# Light Front For Nuclear Physics

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**What, why Light Front Nuclear Physics**

**Uses: pions, deuteron**

**Brodsky, Coester, Frankfurt Strikman, Salme**

# What is light front dynamics?

"time" $t+z = x^0 + x^3 \equiv x^+$, "energy" $p^0 - p^3 \equiv p^-$

$p^-$ is the $x^+$ evolution operator

"space $x^- \equiv t - z$, "momentum $p^+ = p^0 + p^3$

$\frac{p^+}{P+}$ is Bjorken variable. $\mathbf{x}_\perp, \mathbf{p}_\perp$ as usual

$$A \cdot B = A^\mu B_\mu = \frac{1}{2}(A^+ B^- + A^- B^+) - \mathbf{A}_\perp \cdot \mathbf{B}_\perp$$

$$p^\mu p_\mu = m^2 = p^+ p^- - p_\perp^2 \rightarrow p^- = \frac{1}{p^+}(p_\perp^2 + m^2)$$

relativistic but no $\sqrt{\phantom{x}}$

Separate cm from internal degrees of freedom

# I: Form Factors, large $Q^2$

Form factor: probability amplitude for system to absorb four-momentum $q$, remain in ground state $P^2 = M^2 = (P+q)^2,\ Q^2 = -q^2$ N, π, D, $He^3$

**Non-Rel:** $\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R} - iEt}\phi(r),\ \mathbf{r}_{1,2} = \mathbf{R} \pm \frac{1}{2}\mathbf{r}$

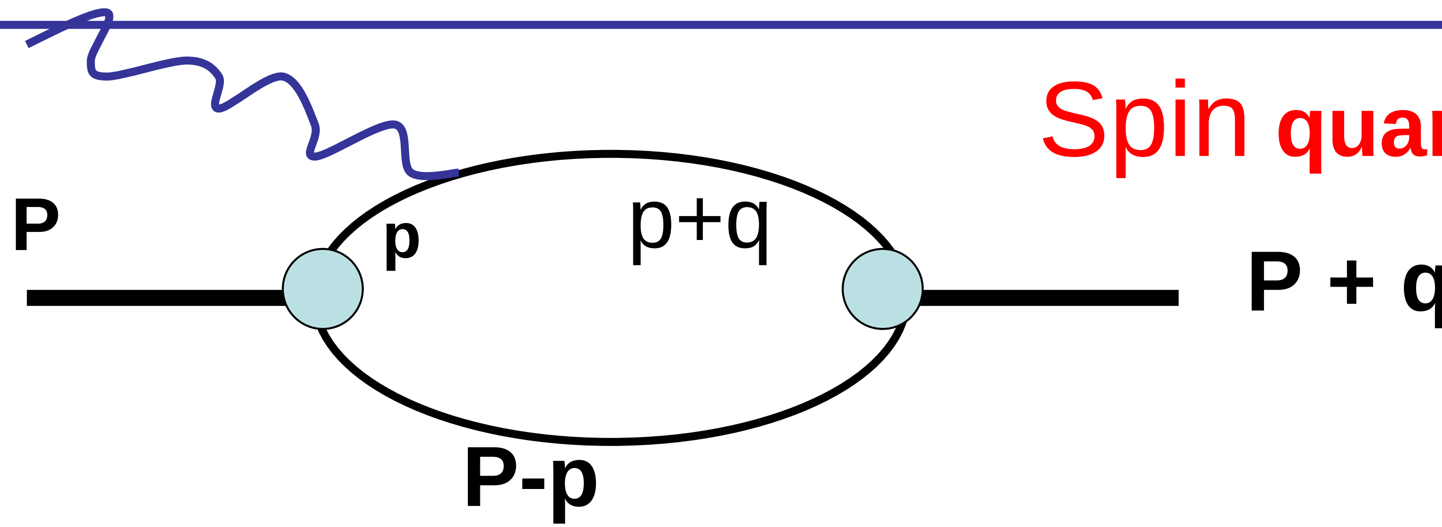
$$F(q^2) = \int d^3r e^{i\mathbf{q}\cdot(\frac{1}{2}\mathbf{r})} |\phi(r)|^2$$

relativistic: $\mathbf{r}_1, t_1, \mathbf{r}_2, t_2, t_1 \neq t_2$, no factorization

$|\Psi(P=0)\rangle,\ |\Psi(P=q)\rangle$ differ - Lorentz

# Almost everything $\pi$

# Pion Form Factor




**Spin quarks**

$$F(Q^2) = \int d^4k \, Tr\left[\frac{\not{k}+m}{k^2-m^2+i\epsilon}\gamma^+ \frac{\not{k}+\not{q}+m}{(k+q)^2-m^2+i\epsilon}\gamma^5 \times \frac{-\not{P}+\not{k}+m}{(P-k)^2+m^2+i\epsilon}\gamma^5\right]$$

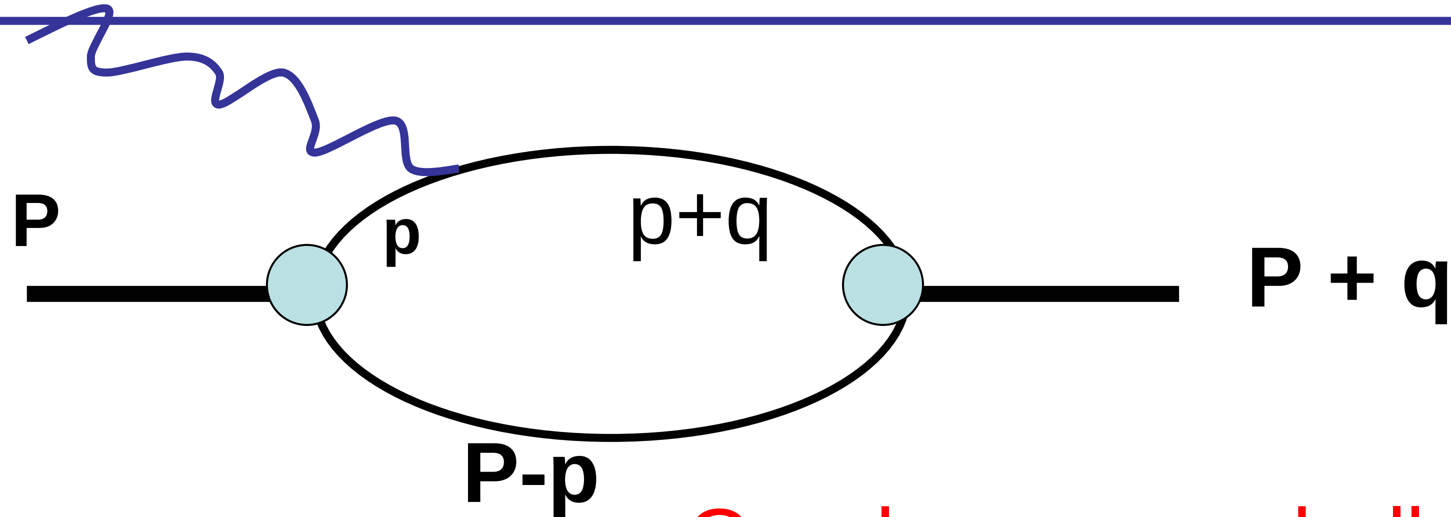
**Integrate over $k^-$**

$$\frac{\not{k}+m}{k^2-m^2+i\epsilon} = \frac{\sum_s u(k,s)\bar{u}(k,s)}{k^2-m^2+i\epsilon} + \frac{\gamma^+}{2k^+}$$

$\gamma^+\gamma^+=0$

Quarks are on shell

# Pion Form Factor

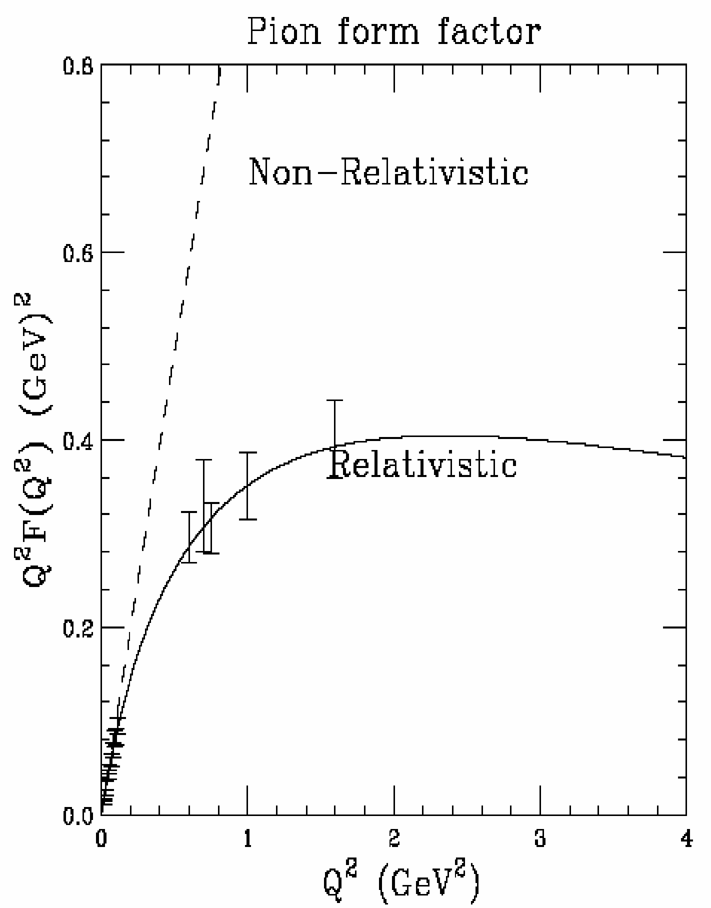


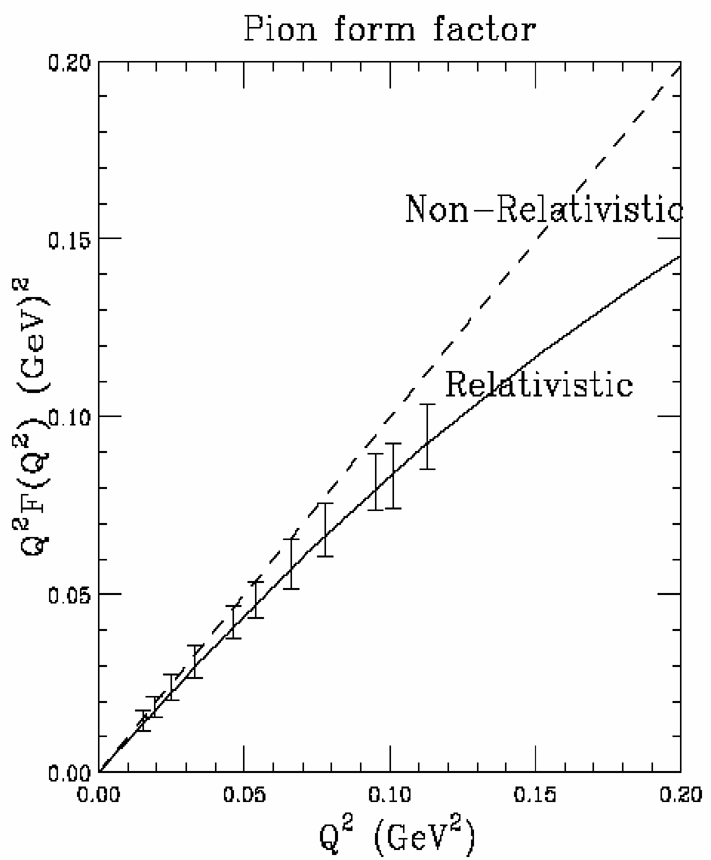

Quarks are on shell $\gamma^+\gamma^+=0$

$$F(Q^2) = \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)} \int d^2k_\perp \psi(\mathbf{k}_\perp + (1-\alpha)\mathbf{q}_\perp)(\text{Spin factor})\ \psi(\mathbf{k}_\perp)$$

