Nuclear Lattice Simulations with Effective Field Theory

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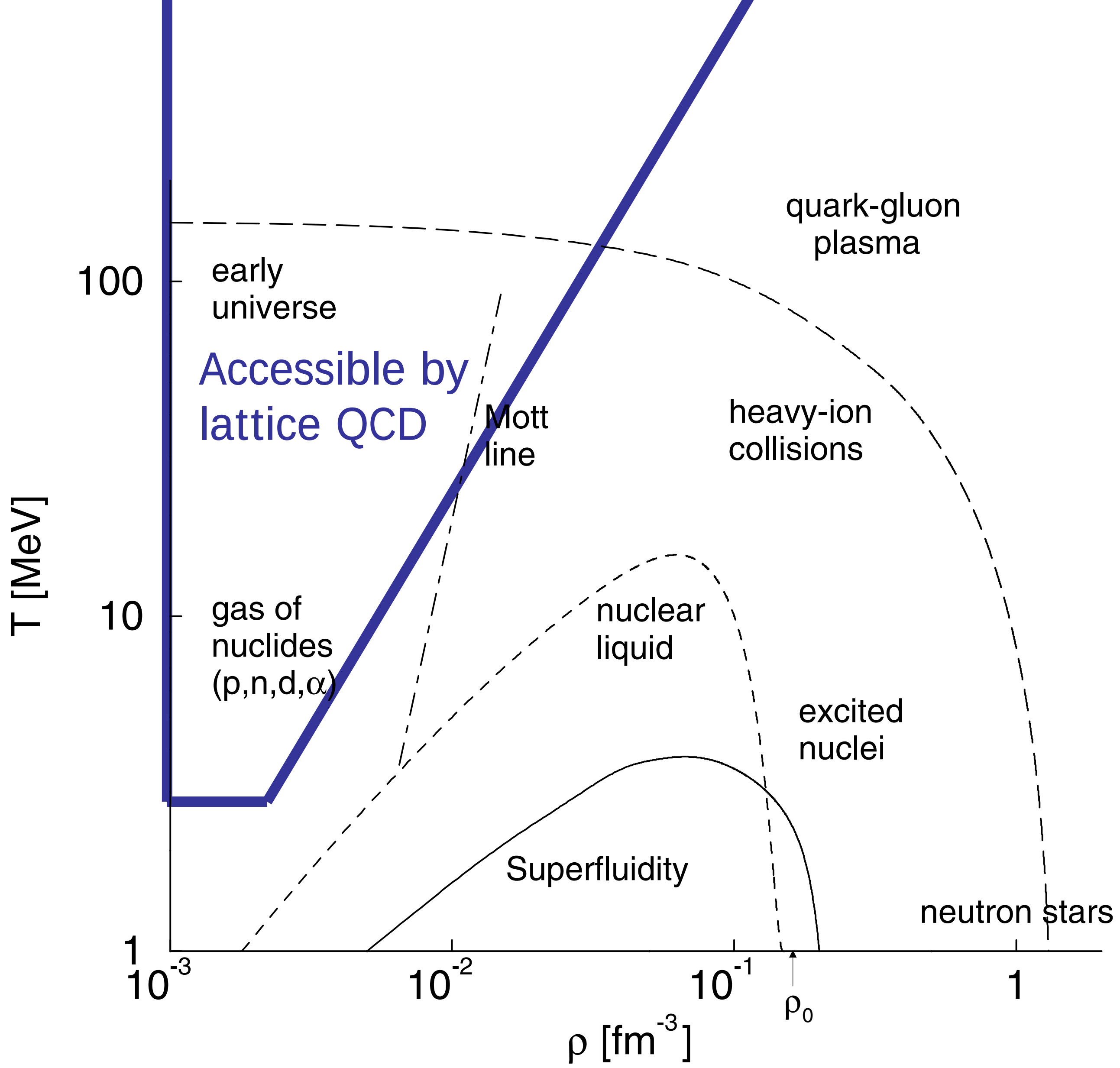
Nuclear Lattice Collaboration

Outline

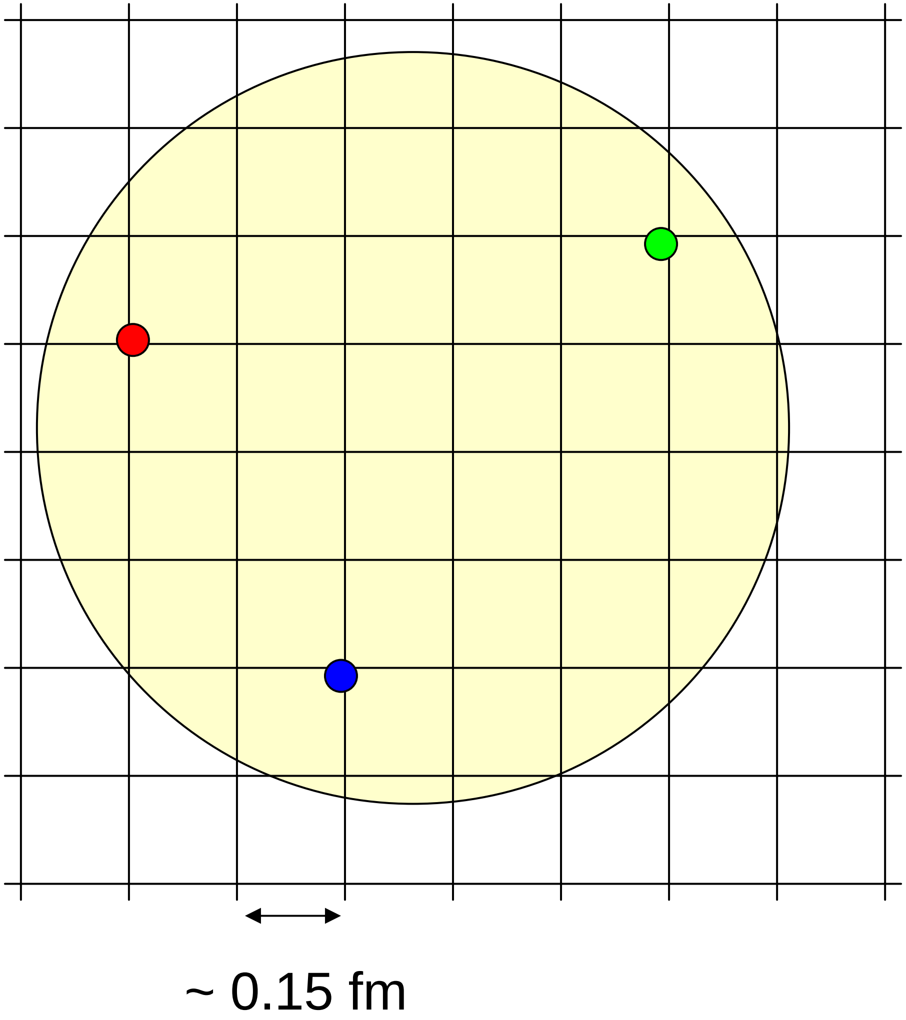
1. Why nuclear lattice simulations?
2. Chiral effective theory and lattice methods
3. Operator coefficients
4. Results for neutron matter – with pion
5. Results for neutron matter – without pion
6. Future directions



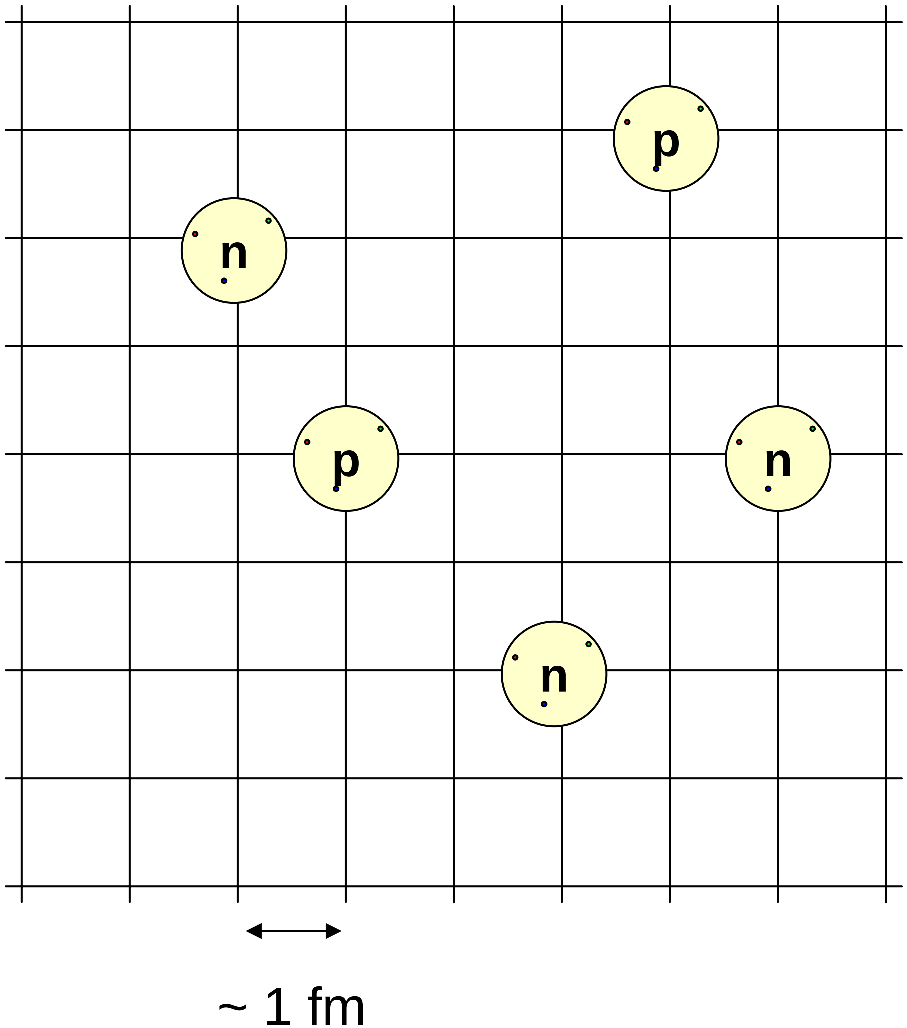
from Ropke and Schell, Prog. Part. Nucl. Phys. 42, 53 (1999)

Why do nuclear lattice simulations?

Nucleon in lattice QCD



Nucleons as point particles on lattice



Nuclear Lattice Simulations

Numerous studies of ground state properties of few nucleon systems using potential models together with variational and/or Green's function Monte Carlo [Wiringa and Pieper, PRL 89 (2002)182501; Carlson and Schiavilla, Rev. Mod. Phys. 70 (1998) 743; etc.]

