Color Transparent GPDs?

or: size does matter

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Motivation

- color transparency (CT): phenomenon where hadron produced inside nucleus at large momentum transfer experiences little final state interaction

$\hookrightarrow$ if CT occurs then cross section should should scale $\sim A$

- pert. QCD has been successful in qualitative description of form factors at large $\mathbf{\Delta}_\perp^2$
- pert. QCD also predicts CT at large $\mathbf{\Delta}_\perp^2$
- exp: no clear evidence for CT @Jlab
- Can GPDs shed some light on physics of form factors and CT? (Can CT shed some light on physics of GPDs?)

Form Factors and Light-Cone Wave Functions

- 2 partons:

$$F(-\boldsymbol{\Delta}_\perp^2) = \int dx \int d^2\mathbf{k}_\perp \psi^*(x, \mathbf{k}_\perp)\psi\left(x, \mathbf{k}_\perp + (1-x)\boldsymbol{\Delta}_\perp\right)$$

- similar convolution formulas hold for 3 or more partons
- hadron form factor obtained by summing over all Fock components
- In general, large $\boldsymbol{\Delta}_\perp$ probes behavior of LC w.f. at large $\mathbf{k}_\perp$

↪ use pQCD to calculate $\psi(x, \mathbf{k}_\perp)$

↪ pQCD predictions for form factors

- However, for example for $x = 0.75$ and $\boldsymbol{\Delta}_\perp^2 = 4\,GeV^2$, momentum transfer from active quark to spectator only $(1-x)\boldsymbol{\Delta}_\perp = 0.5\,GeV$

↪ significant contributions to form factor from nonperturbative scales at these values of $\boldsymbol{\Delta}_\perp$ possible

Generalized Parton Distributions (GPDs)

- GPDs are defined as **nonforward** matrix elements of same operator whose **forward** matrix elements are the usual PDFs

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p\right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$
$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+\gamma_5 q\left(\frac{x^-}{2}\right)\right| p\right\rangle = \tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5 u($$
$$+\tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p$$

where $\Delta = p' - p$ is the momentum transfer and ξ measures the longitudinal momentum transfer on the target $\Delta^+ = \xi(p^+ + p^{+\prime})$.

Parton Interpretation

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- Actually $H = H(x, \xi, \Delta^2, Q^2)$, but will not discuss Q^2 dependence of GPDs today! (will assume $Q^2 > -\Delta^2$)
- x is mean long. momentum fraction carried by active quark
- ξ measures longitudinal momentum transfer $\xi = \frac{p^{+\prime} - p^+}{p^+ + p^{+\prime}}$
- In general no probabilistic interpretation since initial and final state not the same $\longrightarrow$ interpretation as transition amplitude
- $\int dx H(x,\xi,\Delta^2) = F_1(\Delta^2)$ and $\int dx E(x,\xi,\Delta^2) = F_2(\Delta^2)$

$\hookrightarrow$ GPDs provide a decomposition of form factor w.r.t. the momentum fraction (in IMF) carried by the active quark

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$\hookrightarrow$ GPDs provide a decomposition of form factor w.r.t. the momentum fraction (in IMF) carried by the active quark

$\hookrightarrow$ knowledge of GPDs $\Rightarrow$ better understanding of form factors

Overlap Representation for GPDs

$$H(x,0,-\boldsymbol{\Delta}_\perp^2) = \sum_N \sum_j \int [dx]_N \int [d^2\mathbf{k}_\perp]_N \delta(x-x_j) \Psi_N^*(x_i, \mathbf{k}'_{\perp,i}) \Psi_N(x_i, \mathbf{k}_{\perp,i})$$

with

$$\mathbf{k}'_{\perp,i} = \mathbf{k}_{\perp,i} + x_i \boldsymbol{\Delta}_\perp \qquad (i \neq j) \qquad \text{and} \qquad \mathbf{k}'_{\perp,j} = \mathbf{k}_{\perp,j} - (1-x_j)\boldsymbol{\Delta}_\perp$$

$$[dx]_N = \prod_{i=i}^N dx_i \delta\left(1 - \sum_{i=1}^N x_i\right) \qquad [d^2\mathbf{k}_\perp]_N = \prod_{i=i}^N d^2\mathbf{k}_{\perp,i} \delta\left(\sum_{i=1}^N \mathbf{k}_{\perp,i}\right)$$

