Generalized Parton Distributions in Nuclei

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Motivation

• Understand transverse motion in hadrons and its relation to Final State Interactions (problem first studied within QCD, perhaps, by Ellis, Furmanski, and Petronzio, '80s)

• Explore the connection between intrinsic transverse momentum, k_T, and transverse interparton distances, b,

$k_T \Rightarrow$ accessible through azimuthal asymmetries involving transverse polarization,

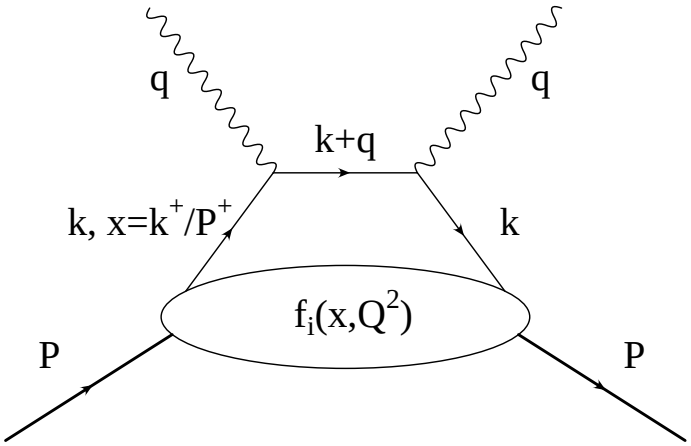
$b \Rightarrow$ accessible through DVCS and related processes.

• Consistent treatment of a number of reactions: SIDIS, DVCS, hadro-production (DY, Λ, ...)

• Use nuclei as laboratories to "trigger" modifications of transverse d.o.f.

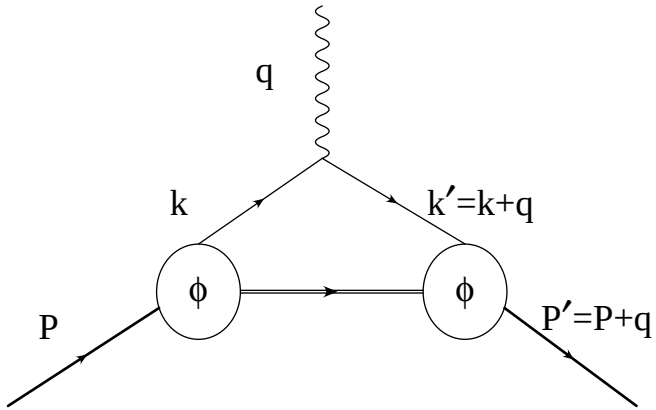
Role and interplay of transverse momentum – k_T – and spatial – b – d.o.f. (1)

PDFs: Hadronic Structure in Momentum Space



$d\sigma_{hadron} \approx d\sigma_{parton} \times f_i(x)$

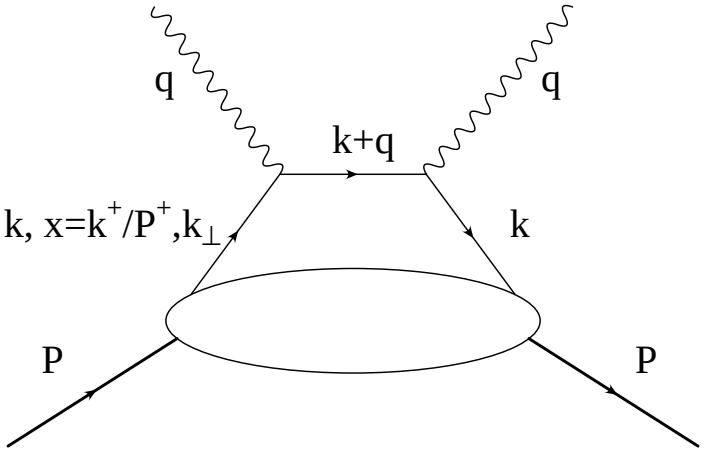
Form Factors: Hadronic Structure in Coordinate Space



$2M_N G_E(Q^2) = \int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r})$

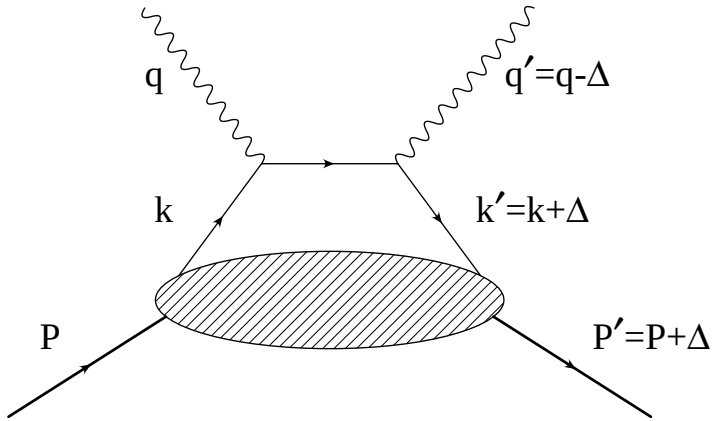
Role and interplay of transverse momentum – k_T – and spatial – b – d.o.f. (2)

UPDs: direct access to intrinsic transverse momenta



$f_i(x) \rightarrow f_i(x, k_\perp^2)$

IPPDFs: direct access to transverse coordinates



$H(x, \Delta) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}} q(x, \mathbf{b})$

PDFs

$$P^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P,S \mid \psi\left(-\frac{\xi^-}{2}\right)\gamma^+\psi\left(\frac{\xi^-}{2}\right) \mid P,S\rangle = \bar{u}(P,S)\gamma^+ u(P,S) f(x)$$

FFs

$$\langle P',S' \mid \psi(0)\gamma^+\psi(0) \mid P,S\rangle = \bar{u}(P',S')\left[\gamma^+ F_1(Q^2) + \frac{i\sigma^{+\nu}q_\nu}{2M}F_2(Q^2)\right]u(P,S)$$

UPDs

$$\int \frac{d\xi^- d^2\xi}{(2\pi)^3} e^{ixP^+\xi^- - i\mathbf{k}_\perp\cdot\xi} \langle P,S \mid \psi\left(-\frac{\xi^-}{2},-\frac{\xi_\perp}{2}\right)\gamma^+\psi\left(\frac{\xi^-}{2},\frac{\xi_\perp}{2}\right) \mid P,S\rangle = f(x,k_\perp^2)$$

GPDs

$$\bar{P}^+ \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \langle P',S' \mid \psi\left(-\frac{\xi^-}{2}\right)\gamma^+\psi\left(\frac{\xi^-}{2}\right) \mid P,S\rangle =$$

$$\bar{u}(P',S')\left[\gamma^+ H(x,\xi,-\Delta^2) + \frac{i\sigma^{+\nu}q_\nu}{2M}E(x,\xi,-\Delta^2)\right]u(P,S) \to \mathcal{F}\left[q(x,\mathbf{b})\right]$$

Beyond naive definition ...

