RF focusing in linear accelerator cavities

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ERL 2005, JLAB
March 22, 2005

RF focusing

- Two sources:
  - Entrance kick (first order)
  - Alternating gradient (second order)
- Studies at UCLA, INFN-Milano, DESY in 1990's (originates in 60's)
  - References:

"Ponderomotive Focusing in Axisymmetric Rf Linacs",
S.C. Hartman and J.B. Rosenzweig, *Physical Review E* **47**, 2031 (1993).
"Transverse Particle Motion In Radio-Frequency Linear Accelerators",
J.B. Rosenzweig and L. Serafini, *Physical Review E* **49**, 1499 (1994).
"Experimental Confirmation of Transverse Focusing and Adiabatic Damping in a Standing Wave Linear Accelerator", S. Reiche, *et al.*. *Physical Review E* **56**, 3572 (1997).
E. Chambers, Stanford High Energy Physics Laboratory Report (1965).
R. Helm and R. Miller, in *Linear Accelerators,* Eds. Pierre M. Lapostolle and Albert L. Septier (North-Holland, Amsterdam, 1969).

Fields in axisymmetric cavity

- Floquet representation (no Bessel functions). Note phase convention…

$$E_z \equiv E_0 \,\mathrm{Im}\left[\sum_{n=-\infty}^{\infty} b_n e^{i(\omega t - k_n z)}\right] \qquad k_n \equiv \left(\psi + 2\pi n\right)/d$$

$$E_z = E_0 \,\mathrm{Im}\left[\sum_{n=-\infty}^{\infty} b_n e^{i(2k_0 \frac{m}{l} nz + \varphi)}\right]$$

- Radial forces (relativistic limit)

$$F_r \cong -\frac{qr}{2}\frac{d}{dz}E_z$$

Average second order radial force

- "Ponderomotive" force is found by averaging over *small* oscillations

$$\overline{F}_r = \frac{\left(qE_0\right)^2}{8\gamma m_0 c^2} r \sum_{n=1}^{\infty} b_n^2 + b_{-n}^2 + 2b_n b_{-n} \sin(2\varphi) \equiv \eta(\varphi) \frac{\left(qE_0\right)^2}{8\gamma m_0 c^2} r$$

- Related to variance of acceleration

$$\overline{F}_r = \frac{\left[\left\langle E_z^2 \right\rangle - \left\langle E_z \right\rangle^2\right]}{\gamma m_0 c^2} r$$

Equation of motion & solution

- Relativistic limit

$$x'' + \left(\frac{\gamma'}{\gamma}\right)x' + \frac{\eta(\varphi)}{8\sin^2(\varphi)}\left(\frac{\gamma'}{\gamma}\right)^2 x = 0$$

$$\gamma' = qE_0 \sin(\varphi)/m_0c^2$$

- Solution

$$x(z) = x_0 \cos\left[\alpha(z)\right] + x_0' \sqrt{\frac{8}{\eta(\varphi)}}\frac{\gamma_0}{\gamma'}\sin(\varphi)\sin\left[\alpha(z)\right],$$

$$\alpha(z) = \left(\frac{\sqrt{\eta(\varphi)/8}}{\sin(\varphi)}\right)\ln\left[\frac{\gamma(z)}{\gamma_0}\right]$$

Matrix in cavity interior

- Assume integer number of cells, w/o transients at ends

$$\mathcal{M}_{\mathrm{acc}} = \begin{bmatrix} \cos\left[\alpha(z)\right] & \sqrt{\dfrac{8}{\eta(\varphi)}}\,\dfrac{\gamma_0}{\gamma'}\sin(\varphi)\sin\left[\alpha(z)\right] \\ \sqrt{\dfrac{\eta(\varphi)}{8}}\,\dfrac{\gamma'}{\gamma(z)}\,\dfrac{\sin\left[\alpha(z)\right]}{\sin(\varphi)} & \dfrac{\gamma_0}{\gamma(z)}\cos\left[\alpha(z)\right] \end{bmatrix}$$

