Transverse Momentum in Hard Exclusive Processes

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Consider the simplest hard exclusive process $\gamma^*\gamma^* \to \pi^o$

Form factor $F_{\gamma^*\gamma^*\pi^o}\left(q_1^2, q_2^2\right)$ of e $\gamma^*\gamma^* \to \pi^o$ transition defined through nonforward matrix element

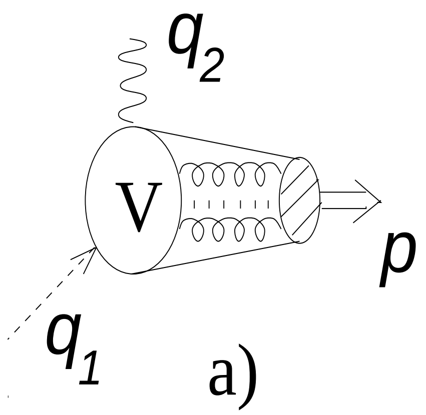
$$4\pi \int \langle \pi, \vec{p}\ | T \left\{ J_\mu(X)\, J_\nu(0) \right\} |0\rangle e^{-iq_1 X} d^4 X$$

$$= ie^2\sqrt{2}\,\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F_{\gamma^*\gamma^*\pi^0}\left(q_1^2, q_2^2\right)$$

J_μ: electromagnetic current of light quarks

$$J_\mu = e_u \bar{u}\gamma_\mu u + e_d \bar{d}\gamma_\mu d$$

$|\pi, \vec{p}\rangle$: one-pion state with 4-momentum p.



Structure of factorization for $F_{\gamma^*\gamma\pi^0}(Q^2)$ at large Q^2.

Large virtuality goes through subgraph V containing both photon vertices.

Leading $1/Q^2$ contribution comes from

$$F_{\gamma^*\gamma^*\pi^0}(q_1,q_2) = \int C(\xi,\eta,q_1,q_2;\mu^2)\,\langle p|\mathcal{O}(\xi,\eta)|0\rangle|_{\mu^2} d^4\xi d^4\eta\,,$$

$C(\xi,\eta,q_1,q_2)$: short-distance amplitude

$\mathcal{O}(\xi,\eta) \sim \bar{q}(\xi)\gamma_5\gamma_\nu E(\xi,\eta;A)q(\eta)$: composite operator

μ^2: factorization scale

$E(\xi,\eta;A)$: path-ordered exponential

$$E(\xi,\eta;A) \equiv P\exp\left(ig\int_\eta^\xi A_\mu(z)dz^\mu\right)$$

Results from summation over external gluon lines of V

For quark propagator:

$$S^c(\xi-\eta) + \int S^c(\xi-z)\gamma^\mu g A_\mu(z) S^c(z-\eta)\,d^4z + \ldots$$

$$= E(\xi,\eta;A)\,S^c(\xi-\eta)[1+O(G)]$$

$O(G)$ depends on gluonic fields through field strength tensor $G_{\mu\nu}$ and its covariant derivatives

Leads to operator product expansion for $J^\alpha(0)J^\beta(z)$

To get the lowest-twist part, expand

$$\bar{q}(\xi)\gamma_5\gamma_\nu E(\xi,\eta;A)q(\eta)$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\Delta^{\nu_1}\Delta^{\nu_2}\ldots\Delta^{\nu_n}\bar{q}(\xi)\gamma_5\gamma_\nu D_{\nu_1}D_{\nu_2}\ldots D_{\nu_n}q(\xi)$$

$\Delta = \eta - \xi$

and pick out symmetric-traceless part $\bar{q}\gamma_5\{\gamma_\nu D_{\nu_1}D_{\nu_2}\ldots D_{\nu_n}\}q$ of each local operator

Traces: operators with contracted covariant derivatives $D^\nu D_\nu$ accompanied by powers of $(\xi-\eta)^2 \Rightarrow$ extra powers of $z^2 \Rightarrow$ extra powers of $1/Q^2$

Twist-2 part: lowest term of expansion over $(\xi-\eta)^2$

$$\mathcal{O}(\xi,\eta) = \mathcal{O}(\xi,\eta)|_{(\xi-\eta)^2=0} + O((\xi-\eta)^2).$$

