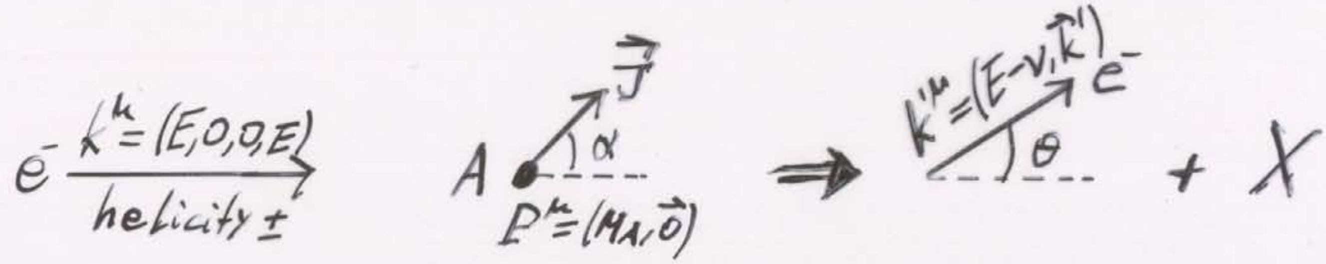
Medium modifications of nucleon structure functions

W. Bentz (Tokai Univ., Japan), I. Cloet (JLab, USA),
A.W. Thomas (JLab, USA)

I) Deep inelastic e-nucleus scattering:



Variables: $x_A = A \cdot \frac{Q^2}{2M_A\nu} = \frac{Q^2}{2\overline{M}_N\nu}$ $(\overline{M}_N = M_A/A)$,

$y = \frac{\nu}{E}$. (e.g., Jaffe, Manohar, NPB321('89)343)

Express polarized nuclear state $|J\,J>_\alpha$ in a basis $|J\,H>$, where $J_z = H$ refers to the direction of $\vec{q}$. [In Bjorken limit (BL): $\vec{q} \,//\, \vec{k} \,//\, \hat{z}$. Reason: $k'^2 = (k-q)^2 \Rightarrow \cos\beta = \nu/|\vec{q}| + Q^2/(2E|\vec{q}|) \overset{BL}{\rightarrow} 1$].

Cross section per nucleon in BL:

$$\frac{1}{A}\left(\frac{\mathrm{d}\,\overline{\sigma}}{\mathrm{d}x_A\,\mathrm{d}y} \pm \frac{\mathrm{d}\,\Delta\sigma}{\mathrm{d}x_A\,\mathrm{d}y}\right)$$

where

$$\frac{1}{A}\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}x_A\,\mathrm{d}y} = \frac{4\pi\alpha^2}{Q^2}\sum_H\left[yF_{1A}^{(H)}(x_A)+\frac{1-y}{x_Ay}F_{2A}^{(H)}(x_A)\right]\left(d^J_{HJ}(\alpha)\right)^2$$

$$\frac{1}{A}\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}x_A\,\mathrm{d}y} = \frac{-4\pi\alpha^2}{Q^2}(2-y)\sum_H g_{1A}^{(H)}(x_A)\left(d^J_{HJ}(\alpha)\right)^2$$

Here the nuclear structure functions per nucleon are:

$$F_{1A}^{(H)}(x_A) = \frac{1}{2}\sum_q e_q^2\, q_A^{(H)}(x_A) = \frac{1}{2}\sum_q\left(q_{A\uparrow}^{(H)}+q_{A\downarrow}^{(H)}\right)$$

$$F_{2A}^{(H)}(x_A) = 2x_AF_{1A}^{(H)}(x_A)$$

$$g_{1A}^{(H)}(x_A) = \frac{1}{2}\sum_q e_q^2\,\Delta q_A^{(H)}(x_A) = \frac{1}{2}\sum_q\left(q_{A\uparrow}^{(H)}-q_{A\downarrow}^{(H)}\right)$$

Interpretation: $q_{A\uparrow}^{(H)}(x_A)\ldots$ Probability to find quark (flavor q) with light cone (LC) momentum fraction x_A and $s_z=+1/2$ in the nucleus with $J_z=H$.

These quark LC momentum distributions (per nucleon) are defined by the quark 2-point functions in the nucleus for fixed $k_-=\overline{M}_N x_A/\sqrt{2}$ of the quark:

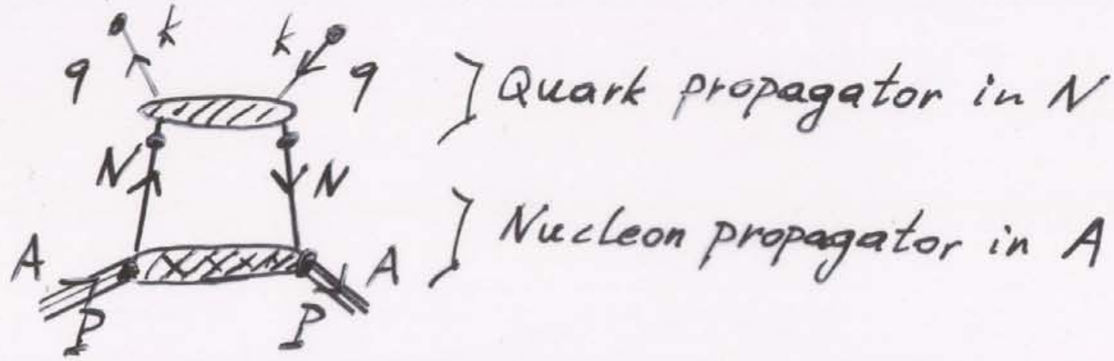
$$q_A^{(H)}(x_A) \equiv \frac{\overline{M}_N}{\sqrt{2}A}\int\frac{\mathrm{d}\omega^-}{2\pi}e^{i(\overline{M}_N/\sqrt{2})x_A\cdot\omega^-} \times <J\,H|\overline{\psi}_q(0)\gamma^+\psi_q(\omega^-)|J\,H> = \frac{-i}{A}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\delta\left(x_A-\frac{k_-}{\overline{M}_N/\sqrt{2}}\right)\mathrm{Tr}\left(\gamma^+G_q(k)\right)$$

and similar for $\Delta q_A^{(H)}(x_A)$ with $\gamma^+\to\gamma^+\gamma_5$. Here $G_q(k)$ is the quark propagator in the nucleus:

$$G_q(k) = -i\int\mathrm{d}^4\omega\, e^{ik\cdot\omega}\,\langle JH|\mathrm{T}\left(\psi_q(\omega)\,\overline{\psi}_q(0)\right)|JH\rangle$$

How to calculate these quark distributions?

In a single particle (mean field) model for the nucleus, the following "convolution" holds for the quark 2-point function:

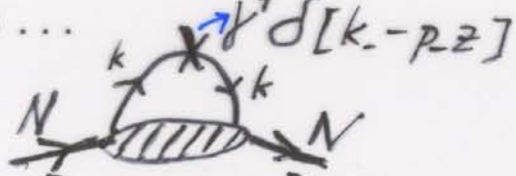


This leads to the familiar forms

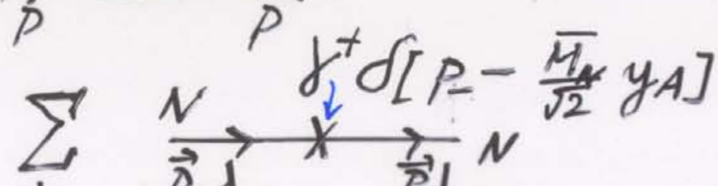
$$q_A^{(H)}(x_A) = \int \mathrm{d}y_A \int \mathrm{d}z\, \delta(x_A - y_A z)\, q_N(z)\, f_{N/A}^{(H)}(y_A)$$
$$\Delta q_A^{(H)}(x_A) = \int \mathrm{d}y_A \int \mathrm{d}z\, \delta(x_A - y_A z)\, \Delta q_N(z)\, \Delta f_{N/A}^{(H)}(y_A)$$

By diagrams ...

$q_N(z) =$



$f_{N/A}^{(H)}(y_A) = \sum_\lambda$



$$= \frac{\sqrt{2}\,\overline{M}_N}{A} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_\lambda n_\lambda^{(H)} \delta\left(p_3 + \epsilon_\lambda - \overline{M}_N y_A\right) \left(\overline{\Phi}_\lambda(\vec{p})\gamma^+ \Phi_\lambda(\vec{p})\right)$$

($\gamma^+ \to \gamma^+\gamma_5$ for the spin dependent distributions.)

What are the **differences to the nucleon case**?