- Overlap rep. in principle exact; all higher order effects (DGLAP et al.) included in light-cone wave functions

GPDs and the $\perp$ size of hadrons

- GPDs allow simultaneous determination of longitudinal momentum and transverse position of partons

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $\mathbf{b}_\perp$ measured relative to the $\perp$ center of momentum $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{b}_{\perp,i}$

$\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \to 1$, since the active quark becomes the $\perp$ center of momentum in that limit! $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp^2$-indep. as $x \to 1$. Confirmed by recent lattice studies (QCDSF, LHPC), which compared form factors for 1^{st}, 2^{nd}, and 3^{rd} x-moment of $H(x, 0, -\mathbf{\Delta}_\perp^2)$

Overlap Representation in Impact Parameter Rep.

$$\Psi_N(x_i, \mathbf{k}_{\perp,i}) = \int [d^2\mathbf{b}_\perp]_N \exp\left[-i\sum_{i=1}^{N} \mathbf{k}_{\perp,i}\cdot\mathbf{b}_{\perp,i}\right] \tilde{\Psi}(x_i, \mathbf{b}_{\perp,i})$$

with

$$[d^2\mathbf{b}_\perp]_N = \prod_{i=i}^{N} dx_i \delta\left(\sum_{i=1}^{N} \mathbf{b}_{\perp,i}\right)$$

$\hookrightarrow$

$$q(x, \mathbf{b}_\perp) = \sum_N \sum_j \int [d^2\mathbf{b}_\perp]_N \delta(\mathbf{b}_\perp - \mathbf{b}_{\perp,j}) \left|\tilde{\Psi}(x_i, \mathbf{b}_{\perp,i})\right|^2$$