Brodsky, Hoyer, Hwang, Ji, Schmidt,...

Because of the different $\xi_\perp$ in the correlation, a Wilson line (gauge link $\Rightarrow$ FSI) appears in the correlation.

Difficult to calculate $i.e.$ to render the gauge link in a partonic language $\Rightarrow$ models (Transversity sessions on May 18th...)

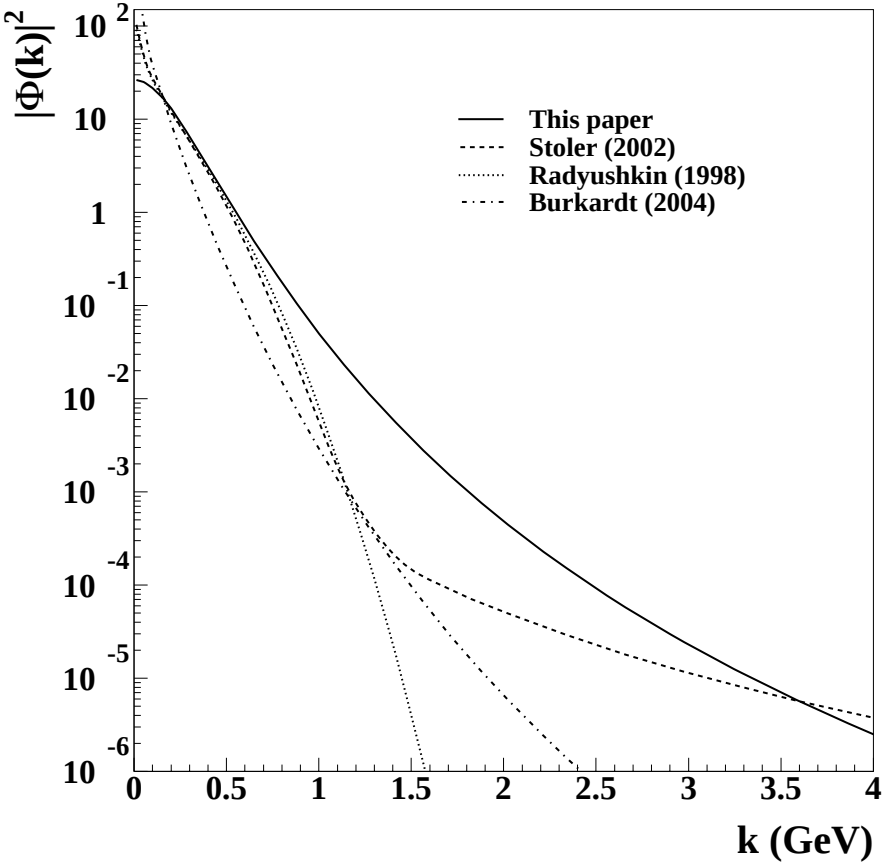
In summary ...

$$\text{IPPDF} \quad \rightarrow \quad q(x,\mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2}\, e^{-i\mathbf{b}\cdot\mathbf{\Delta}} H_q(x,0,-\mathbf{\Delta}^2) \rightarrow \langle \mathbf{b}^2(x)\rangle = \mathcal{N}_b \int d^2\mathbf{b}\, q(x,\mathbf{b})\, \mathbf{b}^2$$

$$\text{GPD} \quad \rightarrow \quad H_q(x,0,-\mathbf{\Delta}^2) = \int d^2\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{\Delta}} q(x,\mathbf{b}) \rightarrow \langle x(\Delta)\rangle = \mathcal{N}_x \int dx x H(x,\mathbf{\Delta})$$

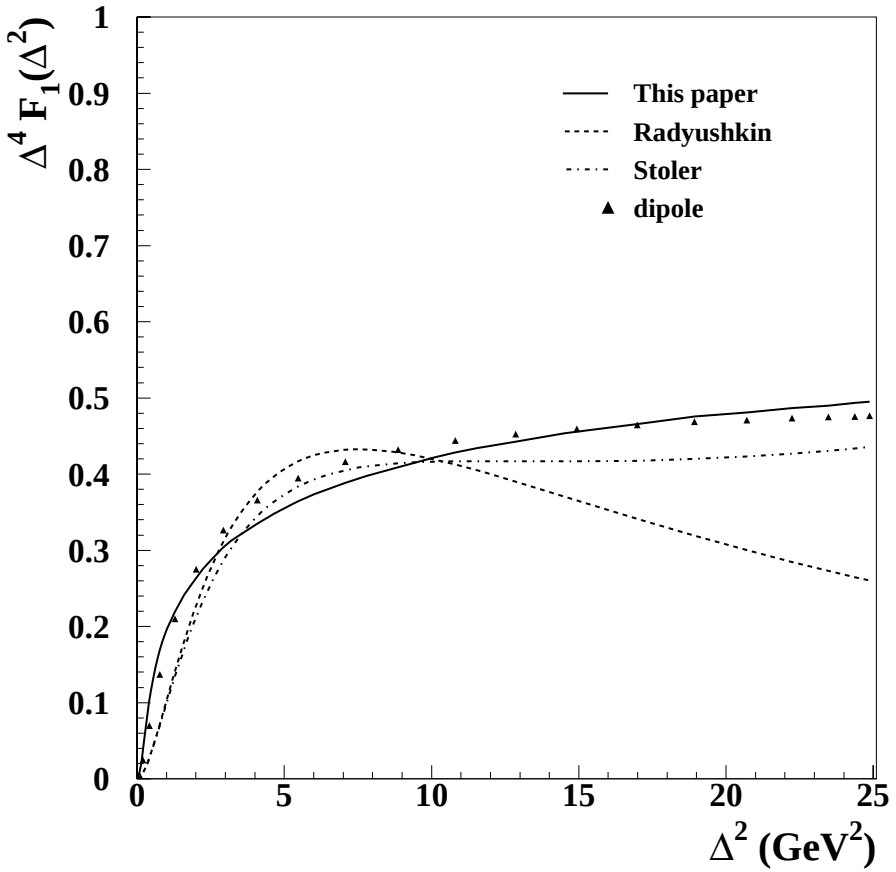
$$\text{UPD} \quad \rightarrow \quad f(x,\mathbf{k}) = \int d^2\mathbf{b} \int d^2\mathbf{b}'\, e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\, q(x,\mathbf{b},\mathbf{b}') \rightarrow \langle \mathbf{k}^2(x)\rangle = \mathcal{N}_k \int d^2\mathbf{k} f(x,\mathbf{k})\mathbf{k}^2$$