Also studies of the liquid-gas transition using classical lattice gas models [Ray, Shamanna, and Kuo, PLB 392 (1997) 7]

First study of quantum many body effects in infinite nuclear matter on the lattice – quantum hadrodynamics on momentum lattice [Brockmann and Frank, PRL 68 (1992) 1830]

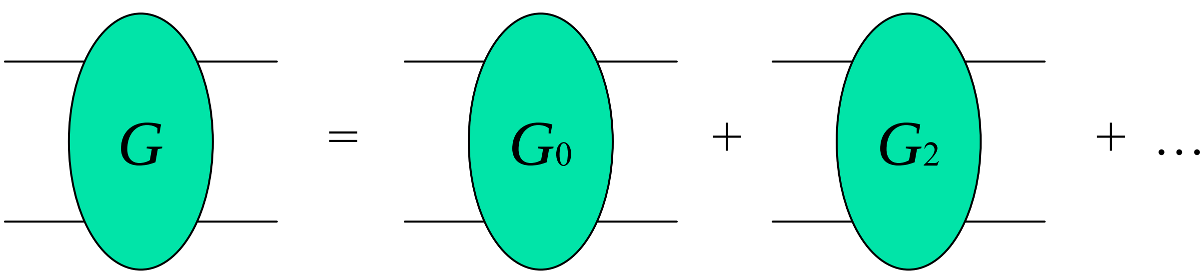
First study on spatial lattice at finite temperature (Nuclear Lattice Collaboration) [Müller, Koonin, Seki, and van Kolck, PRC 61 (2000) 044320]

Simulations with Chiral Effective Theory

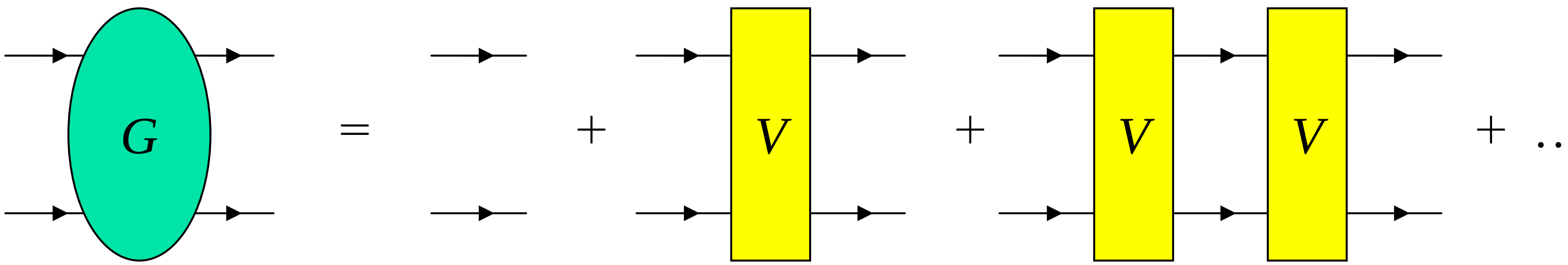
Non-perturbative lattice simulations of effective field theory of low energy pions and nucleons.

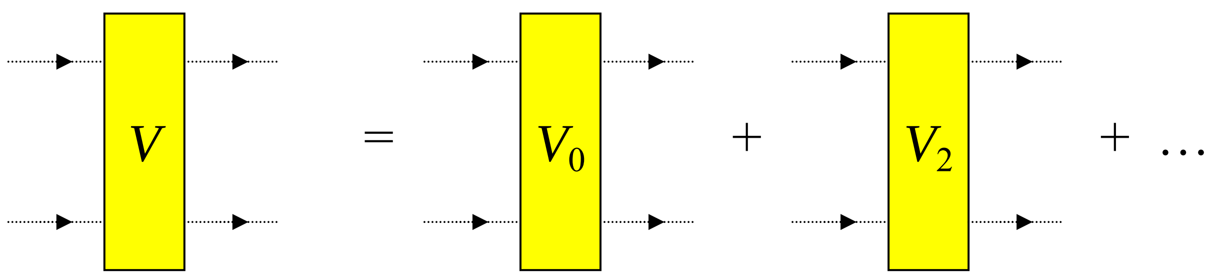
Non-perturbative effective field theory?... but isn't effective field theory based upon an expansion?

For pions the expansion is simple

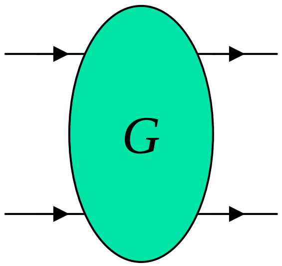


For nucleons we must take care of infrared singularities [Weinberg, PLB 251 (1990) 288, NPB 363 (1991) 3]

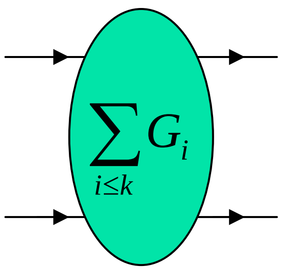




We will iterate “everything”



$$= \frac{\int D\pi DND\bar{N}\; G(\pi, N, \bar{N}) e^{-S[\pi,N,\bar{N}]}}{\int D\pi DND\bar{N}\; e^{-S[\pi,N,\bar{N}]}}$$



$$= \frac{\int D\pi DND\bar{N}\; G(\pi, N, \bar{N}) e^{-\sum_{i\le k} S_i[\pi,N,\bar{N}]}}{\int D\pi DND\bar{N}\; e^{-\sum_{i\le k} S_i[\pi,N,\bar{N}]}}$$

A complete summation of all diagrams involving interaction terms with order $\le k$

All the lowest-order diagrams are included along with counterterms, but we also produce higher-order diagrams without the accompanying counterterms. So we cannot take the continuum limit. Instead we use

$$p_{\text{external}}, m_{\pi} < \Lambda_{\text{cutoff}} < \Lambda_{\chi}$$

We check for cutoff independence in this range.

Our method:

$$N = \begin{bmatrix} p \\ n \end{bmatrix} \otimes \begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix}$$

Following Weinberg [PLB 251 (1990) 288; NPB 363 (1991) 3], we write the most general local Lagrangian involving pions and low-energy nucleons

$$D \equiv 1 + \pi_i^{\,2} / F_\pi^2$$

$$S = S_{\pi\pi} + S_{\overline{N}N} + S_{\pi\overline{N}N} + S_{\overline{N}N\overline{N}N} + S_{\pi\pi\overline{N}N} + \cdots$$

$$S_{\pi\pi} = \int d^3\vec{r}\,dr_4 \left[\tfrac{D^{-2}}{2} \left(\tfrac{\partial \pi_i}{\partial r_4} \right)^2 + \tfrac{D^{-2}}{2} \left(\vec{\nabla} \pi_i \right)^2 + \tfrac{D^{-1}}{2} m_\pi^2 \pi_i^{\,2} \right]$$

$$S_{\overline{N}N} = \int d^3\vec{r}\,dr_4 \left[N^\dagger \tfrac{\partial N}{\partial r_4} - N^\dagger \tfrac{\vec{\nabla}^2 N}{2 m_N} + (m_N - \mu) N^\dagger N \right]$$

$$S_{\pi\bar{N}N} = \int d^3\vec{r}\,dr_4 \left[D^{-1} g_A F_\pi^{-1} N^\dagger \left(\tau_i \vec{\sigma} \cdot \vec{\nabla} \pi_i \right) N \right]$$

$$S_{\bar{N}N\bar{N}N} = \int d^3\vec{r}\,dr_4 \left[\tfrac{1}{2} C_s N^\dagger N N^\dagger N + \tfrac{1}{2} C_t N^\dagger \vec{\sigma} N \cdot N^\dagger \vec{\sigma} N \right]$$

$$S_{\pi\pi\bar{N}N} = \int d^3\vec{r}\,dr_4 \left[i D^{-1} g_A F_\pi^{-2} N^\dagger \tau_i \left(\varepsilon_{ijk} \pi_j \tfrac{\partial \pi_k}{\partial r_4} \right) N \right]$$

Weinberg power counting:

$$\Delta = \#\partial + \tfrac{\# f}{2} - 2$$

We use Hubbard-Stratonovitch transformation for the *NN* contact interaction.

We consider with neutron matter with just neutrons and neutral pions

Operator coefficients

Neutron-neutron contact interaction coefficient determined by s-wave zero-temperature scattering phase shifts on the lattice

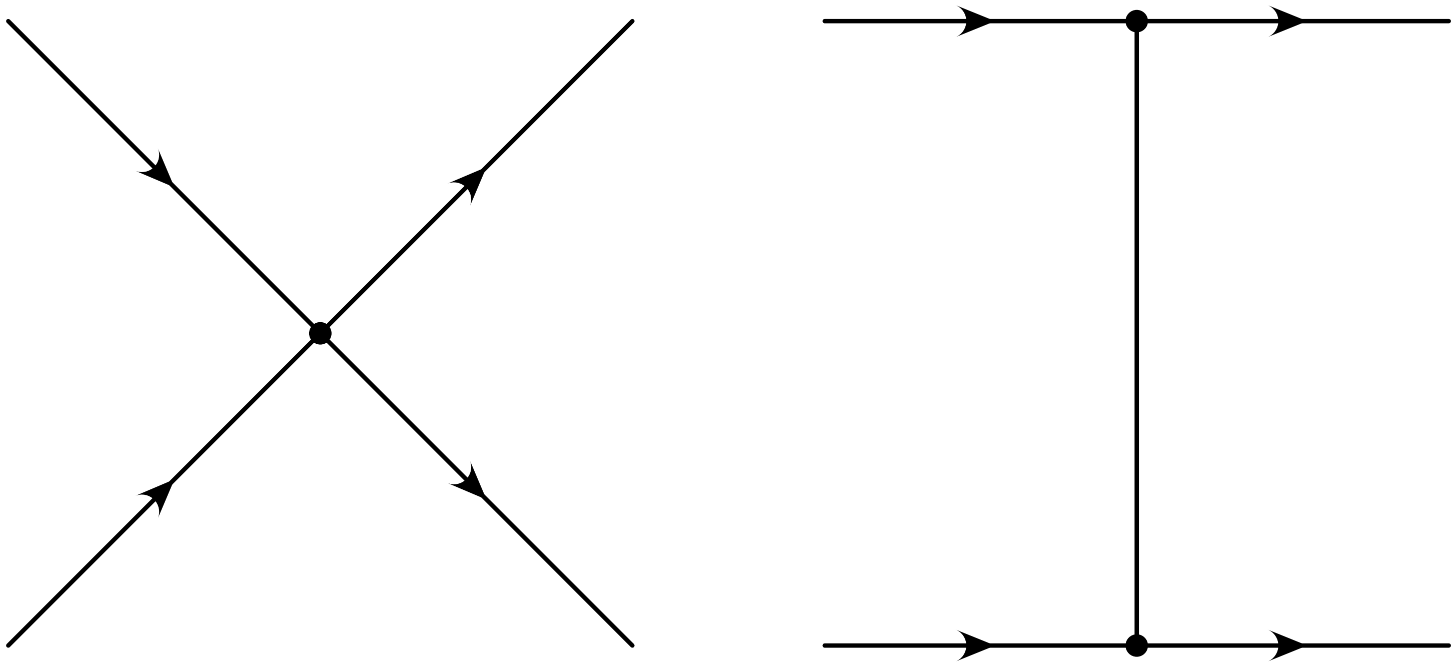
Two possibilities:

Luscher's formula [Luscher, NPB 354 (1991) 531]

