Form Factors and Proton Knockout Processes: the Hadronic View

- Nuclear interactions and spectra of light nuclei

- Nuclear electromagnetic currents and form factors of few-nucleon systems (see also Ingo's talk)

- Relativistic approaches and deuteron form factors

- Proton knock-out processes and FSI effects; case studies:

  $^4\mathrm{He}(\vec{e},e'\vec{p})^3\mathrm{H}$ (E93-049)
  $^3\mathrm{He}(e,e'p)d$ and pn (E89-044)
  $^4\mathrm{He}(e,e'p)^3\mathrm{H}$ (E97-111)

- Summary

Nuclear Interactions and Spectra

- $$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- Argonne v_{18}: $v = v^{\gamma} + v^{\pi} + v^{R} \equiv \sum_p v^p(r_{ij})\, O^p_{ij}$

$$O^p_{ij} = [\mathbb{1}, \sigma_i\cdot\sigma_j, S_{ij}, (L\cdot S)_{ij}, \ldots] \otimes [\mathbb{1}, \tau_i\cdot\tau_j]$$

charge independent

$$= [\mathbb{1}, \sigma_i\cdot\sigma_j, S_{ij}, (L\cdot S)_{ij}] \otimes [\tau_{iz}+\tau_{jz}, T_{ij}]$$

charge dependent

$$T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i\cdot\tau_j$$

i) v^{γ} includes Coulomb, Darwin-Foldy, vacuum polarization, magnetic-moment interactions

ii) v^{π} includes charge dependence due to $\pi^0 - \pi^{\pm}$ mass difference

iii) short range part includes charge dependence fixed by differences in $^1S_0$ pp-np scattering & charge asymmetry fixed by difference in pp-nn scattering length

• Illinois model for V_{ijk}^{*}:

$$V = V^{2\pi} + V^{3\pi} + V^{R}$$

$V^{2\pi} = A^{PW}_{2\pi}$ 

$+ A^{SW}_{2\pi}$ 

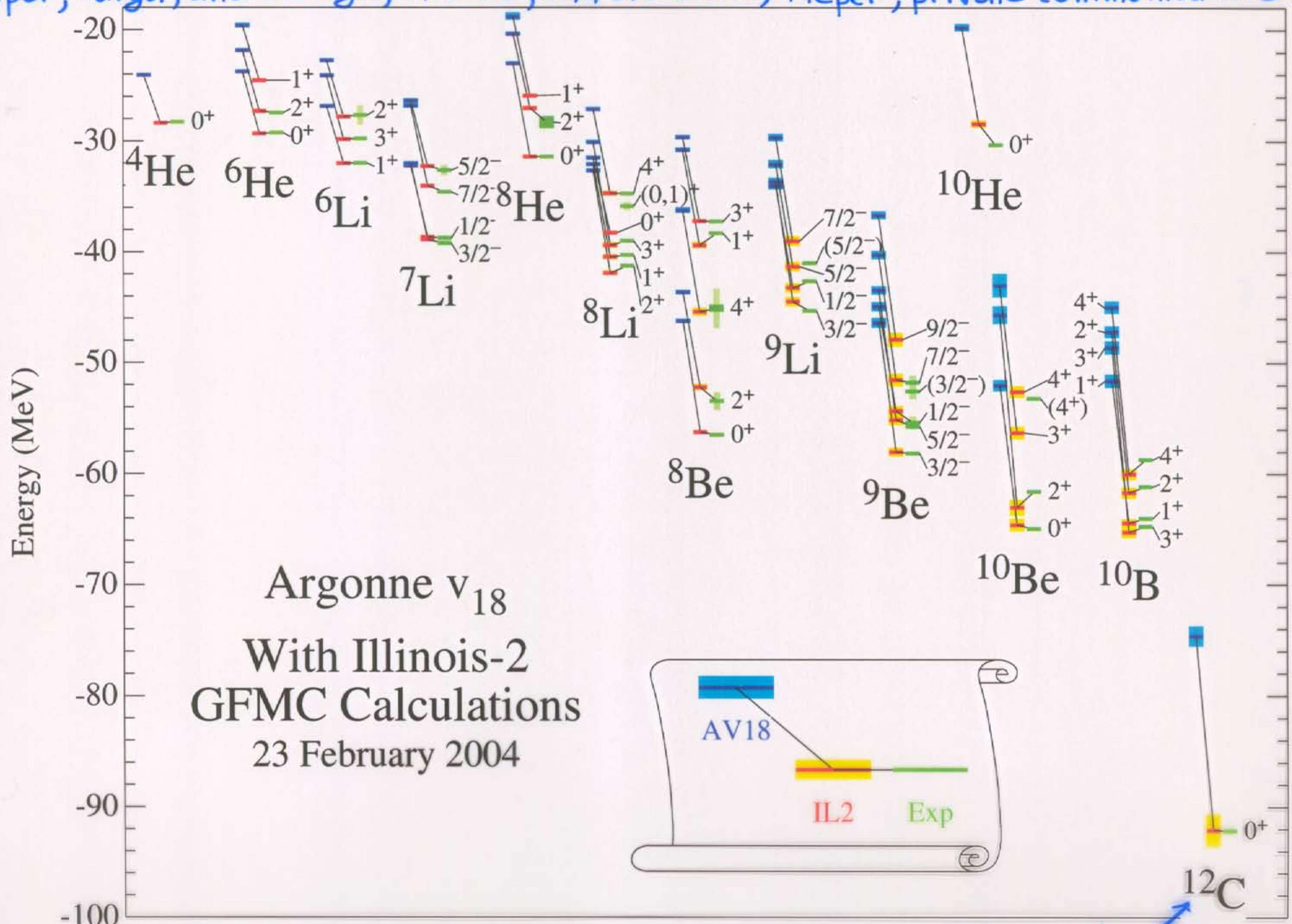
$V^{3\pi} = A_{3\pi}$ [ + ] π-ring diagrams

$$V^{R} = A_{R} \sum_{cyc} T^{2}(r_{ij})\, T^{2}(r_{jk})$$

• parameters (~ 3) determined by fitting energies of 17 low-lying states of $A \leq 8$

* Pieper, Pandharipande, Wiringa, and Carlson, PRC 64 (2001)

Pieper, Varga, and Wiringa, PRC 66, 044310 (2002); Pieper, private communication



very preliminary!

Electromagnetic Current

- One-body terms:

$$\vec{j}_i = G_E \frac{\vec{p}_i + \vec{p}_i^{\,\prime}}{2m} + \frac{i}{2m} G_M \, \vec{\sigma}_i \times \vec{q}$$

$$\rho_i = \frac{G_E}{\sqrt{1+\tau}} + \frac{i}{4m^2}(2G_M - G_E)\,\vec{\sigma}_i \cdot (\vec{p}_i^{\,\prime} \times \vec{p}_i)$$

- Two-body terms:

i) from v_{ij}, implied by current conservation

$$\vec{q} \cdot \vec{j}_{ij} = [v_{ij}, \rho_i + \rho_j]$$

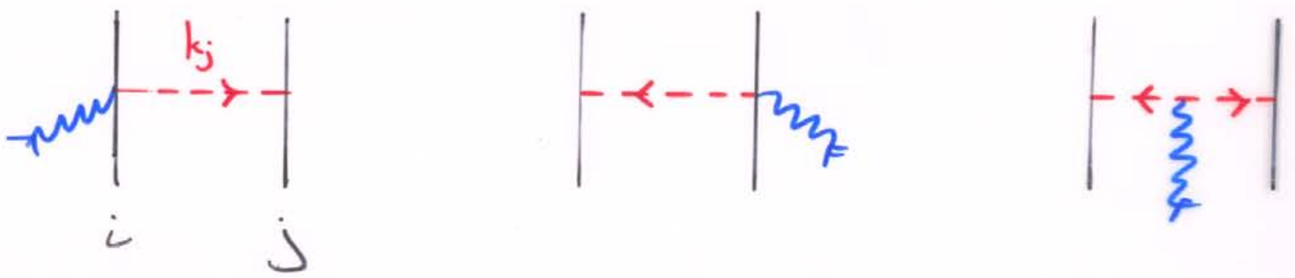
- $v_{ij} = (\underbrace{v_0 + v_{0\tau}\,\vec{\tau}_i \cdot \vec{\tau}_j}_{\text{static}}) + (\underbrace{v_p + v_{p\tau}\,\vec{\tau}_i \cdot \vec{\tau}_j}_{p\text{-dependent}})$

- $v_{0\tau}$ due to π- and ρ-exchange

$$\left\{ \underbrace{\left[D_\pi(k) + 2D_\rho(k) \right]}_{v^{\sigma\tau}(k)} k^2\, \vec{\sigma}_i \cdot \vec{\sigma}_j - \underbrace{\left[D_\pi(k) - D_\rho(k) \right]}_{v^{t\tau}(k)} k^2 S_{ij}(\hat{k}) \right\} \vec{\tau}_i \cdot \vec{\tau}_j$$

- obtain D_π and D_ρ from $v^{\sigma\tau}$ and $v^{t\tau}$

· π- and ρ-exchange currents:



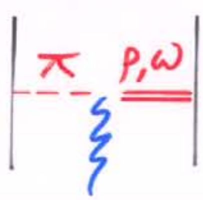
$$j^{\pi}_{ij}(\vec{k}_i\,\vec{k}_j) = 3i\,(\vec{\tau}_i \times \vec{\tau}_j)_z\, G_E^V \Big[D_\pi(k_j)\, \vec{\sigma}_i\, (\vec{\sigma}_j \cdot \vec{k}_j)$$

$$+ \frac{\vec{k}_i - \vec{k}_j}{k_i^2 - k_j^2}\, \underbrace{D_\pi}_{\text{from } v_{\sigma\tau}}(k_i)\, (\vec{\sigma}_i \cdot \vec{k}_i)(\vec{\sigma}_j \cdot \vec{k}_j) \Big] + i \rightleftarrows j$$

· j^π and j^ρ conserved with $v_{\sigma\tau}$

· currents from v_p and $v_{p\tau}$ also taken into account: short range $<< j^\pi_{ij}$

ii) $\rho\pi\gamma$ and $\pi\omega\gamma$ transition currents:



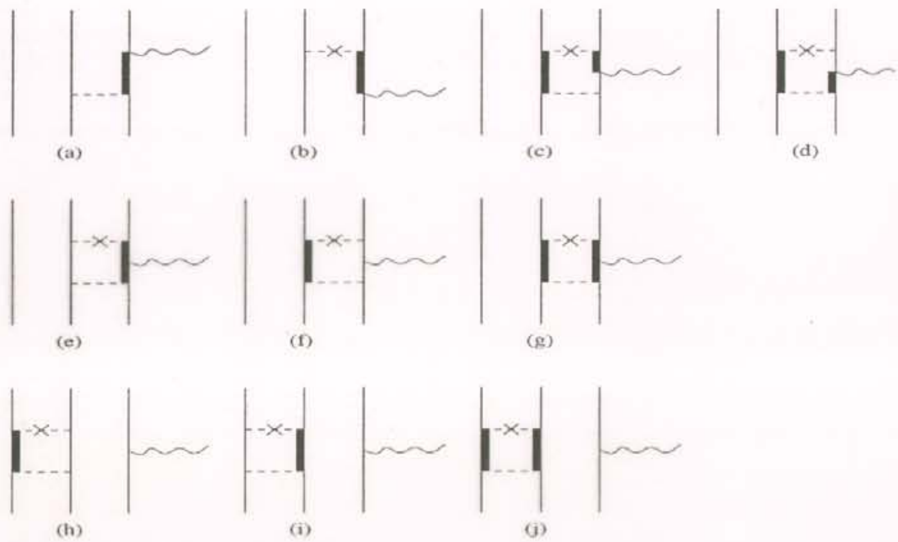
· transverse
· $\rho\pi\gamma$ plays role in B-structure function

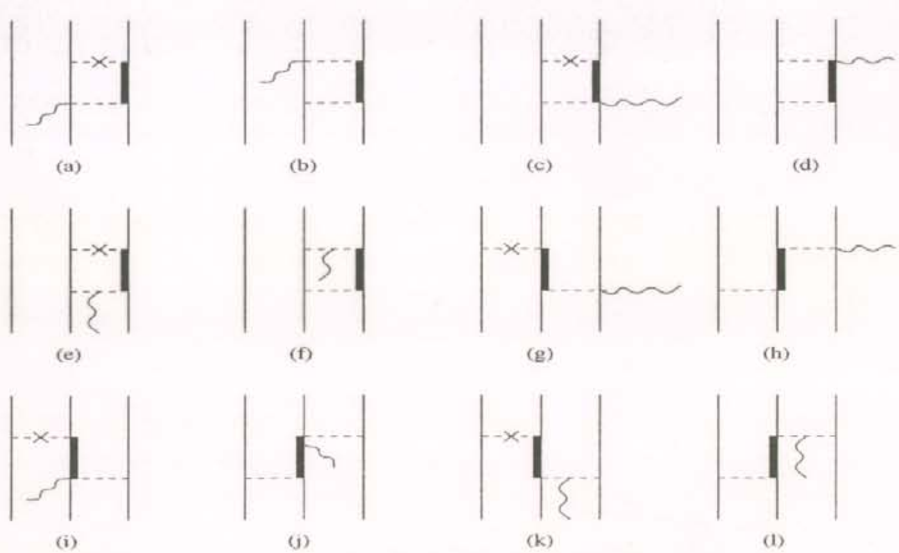
• Δ d.o.f. induce many-body currents*:

$$-\frac{i}{2m}\, G_{\gamma N\Delta}\; \vec{q}\times\vec{S}\; T_z$$

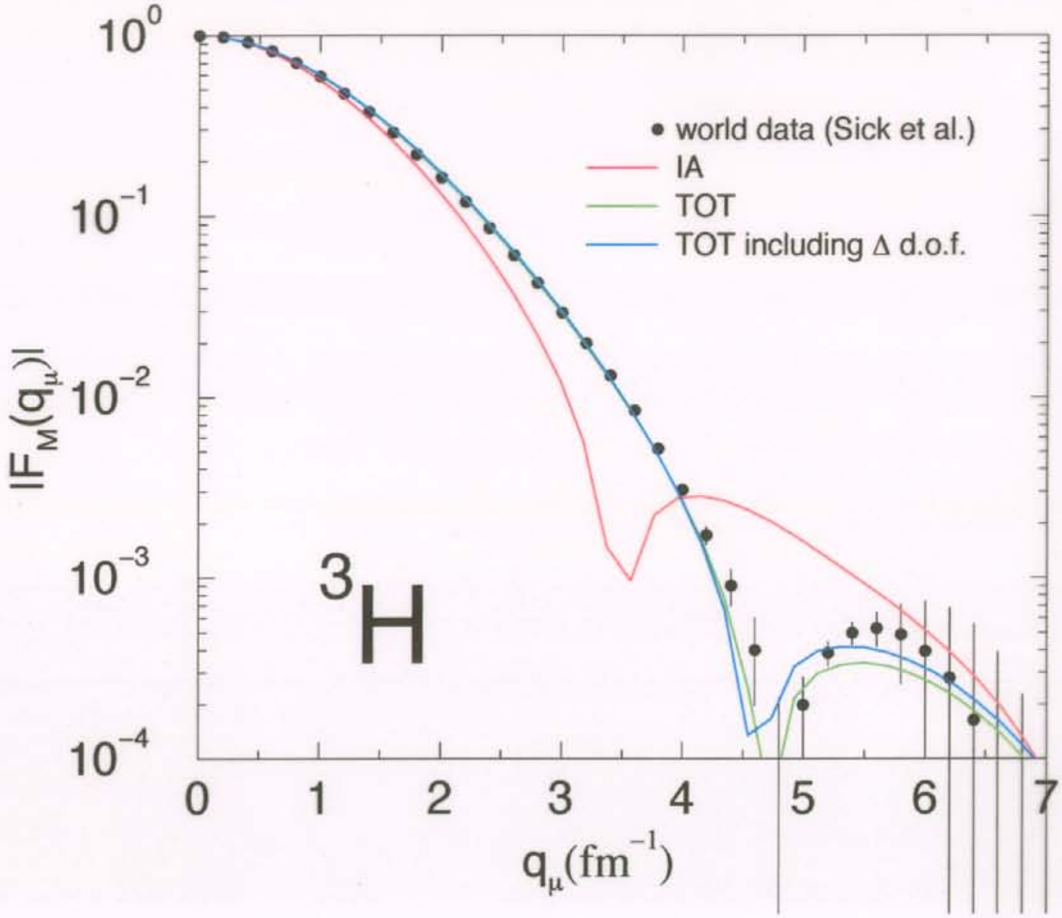
spin (isospin) transition operator

$$-\frac{i}{24m}\, G_{\gamma\Delta\Delta}\; \vec{q}\times\vec{\Sigma}\,(1+\Theta_z)$$

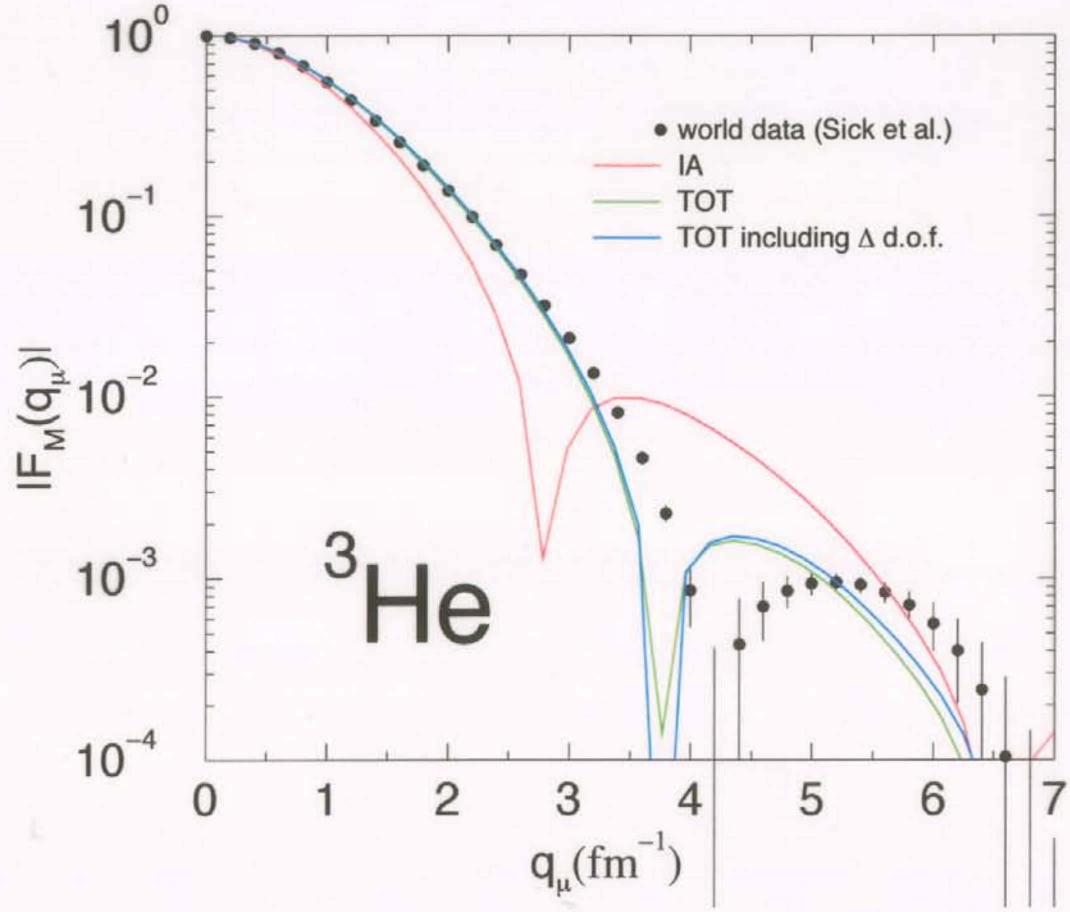




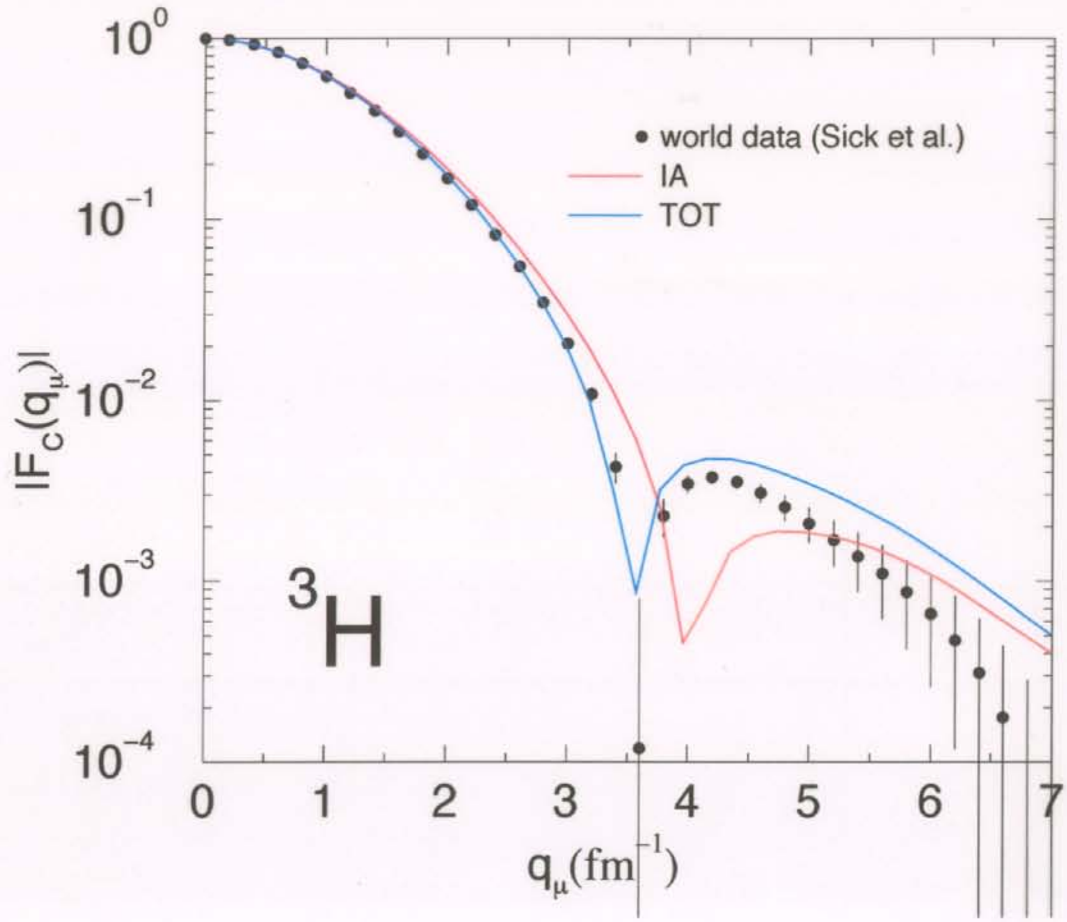
* Sauer et al. (1987) ; Marcucci et al. (1998)



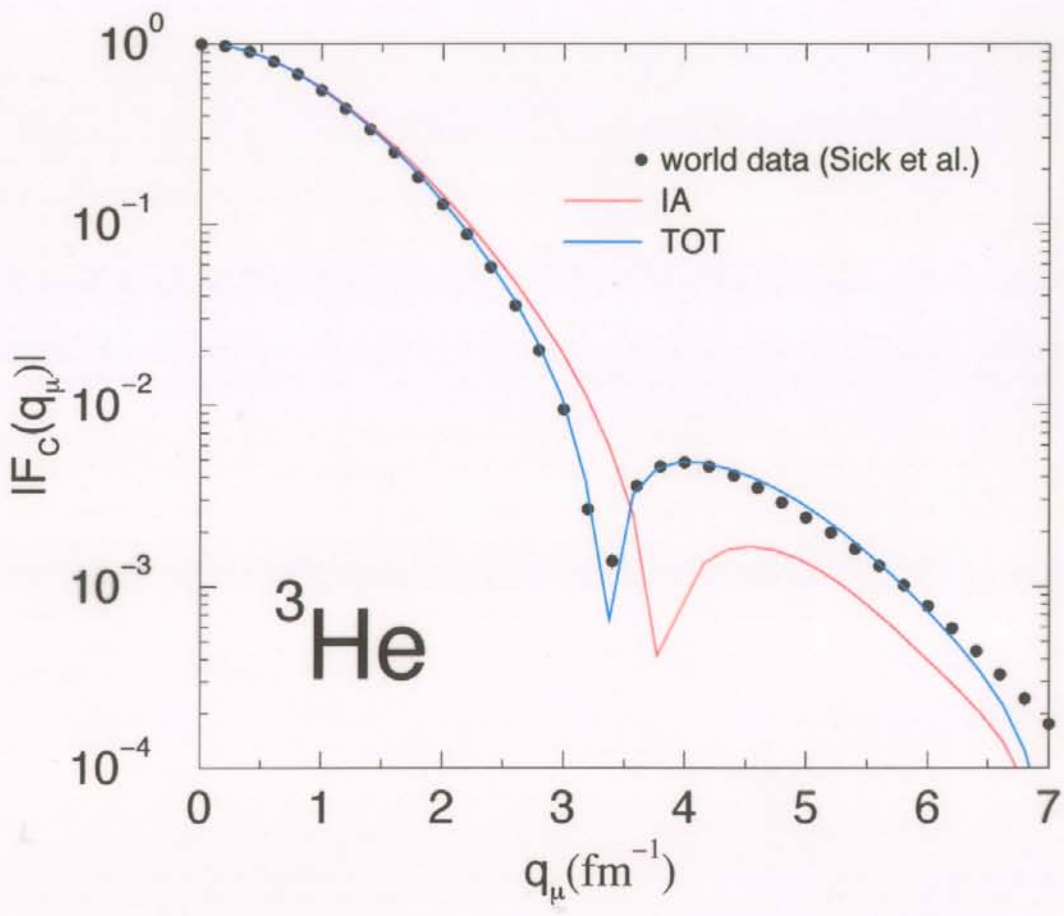
world data (Sick et al.)
IA
TOT
TOT including Δ d.o.f.
^{3}H
$|F_M(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$



world data (Sick et al.)
IA
TOT
TOT including Δ d.o.f.
^{3}He
$|F_M(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$



world data (Sick et al.)
IA
TOT
^{3}H
$|F_C(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$



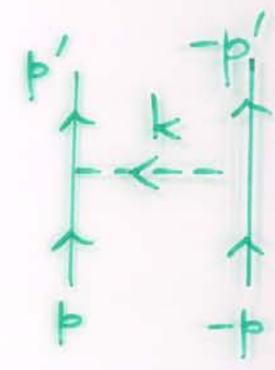
world data (Sick et al.)
IA
TOT
^{3}He
$|F_C(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$

Relativistic Approaches and Form Factors

Hamiltonian dynamics:

- rest frame

$$H^{\mu} = 2\sqrt{p^2+m^2} + v^{\mu}, \quad v^{\mu} = v_R + v_{\pi}^{\mu},$$



$$v_{\pi}^{\mu} = -\frac{f_{\pi NN}^2}{m_{\pi}^2} \cdot \frac{f_{\pi}^2(k)}{m_{\pi}^2+k^2} \cdot \frac{m}{E'} \cdot \frac{m}{E} \Big[\vec{\sigma}_1\cdot\vec{k}\, \vec{\sigma}_2\cdot\vec{k} + \mu\,(E'-E)\Big(\frac{\vec{\sigma}_1\cdot\vec{p}\,'\, \vec{\sigma}_2\cdot\vec{p}\,'}{E'+m} - \frac{\vec{\sigma}_1\cdot\vec{p}\, \vec{\sigma}_2\cdot\vec{p}}{E+m} \Big) \Big]$$

$\mu =$ +1 PV
0 "maximally local"
−1 PS (Bonn potential)

Forest PRC 61 (2000)

- unitary equivalence*

$$H^{\mu} = e^{-i\mu U} H^{\mu=0} e^{i\mu U} \simeq H^{\mu=0} + i\mu \left[H^{\mu=0}, U \right]$$

ignoring 2π and short-range terms

- choice of μ off-energy-shell extrapolation for OPEP irrelevant: $\psi^{\mu} \simeq (1 - i\mu U)\psi^{\mu=0}$

* Friar (1977), Gross

- boosting w.f. to order $(v/c)^2$*:

$$\psi^{\mu}(\vec{p};\vec{V}) \simeq \frac{1}{\sqrt{\gamma}}\left[1 - \frac{i}{4m}\,\vec{V}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}\right]\psi^{\mu}(\vec{p}_{\parallel}/\gamma,\,\vec{p}_{\perp};0)$$

Thomas precession

Lorentz contraction

i) interaction-dependent terms in boost do not contribute to $(v/c)^2$ order in f.f. calculation

- T=0 currents:

i) one-body: $\frac{1}{2}\,\bar{u}'\left[F_1^S\gamma^{\sigma} + \frac{i}{2m}F_2^S\sigma^{\sigma\tau}q_{\tau}\right]u$

ii) two-body: depend on off-shell extrapolation

$$\rho_{\pi}^{\mu}(q) = \frac{(3-\mu)}{8m}\,\frac{f_{\pi NN}^2}{m_{\pi}^2}\,F_1^S\,f_{\pi}^2(k_2)\;\vec{\sigma}_1\cdot\vec{q}\;\vec{\sigma}_2\cdot\vec{k}_2\;\vec{\tau}_1\cdot\vec{\tau}_2 + 1\rightleftharpoons 2$$

- $e^{-i\mu U}(\rho_1 + \rho_{\pi}^{\mu=0})\,e^{i\mu U} \simeq \rho_1 + \rho_{\pi}^{\mu}$

- ρ_{ρ} and $\rho_{\pi\rho\gamma}$ are also included

* Friar (1977); Gross (1986)

Spectator formalism (Gross equation):

- explicit inclusion of $\bar{N}$ d.o.f.
- bound state:

$V = \text{OBE}(\pi, \eta, \delta, \sigma, \rho, \omega)$ constrained to fit NN elastic scattering data*

- rest-frame deuteron w.f.:

$$\psi(\vec{p}) = u(p)\,\mathcal{Y}^{LSJ}_{011} + w(p)\,\mathcal{Y}_{211} + \underbrace{v_t(p)\,\mathcal{Y}_{111} - v_s(p)\,\mathcal{Y}_{101}}_{\text{P-waves}}$$

← because of $\bar{N}$ d.o.f.