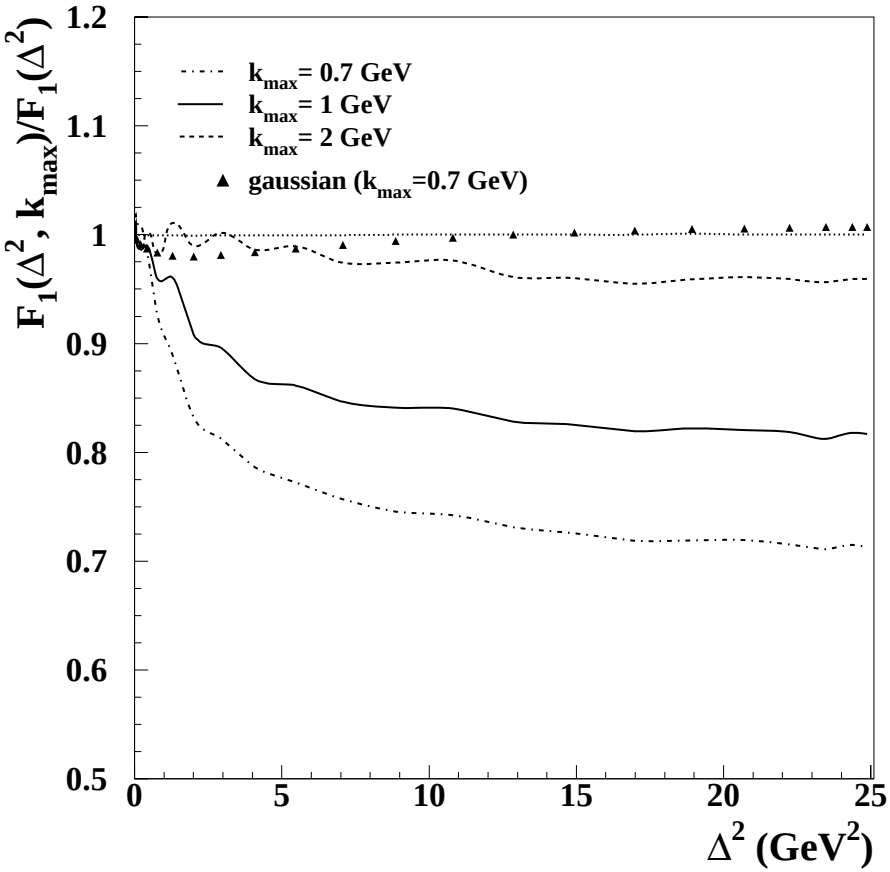
Quark-diquark model $\Rightarrow$ main ingredient



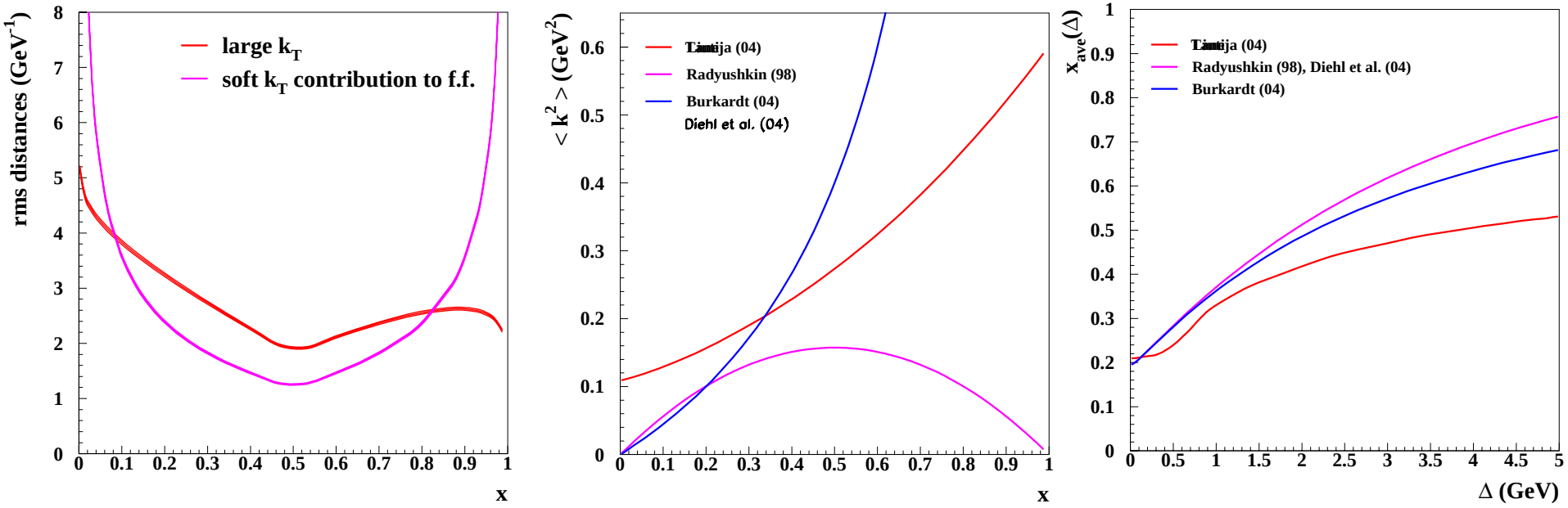
$$\Phi(\mathbf{k}_T) = \int dx\, \Phi(x, \mathbf{k}_T)$$

How models reproduce the proton form factor...





$\langle b^2(x)\rangle^{1/2}$ $\langle k_T^2(x)\rangle^{1/2}$ $\langle x(\Delta)\rangle$



S.L., S.K. Taneja, Phys. Rev. **D70**, 074019, (2004)

$\langle b^2(x)\rangle$ analysis extended and confirmed in Diehl et al., EPJ **C39**, (2005)

Role of nuclei as "laboratories"

- IPPDFs in Nuclear Medium:

1) GPDs from Color Transparency

2) Calculating from scratch Overlapping Volumes in DIS

- GPDs in Nuclear Medium: Off-forward EMC Effect

- UPDs in nuclear medium: F_L

Color Transparency

Brodsky, Muller, Ralston, Pire ...

"Transparency"

Small hadronic configurations, selected at asymptotically high Q^2, interact "weakly" as they pass through the nuclear medium.

"Filtering"

Small hadronic confirgurations can be filtered at finite Q^2, by impeding the passage of large separations because of the strong interactions occurring in a nucleus with $A >> 1$.