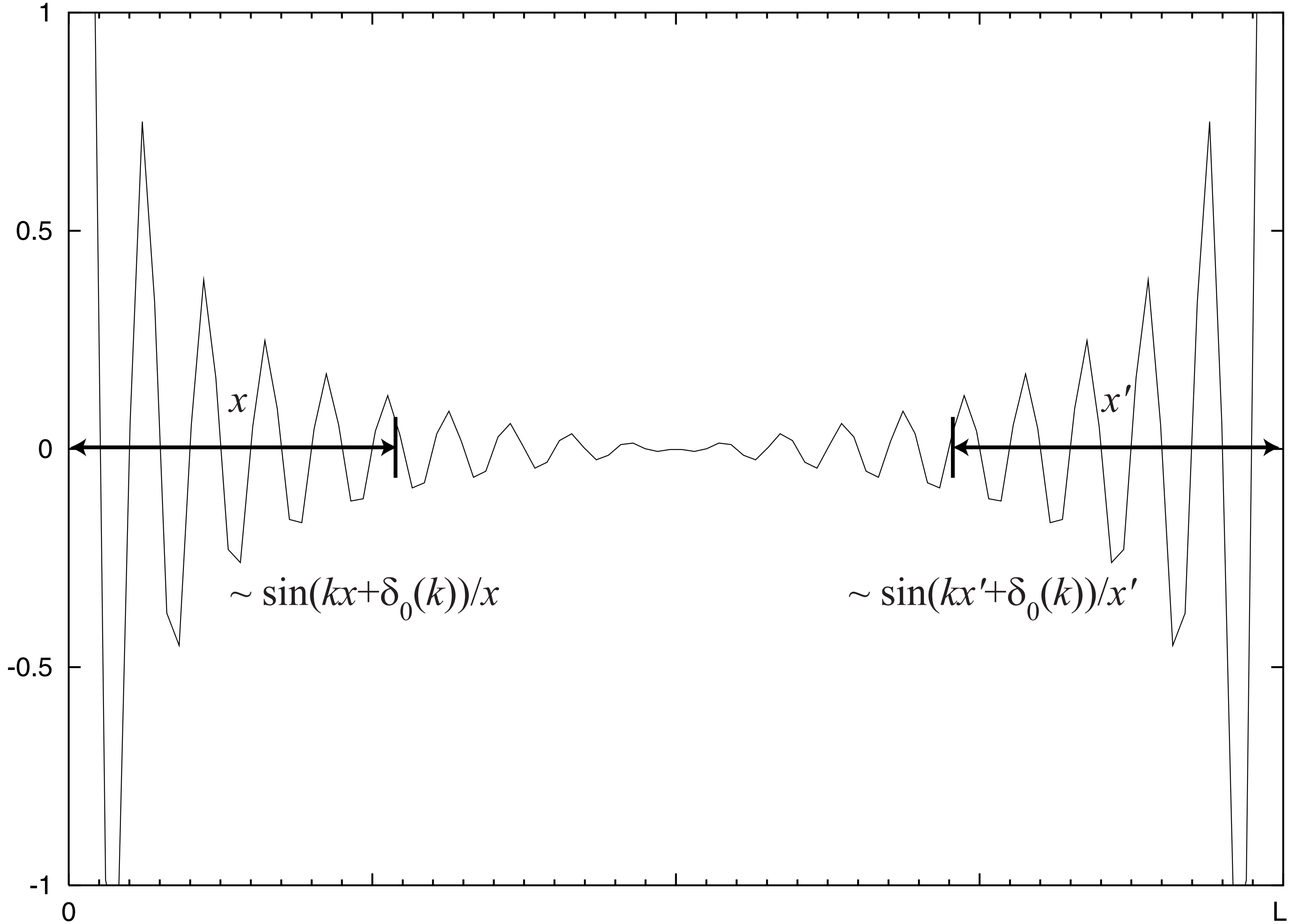
or

Solve lattice Schrodinger equation and find phase shifts from asymptotic wavefunctions of scattering states

Lattice Schrödinger Potential



Scattering states on a 3D periodic spatial lattice

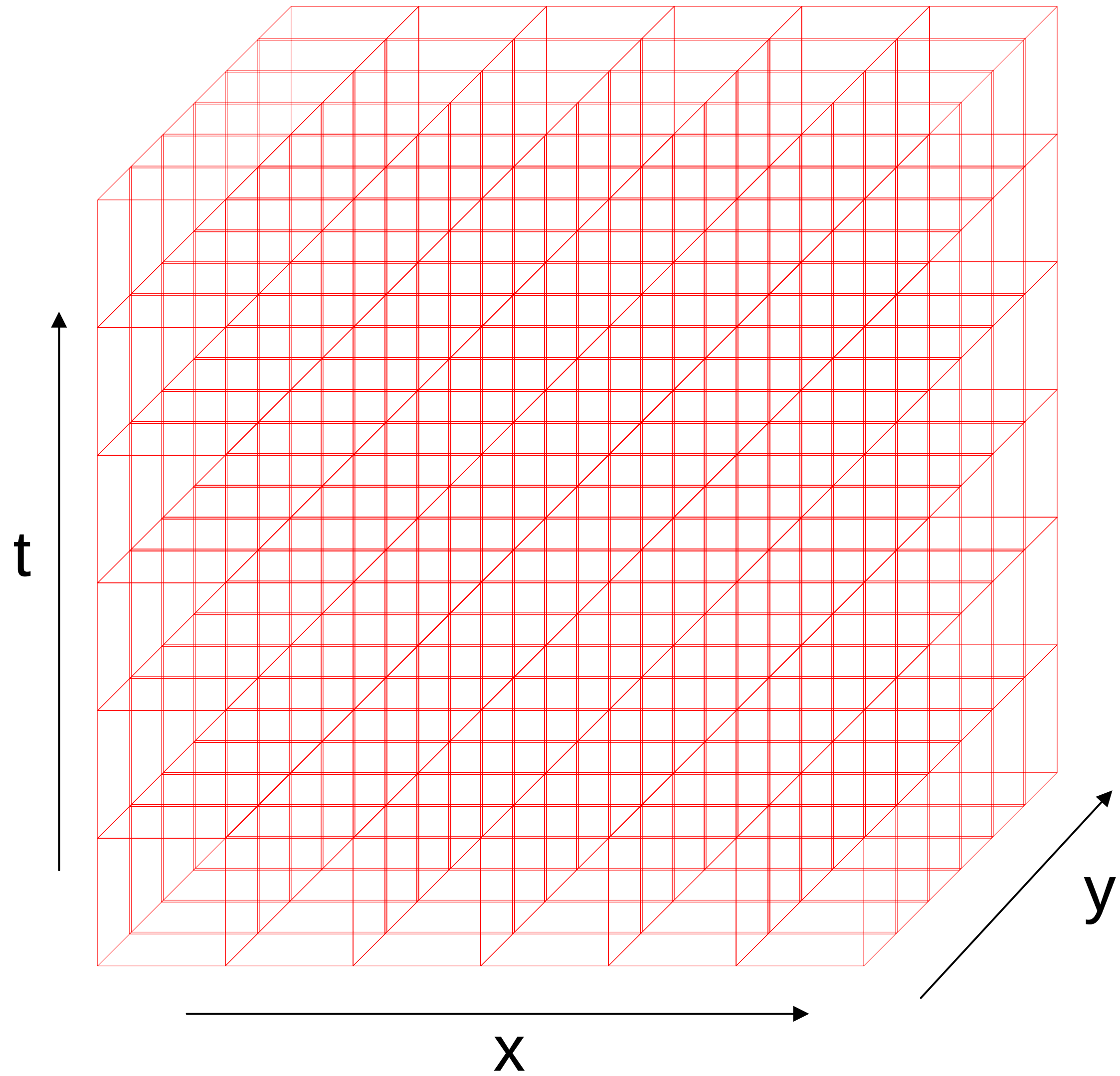


As expected we see significant cutoff dependence

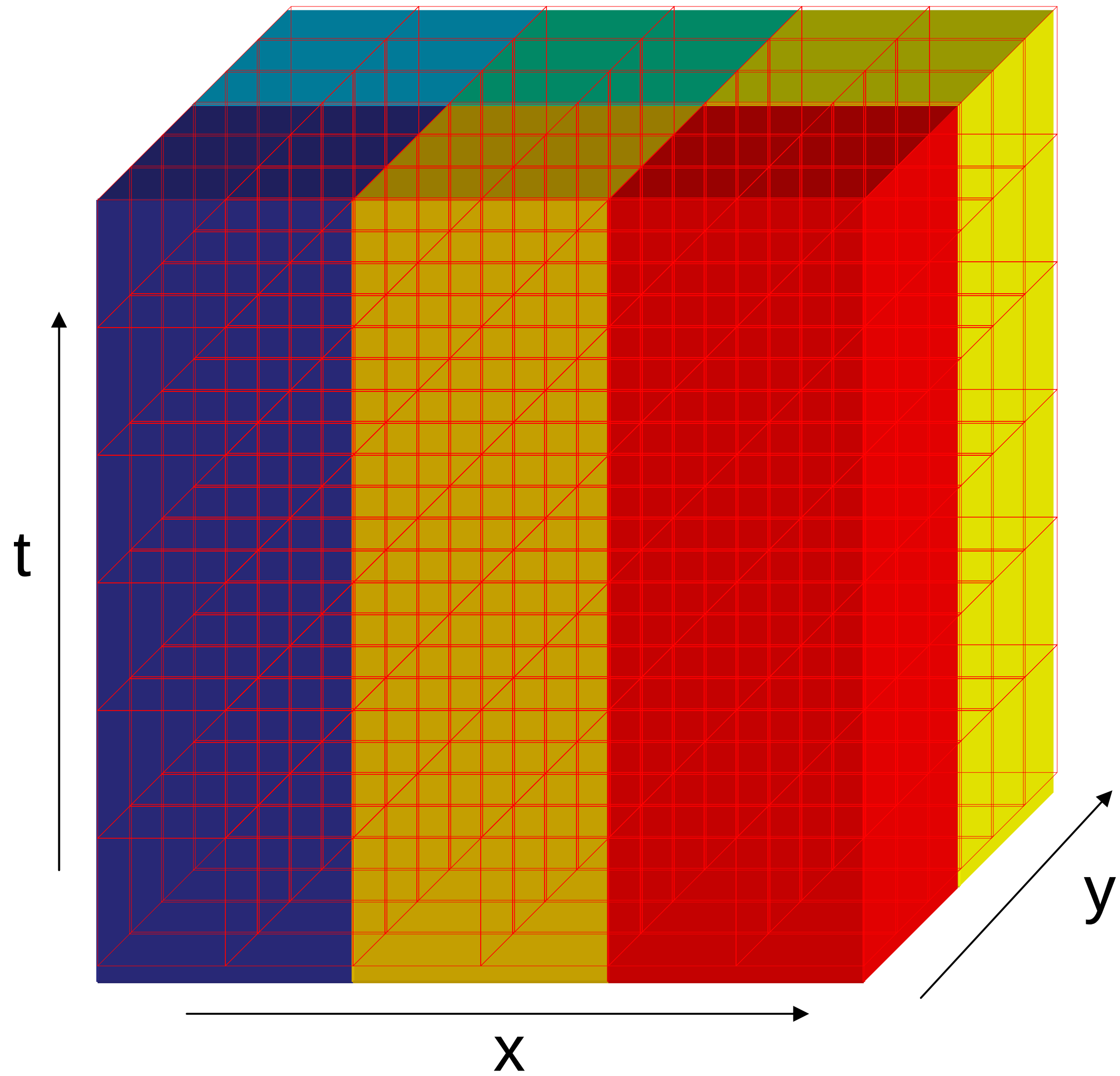
a^{-1} (MeV)	C (MeV^{-2})
150	-0.40E-4
200	-0.34E-4
250	-0.31E-4
300	-0.29E-4

If pion exchange ignored (i.e., only bubble diagrams), we expect $C \sim a$ (lattice spacing)

