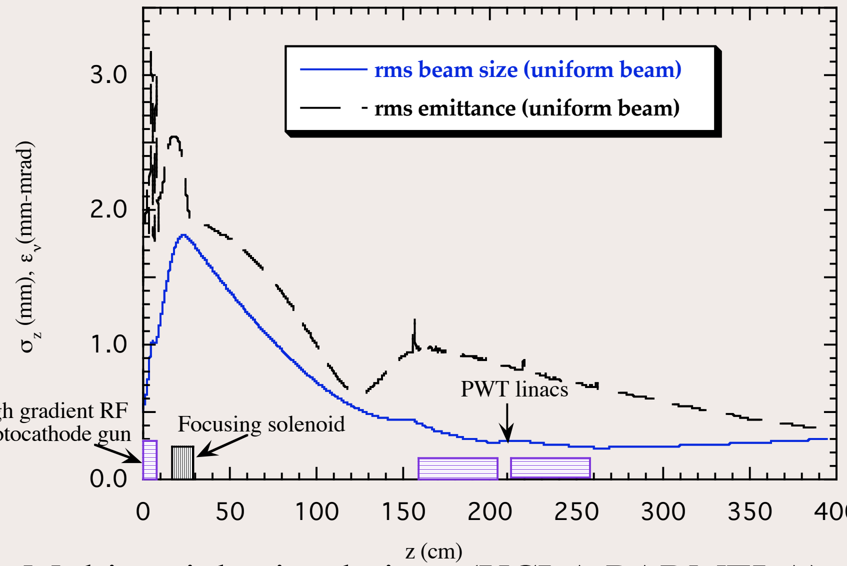
- Thermal emittance limit
  - Small transverse beam size
  - Avoid metal cathodes? $\varepsilon_{n,th} \approx \frac{1}{2}\sqrt{\frac{h\nu - W}{m_0 c^2}}\sigma_x \approx 5\times 10^{-4}\sigma_x(\mathrm{m})$
- RF emittance $\varepsilon_{n,RF} \approx k_{RF}\alpha_{RF}\left(k_{RF}\sigma_z\right)^2 \sigma_x{}^2$
  - Small beam dimensions
  - Small acceleration field? Maybe not…
- Space charge emittance
  - Original K.J.Kim treatment discouraging

$$\varepsilon_{n,sc} \approx \frac{m_e c^2}{(2\pi)^2 e E_0}\frac{10I}{I_0(1+\frac{3}{5}A)} \qquad A = \frac{\sigma_x}{\sigma_z},\ I_0 = \frac{ec}{r_e}$$

$$\varepsilon_{n,sc} \approx 5\ \text{mm-mrad}\ (I = 100\ \text{A},\ E_0 = 100\ \text{MV/m})$$

Space-charge emittance control: Emittance compensation

- Space-charge emittance evolution *not monotonic* in time
- Multiparticle simulations at LLNL (Carlsten) show emittance oscillations, minimization possible: *Emittance compensation*
- Analytical approach
- Scaling laws
- Prescriptions for design
  - LCLS (Ferrario WP)
  - Superconducting version?



Multiparticle simulations (UCLA PARMELA)
Showing emittance oscillations and minimization

Transverse dynamics model

- After initial acceleration, space-charge field is mainly transverse (beam is long in rest frame).
- Force dependent ~ exclusively on local value of current density I/σ^2 (electric field simply from Gauss' law)
- Linear component of self-force most important. *We initially assume that the beam is nearly uniform in* r.
- The linear "slice" model…
  - Extend linear model to include nonlinearities within slices?
- Scaling of design physics with respect to charge, λ_{rf}

The rms envelope equation

- The rms envelope equation for a *cylindrically symmetric, non-accelerating, space-charge dominated* beam

$$\sigma_x''(\zeta,z) + k_\beta^2 \sigma_r(\zeta,z) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,z)} + \cancel{\frac{\varepsilon_{n,x}^2}{\gamma 2 \sigma_x^3(\zeta,z)}} \qquad r_e \lambda(\zeta) = I(\zeta)/I_0$$

- Separate DE for each slice (tagged by ζ), $\zeta = z - v_b t$
- Each slice has different current $I(\zeta) = \lambda(\zeta) v$
- External focusing measured by *betatron wave-number*

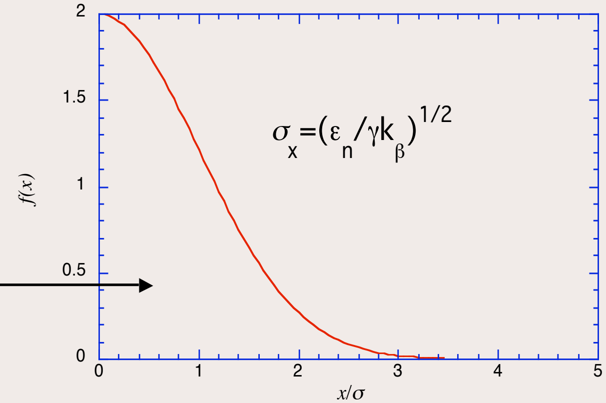
$$k_\beta = eB_z / 2p_0$$

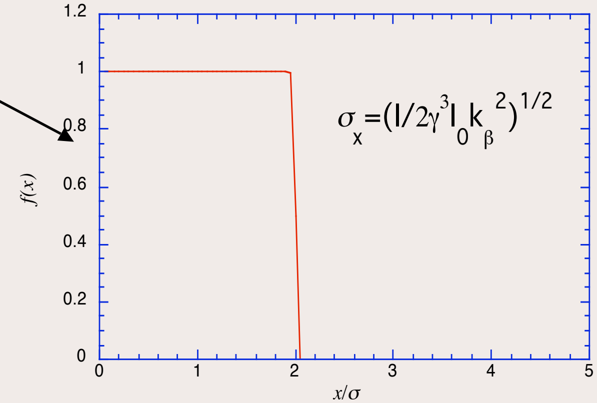
- Envelope coordinates are in Larmor frame
- Rigid rotator equilibrium (Brillouin flow) depends on *I*(ζ). "Pressure" forces negligible

$$\sigma_{eq}(\zeta) = \frac{1}{k_\beta}\sqrt{\frac{r_e \lambda(\zeta)}{2\gamma^3}}$$