Color Transparency is attained when: $T_A(Q^2) = \frac{\sigma_A^{excl.}}{\sigma_h} \to 1$

GPDs from Color Transparency

S.L., S.K. Taneja, Phys. Rev. **D70**, 074019, (2004)

$$T_A(\Delta^2) = \frac{\left[\int_0^1 dx H_A(x,\Delta)\right]^2}{\left[\int_0^1 dx H(x,\Delta)\right]^2}$$

$$H_A(\Delta^2) = \int_0^{b_{max}(A)} db\, b\, q(x,b) J_0(b\Delta)$$

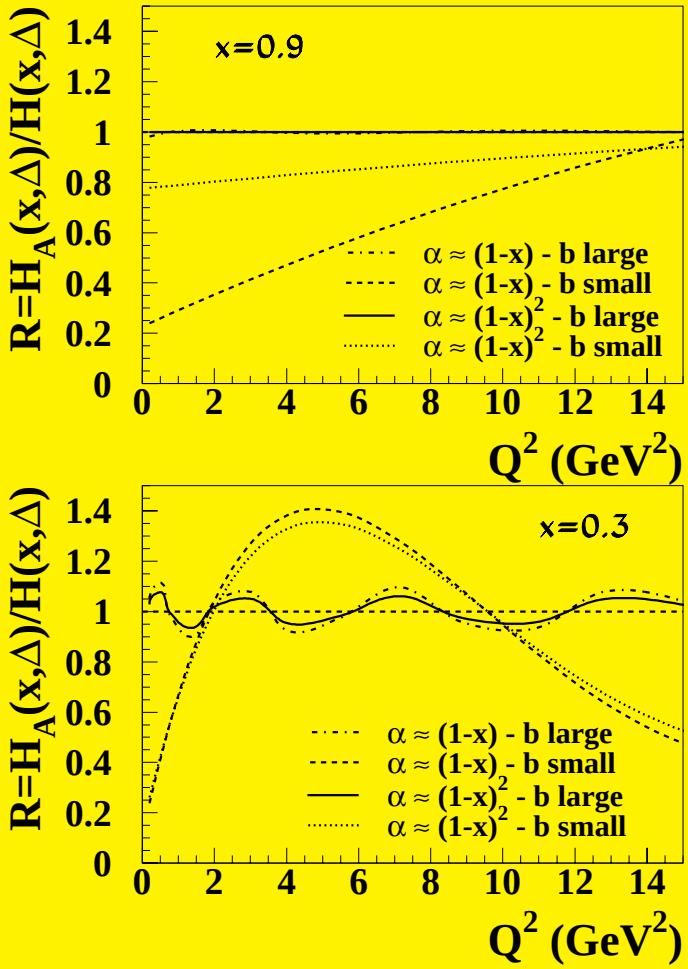
$b_{max}(A)$ = size of the filter given by a square function

$\Rightarrow$ By assuming: $q(x,b) = A(x)\exp(-\alpha(x)\, b)$

i) $\alpha \propto (1-x)/x \Rightarrow$ soft in $k_\perp$

ii) $\alpha \propto (1-x)^2/x \Rightarrow$ hard in $k_\perp$

predict both damping and oscillations of the transparency ratio

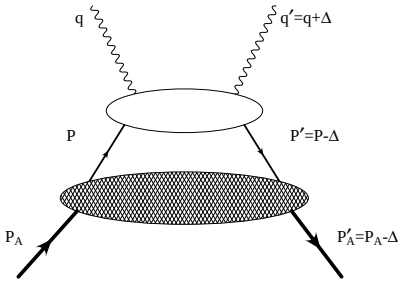


$$R = 1 - \exp(-\alpha\, b_{max})\,[\alpha\, b_{max} J_0(\Delta b_{max}) + \cos(\Delta b_{max})]$$

Off-forward EMC Effect

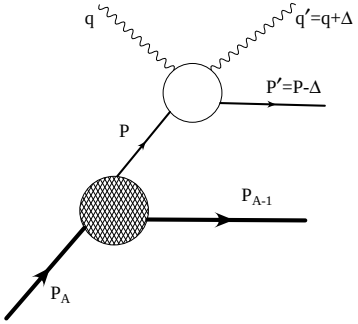
S.L., S.K. Taneja, hep-ph/0504027; hep-ph/0505123

Coherent-DVCS



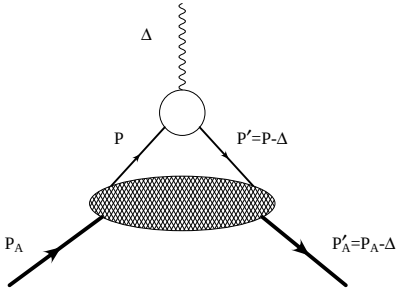
(a)

Incoherent-DVCS



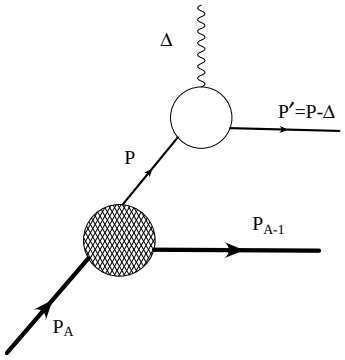
(b)

Coherent-BH



(c)

Incoherent-BH



(d)

Coherent DVCS $\rightarrow$ Off-forward EMC Effect

$$T^A_{\mu\nu}(P_A,\Delta) = \int \frac{d^4P}{(2\pi)^4} T^N_{\mu\nu}(k,P,\Delta)\, \mathcal{M}^A(P,P_A,\Delta),$$

$$\mathcal{M}^A_{ij}(P,P_A,\Delta) = \int d^4y\, e^{iP\cdot y} \langle P'_A|\overline{\Psi}_{A,j}(-y/2)\Psi_{A,i}(y/2)|P_A\rangle.$$

Diquark model with off-forward propagators

Nucleon

$$\mathcal{M}^N_{ij} = \rho_N \left(k^2, k'^2\right) \sum_s u_i(k,s)\overline{u}_j(k',s) 2\pi\delta(k_X^2 - M_X^2),$$

Nucleus

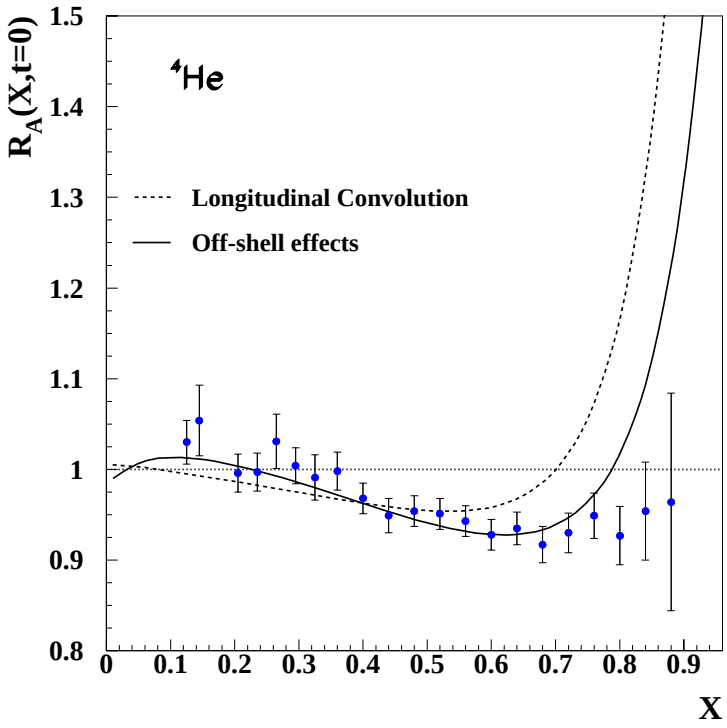
$$\mathcal{M}^A_{ij} = \rho_A(P^2, P'^2) \sum_S U_i(P,S)\overline{U}_j(P',S),$$

Accounting for off-shell effects...