Space-time lattice

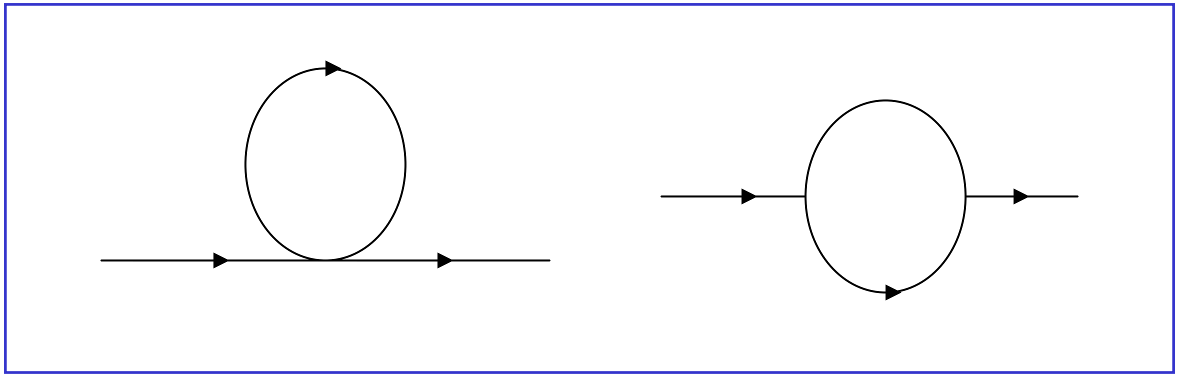


Break up into spatial zones

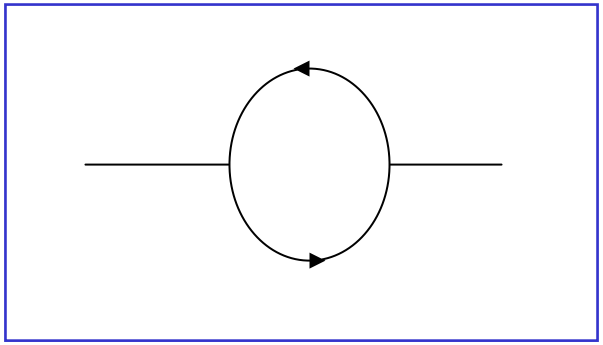


Loop calculations

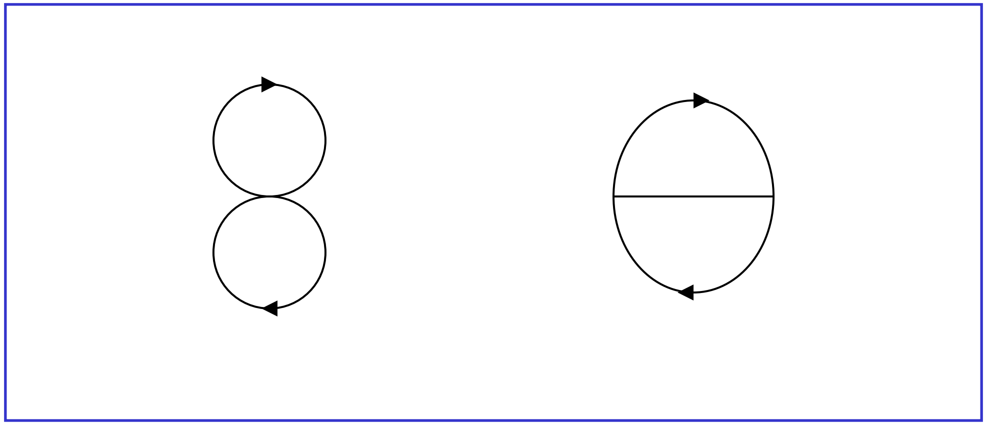
Neutron self-energy



Pion self-energy



Energy diagrams



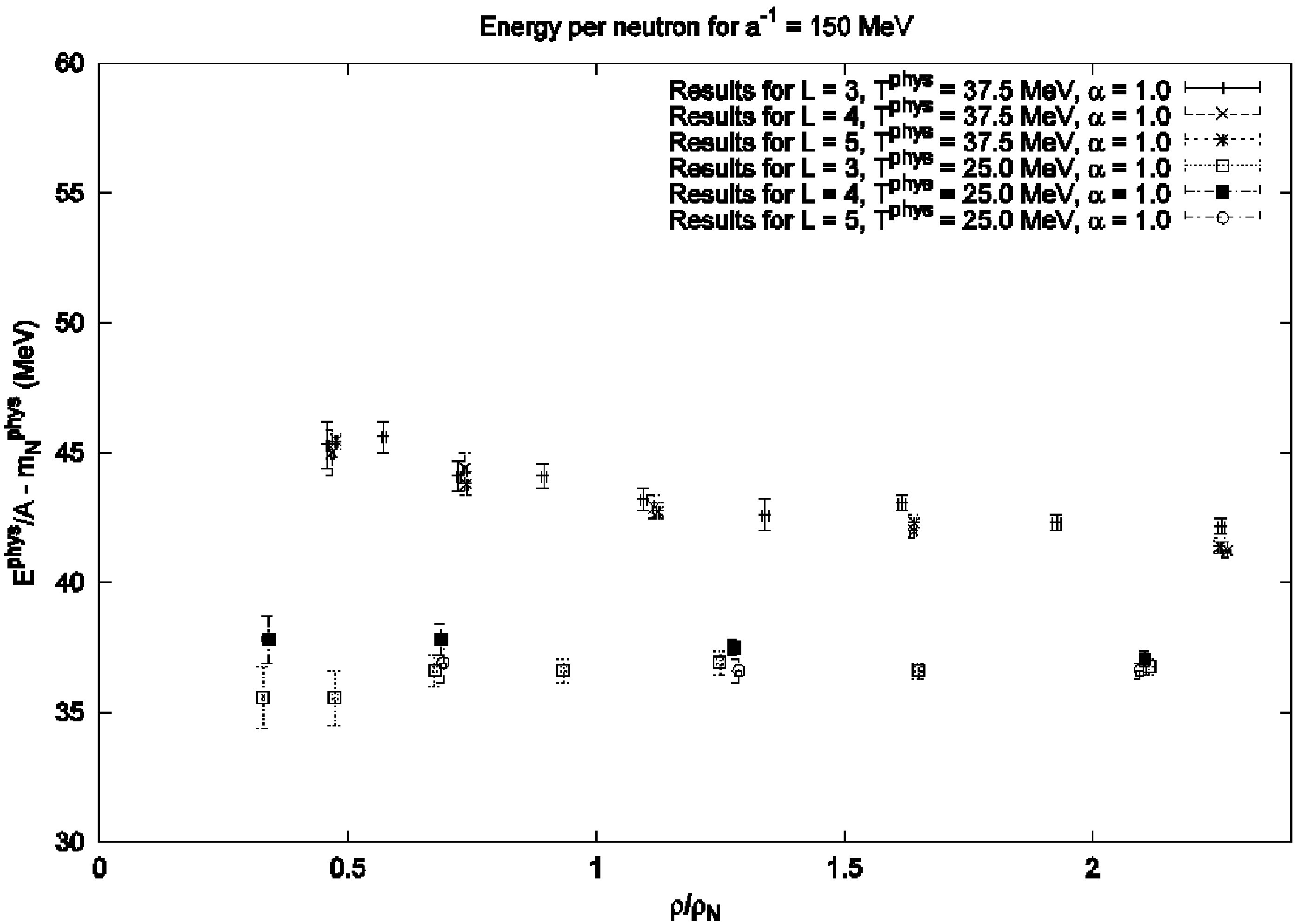
Temporal neutron correlation ($g = 0$, $Ca^2 = -0.135$)

n_t	0	1	2
Free	0.7568	0.5027	0.3444
Loop calc.	0.7453	0.5059	0.3537
Simulation	0.7447(2)	0.5057(3)	0.3537(2)

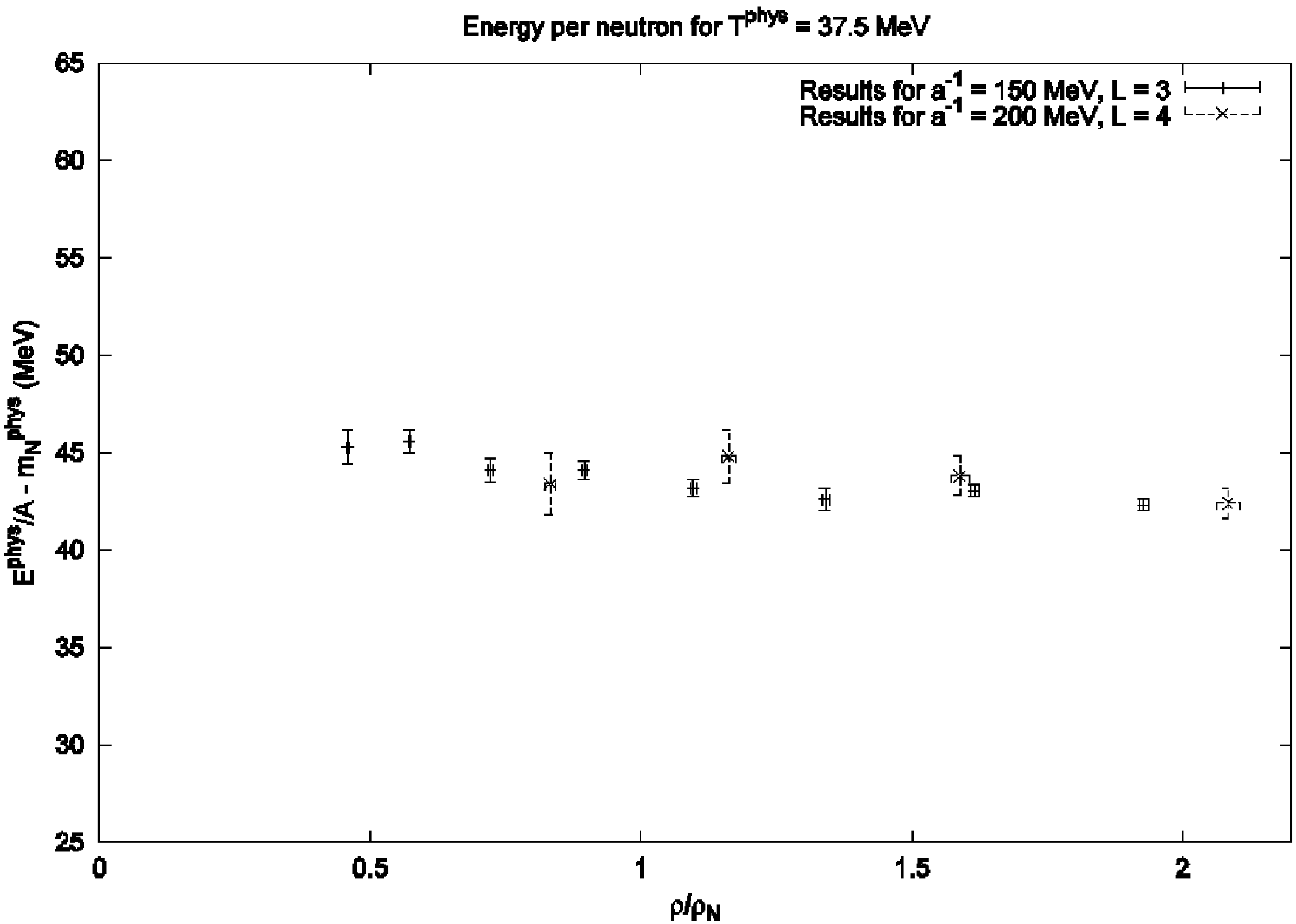
Temporal neutron correlation ($g = 0.750$, $Ca^2 = 0$)

n_t	0	1	2
Free	0.7568	0.5027	0.3444
Loop calc.	0.7496	0.5005	0.3475
Simulation	0.7494(1)	0.5000(1)	0.3472(1)

