The Quest of Relativistic Effects in the Electron Scattering by Few-Nucleon Systems

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Outline

- Motivations & Perspectives
- The em responses of a polarized ^{3}He target with relativistic kinematics
- Two-nucleon and three-nucleon em form factors in the Light-Front Hamiltonian Dynamics
- The extraction of F_{2n}/F_{2p} from DIS by Few-Nucleon Systems: limits and possibilities
- Can the Z-diagram effects be measured in Few-Nucleon Systems?
- Conclusions

Some results already published + work in progress

F. M. Lev, E. Pace and G. Salmè, ”Poincaré covariant current operator and elastic electron-deuteron scattering in the front-form Hamiltonian dynamics”, Phys. Rev. **C 62** (2000) 0640004.

E. Pace, G. Salmè, S. Scopetta and A. Kievsky ”Neutron structure function $F_2^n(x)$ from deep inelastic electron scattering off few-nucleon systems ”, Phys. Rev. **C 64** (2001) 055203.

J.P.B.C. de Melo, T. Frederico, E. Pace and G. Salmè, ”Pair term in the electromagnetic current within the Front-Form dynamics: spin-0 case”, Nucl.Phys. **A 707** (2002) 399.

A. Kievsky, E. Pace and Salmè, ”Transverse asymmetry of $^3\vec{\rm He}$ and the magnetic form factor of the neutron”, Eur. Phys. J. **A 19** (2004) 87.

Motivations & Perspectives

- The *standard model* of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account, is already at a very sophisticated stage, and many efforts are presently carried on in order to retain all the general principles compatible with a theory where a fixed number of constituents is acting.
- On top of this, to include relativity (as much as possible) represents an important goal, in view of the fact that i) the underlying theory is a local relativistic field theory, after all, and ii) the extraction of unambiguous signatures beyond the *standard model* of Few-Nucleon Systems could be affected by relativistic effects.
- The field theoretical approaches based on the Bethe-Salpeter equations have been highly developed for two-nucleon system (Tjon, Gross,...), in particular within the so-called quasi-potential approximation, and for the trinucleon system many efforts are in progress.

★ Aim : to construct a relativistic approach for Few-Nucleon System that i) retains all the *successful phenomenology* already developed and ii) includes, *in a non perturbative way*, relativistic features, requested by Poincaré covariance, and ...iii) allows one affordable calculations.

★★ A role for antinucleons ?

★ ★★ Tool: Electron scattering by Few-nucleon Systems

Caveats: isobar configurations, MEC ....

Em Responses of $^3\vec{\mathrm{He}}$ within Plane Wave Impulse Approximation

Aim: to extract the neutron magnetic form factor, G_M^n

★ A first step: relativistic kinematics and a relativistic one-body current

- Relativistic kinematics, but no Wigner Functions for boosting initial and final wave functions of the three-nucleon system

$$\langle j', j'_z; \epsilon'_{int}, \alpha'_i, \mathbf{q}| J^\mu_{IA}(0) | \frac{1}{2}, j_z; \epsilon_{int}, \alpha_i, \mathbf{0}\rangle =$$

$$= 3 \int d\mathbf{k}_1 d\mathbf{k}_2 \langle j', j'_z; \epsilon'_{int}, \alpha'_i | \mathbf{k}'_1, \mathbf{k}_2\rangle \quad \leftarrow (excited\ state)$$

$$\times \langle \mathbf{q} + \mathbf{k}_1 | J^\mu_{1,free}(0) | \mathbf{k}_1\rangle \quad \leftarrow (1b - current)$$

$$\times \langle \mathbf{k}_1, \mathbf{k}_2 | \frac{1}{2}, j_z; \epsilon_{int}, \alpha\rangle \quad \leftarrow (bound\ state)$$

- Relativistic electron-nucleon cross section $\rightarrow$ cc1 by T. De Forest, NPA **392**, 232 (1983)
- Bound state of ^{3}He $\rightarrow$ variational solution of the Schrödinger Equation by Kievsky, Rosati, Viviani, Nucl. Phys. A 577 (1994) 511.

- Excited state of the three-nucleon system is approximated by (*this is the core of the Plane Wave Impulse Approximation !*)

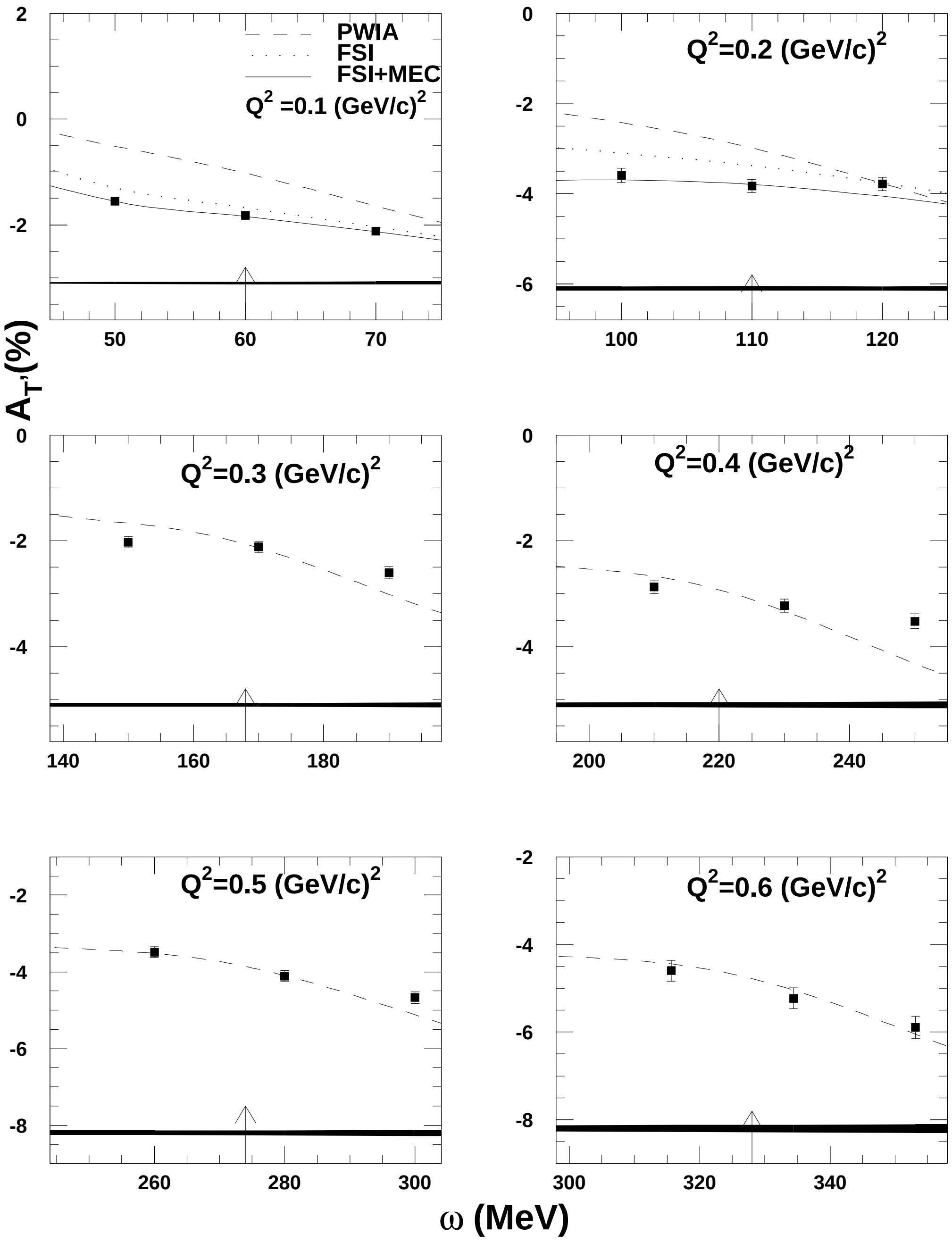
$$|j', j'_z; \epsilon'_{int}, \alpha'; \mathbf{q}\rangle \to \frac{1}{\sqrt{3}}|\mathbf{p}_f, \sigma_f \tau_f\rangle |j_{23}\mu_{23}, \pi_{23}, T_{23}\tau_{23}, \epsilon_{23}; \mathbf{P}_{23}\rangle$$

