

Ab-Initio No Core Shell Model and its Relation to Physics with 12 GeV Electrons

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The Physics of Nuclei with 12 GeV Electrons TJNL, November 1-5, 2004

- I. Ab initio approach to many-fermion/boson systems
- II. Results for light nuclei
- III. Links to the quark cluster model of nuclei
- III. Conclusions and outlook

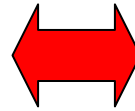
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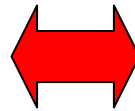
Constructing a non-perturbative bridge between
“Short distance physics”  “Long distance physics”

Asymptotically free current quarks
Chiral symmetry
High momentum transfer processes



Constituent quarks
Broken Chiral symmetry
Meson and Baryon Spectroscopy
Hadron-Hadron interactions

Bare NN, NNN interactions
fitting 2 & 3-body data
Short range correlations &
strong tensor correlations
Bare transition operators



Effective NN, NNN interactions
describing low energy nuclear data
Mean field, pairing, &
quadrupole, etc., correlations
Effective charges, GT quenching, etc.

BOLD CLAIM

**We now have the tools to accomplish this program in
nuclear many-body theory**

The tools are now sufficiently robust to provide precision tests of the Hamiltonians themselves
Argonne-LANL-Urbana (GFMC) pioneered this path

New and Emerging NN, NNN interactions fitting NN and NNN data

- ❖ Traditional meson-exchange theory (Nijmegen **X**, CD Bonn **X**, AV**X**, etc.,)
- ❖ Effective field theory with roots in QCD (EFT **X**, Idaho **X**, N**X**LO, etc.,)
- ❖ Renormalization group reduced bare NN interactions (V-lowk **X**)
- ❖ Off-shell variations of bare NN interactions (INOY-**X**, etc.,)
- ❖ Inverse scattering theory (ISTP, JISP**X**, etc.,)



Once these issues resolved, we have the tools to make high precision predictions for tests of fundamental symmetries in nuclear experiments.

Ab Initio No-Core Shell Model

H and other observables, O,
act in their full infinite Hilbert Space

H_{eff} & O_{eff} of finite subspace

Effective Hamiltonian for A-Particles

Lee-Suzuki-Okamoto Method plus Cluster Decomposition

P. Navratil, J.P. Vary and B.R. Barrett,
Phys. Rev. Lett. 84, 5728(2000); Phys. Rev. C62, 054311(2000)
C. Viazminsky and J.P. Vary, J. Math. Phys. 42, 2055 (2001);
K. Suzuki and S.Y. Lee, Progr. Theor. Phys. 64, 2091(1980);
K. Suzuki, *ibid*, 68, 246(1982);
K. Suzuki and R. Okamoto, *ibid*, 70, 439(1983)

**Preserves the symmetries of the full Hamiltonian:
Rotational, translational, parity, etc., invariance**

$$H_{\mathcal{A}} = T_{rel} + V = \sum_{i < j}^{\mathcal{A}} \left[\frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + V_{ij} \right] + V_{NNN}$$

Select a finite oscillator basis space (P-space) and evaluate an a - body cluster effective Hamiltonian:

$$\mathcal{H} = T_{rel} + \mathcal{V}^{(a)}$$

Guaranteed to provide exact answers as $a \rightarrow A$ or as $P \rightarrow 1$.

Introduce similarity transformation
acting only on the intrinsic coordinates
[Lee-Suzuki-Okamoto]

$$H_A |k\rangle = E_k^A |k\rangle$$

$$\mathcal{H} = e^{-S_A} H_A e^{S_A} = e^{-S_A} [H_A^\Omega - H_{CM}] e^{S_A} = \mathcal{H}^{(1)} + \mathcal{H}^{(A)} - H_{CM}$$

$$\mathcal{H}^{(1)} = H_0 = \sum_i^A h_i = \sum_i^A \left(\frac{p_i^2}{2m} + \frac{1}{2} m \Omega^2 r_i^2 \right)$$

$$|\tilde{k}\rangle = e^{-S_A} |k\rangle$$

$$\mathcal{H} |\tilde{k}\rangle = E_k^A |\tilde{k}\rangle$$

Nothing has happened to the original problem

Define a normalized A - Particle Fock - space basis state:

$$\mathbf{a}_\alpha^+ \mathbf{a}_\beta^+ \cdots \mathbf{a}_\zeta^+ |0\rangle \equiv |\alpha\beta\cdots\zeta\rangle \equiv |\alpha_n\rangle$$

$$H_0 |\alpha_n\rangle = \varepsilon_n |\alpha_n\rangle$$

$$\varepsilon_n \equiv \{\varepsilon_\alpha + \varepsilon_\beta + \cdots + \varepsilon_\zeta\}$$

Define the A - particle basis space (P - space) to consist of all basis states $\{ |\alpha_n\rangle \}$ satisfying:

$$\varepsilon_n \leq \{N_m + 3A / 2\} \hbar\Omega$$

$$N_m \equiv N_{\min} + N_{\max}$$

For A - Fermions $N_{\min} \equiv$ Minimum oscillator quanta $[\sum (2n + 1)]$ satisfying the Pauli Principle.

We characterize the P - space by quoting " $N_{\max} \hbar\Omega$ "

Notation: $\{ |\alpha_n\rangle \}_P \equiv \{ |\alpha_P\rangle \}$

Define the finite basis (P-space) effective Hamiltonian

Introduce A-fermion basis space projectors P and Q

$$P_A = \sum_{P \in \mathcal{P}} |\alpha_P\rangle\langle\alpha_P|$$

$$Q_A = \sum_{Q \in \mathcal{Q}} |\alpha_Q\rangle\langle\alpha_Q|$$

$$P_A + Q_A = 1_A$$

$$P_A^2 = P_A \quad Q_A^2 = Q_A \quad P_A Q_A = 0$$

Require the similarity transform to satisfy decoupling condition
=> separate Hermitian effective Hamiltonians for P&Q spaces

$$[P_{\mathcal{A}} + Q_{\mathcal{A}}] \mathcal{H} [P_{\mathcal{A}} + Q_{\mathcal{A}}] |\tilde{k}\rangle = E_k^{\mathcal{A}} [P_{\mathcal{A}} + Q_{\mathcal{A}}] |\tilde{k}\rangle$$

$$P_{\mathcal{A}} \mathcal{H} Q_{\mathcal{A}} = 0 = Q_{\mathcal{A}} \mathcal{H} P_{\mathcal{A}}$$

$$\Rightarrow P_{\mathcal{A}} \mathcal{H} P_{\mathcal{A}} |\tilde{k}\rangle = E_k^{\mathcal{A}} P_{\mathcal{A}} |\tilde{k}\rangle, \quad k = 1, \dots, d$$

$$\Rightarrow Q_{\mathcal{A}} \mathcal{H} Q_{\mathcal{A}} |\tilde{k}\rangle = E_k^{\mathcal{A}} Q_{\mathcal{A}} |\tilde{k}\rangle, \quad k = d+1, \dots, \infty$$

Still no approximations to the original problem

Formal solution to the full decoupling problem

Instructive for the cluster approximation

$$H_{\mathcal{A}}|k\rangle = E_{\hat{k}}^{\mathcal{A}}|k\rangle$$

$$|\tilde{k}\rangle = e^{-S_{\mathcal{A}}}|k\rangle$$

$$\mathcal{H} = e^{-S_{\mathcal{A}}} H e^{S_{\mathcal{A}}}$$

$$S_{\mathcal{A}} = \operatorname{arctanh}(\omega - \omega^T)$$

$$\omega = Q_{\mathcal{A}} \omega P_{\mathcal{A}}$$

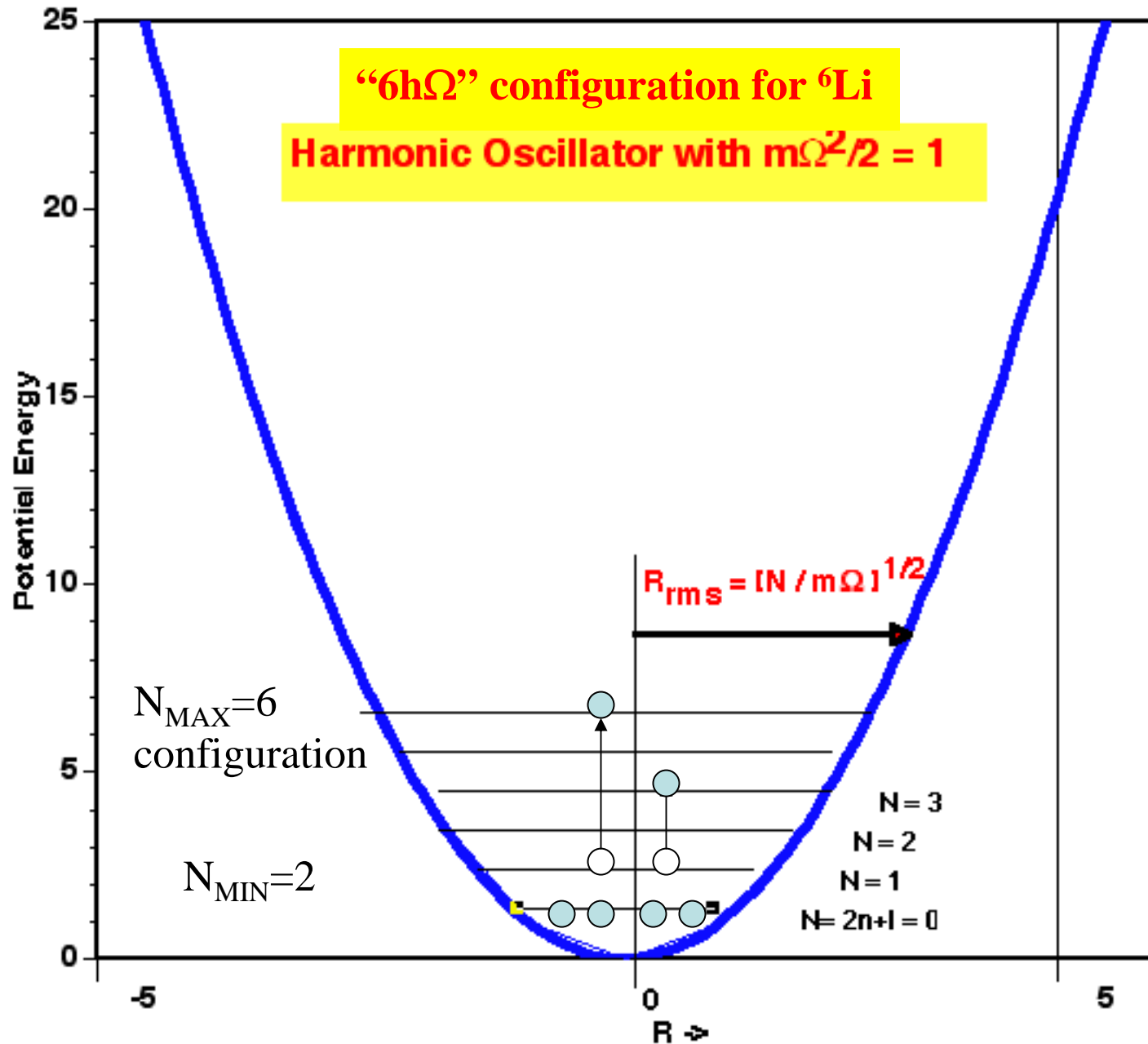
$$\langle \alpha_Q | \omega | \alpha_P \rangle = \sum_{k \in K} \langle \alpha_Q | k \rangle \langle \hat{k} | \alpha_P \rangle$$

$$\text{where: } \langle \hat{k} | \alpha_P \rangle = \text{Inverse}\{\langle k | \alpha_P \rangle\}$$

