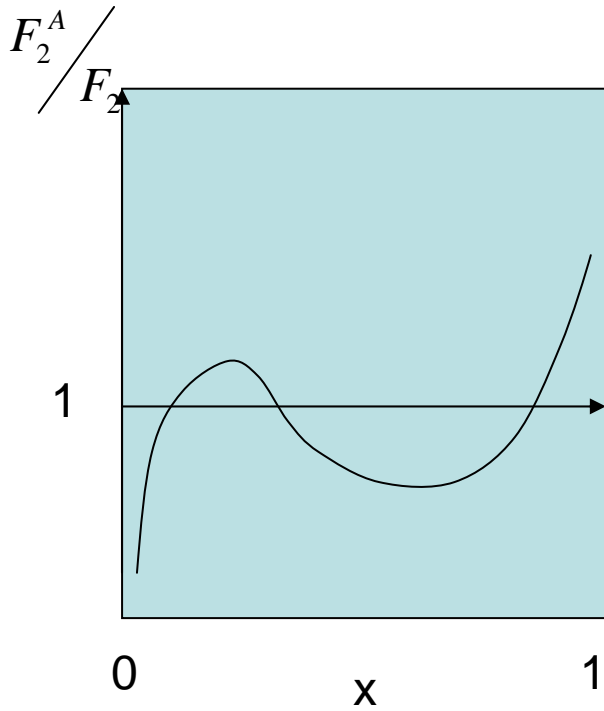


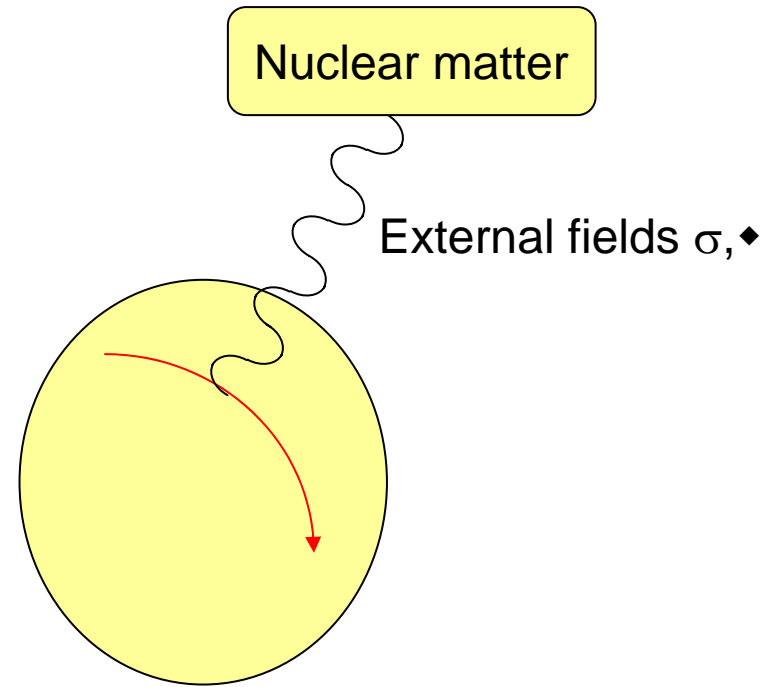
# Quark structure and nuclear effective force

in collaboration with A.W. Thomas  
Phys. Rev. Lett.93, 132502 (2004)

## EMC effect



## A bag in matter

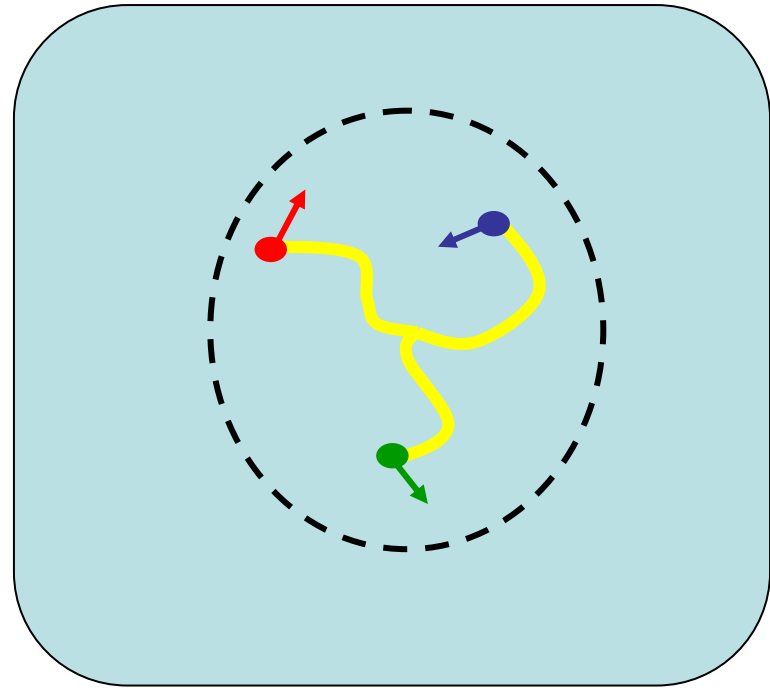
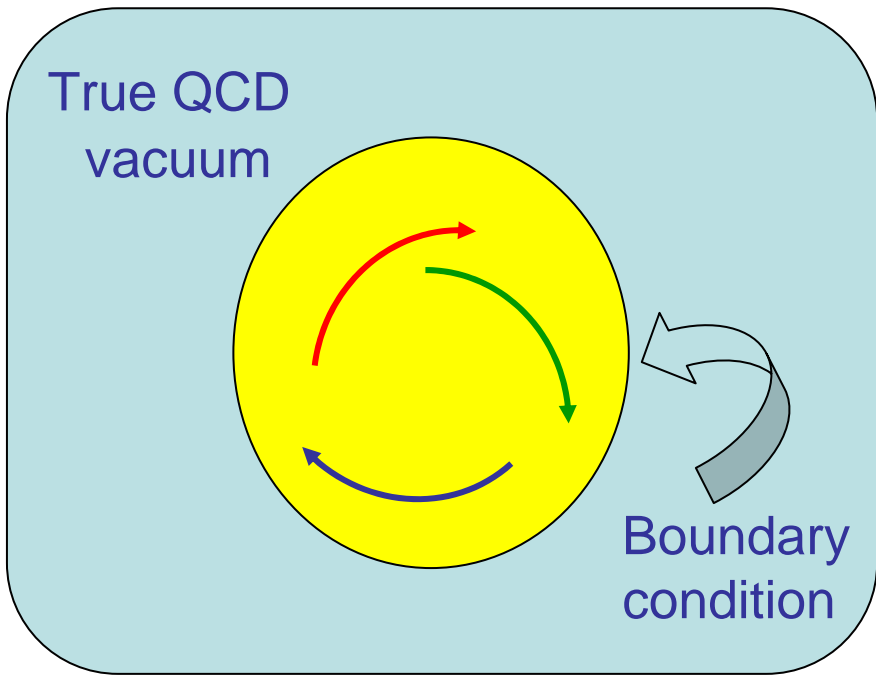