**$F(Q^2)$ not a Fourier transform**

# Pion with CCP parameters (88)



# The Issue: 4 dimensions $\longrightarrow$ 3 dimensions

Bethe-Salpeter equation

$$\Psi_P(k) = \int d^4k' \; S_1(k)S_2(P-k)K(k,k')\Psi_P(k')$$

Instant form: $$\psi_P(\mathbf{k}, t=0) \equiv \int dk^0 \Psi_P(k)$$

+ Gross eq: spect. on shell

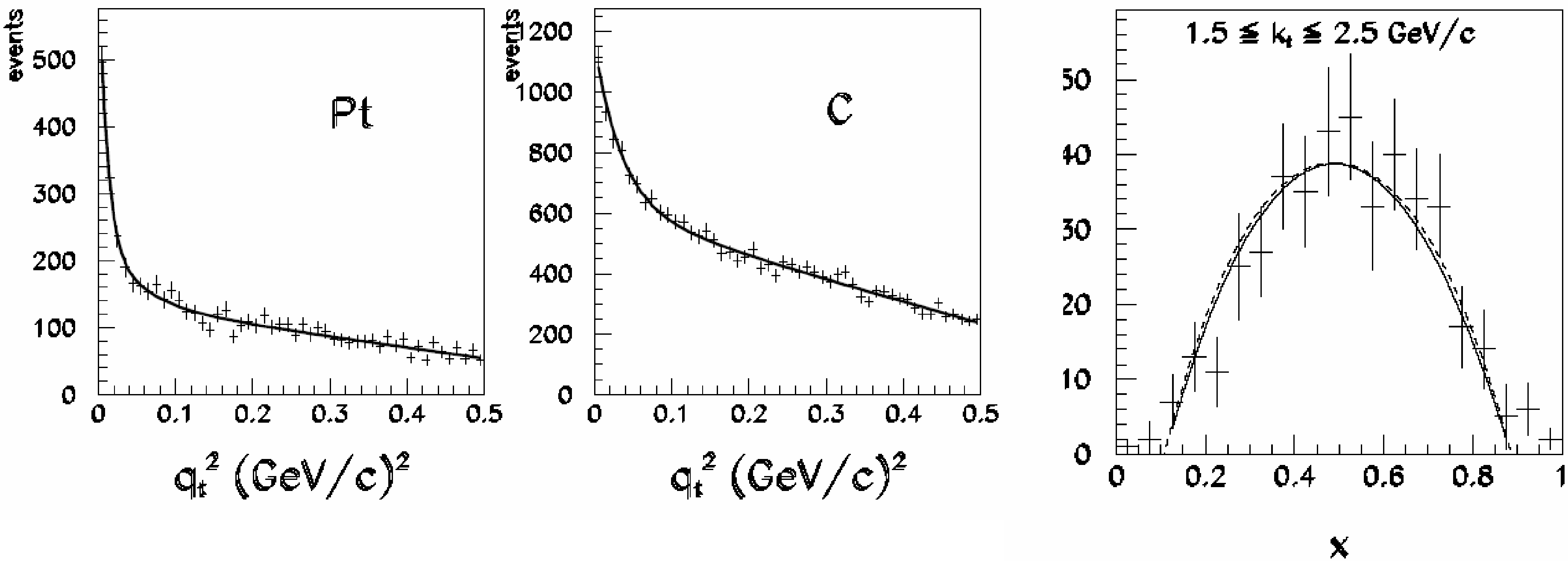
front form: $$\psi_P(k^+, \mathbf{k}_\perp, x^+ = 0) \equiv \int dk^- \Psi_P(k)$$

**Which is better? Need to put 4D effects in K**

**Light front: kill $(\gamma/2k^+)$ term in nucleon $S_F$**
**$k^+/P^+$ enters at high energy**

# Pions &nuclear color transparency

- **Light cone wave function- PLC exists**
- **pi +A to dijets (coherent) FMS93**
- **M(A)≈A M(N)(t=0) Ashery, Fermilab E791**

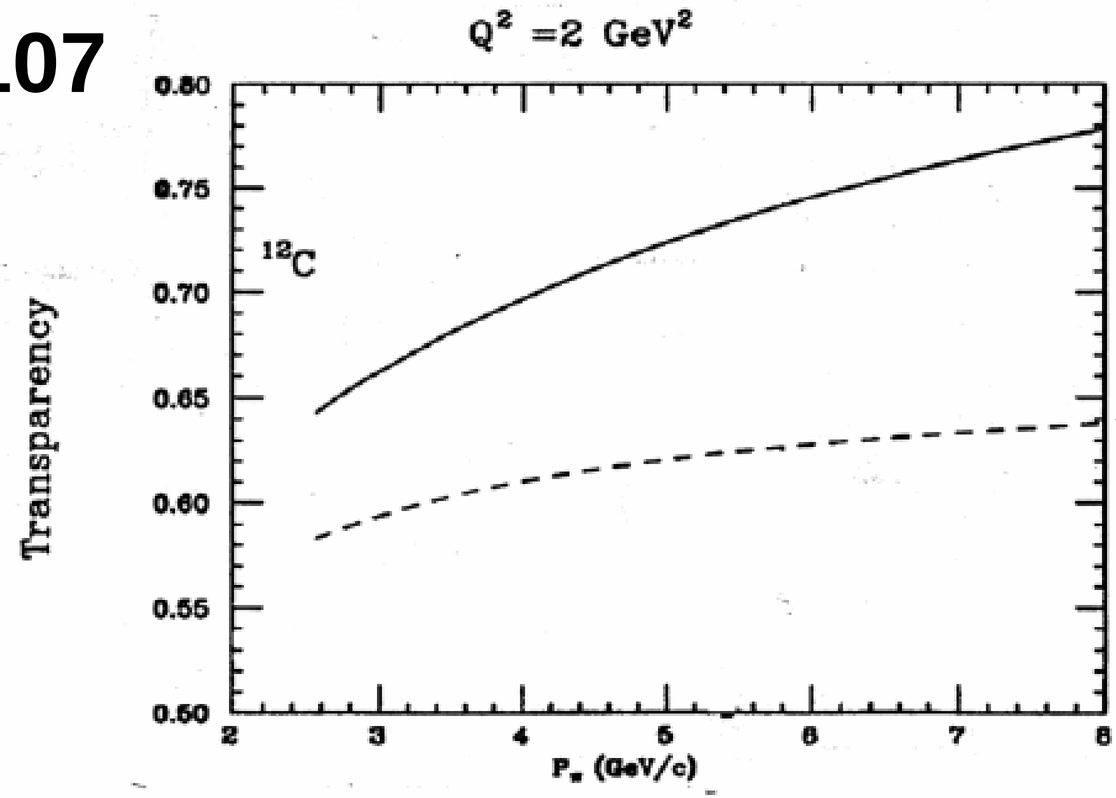



**Color transparency exists**

# Pion color transparency Jlab 6,12

- (e,e', $\pi$): $\gamma^*$ makes pion PLC (expands?)

**E01-107**




**Miller, Strikman**

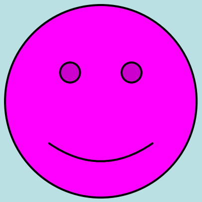
**Good at 6, better at 12**

# Pions and the EMC effect

**Why pions?**

**Too many pions bad**

**Some allowed**



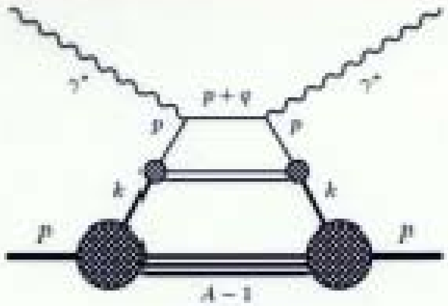
# Deep Inelastic Scattering From Nuclei

**Smith, Miller**

Test Hypothesis: conventional nuclear dynamics explains the EMC effect

binding energy and Fermi motion  Walecka model

manifestly covariant formulation, free nucleon structure function $F_{2N}$

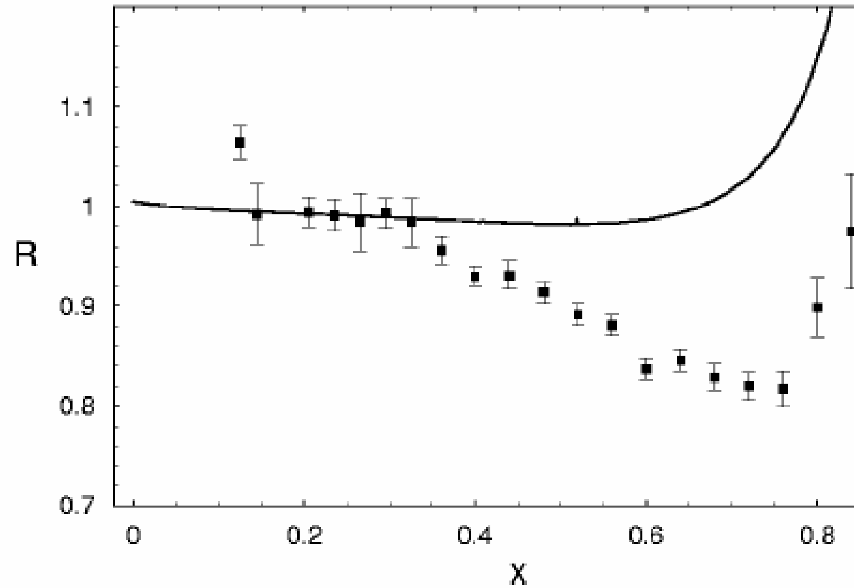



$$\frac{F_{2A}(x_A)}{A} = \int_{x_A}^{\infty} dy f_N(y) F_{2N}(x_A/y)$$

$$f_N(y) = \int \frac{d^4k}{(2\pi)^4} \delta(y - \frac{k^0 + k^3}{\bar{M}}) Tr \left[ \frac{\gamma^+}{2P^+A} \chi_C(k, P) \right]$$