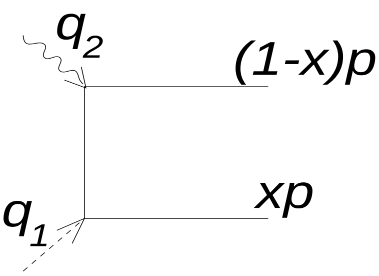
Parametrization in terms of pion distribution amplitude (DA)

$$\langle 0|\mathcal{O}_\nu(\xi,\eta)|\pi^0,p\rangle\big|_{(\xi-\eta)^2=0} = ip_\nu \int_0^1 e^{-ix(\xi p)-i\bar{x}(\eta p)}\varphi_\pi(x)dx$$

$\varphi_\pi(x)$: gives probability amplitude that pion with momentum p is a $\bar{q}q$ pair with momenta xp and $\bar{x}p \equiv (1-x)p$

Hard scattering formula

$$F_{\gamma^*\gamma\pi^0}(q_1,q_2) = \frac{4\pi}{3}\int_0^1 T(q_1,q_2;xp,\bar{x}p)\,\varphi_\pi(x)\,dx \ ,$$



Lowest-order diagram.

Neglect of higher-twist operators with extra D^2 is similar to taking $k^2=0, \bar{k}^2=0$ for quark momenta or collinear approximation $k=k^+=xp^+$

Lowest order amplitude:

$$T_0(x,Q^2) = \frac{1}{-(q_1-xp^+)^2} = \frac{1}{xQ^2}$$

Proposal to include primordial transverse momentum into handbag diagram: $k = xp^+ + k_\perp$. Then

$$\frac{1}{-(q_1-k)^2} = \frac{1}{xQ^2 + k_\perp^2} = \frac{1}{xQ^2} - \frac{k_\perp^2}{(xQ^2)^2} + \ldots$$

$\Rightarrow$ infinite series of $(k_\perp^2/Q^2)^n$ terms that one may wish to interpret as finite size effects

Known fact: Handbag diagram contains only hadron-mass corrrections

Take toy scalar analog

$$F(q_2,p) = \frac{1}{4\pi^2}\int e^{-iq_2 z}\langle 0|\phi(0)\phi(z)|p\rangle \frac{d^4z}{z^2}\,.$$

First term in z^2-expansion for matrix element

$$\langle 0|\phi(0)\phi(z)|p\rangle = \xi_2(zp) + z^2\xi_4(zp) + (z^2)^2\xi_6(zp) + \ldots$$

corresponds to twist-2 distribution amplitude,
further terms to operators with ∂^2's

Twist-2 term produces $1/Q^2$ contribution
Twist-4 and higher terms have z^2-factors
killing $1/z^2$-singularity of “quark” propagator,
$\Rightarrow$ d^4z integration gives $\sim \delta^4(q_2 - \bar{x}p)$
$\Rightarrow$ invisible for large Q^2

$\Rightarrow$ handbag diagram contains only one term with a powerlike behavior for large Q^2:
it cannot generate higher powers of $1/Q^2$ which one could interpret as the $(\langle k^2\rangle/Q^2)^n$ expansion

Paradox:

$$\frac{1}{-(q-k)^2} = \frac{1}{Q^2 + 2(qk) - k^2}$$

apparently generates $(k^2/Q^2)^n$ series

Solution: it also generates $[2(qk)/Q^2]^n$ series, and $(qk)^n \sim (q\partial)^n$ should be expanded over traceless combinations $\{q_{\mu_1} \dots q_{\mu_n}\}\{k^{\mu_1} \dots k^{\mu_n}\}$ producing extra k^2 terms.