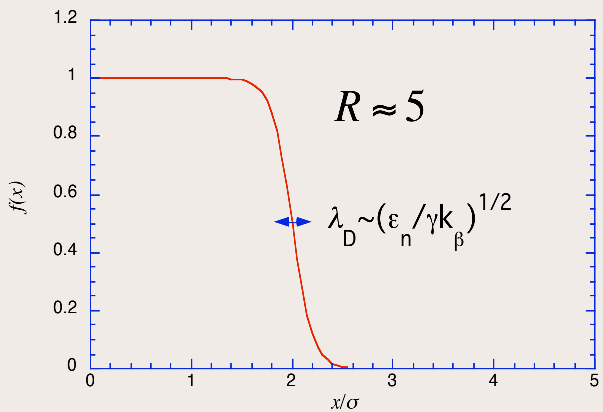
Equilibrium distributions and space charge dominated beams

- Maxwell-Vlasov equilibria have simple asymptotic forms, dependent on parameter
  $$R \equiv I/2\gamma^2 k_\beta \varepsilon_n I_0$$
- Emittance dominated gaussian $R \ll 1$
- Space-charge dominated uniform $R >> 1$





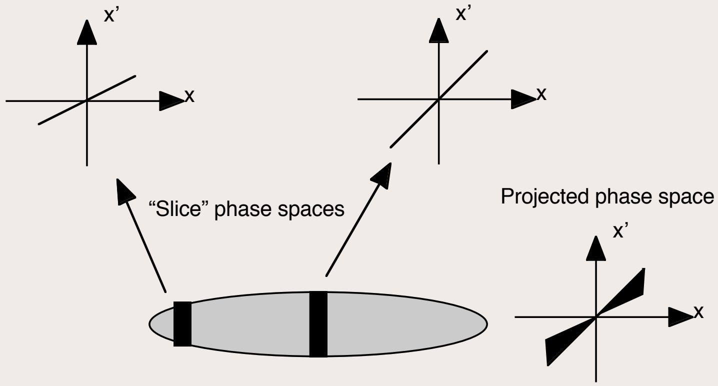
- Uniform beam approximation very useful



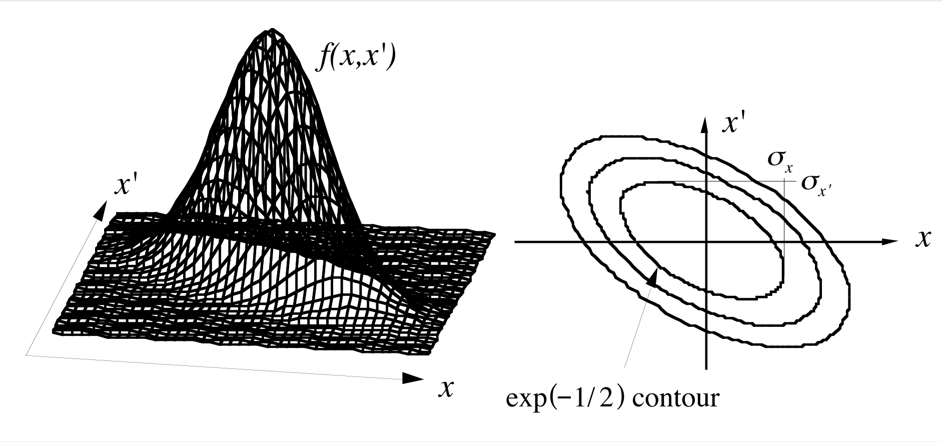
- Nominally uniform has Debye sheath
- High brightness photoinjector beams have
  $R > 1, \ \ \gamma \leq 250!$

The trace space model

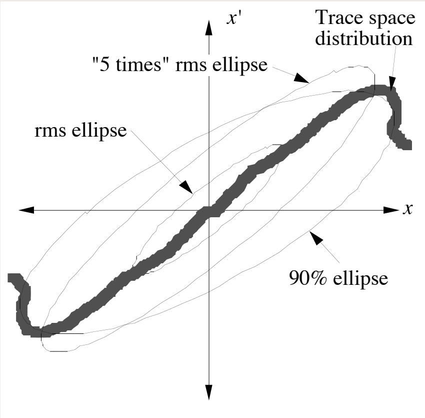
- Each ζ-slice component of the beam is a line in trace space.
- No thermal effects
- No nonlinearities (lines are straight!)



The model



Contrast with thermal trace space…



and nonlinear slice trace space

Envelope oscillations about equilibria

- Beam envelope is *non-equilibrium* problem
- Linearizing the rms envelope equation about equilibrium gives

$$\delta\sigma_x''(\zeta,z) + 2k_\beta^2\sigma_x(\zeta,z) = 0$$

Dependent on betatron wave-number, *not* local beam size or current

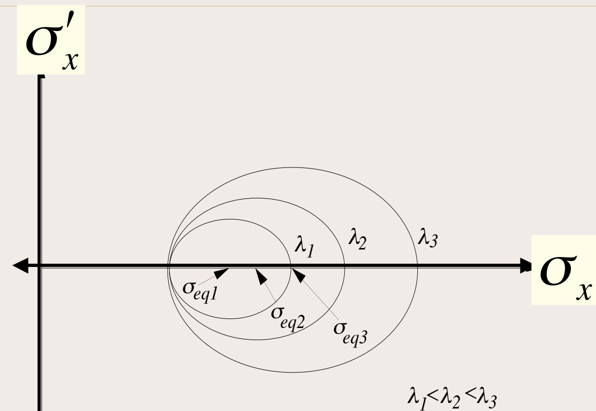
- Small amplitude envelope oscillations proceed at $2^{1/2}$ times the betatron frequency or *assuming uniform beam distribution*

$$k_{\text{env}} = \sqrt{2}k_\beta = \sqrt{\frac{4\pi r_e n_{b,eq}}{\gamma^3}} = k_p$$

