The Hadron-Hadron Interaction from the Lattice

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- What lattice QCD can do to nuclear physics (and what it cannot)
- Problems in two-baryon systems: large volumes, fine tunings, chiral extrapolations
- Some solutions
- Some simpler systems

Some basic facts on nuclear forces

expected :

isospin symmetric, pion dominated at large distances, non-central

unexplained :

S-waves fine tuned: shallow (virtual) bound states

$$B_{deuteron} \simeq 2.2\,\mathrm{MeV}$$

$$a_s \simeq 20\ \mathrm{fm} \simeq \frac{1}{8\ \mathrm{MeV}}$$

as opposed to

$$m_\rho \simeq 790\,\mathrm{MeV}$$

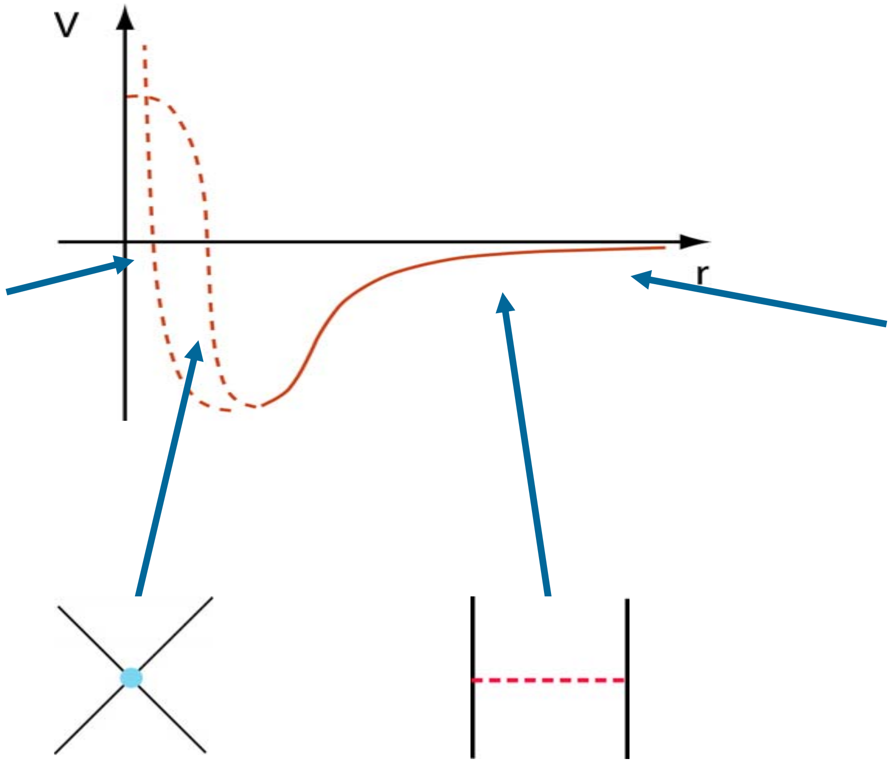
$$\mathrm{m}_\pi \simeq 140\,\mathrm{MeV}$$

$$\frac{\mathrm{m}_\pi^{\;2}}{M} \simeq 20\,\mathrm{MeV}$$

All quantitative details

QCD → NN potential → nuclear physics ? No !

model dependent,
off-shell "ambiguities",
NRQM breaks down here



model-independent
pion exchange

EFT:

Potential is not well defined:
can't measured it on the lattice or
anywhere else

Interesting things to compute in the lattice:

0, 1 baryon

- hadron masses
- decay constants
- weak matrix elements
- pion scattering
- …

2 or more baryons

- NN phase shifts
- decay constants
- pion-nucleon couplings
- hyperon interactions
- electroweak “exchange currents”
- three-body forces
- …

QCD reduced to quadratures

$$<|\bar{q}...q(x) \ \ ... \ \ \bar{q}...q(y)|> = \int DA\, e^{-S[A]} \det(\gamma.(\partial + A) + m) \ \ (\gamma.(\partial + A) + m)^{-1} ... (\gamma.(\partial + A) + m)^{-1}$$

always the same

depends on the operator considered

Path integral can be computed numerically

$$10 \text{ x } 10 \text{ x } 10 \text{ x } 10 \text{ x } 8 \text{ x } 4 \ = \ 10^6 \text{ dimensions}$$