- boosts are kinematical and exactly included (covariance)
- currents:

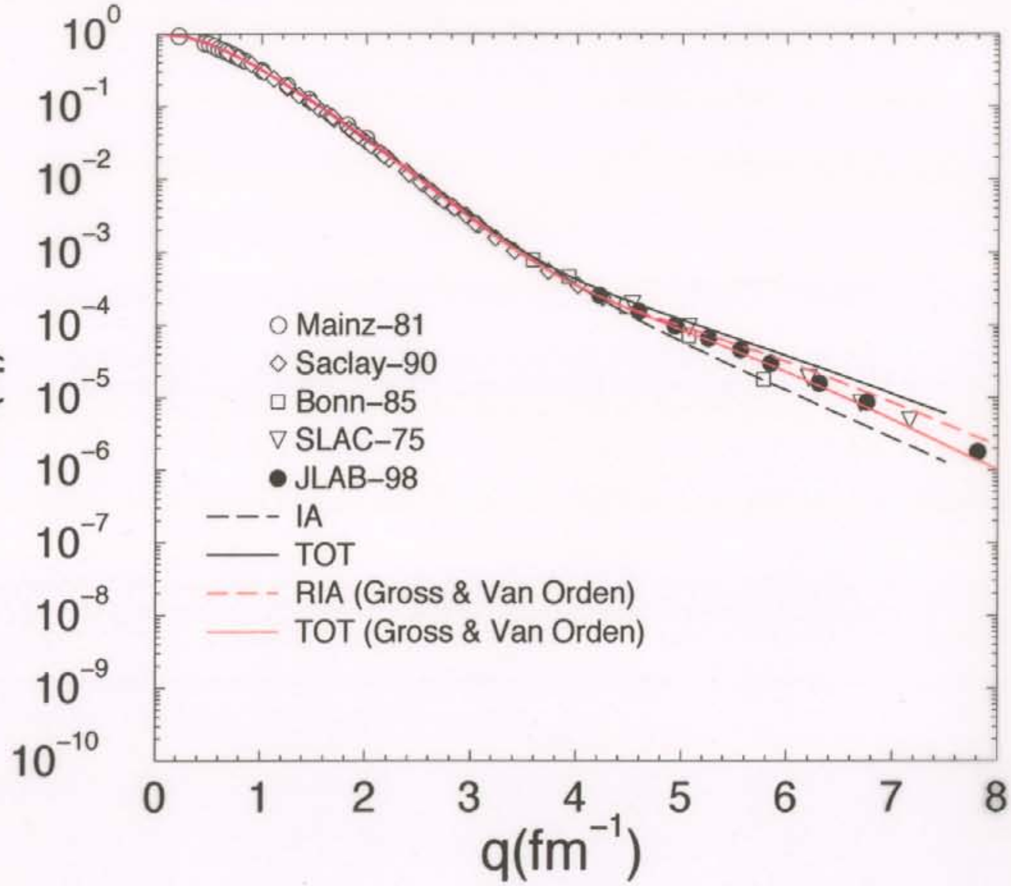
i) one-body: $$j^\mu(p',p) = f_0(p',p)\left[F_1(Q^2)\gamma^\mu + \frac{i}{2m}F_2(Q^2)\sigma^{\mu\nu}q_\nu\right] + g_0(p',p)\,F_3(Q^2)\,\frac{m-\not{p}'}{2m}\,\gamma^\mu\,\frac{m-\not{p}}{2m}$$

fixed by WT identity

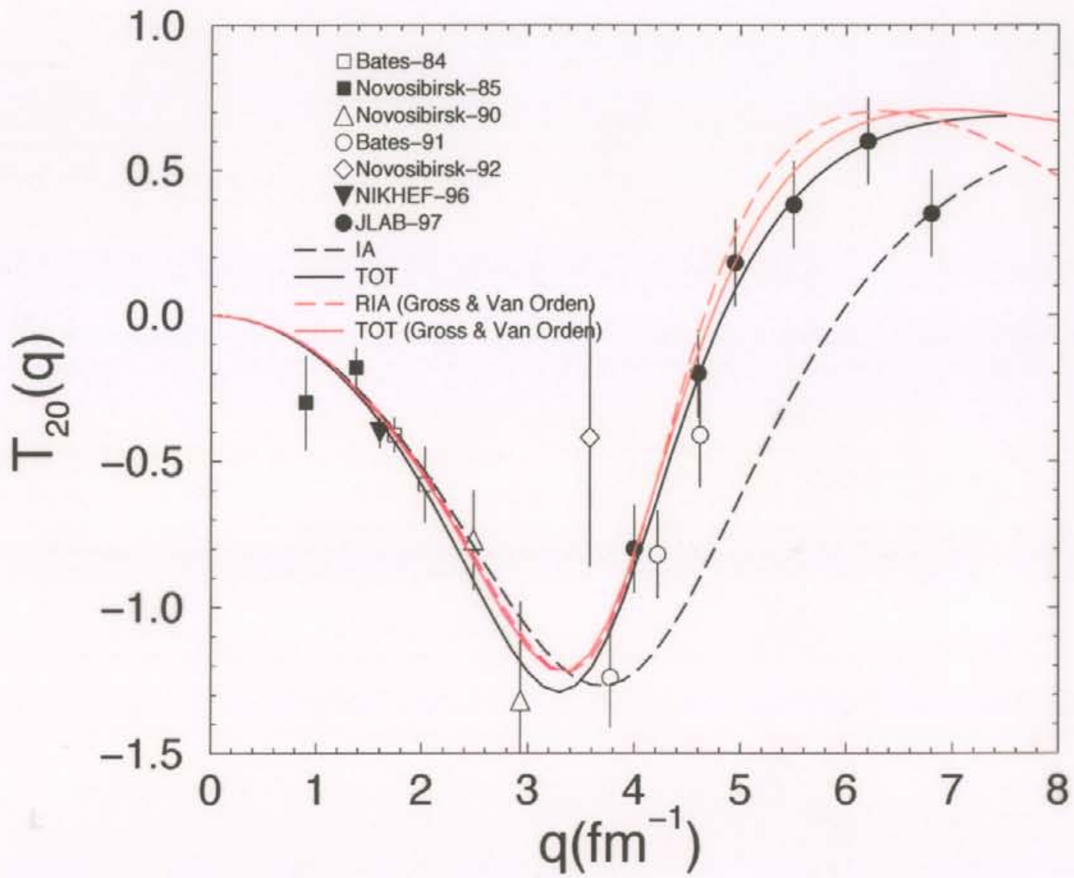
unconstrained

ii) two-body: $j^\mu_{\rho\pi\gamma}$ transition current

* Gross, Van Orden, and Holinde, PRC 45 (1992)



A(q)
q(fm^{-1})
Mainz–81
Saclay–90
Bonn–85
SLAC–75
JLAB–98
IA
TOT
RIA (Gross & Van Orden)
TOT (Gross & Van Orden)



$T_{20}(q)$
q(fm^{-1})
Bates–84
Novosibirsk–85
Novosibirsk–90
Bates–91
Novosibirsk–92
NIKHEF–96
JLAB–97
IA
TOT
RIA (Gross & Van Orden)
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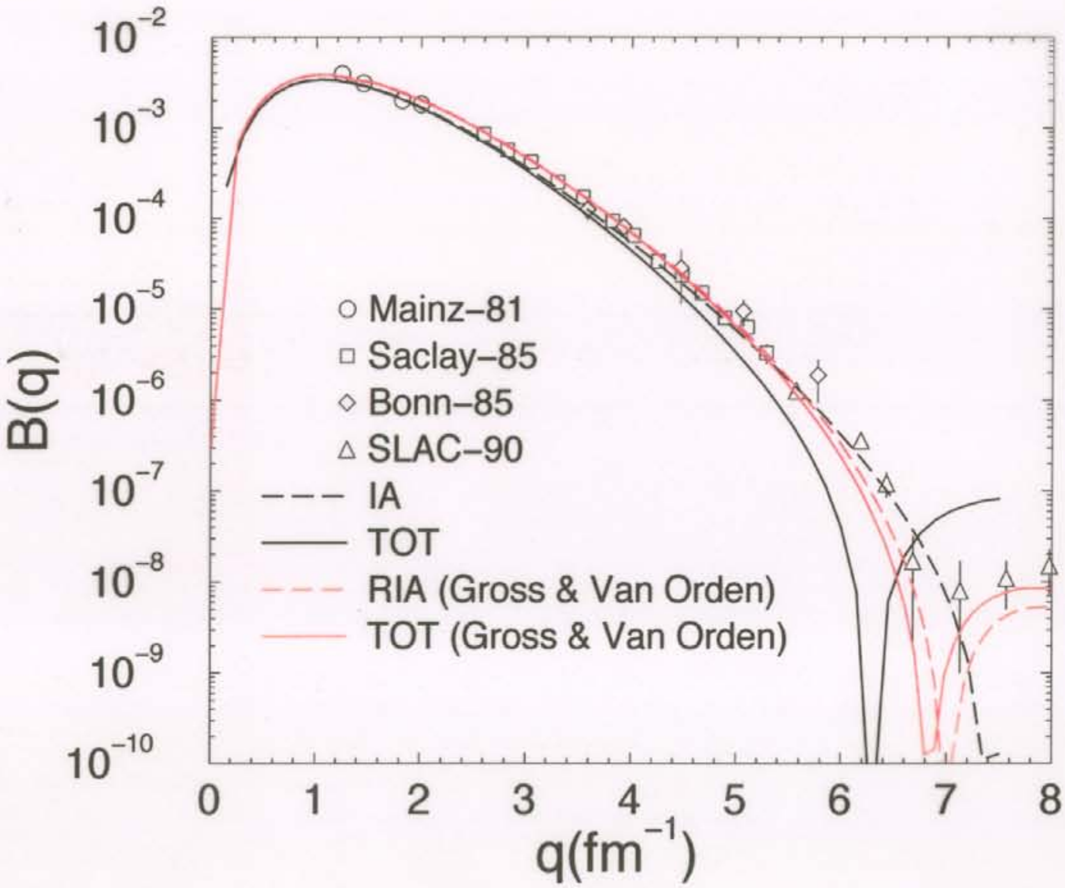
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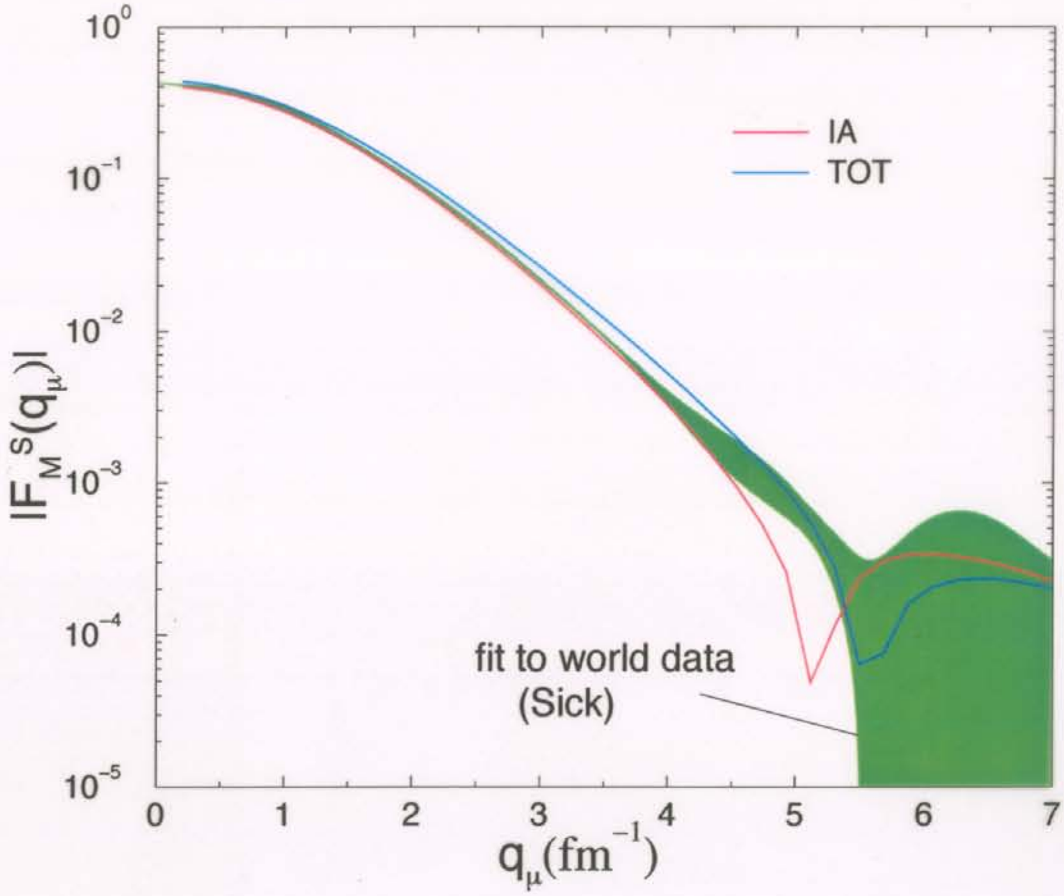
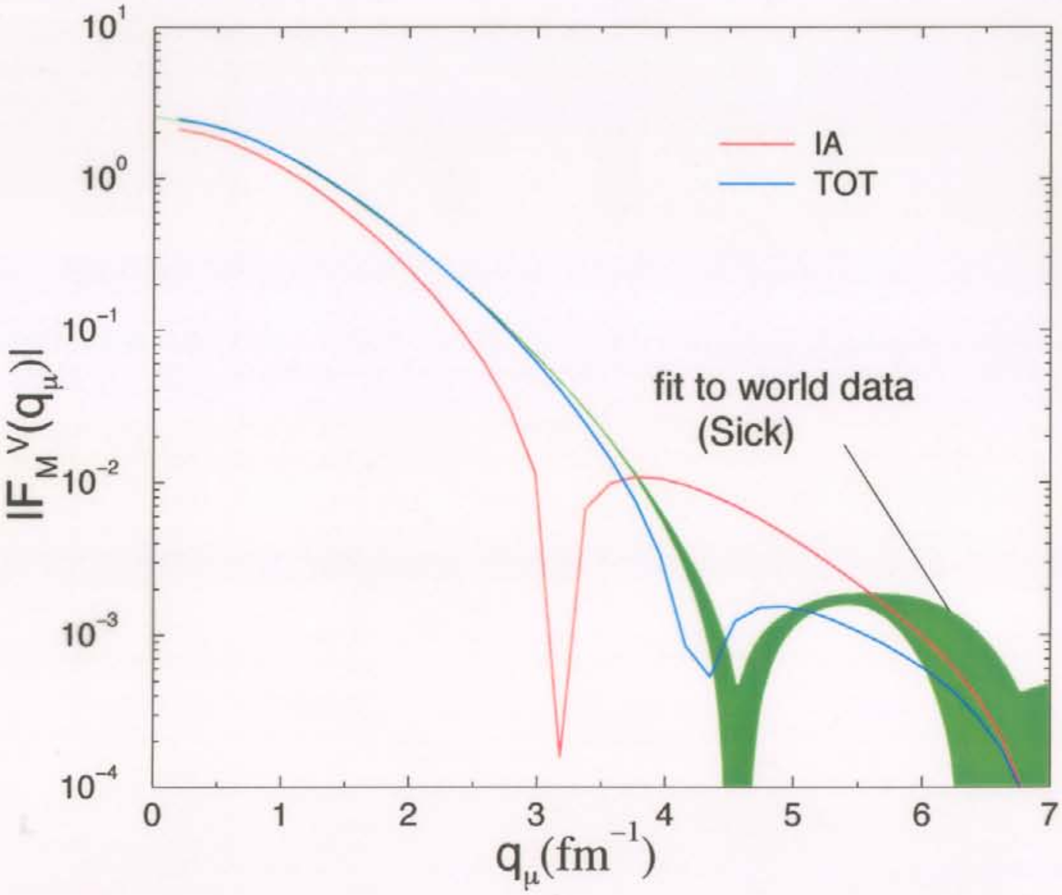


