Fragmentation Functions with Complete Quark Flavour Separation

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- First phenomenological separation of light quark flavour FFs
- AKK is update of KKP

1. Introduction
2. Fitting Method
3. Predictions
4. Conclusions

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1. Introduction

- FFs most reliably obtained by fitting to data from

$$e^+ + e^- \to X + h$$

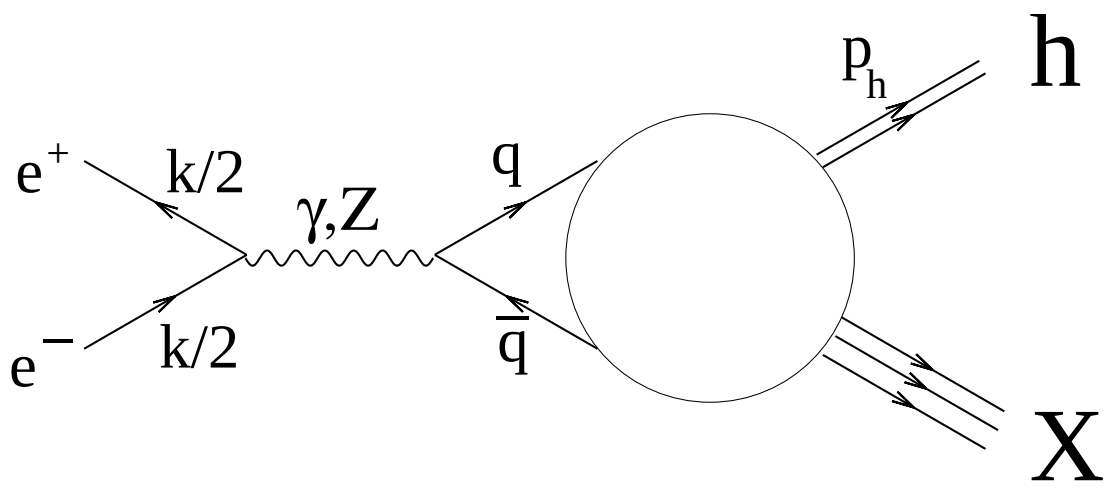

Figure : Diagrammatic representation of $\frac{d\sigma^h}{dx_p}(x_p, s)$

- Cross section is

$$\frac{d\sigma^h}{dx_p}(x_p, s)$$

where

$$s = k^2$$
$$x_p = \frac{2E_h}{\sqrt{s}} \ (0 < x_p < 1)$$

• Cannot completely calculate in perturbative QCD, because of h (No description of h in terms of partons).

• To get the hadron cross section,

1. calculate partonic cross section (perturbative)

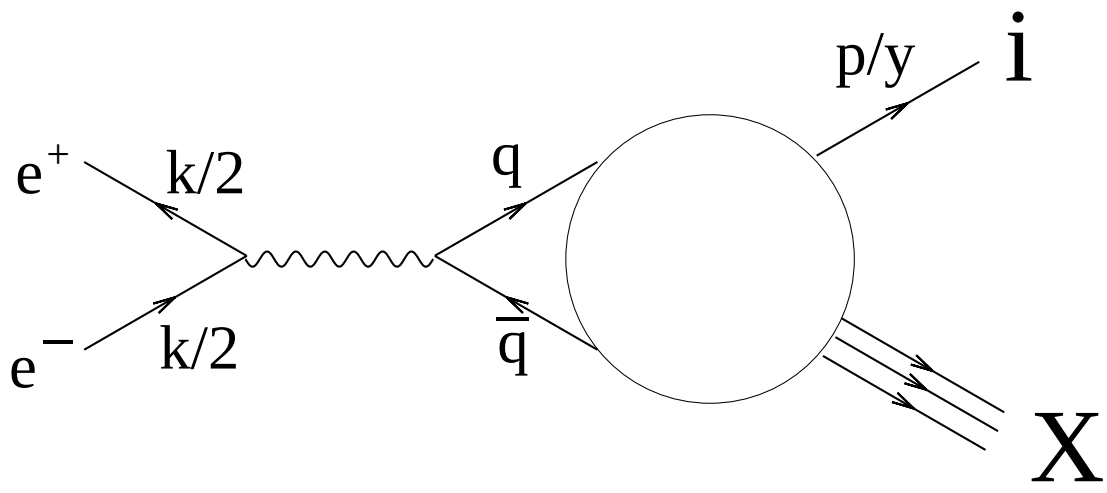

Figure : Diagrammatic representation of $\frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, M_f^2, s\right)$

2. Use the FFs to turn this into the hadron cross section; from statistics (incoherent),

$$\text{XS to produce } h = \text{XS to produce } i \times \text{ Probability that } i \text{ emits } h$$

"Probability that i emits h" is the FF.
Summing over all degrees of freedom, this is

$$\frac{d\sigma^h}{dx_p}(x_p, s) = \sum_i \int_{x_p}^1 \frac{dy}{y} \frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, M_f^2, s\right) D_i^h(y, M_f^2)$$

• All other processes in the cross section are $O(1/\sqrt{s})$ (higher twist terms).