Provides formal solution in a convenient form

$$\mathcal{H} = (P_{\mathcal{A}} + \omega^T \omega)^{-1/2} (P_{\mathcal{A}} + P_{\mathcal{A}} \omega^T Q_{\mathcal{A}}) H_{\mathcal{A}} (Q_{\mathcal{A}} \omega P_{\mathcal{A}} + P_{\mathcal{A}}) (P_{\mathcal{A}} + \omega^T \omega)^{-1/2}$$

*The same transformation applies to other observables.
Still nothing has happened to the original problem except
a similarity transformation which preserves all observables*



Motivation for this definition of P-space

- 1.** All states up to cutoff (N_m) in unperturbed quanta are retained. Thus, the more realistic our choice of H_0 , the more rapidly we expect convergence with cutoff.
- 2.** We can exactly retain translational invariance in this overcomplete basis ($3A$ degrees of freedom vs. $3A-3$ intrinsic) using a Lagrange multiplier (Lipkin-Lawson projection method):

$$H \rightarrow H + \lambda(H_{CM} - \frac{3}{2}\hbar\Omega)$$

With H acting only on intrinsic coordinates and λ chosen sufficiently large, we obtain a spectroscopy:

$$\begin{array}{l}
 \text{===== } N_{cm} = 2 \\
 \updownarrow \lambda\hbar\Omega \\
 \text{===== } N_{cm} = 1 \\
 \updownarrow \lambda\hbar\Omega \\
 \text{===== } N_{cm} = 0
 \end{array}
 \left. \begin{array}{l}
 \right\} \text{States with spurious CM motion} \\
 \\
 \left. \begin{array}{l}
 \right\} \text{Physically relevant states}
 \end{array}
 \right.$$

Cluster Approximation

Assume the model-space \mathbf{A} -body effective Hamiltonian is a superposition of \mathbf{a} -body cluster effective Hamiltonians

$$\mathcal{H} \approx \mathcal{H}^{(1)} + \mathcal{H}^{(a)} - H_{CM}$$
$$= \mathcal{H}^{(1)} + \frac{\binom{\mathcal{A}}{2}}{\binom{\mathcal{A}}{a} \binom{a}{2}} \sum_{i_1 < i_2 < i_3 < \dots < i_a}^{\mathcal{A}} \tilde{V}_{i_1 i_2 i_3 \dots i_a} - H_{CM}$$

$$\tilde{V}_{123\dots a} = e^{-S_a} H_{a,A}^{\Omega} e^{S_a} - \sum_i^a h_i$$

Key equations to solve at the a-body cluster level

Solve a cluster eigenvalue problem in a very large but finite harmonic oscillator basis and retain all the symmetries of the bare Hamiltonian

$$P_a = \sum_{P \in \mathcal{P}} |\alpha_P\rangle\langle\alpha_P|$$

$$Q_a = \sum_{Q \in \mathcal{Q}} |\alpha_Q\rangle\langle\alpha_Q|$$

$$P_a + Q_a \approx 1_a$$

$$H_{a,A}^\Omega |k\rangle = E_k |k\rangle$$

$$\langle\alpha_Q|\omega|\alpha_P\rangle = \sum_{k \in K} \langle\alpha_Q|k\rangle\langle\hat{k}|\alpha_P\rangle$$

$$\text{where } \langle\hat{k}|\alpha_P\rangle = \text{Inverse } \{\langle k|\alpha_P\rangle\}$$

$$\mathcal{H}^{(a)} = (P_a + \omega^T \omega)^{-1/2} (P_a + P_a \omega^T Q_a) H_{a,A}^\Omega (Q_a \omega P_a + P_a) (P_a + \omega^T \omega)^{-1/2}$$

Ab-initio no-core shell model (NCSM) results

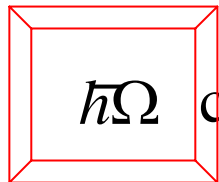
$$a = 2 \text{ for } 2 < A < 16, 47, 48, 49$$
$$a = 3 \text{ for } 3 < A < 16$$

Results in a harmonic oscillator basis space

P_A characterized by:

$$N_m = \sum_{i \in \alpha_P} (2n_i + l_i) \leq N_{\min} + N_{\max}$$

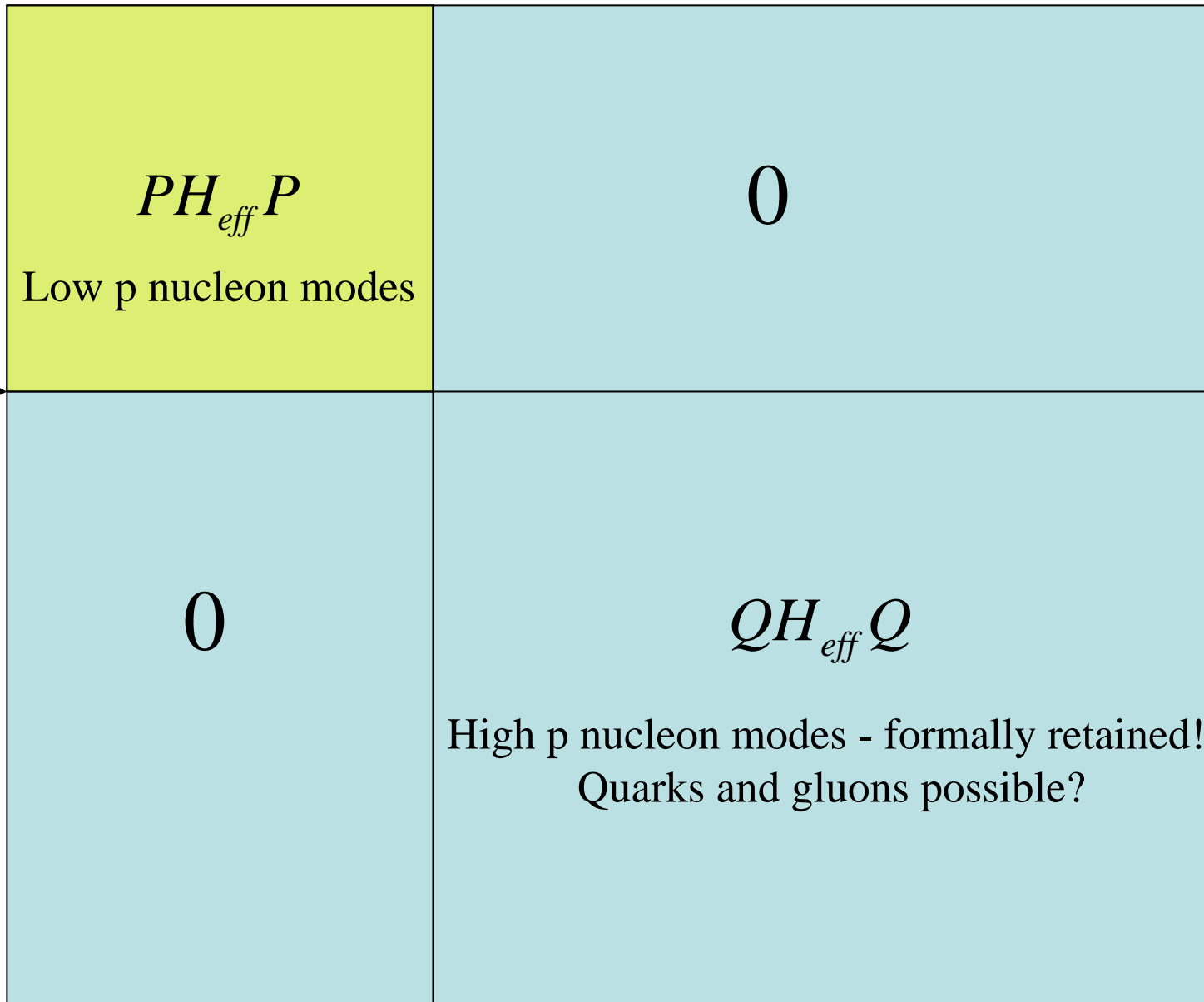
controls
momentum
scale limit

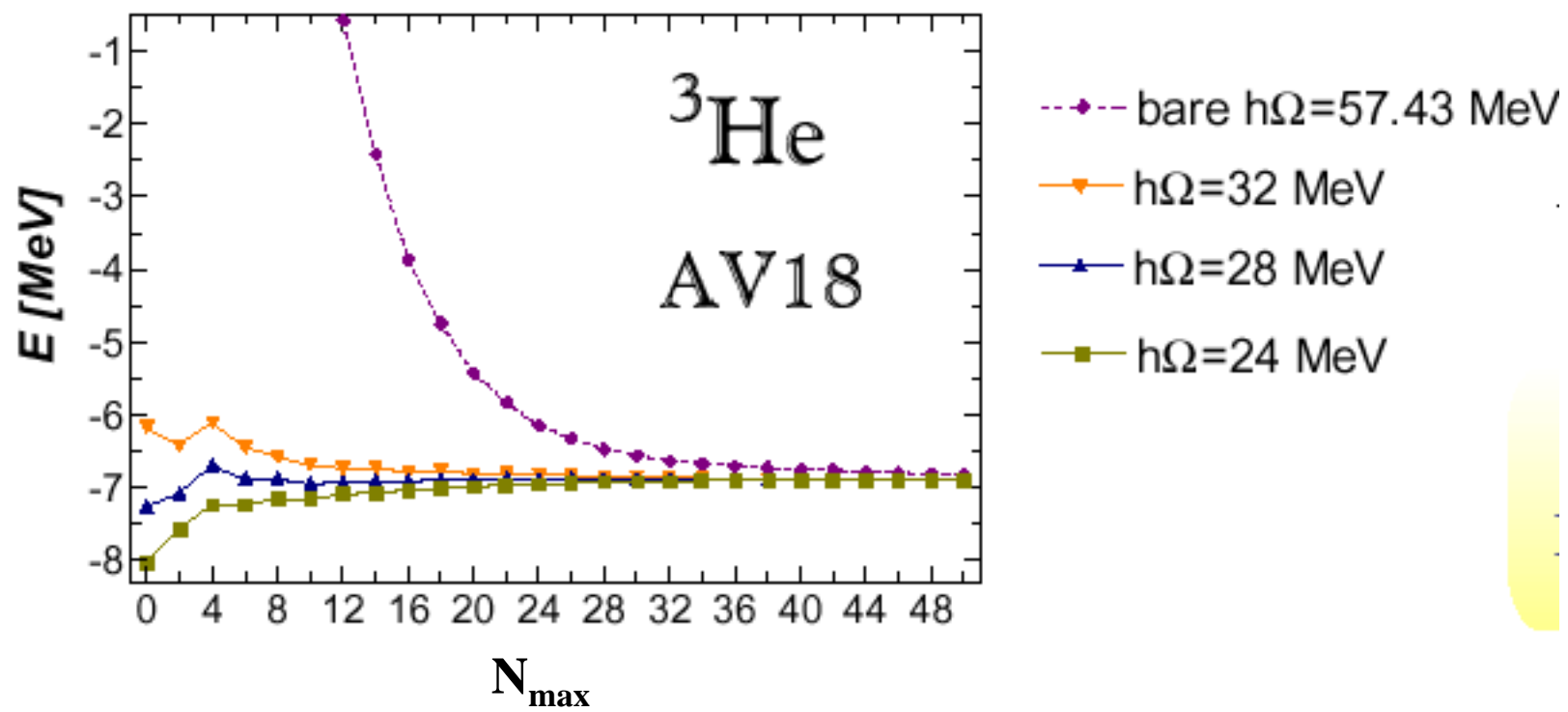


$\hbar\Omega$ controls length scale of basis functions (“box”)

Results compare well with VMC and GFMC results

Transformed system decouples low and high-momentum states



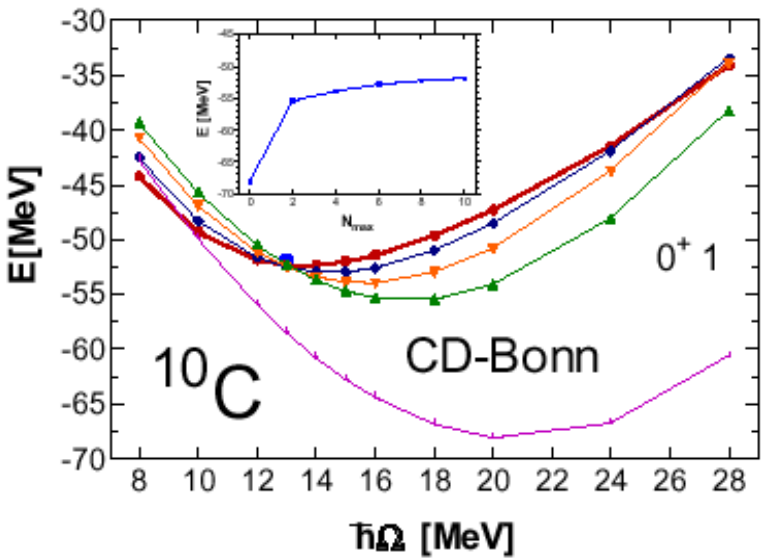


Isospin mixing correction of $^{10}\text{C} \rightarrow ^{10}\text{B}$ Fermi matrix element

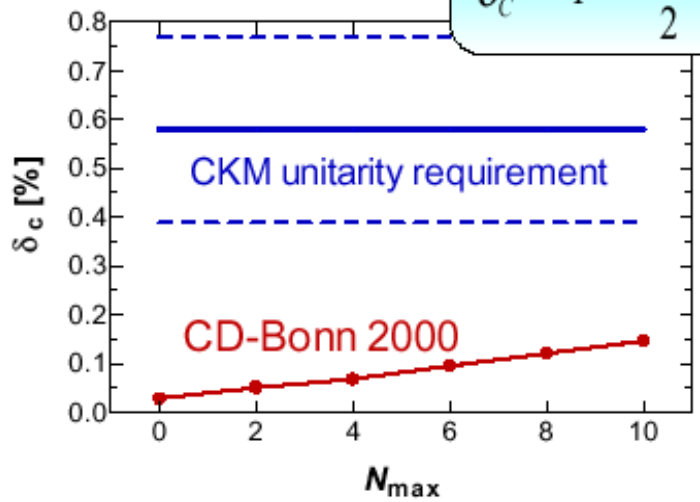


Relevant to CKM matrix unitarity

$$\delta_C = 1 - \frac{M_F^2}{2}$$



- $N_{\max}=0$
- $N_{\max}=2$
- $N_{\max}=4$
- $N_{\max}=6$
- $N_{\max}=8$
- $N_{\max}=10$

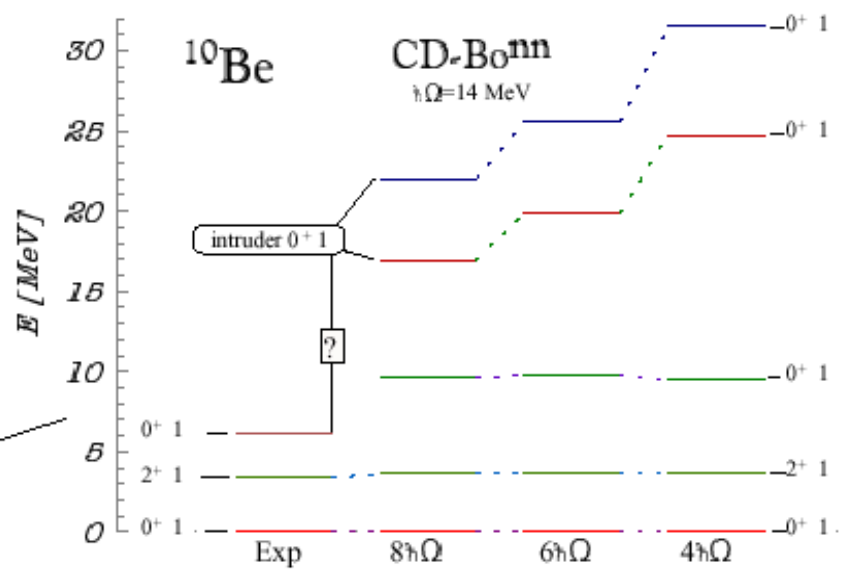


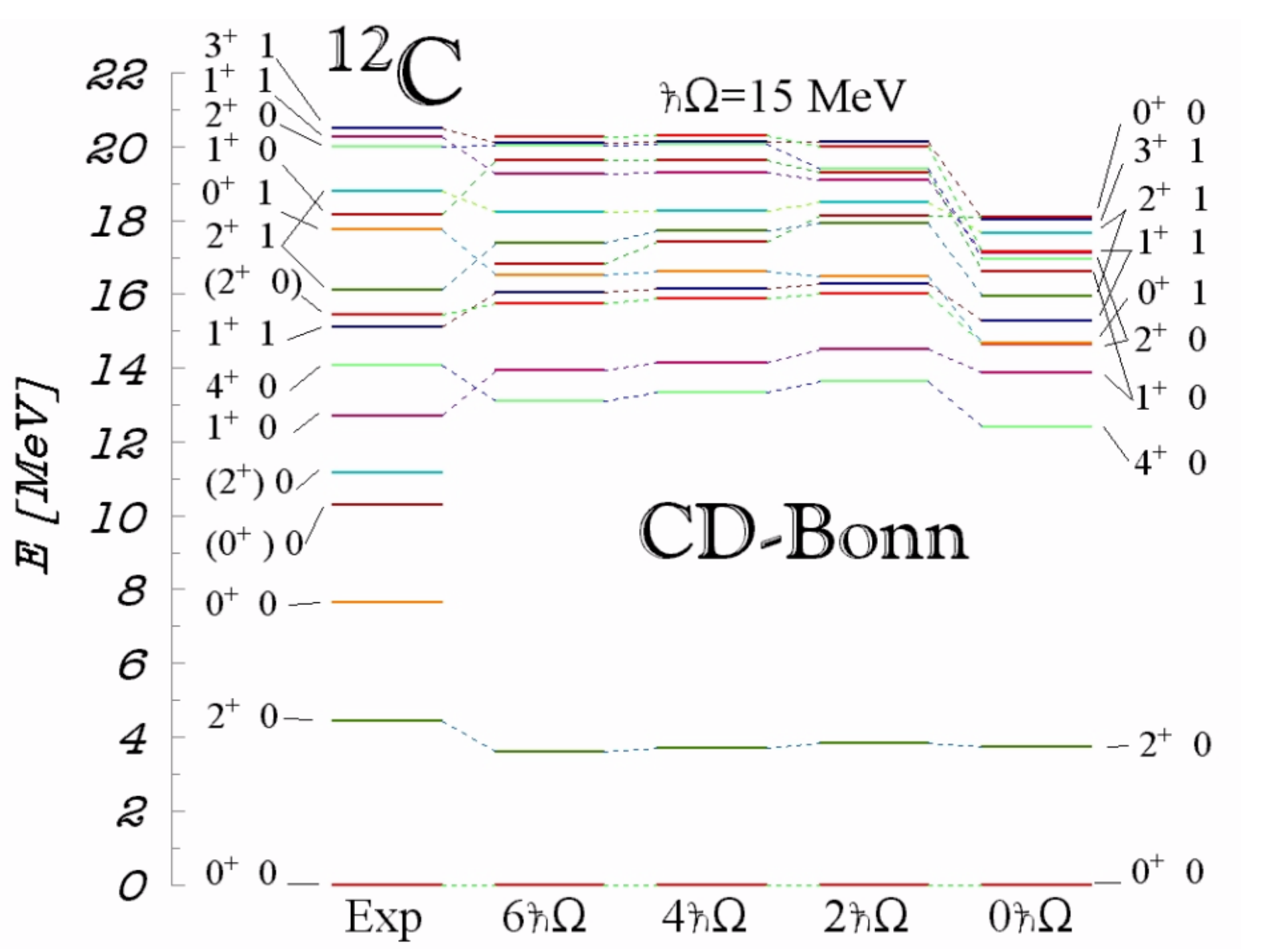
- ^{10}C and ^{10}B $0^+ 1$ calculations up to $10\hbar\Omega$
- ▶ Code Antoine (E. Caurier) - dimension 8×10^8