Effect of the coupling to the (constant) external fields

$$\sigma: \quad m_q \rightarrow m_q - g_\sigma^q \sigma \quad (\textit{attraction})$$

$$\omega: \quad E_q \rightarrow E_q + g_\omega^q \omega \quad (\textit{repulsion})$$

(NB: here  $\diamond$  is a chiral invariant)



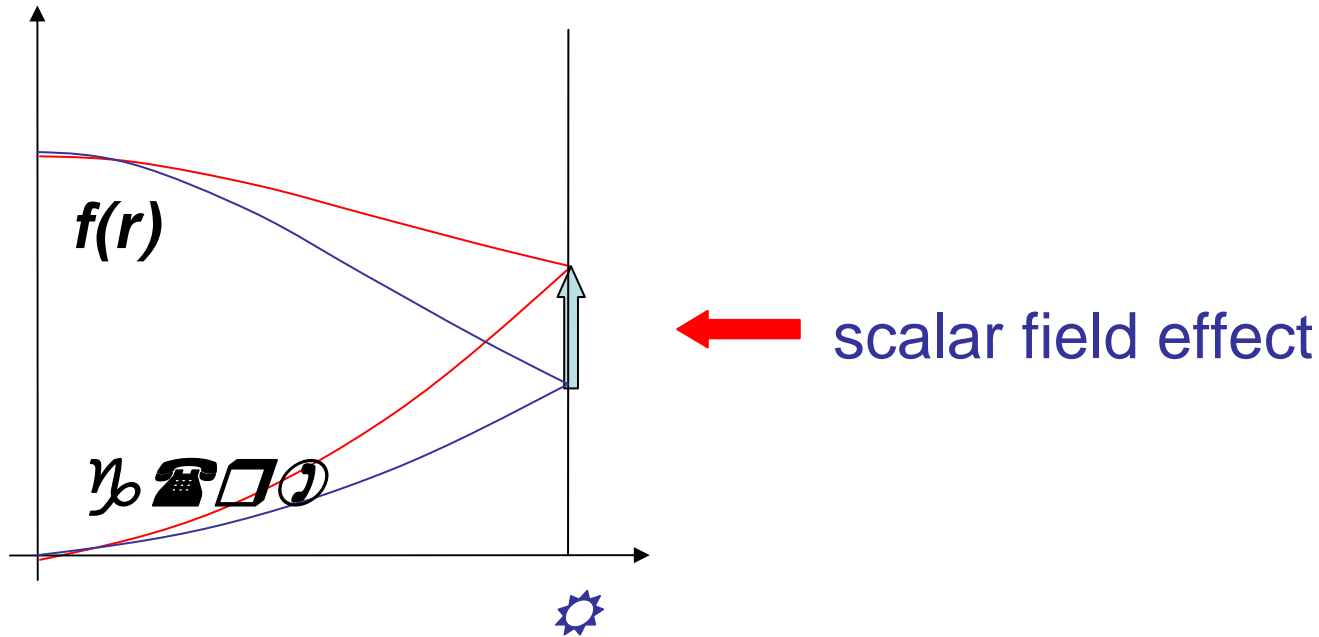
This

is just a schematic representation of

this

In the true picture, the confined quarks are floating in the non perturbative vacuum. They are coupled to every of its fluctuations.