where

  - $\mathbf{p}_f + \mathbf{P}_{23} = \mathbf{q}$
  - $|\mathbf{p}_f, \sigma_f \tau_f\rangle \equiv$ Plane wave describing the struck nucleon
  - $|j_{23}\mu_{23}, \pi_{23}, T_{23}\tau_{23}, \epsilon_{23}; \mathbf{P}_{23}\rangle \equiv$ *fully-interacting* two-body wave function (describing the spectator pair)
  - terms that properly antisymmetrize the three-nucleon wave function are dropped out, and only the direct interaction between the virtual photon and the struck nucleon is taken into account

$$\vec{e} + {}^3\vec{He} \to e' + X$$

$$A = \frac{\sigma(\uparrow\rightarrow) - \sigma(\uparrow\leftarrow)}{\sigma(\uparrow\rightarrow) + \sigma(\uparrow\leftarrow)} \qquad \theta^* = 0^0 \;\to\; A_{T'}$$



■ ≡ TJLAB data (After Xu et al , PRC **67** (2003) 012201)

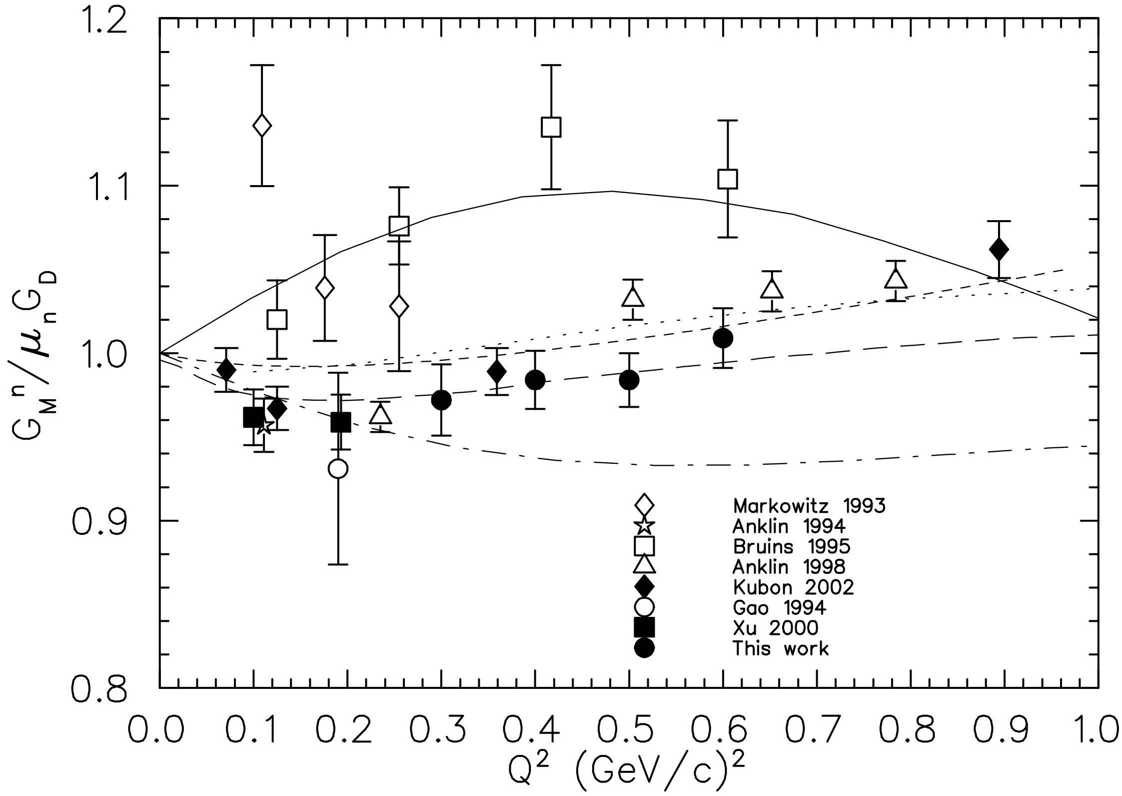
Solid lines: Bochüm calculations with fully-interacting three-nucleon w.f. + two-body currents, *non-relativistic kinematics and not relat.* σ_{eN}

Dashed lines: Rome-Pisa PWIA + *relativistic kinematics and relat.* σ_{eN}

(NN int.: AV18, Wiringa, Smith and Ainsworth, PRC **29**, 1207 (1984))

Neutron Magnetic Form Factor from the exp. transverse asymmetry of $^3\vec{\mathrm{He}}$

The extraction of $G_M^n(Q^2)$ from the exp. transverse asymmetry of $^3\vec{\mathrm{He}}$ needs many efforts for achieving a good description of the nuclear structure, (e.g. FSI, relativity, etc), but also dynamical two-body currents...

