Generalised Parton Distributions and Transversity from Lattice QCD

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Motivation

- Unique opportunity for lattice calculations to compliment experiment
- Assist in the full mapping of the parameter space spanned by GPDs

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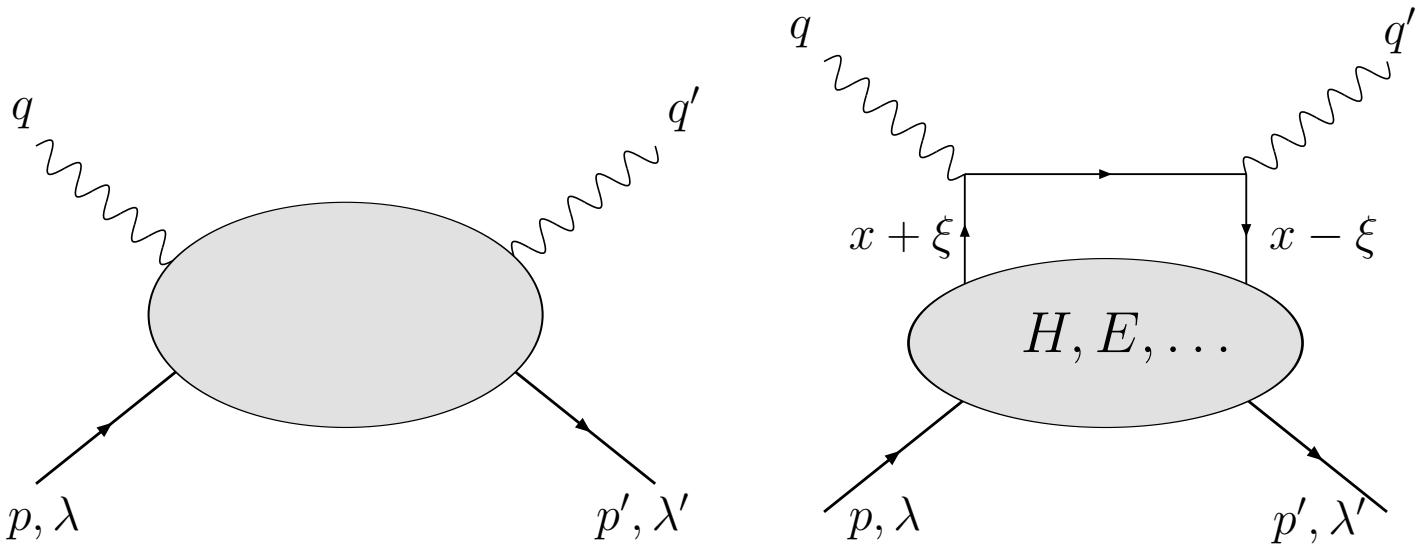


$$\Delta = p' - p,\ t = \Delta^2,\ \overline{p} = \frac{p' + p}{2},\ \xi = -n \cdot \Delta,\ n \cdot \overline{p} = 1$$

Generalised Parton Distributions

- GPDs are defined through nucleon matrix elements of the light cone operators

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle P'|\overline{\psi}(-\frac{\lambda}{2}n)\gamma^\mu \psi(\frac{\lambda}{2}n)|P\rangle = \overline{U}(P')\left(\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} E(x,\xi,t)\right) U(P)$$

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle P'|\overline{\psi}(-\frac{\lambda}{2}n)\gamma_5\gamma^\mu \psi(\frac{\lambda}{2}n)|P\rangle = \overline{U}(P')\left(\gamma_5\gamma^\mu \widetilde{H}(x,\xi,t) + \frac{i\gamma_5\Delta^\mu}{2m} \widetilde{E}(x,\xi,t)\right) U(P)$$

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle P'|\overline{\psi}(-\frac{\lambda}{2}n)\sigma^{\mu\nu} \psi(\frac{\lambda}{2}n)|P\rangle = \overline{U}(P')\left(i\sigma^{\mu\nu} H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m} E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m^2} \widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m} \widetilde{E}_T(x,\xi,t)\right) U(P)$$

Generalised Parton Distributions

- The forward limit $t = \Delta^2 \to 0$ reproduces the parton distributions, $H(x,0,0) = q(x)$, $\widetilde{H}(x,0,0) = \Delta q(x)$ and $H_T(x,0,0) = \delta q(x)$
- $\int dx \ \to$ Dirac, Pauli, axial, pseudo-scalar, tensor etc. form factors, $\int dx H(x,\xi,t) = F_1(t)$, $\int dx E = F_2(t)$, $\int dx \widetilde{H} = g_A(t)$, ...
- FT $\int d\Delta_\perp e^{-ib_\perp \cdot \Delta_\perp}$ GPDs H, $\widetilde{H}$ and H_T ($\xi = 0$) are coordinate space probability densities in the impact parameter $b_\perp$
- $\int dx x E(x,0,0) = B_{20}(0)$, $\to$ quark orbital angular momentum contribution to the nucleon spin, $L^q = 1/2(\langle x \rangle + B_{20} - \Delta q)$, where $\langle x \rangle$ is the quark momentum fraction