• Result follows formally from Factorization Theorem:

1. $\frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, M_f^2, s\right)$ contains processes $E > M_f$
   Finite as parton masses vanish
   (For simplicity, set masses in $\frac{d\sigma^i}{dz}$ to zero)
   Perturbatively calculable,
   i.e. as series in $a_s(\mu^2) = \frac{\alpha_s(\mu)}{2\pi} \propto \frac{1}{\ln \frac{\mu^2}{\Lambda^2_{\rm QCD}}}$
   Best to choose $\mu^2 = O(s)$
   $\therefore$ must have $s \gg \Lambda^2_{\rm QCD}$

2. $D_i^h(y, M_f^2)$ contains processes $E < M_f$
   $\longrightarrow$ not perturbatively calculable
   but can be fitted to data.

• The separator M_f is the *factorization scale*
   Arbitrary, cross section formally independent of it.
   Dependence caused by perturbative approximation.

• *Factorization scheme*:
Can shift terms between $\frac{d\sigma^i}{dz}$ and D_i^h keeping $\frac{d\sigma^h}{dx_p}$ fixed
   Usual choice is $\overline{\rm MS}$

• Partonic cross section takes the form

$$\frac{d\sigma^i}{dz}\left(z, M_f^2, s\right) = A(z) + a_s(s)\left(B(z) + C(z)\ln\frac{M_f^2}{s}\right)$$
$$+ a_s^2(s)\left(D(z) + E(z)\ln\frac{M_f^2}{s} + F(z)\ln^2\frac{M_f^2}{s}\right)$$
$$+ \ldots$$

For convergence, must choose $M_f^2 \simeq s$.

• This is convoluted with $D_i^h(z, M_f^2)$, recall

$$\frac{d\sigma^h}{dx_p}(x_p, s) = \sum_j \int_{x_p}^1 \frac{dy}{y}\frac{d\sigma^i}{d(x_p/y)}\left(\frac{x_p}{y}, M_f^2, s\right) D_i^h(y, M_f^2)$$

Therefore need to know $D_i^h(y, M_f^2)$ as function of M_f.

• Factorization Theorem says
$D_i^h(y, M_f^2)$ in M_f is perturbatively calculable (DGLAP):

$$\frac{d}{d\ln M_f^2} D_i^h(x, M_f^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}, a_s(M_f^2)\right) D_j^h(y, M_f^2)$$

• Enough to know $D_i^h(x, M_0^2)$.
Take low value for M_0, $M_0 = \sqrt{2}$ GeV,
and evolve upwards.

• Choice of M_f: Choose $M_f^2 = k_f s$.
Fit with $k_f = 1/4$, 1, 4 to get theoretical uncertainty.

• In $a_s(\mu^2)$, take renormalization scale $\mu = M_f$

• Recall DGLAP equation

$$\frac{d}{d\ln M_f^2} D_i^h(x, M_f^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}, a_s(M_f^2)\right) D_j^h(y, M_f^2)$$

• Conventional perturbative calculation of $P_{ij}(z, a_s)$ not applicable at small z ($\therefore$ small x).

$\longrightarrow$ soft gluon logarithms $a_s^n \ln^m z$,

give unreliable evolution at low $z \to x \to x_p$.

$\therefore$ Consider only data for which $x_p > 0.1$.

• Evolution procedure:

1. At $M_f = M_0$, we have gluon and light quark flavour FFs.

2. Evolve from $M_f = M_0$ to $M_f = 2m_c$ with just 3 flavours.

3. At $M_f = 2m_c$, introduce charm FF $D_c^h(x, 4m_c^2)$, which must be fitted to data.

4. Evolve from $M_f = 2m_c$ to $M_f = 2m_b$ with 4 flavours.

5. At $M_f = 2m_b$, introduce bottom FF $D_b^h(x, 4m_b^2)$.

etc.