- GPDs can be expressed as Fourier transform of single particle density

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

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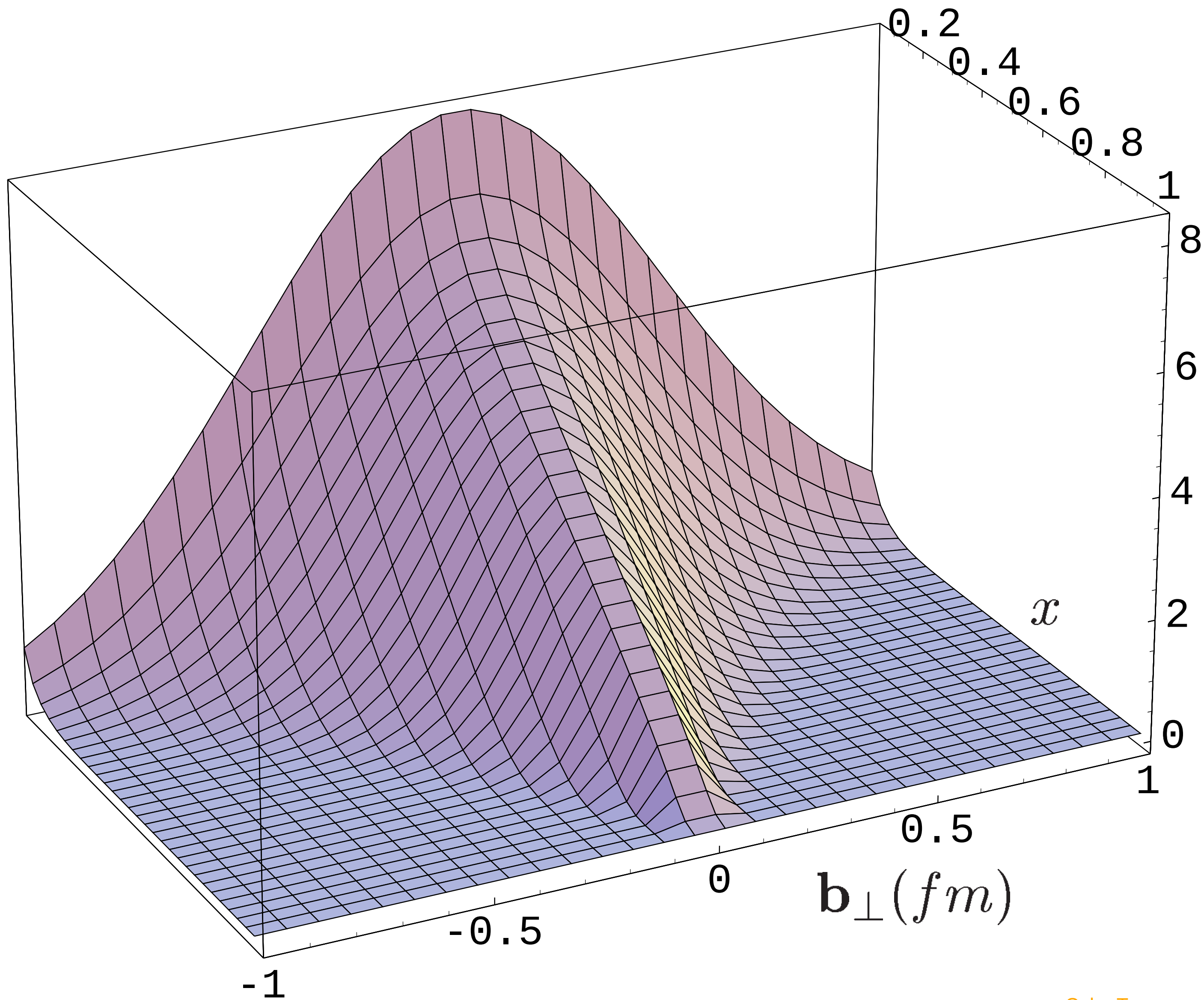
$q(x, \mathbf{b}_\perp)$ in a simple model



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GPDs and the $\perp$ size of hadrons

- $q(x, \mathbf{b}_\perp)$ becomes very narrow for large x

$\hookrightarrow$ conceivable to have significant contribution from large x to
$F_1(-\boldsymbol{\Delta}_\perp^2) = \int dx H(x, 0, -\boldsymbol{\Delta}_\perp^2) = \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) e^{i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$

- Note: distribution becoming narrow in $\mathbf{b}_\perp$ does <u>not</u> have to mean the hadron becomes small
  - $\mathbf{b}_\perp$ measures distance from active quark to center of momentum $\mathbf{R}_\perp = \sum_i x_i \mathbf{b}_{\perp,i}$

  $\hookrightarrow$ distance $\mathbf{r}_\perp$ between active quark and center of momentum of all spectators given by

$$\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$$

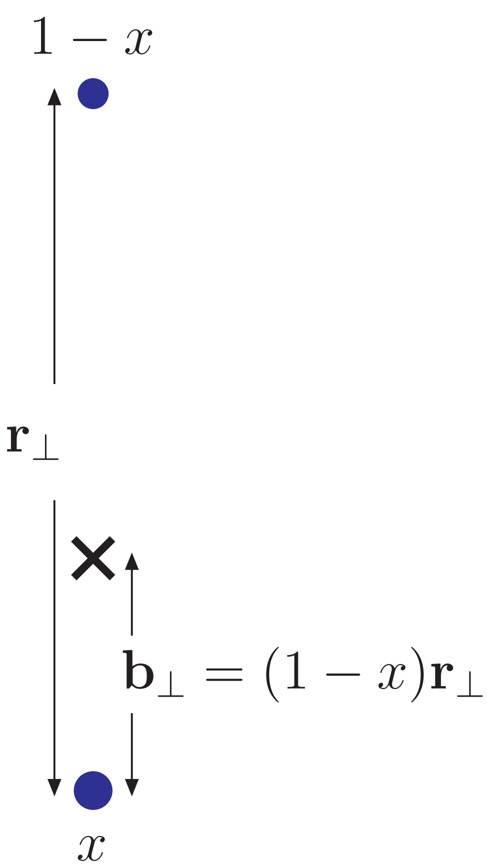
  - example: 2 partons:

$$\mathbf{r}_\perp = \mathbf{r}_{\perp,1} - \mathbf{r}_{\perp,2} \qquad \mathbf{R}_\perp = x\mathbf{r}_{\perp,1} + (1-x)\mathbf{r}_{\perp,2}$$