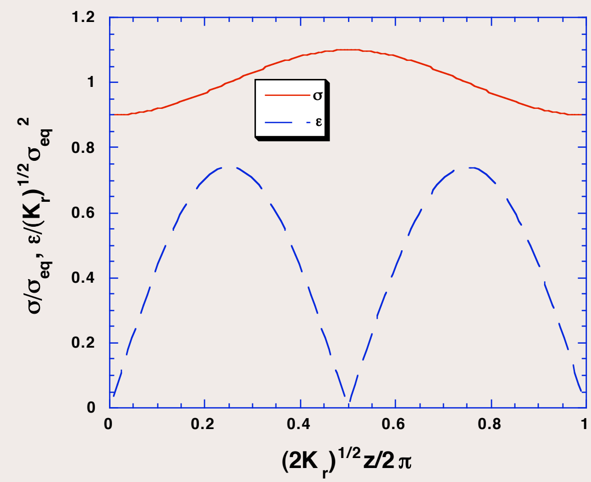
This is the *matched* relativistic plasma frequency

Phase space picture: coherent oscillations

- SC beam envelope oscillations proceed about
  - different equilibria,
  - with different amplitude
  - but at the *same frequency*
- Behavior leads to emittance oscillations…but not damping (yet)
- “1st compensation”, after gun, before linac…

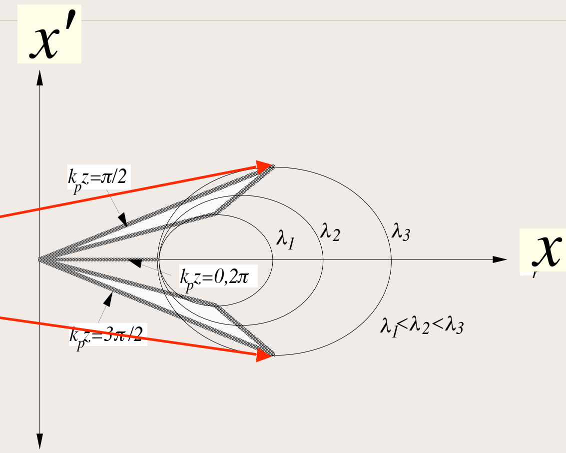


Assume that beam is launched at minimum (e.g. at cathode)



Small amplitude oscillation model

Phase space picture: coherent oscillations



- Emittance (area in phase space) is maximized at

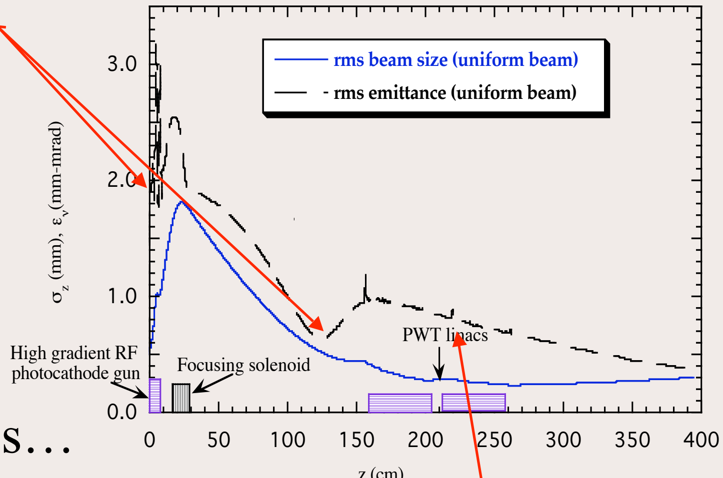
$$k_p z = \pi/2, 3\pi/2$$

- Emittance is locally minimized at

$$k_p z = 0, \pi, 2\pi$$

- the beam extrema!

- Fairly good agreement of simple model with much more complex beamline
- What about acceleration?
  – In the rf gun, in booster linacs…



Damping of e…

Emittance damping: Beam envelope dynamics under acceleration

- Envelope equation (w/o emittance), with acceleration, RF focusing

$$\boxed{\sigma_x''(\zeta,z)+\left(\frac{\gamma'}{\gamma(z)}\right)\sigma_x'(\zeta,z)+\frac{\eta}{8}\left(\frac{\gamma'}{\gamma(z)}\right)^2\sigma_x(\zeta,z)=\frac{r_e\lambda(\zeta)}{2\gamma(z)^3\sigma_x(\zeta,z)}}$$

$\eta \approx 1$ (rf or solenoid focusing)

$\gamma' = eE_0/m_0c^2$ (accel. "wavenumber")

- Particular solution - the "*invariant envelope*" (generalized Brillouin flow), slowly damping "fixed point"-analogue

$$\sigma_{inv}(\zeta,z)=\frac{1}{\gamma'}\sqrt{\frac{r_e\lambda(\zeta)}{(2+\eta)\gamma(z)}}\propto\gamma^{-1/2}$$

- Angle in phase space is *independent of current* $\theta=\frac{\sigma_{inv}'}{\sigma_{inv}}=-\frac{1}{2}\frac{\gamma'}{\gamma}$

- Corresponds *exactly* to entrance/exit kick (matching is naturally at waists)

$$\Delta x_{RF}'=-\frac{1}{2}\frac{\gamma'}{\gamma}x$$

- Matching beam to invariant envelope yields stable *linear* emittance compensation!