- ~~For spin-independent $q_A^{(H)}(x_A)$: All nucleons contribute $\Rightarrow q_A^{(H)}(x_A) \propto \mathcal{O}(1)$. But ...
For spin-dependent $\Delta q_A^{(H)}(x_A)$: Only few valence nucleons contribute $\Rightarrow \Delta q_A^{(H)}(x_A) \propto \mathcal{O}(1/A)$.~~ (cf. Charge form factor $\propto Z$, Magnetic form factor $\propto$ m.m. of valence nucleon.)

- There are more structure functions because of the dependence on $H = -J, \ldots + J$. [$(2J+1)/2$ F_{1A}'s and g_{1A}'s for half-integer J (or $J+1$ for integer J.)] $\Rightarrow$ "<u>Multipole</u> structure funct."

- Informations on both nuclear effects and quark effects. For example, the spin sum in the nucleus $(N_{q\uparrow/A} - N_{q\downarrow/A})/2$ is:

$$
\begin{aligned}
\int \mathrm{d}x_A \, \Delta q_A^{(H)}(x_A) &= \left(\int \mathrm{d}z \, \Delta q_N(z)\right) \times \left(\int \mathrm{d}y_A \, \Delta f_{N/A}^{(H)}(y_A)\right) \\
&= \langle \Sigma_q \rangle_N \times \langle \Sigma_N \rangle_A \\
&= \left(N_{q\uparrow/N\uparrow} - N_{q\downarrow/N\uparrow}\right) \times \left(N_{N\uparrow/A} - N_{N\downarrow/A}\right)
\end{aligned}
$$

↗ spin sum for nucleon

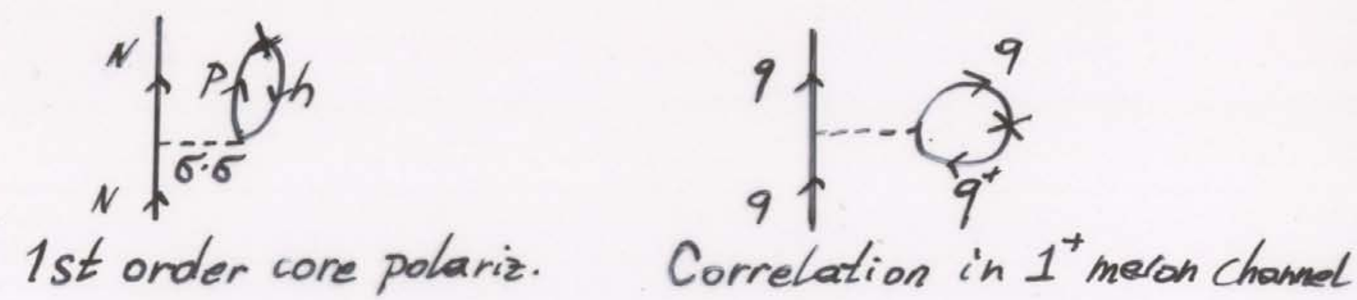
↑ "Polarization factor"

- For a combination $\sum_H q_A^{(H)}(x_A)$: All nucleons contribute $\Rightarrow \mathcal{O}[1]$.
For other combinations, like $\Delta q_A^{(H)}$ etc, only few valence nucleons contribute $\Rightarrow \mathcal{O}[1/A]$

Some comments...

For closed shell $\pm$ 1 nuclei, $\langle \Sigma_N \rangle_A$ is known to be smaller than the single particle (Schmidt) value. ($\Leftrightarrow$ Quenching of nuclear spin matrix elements). Also, $\langle \Sigma_q \rangle_N$ is smaller than the SU(6) value. The theoretical explanations of $\langle \Sigma_N \rangle_A$ and $\langle \Sigma_q \rangle_N$ may be quite similar ... For example,

Nuclear structure | Nucleon structure



(Some spin strength is moved to collective states.)

(Some spin is converted to orbital angular momentum of "sea")

- We can discuss **"medium modifications"**:
  - $q_N(z), \Delta q_N(z) \neq q_{N\text{free}}(z)$
  - Because of nucleon Fermi motion, nuclear structure etc, $f^{(H)}_{N/A}(y_A)$ and $\Delta f^{(H)}_{N/A}(y_A)$ differ from the "static single particle values"

$$f^{(H)}_{N/A}(y_A) \stackrel{\text{stat}}{\Longrightarrow} \delta(y_A - 1)$$
$$\Delta f^{(H)}_{N/A}(y_A) \stackrel{\text{stat}}{\Longrightarrow} \delta(y_A - 1) \times \langle \Sigma_N \rangle_A^{(\text{Schmidt})}$$

One can express these medium modifications by **ratios**:
For example,

– J=0, $N \simeq Z$ case:

$$F_{2A}(x_A) = x_A \left(\frac{4}{9}u_A(x_A) + \frac{1}{9}d_A(x_A)\right)$$
$$= \int_{x_A}^{A} \mathrm{d}y_A \, f_{N/A}(y_A) \cdot F_{2N}\left(\frac{x_A}{y_A}\right)$$

$(F_{2N} \equiv (F_{2p} + F_{2n})/2.)$

⇒ "EMC ratio"

$$R(x) = \frac{F_{2A}(x_A)}{F_{2N,\mathrm{free}}(x)} \qquad \left(x_A = x \cdot \frac{M_N}{\overline{M}_N}\right)$$

– Single (valence) proton case ($H = -j, \dots + j$):

$$g_{1A}^{(H)}(x_A) = \frac{1}{2}\left(\frac{4}{9}\Delta u_A^{(H)}(x_A) + \frac{1}{9}\Delta d_A^{(H)}(x_A)\right)$$
$$= \int_{x_A}^{A} \frac{\mathrm{d}y_A}{y_A} \Delta f_{N/A}^{(H)}(y_A) \cdot g_{1p}\left(\frac{x_A}{y_A}\right)$$

⇒ "Spin dependent EMC ratios"

$$R_s^{(H)}(x) = \frac{g_{1A}^{(H)}(x_A) \,/\, \langle \Sigma_N \rangle_A^{(\mathrm{Schmidt})}}{g_{1p,\mathrm{free}}(x)}$$

II) Model calculations

Aim: Medium modifications of nucleon structure functions.
More general: Role of quark degrees of freedom in nuclear systems.

Input:

- **Effective quark theory for single nucleon**:
  Nambu-Jona-Lasinio (NJL) model,
  Relativistic Faddeev Approach ⇒
  Quark-Diquark Model
- **Method to construct nuclear matter EOS**:
  Mean field approximation

Description of the nucleon:

NJL model:

$$\mathcal{L}_{\rm NJL} = \overline{\psi}\,(i\partial\!\!\!/ - m)\,\psi +$$  $(\mathcal{L}_I$, chir. symm.)

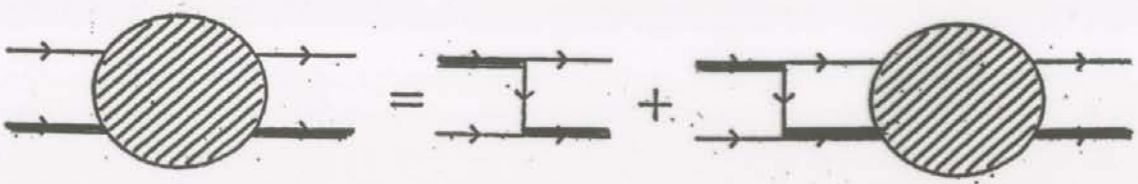
⇒ 2-body problem (→ diquarks, mesons) becomes very simple

⇒ Solve relativistic Faddeev equation

≡ effective BS equation for quark + diquark

(see: N. Ishii et al., NPA **587** (1995) 617)

0^+ ('**scalar diquark**') channel is most important
1^+ ('**axial vector diquark**') channel: Weight is limited by the ratio F_{2p}/F_{2n}. *Not considered in this talk ⇒ Talk of Ian Cloet.*



Simple approximation used for finite density calculation:
"quark-diquark model"
(Momentum dependence of exchanged quark propagator is neglected ⇒ Analytic solution. Very simple and covariant.)

Simple method to avoid unphysical decay thresholds:
Infrared cut-off (in addition to usual UV cut-off) in the **proper time regularization scheme** (Hellstern et al, NPA **625** (1997) 679).

4-Fermi coupling constant in scalar diquark channel is adjusted to reproduce M_N.

Description of nuclear matter (NM)

In the mean field approximation the energy density is simply expressed as

$$\mathcal{E}_N(M) = \mathcal{E}_{\rm vac}(M) + \gamma_N \int^{p_F} \frac{d^3k}{(2\pi)^3}\sqrt{M_N(M)^2 + k^2} + \mathcal{E}_\omega$$

- Constituent quark mass $M \simeq$ scalar field.
- $\mathcal{E}_\omega \ldots$ Arises from mean vector field.
- $\mathcal{E}_{\rm vac}(M) =$ quark loop $\simeq$ Mexican hat potential.
- Effects of nucleon structure are summarized in the function $M_N(M)$ (from quark-diquark equation).