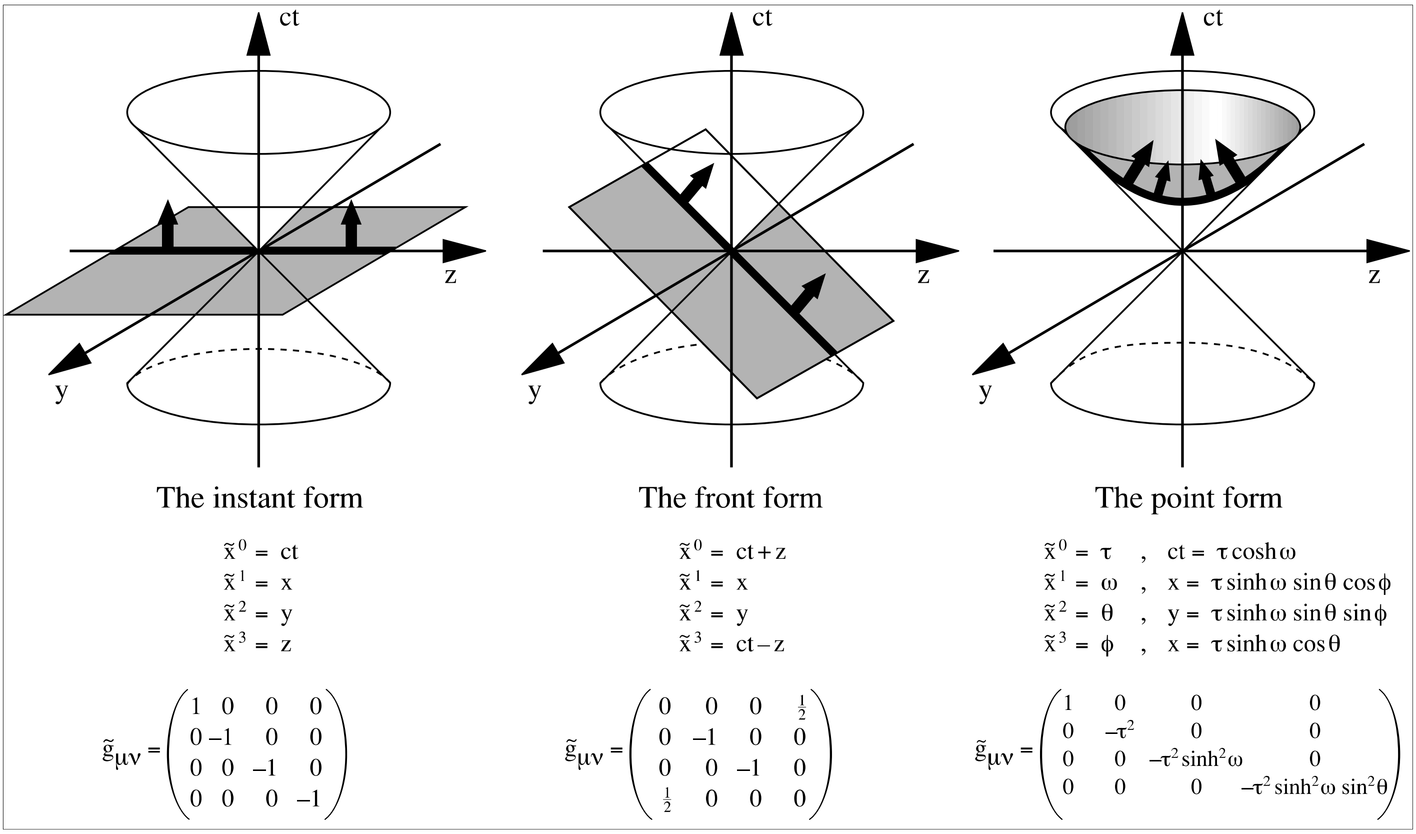


(After Xu et al , PRC **67** (2003) 012201)

The model dependence could be investigated in more detail, once other calculations will be available.

Relativistic Hamiltonian Dynamics: a flash

- A reasonable compromise: i) fulfilling Poincaré covariance in a non perturbative way; ii) embedding the whole successful nonrelativistic phenomenology; iii) affordable numerical calculations; iv) fixed number of constituents; v) large class of permissible interactions.



After S.J. Brodsky, H.C. Pauli and S.S Pinsky, Phys. Rep. **301**, 299 (1998).

- Dirac proposed three forms for the the so-called Relativistic Hamiltonian Dynamics: Instant Form, Front Form or Light-Front Form (most widely adopted, since $\longrightarrow$ light-cone DIS), Point Form (Dirac, Rev. Mod. Phys. 21 (1949) 392)

- Symmetries of the ”initial” hypersurface: the properties of invariance of the hypersurface, where the interacting system is ”sitting” at $\tau = 0$ (τ is the variable that labels the evolution of the system under the action of a Hamiltonian operator, containing the interaction) allows one to separate the 10 generators of the Poincaré group in two sets: the kinematical generators (that leave the initial hypersurface unchanged) and the dynamical ones.
- Explicit construction of 10 generators, given the mass of the interacting system, (see the Bakamjian-Thomas approach PR 92 (1953) 1300)
- Cluster separability can be implemented $\longrightarrow$ macroscopic locality, instead of the microscopic one (e.g. N.N. Sokolov Dokl. Akad. Nauk. 233 (1977) 575)
- All the constituents are on their own mass shell (sharp difference from the explicitly covariant theory, more familiar...) This on-mass-shell constraint allows one to define intrinsic variables as in the non relativistic case.

Summarizing : RHD rigorously fulfills the Poincaré covariance and, in some sense, falls between non-relativistic quantum mechanics and local relativistic field theory

Deuteron em observables

in the Light-Front Hamiltonian Dynamics

In a Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

the matrix elements of the em current operator for the Deuteron, only $|NN\rangle$ state, can be defined in terms of the one-body, free current

$$j^\mu(K\vec{e}_z) = \frac{\mathcal{J}^\mu(K\vec{e}_z)}{2} + L^\mu_\nu[r_x(-\pi)]\, e^{\imath\pi S_x} \frac{\mathcal{J}^\nu(K\vec{e}_z)^*}{2} e^{-\imath\pi S_x}$$

with

$$\mathcal{J}^+(K\vec{e}_z) = \mathcal{J}^-(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P^{'+}|\Pi J^+_{free}(0)\Pi|\vec{P}_\perp = 0, P^+\rangle$$

$$\vec{\mathcal{J}}_\perp(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P^{'+}|\Pi \vec{J}_{free\perp}(0)\Pi|\vec{P}_\perp = 0, P^+\rangle$$