Envelope oscillations near invariant envelope, with acceleration

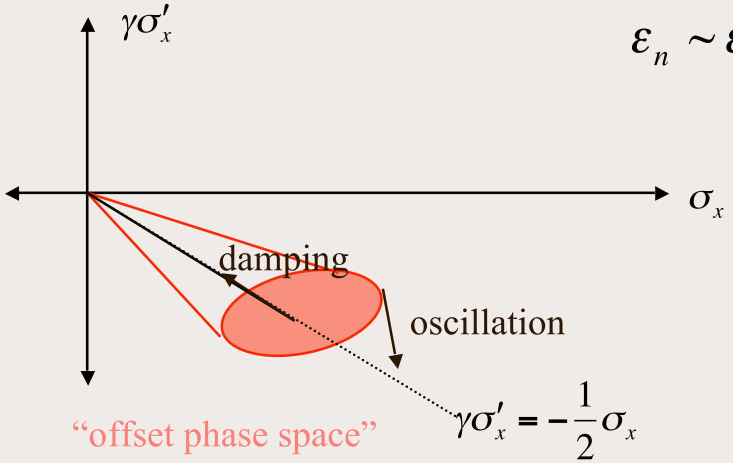
- Linearized envelope equation

$$\delta\sigma_x'' + \left(\frac{\gamma'}{\gamma}\right)\delta\sigma_x' + \frac{1+\eta}{4}\left(\frac{\gamma'}{\gamma}\right)^2 \delta\sigma_x = 0$$

- Homogenous solution (independent of current)

$$\delta\sigma_x = \left[\sigma_{x0} - \sigma_{inv}\right]\cos\left(\frac{\sqrt{1+\eta}}{2}\ln\left(\frac{\gamma(z)}{\gamma_0}\right)\right)$$

- Normalized, projected phase space area oscillates, secularly *damps* as offset phase space (conserved!) moves in…



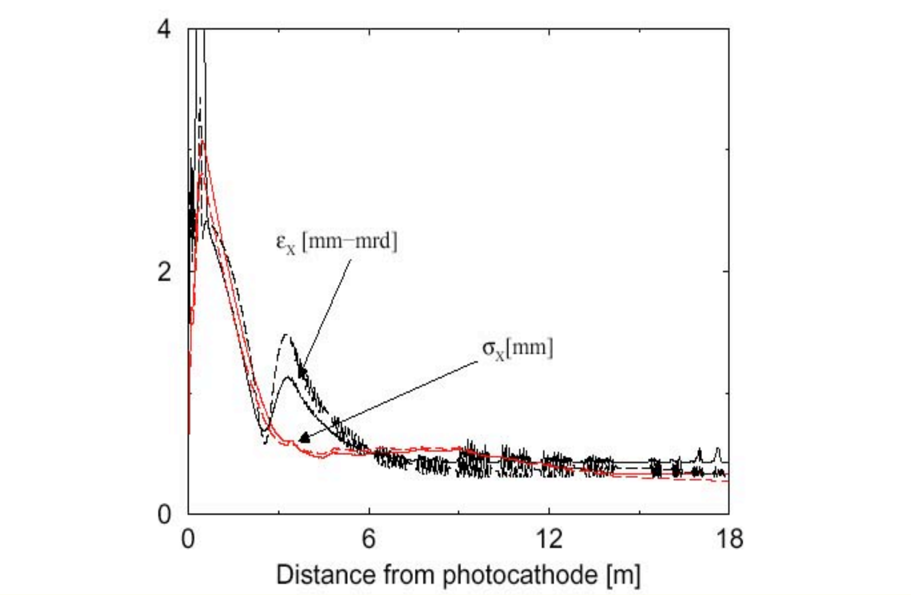
$$\varepsilon_n \sim \varepsilon_{offset}\sigma_{\text{inv}} \sim \gamma^{-1/2}$$

Oscillation (*generalized* matched plasma) frequency damps with energy

$$k_p = \frac{d}{dz}\left(\frac{\sqrt{1+\eta}}{2}\ln(\gamma(z))\right) = \frac{\sqrt{1+\eta}}{2}\frac{\gamma'}{\gamma}$$

Validation of linear emittance compensation theory

- Theory successfully describes "linear" emittance oscillations
  - "Slice" code (HOMDYN) developed that reproduce multiparticle simulations. Much faster! Ferrario will lecture on this..
  - LCLS photoinjector working point found with HOMDYN!



Dash: HOMDYN
Solid: PARMELA

Scaling in photoinjector design

- The envelope equation approach gives rise to powerful scaling laws
- RF acceleration also amenable to scaling
- Scale designs with respect to:
  - Charge (k_p)
  - RF wavelength (k_{RF})
- Change from low charge (FEL) to high charge (LC, wakefield driver) design
- Change RF frequency from one laboratory to another (*e.g.* SLAC X-band, SC L-band)

Charge scaling

- Keep all accelerator/focusing parameters identical
- Density and aspect ratio of the bunch must be preserved

$$\sigma_i \propto Q^{1/3}$$

- Contributions to emittance scale with powers of beam size
- Space-charge emittance $\varepsilon_{x,sc} \propto k_p^2 \sigma_x^2 \propto Q^{2/3}$
- RF/chromatic aberration emittance $\varepsilon_{x,RF} \propto \sigma_z^2 \sigma_x^2 \propto Q^{4/3}$
- Thermal emittance $\varepsilon_{x,th} \propto \sigma_x \propto Q^{1/3}$
- Beam is SC dominated, and emittancs do not affect the beam envelope evolution; compensation is preserved.