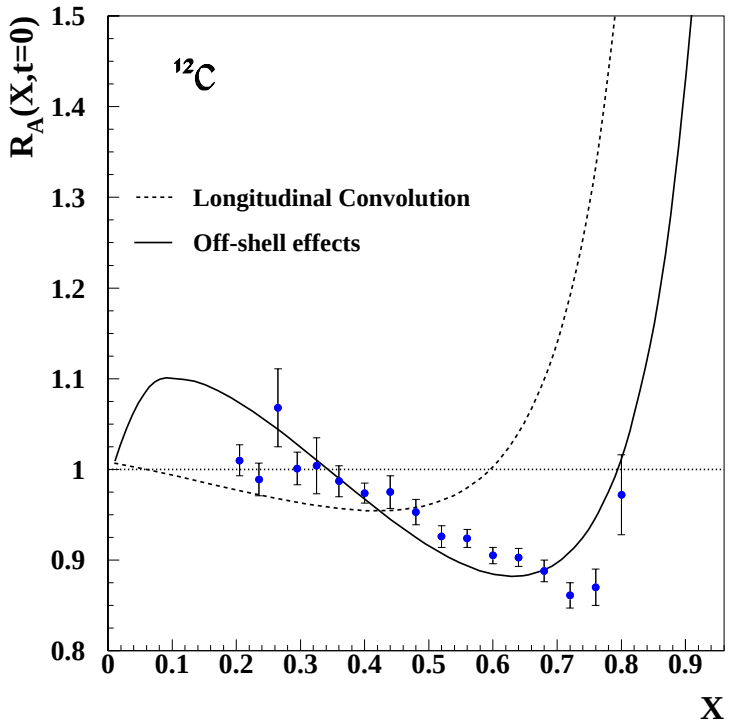
$$H^A(X,\zeta,t) = \int \frac{d^2P_\perp dY}{2(2\pi)^3}\rho_A\left[P^2(Y,P_\perp^2),P'^2(Y,P_\perp^2,\zeta,t)\right] \times\sqrt{\frac{Y-\zeta}{Y}}\left[H^N_{OFF}\left(\frac{X}{Y},\frac{\zeta}{Y},P^2,t\right)-\frac{1}{4}\frac{(\zeta/Y)^2}{1-\zeta/Y}E^N_{OFF}\left(\frac{X}{Y},\frac{\zeta}{Y},P^2,t\right)\right]$$

$$\sqrt{1-\zeta_N}H^N_{OFF}(X_N,\zeta_N,P^2,t)=\frac{X_N}{1-X_N}\sqrt{\frac{X_N-\zeta_N}{X_N}}\int\frac{d^2k_\perp}{2\pi}\widetilde{\rho}_N\left[k^2(P^2),k'^2(P^2)\right],$$

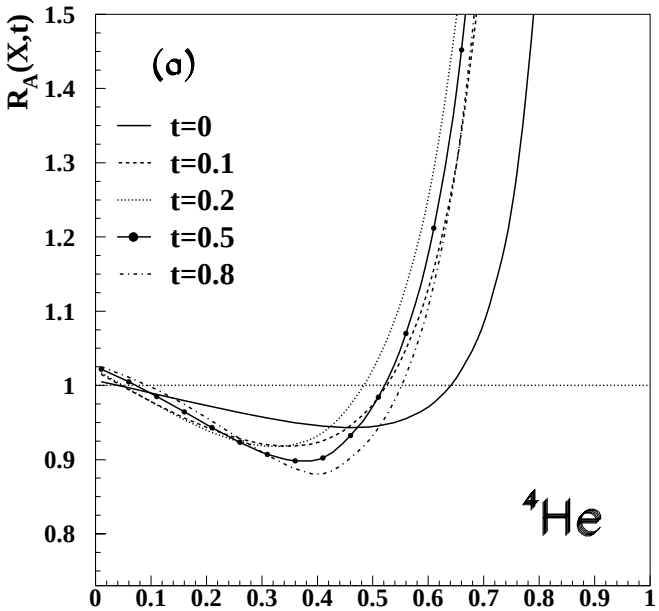
Forward EMC effect



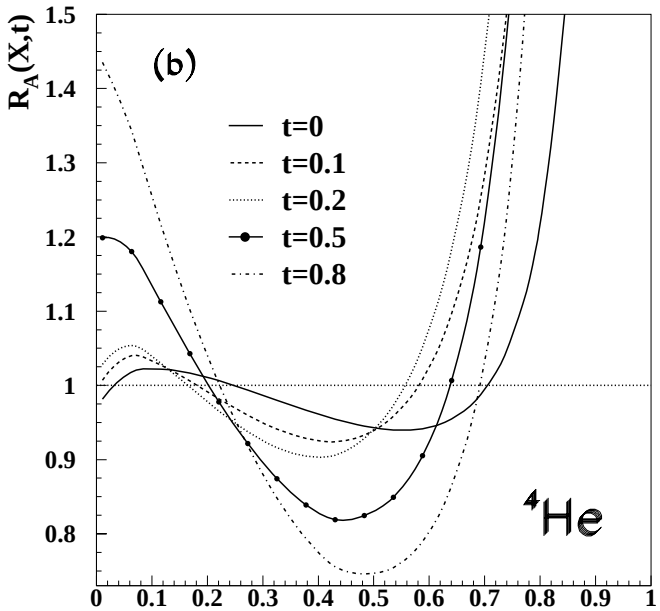
$$R_A = \frac{F_2^A(x,Q^2)}{F_2^N(x,Q^2)}$$