$P$ is nucleus four momentum $x_A \equiv Q^2 A / 2P \cdot q = xAM/M_A =$

$$\chi_C(k, P) \;\; = \;\; -i2P^+\Omega \left( \frac{i\pi}{E^*(k)} \delta(k^0 - E^*(k) - g_v V^0)) \theta(k_F - |\mathbf{k}|) \right]$$




**Binding fails**

## Nucleons and Mesons (m)

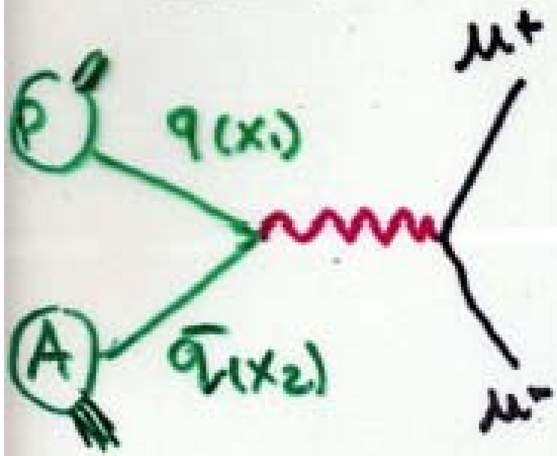
$$P_A^+ = P_N^+ + P_m^+ = M_A \qquad \epsilon \equiv \frac{P_m^+}{M_A}$$

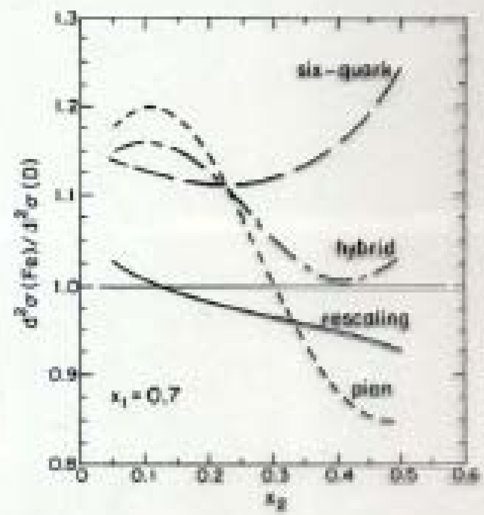
$$\int dy\, y\, f_N(y) = 1 - \epsilon$$

Can get EMC binding effect with $\epsilon \approx 0.05$ (Many authors)

But $\epsilon \approx 0.05$ is a BIG enhancement of nuclear sea

Use Drell-Yan data to disentangle EMC effect
Bickerstaff, Birse and Miller, Phys. Rev. Lett. **53**, 2532 (84).



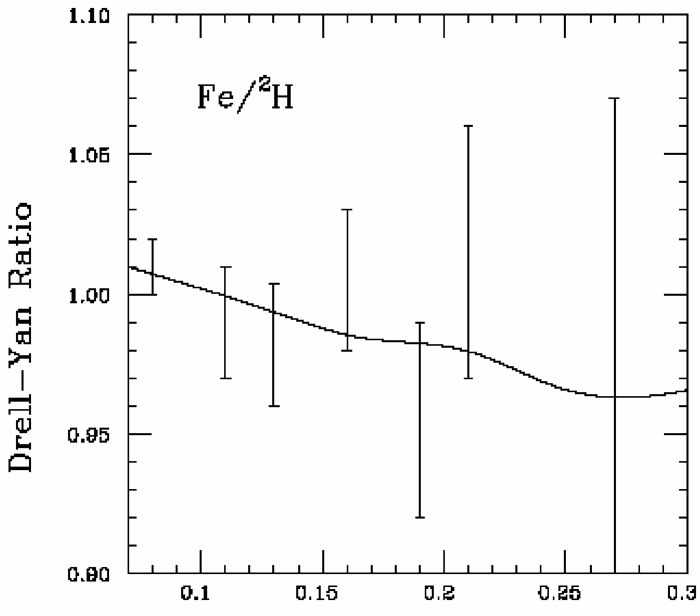

**Pions fail**

Data from E772 -no enhancement, no nuclear effects $x_1 - x_2 \approx 0.3$

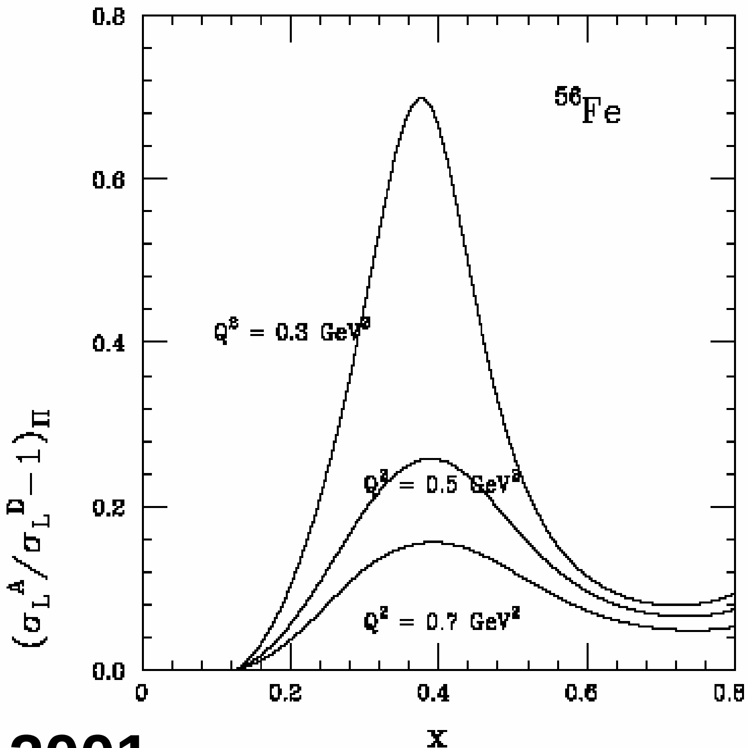
"Crisis in Nuclear Theory"
Bertsch, Frankfurt and Strikman, Science **259**, 773 (1993)

# Pions from nuclear LF wave fun

**DY OK OPE**




**σ (L) enhanced (coherent)**

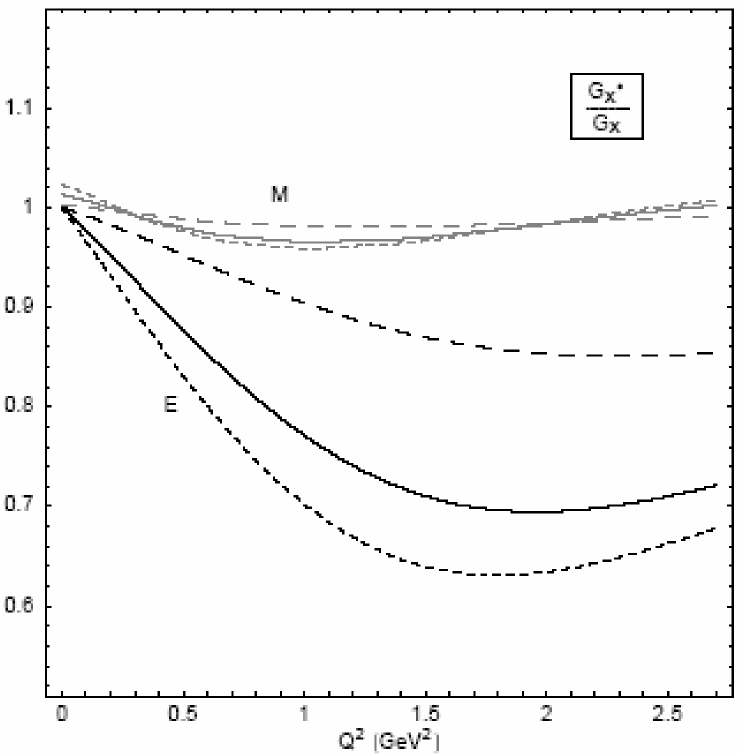


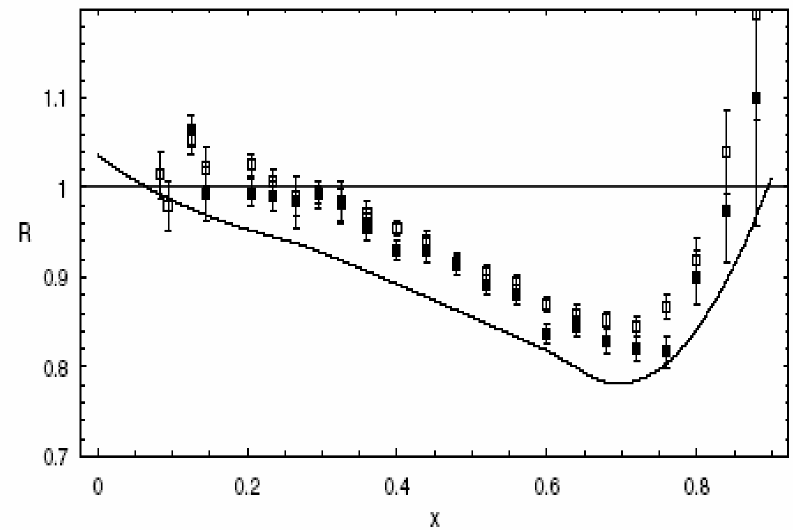

**Miller 2001**

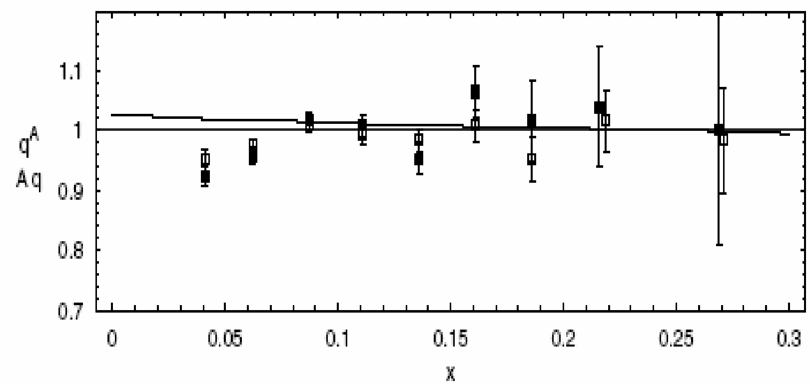
**How to get nuclear LF wave fun ?**

# Chiral instanton soliton Sea!!

- nucleons in nuclei (Smith & M '03,04)