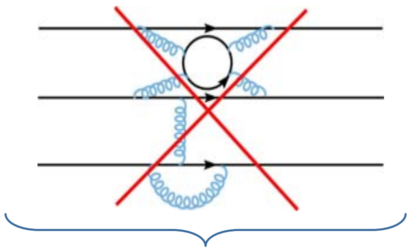
# of space-time points

# of gluons

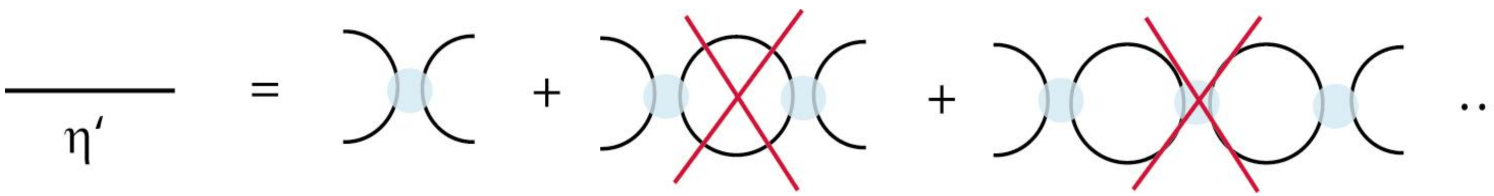
computational cost $\sim \left(\frac{L}{b}\right)^4 \left(\frac{1}{b\, m_q}\right)^2$

Quenched "approximation"

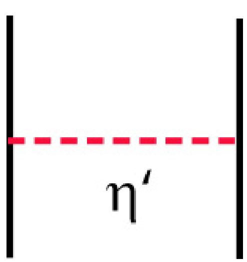
$$\det(\gamma.(\partial + A) + m) \rightarrow 1$$



no internal quark loops



$$= \frac{1}{p^2 - m^2} - \frac{\delta m^2}{(p^2 - m^2)^2} - \frac{(\delta m^2)^2}{(p^2 - m^2)^3} + \ldots$$

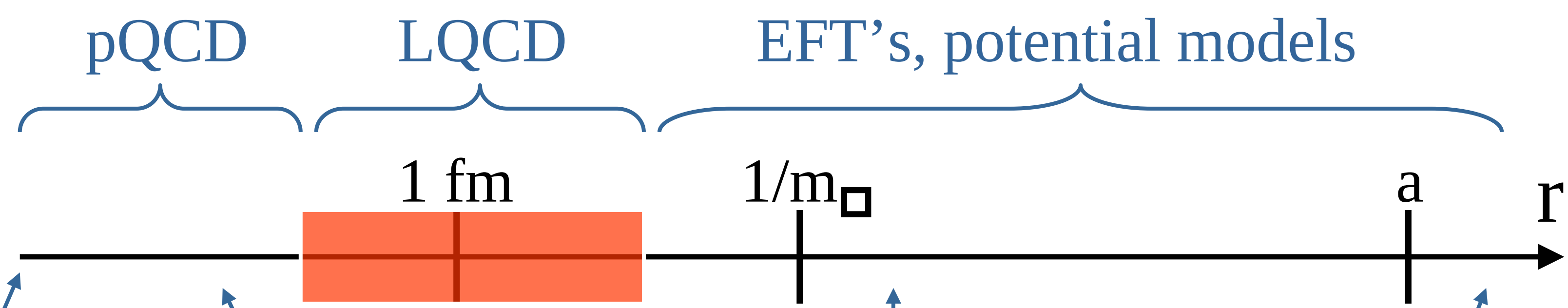


$$\sim - \frac{g^2}{16\pi f^2} \sigma_1.\hat{r}\sigma_2.\hat{r}\; m_\pi\, e^{-m_\pi r}$$

dominates at large distances

Beane & Savage (2003)