Edge matrices

- First order transient (careful to subtract oscillatory component of motion)

$$\Delta x' = \mp \frac{qE_m \sin(\varphi)}{2\gamma_{i(f)} mc^2} x = \mp \frac{q\gamma'}{2\gamma_{i(f)}} \left[\sum_{n=-\infty}^{\infty} a_n \right] x \equiv \mp \frac{q\gamma'}{2\gamma_{i(f)}} gx.$$

$$\theta_{\text{osc}} = \mp \frac{x}{2} \frac{\gamma'}{\gamma} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} a_n \equiv \mp \frac{x}{2} \frac{\gamma'}{\gamma} (g-1)$$

$$\mathcal{M}_{\text{ent/exit}} = \begin{bmatrix} 1 & 0 \\ \mp \dfrac{\gamma'}{2\gamma_{0(f)}} & 1 \end{bmatrix}$$

Full matrix

$$\mathcal{M} = \mathcal{M}_{\mathrm{ex}}\mathcal{M}_{\mathrm{acc}}\mathcal{M}_{\mathrm{ent}}$$

$$= \begin{bmatrix} \cos(\alpha) - \sqrt{\frac{2}{\eta(\varphi)}}\sin(\varphi)\sin(\alpha) & \sqrt{\frac{8}{\eta(\varphi)}}\frac{\gamma_0}{\gamma'}\sin(\varphi)\sin(\alpha) \\ -\frac{\gamma'}{\gamma_f}\left[\sqrt{\frac{2}{\eta(\varphi)}}\sin(\varphi) + \sqrt{\frac{\eta(\varphi)}{8}}\frac{1}{\sin(\varphi)}\right]\sin(\alpha) & \frac{\gamma_0}{\gamma_f}\left[\cos(\alpha) + \sqrt{\frac{2}{\eta(\varphi)}}\sin(\varphi)\sin(\alpha)\right] \end{bmatrix}$$

Adiabatic damping (relativistic approx.):

$$\det \mathcal{M} = \frac{\gamma_i}{\gamma_f}$$

Note: H. Weise has decomposed this matrix into focusing and adiabatic damping components:

$$\mathcal{M} = \mathcal{M}_{\mathrm{F}}\mathcal{M}_{\mathrm{damp}}$$

$$\mathcal{M}_{\mathrm{damp}} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\gamma_0}{\gamma_f} \end{bmatrix}$$

Experimental measurement UCLA Saturnus 1996

- Photoinjector dark current beam source
- PWT high gradient linac matrix measured
  - Nontrivial higher spatial harmonic content
- Energy and phase dependence noted
- Determinant of matrix checked