2. Fitting Method

• Determine FFs for charged light hadrons
($\pi^{\pm}$, $K^{\pm}$ and $p/\overline{p}$)
by fitting to $e^+ + e^- \to h + X$ data.

• For such data, charge conjugation invariance
($D_q^{h^+} = D_{\bar{q}}^{h^-}$ etc.)
gives $D_q^{h^- + h^+} = D_{\bar{q}}^{h^- + h^+} = D_q^h$.

• Take parameterization

$$D_a^h(x, M_0^2) = N_a^h x^{\alpha_a^h} (1-x)^{\beta_a^h}$$

and fit all N_a^h, α_a^h and β_a^h.

• Individual quark FFs well constrained,
$\because$ data is quark tagged:
$\gamma, Z \to q + \bar{q} \to h + X$,
where q is determined.

In KKP analysis (B. A. Kniehl, G. Kramer and B. Pötter),
they used c and b tagged cross sections,
$\longrightarrow D_c^h(x, M_0^2)$ and $D_b^h(x, M_0^2)$ well determined.

But they only used $d + u + s$ tagged cross sections
$\longrightarrow D_d^h(x, M_0^2)$, $D_u^h(x, M_0^2)$ and $D_s^h(x, M_0^2)$
were not determined separately.

• KKP had to impose (at all momentum fractions)

1. valence quark structure
2. SU(3) invariance

$$D_u^{\pi^\pm}(x, M_0^2) = D_d^{\pi^\pm}(x, M_0^2),$$
$$D_u^{K^\pm}(x, M_0^2) = D_s^{K^\pm}(x, M_0^2) \text{ and}$$
$$D_u^{p/\overline{p}}(x, M_0^2) = 2D_d^{p/\overline{p}}(x, M_0^2).$$

• KKP give good description of $p + p(\overline{p}) \to h + X$ data except for $p + p \to K_S^0 + X$ data from STAR

• Expect more disagreements from such data in future (proton is a "ball" of light partons).

• Fit to same data as KKP:
DELPHI, SLD, ALEPH, TPC

• But exclude charged hadron summed data
(where $\pi^{\pm}$, $K^{\pm}$ and $p/\overline{p}$ not distinguished).
Such data are accurate
- but may be contaminated
with other charged particles.
Use only for checking.

• Include OPAL tagging probabilities
G. Abbiendi et al., Eur. Phys. J. C 16 (2000) 407.

$$\eta_a^h(x_{\rm cut}, s) = \int_{x_{\rm cut}}^1 dx_p \frac{\frac{d\sigma_a^h}{dx_p}(x_p, s)}{\sigma_a(s)}$$

$\sqrt{s} = 91$ GeV.

$a = d, u, s.$

• Can now separate
$D_d^h(x, M_0^2)$, $D_u^h(x, M_0^2)$ and $D_s^h(x, M_0^2)$
completely phenomenologically.