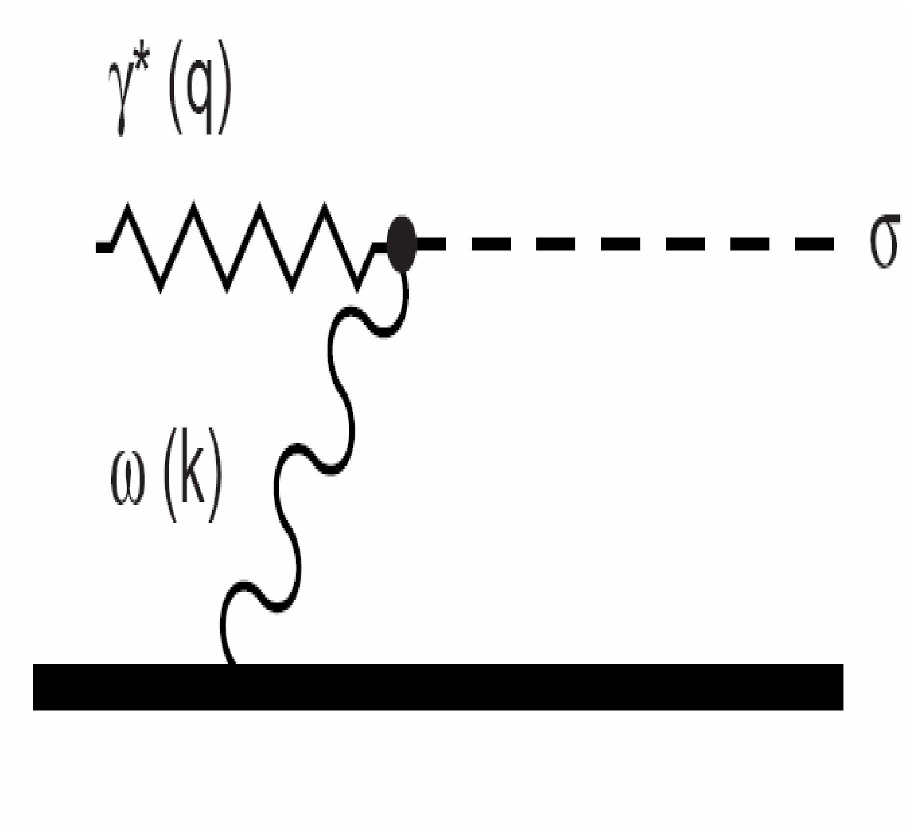


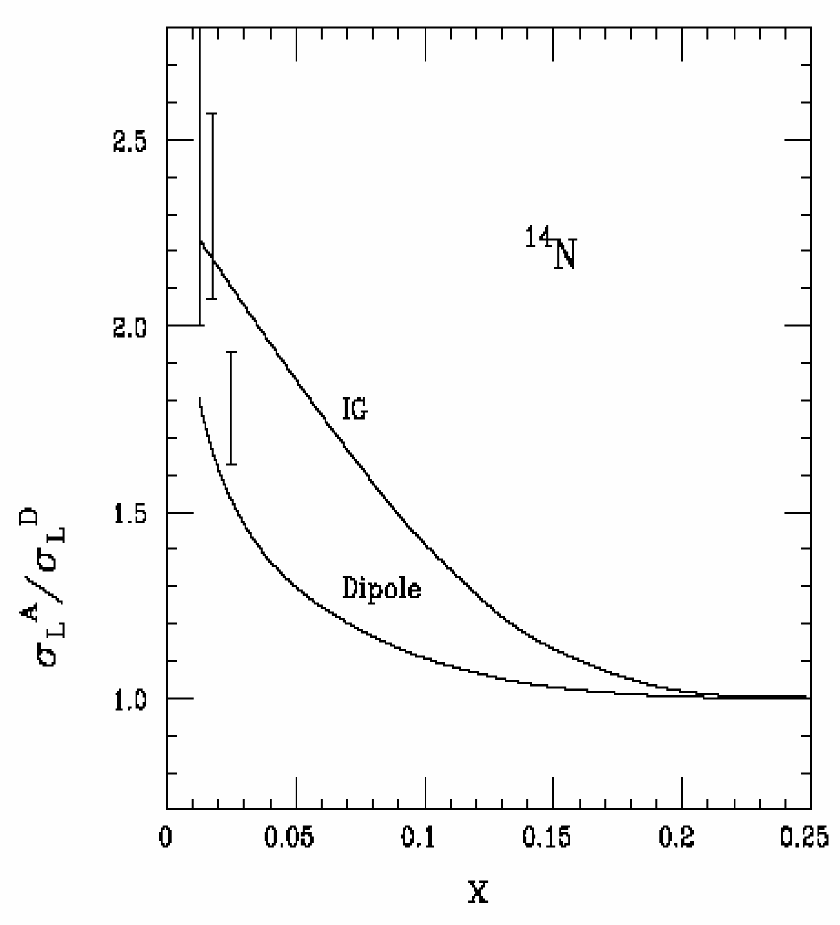




**σ (L) π + gluons**

# More mesons? GAM,SJB,MK (00)




**σ(L) to rule out Walecka model?**

# Summary -Pions

- simple system with PLC
- reproduce form factor
- color transparency observed in coherent nuclear di-jet production
- nuclear pions limited by Drell-Yan
- some pions enhance $\sigma(L)$
- nuclear vector mesons enhance $\sigma(L)$ ?
- nuclear gluons and $\sigma(L)$?

# Dynamics For NN Systems

Symmetries are not enough.

We need $\mathcal{L}$ no matter how bad!

## Light Front Quantization LITE

$$\mathcal{L} \rightarrow T^{\mu\nu} \quad P^{\mu} = \frac{1}{2}\int d^2x_{\perp} dx^{-} T^{+\mu}$$

$T^{+\mu}$ **in terms of independent variables (N,2)**

**easy for massive particles**

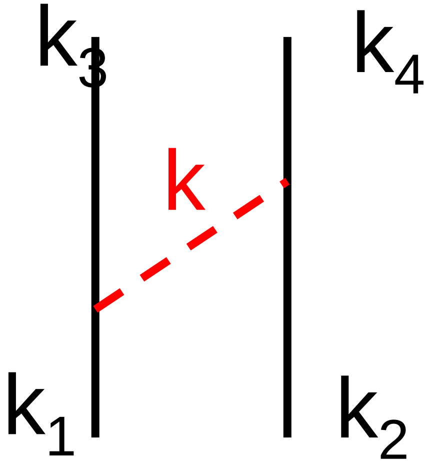
# Light Front Quantization not so LITE

$P^- =$ "Hamiltonian"

Do $x^+$-ordered perturbation theory

energy denominators : $P^- - \sum_i p_i^-$

# Scalar meson exchange



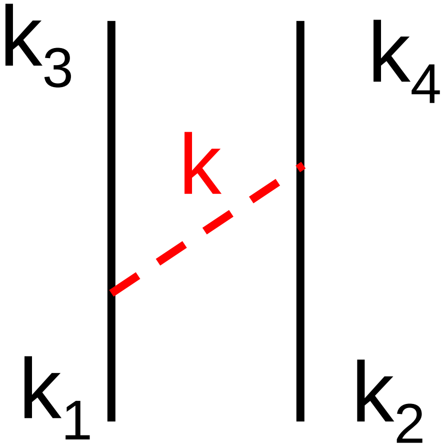

$$q = k_1 - k_3,\; k^+ = q^+, \mathbf{k}_\perp = \mathbf{k} = \mathbf{q}$$

conserved

$$D = k^+(k_1^- - k_3^- - k^-)$$
$$= (k_1^+ - k_3^+)(k_1^- - k_3^-) - k^+k^-$$
$$= q^+q^- - (\mathbf{k}^2 + \mu^2)$$
$$= q^2 - \mu^2$$

rot. inv. Yukawa potential, w. retardation

# Bound States, B




For bound states $B$: $V \sim \frac{1}{k^+(P^- - k_3^- - k_2^- - k^-)}$

RI not manifest, $P^- = M_B$

# Light front One Boson Exchange

## Yukawa propagator with retardation

K= $\sum_{\text{meson}} V(\text{meson}) \quad (\pi\rho, \omega, \sigma, \eta\delta)$