$\Pi \equiv$ projector onto the subspace of the deuteron bound state $|\chi_1\rangle$ of mass M_D and spin 1,
$J^\mu_{free}(0) = \sum_i J^\mu_{pi}(0)(1+\tau_3)/2 + J^\mu_{ni}(0)(1+\tau_3)/2$ with
$J^\mu_N = -F_{2N}(p^\mu + p'^\mu)/2M + \gamma^\mu(F_{1N} + F_{2N})$

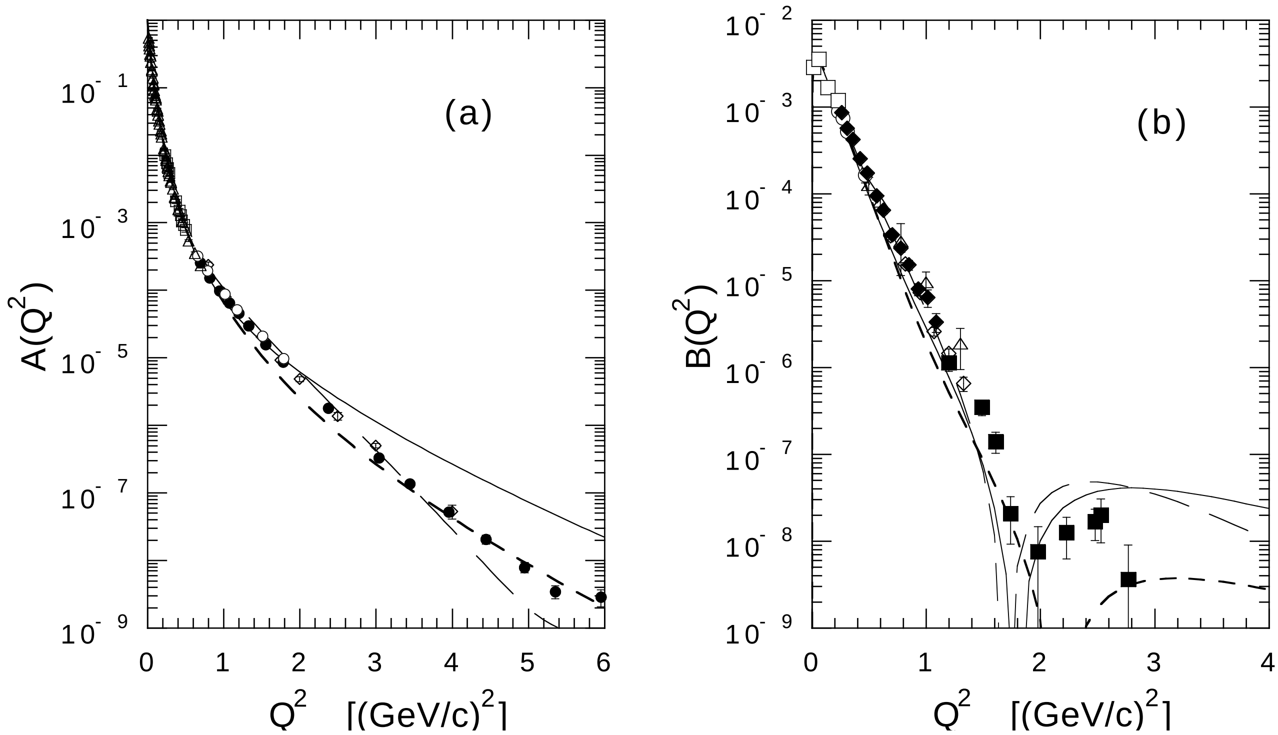
Such a definition allows one to construct a current operator J^μ for the Deuteron that fulfills Poincaré, parity and time reversal covariance, together with hermiticity and the continuity equation.(F. Lev, E. Pace, G.S. Nucl. Phys. **A 641**, 229 (1998)).

The core of the problem: Implementing the Poincaré covariance for J^μ amounts to implementing a rotational covariance for j^μ.

Magnetic moment (in nuclear magnetons) and quadrupole moment (in fm^2) for the Deuteron; P_D is the D-state percentage. LPS PRL **83** (1999) 5250

Interaction	P_D	μ_D^{NR}	μ_D^{LFD}	Q_D^{NR}	Q_D^{LFD}
CD-Bonn	4.83	0.8523	0.8670	0.2696	0.2729
Nijm1	5.66	0.8475	0.8622	0.2719	0.2758
RSC93	5.70	0.8473	0.8637	0.2703	0.2750
Av18	5.76	0.8470	0.8635	0.2696	0.2744

Exp.	0.857406(1)	0.2859(3)

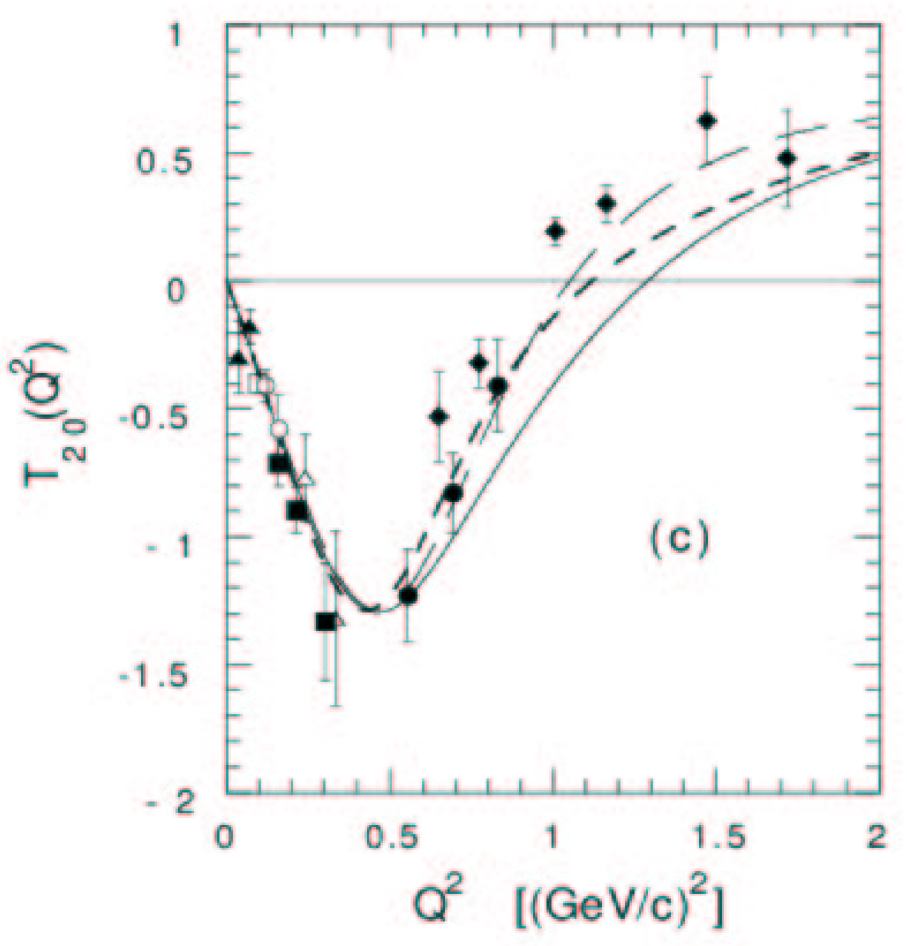


$RSC93$ NN interaction + Gari-Krümpelmann nucleon f. f.