$$M_F = \langle ^{10}\text{B}; 0^+ 1 | T_- | ^{10}\text{C}; 0^+ 1 \rangle$$

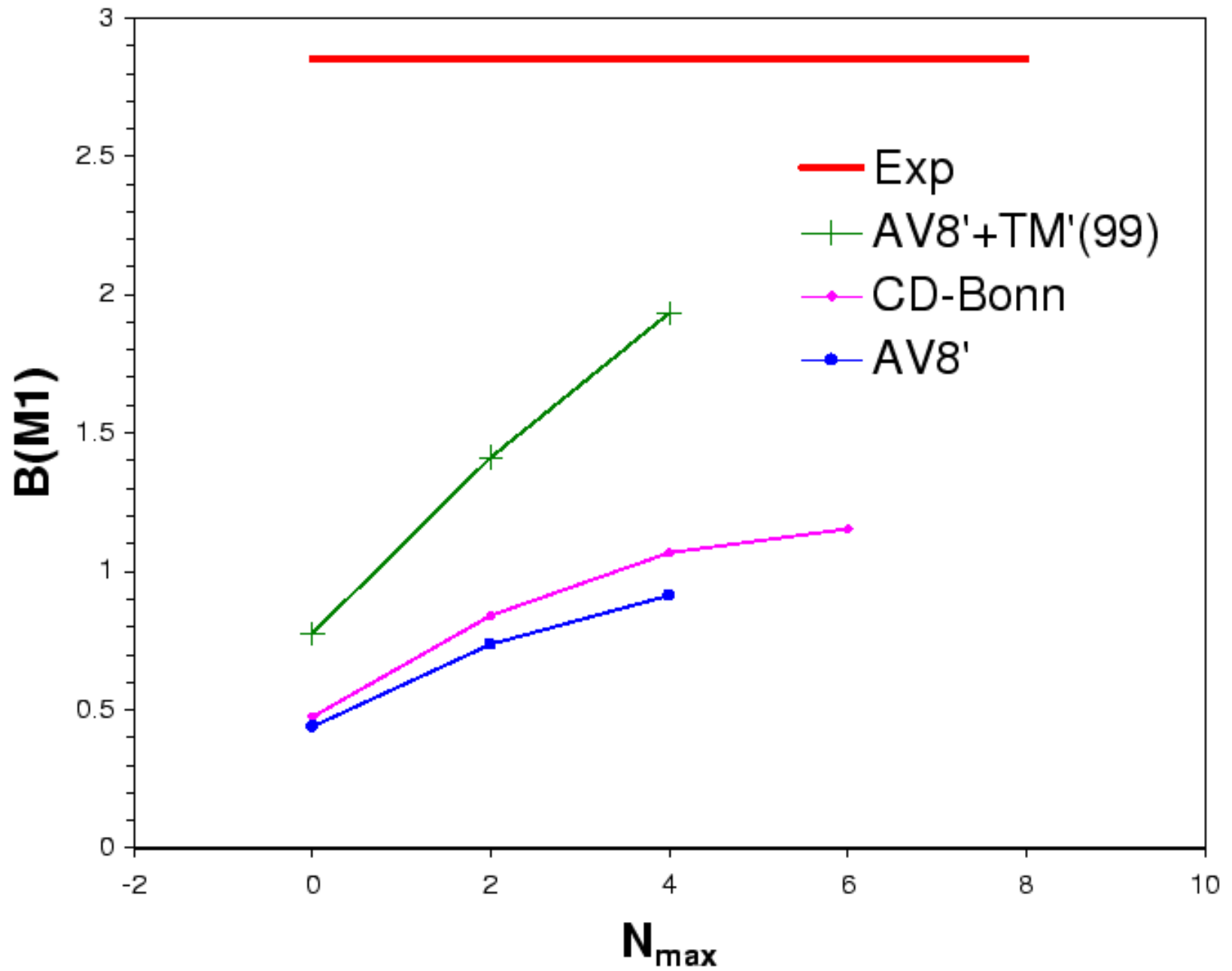
- ▶ $\approx \sqrt{2}$ for an isospin invariant system
- Needed to extract v_{ud} m.e. from experimental Fermi β - decay ft values

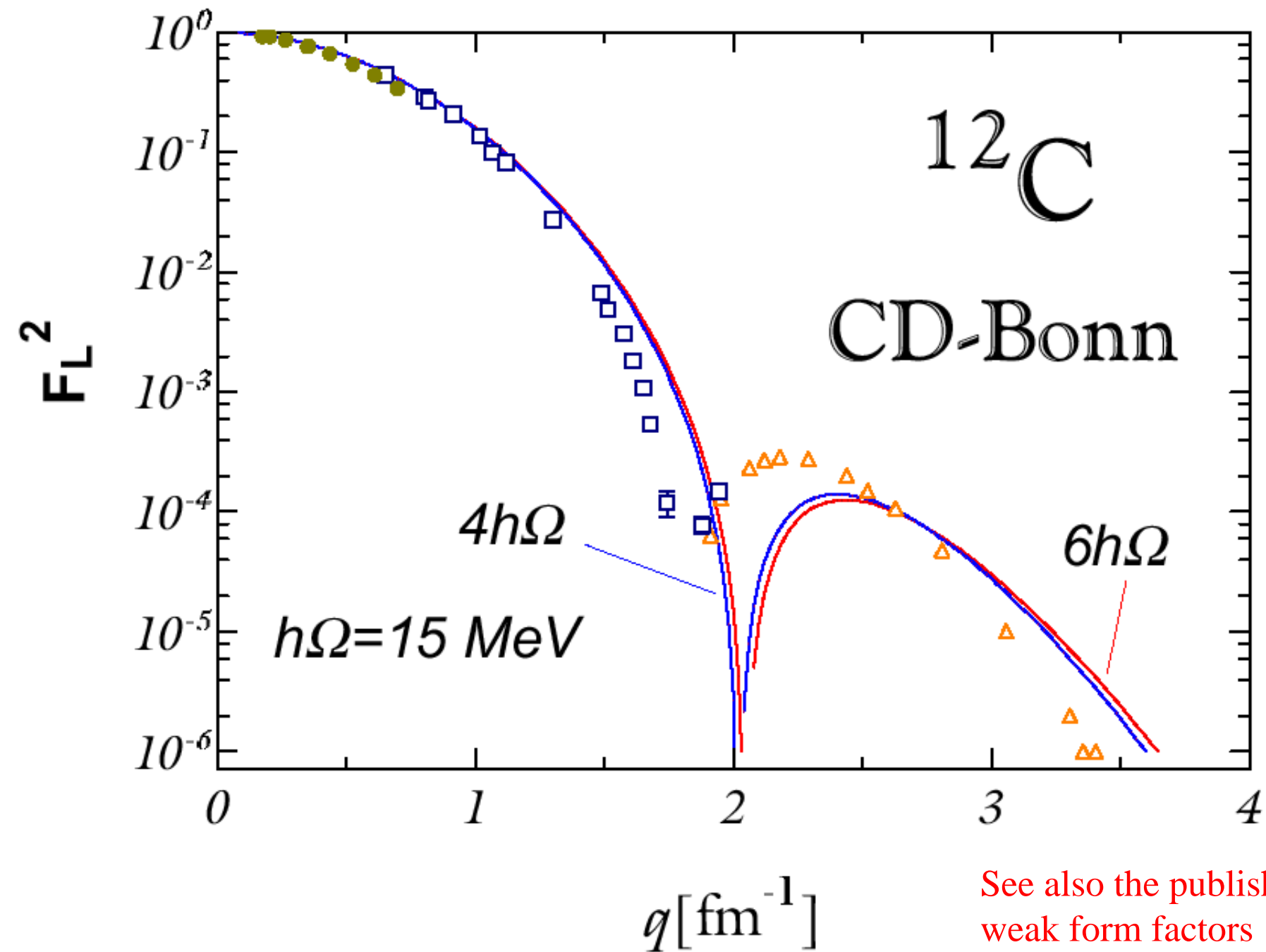
Extrapolated value: $\delta_C \approx 0.19\%$
Insufficient for CKM unitarity