X-dependence



On-shell nucleons



Off-shell nucleons

$$R_A = [H_A(X,t)/F_A(t)]/[H_N(X,t)/F_N(t)]$$

Interpretations of results

Nuclear functions are peaked around $Y = 1$, and $P^2 = M^2$

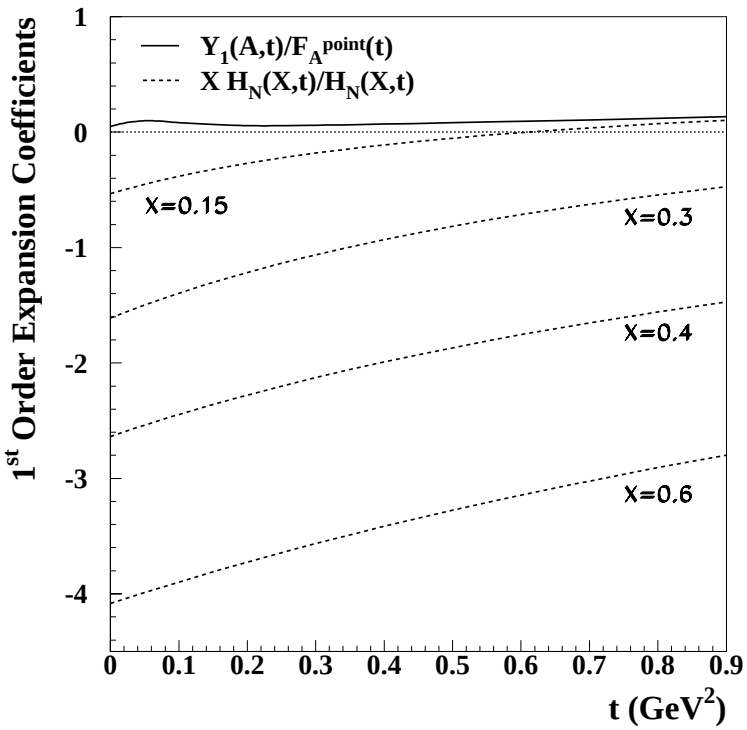
$H^A(X,t) \approx A H_N(X/\langle Y \rangle)$ t-dependent x-rescaling

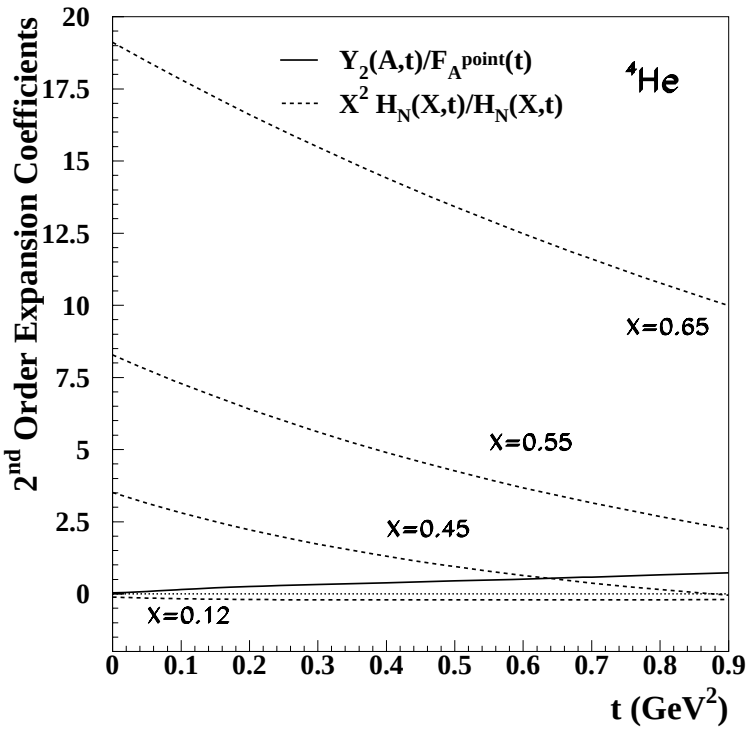
$\langle Y(A,t) \rangle = \int_0^A dY\, \rho_A(Y,t)(1-Y) \approx \frac{\langle E(t) \rangle_A}{F^{A,point}(t)}$

Role of nucleon off-shellness is same as in forward case $\Rightarrow$ affects the k_T dependence !

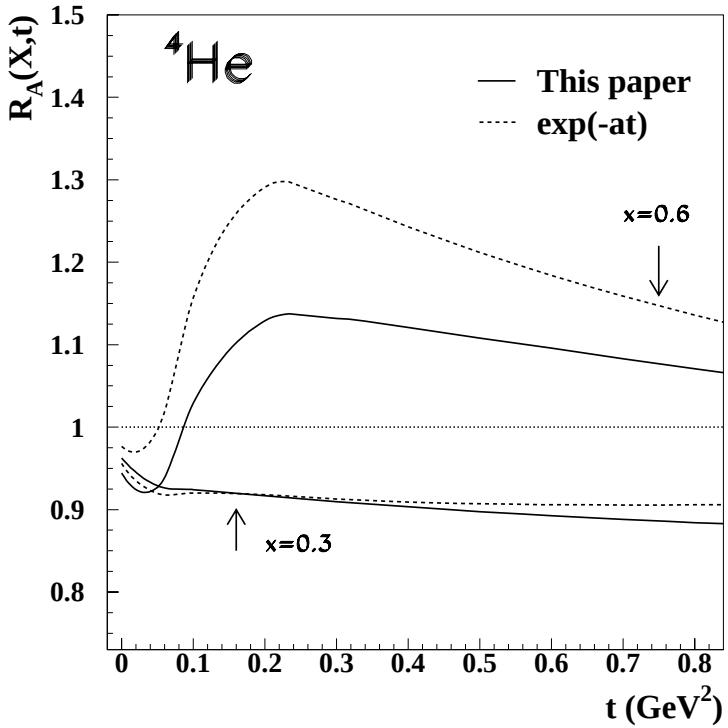
$$R^A(X,t) \approx 1 + \frac{\langle Y_1^A(t) \rangle}{F^{A,point}(t)} \times \left[\frac{1}{H^N(X)} \frac{\partial H^N(X,t)}{\partial X} X \right] + \ldots$$

$$
\begin{aligned}
F^{A,point}(t) &= \int_0^A dY\, \rho_A(Y,t) \\
\langle Y_1(A,t)\rangle &= \int_0^A dY\, \rho_A(Y,t)(1-Y) \\
\langle Y_2(A,t)\rangle &= \int_0^A dY\, \rho_A(Y,t)(1-Y)^2,
\end{aligned}
$$

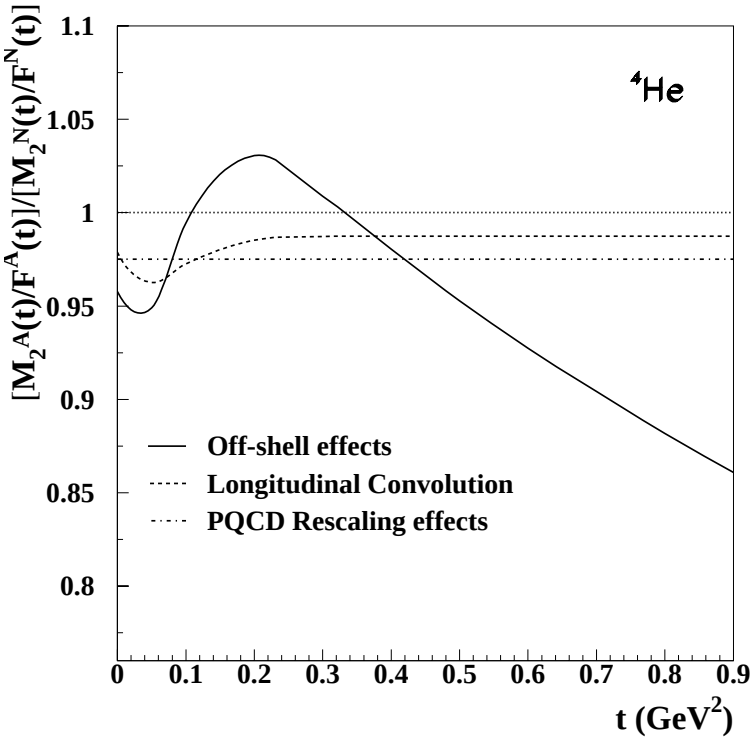

1st Order Expansion Coefficients
$Y_1(A,t)/F_A^{point}(t)$
$X\,H_N(X,t)/H_N(X,t)$
X=0.15
X=0.3
X=0.4
X=0.6
t (GeV^2)