Generalised Form Factors

- Construct Mellin moments
- non-forward MEs of tower of local twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} = \overline{q}\,\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\,q$$

$\longrightarrow$ Generalised Form Factors

$$\langle p',\,s'|\mathcal{O}^{\{\mu_1\cdots\mu_n\}}(\Delta)|p,\,s\rangle =$$

$$\bar{u}(p',\,s')\gamma^{\{\mu_1}u(p,\,s)\sum_{i=0}^{\frac{n-1}{2}}A_{qn,2i}(t)\Delta^{\mu_2}\cdots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\cdots\overline{p}^{\mu_n\}}$$

$$+\bar{u}(p',\,s')\frac{i\sigma^{\{\mu_1\nu}\Delta_\nu}{2m}u(p,\,s)\sum_{i=0}^{\frac{n-1}{2}}B_{qn,2i}(t)\Delta^{\mu_2}\cdots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\cdots\overline{p}^{\mu_n\}}$$

$$+C_{qn}(t)\,\frac{1}{m}\bar{u}(p',\,s')u(p,\,s)\Delta^{\mu_1}\cdots\Delta^{\mu_n}|_{\text{n even}}$$

- Similar for

$$\tilde{A}_{qn,2i}(t),\ \tilde{B}_{qn,2i}(t),\ A^T_{qn,2i}(t),\ B^T_{qn,2i}(t),\ \tilde{A}^T_{qn,2i}(t),\ \tilde{B}^T_{qn,2i}(t)$$

Moments of GPDs

- GPDs are defined through their moments.

$$\int_{-1}^{1} dx\, x^{n-1}\, H_q(x,\xi,t) = H_{qn}(\xi,t)$$
$$\int_{-1}^{1} dx\, x^{n-1}\, E_q(x,\xi,t) = E_{qn}(\xi,t)$$

where $\xi = -n\cdot\Delta$, $n\cdot\bar{p} = 1$, $\bar{p} = \frac{1}{2}(p' + p)$ and

$$H_{qn}(\xi,t) = \sum_{i=0}^{\frac{n-1}{2}} A_{qn,2i}(t)(-2\xi)^{2i} + C_{qn}(t)(-2\xi)^n|_{\text{n even}}$$
$$E_{qn}(\xi,t) = \sum_{i=0}^{\frac{n-1}{2}} B_{qn,2i}(t)(-2\xi)^{2i} - C_{qn}(t)(-2\xi)^n|_{\text{n even}}$$

Moments of GPDs II

- zeroth moment, $n = 1$

$$\int_{-1}^{1} dx\, H_q(x,\xi,t) = F_1(t)$$
$$\int_{-1}^{1} dx\, E_q(x,\xi,t) = F_2(t)$$

- first moment, $n = 2$

$$\int_{-1}^{1} dx\, x\, H_q(x,\xi,t) = A^q_{2,0}(t) + \xi^2 C^q_2(t)$$
$$\int_{-1}^{1} dx\, x\, E_q(x,\xi,t) = B^q_{2,0}(t) - \xi^2 C^q_2(t)$$

Impact Parameter GPDs

- Probabilistic interpretation of $H(x,\xi,t)$ (and $\widetilde{H}(x,\xi,t)$, $H_T(x,\xi,t)$) at $\xi = 0$ due to M.Burkardt

$$q(x,\vec{b}_\perp) = \frac{1}{(2\pi)^2}\int d^2\Delta_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\Delta_\perp} H(x,0,-\Delta_\perp^2)$$

- Also shows $H(x,0,t=-\Delta_\perp^2) \to t$-independent for $x \to 1$

$$\Rightarrow \lim_{x\to 1} q(x,\vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

Ji's Angular Momentum Sum Rule

- Forward limit, $\mathbf{t} = \xi = \mathbf{0}$

$$A^q_{2,0}(0) = \langle x_q \rangle \equiv \int_{-1}^{1} dx\, x \left(q_\uparrow(x) + q_\downarrow(x) \right)$$