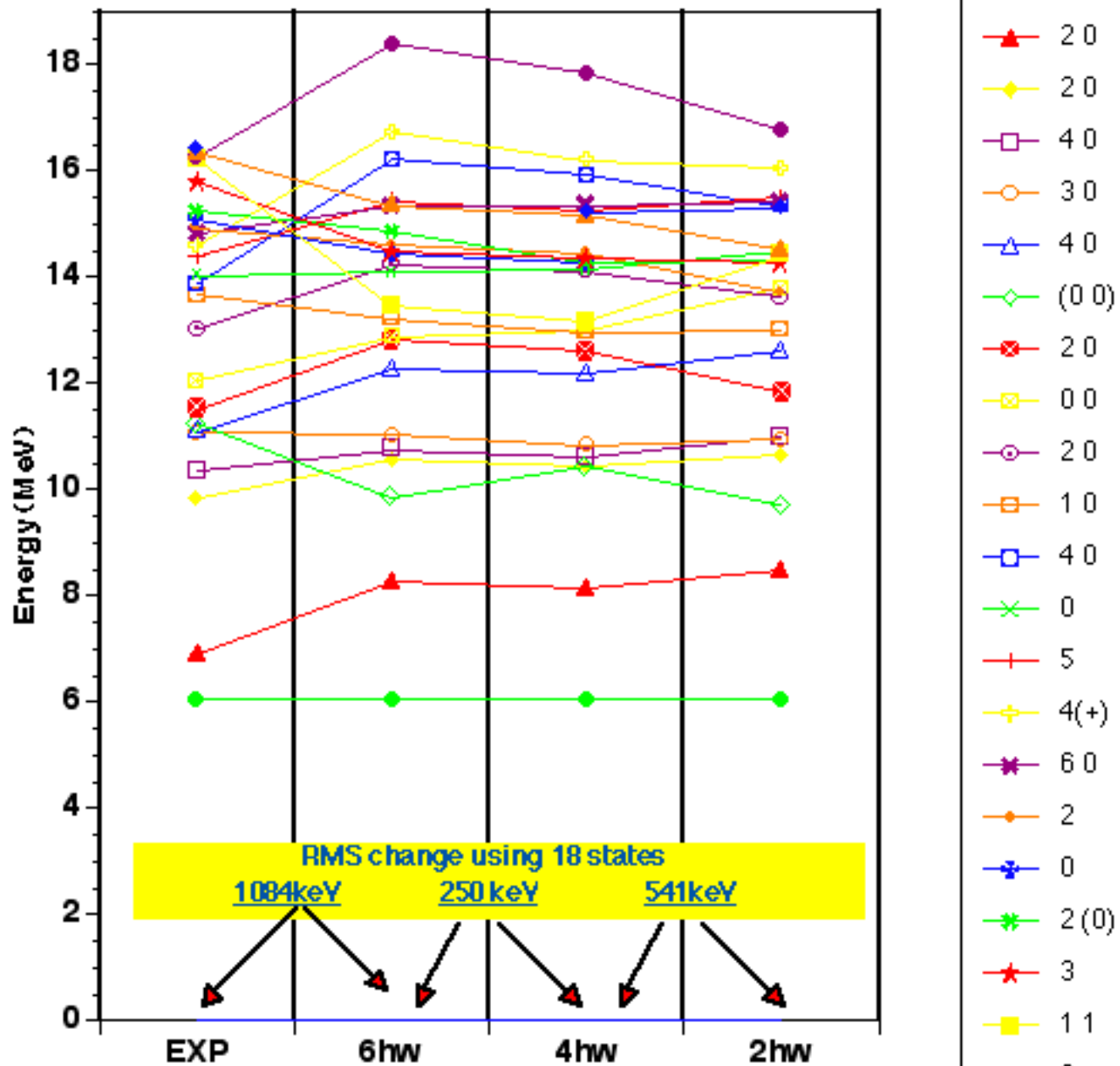
^{12}C B(M1; $0^+ 0 \rightarrow 1^+ 1$)



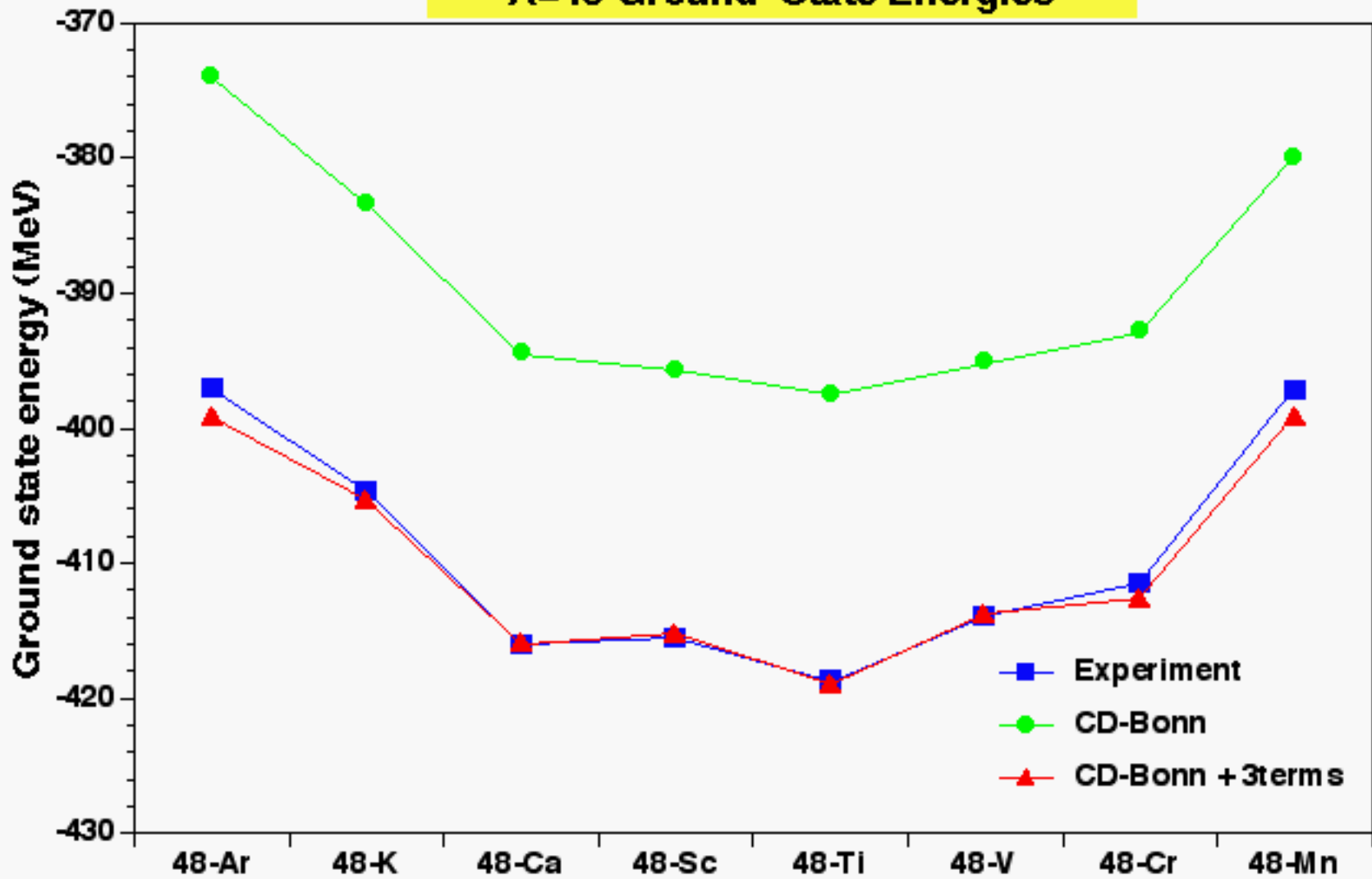


Results from our ^{16}O investigations were shown here. However, as most are not yet published, except for the one spectrum that follows, I refrain from posting them on the web.

16-O Excitation spectrum with CDBonn-2K, hw = 15 MeV
All excitation energies relative to first excited 0+ state



A=48 Ground State Energies

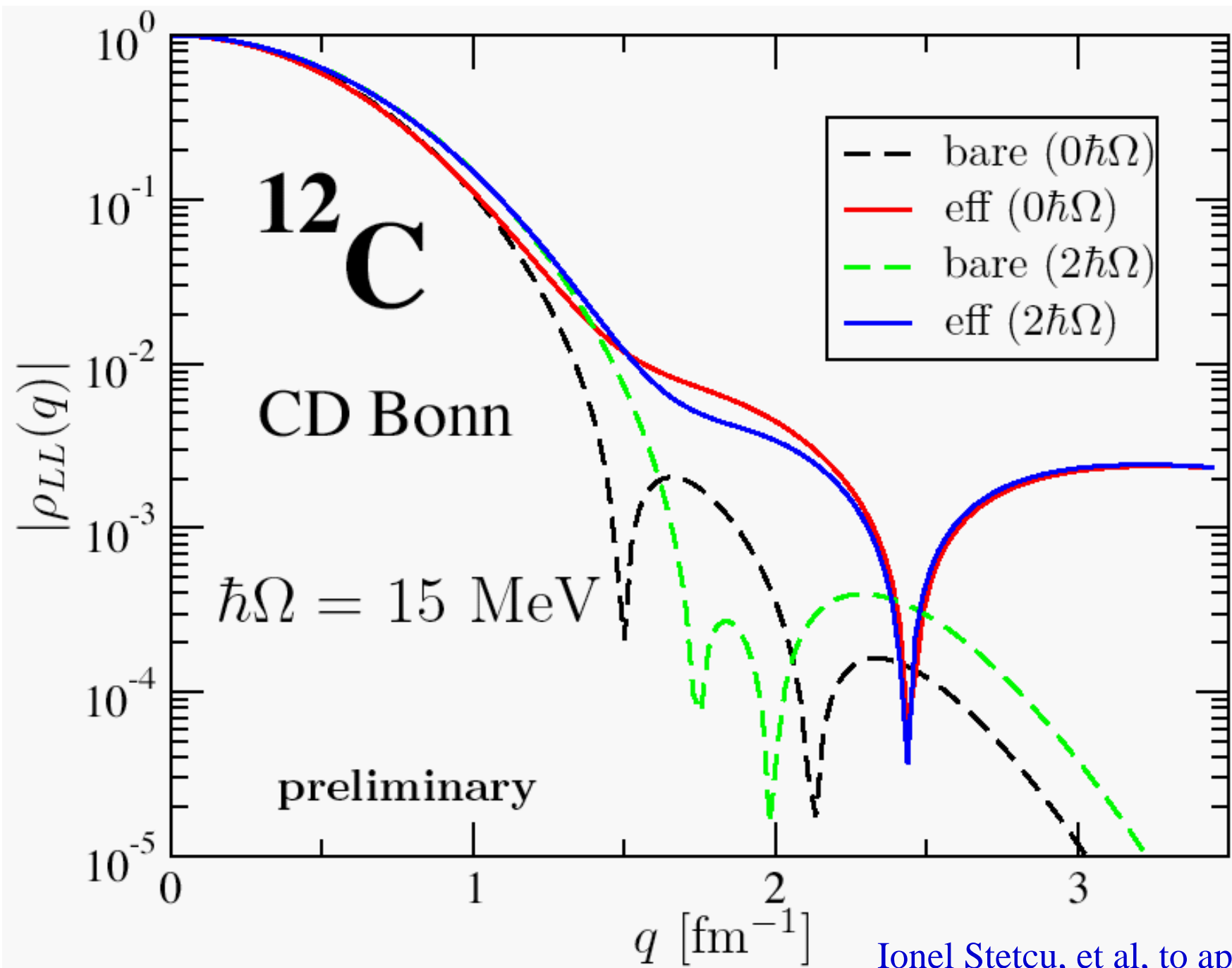


Higher energy observables

Consider the Longitudinal-Longitudinal (LL) response function, sensitive to short-range NN correlations and to our effective operator treatment. Our translational invariance is key attribute for the result.