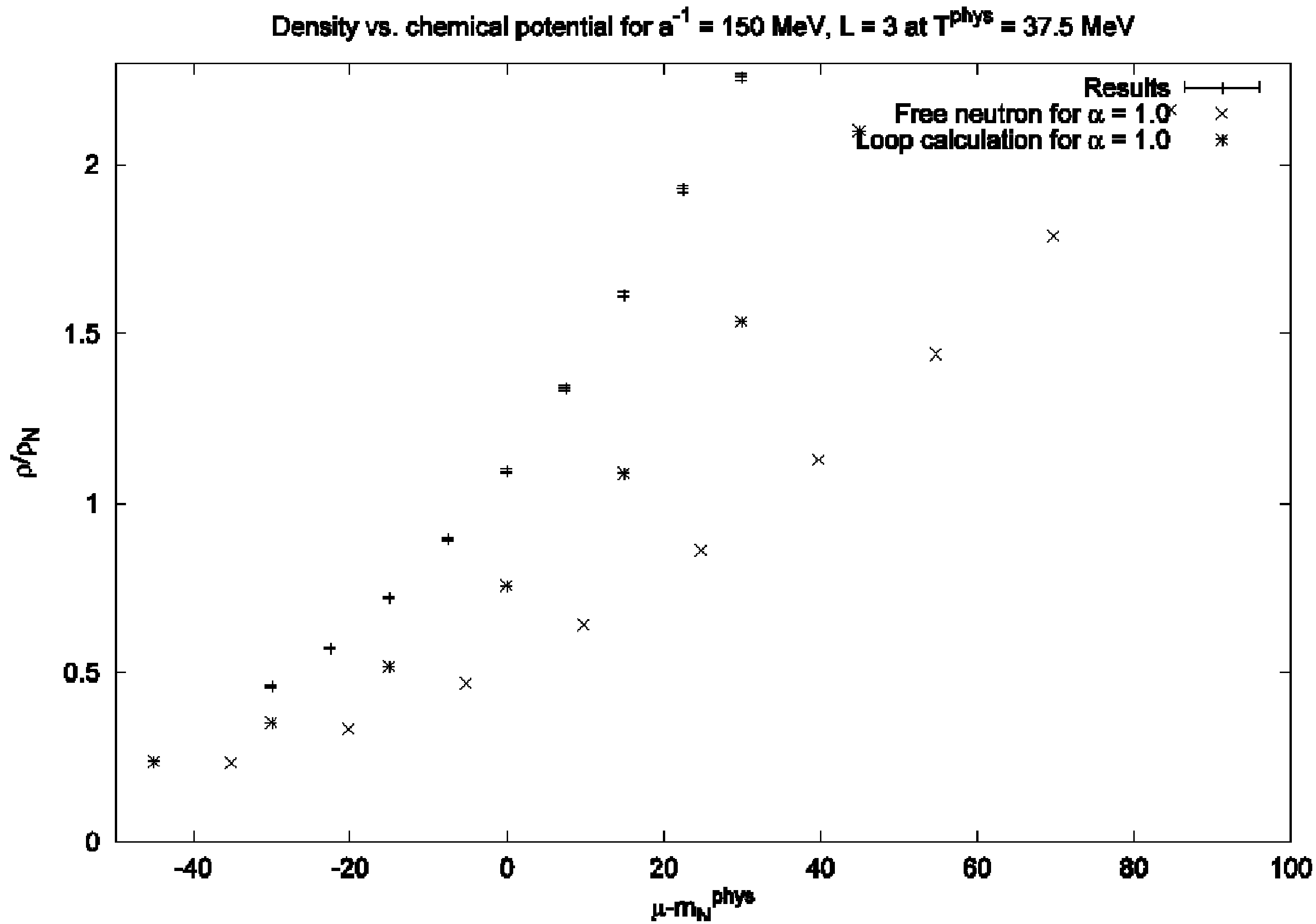
Results



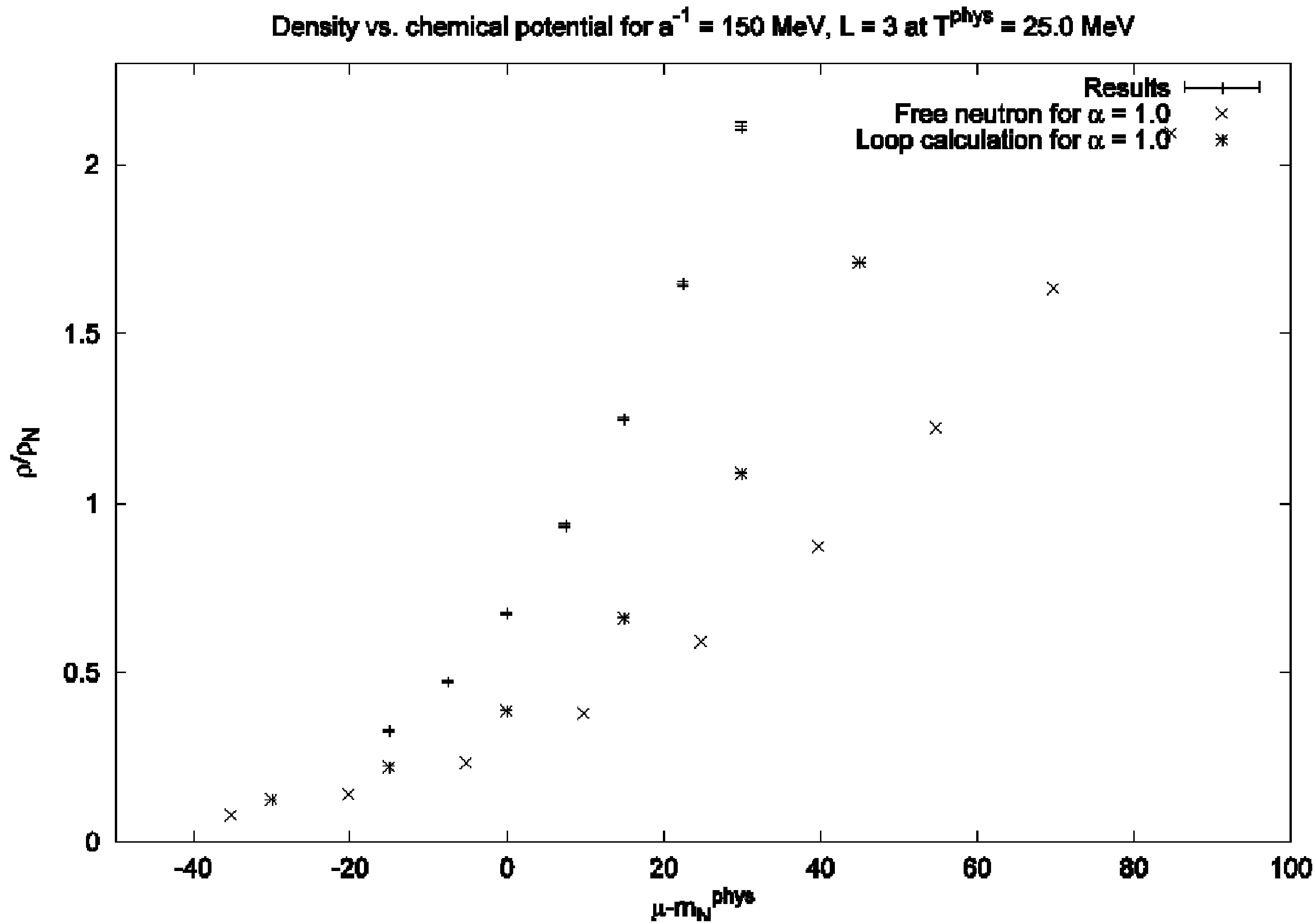
D.L., Borasoy, Schaefer, PRC 70 014007



Energy per neutron for T^{phys} = 37.5 MeV
Results for a^{-1} = 150 MeV, L = 3
Results for a^{-1} = 200 MeV, L = 4
$E^{phys}/A - m_N^{phys}$ (MeV)
ρ/ρ_N
65
60
55
50
45
40
35
30
25
0
0.5
1
1.5
2



Density vs. chemical potential for a^-1 = 150 MeV, L = 3 at T^phys = 37.5 MeV
Results
Free neutron for α = 1.0
Loop calculation for α = 1.0
ρ/ρN
μ-mN^phys
0
0.5
1
1.5
2
-40
-20
0
20
40
60
80
100



Density vs. chemical potential for a^-1 = 150 MeV, L = 3 at T^phys = 25.0 MeV
Results
Free neutron for α = 1.0
Loop calculation for α = 1.0
ρ/ρN
μ-mN^phys
0
0.5
1
1.5
2
-40
-20
0
20
40
60
80
100

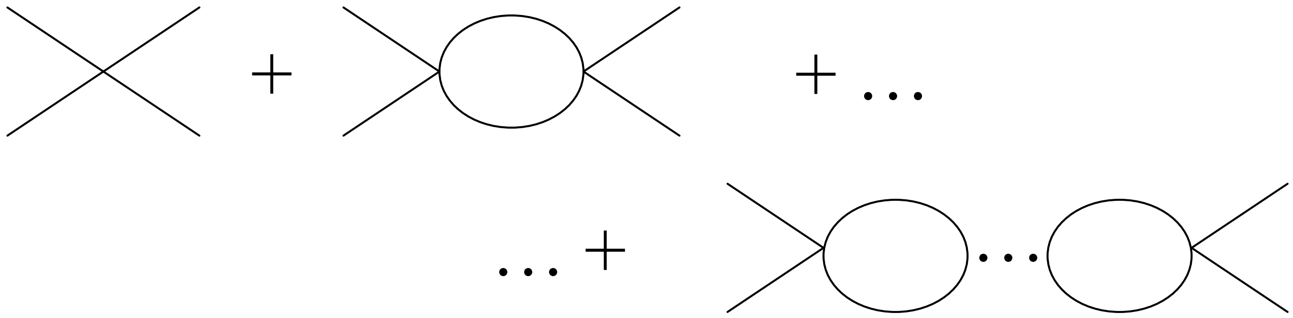
Neutron matter (without pions)

$$S_{\bar{N}N\bar{N}N} = \int d^3\vec{r}\,dr_4 \left[\tfrac{1}{2} C n^\dagger n n^\dagger n\right]$$

No signs or phases, the determinant is positive semi-definite. We can use standard Hybrid Monte Carlo.