Unbelievably, they exactly cancel original k^2 terms:

$$\frac{1}{(q-k)^2} = \theta(|k| < |q|) \sum_{n=0}^{\infty} \frac{2^n}{(q^2)^{n+1}} q^{\mu_1} \dots q^{\mu_n} \{k_{\mu_1} \dots k_{\mu_n}\}$$

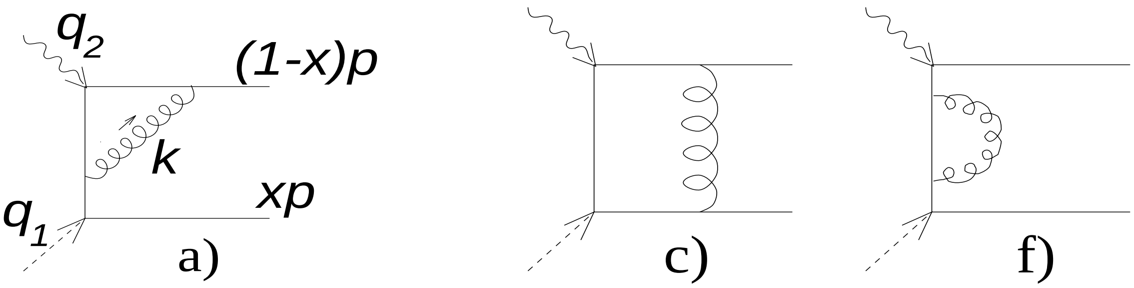
$$+\theta(|k| > |q|)\text{terms}$$

Similar to Legendre polynomial expansion of $1/|\mathbf{r_1} - \mathbf{r_2}|$ in 3 dimensions

$\Rightarrow$ The only source of power corrections in toy model are operators like $\phi(0)\phi(z_1)\phi(z)$ with more than 2 fields

In QCD: infinite tower of $1/Q^2$ corrections comes from $\bar{q}G \dots Gq$ operators
$\Rightarrow$ Their matrix elements are related to average values of field strength, and one may expect that $\langle G \rangle \sim \langle k_\perp^2 \rangle$, but it seems unlikely that the sum is as simple as $1/(xQ^2 + k_\perp^2)$

Perturbative source of transverse momentum: radiative corrections



One-loop diagrams.

Sudakov decomposition:

gluon momentum: $k = (\xi - x)p + \eta q + k_\perp$
quark momentum: $(1 - \xi)p + \eta q + k_\perp$
$d^4k \Rightarrow \quad d^2k_\perp \, d\xi d\eta$

After taking η-integral by residue

$$T_i(x, Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^1 d\xi \int M_i(x, Q^2; \xi, k_\perp) \frac{d^2k_\perp}{2\pi}$$

For diagram a):

$$M_a(x, Q^2; \xi, k_\perp) = \frac{1}{xQ^2} \left\{ - \left(\frac{\bar{\xi}}{\bar{x}} \right) \frac{Q^2 + k_\perp^2/\bar{\xi}}{k_\perp^2 \left[\xi Q^2 + k_\perp^2/\bar{\xi} \right]} \theta(\xi > x) \right.$$
$$\left. + \frac{k_\perp^2 \, \theta(\xi < x)}{\left[\xi Q^2 + k_\perp^2/\bar{\xi} \right] \left[\xi(x - \xi)Q^2 + xk_\perp^2 \right]} \right\}$$

Concentrate on part singular for $k_\perp = 0$

$$M_a^{sing}(x, Q^2; \xi, k_\perp) = -\frac{1}{xQ^2} \frac{Q^2}{k_\perp^2 \left[\xi Q^2 + k_\perp^2/\bar{\xi}\right]} \left(\frac{\bar{\xi}}{\bar{x}}\right) \theta(\xi > x)$$

Collinear divergence may be regulated by $k_\perp^2 \to k_\perp^2 + m^2$ producing evolution logarithm $\ln(Q^2/m^2)$. Using

$$-\frac{1}{\left[\xi Q^2 + k_\perp^2/\bar{\xi}\right] xQ^2} = \left(\frac{1}{\xi Q^2 + k_\perp^2/\bar{\xi}} - \frac{1}{xQ^2}\right) \frac{1}{(\xi - x)Q^2 + k_\perp^2/\bar{\xi}}$$

and taking $k_\perp^2 = 0$ when it is added to $O(Q^2)$ terms gives

$$\frac{1}{\xi Q^2} \left[\left(\frac{\bar{\xi}}{\bar{x}}\right) \frac{\theta(\xi > x)}{\xi - x}\right]_+ \ln\left(\frac{Q^2}{m^2}\right) \quad ,$$

product of the "Born" term $1/\xi Q^2$ and V_a part of ERBL kernel. "Plus" prescription:

$$[F(\xi, x)]_+ = F(\xi, x) - \delta(\xi - x) \int_0^1 F(\zeta, x)\, d\zeta$$

NB: in impact parameter $b_\perp$ representation
[$b_\perp$ is Fourier-conjugate to $k_\perp$]
difference $[\ldots k_\perp \ldots] - [\ldots k_\perp = 0 \ldots]$ produces
$e^{ik_\perp b_\perp} - 1$ factor vanishing at $b_\perp = 0$.