$\hookrightarrow$ $\mathbf{b}_\perp \equiv \mathbf{r}_{\perp,1} - \mathbf{R}_\perp = (1-x)\mathbf{r}_\perp$

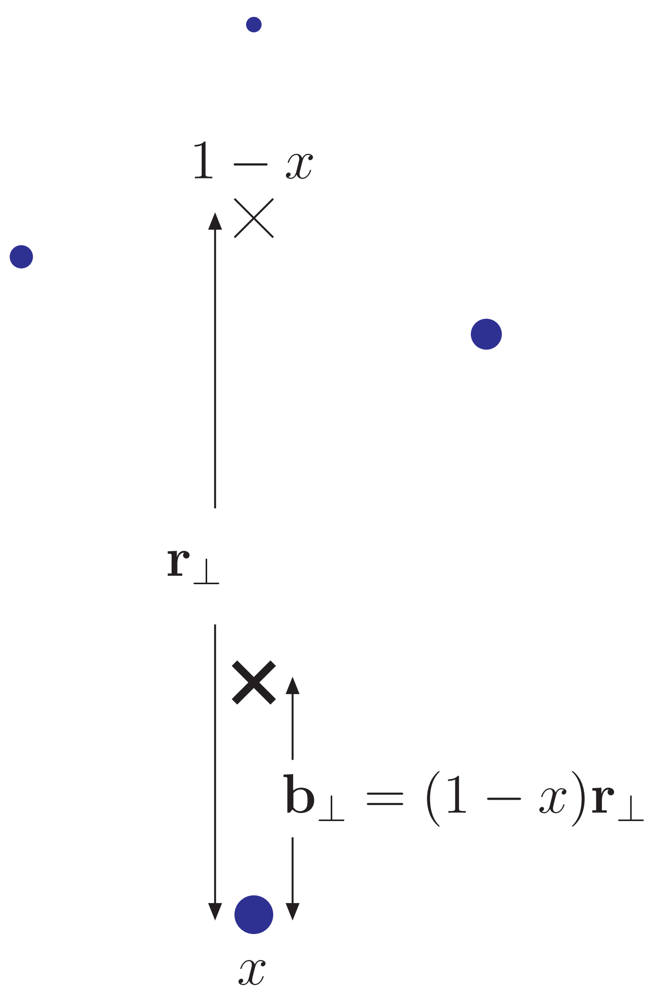
GPDs and the ⊥ size of hadrons

- relation between $\mathbf{b}_\perp$ and the separation $\mathbf{r}_\perp$ between the active quark and the spectator for a 2-parton system



GPDs and the $\perp$ size of hadrons

- relation between $\mathbf{b}_\perp$ and the separation $\mathbf{r}_\perp$ between the active quark and the center of momentum of all spectators for a multi-parton system



GPDs and the $\perp$ size of hadrons

- $q(x, \mathbf{b}_\perp)$ becomes very narrow for large x

$\hookrightarrow$ conceivable to have significant contribution from large x to $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x,0,-\mathbf{\Delta}_\perp^2) = \int dx \int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$

- Note: distribution becoming narrow in $\mathbf{b}_\perp$ does not have to mean the hadron becomes small
  - $\mathbf{b}_\perp$ measures distance from active quark to center of momentum $\mathbf{R}_\perp = \sum_i x_i \mathbf{b}_{\perp,i}$

  $\hookrightarrow$ distance $\mathbf{r}_\perp$ between active quark and center of momentum of all spectators given by

$$\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$$

  $\hookrightarrow$ finite $\perp$ size implies

$$\frac{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)\mathbf{b}_\perp^2}{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)} \propto (1-x)^2$$