Solid line: full result with the Poincaré covariant current operator, in the Breit frame where $\boxed{\mathbf{q}_\perp = 0}$

Long-dashed line: nonrelativistic result in the same Breit frame.

Dashed line: the argument of the Nucleon ff's, $(p_1' - p_1)^2 \to -Q^2$.



RSC $N - N$ interaction + Gari-Krümpelmann nucleon f. f.

Solid line: full result with the Poincaré covariant current operator in the Breit frame where $\boxed{\mathbf{q}_\perp = 0}$.

Long-dashed line: nonrelativistic result in the same Breit frame.

Dashed line: the argument of the Nucleon ff's, $(p'_1 - p_1)^2 \to -Q^2$.

(After LPS PRC **62** (2000) 064004.)

$F_2^n(x)/F_2^p(x)$

in the Light-Front Hamiltonian Dynamics

In the DIS region, the reaction

$$e +^2 \mathrm{H} \rightarrow e' + X$$

could represent a suitable tool for extracting the neutron structure function, $F_2^n(x)$, from the Deuteron structure function, $F_2^D(x)$.

To take under control the model dependence of the extraction method, one should study the relativistic effects, besides the presence of isobar configuration and non standard effects, like, e.g., a 6-quark bag.

In the Bjorken limit, the Deuteron structure function can be approximated in a convolution model as follows

$$F_2^D(x) = \int_x^{M_D/M} [F_2^p(x/z) + F_2^n(x/z)] f^D(z) dz$$

where $x = Q^2/(2M\nu)$, M =nucleon mass and M_D =deuteron mass.

The distribution $f^D(z)$ describes the distribution probability to find a nucleon with light-front momentum z

★ In the usual approach, based on the impulse approximation and an instant-form framework, the distribution $f^D(z)$ has the following expression

$$f^D_{IF}(z) = \int d\vec{p}\, n^D(|\vec{p}|)\, \delta\left(z - \frac{p\cdot q}{M\nu}\right)\, z\, C$$

$n^D(|\vec{p}|) \equiv$ the nucleon momentum distribution inside the Deuteron, C a normalization factor, $q \equiv (\nu, \vec{q})$, and p the four-momentum of an off-mass shell nucleon, i.e., $p \equiv (p^0, \vec{p})$ with $p^0 = M_D - \sqrt{M^2 + |\vec{p}|^2}$. (see, e.g., Frankfurt and Strikman PLB **183B** (1987) 254, Ciofi and Liuti PRC **41** (1990) 110)

★★ Within the Light-Front Hamiltonian Dynamics, the distribution $f^D(z)$ reads as follows

$$f^D_{FF}(z) = \int d\vec{k}\, n^D(|\vec{k}|)\, \delta\left(z - \xi\frac{M_D}{M}\right)$$

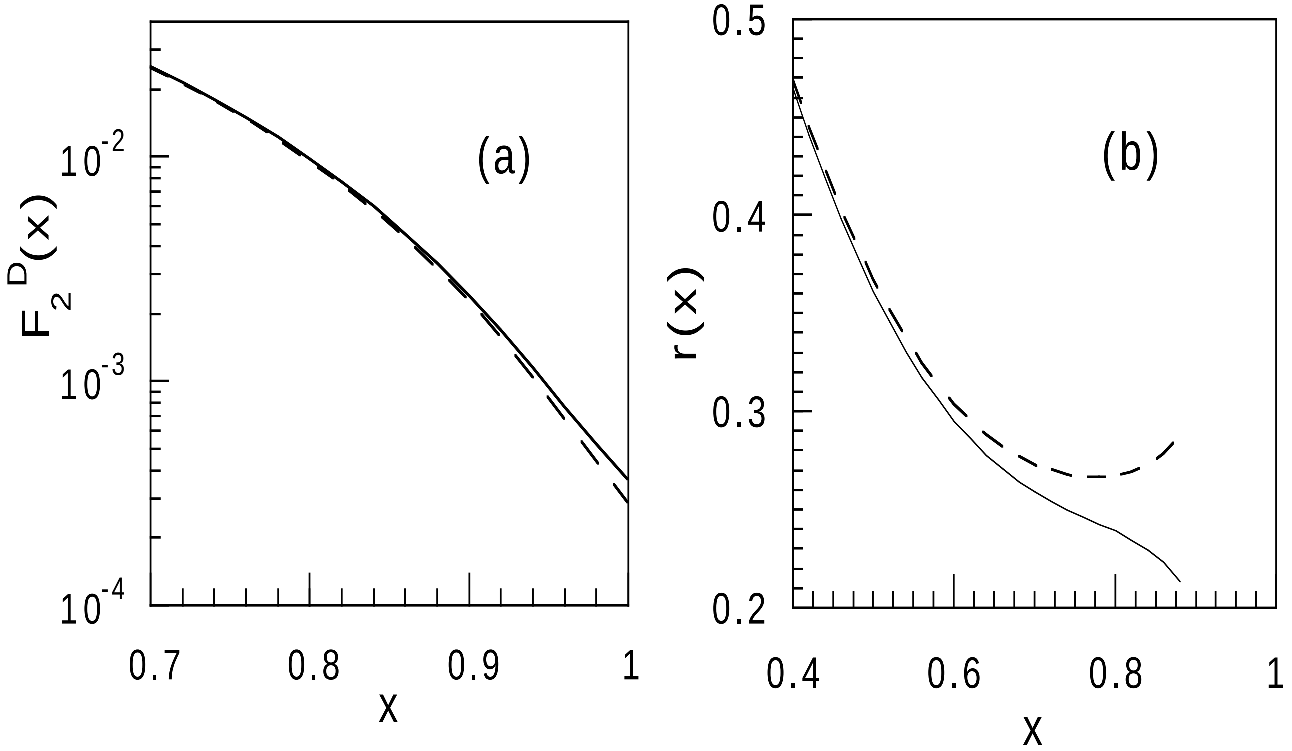
where $\xi = p^+/P^+ = (\sqrt{M^2 + |\vec{k}|^2} + k_z)/2\sqrt{M^2 + |\vec{k}|^2}$.
(see also Oefelke, Sauer and Coester NPA **518** (1990) 593)

★★★ It has been proposed to extract the ratio $r(x) = F_2^n(x)/F_2^p(x)$ from the experimental data for the Deuteron structure function, $F_2^{Dexp}(x)$, by the following recurrence relation

$$r^{(n+1)}(x) = \frac{F_2^{Dexp}(x)[1 + r^{(n)}(x)]}{\int_x^{M_D/M} [1 + r^{(n)}(x/z)] F_2^p(x/z) f^D(z) dz} - 1$$

Let see the effects $\Rightarrow$ ....