Similarly, it makes sense to isolate $x = \xi$ part, where longitudinal momentum does not change

Factorization in $b_\perp$ space

$$M_a^{sing}(x,Q^2) = \frac{\alpha_s}{2\pi} C_F \int_0^1 d\xi \int \; B(\xi; bQ) \; \big[V_a(\xi,x) L(bm)$$

$$+\delta(\xi - x) S_a(x, bQ) + E_a(x,\xi; bQ) \big] \frac{d^2 b_\perp}{2\pi}$$

Born term

$$B(\xi; bQ) = \frac{1}{2\pi} \int \frac{e^{-ik_\perp b_\perp}}{\xi Q^2 + k_\perp^2/\bar{\xi}} d^2 k_\perp = \bar{\xi} K_0 \left(bQ\sqrt{\xi\bar{\xi}} \right)$$

Evolution term

$$V_a(\xi,x) L(bm) \quad , \quad L(bm) = \frac{1}{2\pi} \int \frac{d^2 k_\perp e^{ik_\perp b_\perp}}{k_\perp^2 + m^2} = K_0(bm)$$

"Sudakov" term

$$S_a(x; bQ) = \frac{1}{2\pi} \int d^2 k_\perp \frac{e^{ik_\perp b_\perp} - 1}{k_\perp^2} \int_0^1 \left(\frac{\bar{\zeta}}{\bar{x}} \right) \; \frac{\theta(\zeta > x) d\zeta}{\zeta - x + k_\perp^2/\bar{\zeta} Q^2}$$

NB: singularity of evolution kernel at $\zeta = x$ is regularized here by $k_\perp^2/\bar{\zeta}Q^2$ rather than by $+$ prescription.

Evolution-related term

$$E_a(x,\xi; bQ) = -\left[\frac{\bar{\xi}}{\bar{x}} \; \frac{\theta(\xi > x)}{\xi - x} \; K_0 \left(bQ\sqrt{(\xi - x)\bar{\xi}} \right) \right]_+$$

$b_\perp$ space (or modified) factorization at one loop level

$$F_{\gamma^*\gamma\pi^0}(Q^2) = \frac{4\pi}{3}\int_0^1 \varphi_\pi(x)\,dx\,\left\{\frac{1}{xQ^2} + \frac{\alpha_s}{2\pi}C_F\int_0^1 d\xi \int \frac{d^2b_\perp}{2\pi}\right.$$

$$\times B(\xi;bQ)\Big[V(\xi,x)\;L(bm) + E(\xi,x;bQ) + \delta(\xi - x)S(x,bQ)$$

$$\left. +R(\xi,x;bQ)\Big]\;\right\}$$

involves full evolution kernel $V(\xi,x)$. Terms, nonsingular at $k_\perp = 0$, give $R(\xi,x;bQ)$

Born term $B(\xi;bQ)$ and evolution terms $L(bm)$, $E_a(x,\xi;bQ)$ exponentially decrease at large b: $K_0\,(b\ldots) \sim \exp(-b\ldots)$

Sudakov terms are doubly-logarithmic in b, e.g.,

$$S_a(x;bQ) \approx \frac{1}{2\pi}\int d^2k_\perp \frac{e^{ik_\perp b_\perp}-1}{k_\perp^2}\ln\left(\frac{\bar{x}Q}{k_\perp}\right)$$

$$\approx \int_{1/b} \frac{dk_\perp}{k_\perp}\ln\left(\frac{k_\perp}{\bar{x}Q}\right) \approx -\frac{1}{2}\ln^2(\bar{x}Qb),$$

$\Rightarrow$ Resummation is needed

NB: Derivation was done in perturbation theory
$\Rightarrow$ $d^2b_\perp$ integration gives the same result as standard pQCD factorization