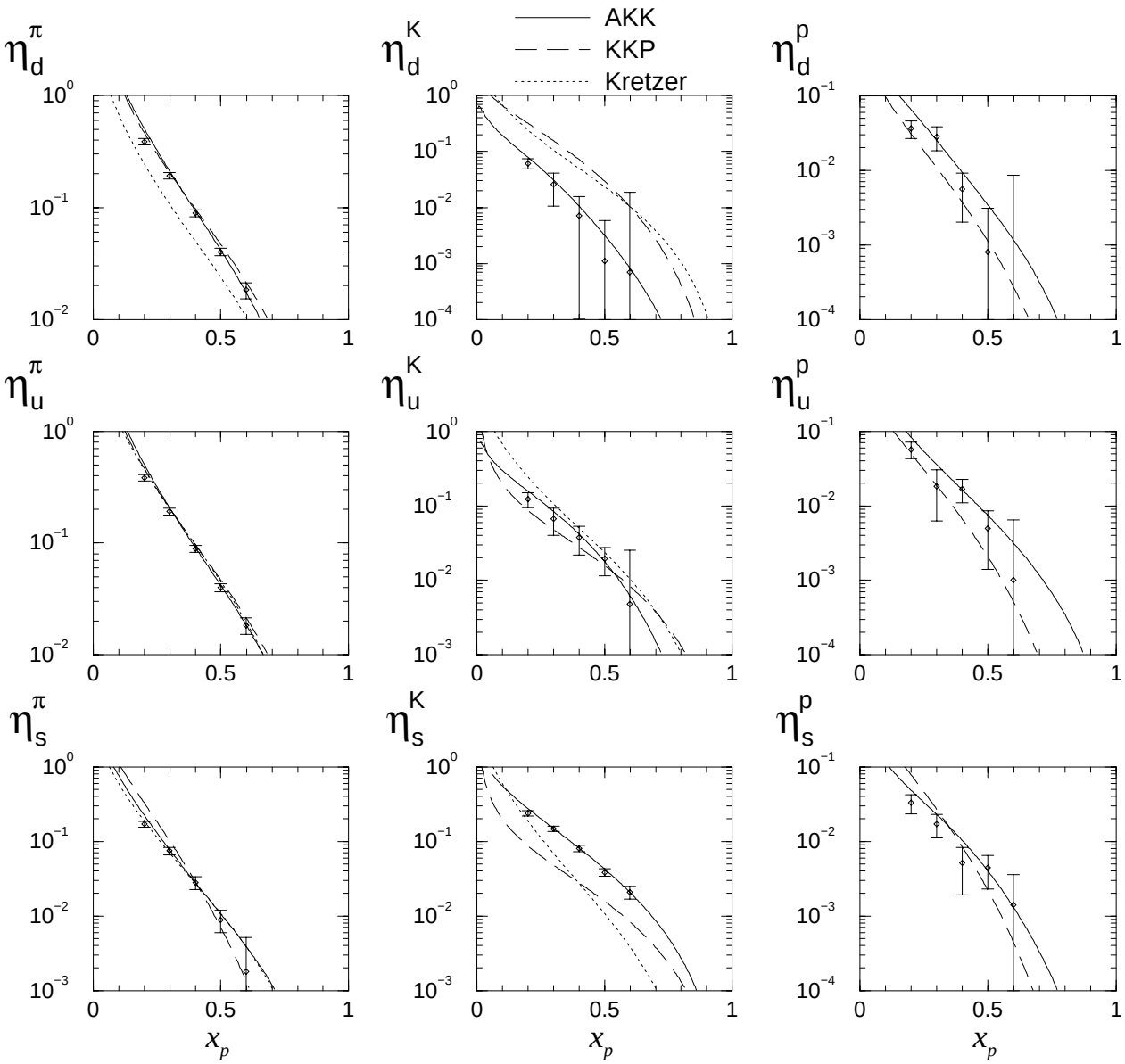

Figure : Light quark probabilities $\eta_a^h(x_p, s)$ at $\sqrt{s} = 91.2$ GeV. The dashed curves are calculated using the AKK FF's, the dotted curves are calculated from the (x, M_f^2) grid of Kretzer's FF's (in which no $p/\overline{p}$ FF's are obtained), and the solid curves are calculated using the FF's obtained in the analysis of this paper (AKK). The corresponding measured OPAL tagging probabilites are also shown.

3. Predictions

Predictions for $p + p(\overline{p}) \to h + X$ data:

• For proton,
choose CTEQ6M parton distribution functions.
• Set $M_f = k_f p_T$.
• Formally, cross section at $x_T = 2p_T/\sqrt{s}$
depends on FFs $D_i^h(z, M_0^2)$ for $x_T < z < 1$.
But in practice,
there is little dependence on FFs
at low z (and low M_f)
$\longrightarrow$ fix FFs for $z < 0.1$, $M_f < M_0$.