GPDs and the $\perp$ size of hadrons

- Whether or not large x yield a significant contribution to $F_1(-\mathbf{\Delta}_\perp^2)$ at large $\mathbf{\Delta}_\perp^2$ depends on details of the shape of $q(x,\mathbf{b}_\perp)$, such as
  - how rapidly $q(x) = \int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)$ goes to zero for large x
  - how rapidly $q(x,\mathbf{b}_\perp)$ becomes narrow for large x
- wave function components where active quark carries momentum fraction x have $\perp$ size larger than

$$d_\perp^2(x) \equiv \frac{1}{(1-x)^2}\frac{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)\mathbf{b}_\perp^2}{\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp)}$$

- Note: commonly used ansatz $\psi(x,\mathbf{k}_\perp) \sim f(x)\exp\left(-const.\frac{\mathbf{k}_\perp^2}{x(1-x)}\right)$ would yield $\tilde{\psi}(x,\mathbf{r}_\perp) \sim f(x)\exp\left(-c\mathbf{r}_\perp^2 x(1-x)\right)$, resulting in a divergent $\perp$ size as $x \to 1$.

Model for GPDs at large x

- at large $\perp$ separations, gluon components in wave-function will become significant

$\hookrightarrow$ want to investigate how large size configurations get dynamically suppressed

- represent gluon string in π by one effective degree of freedom with invariant mass $M_s^2 = (\sigma \mathbf{r}_\perp)^2$
- quarks can interact with ends of "string"

$$\begin{aligned} M^2\psi(x,y,\mathbf{r}_\perp) &= T\psi(x,y,\mathbf{r}_\perp) + G^2\int_0^{1-y} dx' \frac{\psi(x,y,\mathbf{r}_\perp)-\psi(x',y,\mathbf{r}_\perp)}{(x-x')^2} \\ &+ G^2\int_0^{1-x} dy' \frac{\psi(x,y,\mathbf{r}_\perp)-\psi(x,y',\mathbf{r}_\perp)}{(x-x')^2} \end{aligned}$$

where

$$T = \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{y} + \frac{\sigma^2\mathbf{r}_\perp^2}{z} + \frac{\mathbf{k}_\perp^2}{x+\frac{z}{2}} + \frac{\mathbf{k}_\perp^2}{y+\frac{z}{2}}$$

Model for GPDs at large x

- Main result: $\perp$ size $d_\perp(x)$ decreses with x but remains finite for $x \longrightarrow 1$
- physics: both kinetic $\frac{\mathbf{k}_\perp^2}{y+z/2}$ and potential $\frac{\mathbf{r}_\perp^2}{z}$ have divergent coefficients when $x \longrightarrow 1$

$\hookrightarrow$ energy minimized if both $\mathbf{k}_\perp$ and $\mathbf{r}_\perp$ stay finite.

- will in the following assume that $d_\perp(x)$ remains finite as $x \rightarrow 1$

PDFs at large x

- nucleon
  - (exp.) $q(x) = c(1-x)^\beta$, where $\beta \approx 3$
  - However, not well enough known for very large x
  - pQCD: $q(x) \sim (1-x)^3$, but suppression due to higher order corrections expected at very large x

$\hookrightarrow$ will make ansatz $q(x) = c(1-x)^3$, plus possible correction (suppression) for very large x

Example for $x \to 1$ contribution to form factor

- assume (conjecture) hadron has finite size $1/\Lambda$ even when $x \to 1$

$\hookrightarrow$ $q(x, \mathbf{b}_\perp) = 0$ for $\mathbf{b}_\perp^2 > \frac{(1-x)^2}{\Lambda^2}$

$\hookrightarrow$ significant contribution to form factor from quarks with $x > 1 - \frac{\Lambda}{|\mathbf{\Delta}_\perp|}$, where $|\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp| < 1$

$$F_{large\,x}(\mathbf{\Delta}_\perp^2) \equiv \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^1 dx\, H(x, 0, \mathbf{\Delta}_\perp^2) = \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^1 dx \int d^2\mathbf{b}_\perp e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} q(x, \mathbf{b}_\perp)$$

$$= \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^1 dx q(x) = \frac{c}{4} \frac{\Lambda^4}{\mathbf{\Delta}_\perp^4} \qquad \text{for} \quad q(x) = c(1-x)^3.$$