NB: Except $L(bm) = L(bQ * m/Q)$, $b_\perp$ everywhere appears through bQ. After $d^2 b_\perp$ integration

$$F_{\gamma^*\gamma\pi}(Q^2) = \frac{4\pi}{3}\int_0^1 dx\, \frac{\varphi_\pi(x)}{xQ^2}\left\{1 + C_F \frac{\alpha_s}{2\pi}\left[\left(\frac{3}{2} + \ln x\right)\ln\left(\frac{Q^2}{m^2}\right) + \frac{1}{2}\ln^2 x - \frac{x\ln x}{2(1-x)} - \frac{9}{2}\right]\right\}$$

mass logarithm $\ln(Q^2/m^2)$ is accompanied by

$$\frac{1}{x}\left(\frac{3}{2} + \ln x\right) = \int_0^1 \frac{d\xi}{\xi} V(\xi, x)$$

forming standard evolution combination

$$\left[\delta(\xi - x) + \frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{m^2}\right) V(\xi, x)\right]\varphi_\pi(x)$$

suggesting the change $\varphi_\pi(x) \to \varphi_\pi(\xi, Q^2)$

In the impact parameter representation,

$$\varphi_\pi(\xi) - \frac{\alpha_s}{2\pi}\ln(bm)\int_0^1 V(\xi, x)\,\varphi_\pi(x)\,dx$$

suggesting the change $\varphi_\pi(x) \to \varphi_\pi(\xi, 1/b^2)$
Symbolically:

$$\varphi(\xi, 1/b^2) = \exp\left[-\frac{\alpha_s}{2\pi}\ln(bm)V\right](\xi, x) \otimes \varphi(x)$$

NB: $\varphi(\xi, 1/b^2)$ is usual ("collinear") pion DA, with $1/b^2$ serving as factorization scale

NB: Transverse momentum appeared only as integration variable $k_\perp$: external momenta q_1, q_2 and $p = q_1 + q_2$ are "longitudinal" in Sudakov parametrization.

In light-cone (IMF) formalism $p^+ \to \infty$, and q_1 is transverse $q_1 = q_\perp$, $q_\perp^2 = Q^2$, $(k_\perp q_\perp) = |k_\perp| Q \cos\phi$,

$$(\epsilon_\perp \times q_\perp) F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi^2\sqrt{6}} \int_0^1 dx \int \Psi(x, k_\perp)\, d^2k_\perp$$

$$\frac{(\epsilon_\perp \times (xq_\perp + k_\perp))}{(xq_\perp + k_\perp)^2 - i\epsilon}$$

Expansion over $k_\perp$ gives only traceless combinations:

$$q_\perp \frac{xq_\perp + k_\perp}{(xq_\perp + k_\perp)^2} = \frac{xQ^2 + |k_\perp| Q\cos\phi}{x^2Q^2 + 2x|k_\perp|Q\cos\phi + k_\perp^2}$$

$$= \frac{1}{x}\left\{\theta(|k_\perp| < xQ) + \sum_{n=1}^{\infty}(-1)^n \left[\left(\frac{|k_\perp|}{xQ}\right)^n \theta(|k_\perp| < xQ)\right.\right.$$

$$\left.\left. - \left(\frac{xQ}{|k_\perp|}\right)^n \theta(|k_\perp| > xQ)\right] \cos(n\phi)\right\}$$

Chebyshev polynomials $\cos(n\phi) = T_n(\cos\phi)$ correspond to traceless combinations in two dimensions.
If $\Psi(x, k_\perp) = \psi(x, k_\perp^2)$,

$$F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{2}{\pi\sqrt{6}} \int_0^1 \frac{dx}{xQ^2} \int_0^{xQ} \psi(x, k_\perp^2)\, k_\perp dk_\perp$$

No modification of $1/xQ^2$ factor by $k_\perp$ effects

Gaussian model wave function

$$\Psi^{(G)}(x,k_\perp) = \frac{4\pi^2}{\sigma\sqrt{6}}\,\frac{\varphi_\pi(x)}{x\bar{x}}\,\exp\left(-\frac{k_\perp^2}{2\sigma x\bar{x}}\right)$$

can be integrated explicitly

$$F^{\bar{q}q,G}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2}\left[1-\exp\left(-\frac{xQ^2}{2\bar{x}\sigma}\right)\right]dx$$