• Predict $h = \pi^0$ and K_S^0 production:
use SU(2) flavour symmetry

$$D_a^{\pi^0}(x, M_f^2) = \frac{1}{2} D_a^{\pi^\pm}(x, M_f^2)$$

$$D_a^{K_S^0}(x, M_f^2) = \frac{1}{2} D_b^{K^\pm}(x, M_f^2)$$

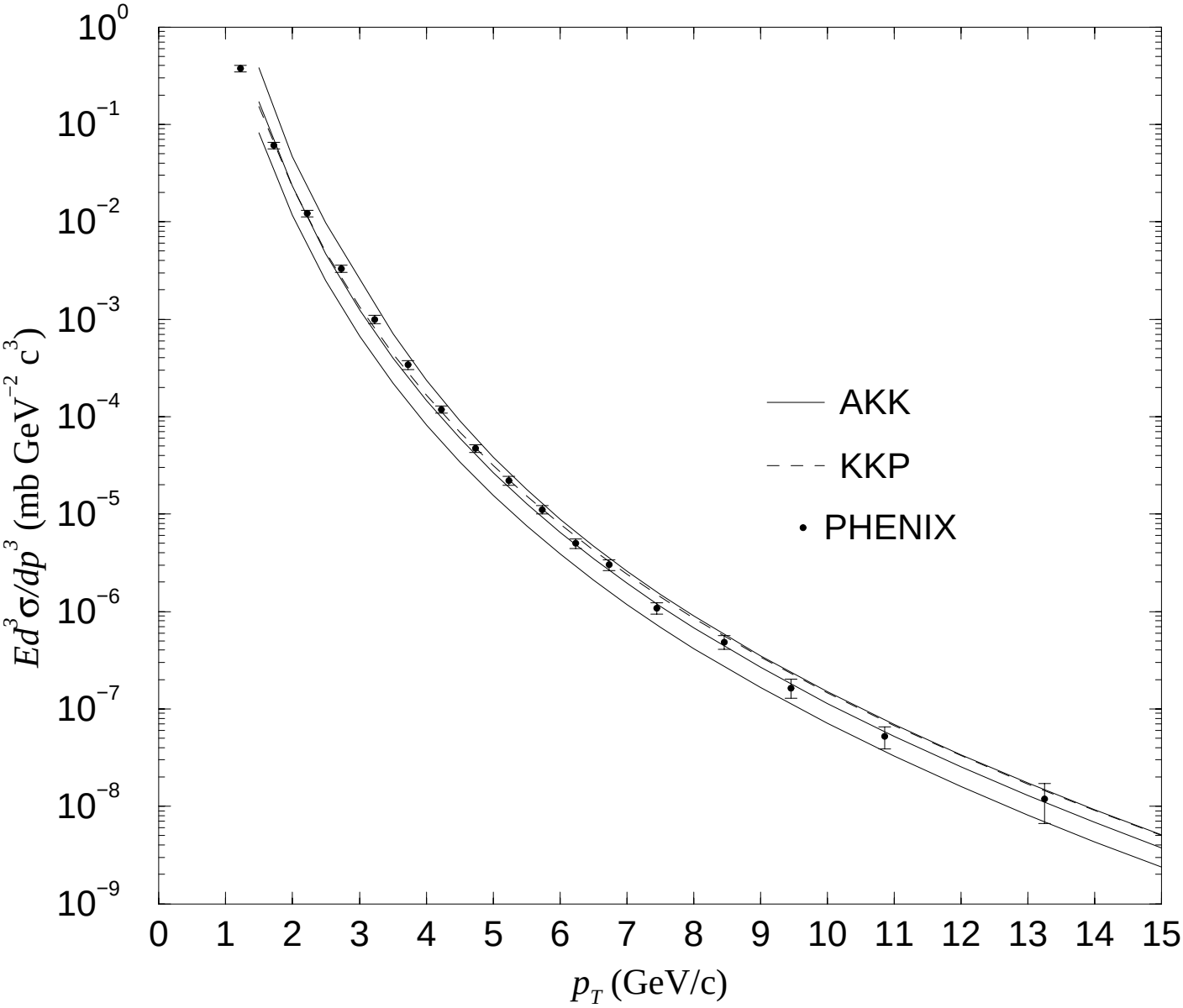

Figure : The invariant differential cross section for inclusive π^0 production in $p+p$ collisions at $\sqrt{s} = 200$ GeV. Data from the PHENIX Collaboration are shown, without the absolute 9.6% normalization error. Compared with this data are the cross sections calculated from the FF's obtained in this paper (labelled AKK) and that from the AKK FF's (labelled KKP). The upper, central and lower AKK curves are calculated with $k = 1/4$, 1 and 4 respectively.