Experimental layout

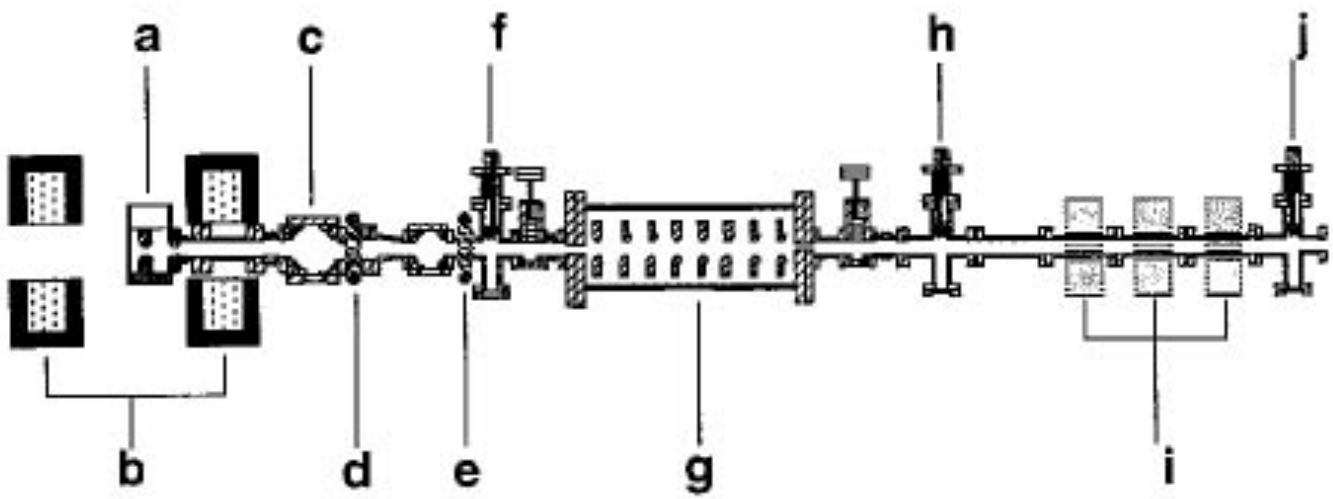
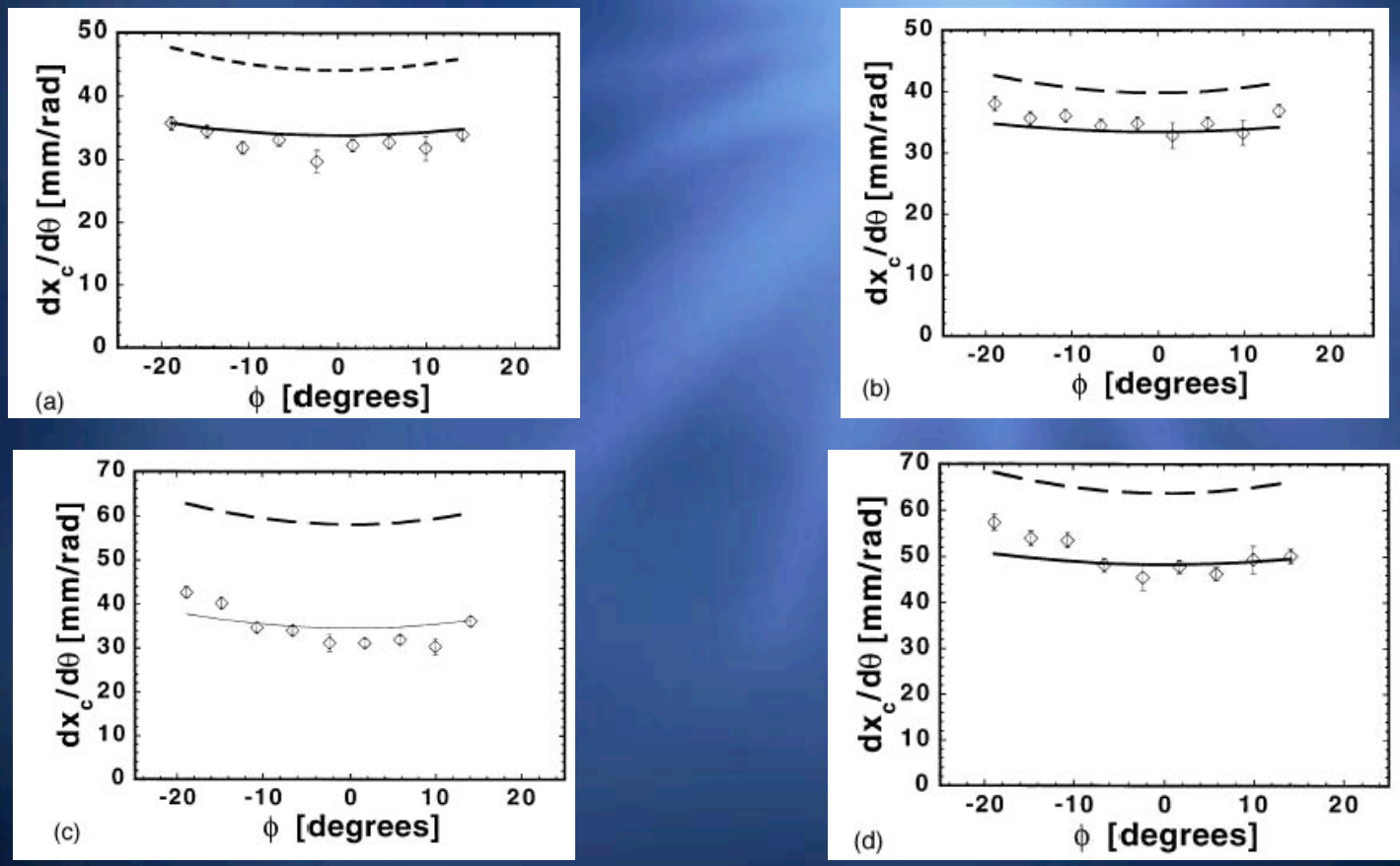