Take simple test case - spin & isospin averaged LL response:

$$\begin{aligned} |\rho_{LL}(q)| &= \left| \frac{2}{A(A-1)} \sum_{i < j} \int d^3 r_i d^3 r_j \langle \Psi_0 | e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} | \Psi_0 \rangle \right| \\ &= \left| \frac{2}{A(A-1)} \sum_{i < j} \int d^3 r_i d^3 r_j \langle \tilde{\Psi}_0 | e^{-S_a} e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} e^{S_a} | \tilde{\Psi}_0 \rangle \right| \end{aligned}$$



How can we imagine moving to even higher momenta and energies as appropriate to JLAB with high intensity 12 GeV beams?

Return to basic theory, e.g. QCD, and start all over - Hard
OR

Retain the P-Q division with the dynamics in the Q-space derived in “more fundamental” degrees of freedom - Hard

***** But SJB 8/20/2004: “Just do it” *****

How do we use successful low-energy theory to constrain/complement higher-energy theory/experiment?

Until one of these hard problems is solved by a community-wide effort, let us examine a model, the “Quark Cluster Model” which was developed for use with NCSM wavefunctions.

Consistency requires that we work with intrinsic coordinates exploiting our translational invariance and all observables are treated as local operators - coordinate space many-body wavefunctions have a new P & Q space interpretation as shown in the following pictures.

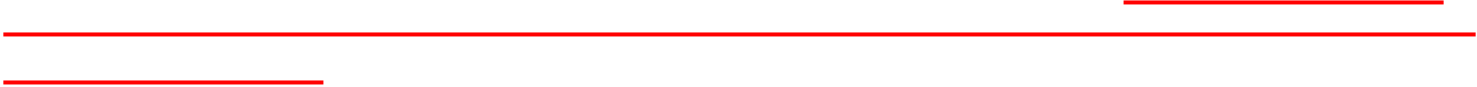
Additional motivation comes from the uncertainty at short distances which has been integrated out in the effective Hamiltonian method

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.



QuickTime™ and a
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are needed to see this picture.

QuickTime™ and a
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are needed to see this picture.



QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Each 3q cluster is separated from another by $d < 2R_c$

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are needed to see this picture.

Snapshots of the A=3 probability density

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TIFF (Uncompressed) decompressor
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Orthonormal state decomposition from configurations to single quark amplitudes:
from configurations in A=3 system:

$$p_A = \frac{A}{A+B+C}, \quad p_B = \frac{B}{A+B+C}, \quad p_C = \frac{C}{A+B+C}$$

$$\tilde{p}_3 = p_A + \frac{1}{3} p_B, \quad \tilde{p}_6 = \frac{2}{3} p_B, \quad p_9 = p_C$$

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

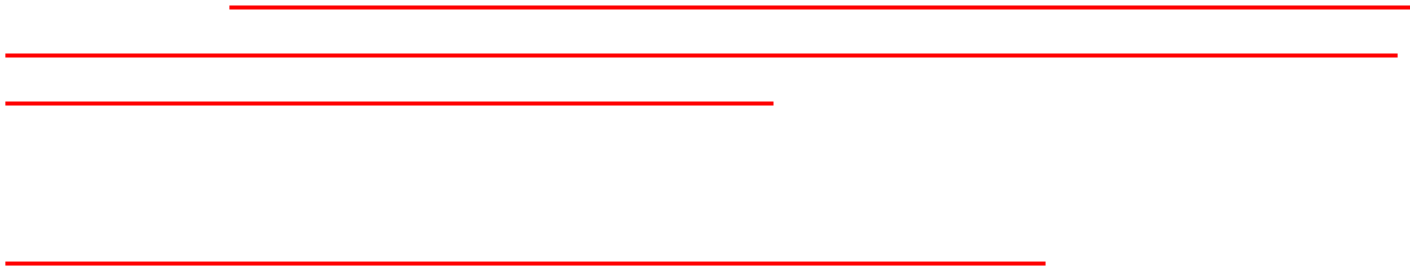


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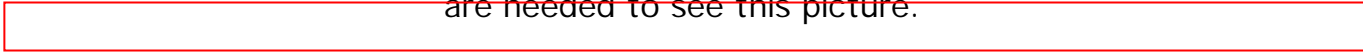
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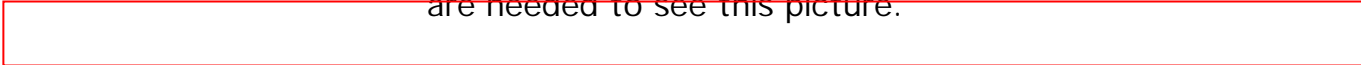
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We will need to update this table with the new NCSM results to make more precise comparisons with new and future JLAB data!

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are needed to see this picture.

Signature of the Quark Cluster Model (QCM) in the scaling regime
“Steps” of ratios are simple ratios of quark cluster probabilities



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are needed to see this picture.

1

2

3

4

Known for more than 20 years as a signature of the QCM

SLAC data - NE3

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Constituent Quark Models of Exotic Mesons

R. Lloyd, PhD Thesis, ISU 2003

Phys. Rev. D 70: 014009 (2004)

$$H = T + V(\text{OGE}) + V(\text{confinement})$$

Solve in HO basis as a bare H problem & study dependence on cutoff

Symmetries:

Exact treatment of color degree of freedom <-- Major new accomplishment

Translational invariance preserved

Angular momentum and parity preserved

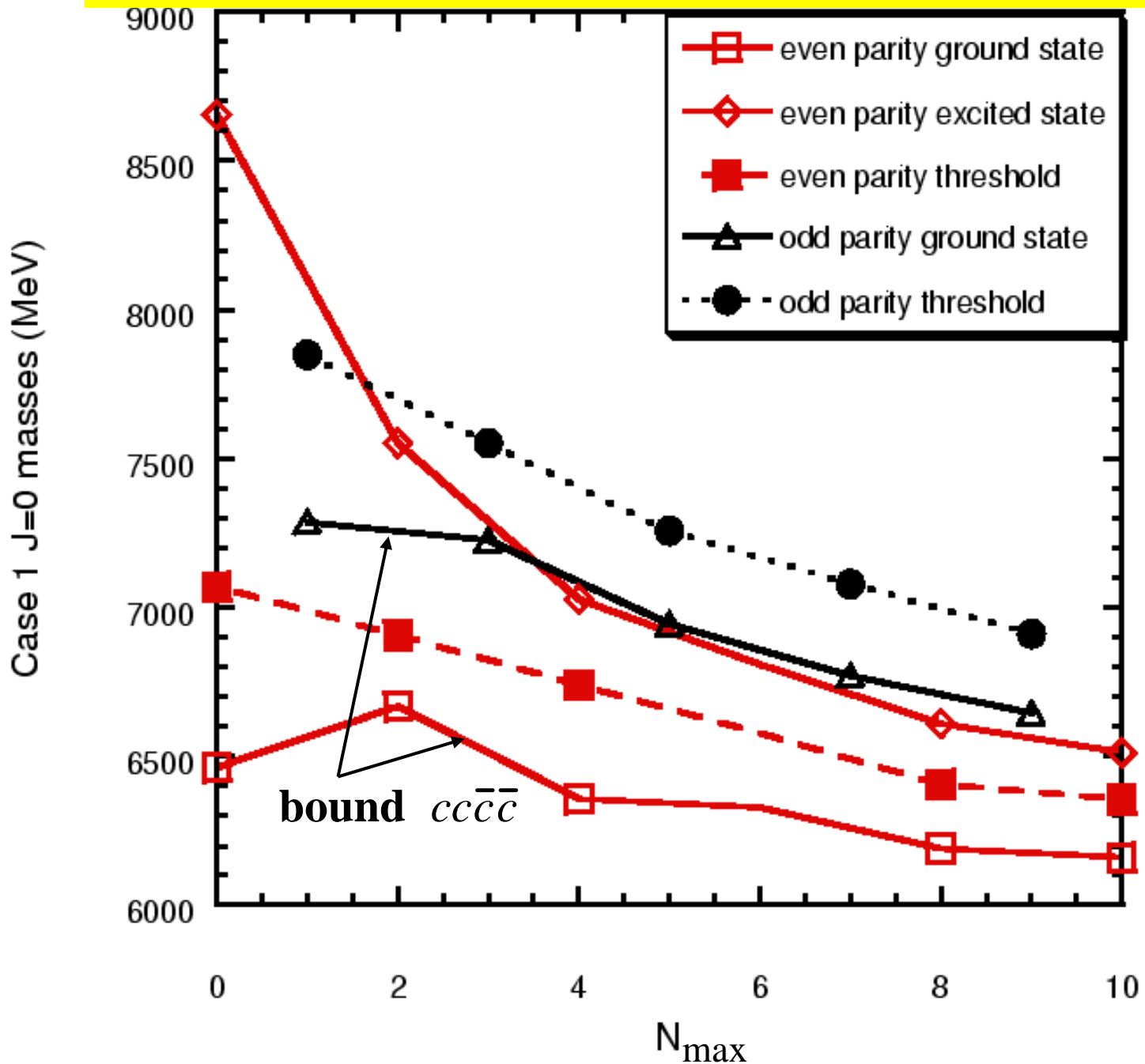
Next generation:

More realistic H fit to wider range of mesons and baryons

Beyond that generation:

H_{eff} derived from QCD

All-charm tetraquarks with bare phenomenological interaction



One step towards “Just do it”

Φ^4 in 1+1 Dimensions

Burning issues

Demonstrate degeneracy - Spontaneous Symmetry Breaking

Topological features - soliton mass and profile (Kink, Kink-Antikink)