Minimize lattice points, maximize quark masses, minimize volume



$$-\frac{1}{4}(F_{\mu\nu})^2+\overline{\psi}(D+m)\psi$$

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$$+bc\,\overline{\psi}F_{\mu\nu}\sigma_{\mu\nu}\psi+b^2d(F_{\mu\nu})^4+\cdots$$

$$N^\dagger(i\partial_0+\frac{\nabla^2}{2M}+\cdots)N+g_A N^\dagger\tau^a\nabla\pi^a N+\cdots$$

$$N^\dagger(i\partial_0+\frac{\nabla^2}{2M}+\cdots)N-C_0(N^\dagger N)^2+\cdots$$

 is analytically intractable

Extracting physics from euclidean space : masses are "easy"

$$< 0 | \pi(t,\vec{k}=0)\, \pi^\dagger(0,\vec{k}=0) | 0 > = \sum_n e^{-Ht} < 0 | \pi(0,\vec{0}) | n >< n | \pi^\dagger(0,\vec{0}) | 0 >$$

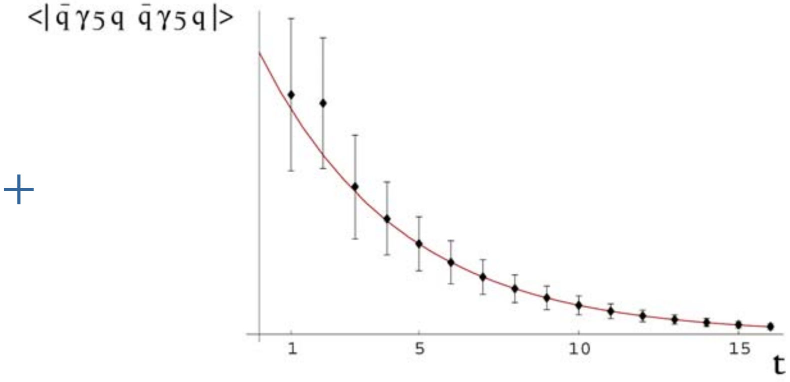
$$\underset{t\to\infty}{\to} e^{-m_\pi t} < 0 | \pi(0,\vec{0}) | \pi >< \pi | \pi^\dagger(0,\vec{0}) | 0 >$$

some operator with quantum numbers of the pion, made of quarks and gluons, for instance:

$$\bar{\psi}(0,-\vec{p})\gamma_5\tau^a\psi(0,\vec{p})$$

lowest energy state with the quantum numbers of the pion

MILC configurations (2+1 staggered quarks) + JLab Domain Wall propagators



Extracting physics from euclidean space : scattering

$$<0|\pi(t,-\vec{k})\,\pi(t,\vec{k})\,\pi^{\dagger}(0,-\vec{k})\,\pi^{\dagger}(0,\vec{k})|0> = \sum_{n} e^{-Ht} <0|\pi(0,-\vec{k})\,\pi(0,\vec{k})|n><n|\pi^{\dagger}(0,-\vec{k})\,\pi^{\dagger}(0,\vec{k})|0>$$

$$\underset{t\to\infty}{\rightarrow} e^{-2m_{\pi}t} \underbrace{<0|\pi(0,-\vec{k})\,\pi(0,\vec{k})|\pi\pi\,\mathrm{at\,rest}><\pi\pi\,\mathrm{at\,rest}|\pi^{\dagger}(0,-\vec{k})\,\pi^{\dagger}(0,\vec{k})|0>}$$

uninteresting off-shell
amplitude

No scattering from infinite volume euclidean amplitudes

(Maiani-Testa “theorem”)

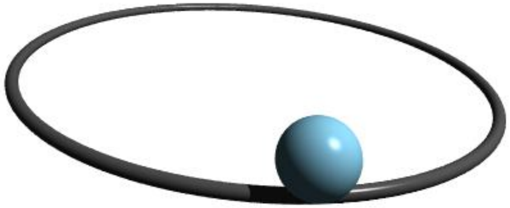
Scattering through finite volumes: the "Luscher method"