Wavelength scaling

- First, make acceleration dynamics scale: $\alpha_{RF} \propto E_0\lambda = \text{constant}$ and $\boxed{E_0 \propto \lambda^{-1}}$
- Focusing (betatron) wavenumbers must also scale (RF is naturally scaled, $\lambda_{\beta,rf} \propto E_0$). Solenoid field scales as $\boxed{B_0 \propto \lambda^{-1}.}$
- Correct scaling of beam size, and plasma frequency: $\sigma_i \propto \lambda$ $\boxed{Q \propto \lambda}$
- All emittances scale rigorously as $\boxed{\varepsilon_n \propto \lambda}$

Brightness, choice of charge and wavelength

- Charge and pulse length scale together as λ
- Brightness scales strongly with λ, $B_e = 2I/\varepsilon_n^2 \propto \lambda^{-2}$
- This implies low charge for high brightness
- What if you want to stay at a certain charge (e.g. FEL energy/pulse)
- Mixed scaling:

thermal charge RF/chromatic

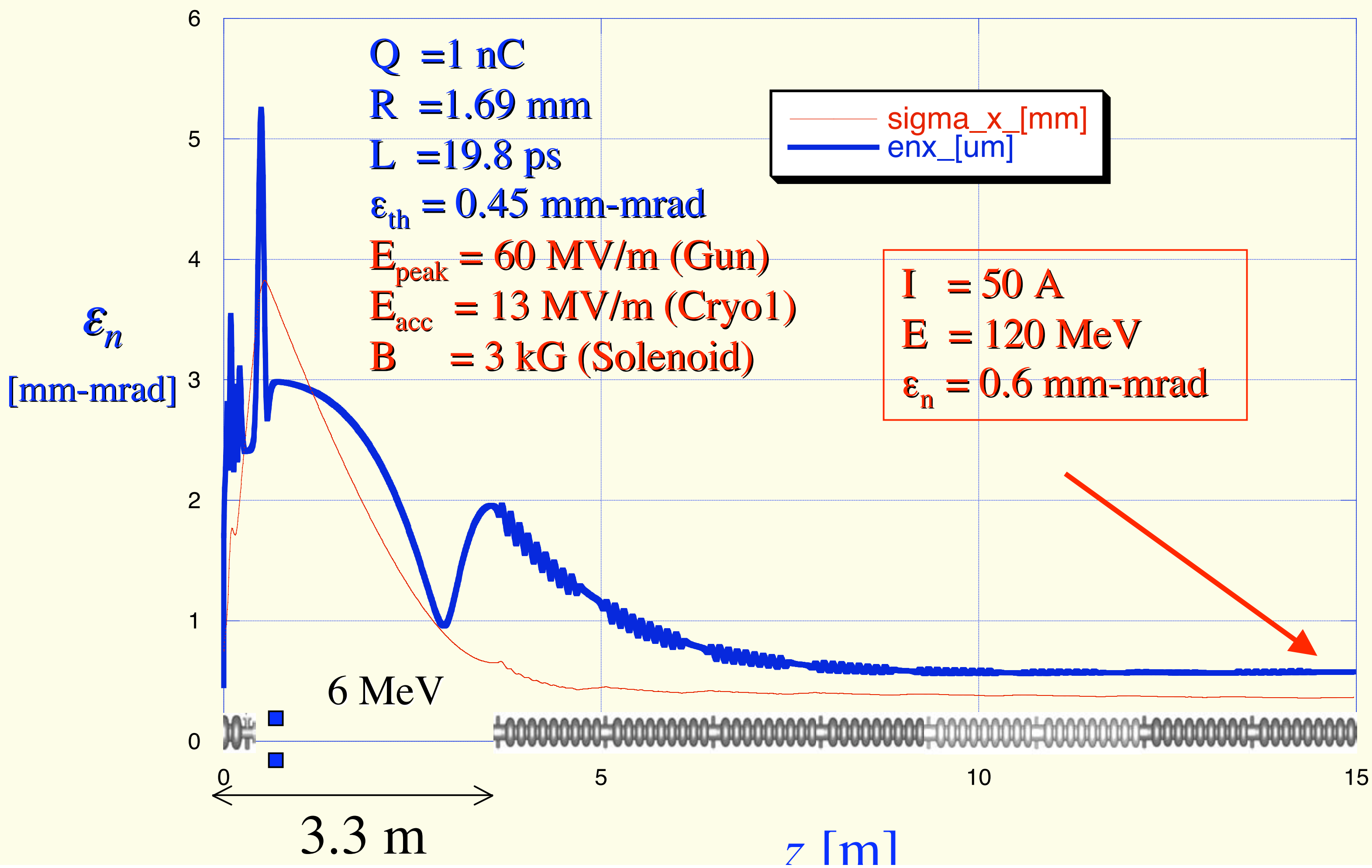
$$\varepsilon_n(\text{mm-mrad}) = \lambda_{rf}(\text{m})\sqrt{a_1\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{2/3} + a_2\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{4/3} + a_3\left(\frac{Q(\text{nC})}{\lambda_{rf}(\text{m})}\right)^{8/3}}$$

- For Ferrario scenario, constants from simulation:

$a_1 = 1.5 \qquad a_2 = 0.81 \qquad a_3 = 0.052$

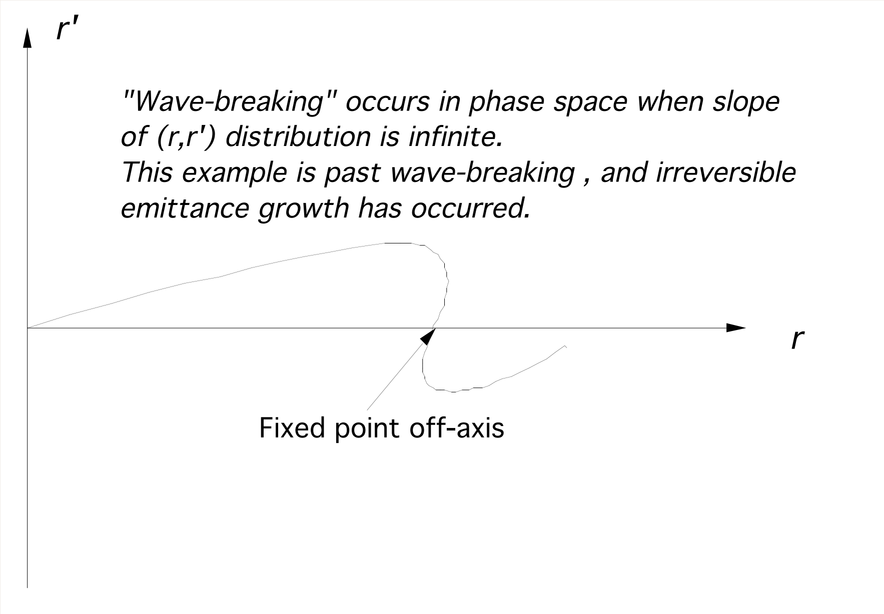
Example: SC gun

- Scale Ferrario scenario to L-band, SC
- 60 MV/m peak (30 average) gun field!