$AV18$ NN interaction and the model of Aubert et al for the nucleon structure functions



Dashed lines - $F_2^D(x)$ from the standard approach:

Solid lines - $F_2^D(x)$ from Light-Front calculations:

$$f_{FF}^D(z) = \int d\vec{k}\, n^D(|\vec{k}|)\, \delta\left(z - \xi \frac{M_D}{M}\right)$$

$$f_{IF}^D(z) = \int d\vec{p}\, n^D(|\vec{p}|)\, \delta\left(z - \frac{p \cdot q}{M\nu}\right)\ z\, C$$

$$r(x) = F_2^n(x)/F_2^p(x)$$

(After E. Pace, G.S. nucl-th/0106004 and Pace, Kievsky, Scopetta,S. PRC C64 (2001) 055203)

Trinucleon , $|NNN\rangle$, em form factors in the Light-Front Hamiltonian Dynamics

Ingredients

★ A Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

★★ A Poincaré covariant current

$$j^\mu(K\vec{e}_z) = \frac{\mathcal{J}^\mu(K\vec{e}_z)}{2} + L^\mu_\nu[r_x(-\pi)]\; e^{\imath\pi S_x}\, \frac{\mathcal{J}^\nu(K\vec{e}_z)^*}{2} e^{-\imath\pi S_x}$$

with

$$\mathcal{J}^+(K\vec{e}_z) = \mathcal{J}^-(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P'^+|\Pi J^+_{free}(0)\Pi|\vec{P}_\perp = 0, P^+\rangle$$

$$\vec{\mathcal{J}}_\perp(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P'^+|\Pi \vec{J}_{free\perp}(0)\Pi|\vec{P}_\perp = 0, P^+\rangle$$

$\Pi \equiv$ projector onto the subspace of a trinucleon bound state $|\chi_{\frac{1}{2}}\rangle$ of mass M_T and spin $1/2$,
$J^\mu_{free}(0) = \sum_i J^\mu_{pi}(0)(1+\tau_3)/2 + J^\mu_{ni}(0)(1+\tau_3)/2$ with
$J^\mu_N = -F_{2N}(p^\mu + p'^\mu)/2M + \gamma^\mu(F_{1N} + F_{2N})$.

★★★ A trinucleon bound state, obtained through a variational technique by Kievsky, Rosati, Viviani (NPA 577 (1994) 511) with AV18.

Preliminary calculation with only S+S' waves:

$\mathcal{P}_S(Av18) = \ 90.1\%\ \mathcal{P}_{S'}(Av18) = \ 1.29\%$

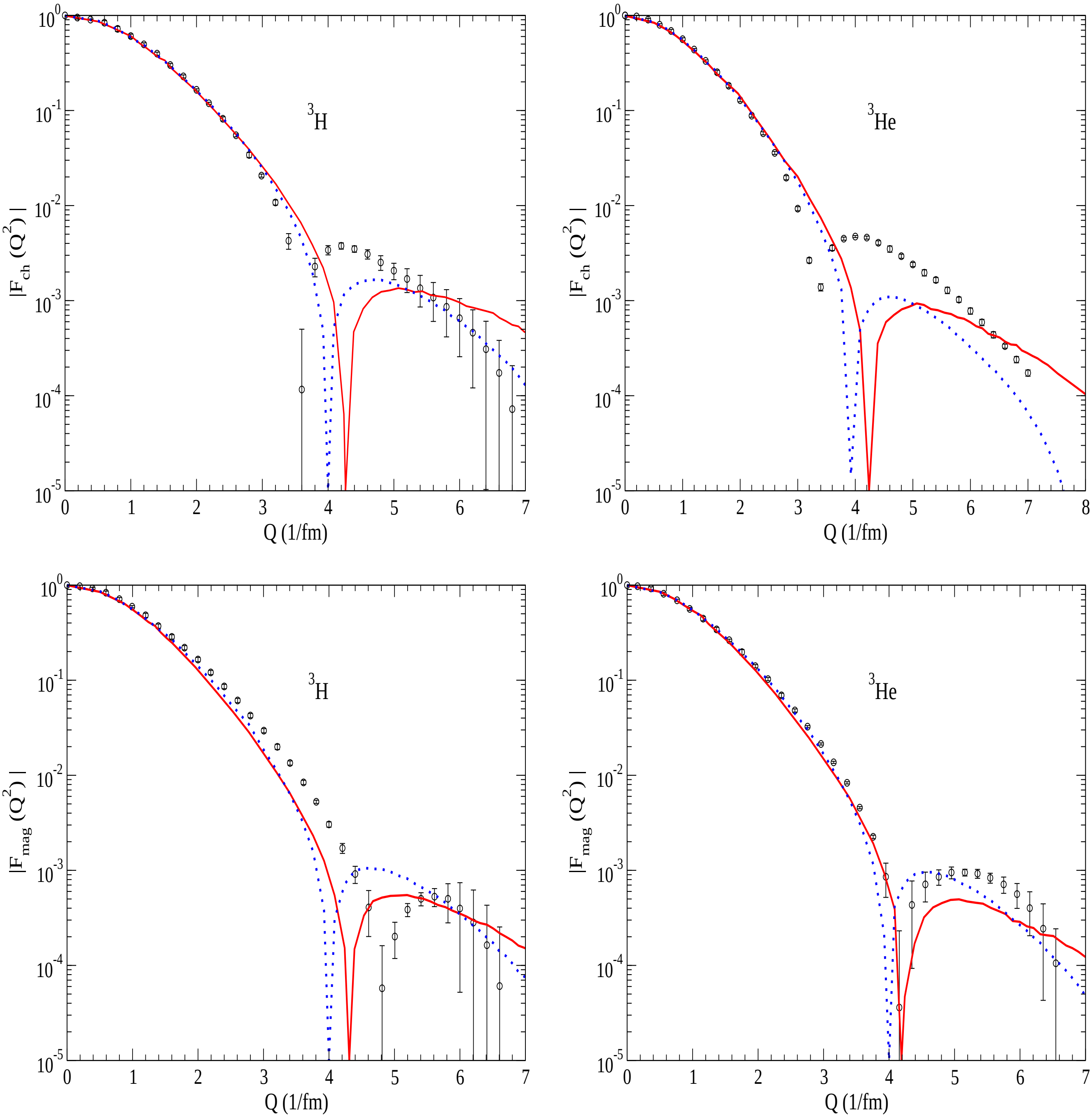
Trinucleon magnetic moments

Theory	^{3}He	^{3}H
NR(S)	-1.723	2.515
LFD(S)	-1.778	2.597
NR(S+S')	-1.707	2.518
LFD(S+S')	-1.759	2.652
Exp.	-2.1276	2.9789

Charge and Magnetic form factors of ^{3}H and ^{3}He

For the first time in LFD !