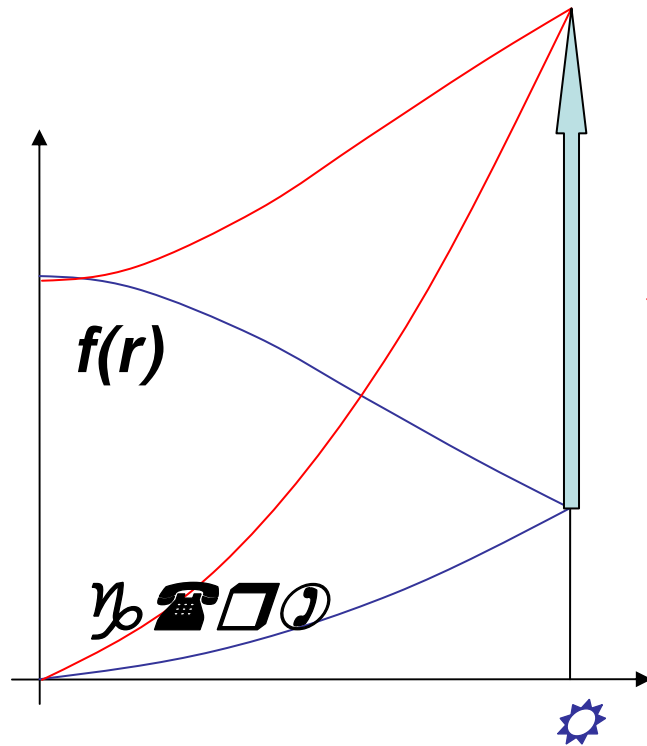
Quark field  $\psi_q(x) = e^{iE_q t} \begin{pmatrix} f(r) \\ i\vec{\sigma} \cdot \hat{r} g(r) \end{pmatrix} \chi_{1/2}$



$$E_q(\sigma) + g_\omega^q \omega \approx E_q(0)$$

Quark structure has changed but not the nucleon energy (binding~0)

# Try the couplings constant from Walecka model (QHD)



$$g_{\sigma} = 3g_{\sigma}^q \int d\vec{r} \bar{\psi} \psi, \quad g_{\omega} = 3g_{\omega}^q$$

← scalar field effect  
at nuclear matter density

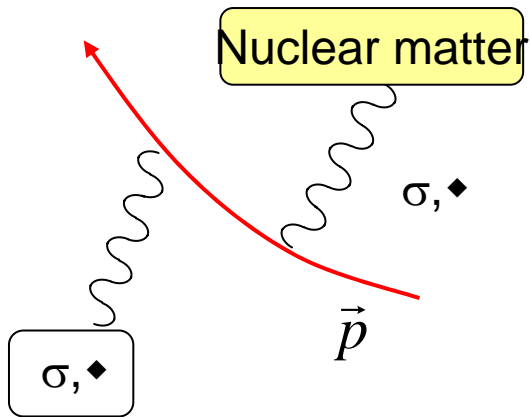
Absurd because (for instance)

$$\mu \approx f(R)g(R)$$

Something wrong somewhere....

# QHD simplified

Point like nucleon



$$M \rightarrow M - g_\sigma \sigma, \quad E \rightarrow E + g_\omega \omega$$

$$\sigma(\rho) = g_\sigma \rho \int_0^{k_F} d\vec{p} \sqrt{1 - v^2} / \int_0^{k_F} d\vec{p}$$

$$\omega(\rho) = g_\omega \rho$$

(non relativistic to simplify presentation)

$$v \approx \frac{p}{M} \rightarrow v \approx \frac{p}{M - g_\sigma \sigma}$$

As  $v \rightarrow 1$ ,  $\sqrt{1 - v^2} \rightarrow 0$  So  $\sigma$  less than  $\rho$  while  $\omega$

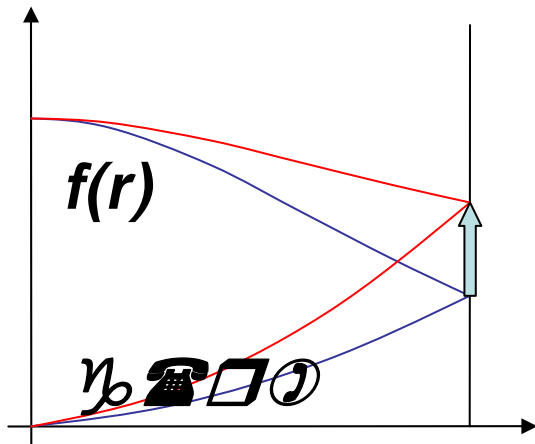
At some point repulsion overcomes attraction  $\Rightarrow$  saturation

In QHD, nuclear saturation is due to the Lorentz contraction factor  $\sqrt{1 - v^2} \Rightarrow$  Efficient only if  $g_\sigma \sigma$  is large

Consider a uniform distribution of *non overlapping static bags* :

$$\sigma(\rho) = \rho(3g_\sigma^q) \int_{Bag} d\vec{r} \bar{\psi}\psi|_\sigma$$

$$\omega(\rho) = \rho(3g_\omega^q) \int_{Bag} d\vec{r} \psi^+\psi|_\sigma = \rho g_\omega$$



$$\int_B d\vec{r} \bar{\psi}\psi|_\sigma = \int_B d\vec{r} (f^2 - g^2) \quad \text{⑩ when } \blacklozenge \text{ ⑨}$$