Reproduces LO pQCD formula
No power corrections under x-integral
Gives finite result for $Q^2=0$

$$F^{\bar{q}q,G}_{\gamma^*\gamma\pi^0}(Q^2=0) = \frac{2\pi}{3\sigma}\int_0^1 \frac{\varphi_\pi(x)}{\bar{x}}dx$$

Value of $F_{\gamma^*\gamma\pi^0}(Q^2=0)$ is restricted by axial anomaly

$$F_{\gamma^*\gamma\pi^0}(0) = \frac{1}{\pi f_\pi}$$

Can be used to fix σ for given $\varphi_\pi(x)$.
Subtlety: $F^{\bar{q}q}_{\gamma^*\gamma\pi^0}(0)$ gives only 1/2 of $F_{\gamma^*\gamma\pi^0}(0)$.
Constraint:

$$\int_0^1 \Psi(x,k_\perp=0)\,dx \;=\; \frac{\sqrt{6}}{f_\pi}$$

or, for Gaussian ansatz

$$\frac{1}{f_\pi}\int_0^1 \varphi_\pi(x)\frac{dx}{x} = \frac{3\sigma}{4\pi^2 f_\pi^2}$$

Remaining 1/2 of $F_{\gamma^*\gamma\pi^0}(Q^2 = 0)$ comes from $\bar{q}q\gamma$ Fock component: gluons attached between two γ vertices. At large Q^2, this term is given by higher twist $\bar{q}G\ldots q$ operators $\Rightarrow$ nonperturbative constraint on their $Q^2 = 0$ contribution

Brodsky-Lepage interpolation

$$F^{int,BL}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi f_\pi \left(1 + \frac{Q^2}{4\pi^2 f_\pi^2}\right)} \equiv \frac{1}{\pi f_\pi (1 + Q^2/s_0)}$$

satisfies both pQCD asymptotics for large Q^2

$$F^{as}_{\gamma^*\gamma\pi^0}(Q^2 \to \infty) = \frac{4\pi f_\pi}{Q^2}$$

and $Q^2 = 0$ constraint $F_{\gamma^*\gamma\pi^0}(0) = 1/\pi f_\pi$

Important scale: $s_0 = 4\pi^2 f_\pi^2 \approx 0.67\, GeV^2 \sim m_\rho^2$

Appears also as pion duality interval in quark-hadron duality relation for a correlator of two axial currents

$$\int_0^{s_0} \rho^\pi(s)\, ds = \int_0^{s_0} \rho^{quark}(s)\, ds$$

with $\rho^\pi(s) = \pi f_\pi^2 \delta(s - m_\pi^2)$ and $\rho^{quark}(s) = 1/4\pi$

Quark spectral density is obtained from cut one-loop diagram

$$\rho^{quark}(s) = \frac{3}{2\pi^2}\int_0^1 dx \int \delta\left(s - \frac{k_\perp^2}{x\bar{x}}\right) d^2k_\perp$$

Duality relation then gives

$$f_\pi^2 = \frac{3}{2\pi^3}\int_0^1 dx \int \theta\left(k_\perp^2 \leq x\bar{x}s_0\right) d^2k_\perp$$

Structure similar to expression for f_π in the light-cone formalism

$$f_\pi = \sqrt{6}\int_0^1 dx \int \Psi(x,k_\perp)\frac{d^2k_\perp}{8\pi^3}$$

Introduce "local duality" wave function for pion:

$$\Psi^{LD}(x,k_\perp) = \frac{2\sqrt{6}}{f_\pi}\theta(k_\perp^2 \leq x\bar{x}s_0)$$

$b_\perp$-space version

$$\widetilde{\Psi}^{LD}(x,b_\perp) = \frac{\sqrt{6}}{\pi f_\pi b_\perp}\sqrt{x\bar{x}s_0}\, J_1(b_\perp\sqrt{x\bar{x}s_0})$$