2nd Order Expansion Coefficients
$Y_2(A,t)/F_A^{point}(t)$
$X^2\,H_N(X,t)/H_N(X,t)$
^{4}He
X=0.65
X=0.55
X=0.45
X=0.12
t (GeV^2)

t-dependence



Moments in Nuclei



Disentangle the origin of the effect

$$\int_{-A}^{A} dx H^A(x,\xi,t) = F^A(t)$$

$$\int_{-A}^{A} dx x H^A(x,\xi,t) = M_2^A(t) + \frac{4}{5} d_1^A(t)\xi^2,$$

In medium form factor

$$F^A(t) \quad = \quad F^{A,point}(t) F^N(t)$$

New component

$$M_2^A(\xi, t) \quad = \quad M_2^{A,point}(t) M_2^N(t) + M_0^{A,point}(t) \frac{4}{5} d_1^N(t) \xi^2$$

$d_1^A(0) \equiv$ component of the energy-momentum tensor describing the shear forces in the hadronic system

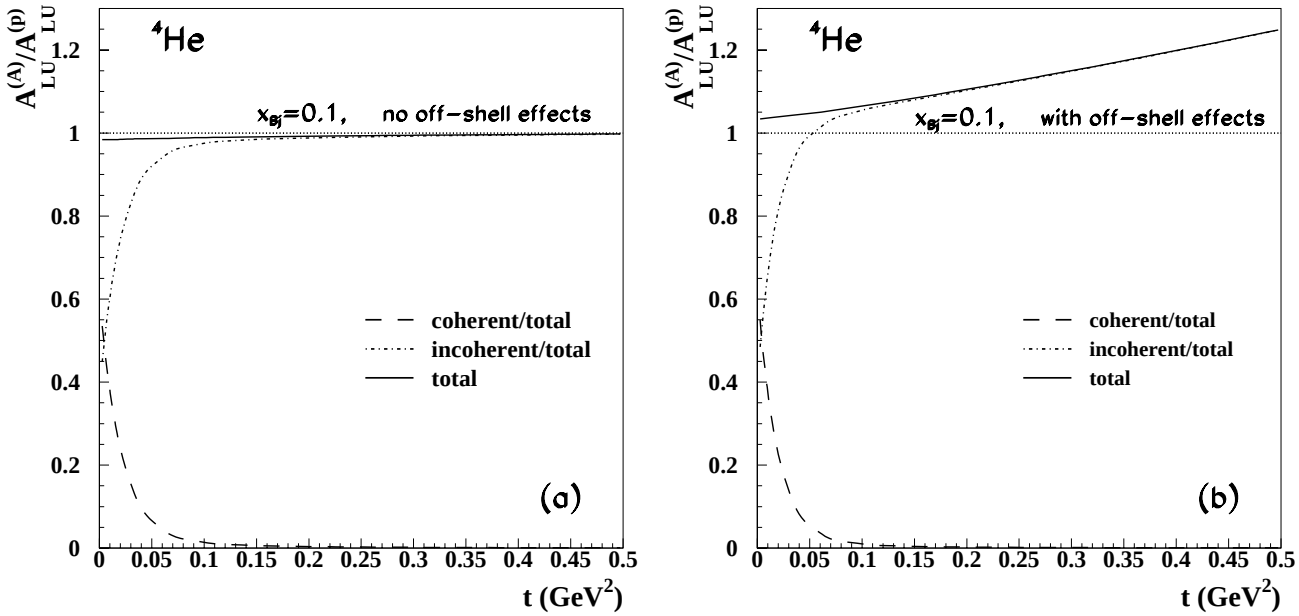
Predictions for large nuclei:

"Liquid Drop" M. Polyakov, PLB **555**, 57 (2003) $\Rightarrow d_1^A \propto A^{7/3}$

"Microscopic Model" $\Rightarrow d_1^A \propto A^\alpha$, $\alpha \approx 1.4$

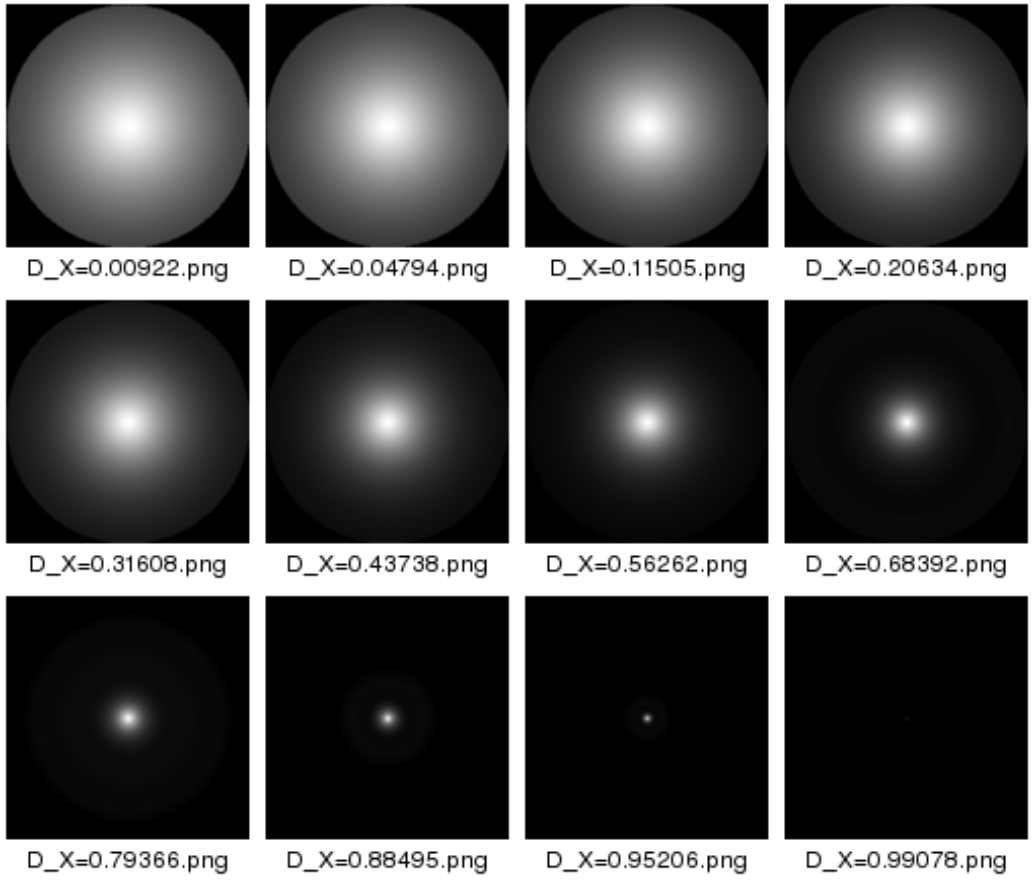
Experiments?