$\blacklozenge$  ☎ ☐ Ⓜ ⑨ less than ☐, while  $\blacklozenge$  (☐)''☐



Saturation mechanism due to the change of the quark structure

More efficient than the QHD mechanism because

At quark level:  $m_q^* = (m_q \sim 0) - g_\sigma \sigma$

At nucleon level:  $M^* = M - g_\sigma \sigma$

# The quark meson coupling model

(Phys. Lett B200,235,1988)

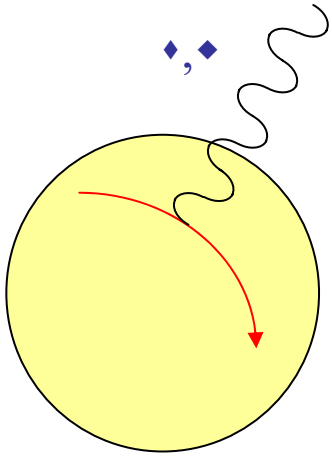
Hypothesis : (red boxes are relaxed later)

- Uniform nuclear matter (N=Z)
- In a time average sense, nuclear matter (at moderate density) is a collection of non overlapping quark bags (quasi-nucleons)
- The quarks in **different** bags interact by the exchange of mesons.
- Fermi motion and Pauli blocking are neglected
- The meson fields are replaced by their mean value

$$\sigma(r) \rightarrow \frac{1}{A} \sum_i \sigma_i(r) = \sigma$$

Only the  $\sigma$  and  $\omega$  (time component) need to be considered.





For one particular bag :  $E = M(\sigma) + g_\omega \omega$

$$M(\sigma) = \frac{3\Omega(\sigma)}{R_B} + BV - \frac{Z}{R_B}$$

$\Phi(\diamond)$  and  $R_B$  are given by the bag equations :

$$\left\{ \begin{array}{l} -i\gamma^0 \vec{\gamma} \cdot \vec{\nabla} \psi + (m_q - g_\sigma^q \sigma) \psi = \frac{\Omega(\sigma)}{R_B} \quad (r < R_B) \\ (1 + i\vec{\gamma} \cdot \hat{r}) \psi(R_B) = 0, \quad \frac{\partial M(\sigma)}{\partial R_B} = 0 \end{array} \right.$$

Numerical study  $\Rightarrow$

$$M(\sigma) \approx M(0) - g_\sigma \sigma + \frac{d}{2} (g_\sigma \sigma)^2, \quad d = 0.22 R_B$$

$$g_\sigma = 3 g_\sigma^q \int_B d\vec{r} \bar{\psi} \psi |_{\sigma=0}$$

provided  $g_\sigma \sigma \leq 400 \text{ MeV}$  (enough for density  $\sim$  normal)

## Meson fields :

$$E_{tot} = A[M(\sigma) + g_\omega \omega] + E_{mesons}$$

$$E_{mesons} = \frac{1}{2} \int d\vec{r} \left[ (\nabla \sigma)^2 + m_\sigma^2 \sigma^2 - (\nabla \omega)^2 - m_\omega^2 \omega^2 \right]$$

$$= \frac{1}{2} V_\infty (m_\sigma^2 \sigma^2 - m_\omega^2 \omega^2), \quad (V_\infty = A / \rho)$$

Equation for  $\sigma, \omega$

$$\frac{\delta E_{tot}}{\delta \sigma} = \frac{\delta E_{tot}}{\delta \omega} = 0$$

A little algebra

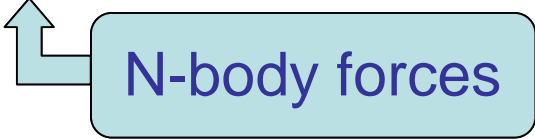


$$g_\sigma \sigma = \frac{G_\sigma \rho}{1 + d\rho G_\sigma}, \quad g_\omega \omega = G_\omega \rho$$

$$G_\sigma = g_\sigma^2 / m_\sigma^2 \quad G_\omega = g_\omega^2 / m_\omega^2$$

Inject  $\diamond, \diamond$  in  
the total energy:

$$\longrightarrow \frac{E_{tot}}{A} = M - \frac{1}{2} \frac{G_\sigma \rho}{1 + d\rho G_\sigma} + \frac{1}{2} G_\omega \rho$$

 N-body forces

Fix  $G_\sigma, G_\omega$

$$\left. \frac{\partial}{\partial \rho} \frac{E_{tot}}{A} \right|_{\rho_0} = 0, \quad \rho_0 = 0.16 \text{ fm}^{-3}, \quad \left. \frac{E_{tot}}{A} \right|_{\rho_0} = M - 16 \text{ MeV}$$

$$\longrightarrow G_\sigma = 7.2 \text{ fm}^2, \quad G_\omega = 5 \text{ fm}^2$$

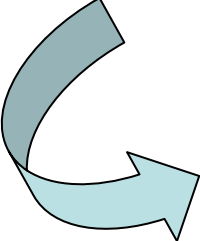
Nucleon structure is little changed :

$$\frac{g_A(\rho_0)}{g_A(0)} = 0.92, \quad \frac{R_B(\rho_0)}{R_B(0)} = 0.99, \quad \frac{\mu(\rho_0)}{\mu(0)} = 1.07$$

Good surprise: compressibility  $K=220 \text{ MeV}$

## Including Fermi motion and blocking

$$E_{tot} = A[M(\sigma) + g_\omega \omega] + E_{mesons}$$


$$E_{tot} = A \left[ \frac{\int_0^{k_F} d\vec{p} \sqrt{p^2 + M^2(\sigma)}}{\int_0^{k_F} d\vec{p}} + g_\omega \omega \right] + E_{mesons}$$

and redo the stuff.