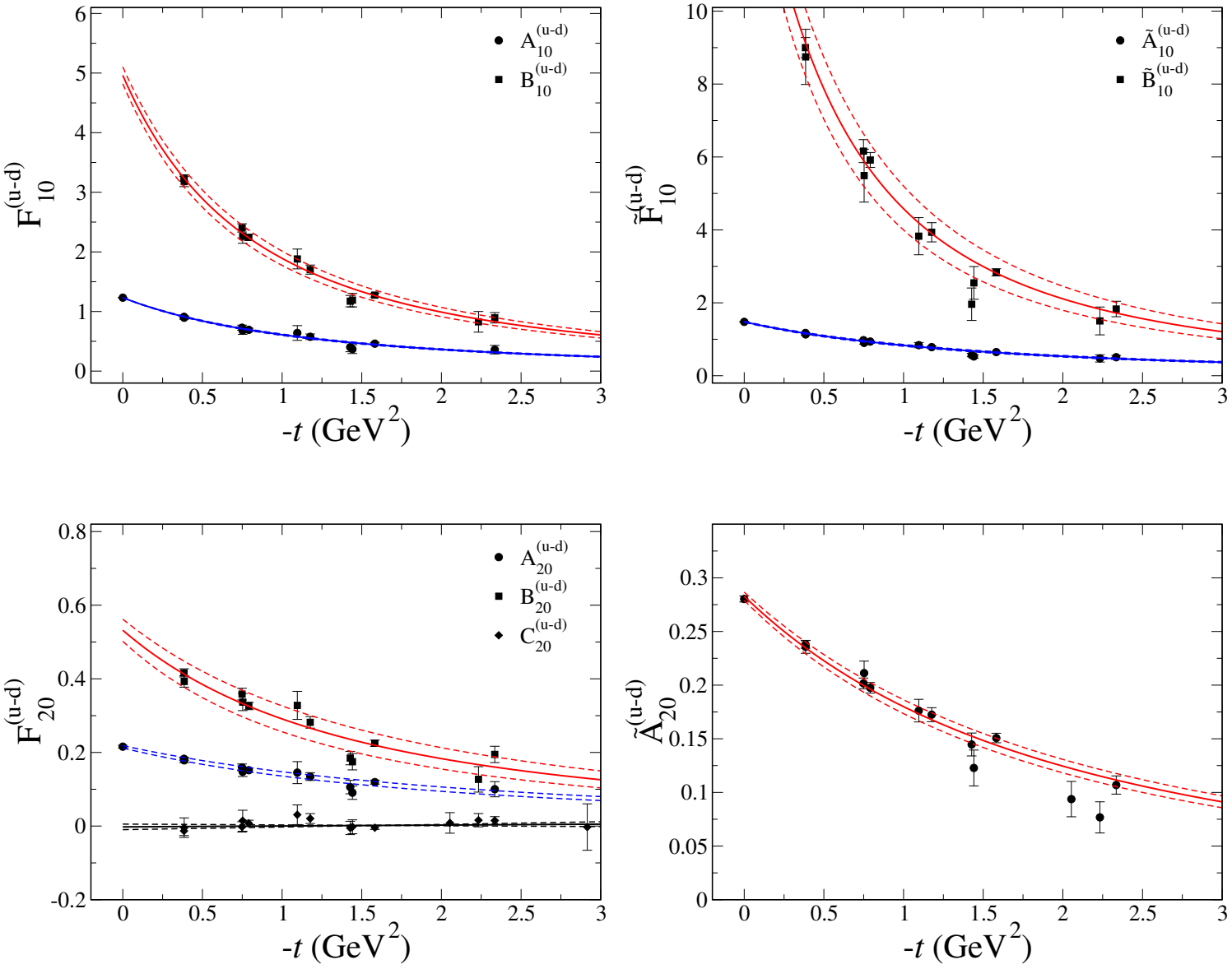
$$\frac{1}{2}\left(A^q_{2,0}(0) + B^q_{2,0}(0) \right) = J_q$$

- $J_q = L_q + S_q$ - angular momentum of quark, q
- L_q - orbital angular momentum of q
- S_q - spin of q

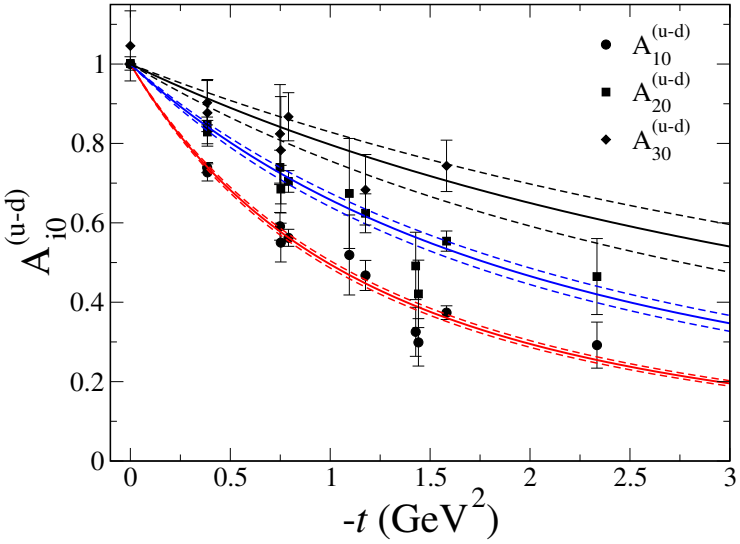
$$S_q = \frac{1}{2}\Delta q \equiv \frac{1}{2}\int_{-1}^{1} dx \left(q_\uparrow(x) - q_\downarrow(x) \right)$$