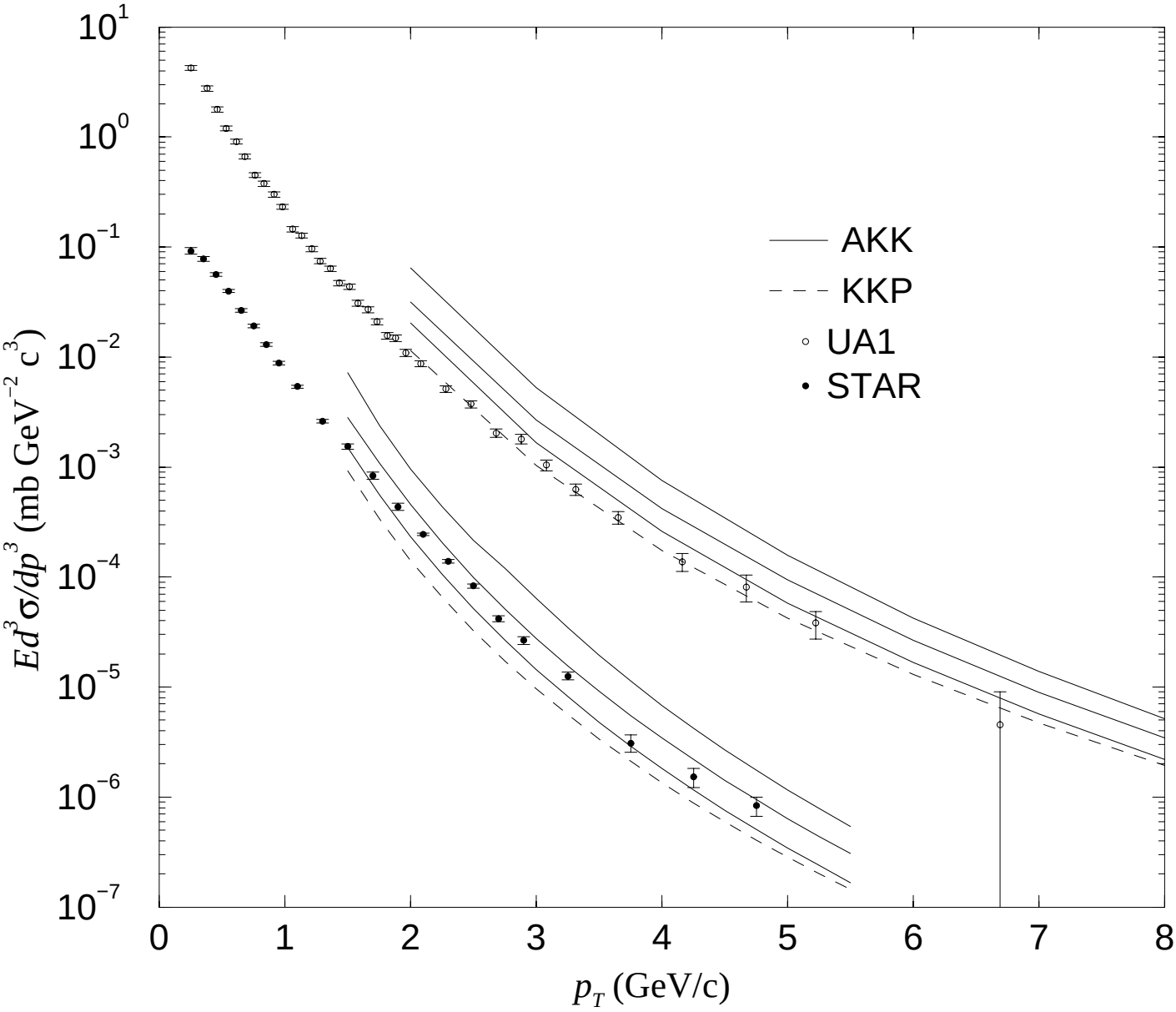

Figure : For the invariant differential cross section for inclusive K_S^0 production in $p+p$ collisions at $\sqrt{s} = 200$ GeV compared with data from the STAR Collaboration, and in $p+\overline{p}$ collisions at $\sqrt{s} = 630$ GeV compared with data from the UA1 Collaboration. For clarity, the former results have been divided by a factor of 30.

• Also fit $\alpha_s(M_Z)$.

• Experimental errors determined by varying $\alpha_s(M_Z)$ until $\chi^2_{\rm DF}$ increased by 1.

$$\alpha_s(M_Z) = 0.1176^{+0.0053}_{-0.0067}[\text{exp}]^{+0.0007}_{-0.0009}[\text{theo}] = 0.1176^{+0.0053}_{-0.0068}$$

• For KKP,

$$\alpha_s(M_Z) = 0.1170^{+0.0058}_{-0.0073}$$

• Both consistent with Particle Data Group

$$\alpha_s(M_Z) = 0.1187 \pm 0.002$$

4. Conclusions

- Update of KKP analysis

- Do not use charged data

- AKK also fitted OPAL data on
individual light quark flavour tagging probabilities,
  $\longrightarrow$ light quark flavour FFs
  determined phenomenologically for first time

  Improved determination of $s, d \to K^{\pm}$ transition.

- Find only slight shift towards PHENIX data

- Big difference between AKK and KKP
for K_S^0 $(= K^{\pm})$ production:
  Find shift towards STAR data
  but away from UA1 data

- AKK's $\alpha_s(M_Z)$ consistent with KKP's and PDG