Then QHD becomes a special case (corresponding to  $d=0$ )

With  $d=0.22R_B$  the quark mechanism dominates over the relativistic mechanism.

This was still for uniform matter. Wait for 6 years...

## Finite nuclei

(PAMG, K.Saito, E. Rodionov, AW Thomas, NPA601,349)



Look there  
for details

Framework : Born Oppenheimer Approximation

Nucleon velocity  $\ll$  Quark velocity

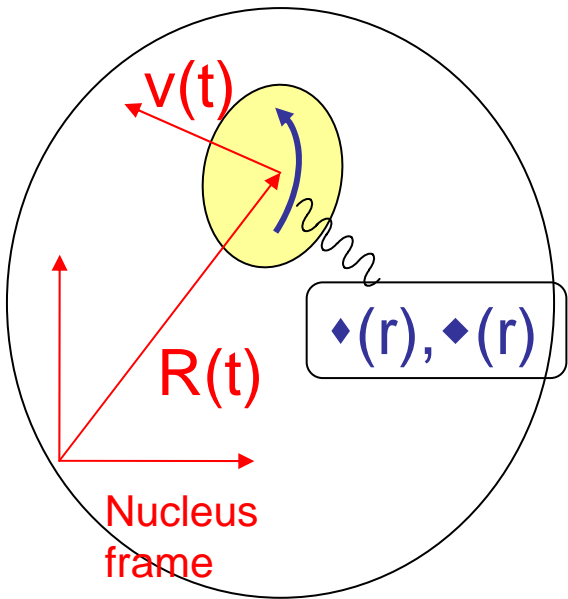
*The characteristic time ( $\blacklozenge$ ) it takes to the quark to adjust its motion is small enough that, during this time, the nucleon motion is locally inertial (rather than frozen in the strict BOA)*

$$\vec{R}(t) \approx R(0) + \dot{\vec{R}}(0)t \quad \text{for } t < \tau$$

Estimate:  $\dot{R} \sim 0.3$ ,  $\tau \sim 1/400 \text{ MeV} \approx 0.5 \text{ fm} \rightarrow \dot{R}\tau \sim 0.15 \text{ fm}$

which is small before the typical distance over which the density varies appreciably.

(NB: the motion of the nucleon is classical. Quantization is done at the end.)



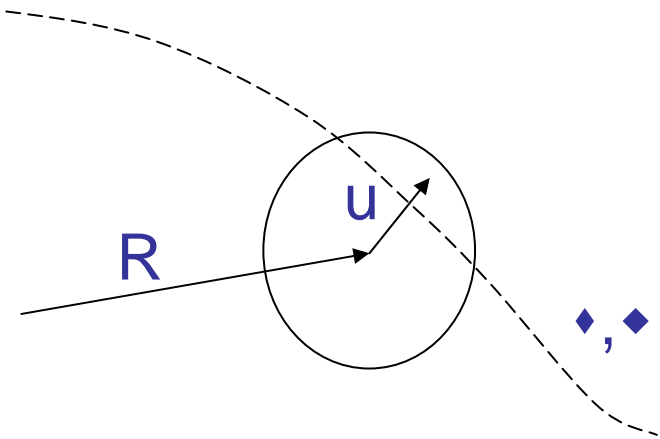
At each time  $t$  one goes to the instantaneous rest frame (IRF):

$$\text{IRF}(t) = L(v) \text{ Nucleus frame}$$

and one evaluates the energy momentum of the bag in the fields as seen in the IRF

Nucleus	IRF
$\sigma$	$\sigma$
$\omega^0 \equiv \omega$	$\omega \text{ch } \xi$
$\vec{\omega} = 0$	$-\omega \hat{v} \text{sh } \xi$

$$(th \xi = v = \dot{R})$$



$$\sigma(\vec{R} + \vec{u}) \approx \sigma(\vec{R}) + \vec{u} \cdot \vec{\nabla} \sigma \Big|_R$$

Constant in the bag volume:  
Treated exactly

Perturbation

## Constant part of the fields

$$\begin{cases} E' = M(\sigma) + g_\omega \omega(\vec{R}) ch \xi \\ \vec{P}' = -g_\omega \omega(\vec{R}) ch \xi \hat{v} \end{cases}$$

$$\begin{cases} E = \sqrt{P^2 + M^2(\sigma)} + g_\omega \omega(\vec{R}) \\ \vec{P} = M(\sigma) ch \xi \hat{v} \end{cases}$$

IRF

Nucleus

Non relativistic  
approximation  
(moderate  
density!)

$$E(\vec{R}, \vec{P}) = \frac{P^2}{2M(\sigma)} + M(\sigma) + g_\omega \omega(\vec{R})$$

LO relativistic  
saturation

Quark  
saturation

$$M(\sigma) \approx M(0) - g_\sigma \sigma + \frac{d}{2} (g_\sigma \sigma)^2$$

## Perturbative effect of the variable part of the fields

$$\Delta E' = \frac{\mu_s}{M(\sigma)} \vec{S} \cdot \vec{\nabla} (g_\omega \omega) \times \vec{v} + O(v^3) \quad (\mu_s \approx 0.9)$$

describes the spin precession in the “magnetic” field

$$\vec{B} = \vec{\nabla} \times \vec{\omega}_{IRF} = \vec{\nabla} \times (-\omega \vec{v})$$

seen in the IRF.