Local duality relation for $F_{\gamma^*\gamma\pi^0}(Q^2)$ form factor

$$F_{\gamma^*\gamma\pi^0}^{LD}(Q^2) = \frac{1}{\pi f_\pi}\int_0^{s_0} \rho^{quark}(s,Q^2)\, ds$$

Spectral density

$$\rho^{quark}(s,Q^2) = 2\int_0^1 \frac{x\bar{x}(xQ^2)^2}{[sx\bar{x}+xQ^2]^3}\,dx$$

Substituting $s = k_\perp^2/(x\bar{x})$:

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{2}{\pi^2 f_\pi}\int_0^1 dx \int \frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\,\theta(k_\perp^2 \le x\bar{x}s_0)\,d^2k_\perp$$

Explicit calculation gives

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi f_\pi(1+Q^2/s_0)}$$

i.e., Brodsky-Lepage interpolation formula

Connection with pQCD Born term

$$\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3} = \frac{1}{xQ^2+k_\perp^2} - \frac{2k_\perp^2}{(xQ^2+k_\perp^2)^2} + \frac{k_\perp^4}{(xQ^2+k_\perp^2)^3}$$

Differ only by $O(k_\perp^2)$ terms invisible in the analysis of effects induced by the $1/k_\perp^2$ singularity at small $k_\perp$. Difference is very essential when one extrapolates into the region of small Q^2.

Finite LD value at $Q^2 = 0$ is due to

$$\left.\frac{\mu^4}{(\mu^2+k_\perp^2)^3}\right|_{\mu^2\to 0} \to \frac{1}{2}\,\delta(k_\perp^2)$$

Taking only the first, "natural" term gives

$$F_{\gamma^*\gamma\pi^0}(Q^2) = \frac{2}{\pi^2 f_\pi}\int_0^1 dx \int \frac{\theta(k_\perp^2 \leq x\bar{x}s_0)\, d^2k_\perp}{(xQ^2 + k_\perp^2)}$$

that diverges for $Q^2 = 0$.

In terms of effective LD wave function

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\pi^2\sqrt{6}}\int_0^1 dx \int \frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\,\Psi^{LD}(x,k_\perp)\, d^2k_\perp$$

In the impact parameter representation

$$F^{LD}_{\gamma^*\gamma\pi^0}(Q^2) = \frac{1}{\sqrt{6}}\int_0^1 dx \int xQ^2b^2\, K_2\left(\sqrt{x}bQ\right)\widetilde{\Psi}^{LD}(x,b_\perp)\,\frac{d^2b_\perp}{2\pi}$$

$K_2\left(\sqrt{x}bQ\right)$: from LD Born term written in the b-space

$$\frac{1}{2\pi}\int e^{-ik_\perp b_\perp}\frac{(xQ^2)^2}{(xQ^2+k_\perp^2)^3}\, d^2k_\perp = \frac{1}{4}xQ^2b^2K_2\left(\sqrt{x}bQ\right)$$

Local duality formula exactly reproduces pQCD asymptotics and also $F_{\gamma^*\gamma\pi^0}(0)$ value dictated by axial anomaly.

In this sense $\Psi^{LD}(x,k_\perp)$ is effective wave function describing soft content of all $\bar{q}G\ldots Gq$ Fock components of the usual light-cone approach

SUMMARY

We analyzed the simplest hard exclusive process $\gamma^*\gamma \to \pi^0$

Trying to express the structure of $F_{\gamma^*\gamma\pi^0}(Q^2)$ in terms of transverse degrees of freedom, we

- Established that handbag diagram is rather insensitive to $k_\perp$ and gives the main result in terms of longitudinal distribution $\varphi(x)$
- One-loop perturbative corrections generate evolution of pion DA $\varphi(x) \to \varphi(x, Q^2)$ or $\varphi(x) \to \varphi(x, 1/b^2)$ in impact parameter space
- Recovered $b_\perp$ space factorization, with soft Sudakov factor, and established that $b_\perp$ integration reproduces the results of the standard factorization (which only uses longitudinal distributions)
- Found that nonperturbative terms are crucial to produce correct behavior in low Q^2 region

Future: similar techniques should be used for DVCS

Lesson: small-$p_\perp$ behavior in SIDIS may be tricky, and definitely nonperturbative