(Marinari, Hamber, Parisi, Rebbi)

one particle

Periodic boundary conditions: box is a torus

Energy levels at $E_n = \frac{1}{2M}\left(\frac{2\pi n}{L}\right)^2$



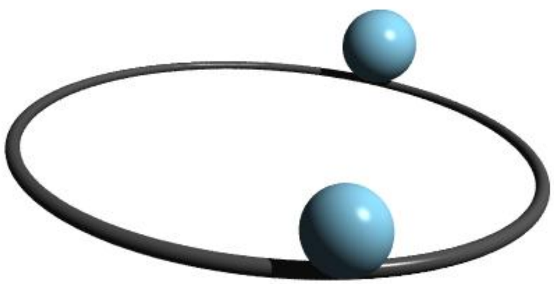
two particles

for L >> a

$$E_0 = \frac{4\pi a}{ML^3}\left[1 - \frac{c_1 a}{L} + c_2\left(\frac{a}{L}\right)^2 + \cdots\right]$$

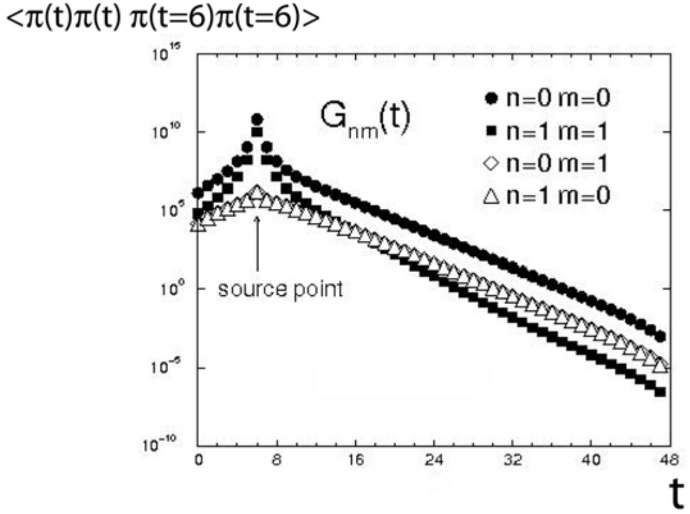
$$E_1 = \frac{4\pi^2}{ML^2} - \frac{12\tan\delta}{ML^2}\left[1 + c'_1 \tan\delta + \cdots\right]$$

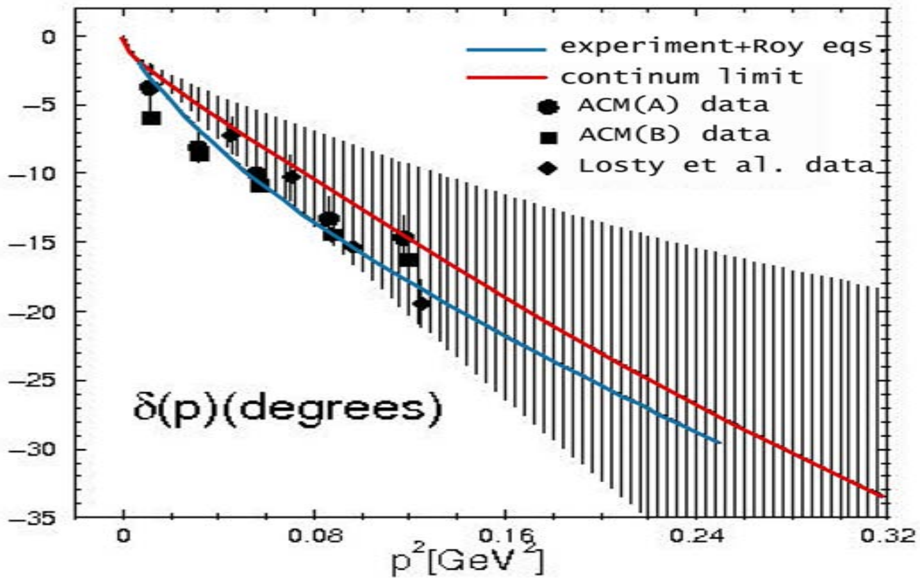
phase shift at p=2π/L



I=2 ππ scattering

CP-PACS collaboration
(hep-lat/0402025)
2 flavors improved Wilson quarks
$m_\pi > 0.55\ m_\rho$