The corresponding equation of motion (in the IRF)

$$\frac{d\vec{S}}{dt} = -\frac{\mu_s g_\omega}{M(\sigma)} \vec{S} \times \vec{B}$$

must be written in the nucleus frame. Use BMT equation...



## Bargman Michel Telgdi covariant spin equation

$$\frac{d\Sigma^\mu}{d\tau} = -F^{\mu\nu}\Sigma_\nu + u^\mu u_\rho F^{\rho\sigma}\Sigma_\nu - u^\mu\Sigma_\nu \frac{du^\nu}{d\tau}$$

with  $\Sigma, \tau$  covariant spin and proper time.

Apply to the nucleus frame:

$$\frac{d\vec{S}}{dt} = \frac{g_\omega \mu_s}{M(\sigma)} (\vec{\nabla} \omega \times \vec{v}) \times \vec{S} - \frac{1}{2} \left( \frac{d\vec{v}}{dt} \times \vec{v} \right) \times \vec{S} + O(v^3)$$

*Eq. of motion*  $\rightarrow \frac{d}{dt} (M(\sigma) \vec{v}) = -\vec{\nabla} (g_\omega \omega + M(\sigma))$

$$\frac{d\vec{S}}{dt} = \frac{1}{2M(\sigma)} \vec{\nabla} [(2\mu_s - 1) g_\omega \omega - M(\sigma)] \times \vec{v} \cdot \vec{S}$$

In terms of potential energy

$$V_{SO} = \frac{1}{2M(\sigma)} \vec{\nabla} \left[ (2\mu_s - 1) g_\omega \omega - M(\sigma) \right] \times \vec{v} \cdot \vec{S}$$

Remark: assume  $M(\sigma) = M - g_\sigma \sigma$  ( $d = 0$ )

$$V_{SO} = \frac{1}{2M(\sigma)} \vec{\nabla} \left[ (2\mu_s - 1) g_\omega \omega + g_\sigma \sigma \right] \times \vec{v} \cdot \vec{S}$$

For a Dirac particle  $V_{SO} = \frac{1}{2M(\sigma)} \vec{\nabla} \left[ g_\omega \omega + g_\sigma \sigma \right] \times \vec{v} \cdot \vec{S}$


This is approximatively correct because  $2\mu_s - 1 \approx 1$  ( $\mu_s \approx 0.9$ )

Completely wrong for an isovector interaction since  $\mu_V = 4.7$

**Summary:** the (classical) energy of the (quasi) nucleon is

$$E(\vec{R}, \vec{P}) = \frac{P^2}{2M(\sigma)} + M(\sigma) + g_\omega \omega(\vec{R}) + V_{so}$$

$$M(\sigma) = M - g_\sigma \sigma + \frac{d}{2} (g_\sigma \sigma)^2$$

$$\vec{v} = \frac{\vec{P}}{M(\sigma)} + O(v^3)$$


$$V_{so} = \frac{1}{2M^2(\sigma)} \vec{\nabla} \left[ (2\mu_s - 1) g_\omega \omega(\vec{R}) - M(\sigma) \right] \times \vec{P} \cdot \vec{S}$$

Add the effect of the rho meson (*field*  $b^\alpha$ ,  $\alpha = 1, 2, 3$ )

$$g_\omega \omega \rightarrow g_\omega \omega + g_\rho \frac{\vec{\tau}}{2} \cdot \vec{b}$$

$$(2\mu_s - 1) g_\omega \omega \rightarrow (2\mu_s - 1) g_\omega \omega + (2\mu_V - 1) g_\rho \frac{\vec{\tau}}{2} \cdot \vec{b}$$

# Remarks:

- The fields  $\sigma(\vec{R}), \omega(\vec{R})$  acting on the nucleon at  $\vec{R}$  are those created by the OTHER nucleons.
- The spin-orbit interaction has been obtained as a **first order** perturbation. By consistency  $M(\sigma) \rightarrow M$

- Quantization  $\vec{P} \rightarrow -i\vec{\nabla}_R$  allows 2 orderings in the kinetic energy

$$T_1 = \vec{P} \cdot \frac{1}{2M(\sigma)} \vec{P}, \quad T_2 = P^2 \frac{1}{4M(\sigma)} + \frac{1}{4M(\sigma)} P^2$$

however  $T_1 - T_2 \sim \nabla^2 \sigma$  which can be absorbed in a change of  $m_\sigma$ . In practice

$$T_1 \leftrightarrow T_2 \quad \text{amounts to } \Delta m_\sigma \sim 50 \text{ MeV}$$