IA
TOT
fit to world data
(Sick)
$|F_M{}^S(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$



IA
TOT
fit to world data
(Sick)
$|F_M{}^V(q_\mu)|$
$q_\mu(\mathrm{fm}^{-1})$

A(e,e'p) Reactions at JLab

- JLab kinematics typically have $T_p \gtrsim 0.5$ GeV, and NN scattering is mostly inelastic
- To describe FSI between fast struck nucleon and recoiling A-1 system:

  i) optical potential (A=4)

  ii) Glauber approach and derivatives

  iii) "diagrammatic approach" (Laget)

- Currents:

  i) one-body*:

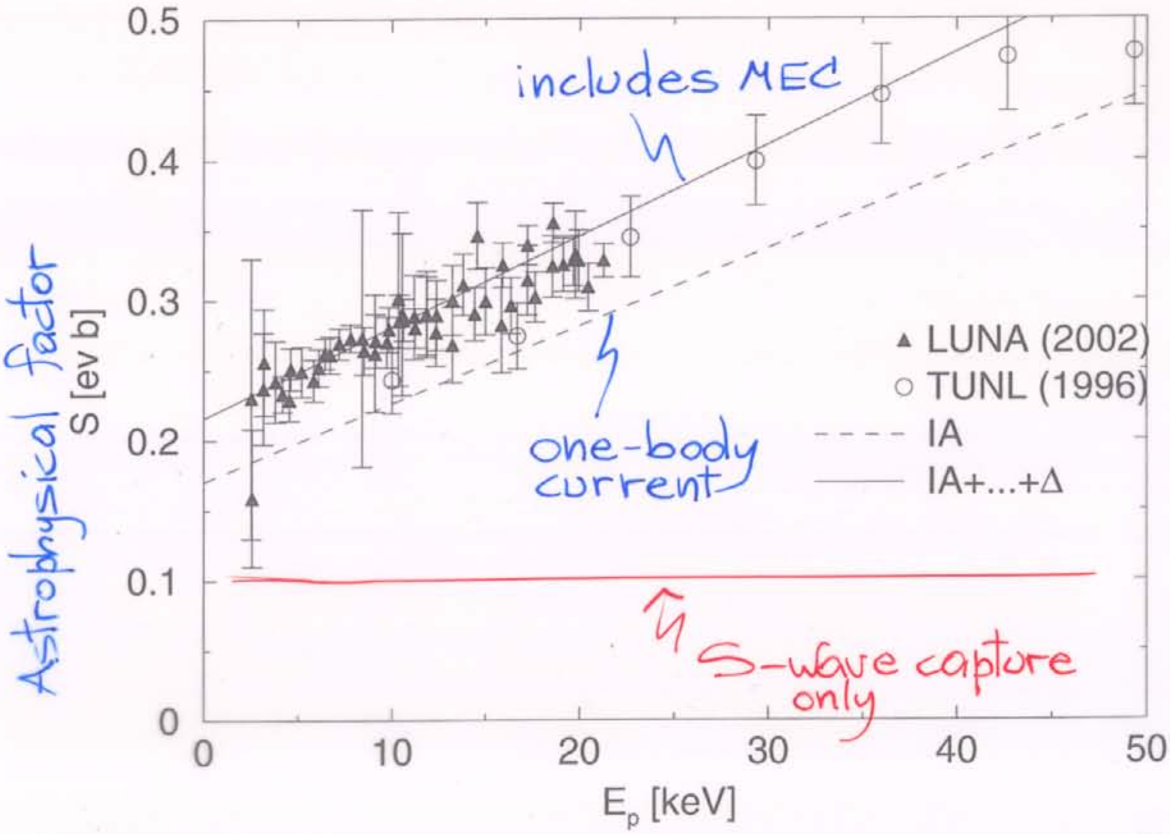
$$\rho = \frac{q}{Q} G_E + \frac{i}{\sqrt{1+\tau}} \frac{q}{2m^2}\left(G_M - \frac{1}{2} G_E\right) \vec{\sigma}\cdot(\hat{q}\times\vec{p})$$

$$\vec{j} = -\frac{Q}{q}\left[i\, G_M \left(\frac{\vec{q}\times\vec{\sigma}}{2m} + \frac{\omega}{4m^2}\, \hat{q}\cdot\vec{\sigma}\ \hat{q}\times\vec{p} \right) - \frac{\vec{p}}{m}\left(G_E + \frac{1}{2}\tau\, G_M \right)\right], \quad \tau = \frac{Q^2}{4m^2}$$

  ii) many-body: same as in f.f. calculations

* Jeschonnek and Donnelly, PRC 57 (1998)

pd radiative capture at $E \lesssim 50$ keV *
(S- and P-wave channels important)



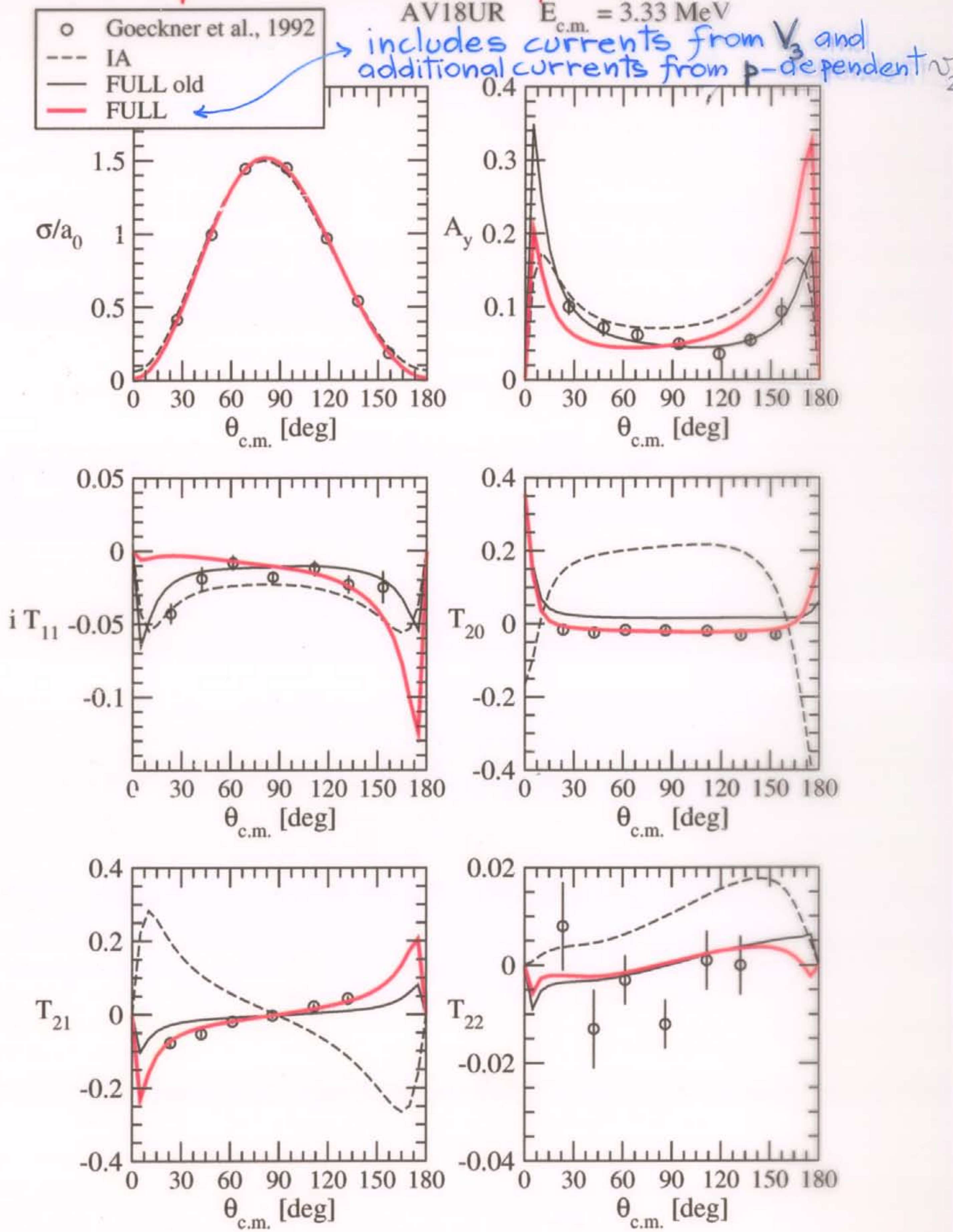
AV18/UIX and CHH (bound and scattering) w.f.'s

	S(0) (eV.b)
SNPA	0.219
LUNA-exp	0.216 ± 0.010

* Viviani et al., PRC 61 (2000)

But $\sigma(nd \rightarrow {}^3H\gamma)$ at thermal energy overestimated by 14% !

pd radiative capture*



Thu May 8 10:35:20 2003 * Marcucci, Kievsky, Viviani et al. (2003)

preliminary!

FSI-I: Optical Potential

- N-3N (bound cluster) complex, energy-dependent potential:

relative energy

$$v^{OPT}_{3+1}(r;T_k) = v^{c}(r;T_k) + v^{b}(r;T_k)\,\vec{\ell}\cdot\vec{s} + \left[v^{c\tau}(r;T_k) + v^{b\tau}(r;T_k)\,\vec{\ell}\cdot\vec{s}\right]\vec{\tau}\cdot\vec{\tau}_3$$

i) $v^{x}(r;T_k)$ are Woods-Saxon type functions: parameters constrained by fits to

$p\,{}^3H \longrightarrow p\,{}^3H \quad T_k = (110-450)\,\mathrm{MeV}$ (Van Oers et al., PRC 25)

$\longrightarrow n\,{}^3He \quad T_k = (43\ \&\ 117)\,\mathrm{MeV}$ (Schiavilla, PRL 65)

ii) charge exchange terms play important role in ${}^4He(\vec{e}, e'\vec{p})^3H$

iii) spin-orbit terms are not well constrained: only $d\sigma/d\Omega$ data were fitted

- 3+1 wave function:

cluster w.f.

$$\psi^{(-)}_{\vec{k}\,\sigma_3\tau_3,\sigma\tau}(3+1) = e^{i\vec{P}\cdot\vec{R}} \sum_{P} \frac{(-)^{P}}{\sqrt{4}}\, \psi_{\sigma_3\tau_3}(ijk)\, \eta^{(-)}_{\vec{k}\sigma\tau}(\ell)$$

scattering solution with v^{OPT}

- ${}^{(-)}\langle {}^3H+p|\, j^{\mu}\, |{}^4He\rangle$ m.e. calculated with QMC methods (no approximations made)

FSI-II: Glauber and Generalized Glauber Approaches

- $$\psi^{(-)}_{(A-1)+1}(\vec{p}) = \Omega^{(-)} \phi_{(A-1)+1}(\vec{p}; PW)$$

  fast struck nucleon

  $$\Omega^{(-)} = \lim_{t \to \infty} \hat{T}\, e^{-i\int_0^t dt'\, H_I(t')}$$

- Glauber approximation:

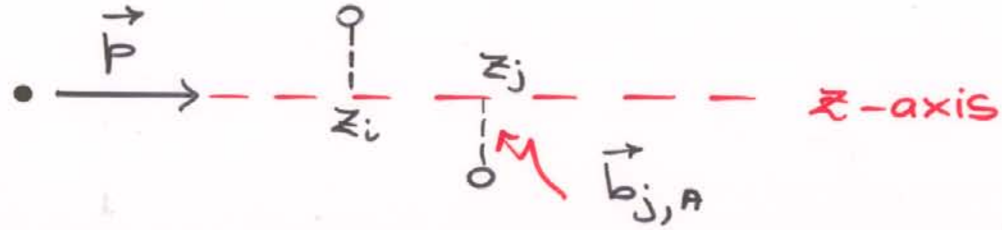
  i) straight-line trajectory for fast nucleon
  ii) nucleons in cluster as fixed scattering centers

$$\Omega^{(-)}_{GA} = \mathbb{1} - \underbrace{\sum_{i=1}^{A-1} \Theta(z_i - z_A)\, \Gamma_p(\vec{b}_{i,A})}_{\text{single rescattering } G^{(1)}}$$

$$+ \underbrace{\sum_{i \neq j=1}^{A-1} \Theta(z_i - z_A)\Theta(z_j - z_i)\, \Gamma_p(\vec{b}_{j,A})\, \Gamma_p(\vec{b}_{i,A})}_{\text{double rescattering } G^{(2)}}$$

$$- \;\ldots$$

$$= \mathbb{1} + \sum_{n=1}^{A-1} (-)^n\, G^{(n)} \quad \text{up to } A\text{-}1 \text{ rescattering}$$



- $$\Gamma_p(\vec{b}) = -\frac{i}{2\pi p}\int d^2k\, e^{i\vec{k}\cdot\vec{b}} f_p(\vec{k})$$

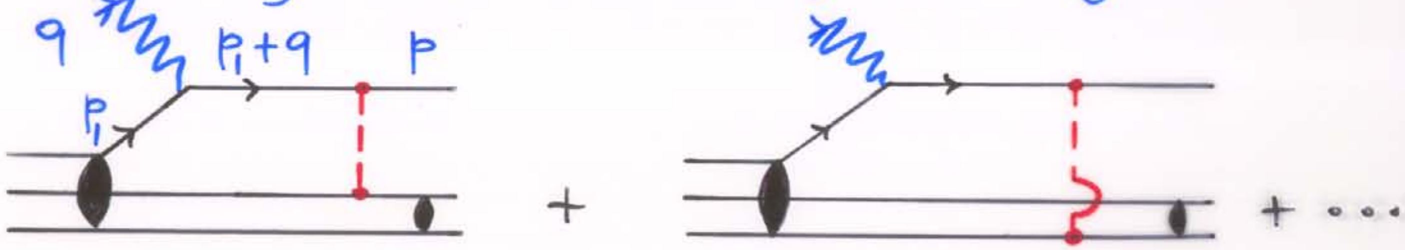
pN scattering amplitude

$$f_p(\vec{k}) = f_p^c(k) + f_p^b(k)(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{k}\times\hat{p} + \cdots$$

$$f_p^c(k) = \frac{p}{4\pi}\sigma_{TOT}^{NN}(i+\alpha_c)\, e^{-\beta_c k^2/2}$$, and similarly for $f_p^b(k)$

from experiment

- Generalized Glauber approximation* (GGA): derived from an analysis of relevant Feynman diagrams



i) struck nucleon propagator:

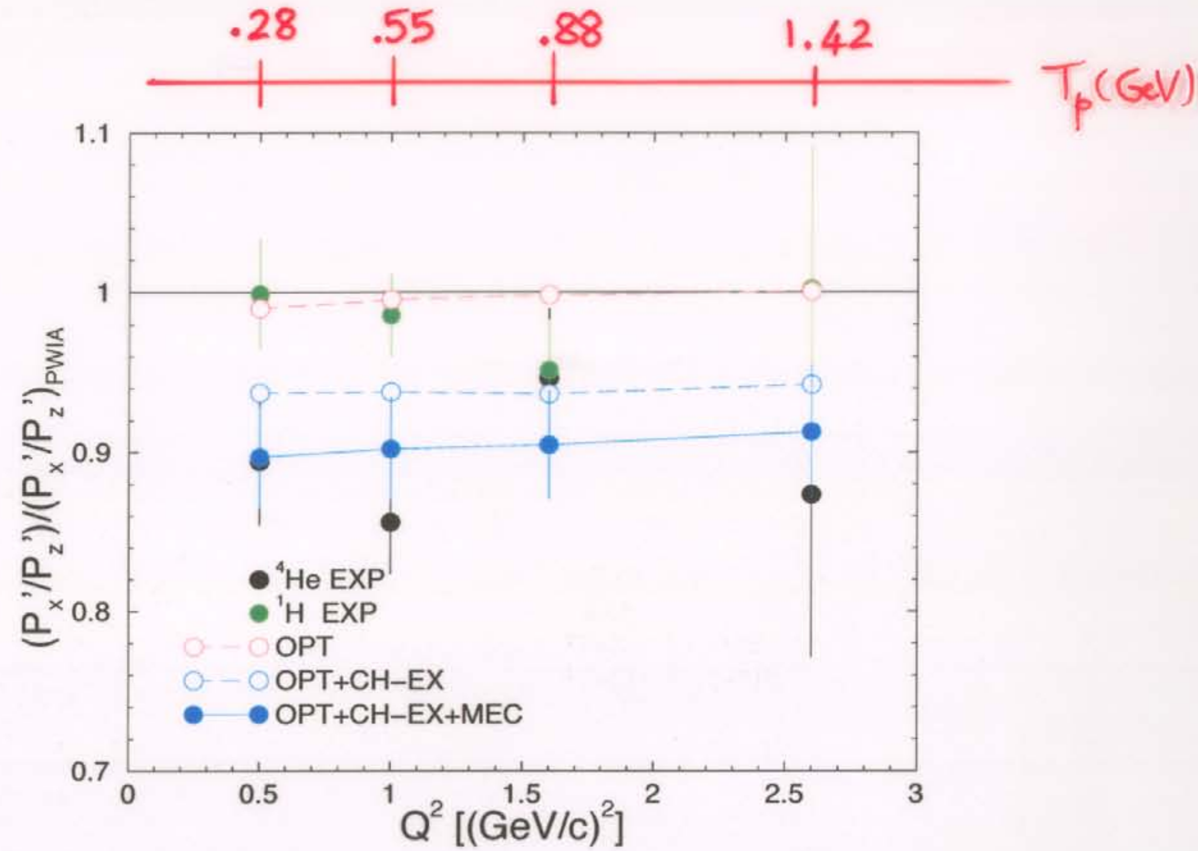
$$-D^{-1}(p_1+q) = (p_1+q)^2 - m^2 + i\varepsilon \simeq 2q(p_{mz} + \Delta_0 - p_{1z} + i\varepsilon)$$

fixed by kinematics $\frac{\omega}{q}E_m$

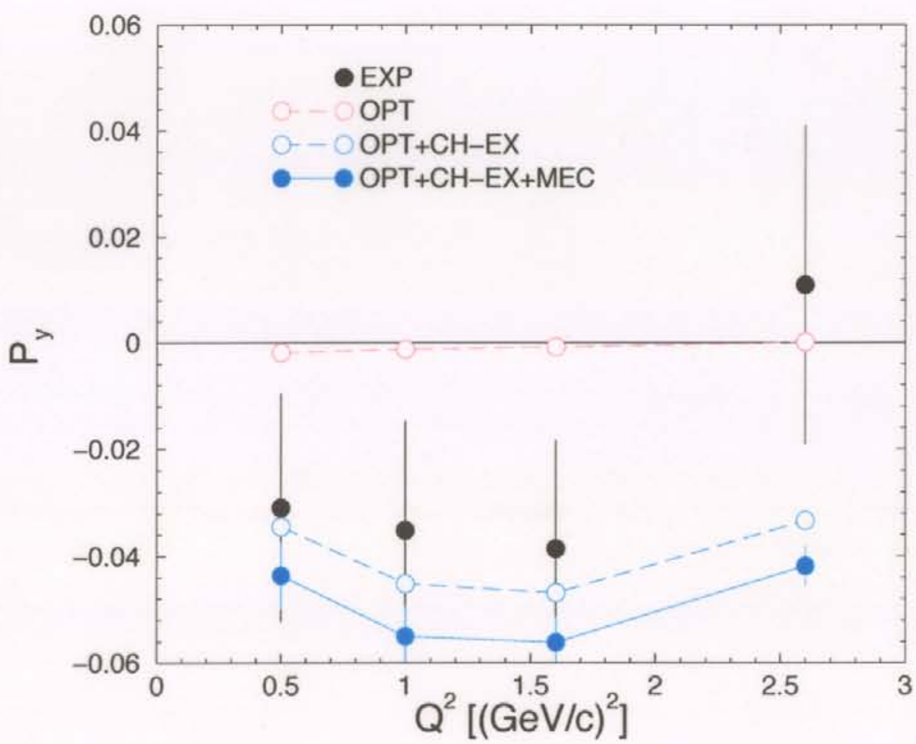
ii) profile function:

$$\Gamma_p(\vec{b}) \longrightarrow e^{i\Delta_0 z}\,\Gamma_p(\vec{b})$$

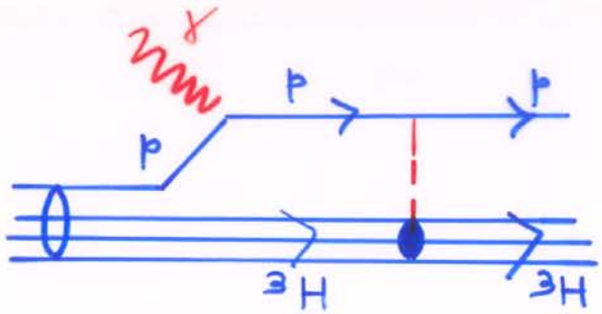
* Gribov (1969); Bertocchi (1972); Frankfurt, Sargsian, and Strikman (1997)

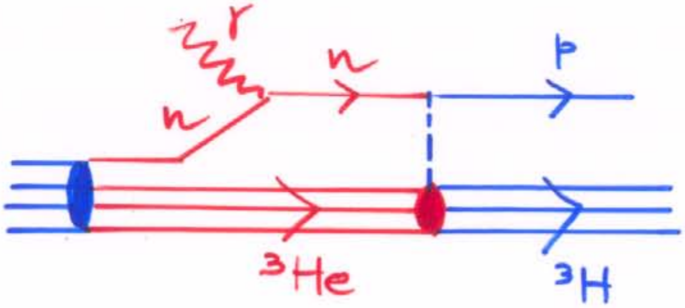


.28
.55
.88
1.42
Tp(GeV)
(Px'/Pz')/(Px'/Pz')PWIA
4He EXP
1H EXP
OPT
OPT+CH-EX
OPT+CH-EX+MEC
Q2 [(GeV/c)2]



Py
EXP
OPT
OPT+CH-EX
OPT+CH-EX+MEC
Q2 [(GeV/c)2]