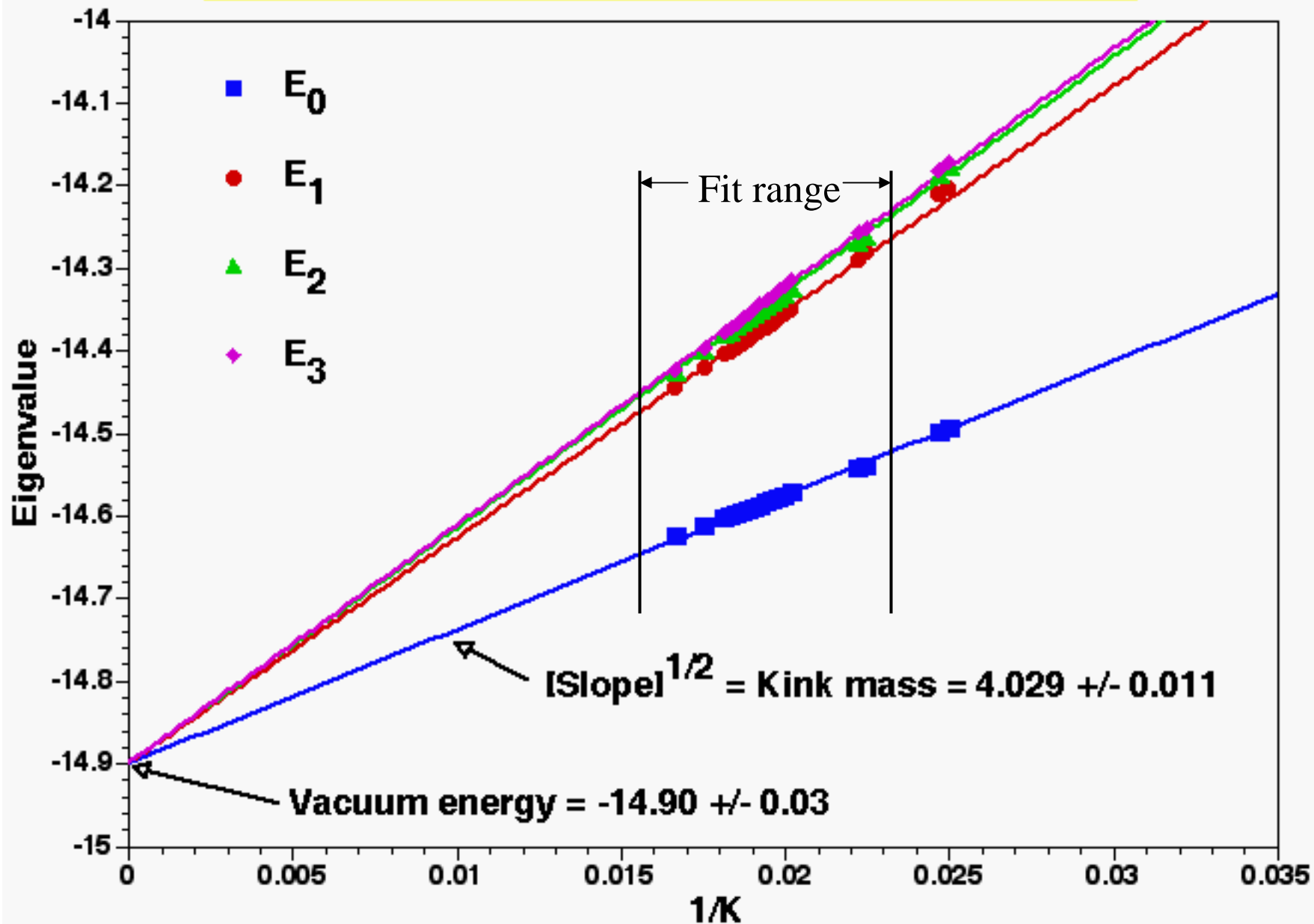
Quantum modes of kink excitation

Phase transition - critical coupling, critical exponent and
the physics of symmetry restoration

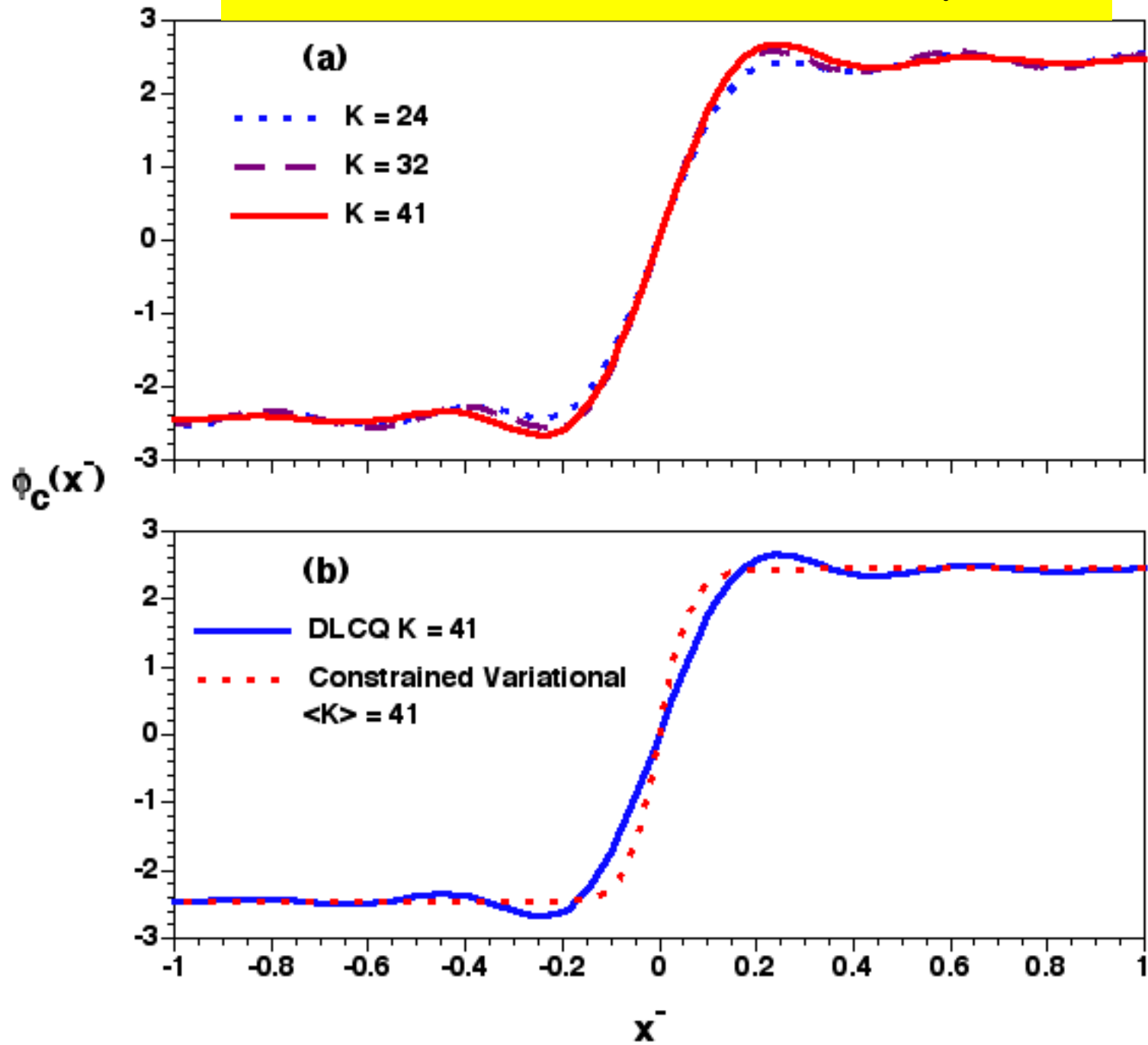
Role and proper treatment of the zero mode constraint

Chang's Duality

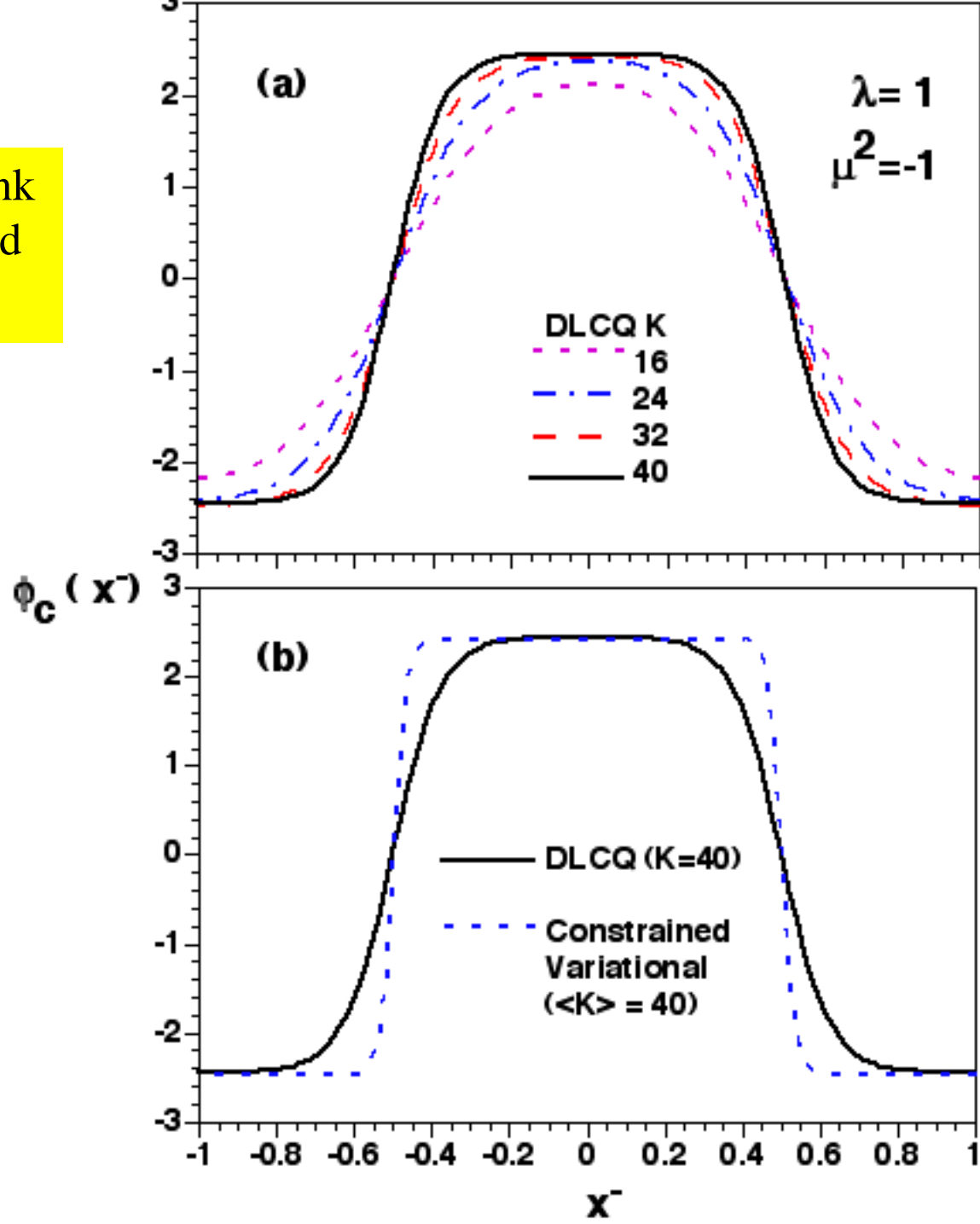
Lambda = 1.25, BP, APBC, Lowest 4 eigenvalues