For definiteness we chose the ordering 1

Total energy

$$E_{tot} = \sum_i E(i) + \frac{1}{2} \int d\vec{r} \left[ (\nabla \sigma)^2 + m_\sigma^2 \sigma^2 - (\nabla \omega)^2 - m_\omega^2 \omega^2 \right]$$

with

$$E(i) = \vec{P}_i \cdot \frac{1}{2M[\sigma(\vec{R}_i)]} \vec{P}_i + M[\sigma(\vec{R}_i)] + \dots$$

$$\frac{\delta E_{tot}}{\delta \sigma(\vec{R})} = \frac{\delta E_{tot}}{\delta \omega(\vec{R})} = 0 \quad \text{determine } \sigma(\vec{R}), \omega(\vec{R})$$

$$-\nabla^2 \omega(\vec{r}) + m_\omega^2 \omega(\vec{r}) = g_\omega \sum_i \delta(\vec{r} - \vec{R}_i)$$

$$-\nabla^2 \sigma(\vec{r}) + m_\sigma^2 \sigma(\vec{r}) =$$

$$g_\sigma (1 - dg_\sigma \sigma(\vec{r})) \sum_i \left[ \delta(\vec{r} - \vec{R}_i) - \vec{P}_i \cdot \frac{\delta(\vec{r} - \vec{R}_i)}{M^2(\sigma)} \vec{P}_i \right]$$

First attempt (1995): mean field approximation

- Replace:  $\sigma(\vec{r}) \rightarrow \langle \sigma(\vec{r}) \rangle \dots$
- Solve the equations self consistently

Using the coupling  
fixed on nuclear matter:

Sensible results for charge  
density ( $A=16,40$ )

Spin orbit splitting much too small ( $1/2$ )

However it is known that for the spin-orbit interaction  
the exchange (Fock) term adds to the direct term,  
can make a factor  $\sim 2$ !

But we had no Fock term... 😞

8 years later...

$$\text{Step 1: } E_{tot} = \sum_i E(i) + E_{mesons}$$

$$\text{Step 2: } \frac{\delta E_{tot}}{\delta \sigma} = \frac{\delta E_{tot}}{\delta \sigma} = 0 \rightarrow \sigma(\vec{r}) = \sigma_{sol}(\vec{r}, \vec{R}_1 \dots \vec{R}_A) \quad (\text{analytical!})$$
$$\omega(\vec{r}) = \omega_{sol}(\vec{r}, \vec{R}_1 \dots \vec{R}_A)$$

$$\text{Step 3: } E_{tot}(\sigma \rightarrow \sigma_{sol}, \omega \rightarrow \omega_{sol}) = H_{eff}(\vec{R}_1, \dots, \vec{R}_A)$$

Allows antisymmetrization

Makes contact with conventional nuclear physics

$H_{eff}(1, 2, \dots, A)$  is pretty complicated: N-body finite range forces  
Can't be used as such (according to many-body practitioners)

$$-\nabla^2 \sigma(\vec{r}) + m_\sigma^2 \sigma(\vec{r}) = \underbrace{\rho_S(\vec{r})}_{\substack{g_\sigma (1 - dg_\sigma \sigma) \sum_i \left[ \delta(\vec{r} - \vec{R}_i) - \vec{p}_i \cdot \frac{\delta(\vec{r} - \vec{R}_i)}{M^2(\sigma)} \vec{p}_i \right]}}$$

Approximation  
scheme: STEP 1

$$\hookrightarrow S(\vec{r}) = g_\sigma (1 - dg_\sigma \sigma) \rho_S$$

Write this as  $\sigma(\vec{r}) = \frac{S(\vec{r})}{m_\sigma^2} + \frac{\nabla^2 \sigma}{m_\sigma^2}$

Grosso modo  $\sigma(\vec{r})$  follows the nuclear density  $\Rightarrow \frac{\nabla^2 \sigma}{m_\sigma^2} \sim \frac{\sigma}{a^2 m_\sigma^2}$   
with  $a \sim 1 \text{ fm}$  the nuclear skin. Take  $m_\sigma \sim 600 \text{ Mev}$

$$\frac{1}{a^2 m_\sigma^2} \sim \frac{1}{10} \ll 1 \Rightarrow \text{In the term } \nabla^2 \sigma / m_\sigma^2 \text{ one can replace } \sigma(\vec{r}) \text{ by its approximate value } \sigma(\vec{r}) \approx S(\vec{r}) / m_\sigma^2$$

New field equation  
(idem for  $\omega(\vec{r})$ )

$$\sigma(\vec{r}) = \frac{S(\vec{r})}{m_\sigma^2} + \frac{\nabla^2 S(\vec{r})}{m_\sigma^4}$$



$$\sigma(\vec{r}) = \frac{S(\vec{r})}{m_\sigma^2} + \frac{\nabla^2 S(\vec{r})}{m_\sigma^4}$$

Approximation  
scheme: STEP 2

$$S(\vec{r}) = g_\sigma (1 - dg_\sigma \sigma) \rho_S \quad \rho_S(\vec{r}) = \sum \delta(\vec{r} - \vec{R}_i) - \text{relat. corr.}$$

Nuclear matter  
estimate

$$g_\sigma \sigma \sim 300 \text{ MeV at } \rho = 0.16 \text{ fm}^{-3}$$

$$d = 0.22 R_B, R_B = 0.8 \text{ fm} \rightarrow dg_\sigma \sigma \sim 0.27$$

→  $dg_\sigma \sigma$  is a reasonable expansion parameter  
(*at normal density*)

Iterative solution  
of the field equation:

$$\sigma = \frac{g_\sigma \rho_S}{m_\sigma^2} - \frac{g_\sigma \rho_S (dg_\sigma \sigma)}{m_\sigma^2} + \frac{\nabla^2 S(\sigma)}{m_\sigma^4}$$