Iterate (in any frame) integrate $k^-$ get

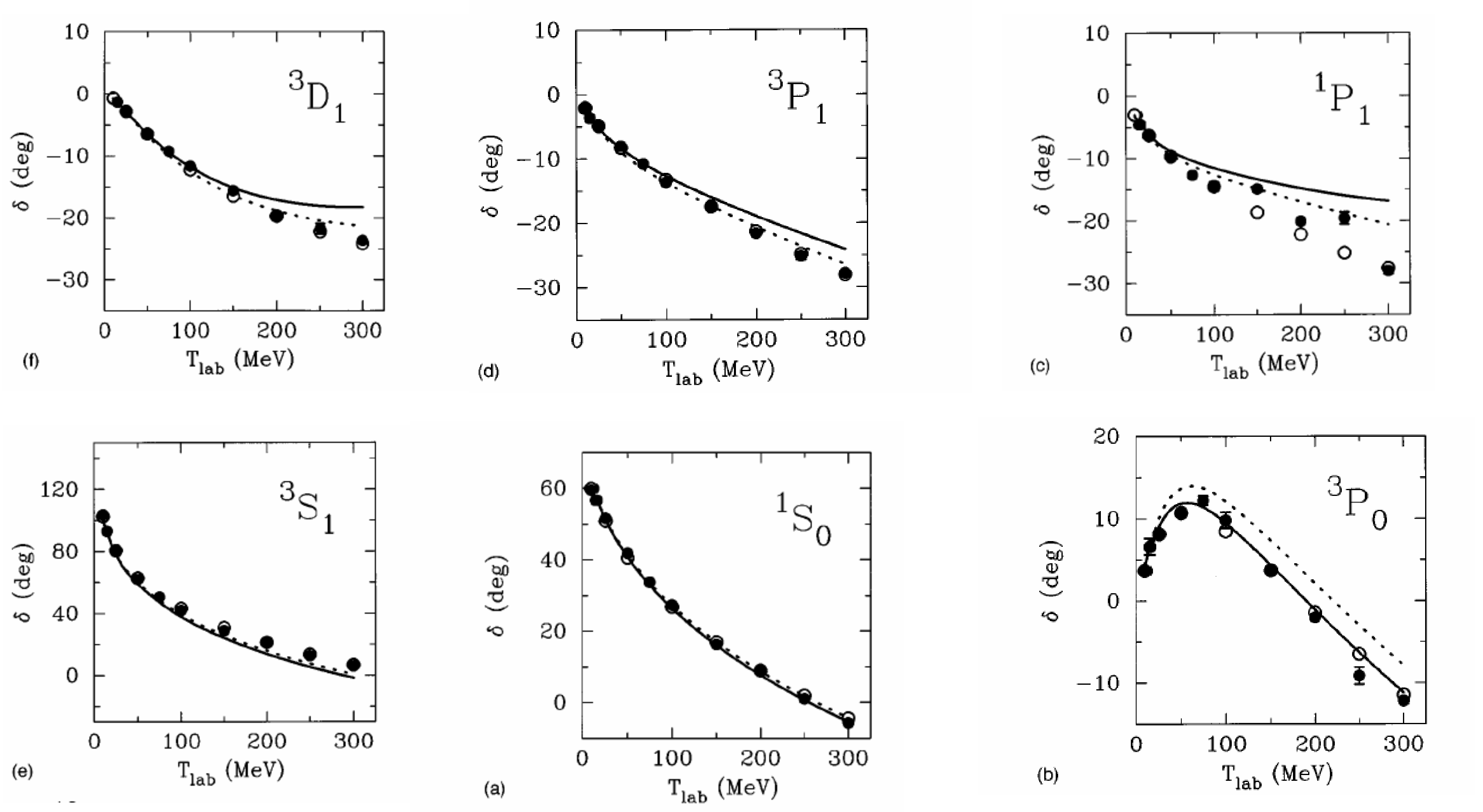
$$\mathcal{M} = K + \int \frac{d^2p_\perp d\alpha}{\alpha(1-\alpha)} \frac{2M^2}{P^2 - \frac{\mathrm{p}^2+M^2}{\alpha(1-\alpha)} + i\epsilon} \mathcal{M}$$

Rotational invariance?

$$\alpha \rightarrow \frac{E(p)+p^3}{2E(p)}, \; E(p) = \sqrt{p^2+M^2}$$

$$\frac{d^2p_\perp d\alpha}{\alpha(1-\alpha)} \frac{2M^2}{P^2 - \frac{\mathrm{p}^2+M^2}{\alpha(1-\alpha)} + i\epsilon} \rightarrow \frac{M^2}{E(p)} \frac{d^3p}{P^2/4 - E^2(p)}$$

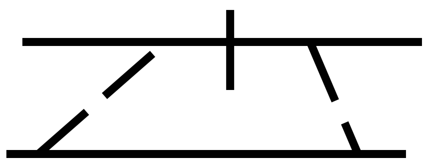
# Machleidt, Miller light front OBE



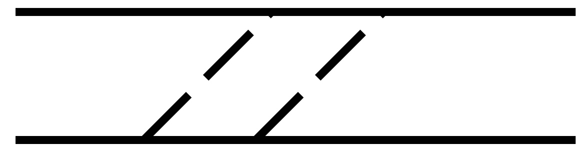
# Deuteron –w. J. Cooke

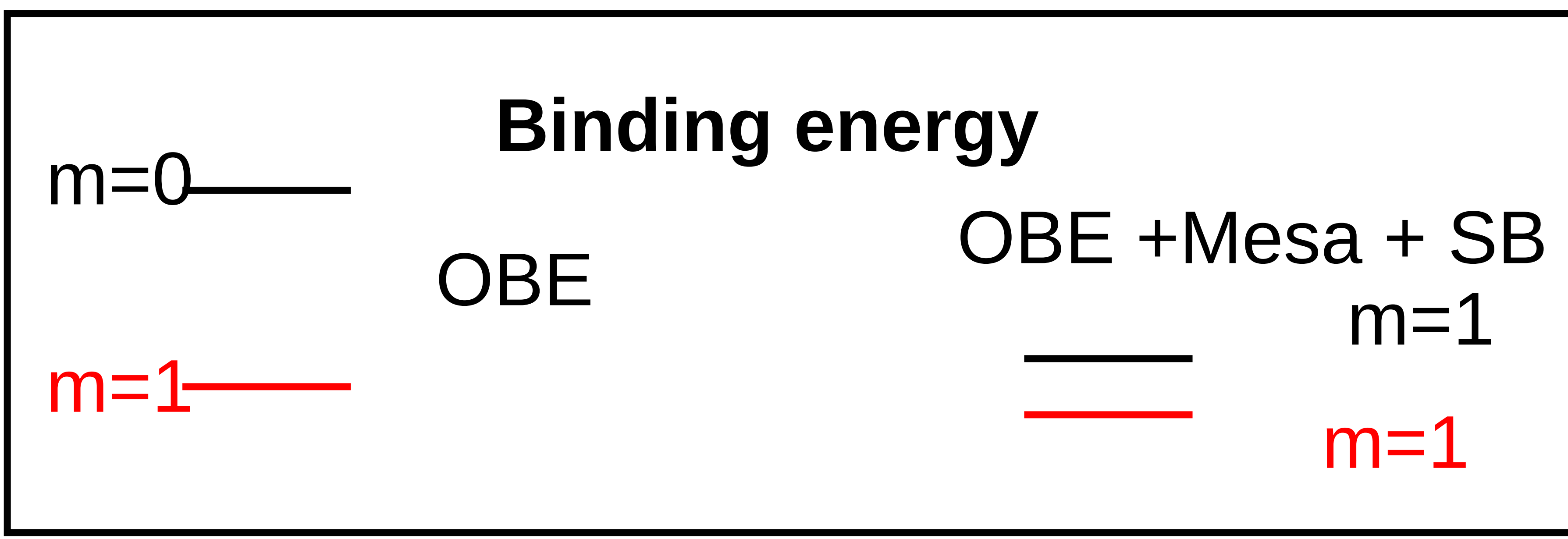
**OBEP violates rotational invariance first fix – add Mesa + SB**

Mesa



SB





# Rotational Invariance –Form Factors

$$I_{\lambda'\lambda} \equiv \langle \lambda' | J^+ | \lambda \rangle$$

4 values of $\lambda', \lambda$ 3 form factors

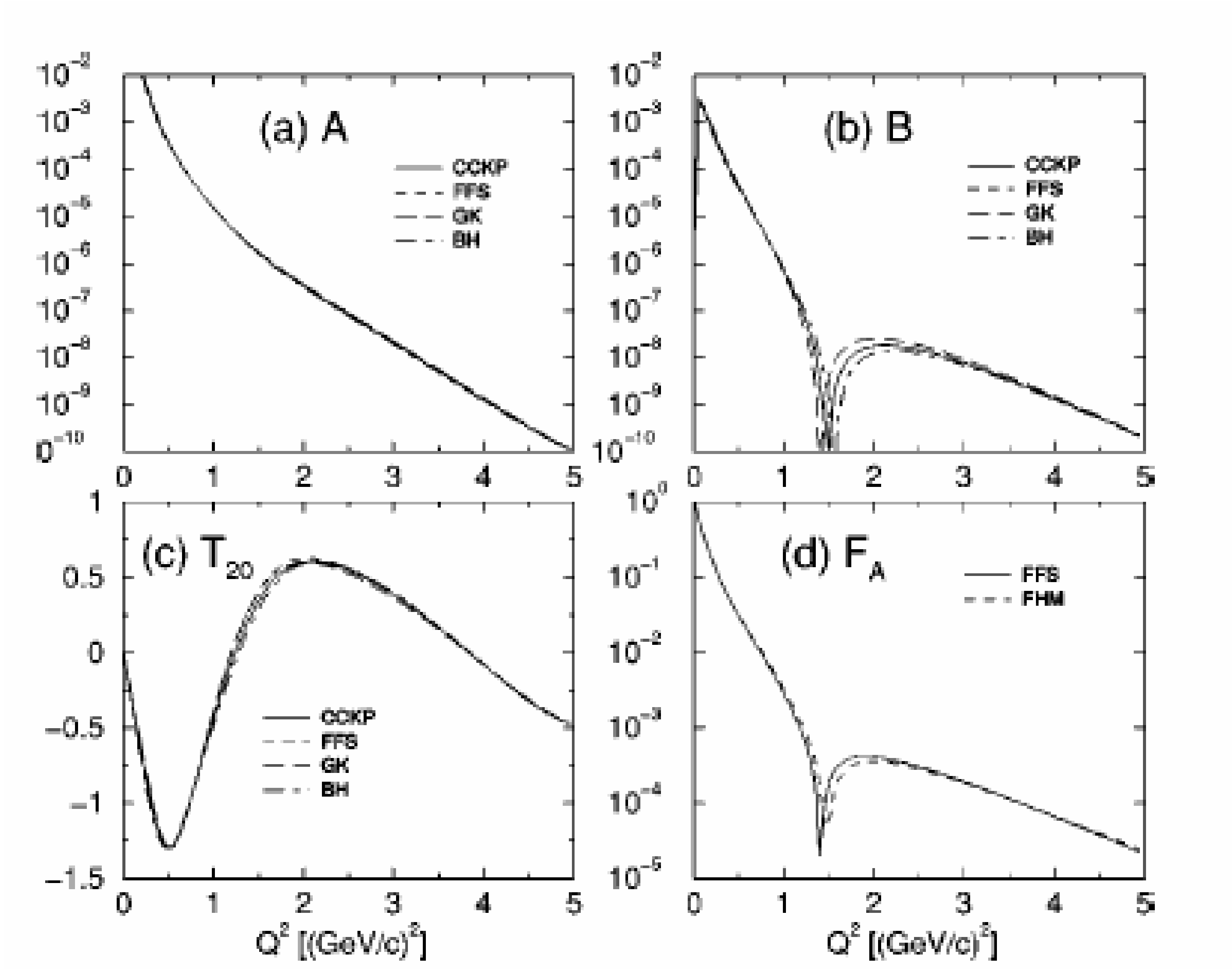
Rotational invariance of charge density $\rightarrow$

$$(1+2\eta)I_{11} + I_{1-1} - \sqrt{8\eta} I_{10} = 0, \ \eta = \frac{Q^2}{4M_d^2}$$