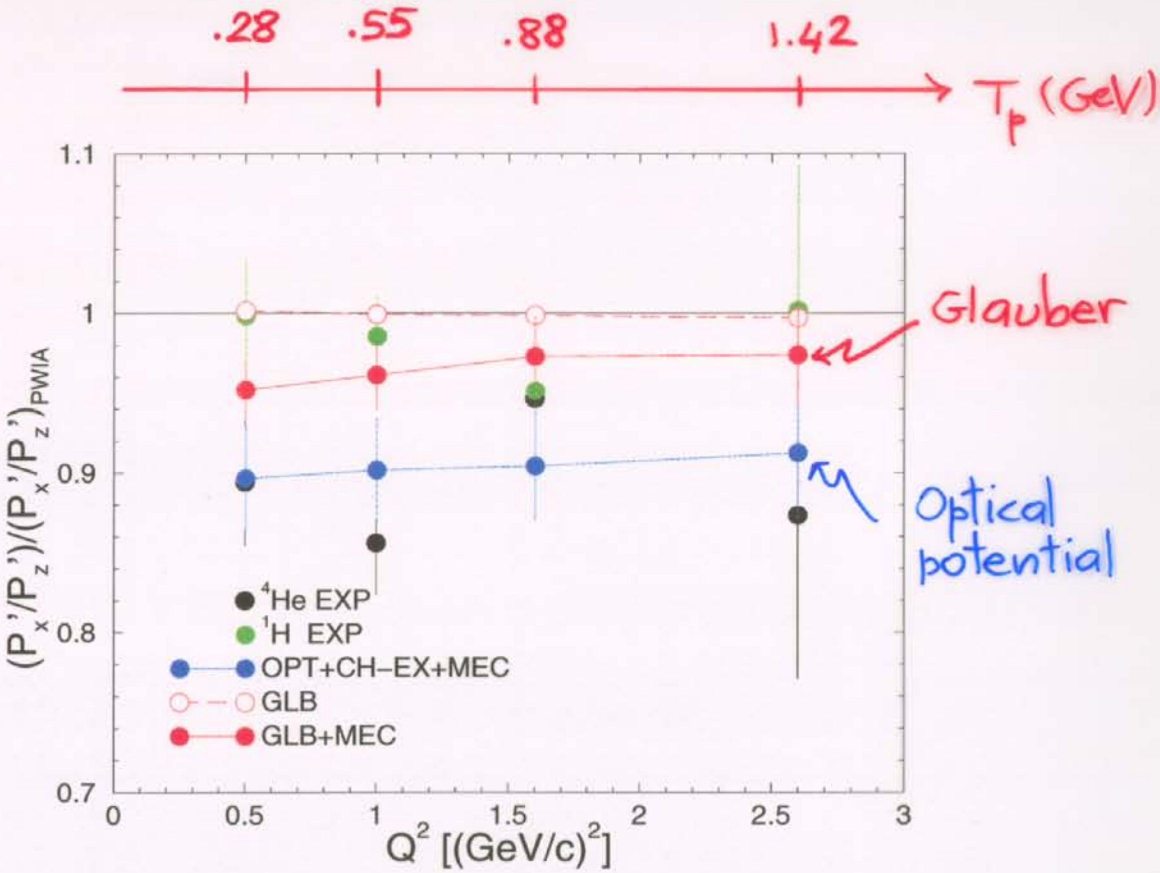


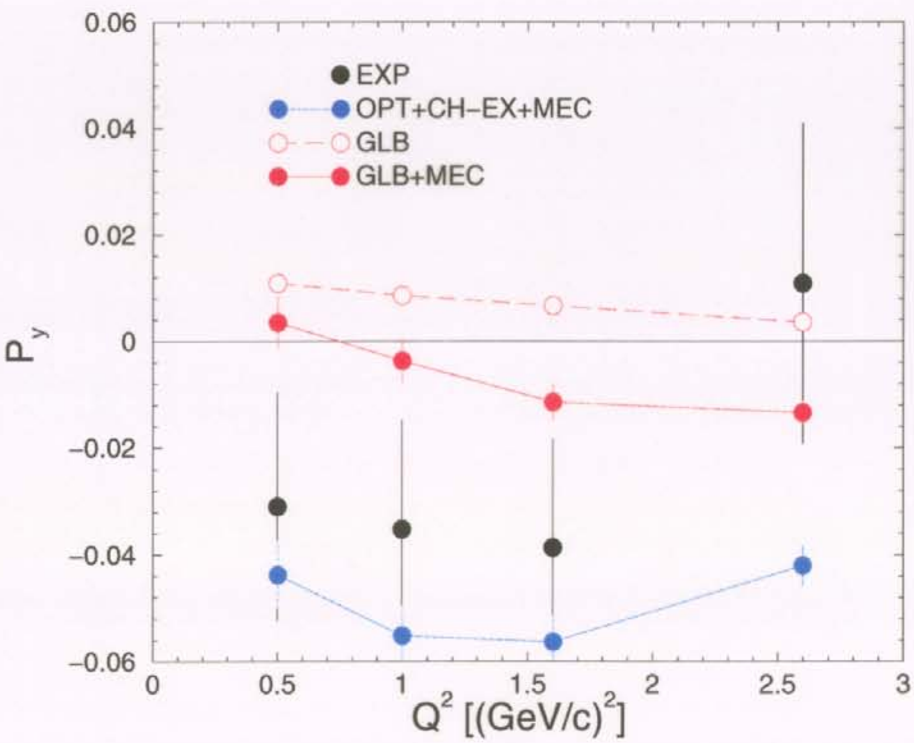


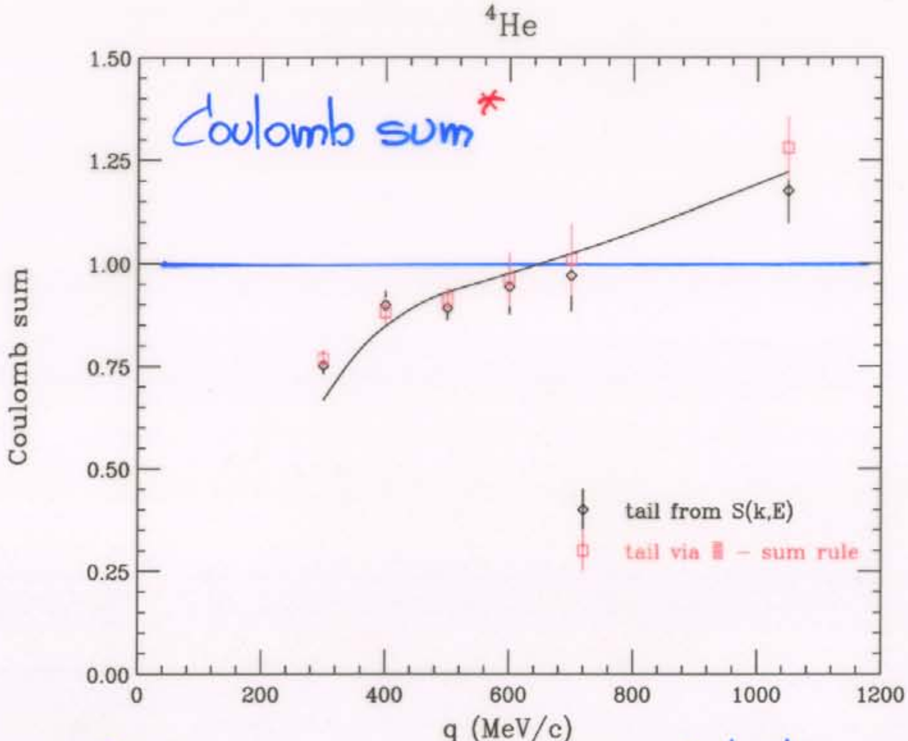
$$P'_x \propto R_{LT'} \sim \rho^*_{fi} \cdot j_{fi}$$
$$P'_z \propto R_{TT'} \sim j^*_{fi} \cdot j_{fi}$$

$Q^2 = 0.5\ (\mathrm{GeV}/c)^2$

	OPT	OPT+CH-EX	... + MEC
P'_x	−.116	−.115	−.114
P'_z	+.130	+.136	+.142


.28
.55
.88
1.42
T_p (GeV)
Glauber
Optical potential
$(P_x'/P_z')/(P_x'/P_z')_{PWIA}$
Q^2 [(GeV/c)^2]
^{4}He EXP
^{1}H EXP
OPT+CH−EX+MEC
GLB
GLB+MEC

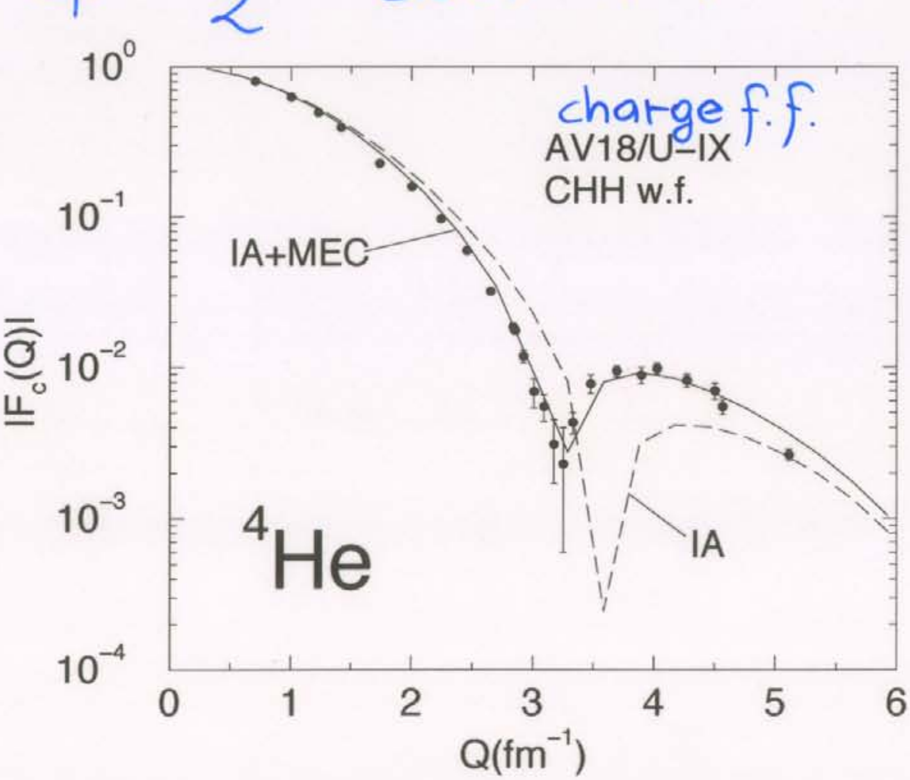

P_y
Q^2 [(GeV/c)^2]
EXP
OPT+CH−EX+MEC
GLB
GLB+MEC



$$S_L = \frac{1}{Z}\int d\omega \frac{R_L}{\tilde{G}_E^2}$$ ← note normalization

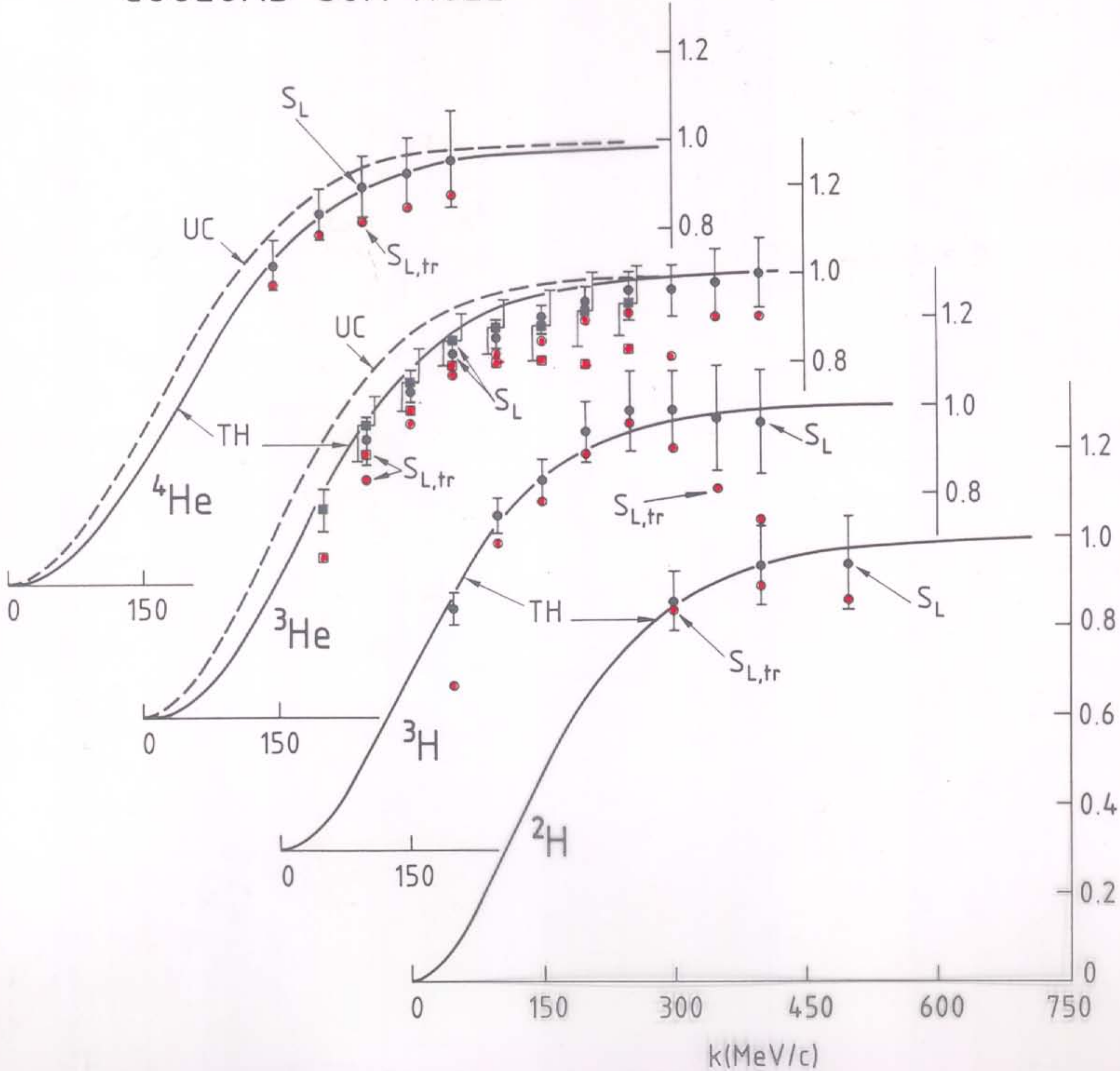
$$S_L^{\text{1-body}} \underset{q\ \text{large}}{\sim} 1$$