Zeroth order solution  $\sigma_0$

Insert the field solutions in  $E_{tot} = \sum_i E(i) + E_{mesons}$

Remember that  $\sigma(\vec{R}_i)$  is the field due to  $j \neq i$

$$\sum_i \sum_j \dots \rightarrow \sum_{i \neq j} \dots \text{ etc...}$$

Simplify...

$$H_{QMC} = \sum_i \frac{\vec{\nabla}_i \cdot \vec{\nabla}_i}{2M} + \frac{G_\sigma}{2M^2} \sum_{i \neq j} \vec{\nabla}_i \cdot \delta(\vec{R}_{ij}) \vec{\nabla}_i$$

Kinetic and non local terms

$$+ \frac{1}{2} \sum_{i \neq j} \left( \frac{G_\omega}{m_\omega^2} - \frac{G_\sigma}{m_\sigma^2} + \frac{G_\rho}{m_\rho^2} \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4} \right) \nabla_i^2 [\delta(\vec{R}_{ij})]$$

Finite range

$$+ \frac{1}{2} \sum_{i \neq j} \left( G_\omega - G_\sigma + G_\rho \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4} \right) \delta(\vec{R}_{ij})$$

2 body

3,4 body

$$+ \frac{dG_\sigma^2}{2} \sum_{i \neq j \neq k} \delta(\vec{R}_{ij}) \delta(\vec{R}_{jk}) + \frac{d^2 G_\sigma^3}{2} \sum_{i \neq j \neq k \neq l} \delta(\vec{R}_{ij}) \delta(\vec{R}_{jk}) \delta(\vec{R}_{kl})$$

+ 5...N body forces

$$+ \frac{i}{4M^2} \sum_{i \neq j} \vec{\nabla}_i \delta(\vec{R}_{ij}) \times \vec{\nabla}_i \cdot \vec{\sigma}_i \left[ G_\sigma + (2\mu_S - 1)G_\omega + (2\mu_V - 1)G_\rho \frac{\vec{\tau}_i \cdot \vec{\tau}_j}{4} \right]$$

Spin orbit

Done...

Antisymmetry  $\longrightarrow$  forces with  $N > 4$  do not contribute.

Set:  $R_B = 0.8 \text{ fm},$

$$m_\omega = 780 \text{ MeV}, m_\rho = 770 \text{ MeV}, m_\sigma = 500 \div 600 \text{ MeV}$$

Fix:  $G_\sigma \equiv \frac{g_\sigma^2}{m_\sigma^2}, G_\omega \equiv \frac{g_\omega^2}{m_\omega^2}, G_\rho \equiv \frac{g_\rho^2}{m_\rho^2}$  on nuclear matter:

$$\frac{E_B}{A} (\rho = 0.16 \text{ fm}^{-3}) = -15.85 \text{ MeV} + \left( \frac{N-Z}{A} \right)^2 30 \text{ MeV}$$

Get:  $G_\sigma = 12.6 \text{ fm}^2, G_\omega = 9.6 \text{ fm}^2, G_\rho = 9.7 \text{ fm}^2$  ( $m_\sigma = 600 \text{ MeV}$ )

$H_{QMC}$  is completely determined.

$H_{QMC}$  has about the same form as the successful Skyrme force.



Consider the Skyrme force as the experience

$$V_{skyrme} = \sum_{i < j} v_{ij} + \sum_{i < j < k} v_{ijk} \quad v_{123} = t_3 \delta(\vec{R}_{12}) \delta(\vec{R}_{23})$$

$$\begin{aligned} v_{12} = & t_0 (1 + x_0 P_\sigma) \delta(\vec{R}_{12}) \\ & + \frac{t_2}{4} \vec{\nabla}_{12} \delta(\vec{R}_{12}) \cdot \vec{\nabla}_{12} - \frac{t_1}{8} \left[ \delta(\vec{R}_{12}) \vec{\nabla}_{12}^2 + \vec{\nabla}_{12}^2 \delta(\vec{R}_{12}) \right] \\ & + \frac{i}{4} W_0 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{\nabla}_{12} \times \delta(\vec{R}_{12}) \vec{\nabla}_{12} \quad (\nabla_{12} = \nabla_1 - \nabla_2) \end{aligned}$$

Compare the Hartree-Fock  
hamiltonians to get

$x_0, t_{0,1,2,3},$  and  $W_0$

	QMC	QMC	SkIII	QMC(N=3)
$m_\sigma (MeV)$	500	600		600
$t_0 (MeV fm^3)$	-1071	-1082	-1129	-1047
$x_0$	0.89	0.59	0.45	0.61
$t_3 (MeV fm^6)$	16620	14926	14000	12996
$M_{eff} / M$	.915	.814	.763	.821
$5t_2 - 9t_1 (MeV fm^5)$	-7622	-4330	-4030	-4036
$W_0 (MeV fm^5)$	118	97	120	91
$K (MeV)$	327	327	355	364

$$\frac{M_{eff}}{M} = \left( 1 + \frac{(3t_1 + 5t_2)M\rho_0}{8} \right)^{-1}$$

## CONCLUSION

We get a pretty good *caricature* of low energy nuclear physics.  
(20% deviation with respect to SkIII)

### Essential ingredients:

- Response of the quark structure to the nuclear medium (N-body forces)
- Minimal relativistic effects (spin orbit)
- Formulation as a many body problem (antisymmetrization)

### To improve

- Limitation to about normal density (field solution)
- Include pion effects (improves K and effective mass)



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VRP