Elementary nucleon: M_N drops linearly with scalar field.
Composite nucleon: $M_N(M)$ has a **curvature** ($\Leftrightarrow$ scalar polarizability), which is important for **saturation**.
(See: W. Bentz, A.W. Thomas, NPA **696** (2001) 138.)

Spin-independent structure functions:

EMC ratio:

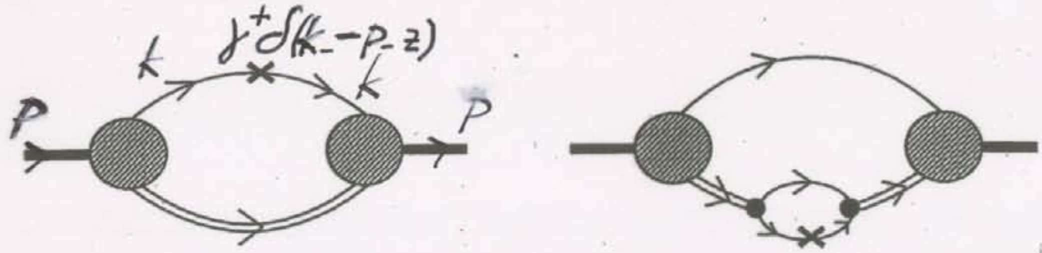
$$R(x) = \frac{F_{2A}(x_A)}{F_{2N}(x)} = \frac{x_A\, q_A(x_A)}{x\, q_N(x)}$$

$x_A \ldots$ Bjorken variable for nucleus $\times A$,
$x \ldots$ Bjorken variable for nucleon $\Rightarrow x_A = x\, M_{N0}/(M_A/A) = x\,(940/925)$.

Spin-independent (isoscalar) quark distribution in nucleus:

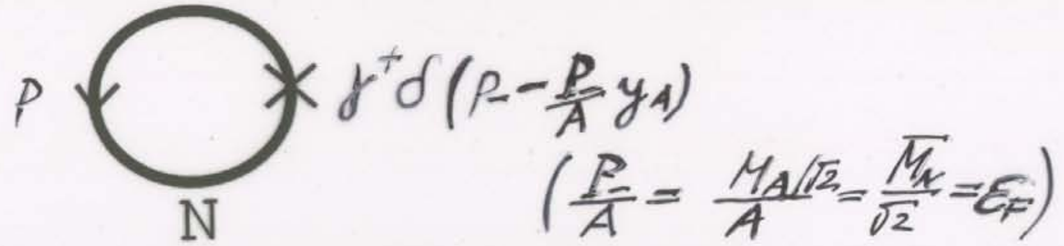
$$q_A(x_A) = \int_{x_A}^{A} \frac{\mathrm{d}y_A}{y_A} q_N\left(x_A/y_A\right) f_{N/A}(y_A)$$

- Quark distribution in nucleon ($q_N(z)$): Calculate the following diagrams in the presence of scalar and vector mean fields:



(For free nucleon, see: H. Mineo et al, PRC **60** (1999) 065201.)

- Nucleon distribution in medium ($f_{N/A}(y_A)$): Calculate



(see: G.A. Miller and J.R. Smith, PRC **65** (2002) 015211.)

This covariant approach satisfies all sum rules.

Binding effects on level of point nucleons cannot explain the EMC effect, but ...

Mean vector field has direct effect on $q_A(x_A)$.

$$q_A(x_A) = \frac{\epsilon_F}{E_F} q_{A0}\left(x'_A = \frac{\epsilon_F}{E_F} x_A - \frac{V_0}{E_F}\right)$$

- $\epsilon_F = \sqrt{k_F^2 + M_N^2} + 3V_0 \equiv E_F + 3V_0$ ... Fermi energy of nucleon. (V_0 ... mean vector field acting on a quark.)
- $q_{A0}(x'_A)$ ... distribution without direct effect of vector field (i.e., only scalar field and Fermi motion).

- Origin of this relation: Including the vector field, we have

$$x_A = k/(P/A) = k/\epsilon_F \,, \qquad (k \equiv k_-,\ P \equiv P_- = P_0/\sqrt{2}, \text{ etc})$$

while without the vector field we have

$$x'_A = k'/(P'/A) = k'/E_F \,.$$

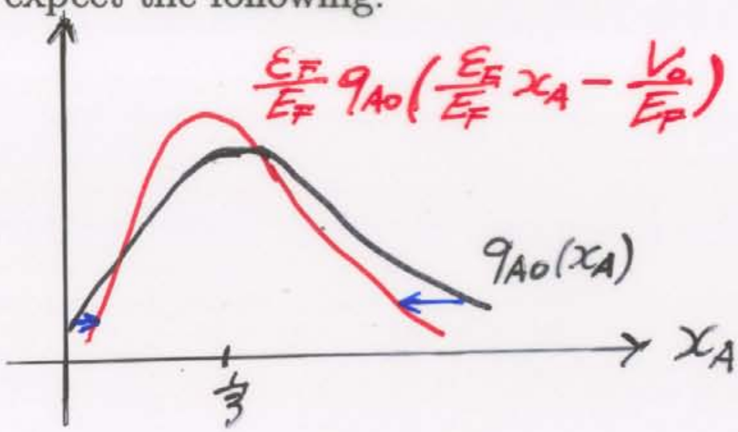
Here the quark (light cone) momentum is shifted by V_0 ($\Rightarrow k = k' + V_0$), and the total (light cone) momentum per nucleon (=chemical potential) by $3V_0$ ($\Rightarrow \epsilon_F = E_F + 3V_0$). Then

$$x'_A = \frac{k'}{P'/A} = \frac{k - V_0}{E_F} = \frac{k}{\epsilon_F}\frac{\epsilon_F}{E_F} - \frac{V_0}{E_F} = x_A \frac{\epsilon_F}{E_F} - \frac{V_0}{E_F}$$

- This rescaling of Bjorken variable implies:

$$x_A > \frac{1}{3} \Rightarrow x'_A > x_A \,, \qquad x_A < \frac{1}{3} \Rightarrow x'_A < x_A$$

and we can expect the following:



Result:

- Depletion of $R(x)$ for $0.4 < x < 0.8$
- Enhancement of $R(x)$ for $x \simeq 0.2$

Consistent with observed EMC effect, except for the small x region ($\Rightarrow \overline{q}$ should also be included.)