Nonlinear Emittance Growth

- Nonuniform beams lead to nonlinear fields and emitance growth
- Propagation of non-uniform distributions in *equilibrium* leads to *irreversible* emittance growth (wave-breaking in phase space).



Fixed point is where space-charge force cancels applied (solenoid) force. It is in the middle of the Debye sheath region.

Heuristic slab-model of non-equilibrium laminar flow

- Laminar flow=no trajectory crossing, no wavebreaking in phase space
- Consider first free expansion of slab (infinite in *y, z*) beam (very non-equilibrium)

$$n_b(x_0) = n_0 f(x_0),\ f(0) = 1$$

- Under laminar flow, charge inside of a given electron is conserved; one may mark trajectories from initial offset x_0. Equation of motion

$$x'' = k_p^2 F(x_0),\ F(x_0) = \int_0^{x_0} f(\tilde{x}_0) d\tilde{x}_0 = const.$$

Note, with normalization $F(x_0) \propto x$

Free-expansion of slab beam

- Solution for electron positions:

$$x(x_0) = x_0 + \frac{(k_p z)^2}{2} F(x_0)$$

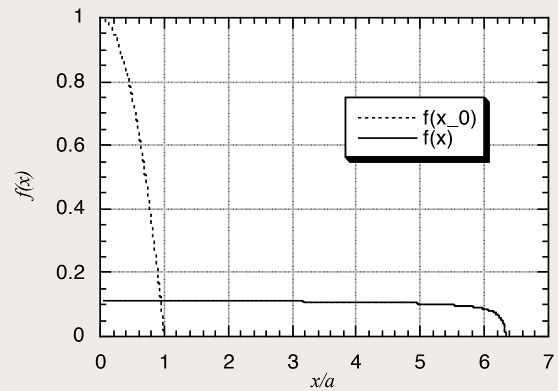
- Distribution becomes more linear in density with expansion

$$f(x(x_0)) = \frac{f(x_0)}{1 + \frac{(k_p z)^2}{2} f(x_0)} \Rightarrow \frac{2}{(k_p z)^2}$$

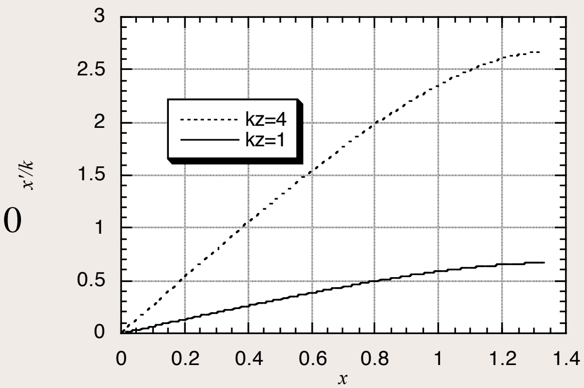
- Example case $f(x_0) = 1 - \left(\frac{x_0}{a}\right)^2$

- Wavebreaking will occur when final x is independent of initial x_0, $\frac{dx}{dx_0} = 0$
- In free-expanding slab, there is no wave-breaking for any profile

$$\frac{dx}{dx_0} = 1 + \frac{(k_p z)^2}{2} f(x_0) > 1 > 0$$



Initially parabolic profile becomes more uniform at $k_p z = 4$



Phase space profile becomes more linear for $k_p z >> 1$

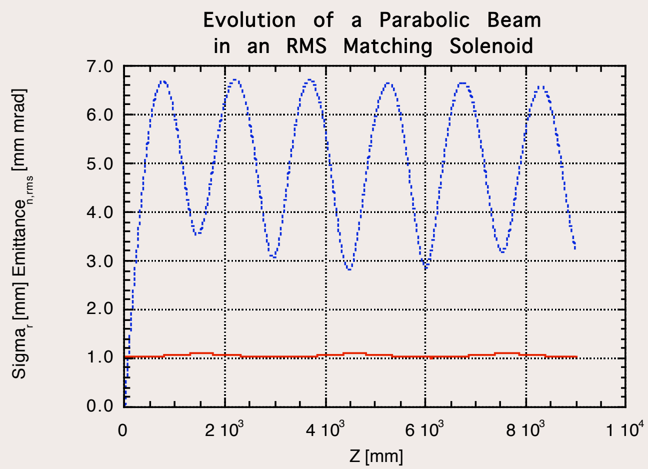
Slab-beam in a focusing channel

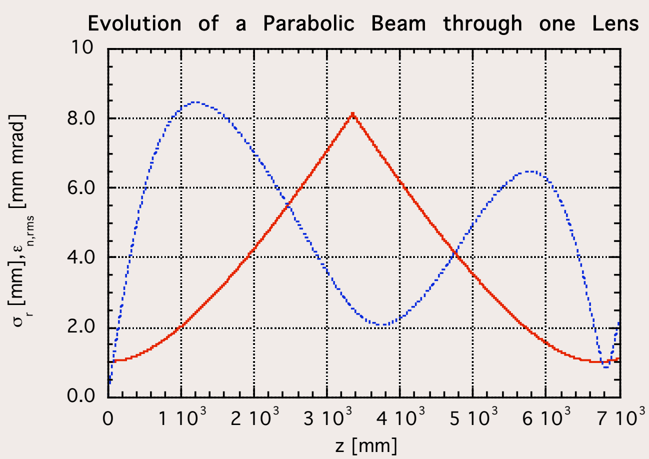
- Add uniform focusing to equation of motion,