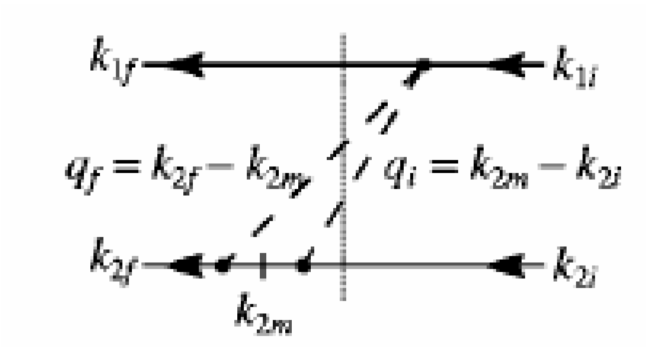
**Impulse approx does not get 0. Choose 1 $I_{\lambda\lambda}$ as dependent, compute other 3**

**4 prescriptions, at least**

# Rotational Inv. of form factors OME



# Two meson exchange w. chiral symmetry

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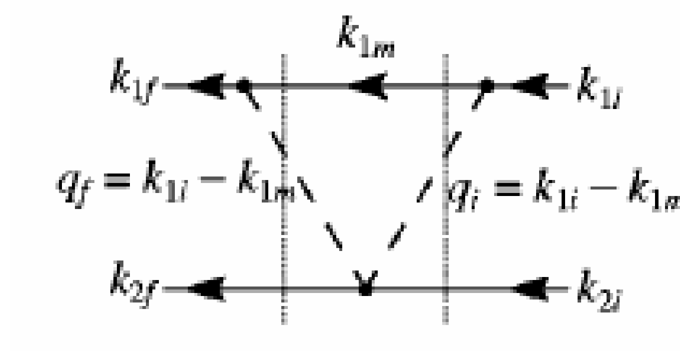


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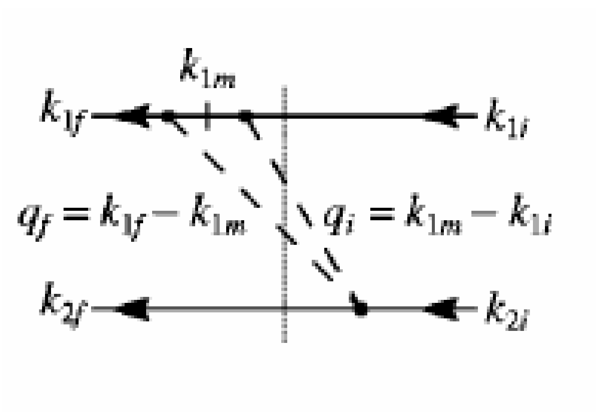


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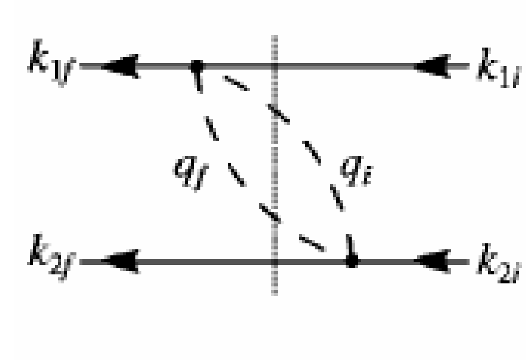


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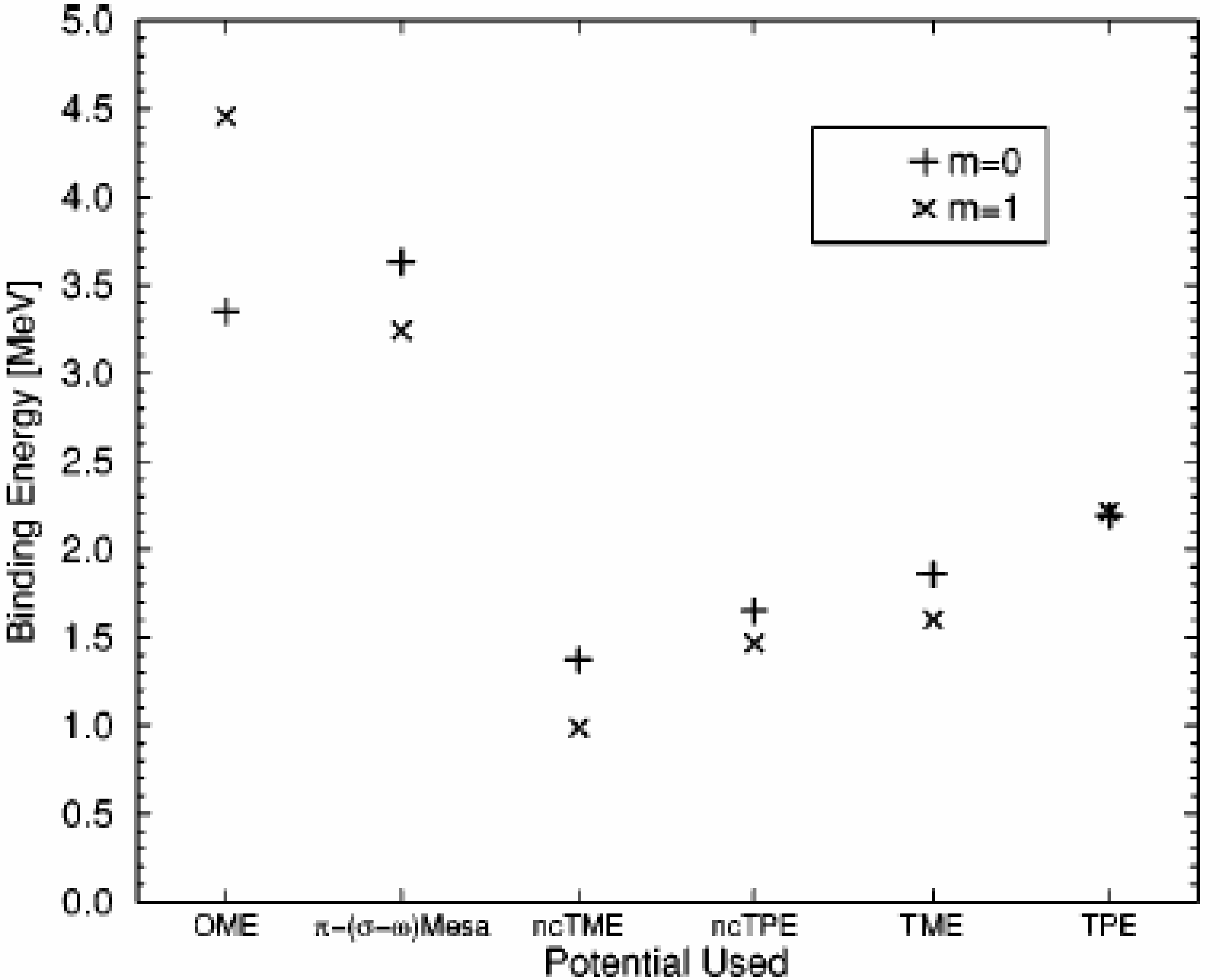
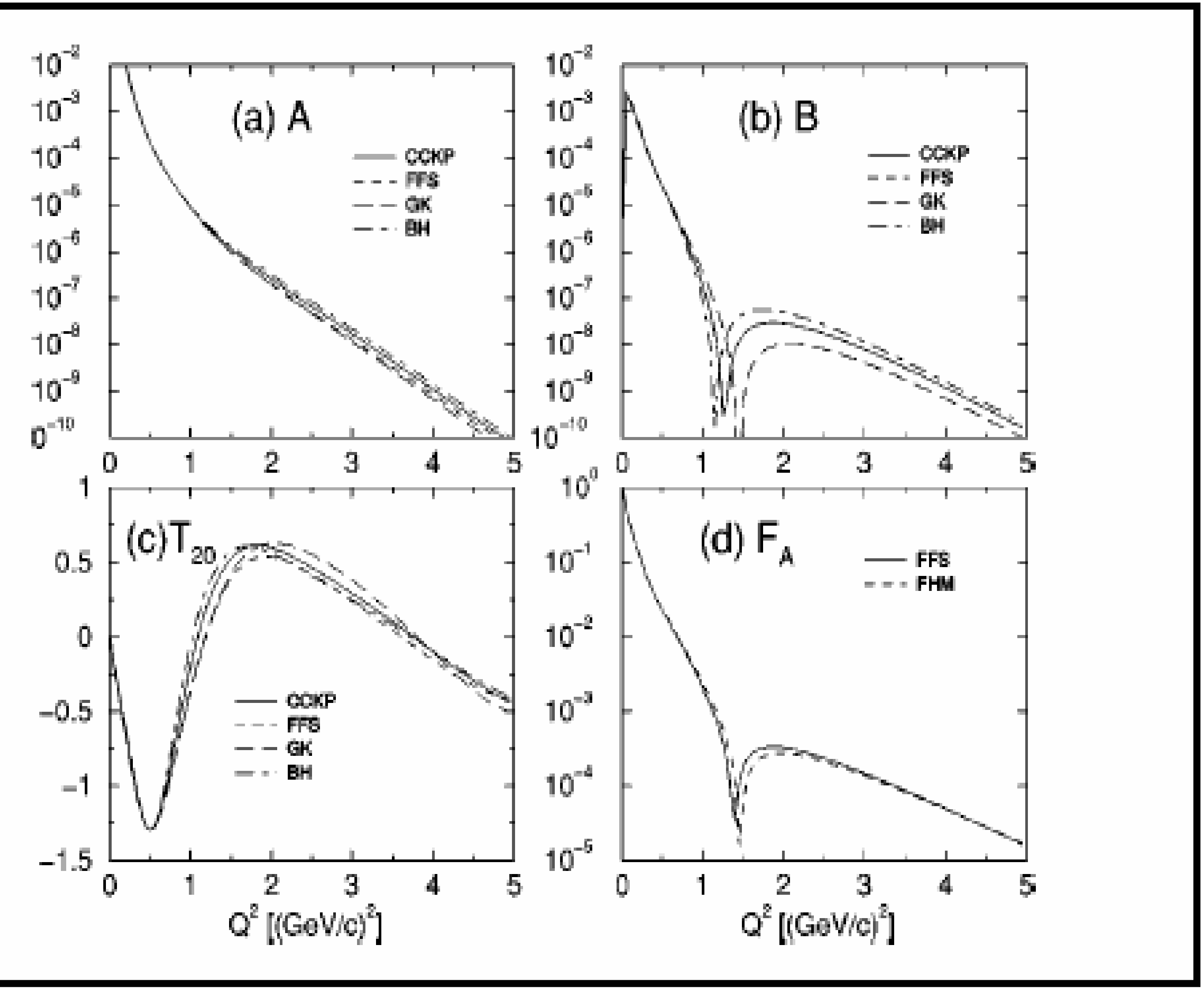
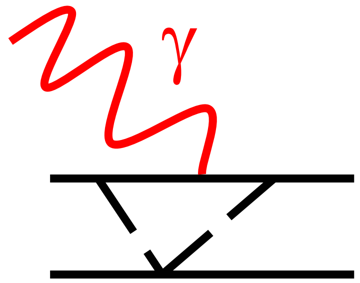
# Recovery of rotational invariance binding energy