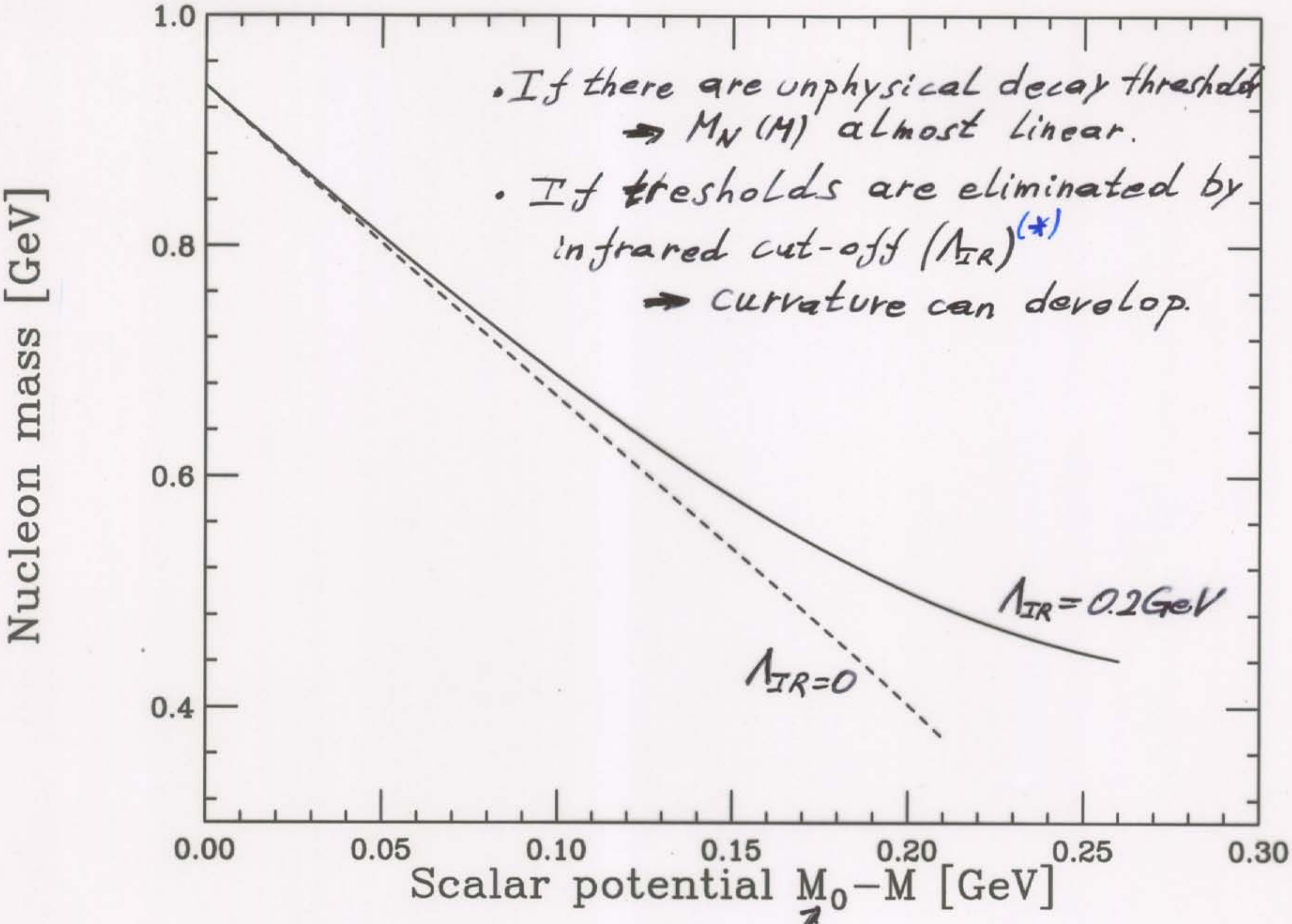
Summary

Effective chiral quark theories provide a simple tool to incorporate nucleon structure into many-body physics.

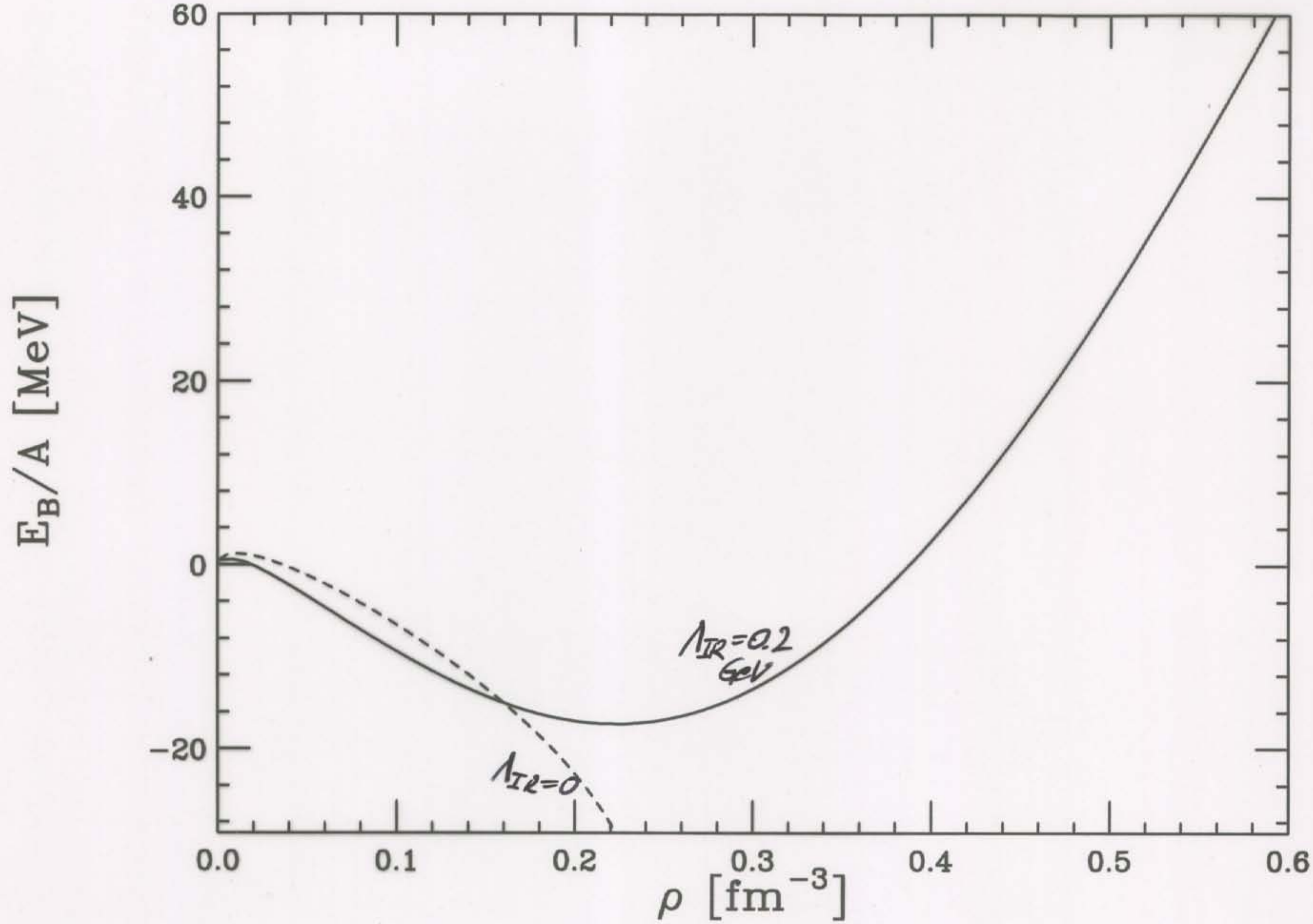
In our investigation, we found the following points:

- Binding effects on quark level (i.e., scalar and vector fields acting on quarks in the nucleons) can largely reproduce the EMC effect.
  ($\Leftrightarrow$ Direct effect of vector field on $q_A(x_A)$.)

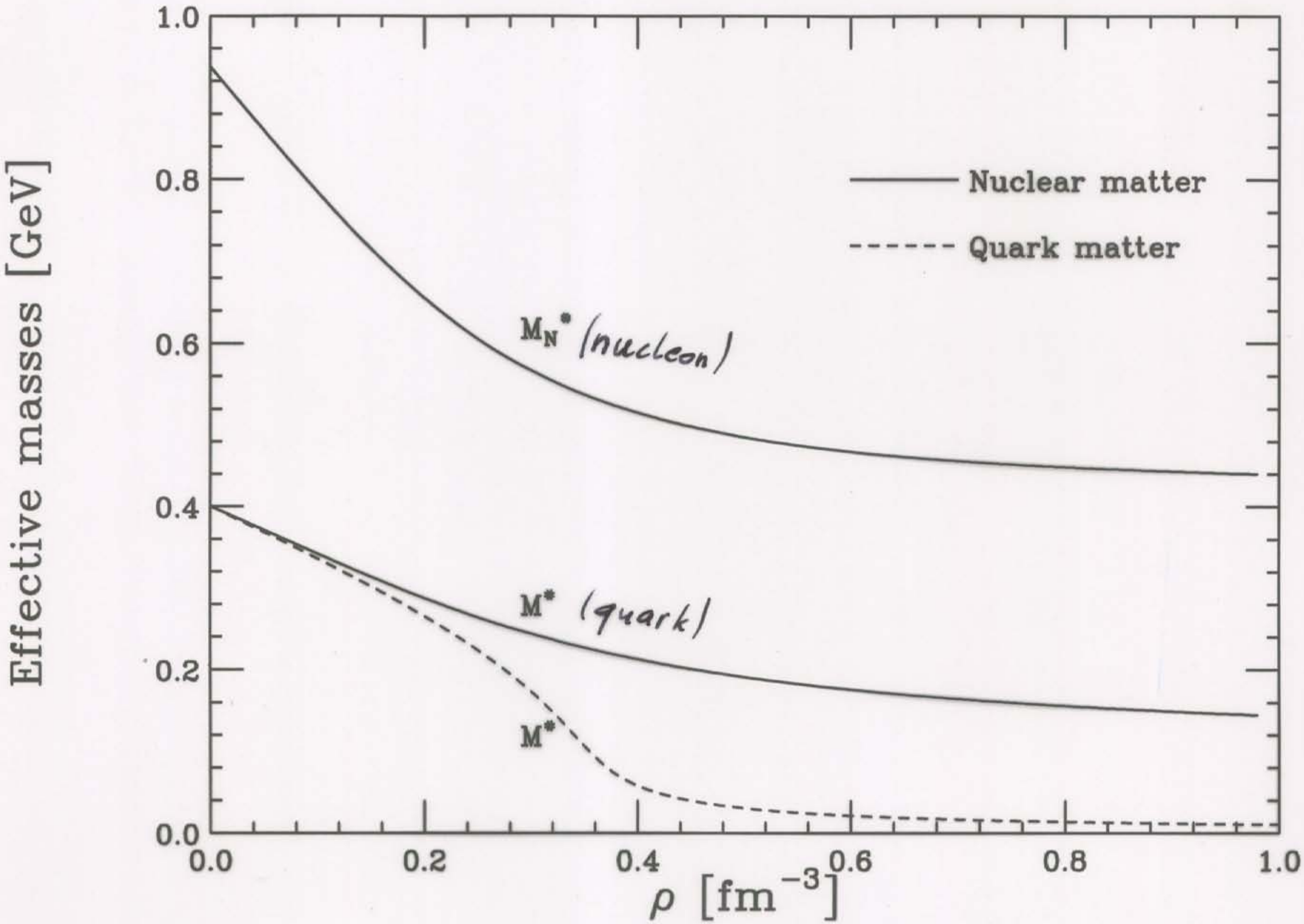
- Medium modification of spin-dependent structure function ($g_1(x)$) seems to be larger than spin-independent case.