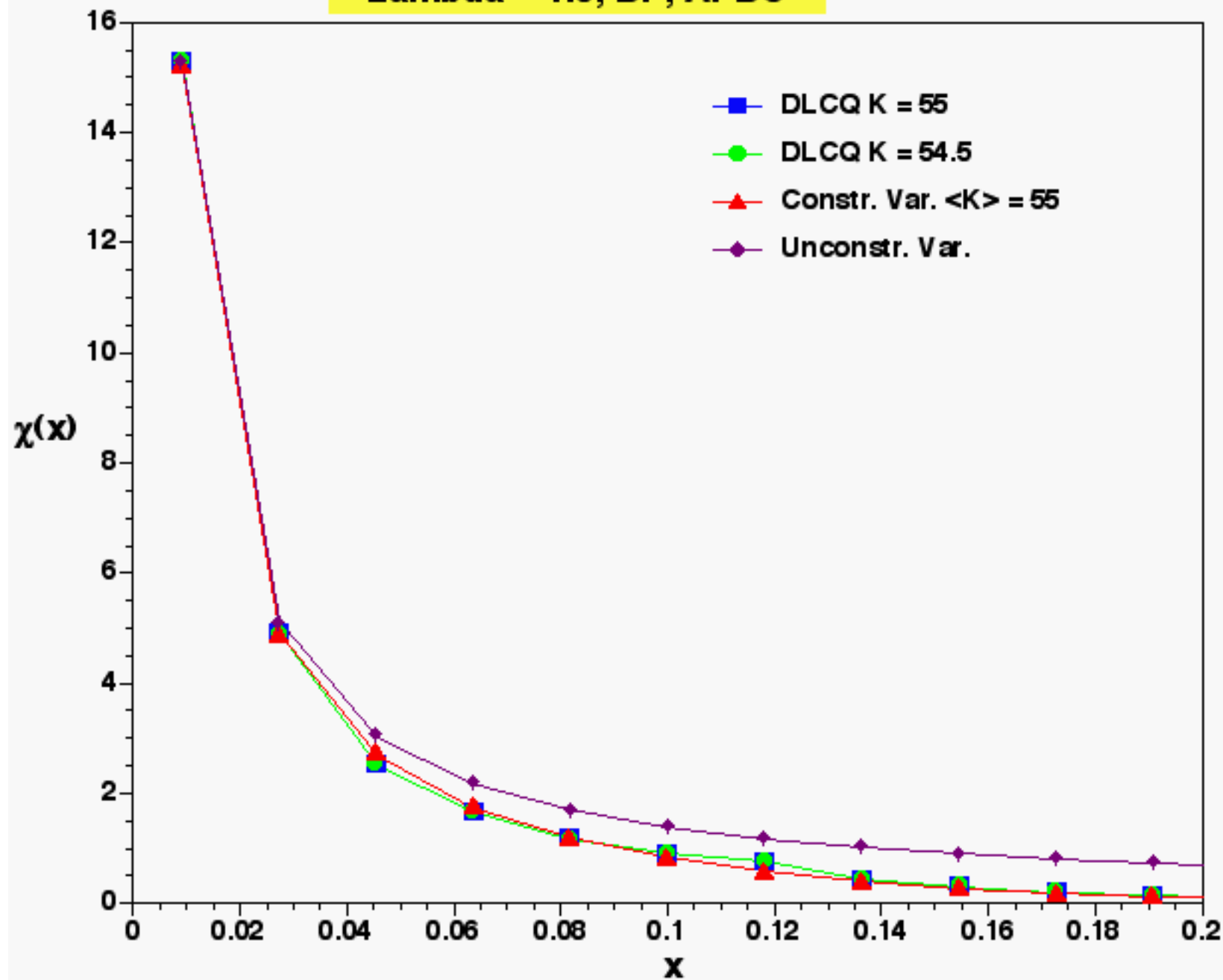
Quantum kink (soliton) in scalar field theory at $\lambda = 1$



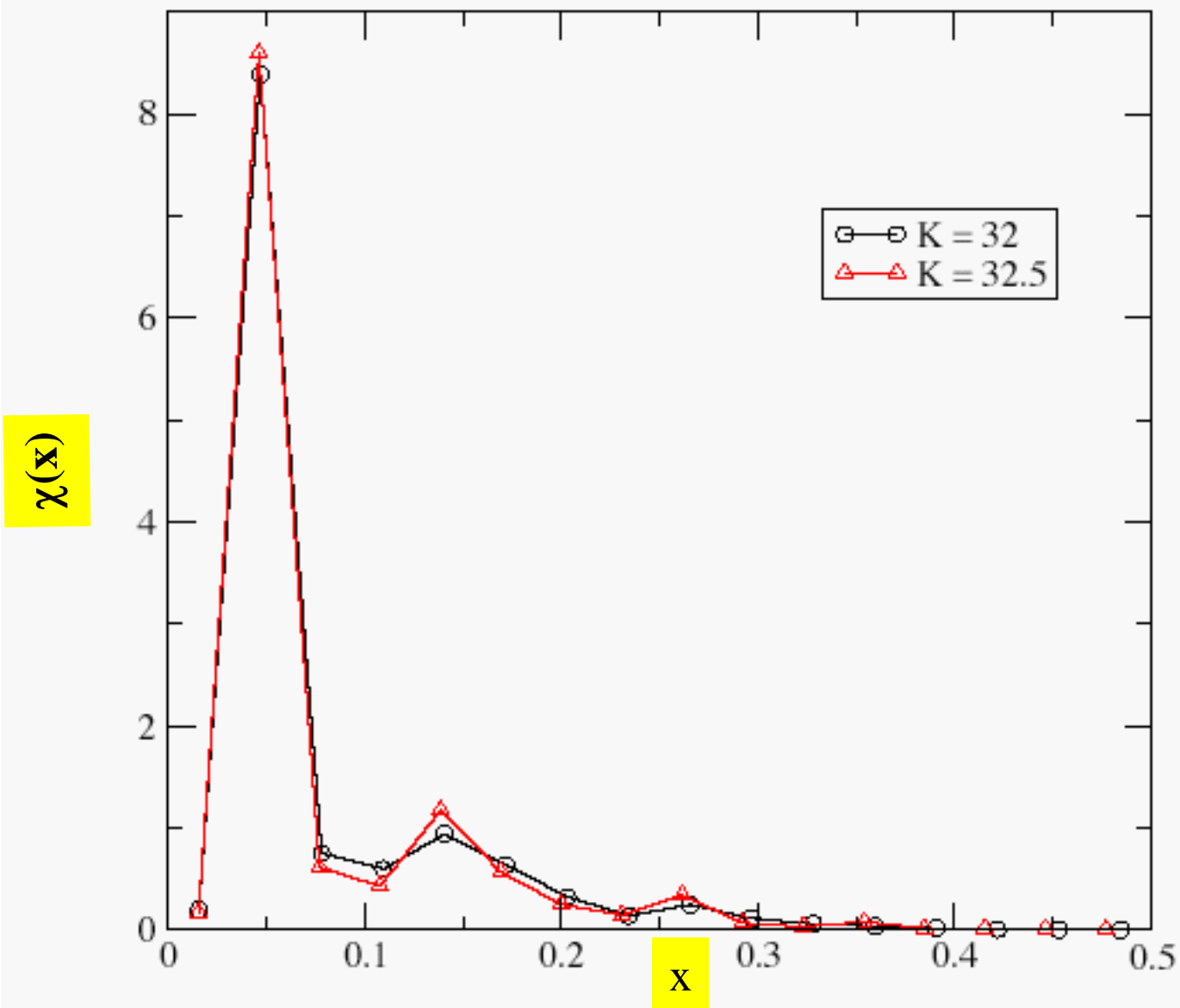
Quantum kink-antikink
(soliton) in scalar field
theory at $\lambda = 1$



Lambda = 1.0, BP, APBC



APBC, BP, $\lambda = 5.0$



Conclusions

- **Similarity of “two-scale” problems in many-particle quantum systems**
- **Ab-initio theory is convergent exact method for solving many-particle Hamiltonians**
- **Method has been demonstrated as exact in the nuclear physics applications**
- **Realistic V_{NN} 's underbind ^{12}C 1.2 and ^{16}O by 0.5 - 1.5 MeV/A**
- **Confirm need for NNN forces to achieve high quality description of light nuclei when local NN interactions used**
- **Some advantages seen with “soft” NN interactions (V-lowk, JISP6, INOY) where ab-initio NCSM is now used to help resolve off-shell freedom**
- **First applications to heavier systems ($A = 47 - 48 - 49$) - new Hamiltonian**
- **Critical properties of quantum field theory emerging**
- **Posit link between ab-initio NCSM and quark cluster model (P-space $\sim [r_{ij} > 2R_c]$)**
- **Advent of low-cost parallel computing has made new physics domains accessible: we have achieved a fully scalable and load-balanced algorithm.**

Outlook

With five examples - our new ability to determine:

- **Three nucleon forces**
- **v_{ud} for CKM mass matrix unitarity**
- **Majorana mass of neutrino through double β decay**
- **Critical properties of quantum field theory**
- **Foundation for a quark cluster model of nuclei**

We Have a New Physics Discovery Engine

ANNOUNCEMENT

New Tenure-Track Assistant Professor position
in Nuclear Theory at Iowa State University
(tenure track) and RIKEN Fellow position.

“Strong Interaction Theory”

Applications now being accepted

See me for details