Do we need to fit the deuteron ($^1S_0$) in the box ? No, but …

$p = \sqrt{ME}$	$^1S_0$ p in MeV		$^3S_1$ p in MeV	
lattice size (fm)	1st	2nd	1st	2nd
100	2.6 i	10.52	45.5i	1.76
25	13.8i	39.3	45.8i	18.25
10	39.0i	104	61.3i	83.1
5	94.4i	224	116i	206

need 8-10 fm lattice to extract scattering length

negative and outside the effective range region

P.B., S. Beane, A. Parreno & M. Savage, PLB, 2004

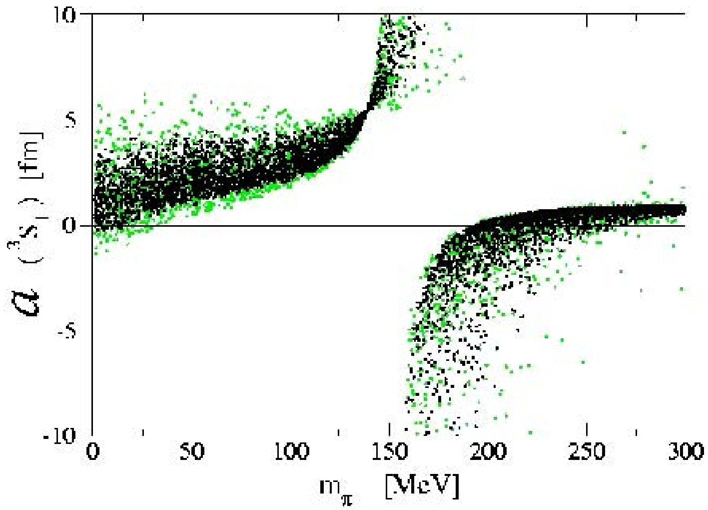
Solution 1: change m_q, match to the 'pionfull" theory and determine low energy constants

Very hard to determine either experimentally or numerically

$$\sim D_2\,(NN)^{\dagger}\,N m_q N$$

Beane & Savage

Epelbaum, Meissner, Gloeckle



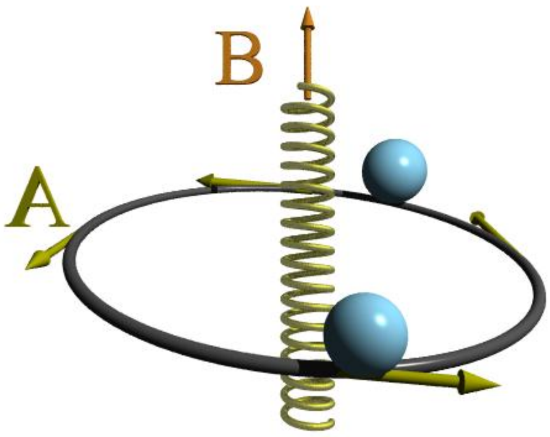
Hinges on the convergence of the chiral expansion

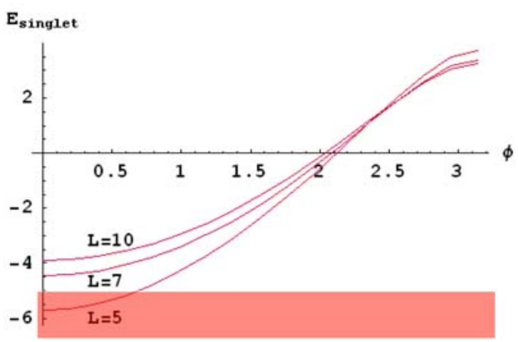
$\Lambda\Lambda$, ΛN is less sensitive to quark masses