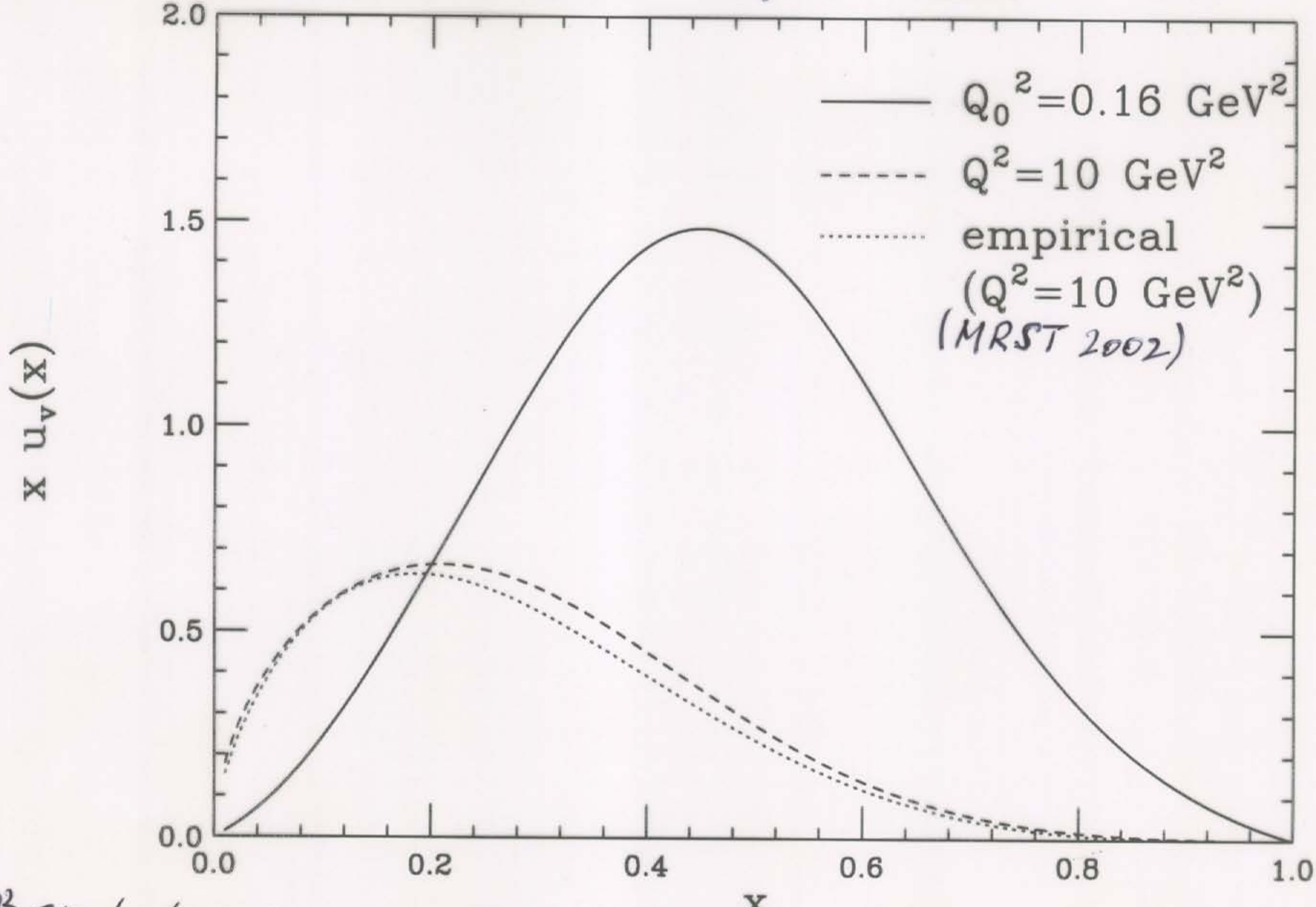
(*) in proper-time regularization:
see G. Hellstern, R. Alkofer, H. Reinhardt, NPA 625 ('97) 697.



60
40
20
0
−20
0.0
0.1
0.2
0.3
0.4
0.5
0.6
E_B/A [MeV]
ρ [fm^-3]
Λ_IR=0.2 GeV
Λ_IR=0

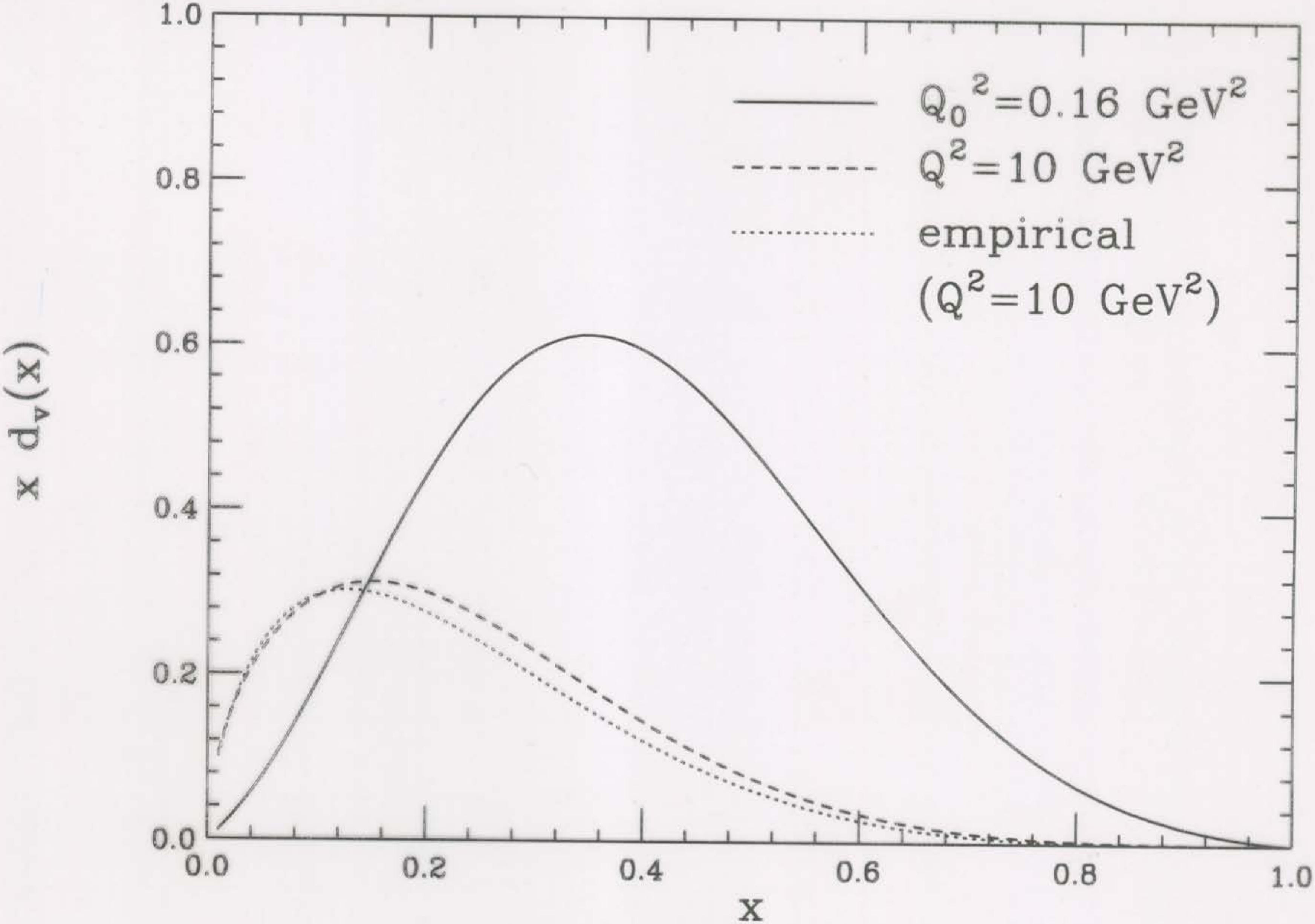

Effective masses [GeV]
ρ [fm^-3]
0.0
0.2
0.4
0.6
0.8
1.0
Nuclear matter
Quark matter
M_N* (nucleon)
M* (quark)
M*