FIG. 1. Layout of the UCLA photoinjector with a (a) 1.5 cell rf gun, (b) focusing and bucking solenoid, (c) mirror box, steering magnets (d) $K1$ and (e) $K2$, (f) phosphor screen $P1$, (g) PWT linac, (h) phosphor screen $P2$, (i) quadrupole triplet, and (j) phosphor screen $P3$.

Matrix element determination

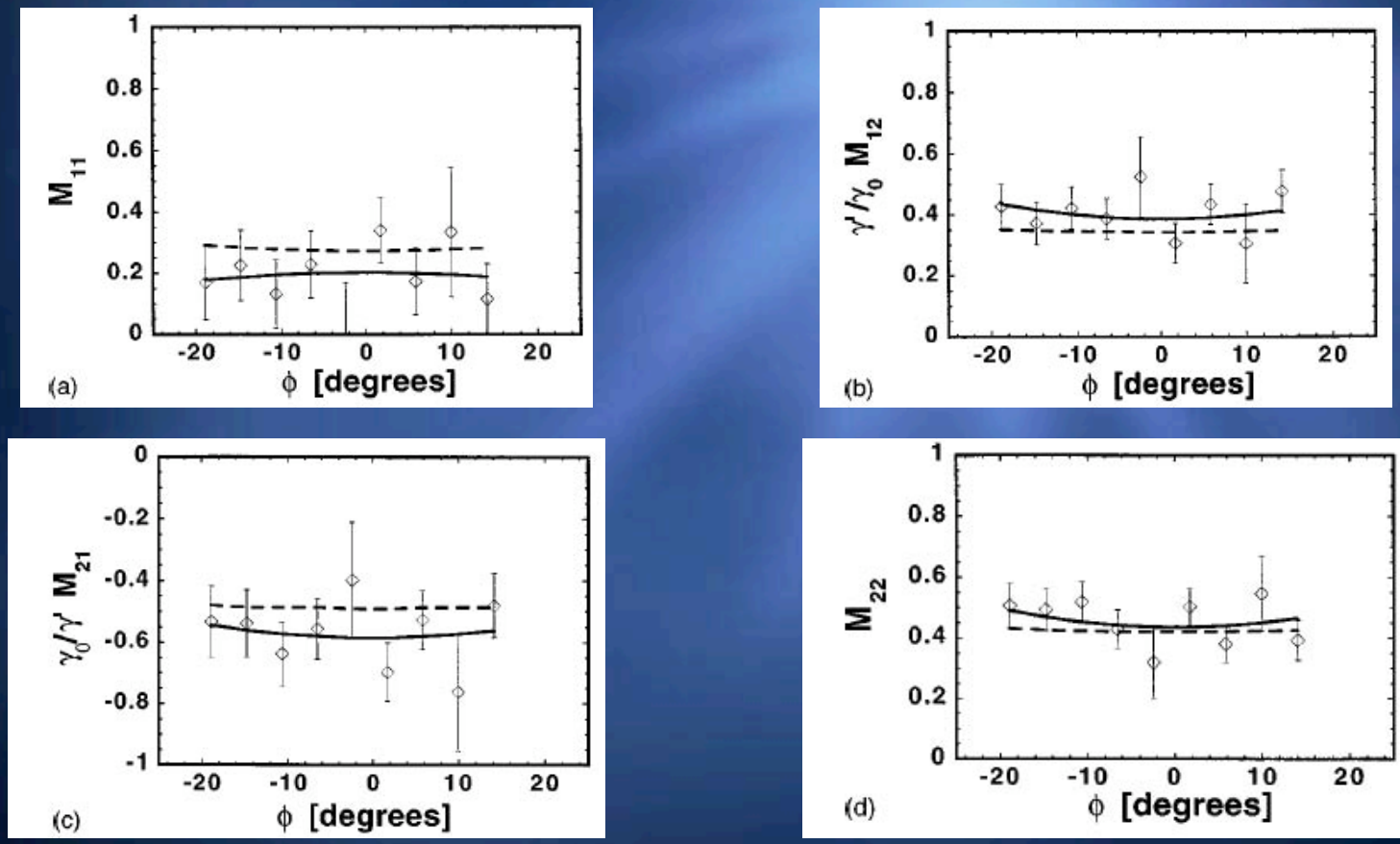
- Two kickers, calibrated
- Two profile monitors
- Variation of phase (revert to $\phi = \varphi - \pi/2$)
- Variation of amplitude