Only S+S' waves + AV18 + Gari-Krümpelmann nucleon form factors



Solid line: LFD calculation in a Breit frame where $\mathbf{q}_{\perp} = 0$

Dotted line: non relativistic calculation.In cartesian coordinates, the chosen frame corresponds to a Breit frame where $\hat{q}_z = \hat{e}_z$.

Z-diagram for a mock Deuteron

Besides the important issues represented by MEC & Isobar contributions another relevant topic to be explored is the so-called Z-diagram or pair term contribution ($N\bar{N}$).

This amounts to explore the em observables in a framework beyond the Poincaré covariance, focusing on the contribution from antinucleons. E.g.

$$|D\rangle = |NN\rangle \; + \; |NN \; N\bar{N}\rangle \; + \; \text{other Fock states}$$

A simple framework to start such an investigation is represented by the Mandelstam formalism applied to the Deuteron form factors.

The Mandelstam formalism is a fully covariant formalism. Therefore we can calculate the observables in any reference frame we like, but we should be aware that the probability of the Fock states can change from a reference to another.

A remark: the study of the Z-diagram is fundamental for describing the em properties of Hadrons in the timelike region. Could this produce synergies between the spacelike and timelike communities in the next future?

The field theoretical approach proposed by Mandelstam for the matrix elements of the em current, when the target is a bound state, reads

$$\langle P'|j^\mu|P\rangle = -\imath \int \frac{d^4p}{(2\pi)^4} \Lambda^*(p,P')\,\Lambda(p,P) \times$$
$$Tr[S(p)\mathcal{V}^\dagger(p,P')S(p-P')\Gamma^\mu S(p-P)\mathcal{V}(p,P)]$$

$\Gamma^\mu \equiv$ the photon vertex. Taken bare in this preliminary calculation

$S(p) \equiv$ the constituent (fermion) propagator

$$S(p) = \frac{\not p + m}{p^2 - m^2 + \imath\epsilon}.$$

For a $^3S_1$-Deuteron with mass M_D, $\mathcal{V} \equiv$ constituent-system vertex,

$$\mathcal{V}^\mu = \gamma^\mu - \frac{M_D}{2}\frac{p^\mu + p'^\mu}{P_D \cdot p + M_D m - \imath\epsilon}$$

Only γ^μ in this preliminary calculation. $\Lambda(p,P) \equiv$ the momentum dependence of the constituent-system vertex (in a dynamical model it should be obtained through a Bethe-Salpeter equation). (Note that $\Lambda(p,P)$ is expected to regularize the integral).

A more general expression for the Bethe-Salpeter vertex could be used, viz

$$\mathcal{V}^\mu\,\Lambda(p,P) \to \sum_i \mathcal{V}^\mu_i\,\Lambda_i(p,P)$$

The Mandelstam formula leads to the following picture for the elastic scattering in a reference frame where

$$\boxed{q^+ \neq 0}$$

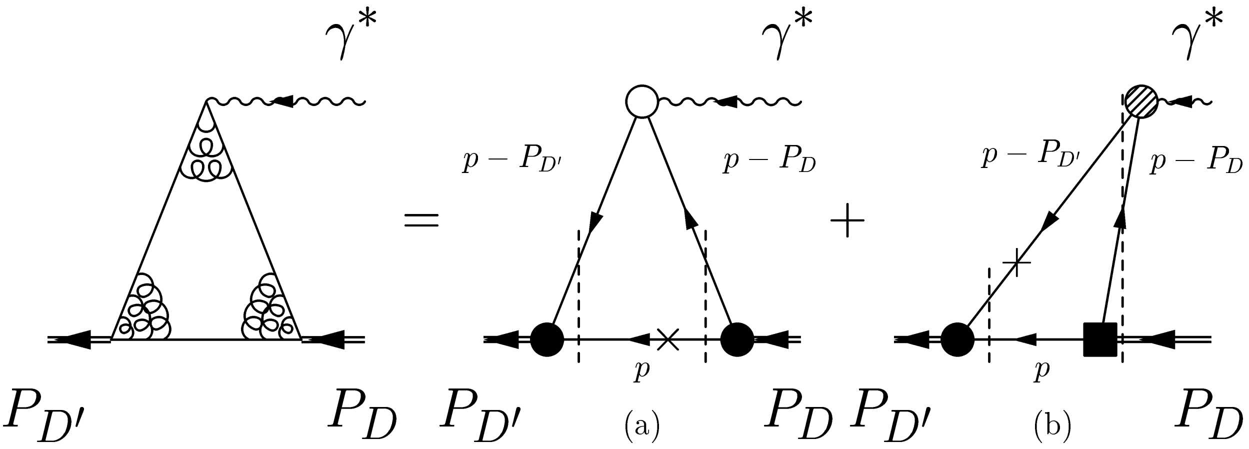


Diagram (a)⇒ Triangle contribution or spectator pole contribution. Within the Light-Front Hamiltonian Dynamics, this diagram is approximated by using a Deuteron wave function ($|NN\rangle$ only) instead of adopting a Bethe-Salpeter vertex function.