$$\tilde{G}_E^2 = \left[G_{Ep}^2 + \frac{(A-Z)}{Z} G_{En}^2\right] / (1+\tau)$$

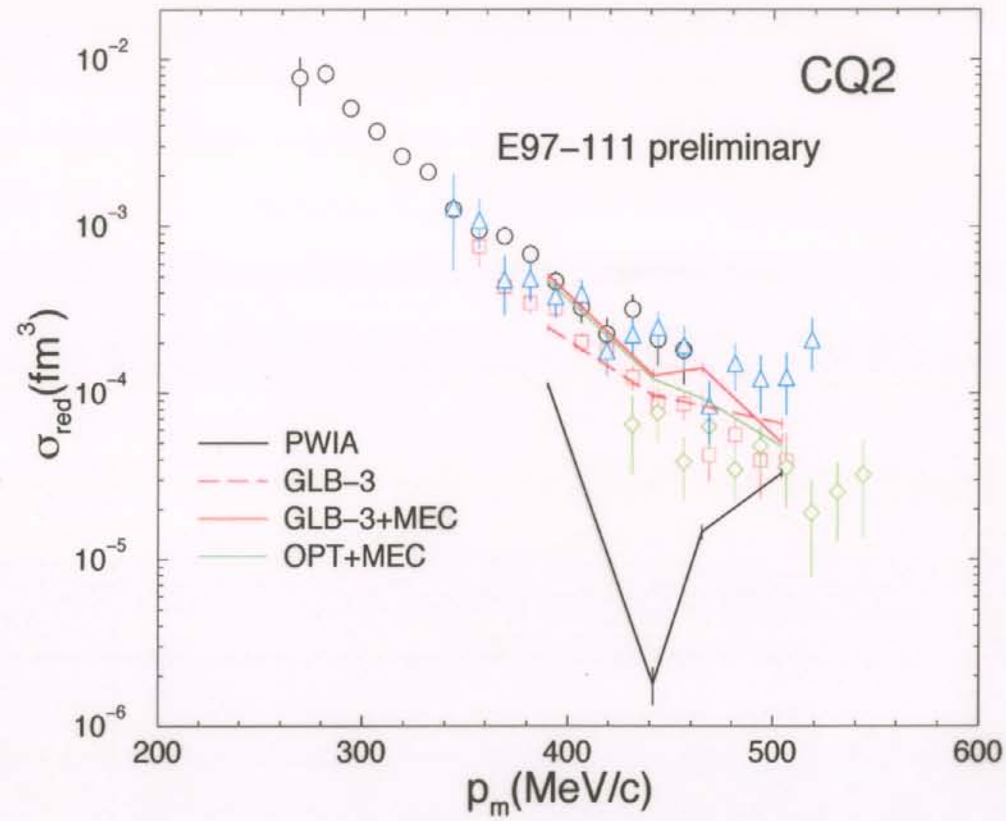


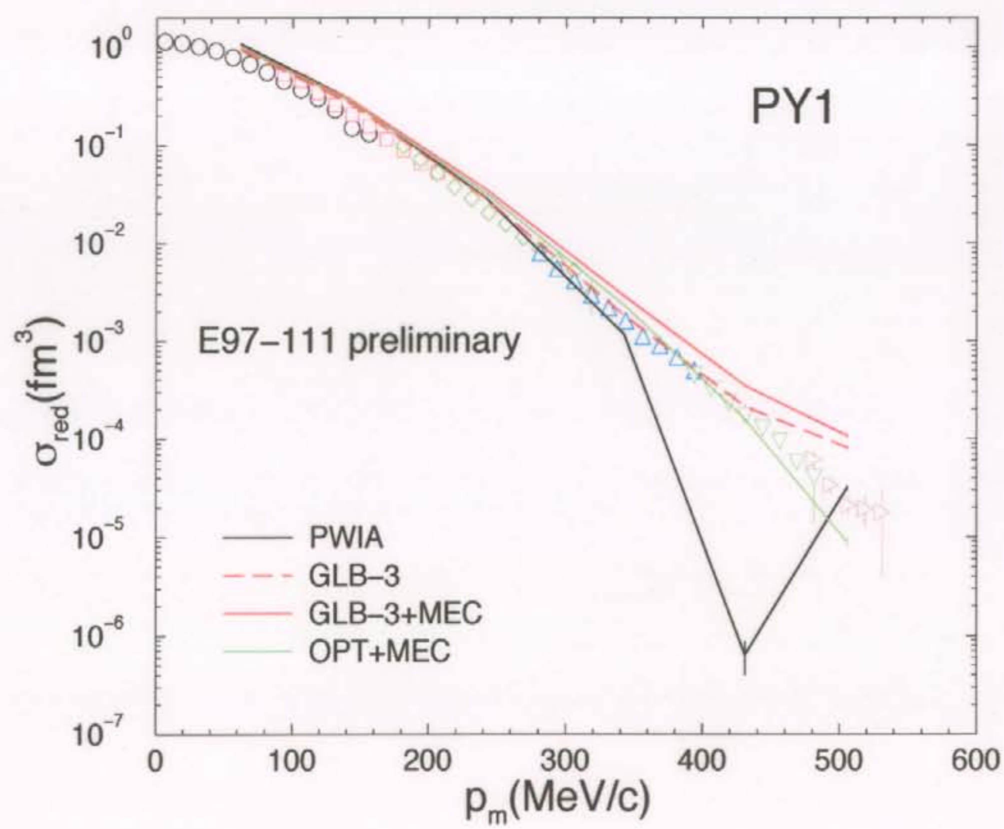
* Carlson, Jourdan, Schiavilla, and Sick, PLB 553 (2003)

COULOMB SUM RULE



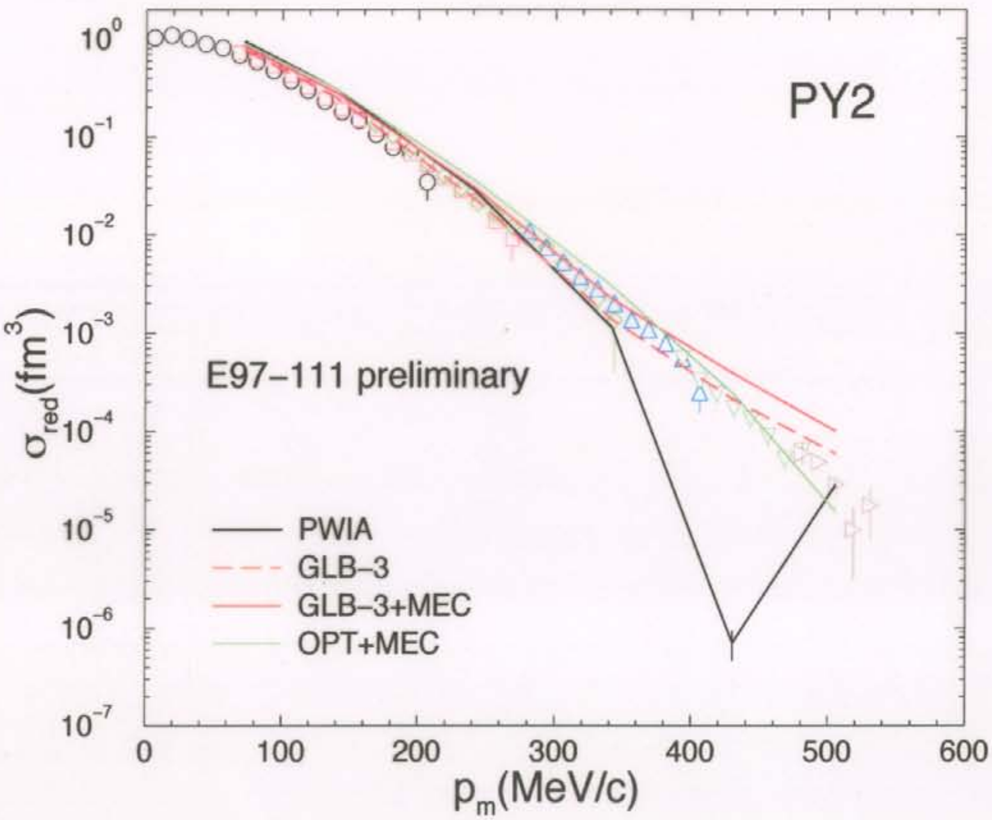
$^4He(e,e'p)^3H$





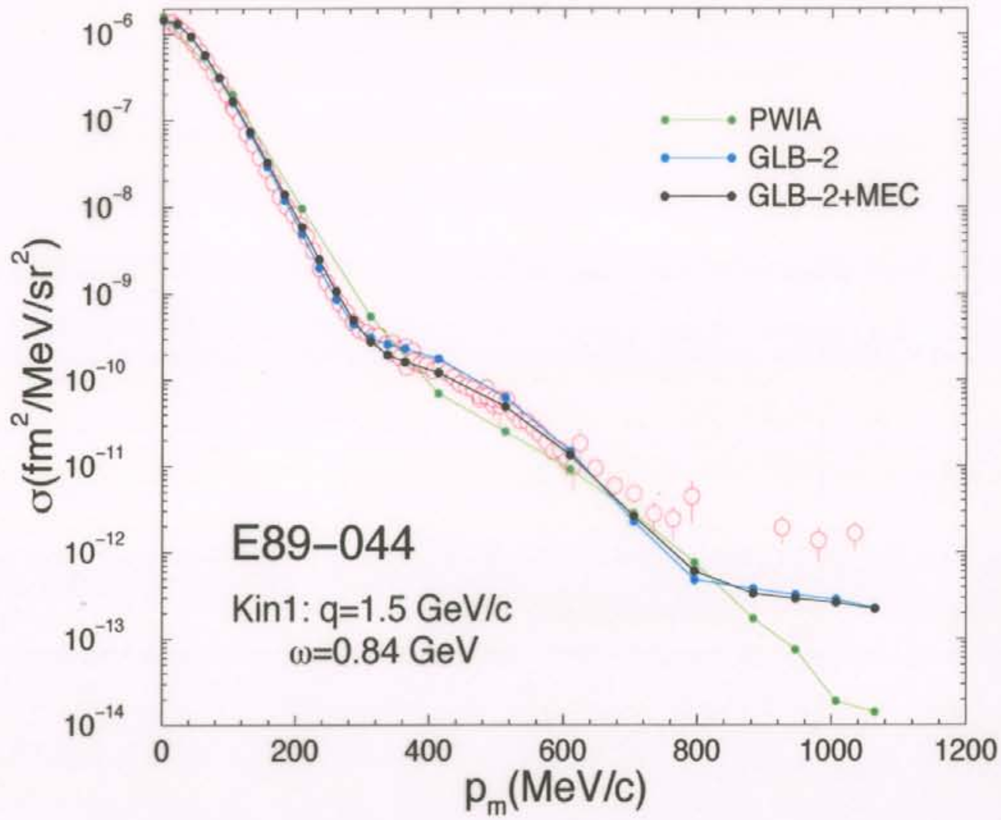
$$\sigma_{red} = \sigma / (p \cdot E_p \cdot \text{recoil} \cdot \sigma_{cc1})$$

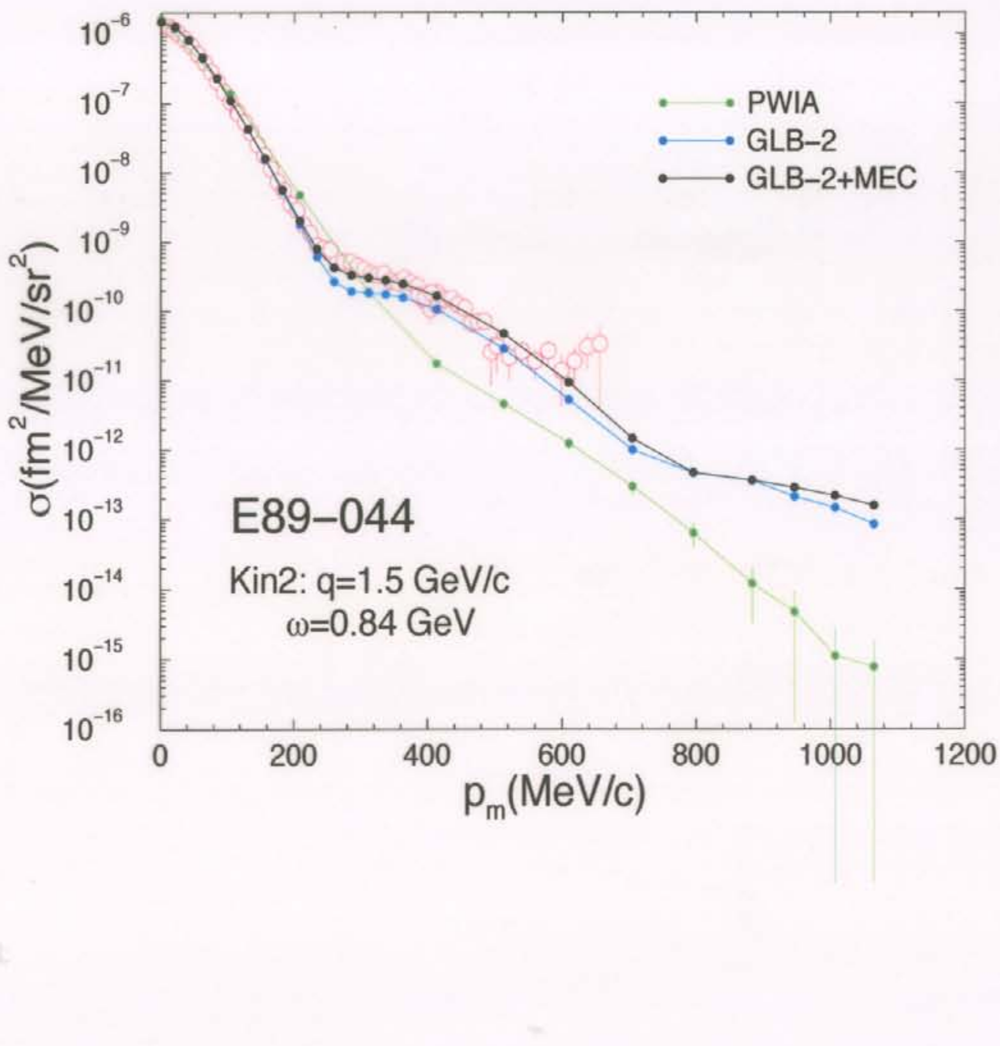
$^4He(e,e'p)^3H$



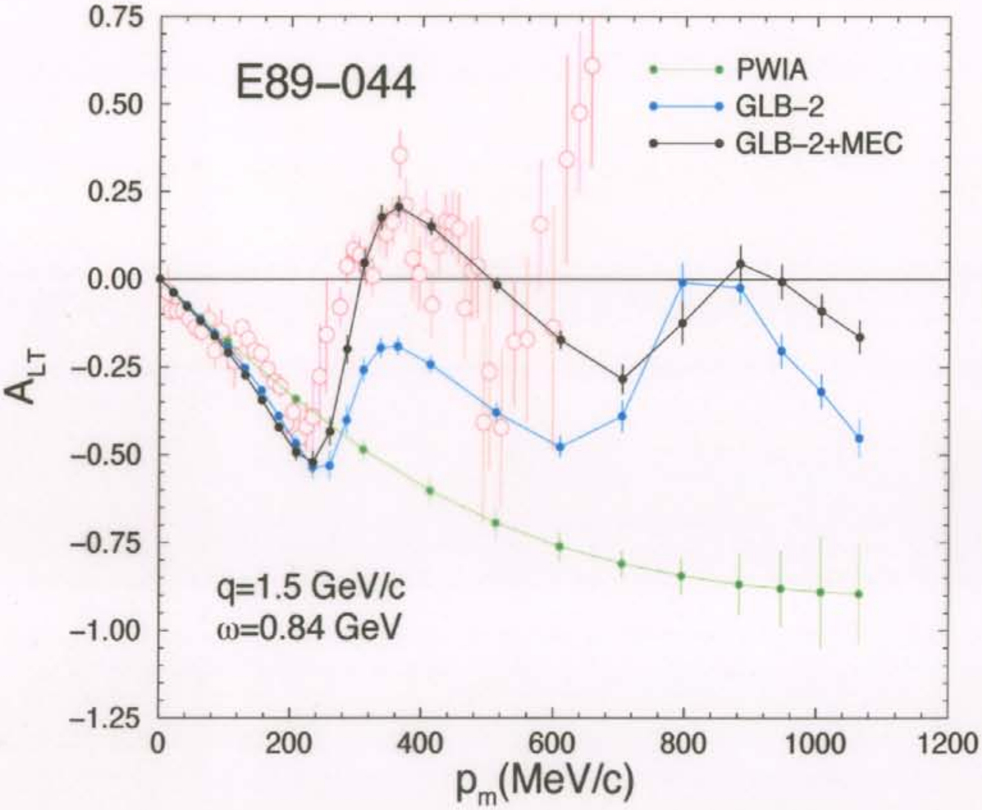
$$\sigma_{red} = \sigma / (p \cdot E_p \cdot \text{recoil} \cdot \sigma_{cc1})$$