# Rot. Inv. form factors TME

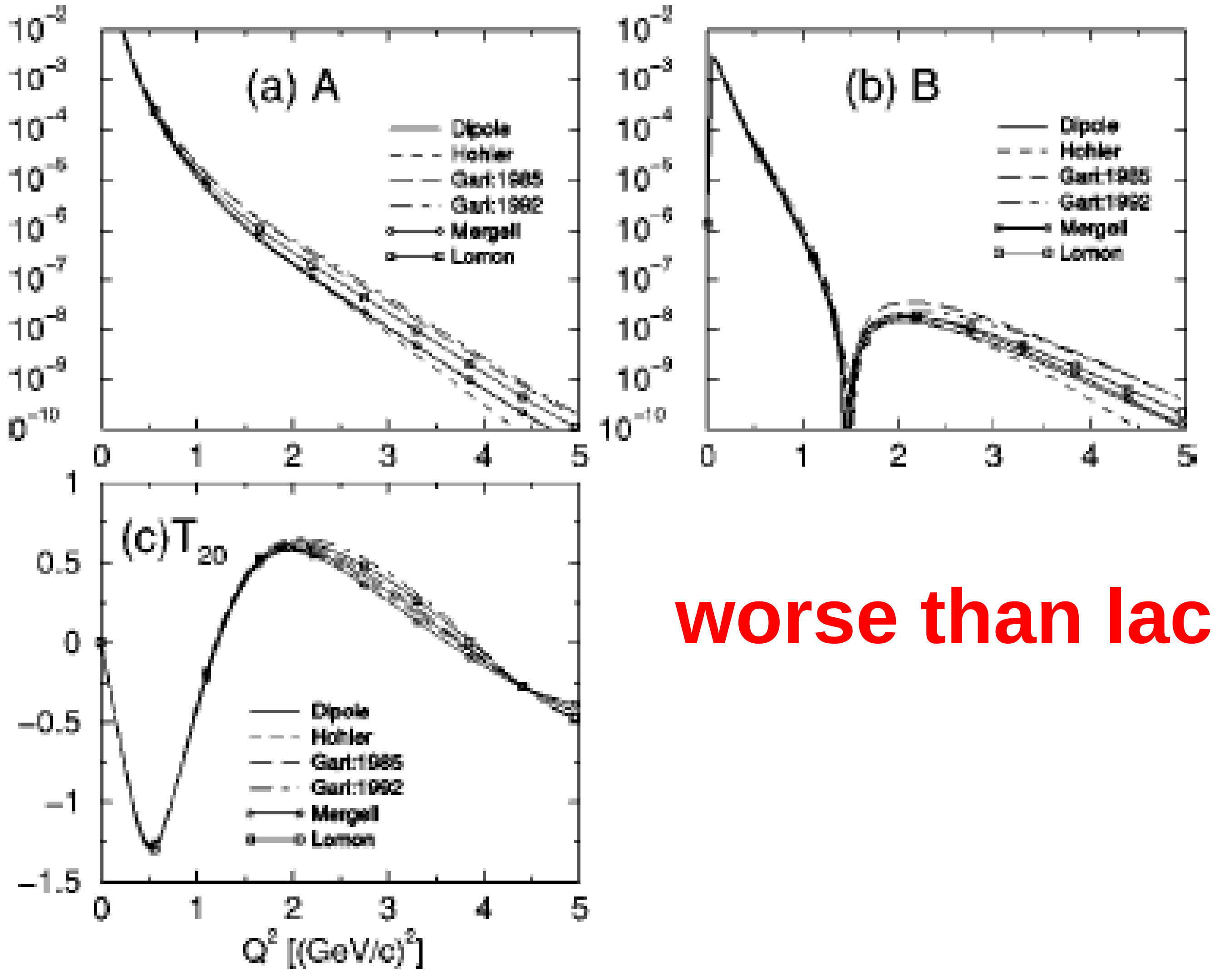



worse! no mec



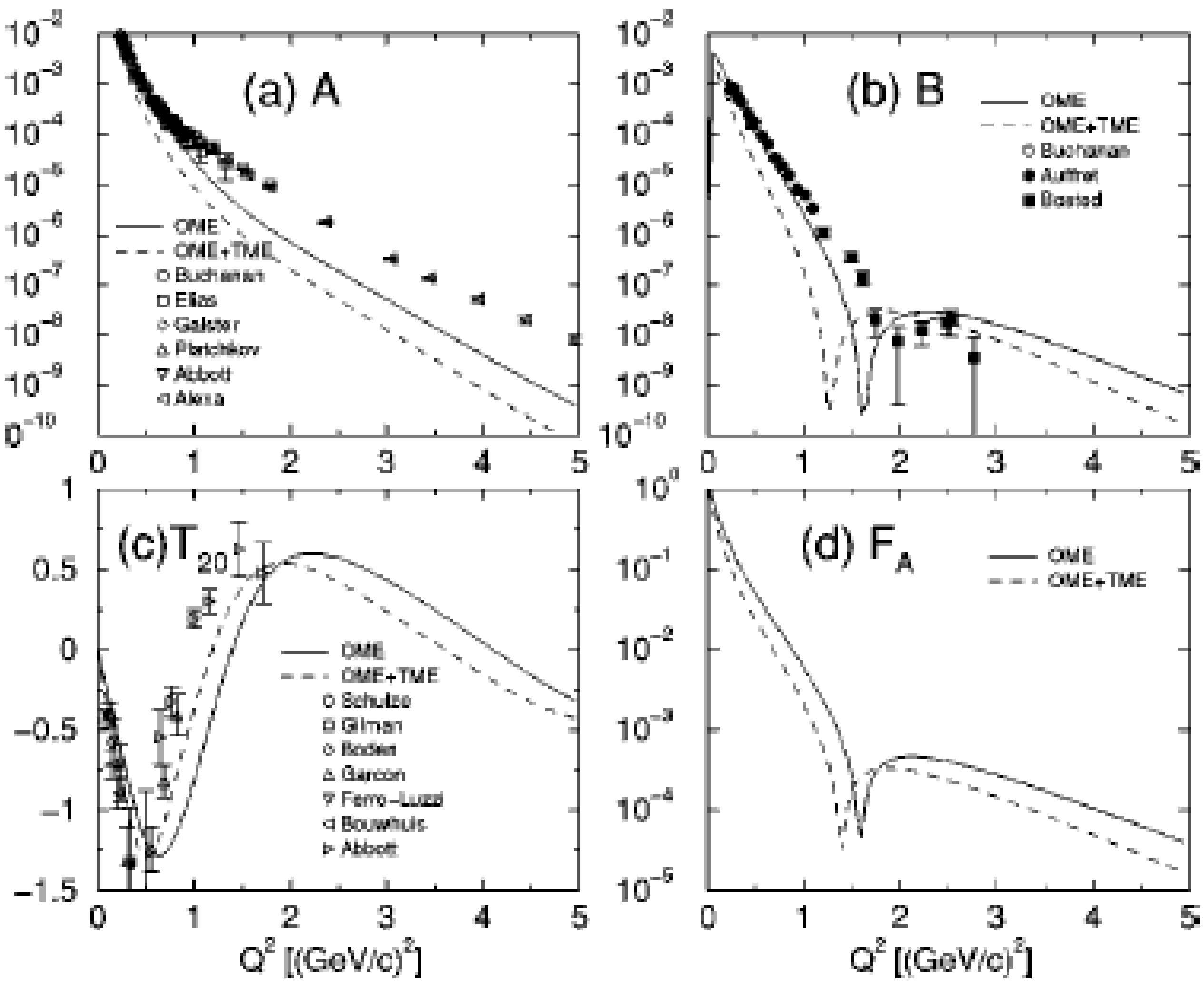

interaction
not consistent
with dynamics

# Sensitivity to isoscalar nucleon ff




**worse than lack of RI**

# Comparison with data




**Price of pure theory**

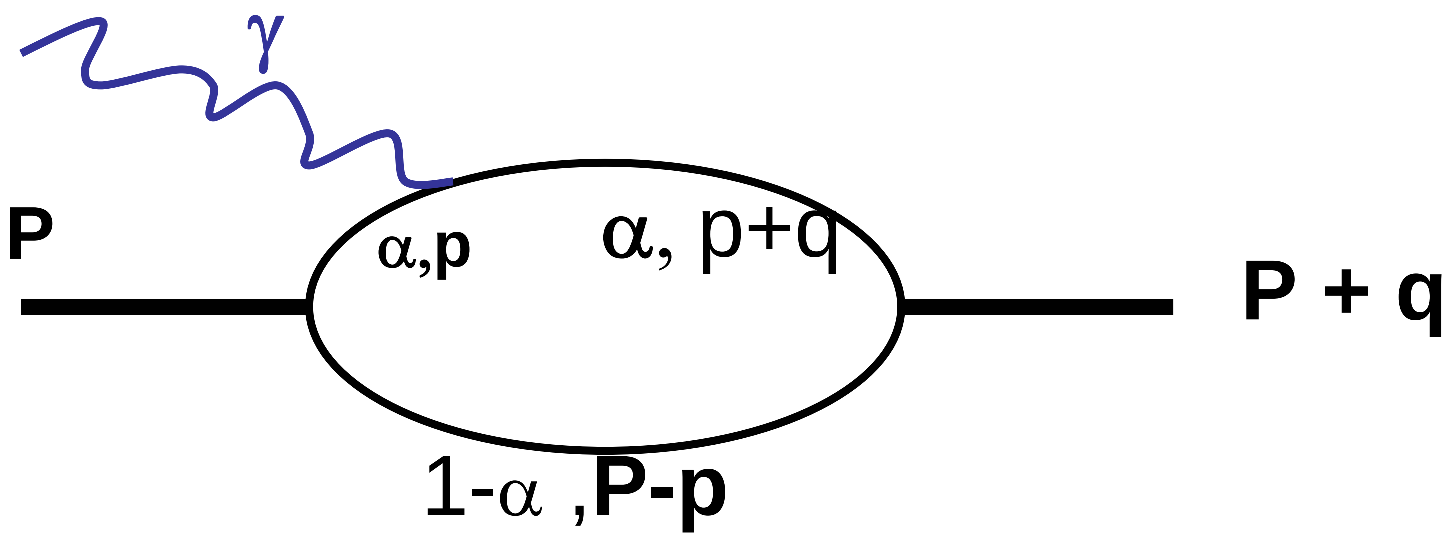
# Summary- Deuteron

- **relativity**
- **4 D to 3 D reduction defines dynamics**
- **light front - need to keep influence of ($\gamma/2k^+$) every where**
- **other theories- other requirements**
- **without consistency, comparison between theory & experiment means little**