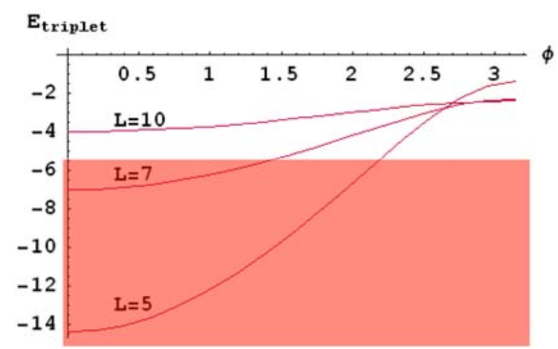
- Λ is an isoscalar, no one-pion exchange
- no reason to expect fine tuning
- less is known, anything learned is useful

Solution 2: Aharonov-Bohm

add a background magnetic potential coupled to baryon number with zero curl







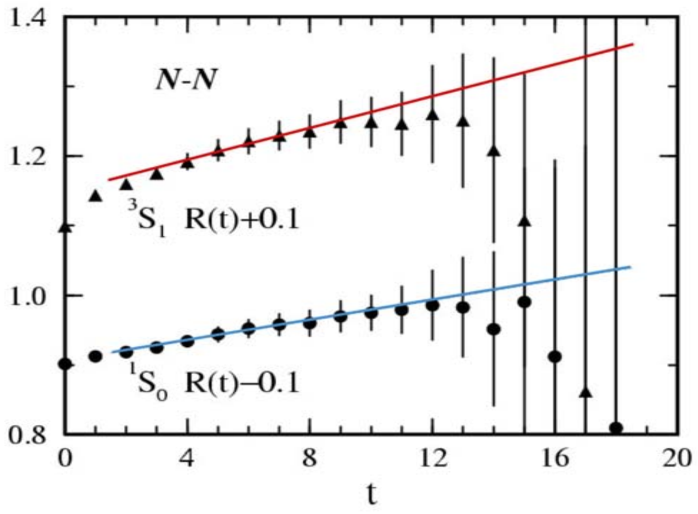
- smaller boxes
- access to any phase shift from a given lattice size
- only valence quarks need to be twisted

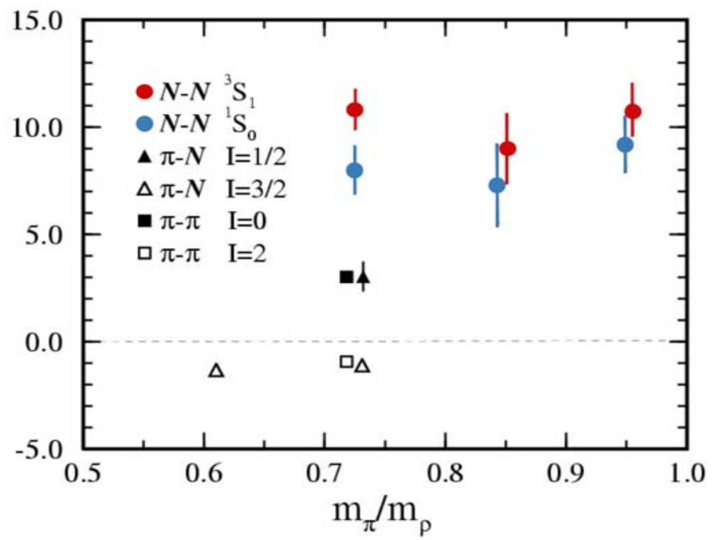
P.B., PLB (2004)

Fukugita *et al.* (1994 !)

Quenched, Wilson quarks, Wilson glue action

Very heavy quarks, b=0.14 fm





Conclusion & Prospects

- LQCD won’t compute the NN potential
- QCD-nuclear physics connection made through observables and/or EFT low energy constants
- NN is particularly sensitive to quark masses
- Some difficulties of principle were addressed, the hard, numerical work barely touched
- simple systems (Ql-Ql) are useful stepping stones
- hypernuclear physics on the lattice first