Sum bubble chain contributions to nucleon-nucleon scattering and use Lüscher's formula to set coefficient to give physical s-wave scattering length

Bubble chain



a^{-1} (MeV)	C (MeV^{-2})
20	-1.86×10^{-4}
40	-9.89×10^{-5}
60	-6.73×10^{-5}
80	-5.10×10^{-5}

Various lattice spacings

1

$$a = (60\ \text{MeV})^{-1}$$

$$a_t = (32\ \text{MeV})^{-1}$$

3

$$a = (70\ \text{MeV})^{-1}$$

$$a_t = (64\ \text{MeV})^{-1}$$

2

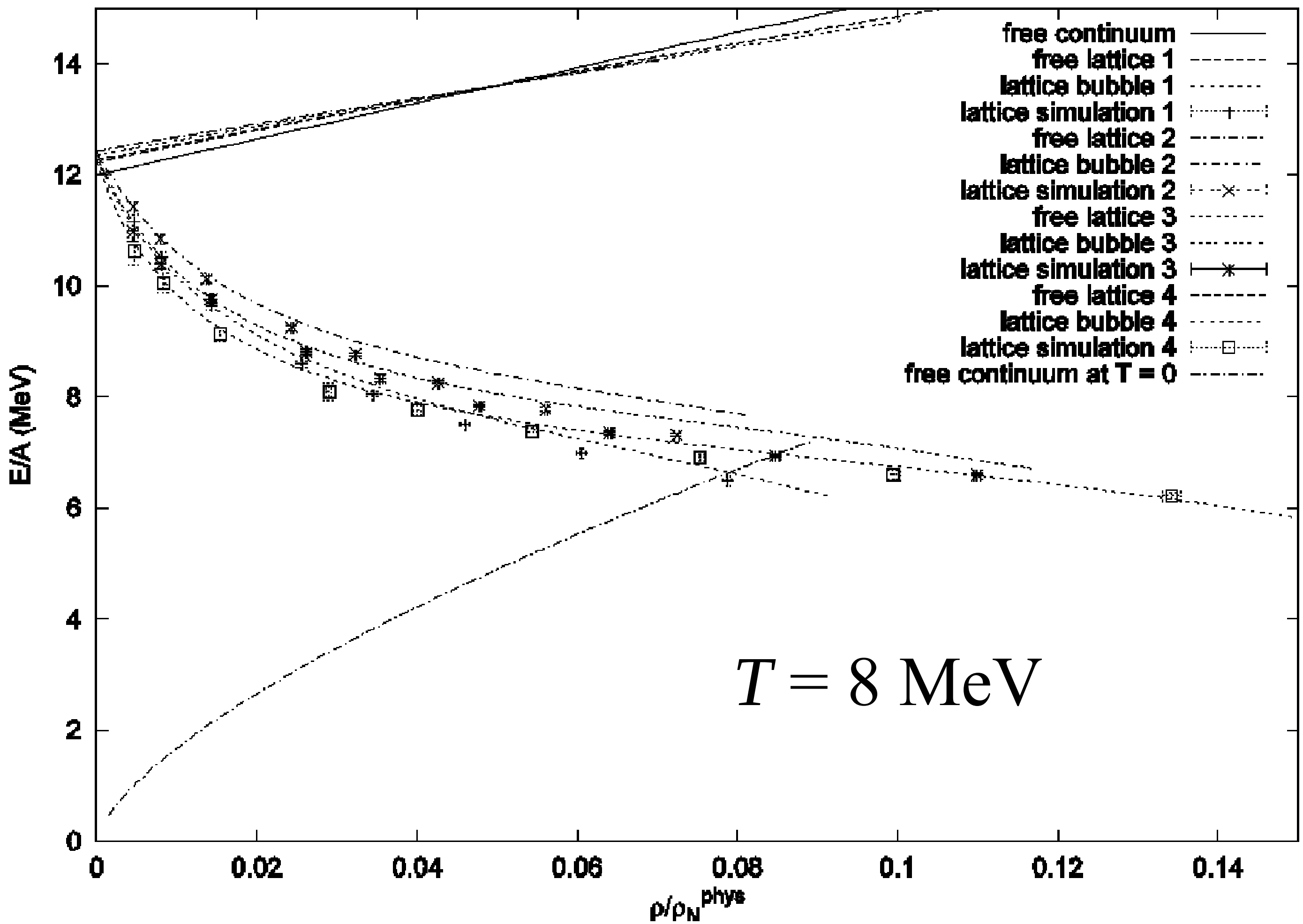
$$a = (60\ \text{MeV})^{-1}$$

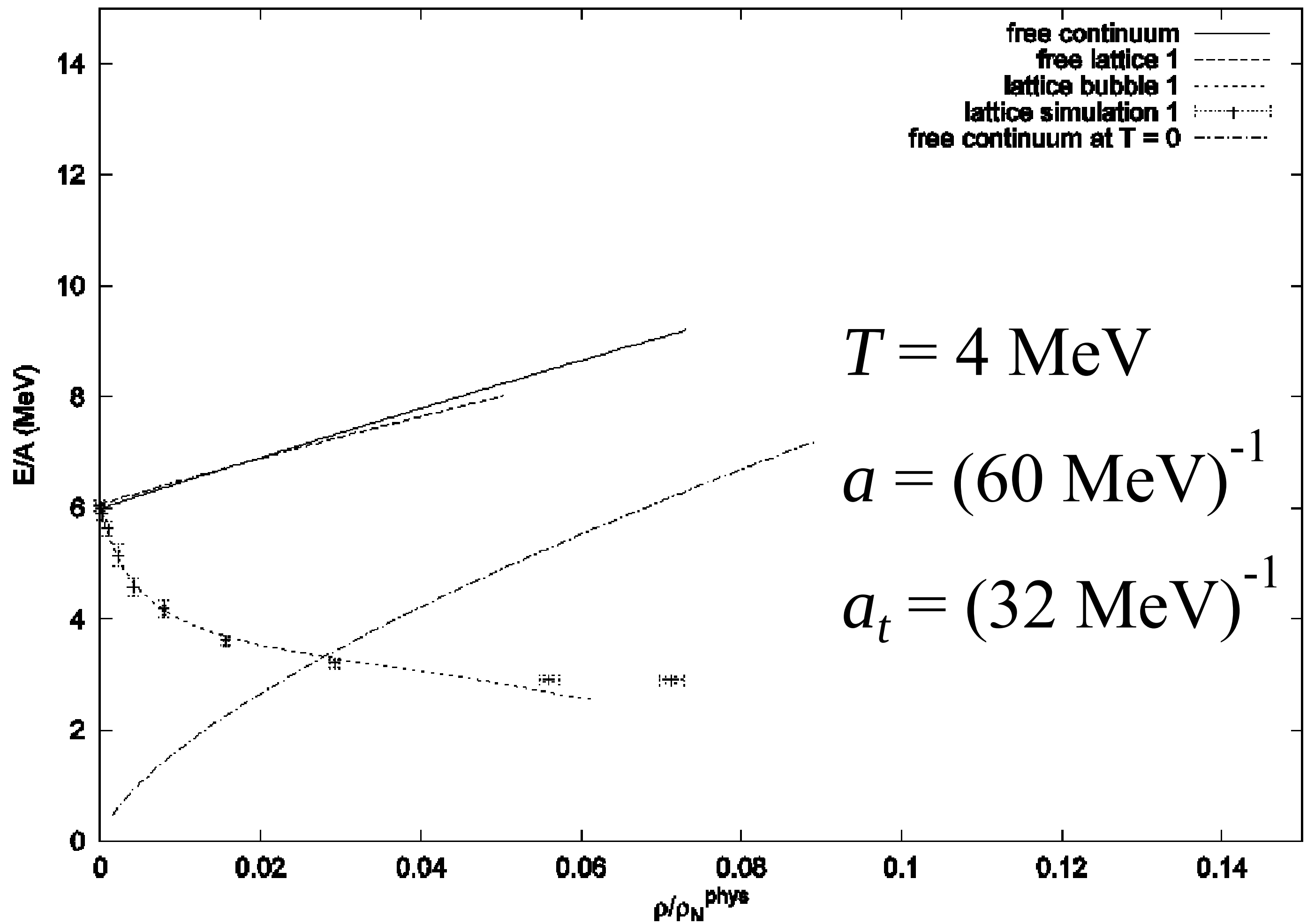
$$a_t = (48\ \text{MeV})^{-1}$$

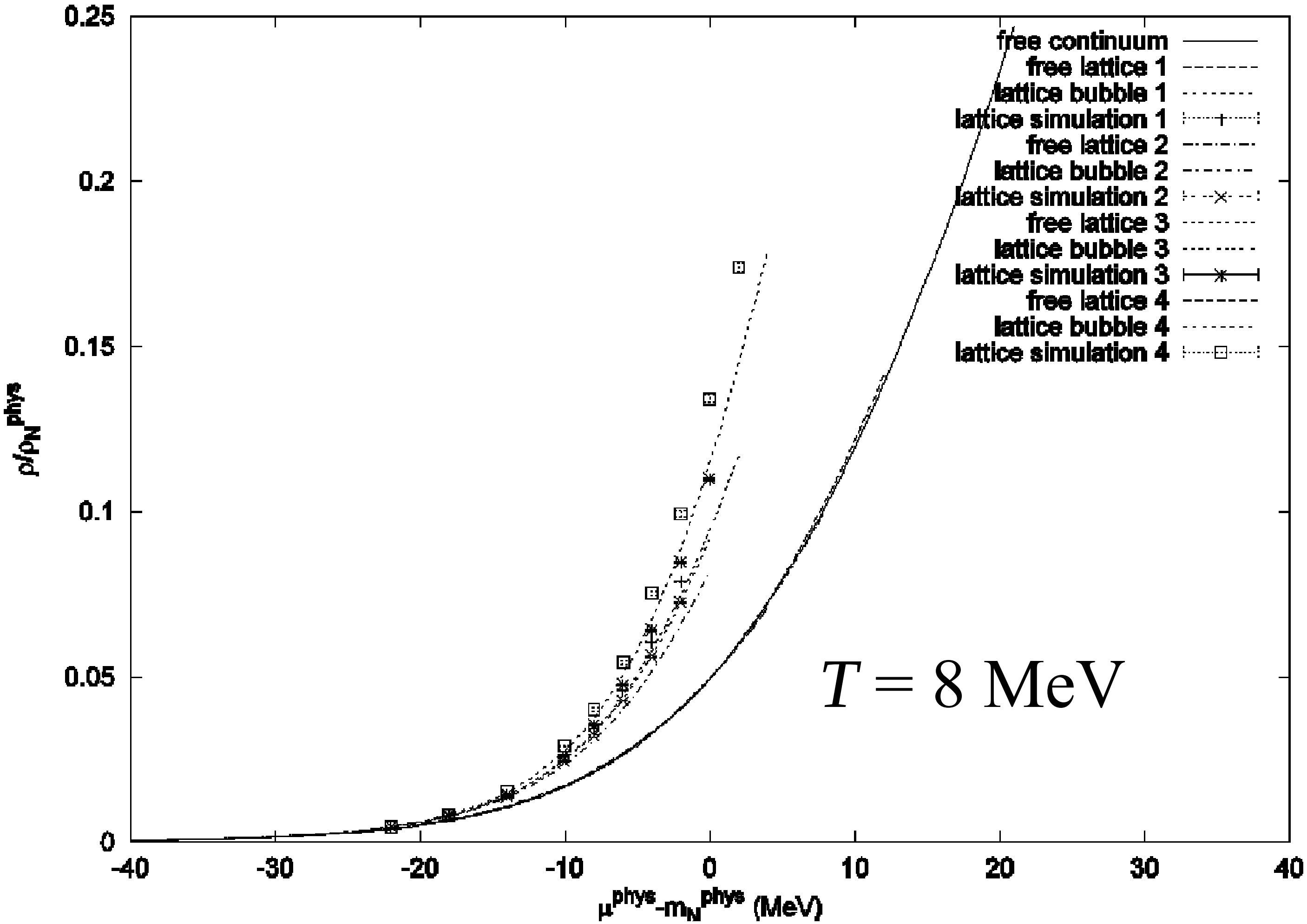
4

$$a = (80\ \text{MeV})^{-1}$$

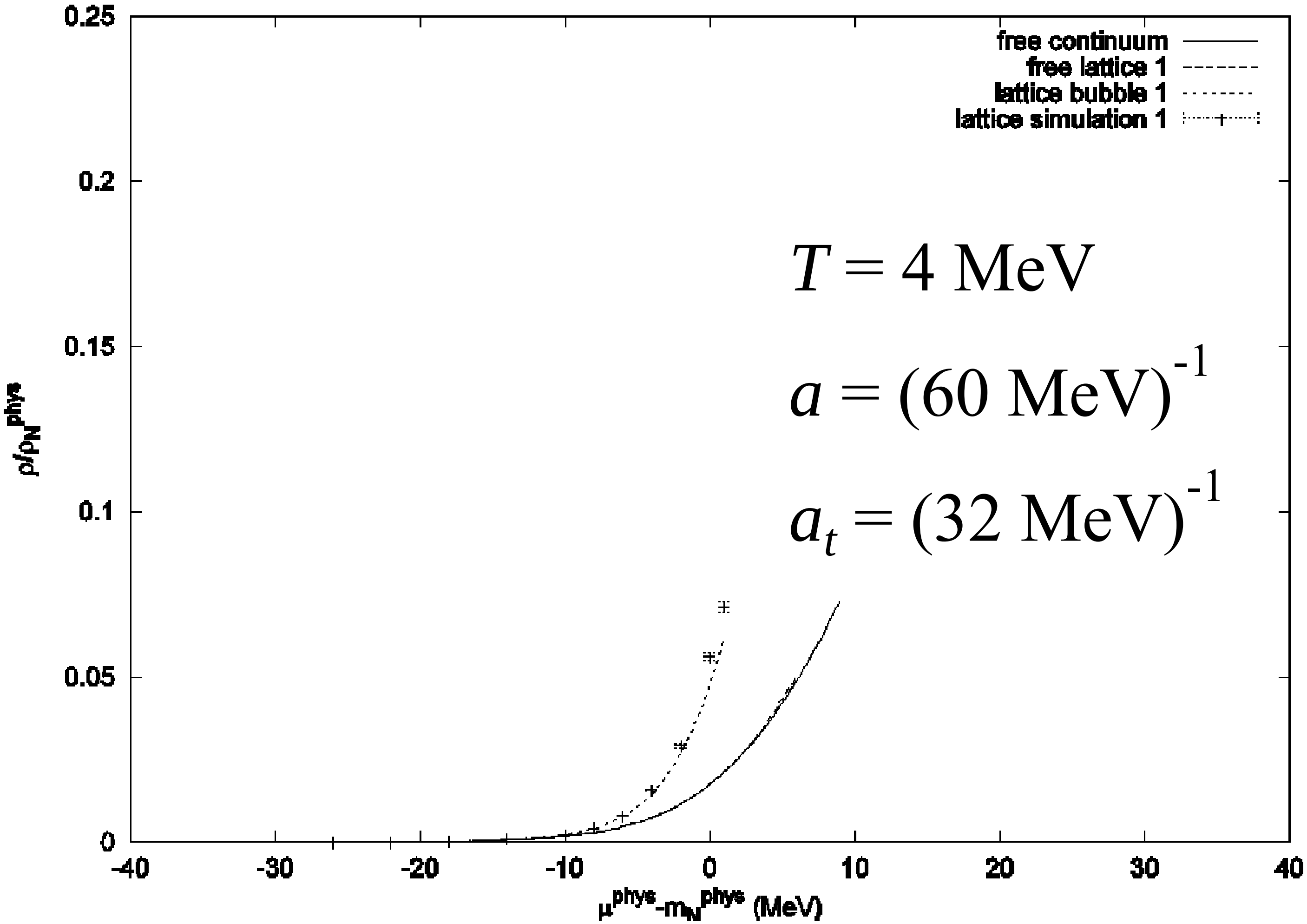
$$a_t = (72\ \text{MeV})^{-1}$$


free continuum
free lattice 1
lattice bubble 1
lattice simulation 1
free lattice 2
lattice bubble 2
lattice simulation 2
free lattice 3