Data: Phase variation



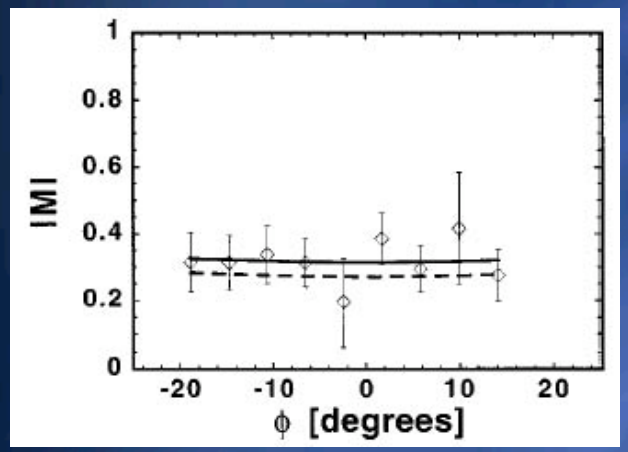
(a)-(d): K1P2, K1P3, K2P2, K2P3
Solid line is theory, dash w/o focusing

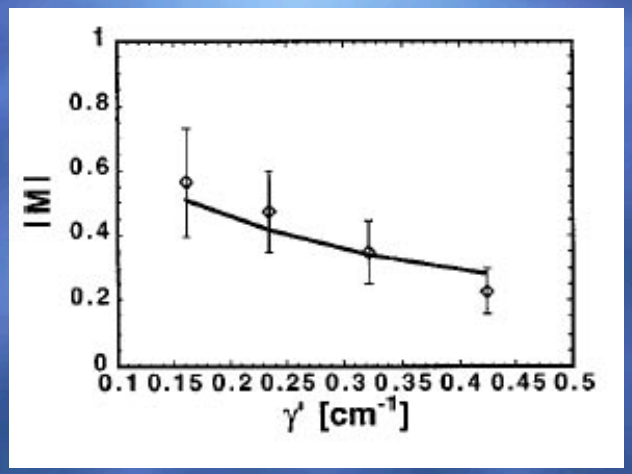
Matrix: Phase variation



Solid line is theory, dash w/o focusing

Matrix determinant





- Check on measurement and physics
- Solid line is theory
- Dashed is overall fit of data, inverted
- Difficult, but not bad

Conclusions

- Good first test of matrix transport
  - Agreement obtained within experimental error
- Critical for high gradient SW linacs
- Also is the basis of *invariant envelope* theory in emittance compensation so…
- *Better measurements* with good beam desirable for LC, ERL community
- Look for
  - Non-linear (higher order mod. Bessel function dependences)
  - Dynamics in emittance compensation and BBU