$\hookrightarrow$ obtain obtain (fake) pQCD-like behavior from $x \to 1$ (Feynman mechanism)

- Note: if $q(x) = c(1-x)^{2N_s-1}$ up to $x = 1$, this will always yield $F_{large\,x}(\mathbf{\Delta}_\perp^2) \propto \frac{1}{\mathbf{\Delta}_\perp^{2N_s}}$

Example for $x \to 1$ contribution to form factor ($x < 1 -$

- assume (conjecture) hadron has finite size $1/\Lambda$ even when $x \to 1$

$\hookrightarrow$ $q(x, \mathbf{b}_\perp) = 0$ for $\mathbf{b}_\perp^2 > \frac{(1-x)^2}{\Lambda^2}$

$\hookrightarrow$ significant contribution to form factor from quarks with $1 - \varepsilon > x > 1 - \frac{\Lambda}{|\mathbf{\Delta}_\perp|}$

$$\begin{aligned} F_{large\,x}(\mathbf{\Delta}_\perp^2) &\equiv \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^{1} dx\, H(x, 0, \mathbf{\Delta}_\perp^2) = \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^{1} dx \int d^2\mathbf{b}_\perp e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} q(x, \mathbf{b}_\perp) \\ &= \int_{1-\frac{\Lambda}{|\mathbf{\Delta}_\perp|}}^{1-\varepsilon} dx q(x) = \frac{c}{4}\left[\frac{\Lambda^4}{\mathbf{\Delta}_\perp^4} - \varepsilon^4\right] \qquad \text{for} \quad q(x) = c(1-x)^3. \end{aligned}$$

- still exhibits $\frac{1}{\mathbf{\Delta}_\perp^4}$ behavior for $\mathbf{\Delta}_\perp^2 < \frac{\Lambda^2}{\varepsilon^2}$

(That's $\mathbf{\Delta}_\perp^2 < 16 GeV^2$ for $\Lambda = 0.4\,GeV$ and $\varepsilon = 0.1$

$\hookrightarrow$ obtain (fake) pQCD like behavior from $x \to 1$ (Feynman mechanism)

Implications for form factor, CT,...

- part of what looks like pQCD at a few GeV^2 could perhaps be from different mechanism ($x \to 1$)
  - requires $q(x) \propto (1-x)^3$ up to $x = 1 - \frac{\Lambda}{\mathbf{\Delta}_\perp}$ as well as a finite $\perp$ size

$\hookrightarrow$ could provide additional eplanation for lack of color transparency at Jlab

- $x \to 1$ contribution would not become color transparent

$\hookrightarrow$ observation of CT would place constraints on large x behavior of GPDs (M.B.+G.A.Miller, hep-ph/0312190; S.Liuti+S.K.Taneja, hep-ph/0405014)

Summary

- GPDs provide decomposition of form factors w.r.t. the momentum of the active quark

$\hookrightarrow$ GPDs could clarify mechanism for form factor at large $\boldsymbol{\Delta}_\perp^2$

- discussed how hadron confiurations with size $R \sim 1/\Lambda$ can contribute significantly to form factor
- contribution to nucleon form factor can be $\sim \frac{1}{\boldsymbol{\Delta}_\perp^4}$ until suppression of PDFs due to higher order corrections sets in

$\hookrightarrow$ could provide additional explanation for lack of CT @JLab

- large x behavior of PDFs & GPDs critical for contribution from large size configurations to form factor
  - how rapidly does $q(x)$ go to zero as $x \to 1$
  - how does $\perp$ size $\mathbf{d}_\perp(x)$ behave for $x \to 1$
- JLab@12GeV could illuminate physics of form factors from three different angles: CT, PDFs@large x, and GPDs.