$$x'' + k_\beta^2 x = k_p^2 F(x_0).$$

- Solution $x(x_0) = x_{eq}(x_0) + \left[x_0 - x_{eq}(x_0)\right]\cos(k_\beta z)$
  with $x_{eq}(x_0) = \frac{k_p^2}{k_\beta^2} F(x_0)$
- Wavebreaking occurs in this case for $f(x_0) = -\frac{k_\beta^2}{k_p^2} \frac{\cos(k_\beta z)}{2\sin^2\left(\frac{k_\beta z}{2}\right)}$
- For physically meaningful distributions, $f(x_0) \to 0$ smoothly, and wavebreaking occurs when $k_\beta z > \pi/2$
- For matched beam, $k_p^2 = k_\beta^2$ half of the beam wave-breaks.
- Stay away from equilibrium! When $k_p^2 >> k_\beta^2$ there is little wavebreaking, and irreversible emittance growth avoided.

Extension to cylindrical symmetry: 1D simulations



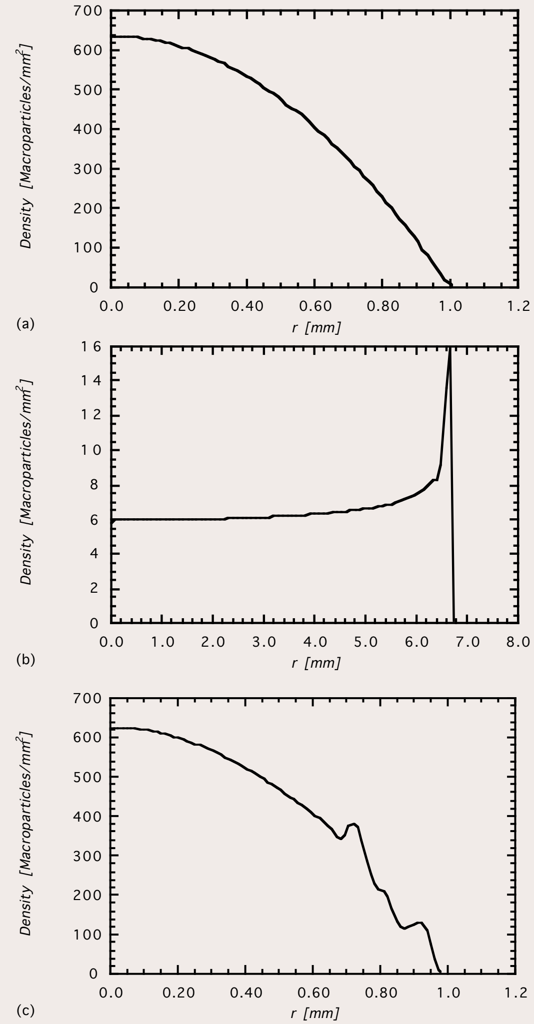


RMS beam size in red, emittance in blue

- Matched parabolic beam shows irreversible emittance growth after single betatron period
- Grossly mismatched single thin lens show excellent *nonlinear* compensation
- Explains robustness of "first compensation"

Emittance growth and entropy

- Irreversible emittance growth is accompanied by entropy increase
- Far-from-equilibrium thin-lens case shows ~uniform beam at maximum
- Near perfect reconstruction of initial profile
- Small wave-breaking region in beam edge

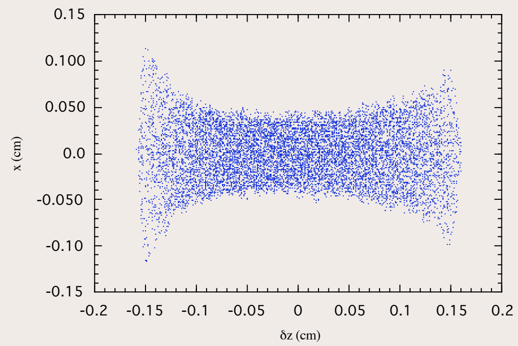


Beam profile at launch

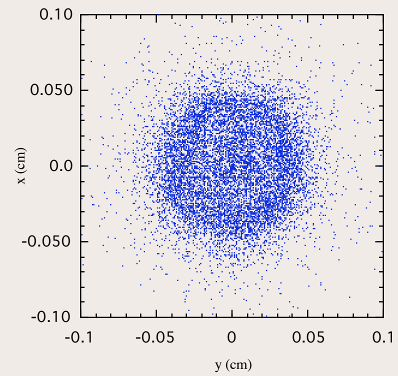
Beam profile at maximum

Beam profile, return to min.

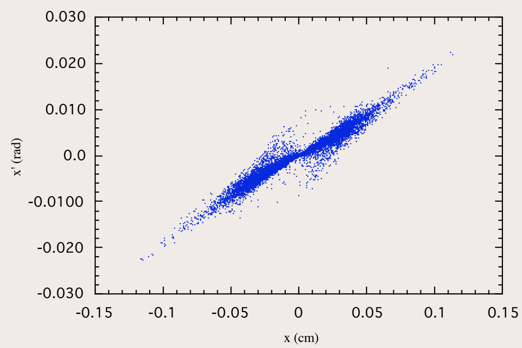
Multiparticle simulation picture: LCLS case (Ferrario scenario)