## LF Boosts and form factor –Quantum Mechanics

$$q^+ = 0 \qquad Q^2 = -q_\perp^2$$

$$\mathbf{p}_{1\perp} \rightarrow \mathbf{p}_{1\perp} + \mathbf{q}_\perp, \qquad \mathbf{p}_{2\perp} \rightarrow \mathbf{p}_{2\perp}$$
$$\mathbf{p}_\perp \rightarrow \mathbf{p}_\perp + (1-\alpha)\mathbf{q}_\perp$$




$$F(Q^2) = \int \frac{d\alpha\, d^2 p_\perp}{\alpha(1-\alpha)} \psi^*(\alpha, p_\perp)\psi(\alpha, |\mathbf{p}_\perp + (1-\alpha)\mathbf{q}_\perp|)$$

**FORM FACTOR IS NOT A FOURIER TRANSFORM OF DENSITY**

# Light Front Boosts & Separation of Variables-Quantum Mechanics

$$P^- = \frac{{\mathbf{p_1}}_\perp^2 + m^2}{p_1^+} + \frac{{\mathbf{p_2}}_\perp^2 + m^2}{p_2^+}$$

$$P^+ = p_1^+ + p_2^+ \qquad \mathbf{P}_\perp = \mathbf{p_1}_\perp + \mathbf{p_2}_\perp$$

**Relative variables**

$$\alpha = \frac{p_1^+}{P^+} \qquad 1 - \alpha = \frac{p_2^+}{P^+} \qquad \mathbf{p}_\perp = (1-\alpha)\mathbf{p_1}_\perp - \alpha \mathbf{p_2}_\perp$$

$$\mathbf{p_1}_\perp = \mathbf{p}_\perp + \alpha \mathbf{P}_\perp \qquad \mathbf{p_2}_\perp = -\mathbf{p}_\perp + (1-\alpha)\mathbf{P}_\perp$$

$$P^+P^- - \mathbf{P}_\perp^2 = P^2 = \frac{p_\perp^2 + m^2}{\alpha(1-\alpha)}$$

$$\widehat{M}^2 \equiv P^2 + W \to \frac{p_\perp^2 + m^2}{\alpha(1-\alpha)} + W$$

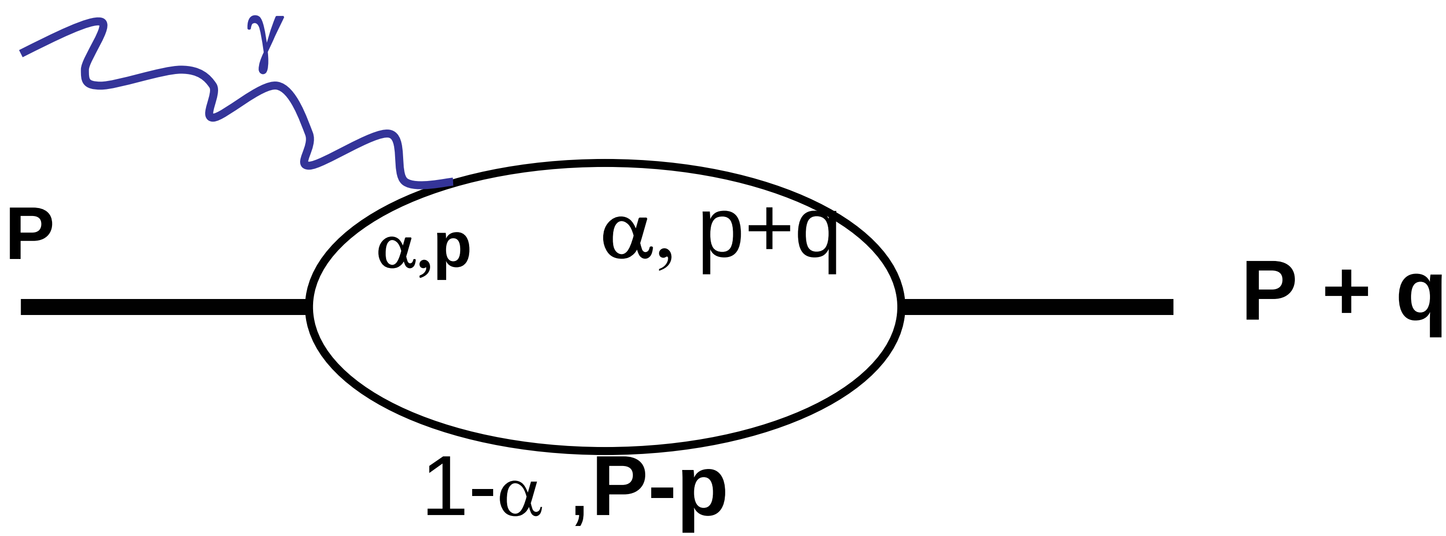
$$\widehat{M}^2\psi(\alpha, \mathbf{p}_\perp) = M^2\psi(\alpha, \mathbf{p}_\perp)$$

$$\psi = \tfrac{1}{\widehat{M}^2 - P^2} W\psi \equiv \tfrac{1}{\widehat{M}^2 - P^2}\Gamma$$

# LF Boosts and form factor –Quantum Mechanics

$$q^+ = 0 \qquad Q^2 = -q_\perp^2$$

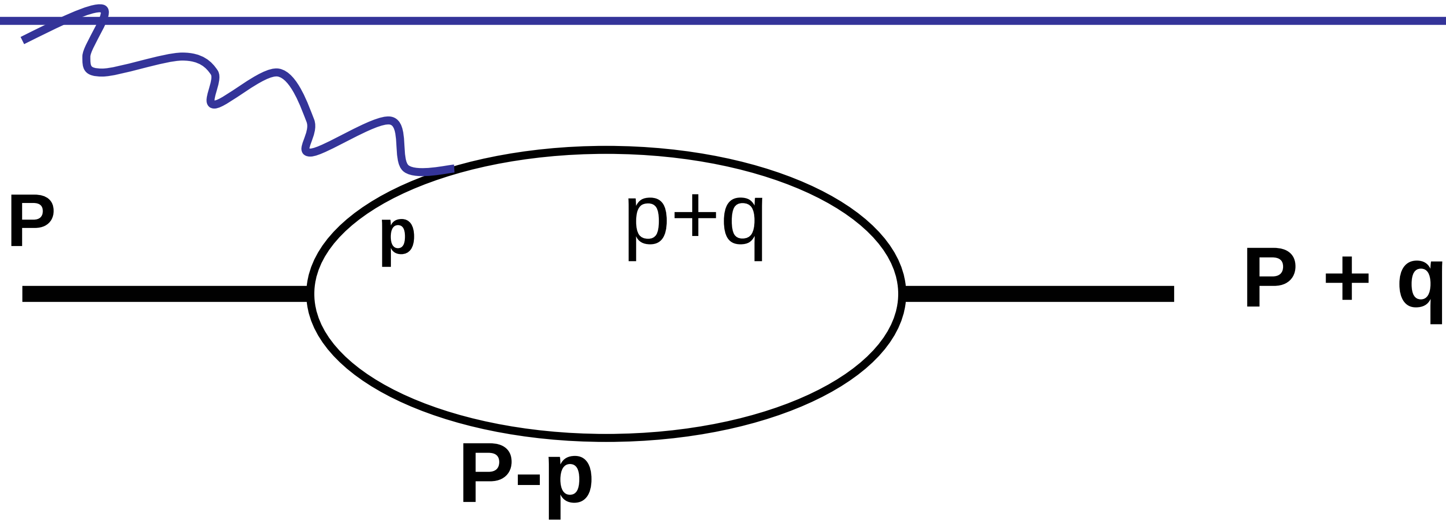
$$\mathbf{p}_{1\perp} \to \mathbf{p}_{1\perp} + \mathbf{q}_\perp, \qquad \mathbf{p}_{2\perp} \to \mathbf{p}_{2\perp}$$
$$\mathbf{p}_\perp \to \mathbf{p}_\perp + (1-\alpha)\mathbf{q}_\perp$$




$$F(Q^2) = \int \frac{d\alpha\, d^2p_\perp}{\alpha(1-\alpha)} \psi^*(\alpha, p_\perp)\psi(\alpha, |\mathbf{p}_\perp + (1-\alpha)\mathbf{q}_\perp|)$$

**FORM FACTOR IS NOT A FOURIER TRANSFORM OF DENSITY**

# Boosts, Separation of Variables –Field Theory




$$F \sim \int \frac{d^4p}{p^2 - m^2 + i\epsilon} \frac{2p^+}{(P-p)^2 - m^2 + i\epsilon} \frac{1}{(p+q)^2 - m^2 + i\epsilon} \rightarrow$$

$$\int \frac{dp^+ dp^- d^2p_\perp}{p^+p^- - p_\perp^2 - m^2 + i\epsilon} \frac{2p^+}{(P-p)^+(P-p)^- - (P-p)_\perp^2 - m^2 + i\epsilon}$$

$\times \frac{1}{p^+(p+q)^- - (p+q)_\perp^2 - m^2 + i\epsilon}$ Poles of $p^-$, U=Upper Half plane

$p^+ < 0$ $\quad UUU$ $\quad p^+ > P^+$ $\quad LLL$

$0 \leq p^+ \leq P^+$ $\quad LUL$ **Integrate UHP**