Diagram (b)⇒ Z-diagram or pair contribution

The Z-diagram effects could be theoretically investigated by analyzing the experimental elastic form factors of the Deuteron, at high momentum transfer ($Q^2 > 1\ (GeV/c)^2$) in a Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

(de Melo, Frederico, Pace, G.S, work in progress)

The macroscopic form factors, $G_0(Q^2)$, $G_1(Q^2)$, $G_2(Q^2)$ are defined through the macroscopic current

$$\langle S'_z 1, P_f | J^\mu | P_i, 1 S_z \rangle = \epsilon'^*_\alpha \epsilon_\beta \ \times$$
$$\left\{ (P_f^\mu + P_i^\mu) \left[F_0(Q^2) g^{\alpha\beta} - \frac{q^\alpha q^\beta}{2M_D^2} F_2(Q^2) \right] \right.$$
$$\left. - F_1(Q^2) \, (q^\alpha \, g^{\beta\mu} - q^\beta \, g^{\alpha\mu}) \right\}$$

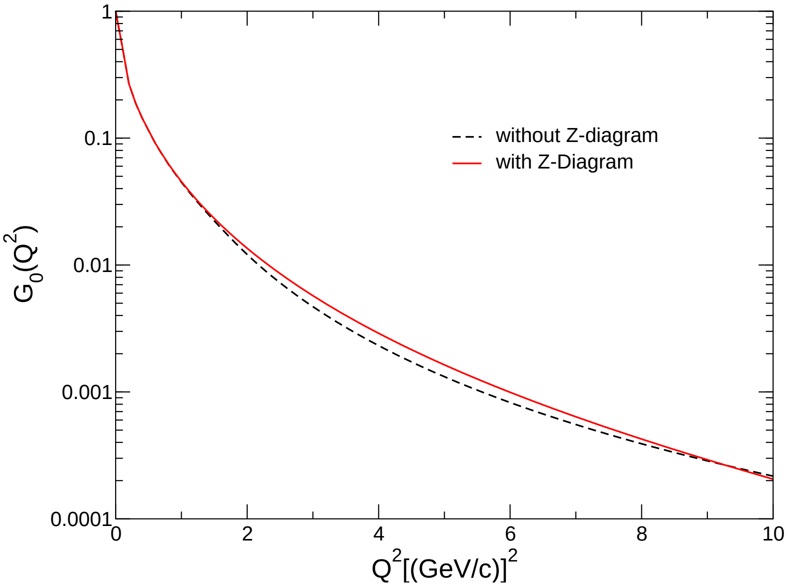
where $Q^2 = -q \cdot q$. Then

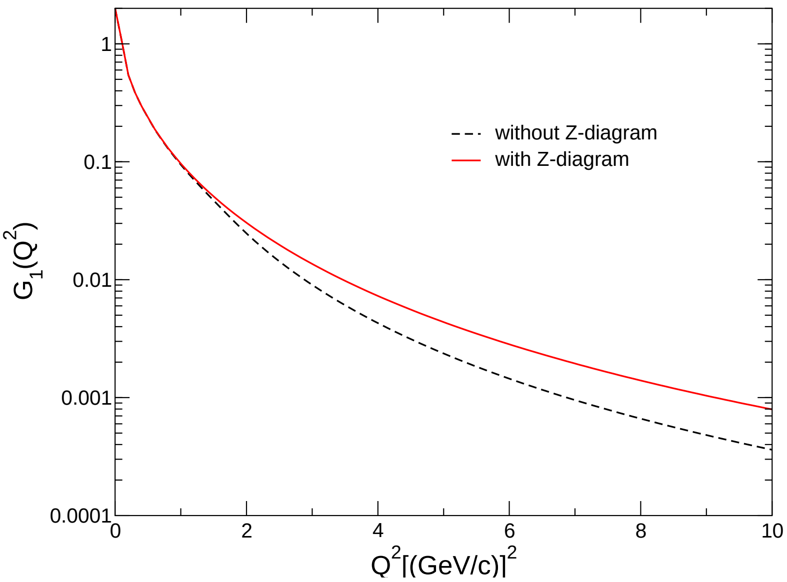
$$\frac{G_0(Q^2)}{\sqrt{1+\eta}} = G_C(Q^2) =$$
$$= -F_0(Q^2) - \frac{2\eta}{3} \left[F_0(Q^2) + (1+\eta) F_2(Q^2) + F_1(Q^2) \right]$$
$$\frac{G_1(Q^2)}{\sqrt{1+\eta}} = G_M(Q^2) = F_1(Q^2)$$
$$\frac{G_2(Q^2)}{\sqrt{1+\eta}} = \frac{2}{3} \sqrt{2} \eta \, G_Q(Q^2) =$$
$$= -\frac{2}{3} \sqrt{2} \eta \, \left[F_0(Q^2) + (1+\eta) F_2(Q^2) + F_1(Q^2) \right]$$

with $\eta = Q^2 / 4M_D^2$.

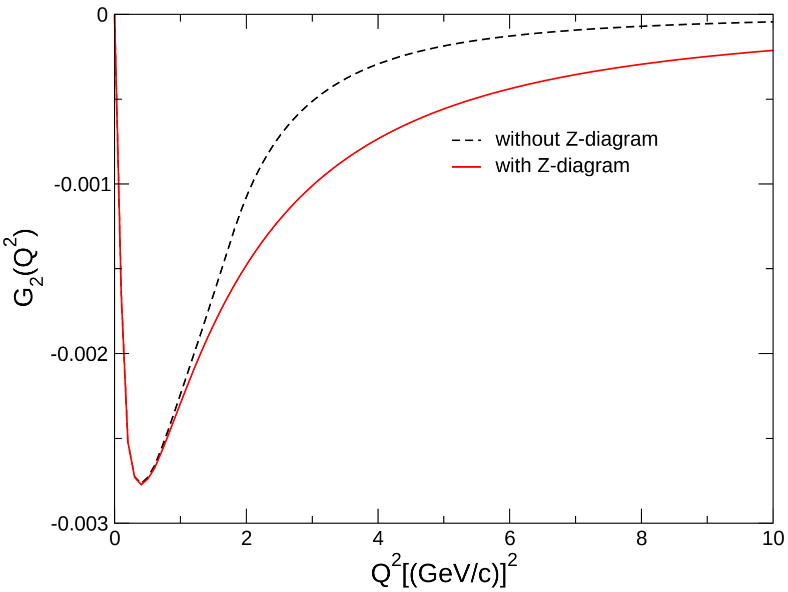
The standard definition for the magnetic moment and the quadrupole one

$$\mu_D = \frac{m_p}{M_D} \, G_M(0)$$
$$Q_D = \, G_Q(0)$$





At high momentum tranfer the Z-diagram becomes more and more important



At high momentum tranfer the Z-diagram becomes more and more important

Conclusions

In order to construct a *Standard Model* for Few-Nucleon Systems it is necessary to take into account relativistic effects

- We have analyzed the em observables for two and three-nucleon systems within the Light-Front Hamiltonian Dynamics, in order to take profit of the successful phenomenology developed for Few-Nucleon Systems
- We have started a systematic analysis of the pair contribution to the em processes
- Aim: i) to develop a phenomenological guidance for more fundamental approaches; ii) to explore the model dependence in the extraction of *non standard* effects in Few-Nucleon Systems