Uv distribution in free nucleon

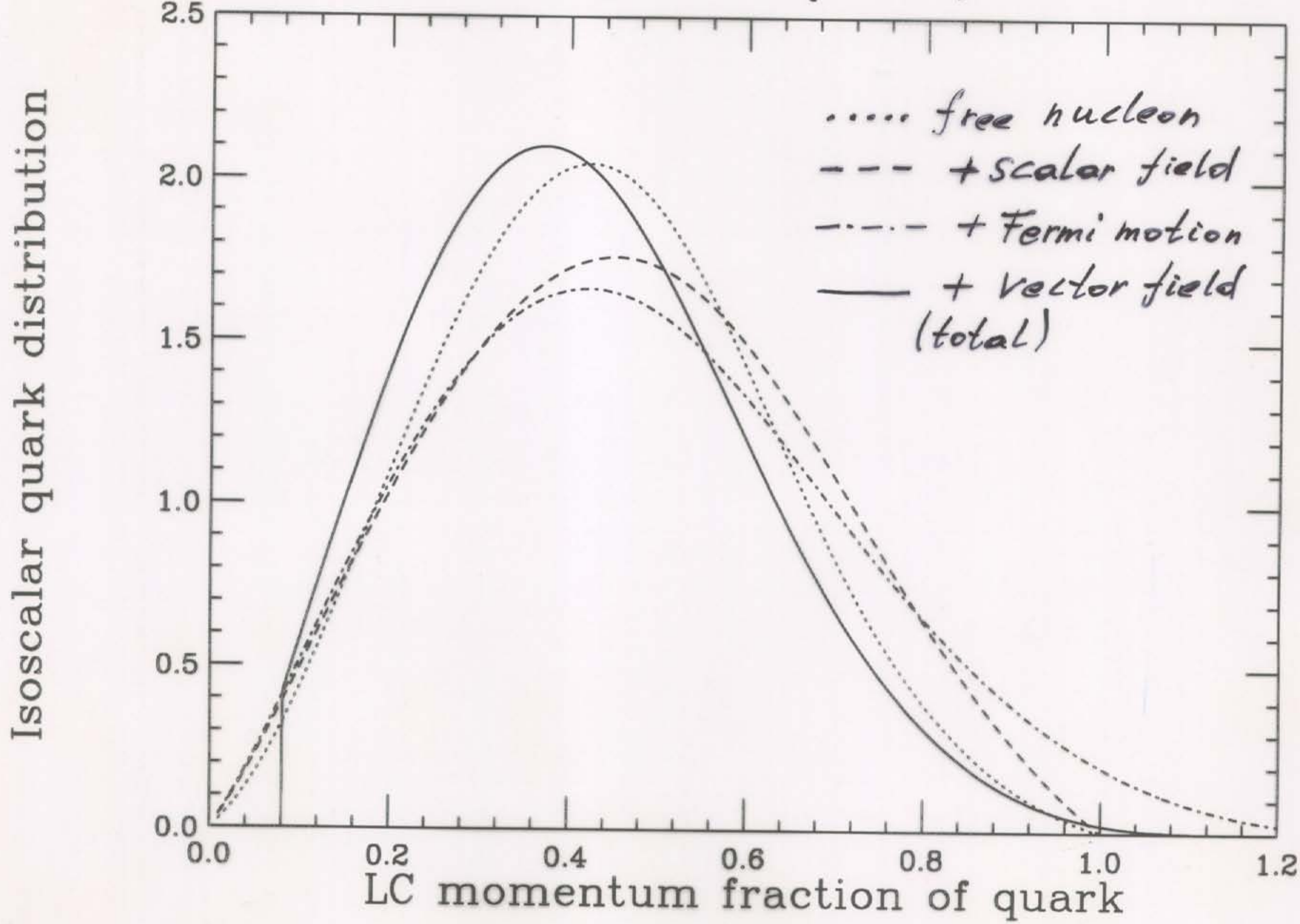


Q^2 evolution in NLO : M. Miyama, S. Kumano, Comp. Phys. Commun. 94 (1996) 185.