First "Preliminary" Data: F. Ellinghaus, R. Shanidze, and J. Volmer, hep-ex/0212019;

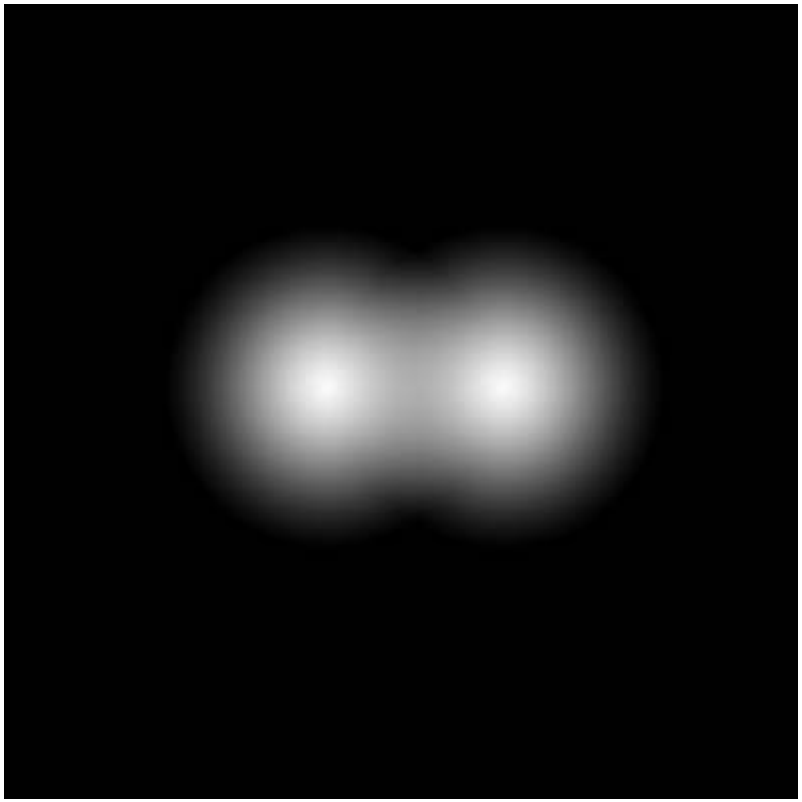


S.L and S.Taneja, hep-ph/0505123

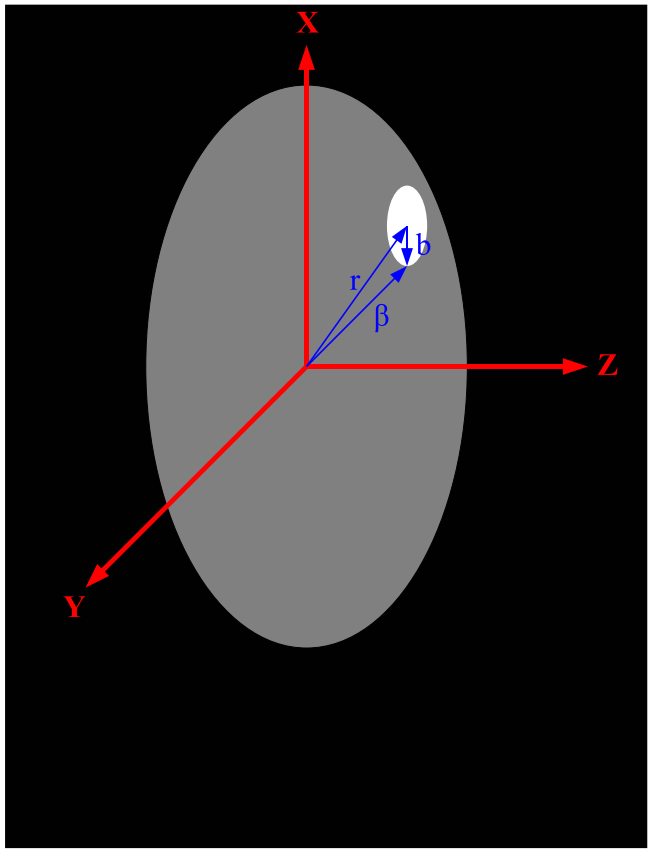
Overlapping volumes



Single Nucleon



... in a nucleus



$$q_A(X,\mathbf{r}) = \int_X^A \int dY \int d^2\mathbf{b}\, \widetilde{\rho}_A(Y,\mathbf{r}-\mathbf{b})\, q_N(X/Y,\mathbf{b})$$

$$
\begin{aligned}
X &= k^+/P^+ \\
Y &= P^+/P_A^+ \\
\beta &= \mathbf{r} - \mathbf{b}
\end{aligned}
$$

$q_N(X/Y, \mathbf{b})$ "in medium" nuclear GPD

$$\widetilde{\rho}_A(Y, \beta) = \frac{2}{\pi B^2} f_A(Y) h_A(\beta/B)$$

Overlap transverse area for a hard nuclear process

$$\begin{aligned}\langle b_A^2(X)\rangle \quad = \quad & \int_X^A dY \langle b_N^2(X/Y)\rangle q_N(X/Y) f_A(Y) + \\ & \int_X^A dY \langle \beta^2(Y)\rangle q_N(X/Y) f_A(Y) + \\ & \int_X^A 2dY \langle b_N(X/Y)\rangle \langle \beta(Y)\rangle q_N(X/Y) f_A(Y)\end{aligned}$$

$$A_{op} = \frac{\langle b_N^2(X)\rangle}{\langle b_A^2(X)\rangle}$$

Overlap volume previously calculated with "empirical" models!

Conclusions

• GPDs provide a whole new dimension for studying transverse d.o.f in hadrons through the interplay between k_T and b

• More constraints on GPDs from nuclei ...

• ... and at the same time: New insight on nuclear medium modifications from GPDs

• Our exploratory study includes both k_T and b dependent observables, vast phenomenology!

Outlook

• More sistematic studies are needed:

– Color Transparency

– Merging Nuclear Moments from Models and Lattice Results

– ...

• Treatment of Longitudinal Distances (P. Hoyer's talk) $\Rightarrow$ modeling dependence on skewedness ζ

• Model low x dependence

• Q^2 Evolution of $\langle b^2 \rangle$, $\langle k_T \rangle$.

- Extend studies to other GPDS: H_T E_T (connected to transversity) and their "coordinate space" counterparts.