lattice bubble 3
lattice simulation 3
free lattice 4
lattice bubble 4
lattice simulation 4
free continuum at T = 0
E/A (MeV)
ρ/ρ_N^phys
T = 8 MeV
0
2
4
6
8
10
12
14
0.02
0.04
0.06
0.08
0.1
0.12
0.14

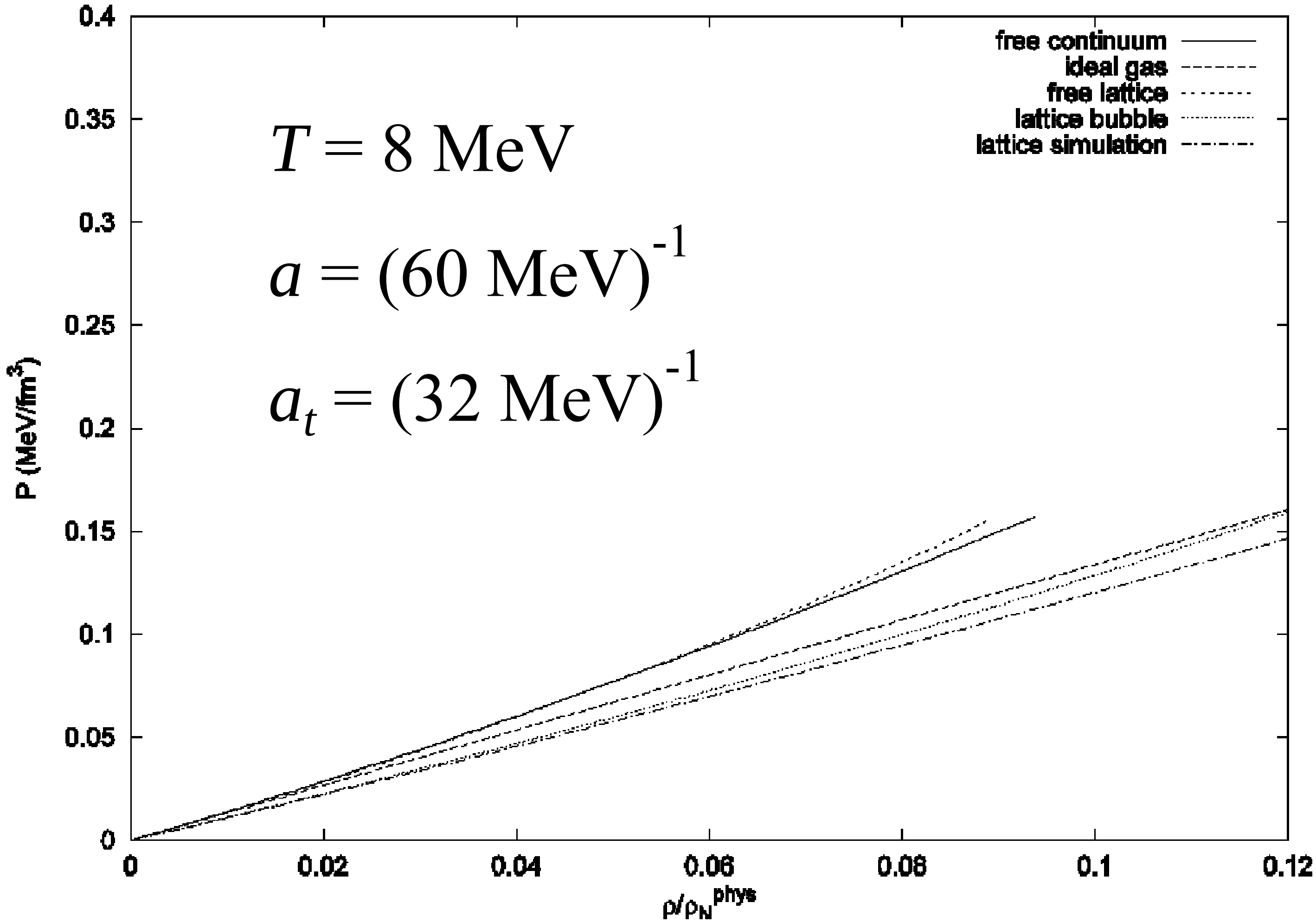

free continuum
free lattice 1
lattice bubble 1
lattice simulation 1
free continuum at T = 0
T = 4 MeV
a = (60 MeV)^-1
a_t = (32 MeV)^-1
E/A (MeV)
ρ/ρ_N^phys
14
12
10
8
6
4
2
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14



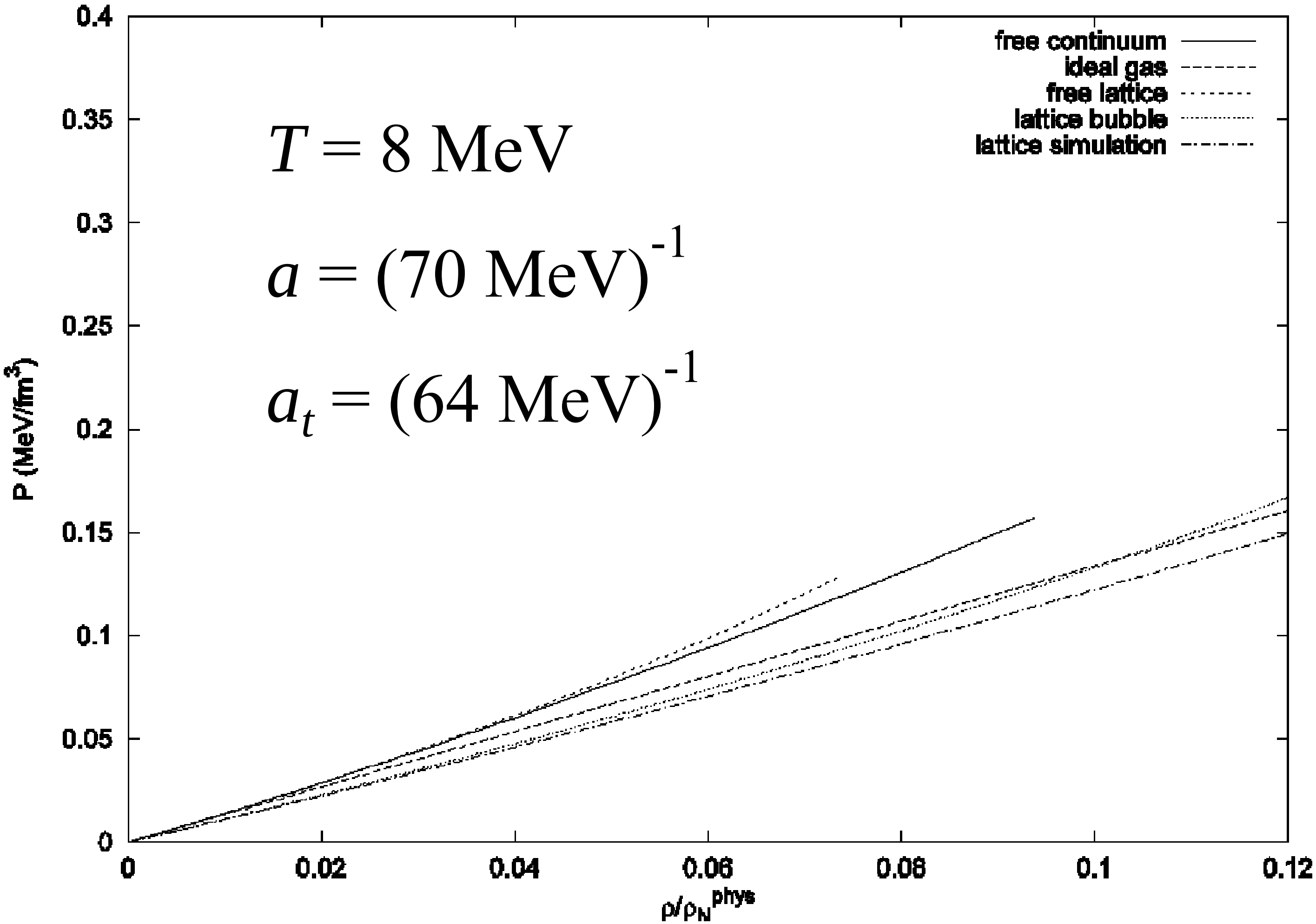
0.25
0.2
0.15
0.1
0.05
0
-40
-30
-20
-10
0
10
20
30
40
ρ/ρN phys
μ phys -mN phys (MeV)
free continuum
free lattice 1
lattice bubble 1
lattice simulation 1
free lattice 2
lattice bubble 2
lattice simulation 2
free lattice 3
lattice bubble 3
lattice simulation 3
free lattice 4
lattice bubble 4
lattice simulation 4
T = 8 MeV