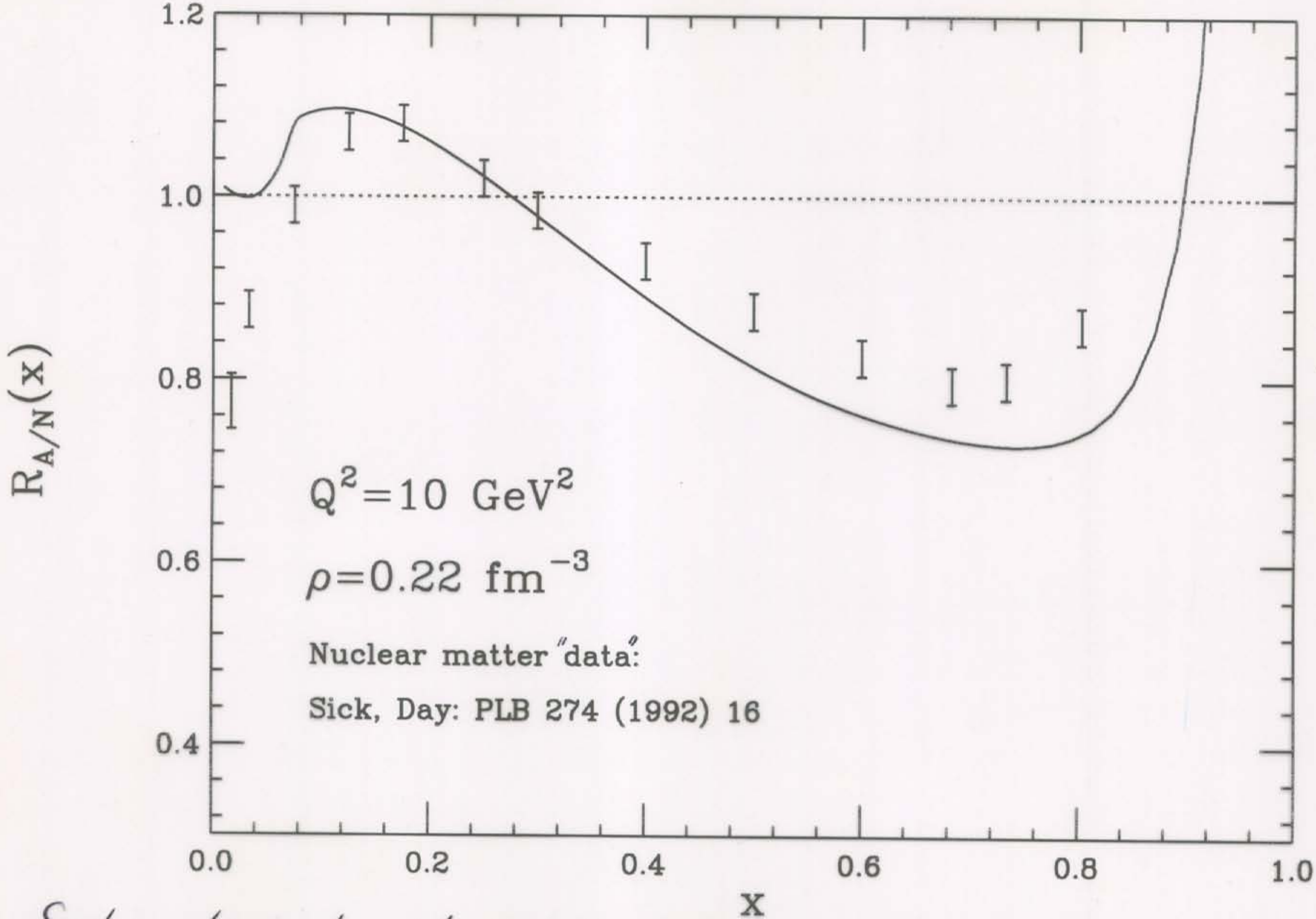
d_v distribution in free nucleon



Medium effects on $(u_v + d_v)$

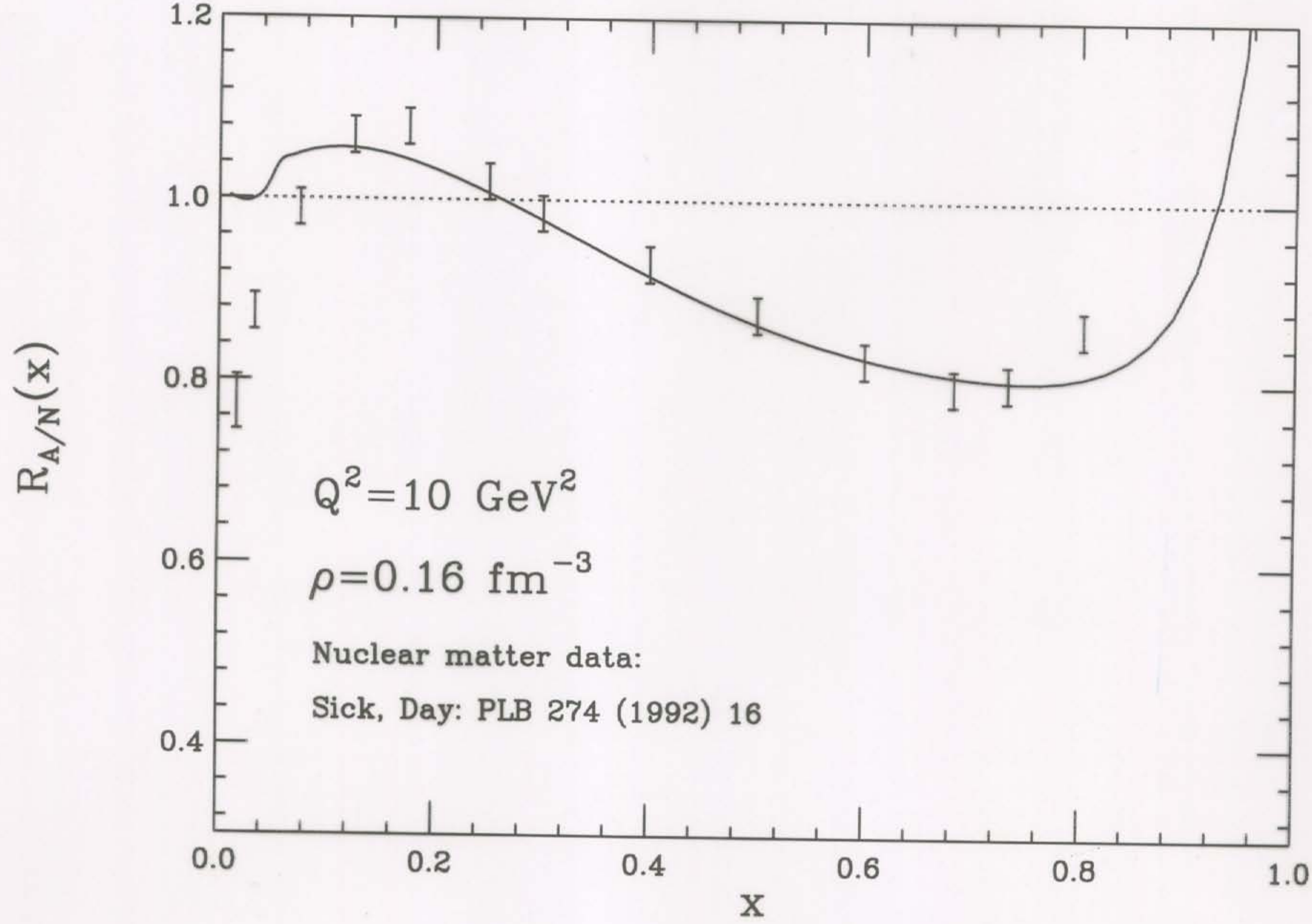


EMC ratio



Saturation density is too high ⇒ EMC effect too large.

EMC ratio



1.2
1.0
0.8
0.6
0.4
$R_{A/N}(x)$
$Q^2=10\ GeV^2$
$\rho=0.16\ fm^{-3}$
Nuclear matter data:
Sick, Day: PLB 274 (1992) 16
0.0
0.2
0.4
0.6
0.8
1.0
X

Spin-dependent structure functions

(First estimates for single proton states ...)

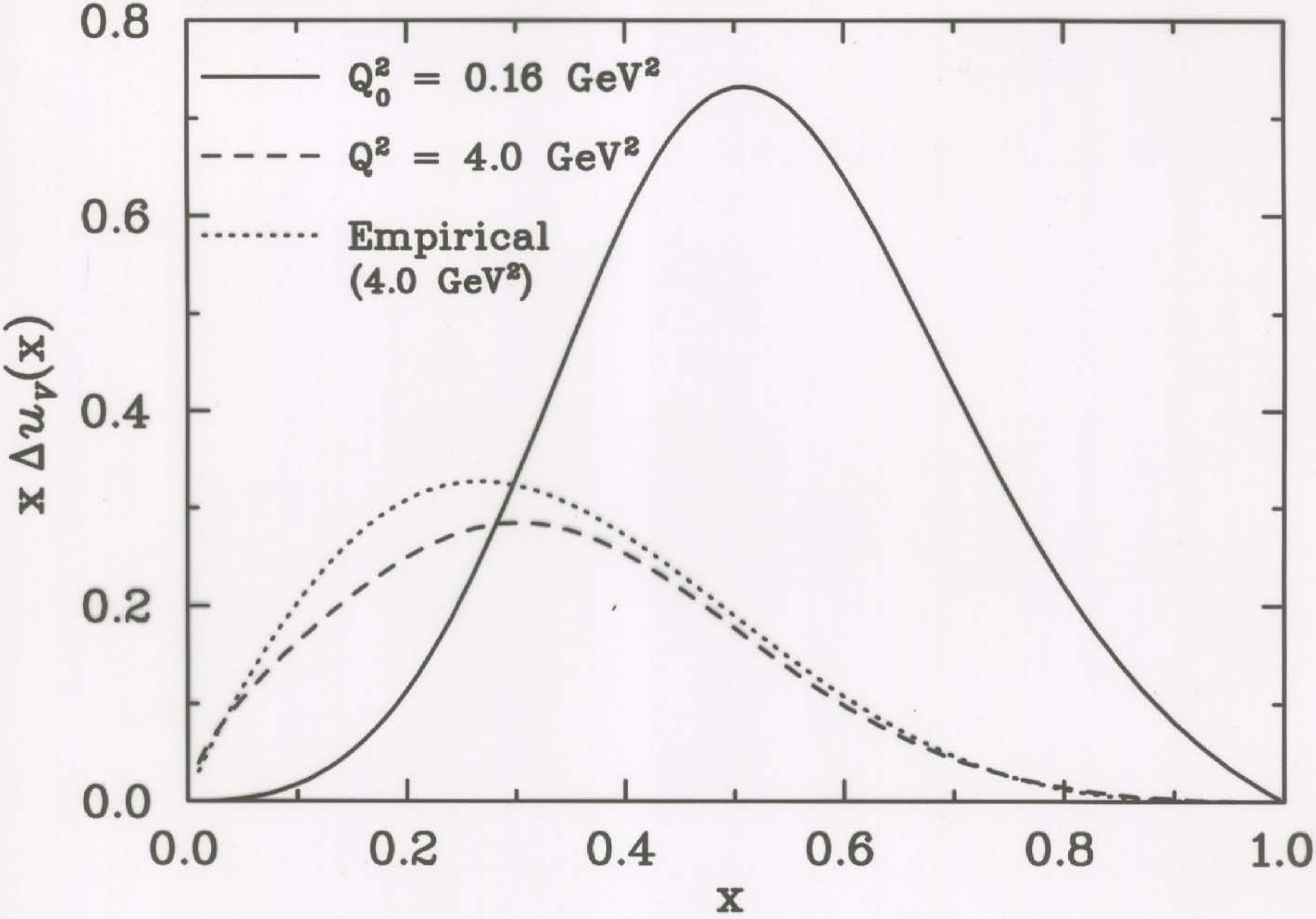
- $\Delta\, q_N(z)$ obtained from same diagrams as for spin-independent case.
  If only 0^+ diquark is included $\Rightarrow \Delta d(x) = 0$. (In this calculation: $\Delta u = 0.69$ for free proton ($\Leftrightarrow$ *empirically* : $\Delta u_V = 0.94$).
  To get also $\Delta d(x)$, the 1^+ diquark should be included ($\Rightarrow$ *Talk of Ian Cloet*).
- The nucleon distribution $\Delta f^{(H)}_{N/A}(y_A)$ should refer to a valence nucleon state, and should be calculated in a finite nucleus (*in progress...*)
  Here: Use the spin independent distribution $f_{N/A}(y_A)$ from nuclear matter to estimate the Fermi motion effects. (That is, replace $\Delta f^{(H)}_{N/A}(y_A) \to f_{N/A}(y_A) \times \langle \Sigma_N \rangle_A^{\text{Schmidt}}$.)
- Mean vector field has the same direct effect as for the spin-independent case:

$$\Delta q_A(x_A) = \frac{\epsilon_F}{E_F}\, \Delta q_{A0}\left(x'_A = \frac{\epsilon_F}{E_F} x_A - \frac{V_0}{E_F}\right)$$