$^3He(e,e'p)d$

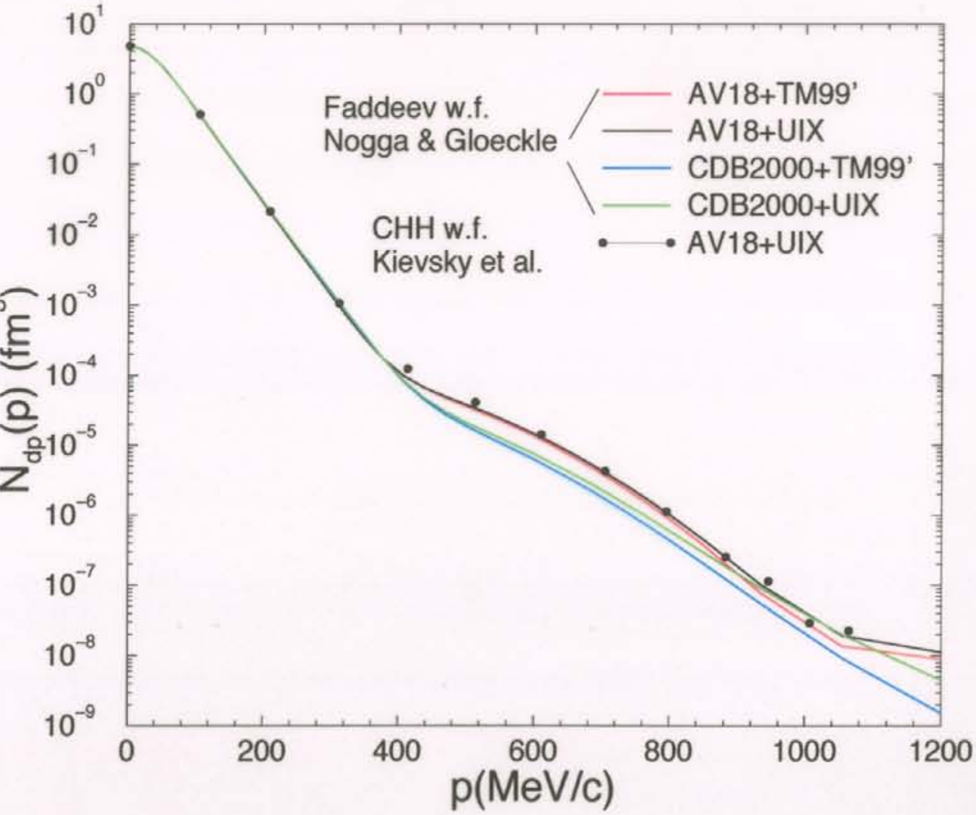

PWIA
GLB-2
GLB-2+MEC
E89-044
Kin1: q=1.5 GeV/c
ω=0.84 GeV
σ(fm²/MeV/sr²)
p_m(MeV/c)


PWIA
GLB-2
GLB-2+MEC
E89-044
Kin2: q=1.5 GeV/c
ω=0.84 GeV
σ(fm²/MeV/sr²)
p_m(MeV/c)

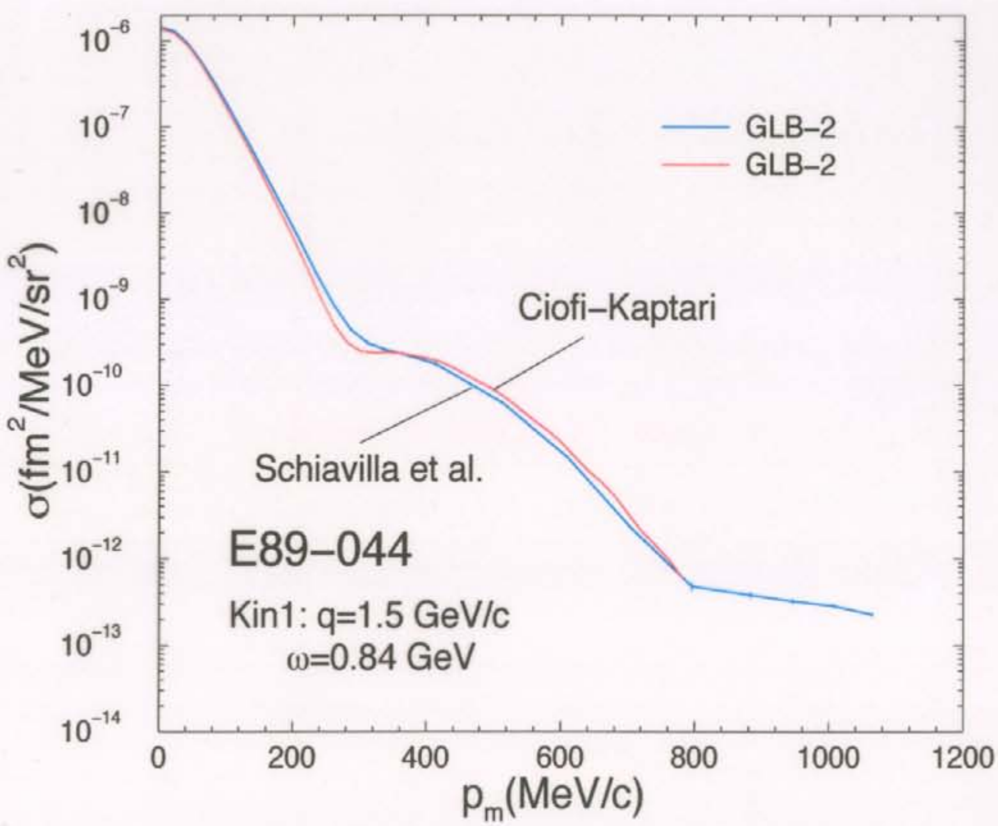
$^{3}He(e,e'p)d$



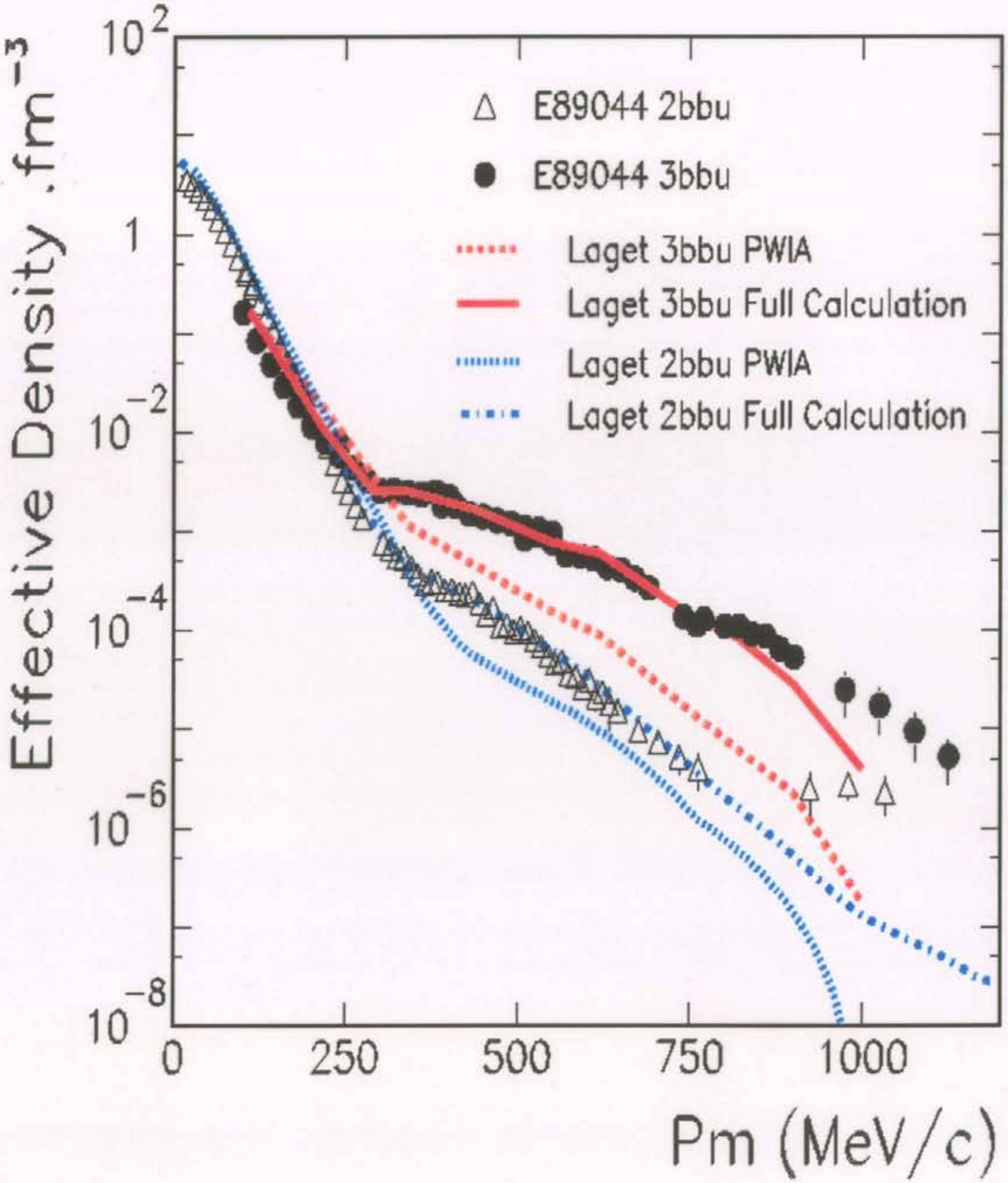
$$A_{LT} = \frac{\sigma(\phi=0) - \sigma(\phi=180°)}{\sigma(\phi=0) + \sigma(\phi=180°)} \propto R_{LT}$$



Faddeev w.f.
Nogga & Gloeckle
AV18+TM99'
AV18+UIX
CDB2000+TM99'
CDB2000+UIX
CHH w.f.
Kievsky et al.
AV18+UIX
$N_{dp}(p)$ (fm^3)
p(MeV/c)



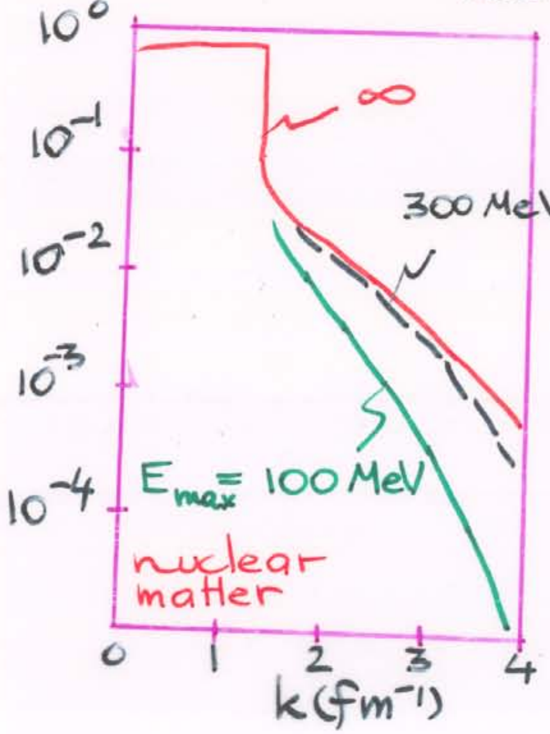
GLB-2
GLB-2
Ciofi–Kaptari
Schiavilla et al.
E89–044
Kin1: q=1.5 GeV/c
ω=0.84 GeV
σ(fm^2/MeV/sr^2)
p_m(MeV/c)

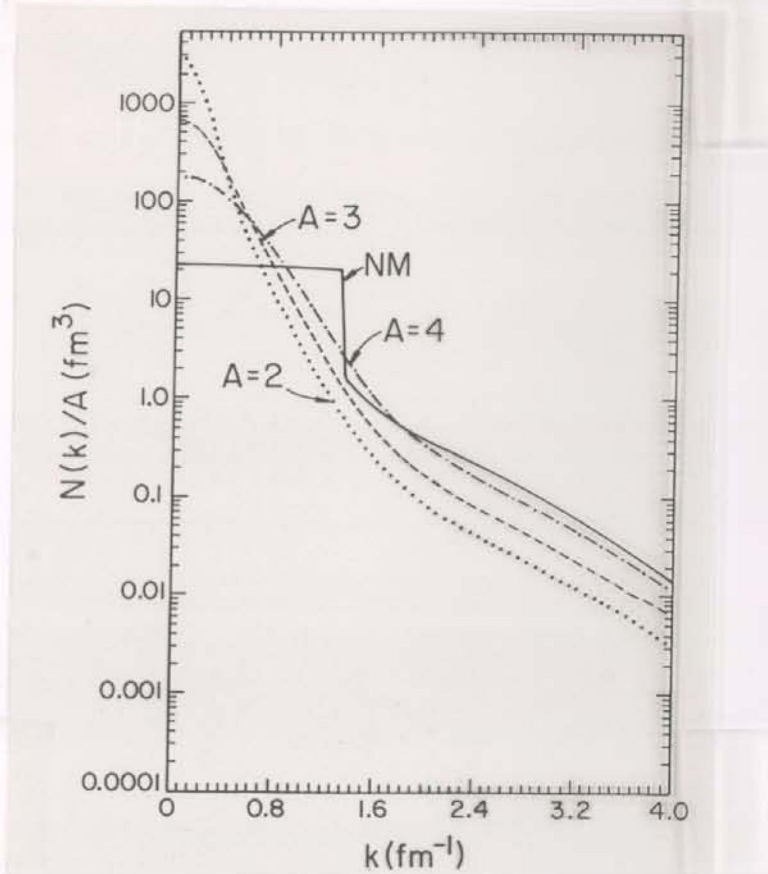


Effective Density .fm^{-3}
10^2
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
250
500
750
1000
Pm (MeV/c)
E89044 2bbu
E89044 3bbu
Laget 3bbu PWIA
Laget 3bbu Full Calculation
Laget 2bbu PWIA
Laget 2bbu Full Calculation

Momentum distribution: $N(k) = \langle 0 | a_k^\dagger a_k | 0 \rangle$

$$N(k) = \lim_{E_{max} \to \infty} \int_{E_{th}}^{E_{max}} dE \, S(k,E)$$





- high momentum strength is located at large removal energy
- high momentum tails of $N(k)$ scale

Summary

- Electromagnetic ground-state structure of few-nucleon systems well reproduced by theory

Will this agreement persist as Q^2 is increased?

Magnetic structure	Charge structure
$B(Q^2)$	$A(Q^2)$, $T_{20}(Q^2)$
$d(e,e')pn$ at threshold	Helium isotopes
$^3H/^3He$	

- FSI effects seem to be under control

i) Charge-exchange mechanism explains low Q^2 measurements of polarization transfer in ^4He

ii) no medium modification of nucleon form factors is required; this is in line with Coulomb sum rule analysis in few-body nuclei

iii) Calculations of FSI effects in quasi-elastic kinematics provide reliable baseline for studying their Q^2 evolution