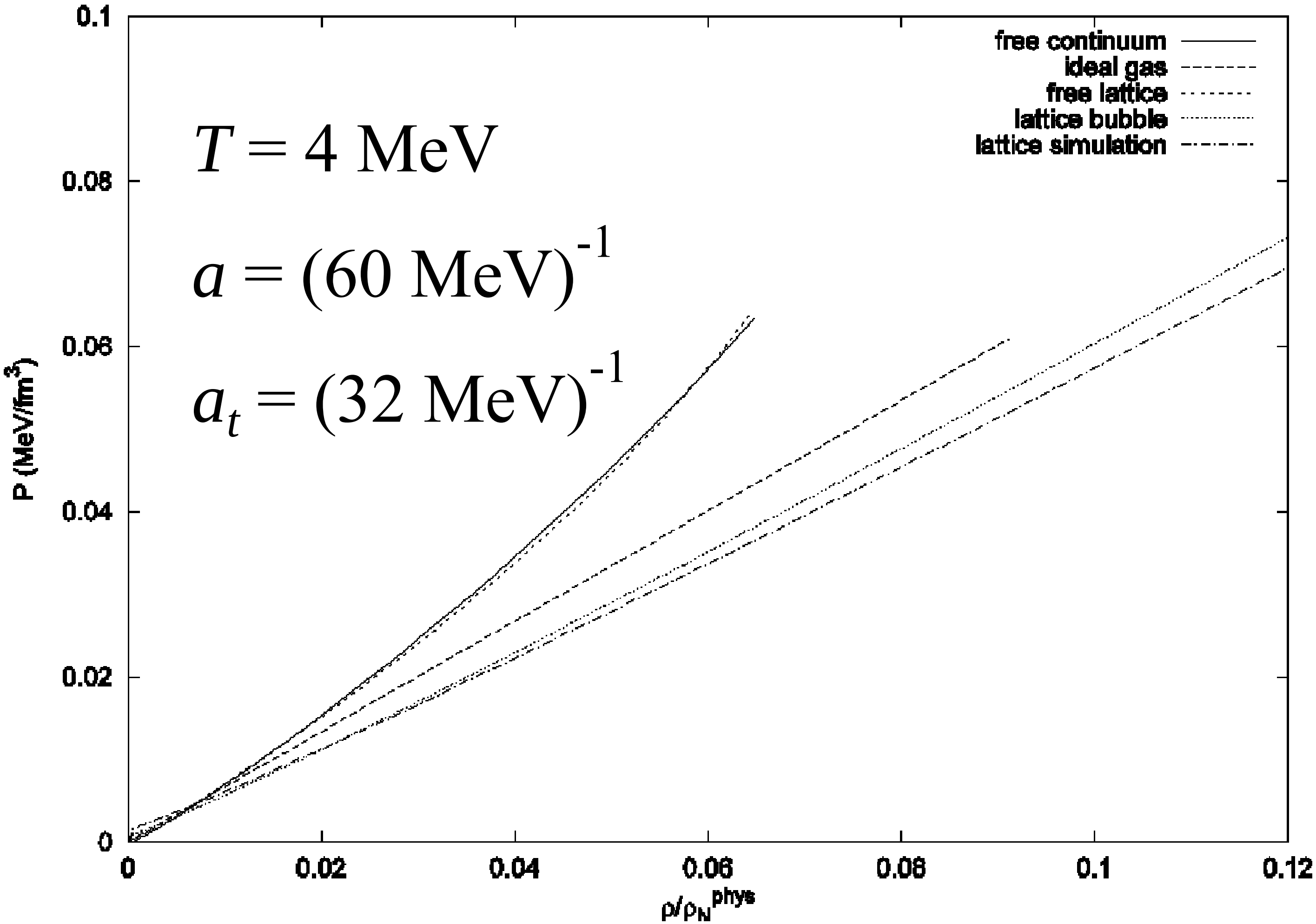
free continuum
free lattice 1
lattice bubble 1
lattice simulation 1
$T = 4$ MeV
$a = (60 \text{ MeV})^{-1}$
$a_t = (32 \text{ MeV})^{-1}$
ρ/ρ_N^{phys}
$\mu^{phys} - m_N^{phys}$ (MeV)
0
0.05
0.1
0.15
0.2
0.25
-40
-30
-20
-10
0
10
20
30
40



free continuum
ideal gas
free lattice
lattice bubble
lattice simulation
$T = 8$ MeV
$a = (60\ \text{MeV})^{-1}$
$a_t = (32\ \text{MeV})^{-1}$
P (MeV/fm^3)
ρ/ρ_N^{phys}
0
0.02
0.04
0.06
0.08
0.1
0.12
0.05
0.15
0.2
0.25
0.3
0.35
0.4



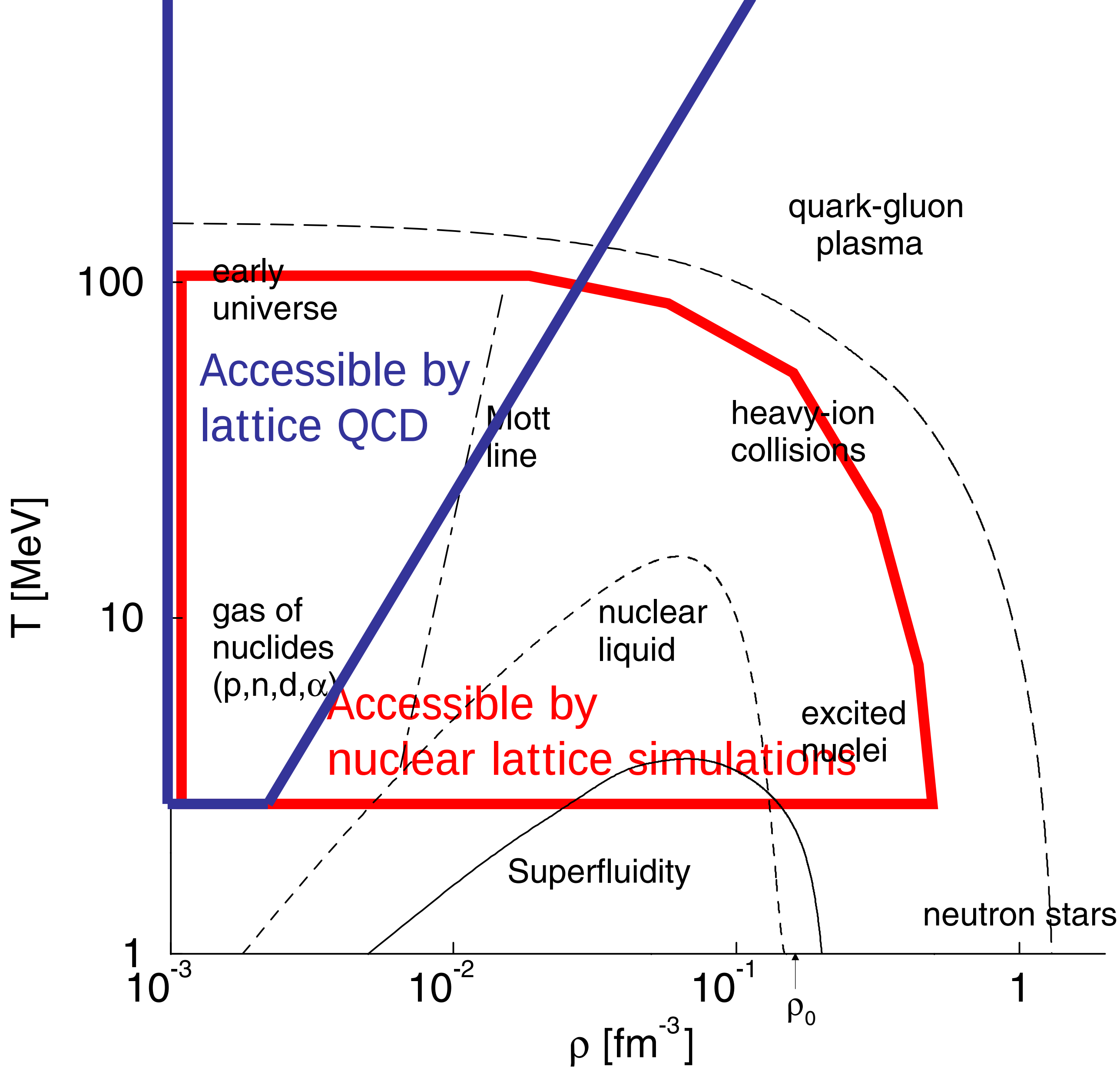
free continuum
ideal gas
free lattice
lattice bubble
lattice simulation
$T = 8$ MeV
$a = (70 \text{ MeV})^{-1}$
$a_t = (64 \text{ MeV})^{-1}$
P (MeV/fm^3)
ρ/ρ_N^{phys}
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.02
0.04
0.06
0.08
0.12



free continuum
ideal gas
free lattice
lattice bubble
lattice simulation
$T = 4$ MeV
$a = (60\ \mathrm{MeV})^{-1}$
$a_t = (32\ \mathrm{MeV})^{-1}$
P (MeV/fm^3)
ρ/ρ_N^{phys}
0
0.02
0.04
0.06
0.08
0.1
0.12

Road map

1. Symmetric nuclear and neutron matter without pions
2. Few body simulations (quenched simulations)
3. Algorithm improvement for determinant phase calculations
4. Asymmetric nuclear matter without pions
5. Nuclear and neutron matter with pions
6. Improved actions
7. Inequalities?



quark-gluon plasma
early universe
Accessible by lattice QCD
Mott line
heavy-ion collisions
gas of nuclides (p,n,d,α)
nuclear liquid
Accessible by nuclear lattice simulations
excited nuclei
Superfluidity
neutron stars
T [MeV]
100
10
1
10^{-3}
10^{-2}
10^{-1}
1
ρ_0
ρ [fm^{-3}]