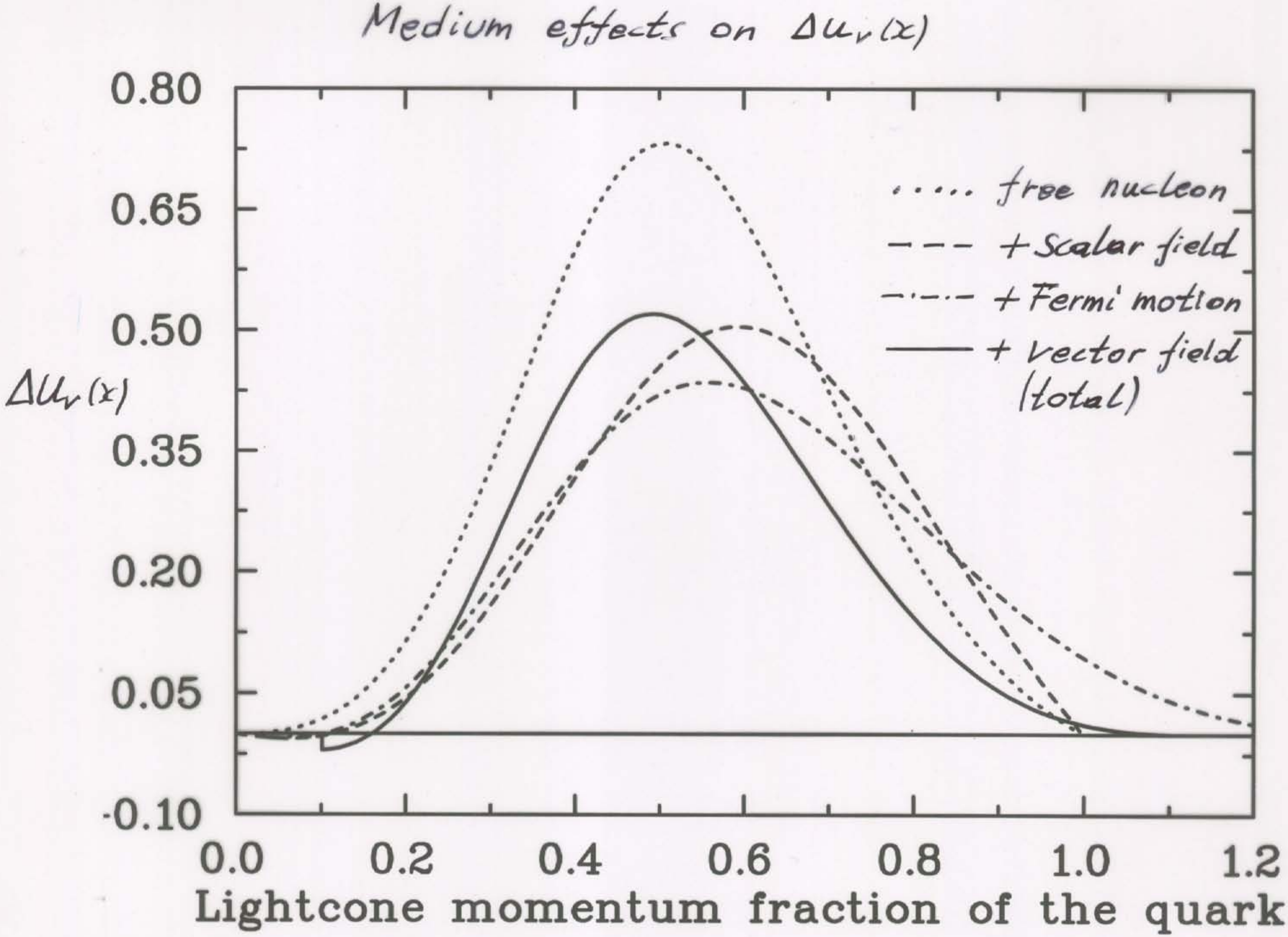
  where Δq_{A0} includes only the effect of the scalar mean field and Fermi motion.

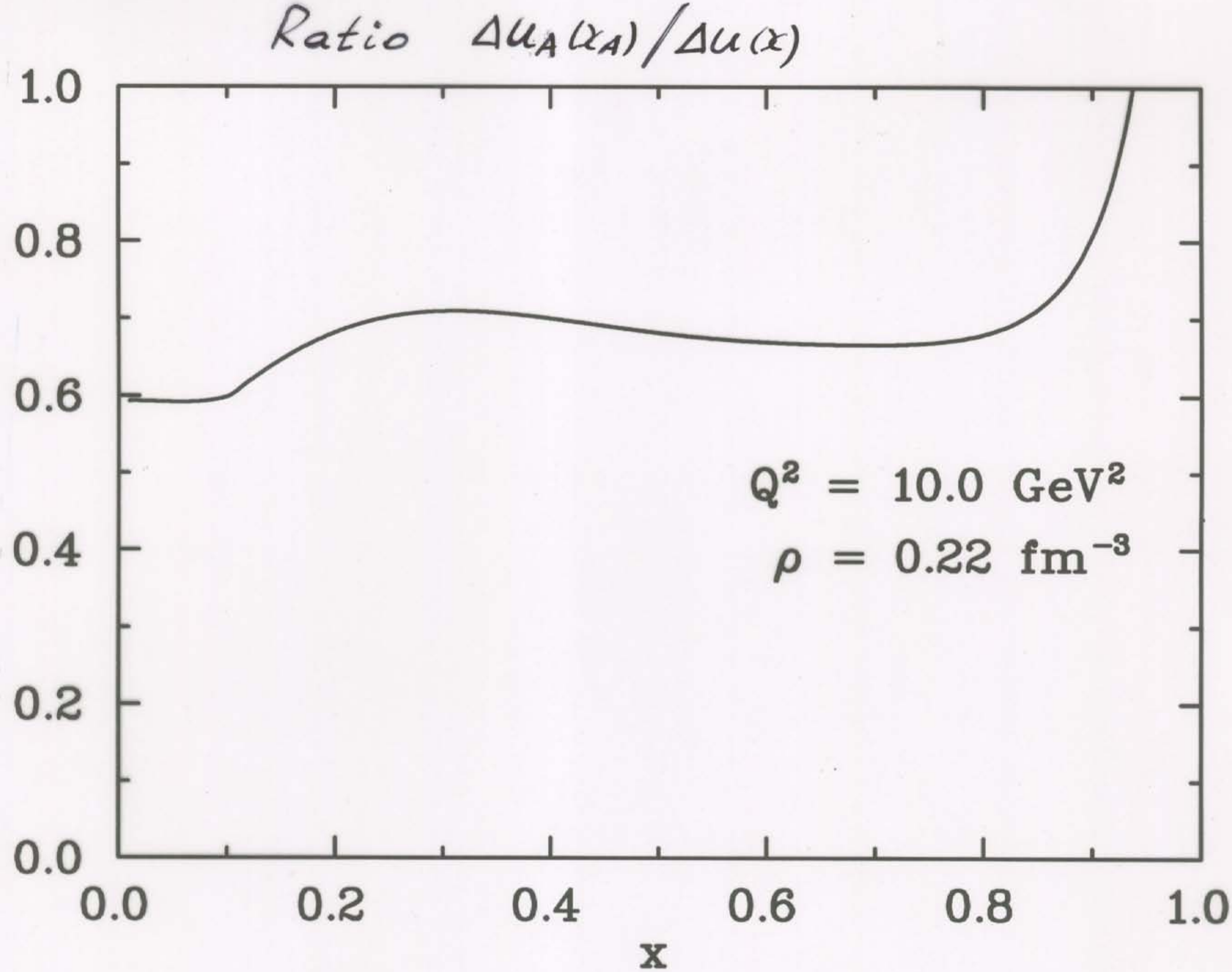
Results:

- Empirical $\Delta u_V(x)$ in free proton can be reproduced reasonably.
- The ratio $\dfrac{\Delta u_A(x_A)}{\Delta u(x)}$ shows a plateau for $0.2 < x < 0.8$ with an average value of about 0.7.



0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
x Δu_v(x)
Q_0^2 = 0.16 GeV^2
Q^2 = 4.0 GeV^2
Empirical
(4.0 GeV^2)


Medium effects on $\Delta u_v(x)$
$\Delta u_v(x)$
0.80
0.65
0.50
0.35
0.20
0.05
-0.10
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Lightcone momentum fraction of the quark
free nucleon
+ Scalar field
+ Fermi motion
+ Vector field (total)



Ratio ΔuA(xA)/Δu(x)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
Q² = 10.0 GeV²
ρ = 0.22 fm⁻³