Spatial (*x*-*z*) distribution



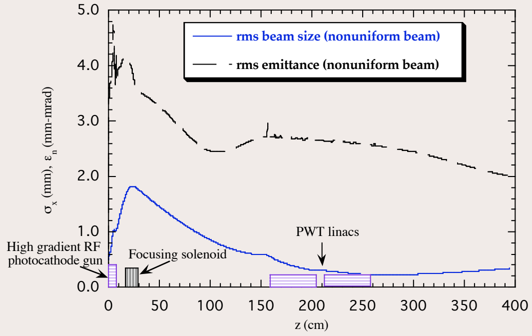
Spatial (*x*-*y*) distribution



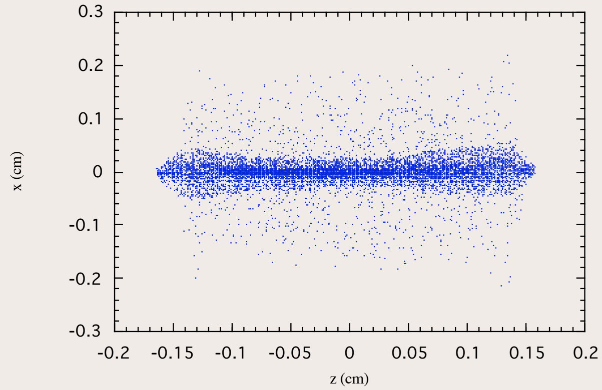
Trace-space distribution

- Case I: initially uniform beam (in *r* and *t*)
- Spatial uniformity reproduced after compensation
- High quality phase space
- Most emittance is in beam longitudinal tails (end effect)

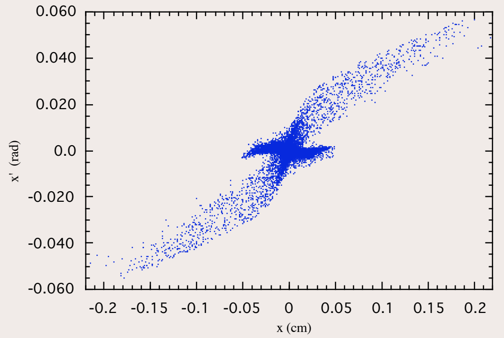
Multiparticle simulation picture: Nonuniform beam



Larger emittance obtained



Spatial (*x*-*z*) distribution



Trace-space distribution

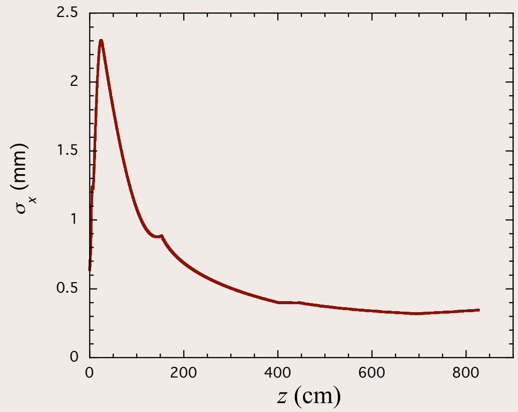
- Case II: Gaussian beam
- Most emittance growth due to nonlinearity
- Large halo formation

Have we chosen the right beam shape?

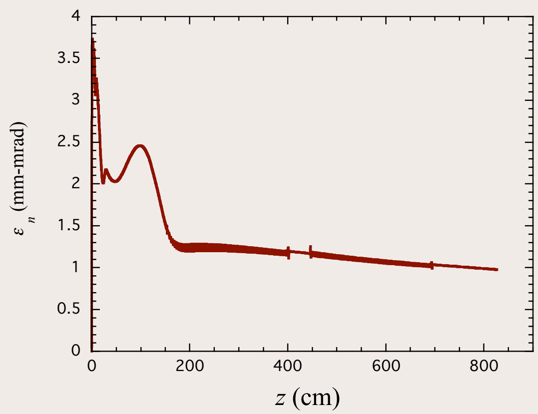
The first beer can

- "Beer-can" beam suffers from
  - Edge erosion, non-uniform distr.
  - Nonlinear fields at edges
  - Practical difficulties with laser
- Luiten (Serafini) proposal:
  - Use *any* ultra-short pulse
  - Longitudinal expansion of radial parabolic profile $I(r) = I_0\left(1-(r/a)^2\right)$
  - Uniform *ellipsoidal* beam created!
  - Linear space-charge fields (3D)

New direction: marry Luiten proposal to emittance compensation

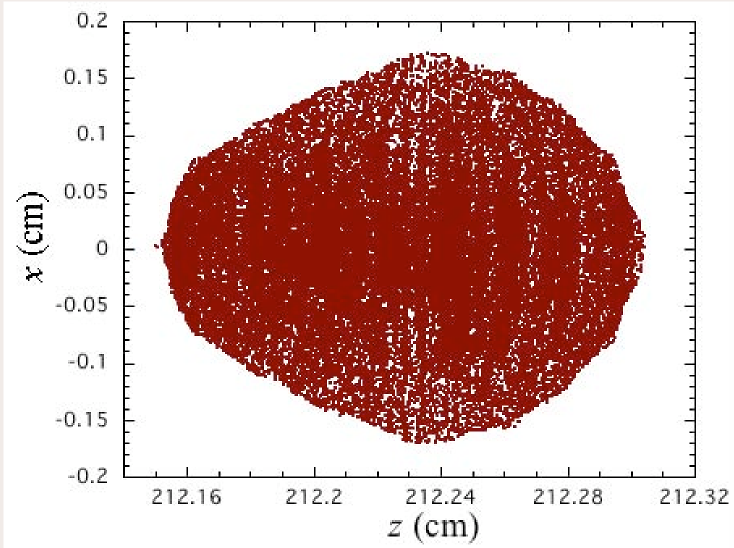


Beam envelope evolution



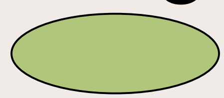
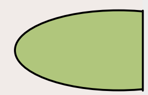
Emittance evolution

- Initial (not-too-optimized) PARMELA study
- Standard LCLS injector conditions
- Q=0.33 nC, initial long. Gaussian σ_t =33 fs (cutoff at 3 σ) trans. Gaussian with σ_x =0.77 mm (cutoff at 2 σ).
- Final bunch length 1.43 mm (full), 104 A.



Beam distribution showing ellipsoidal boundary (12.5 MeV)

Challenges and advantages

- Laser *very* forgiving
- Shorter pulses possible?
- Cathode image charges drive incorrect final state, not  but 
- Excessive energy spread during compensation
- Charge fluctuations
- Experiment at LLNL, ORION or SPARC