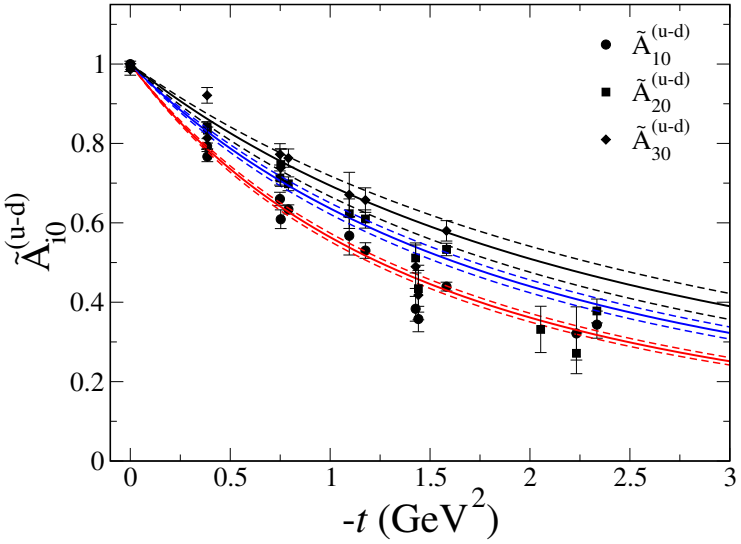
$$J = \sum_q J_q$$

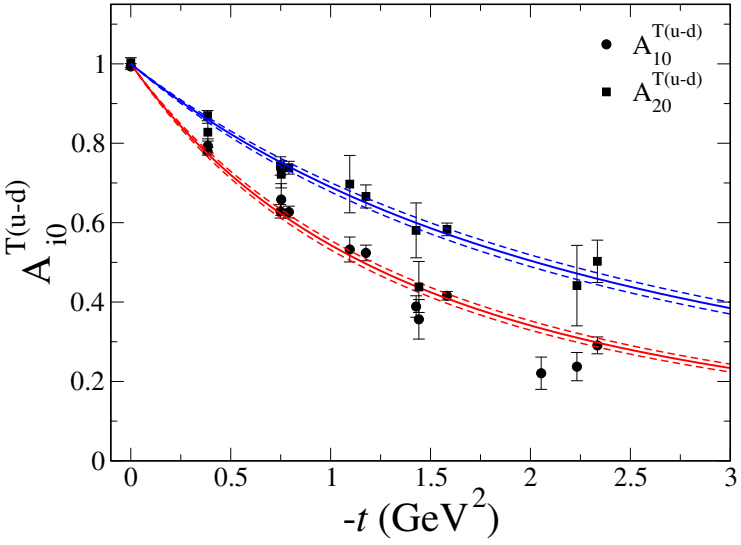
Generalised Form Factors, ($m_\pi \approx 950\mathrm{MeV}$)

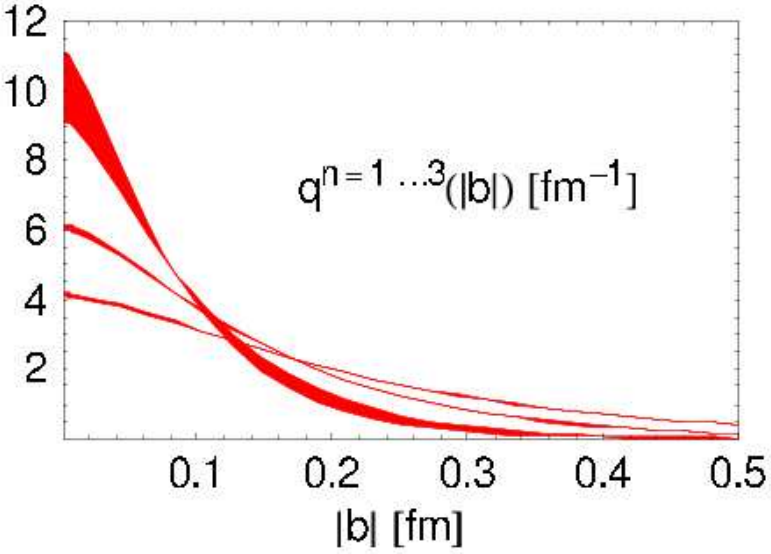


Impact Parameter Space, ($m_\pi \approx 950\mathrm{MeV}$)







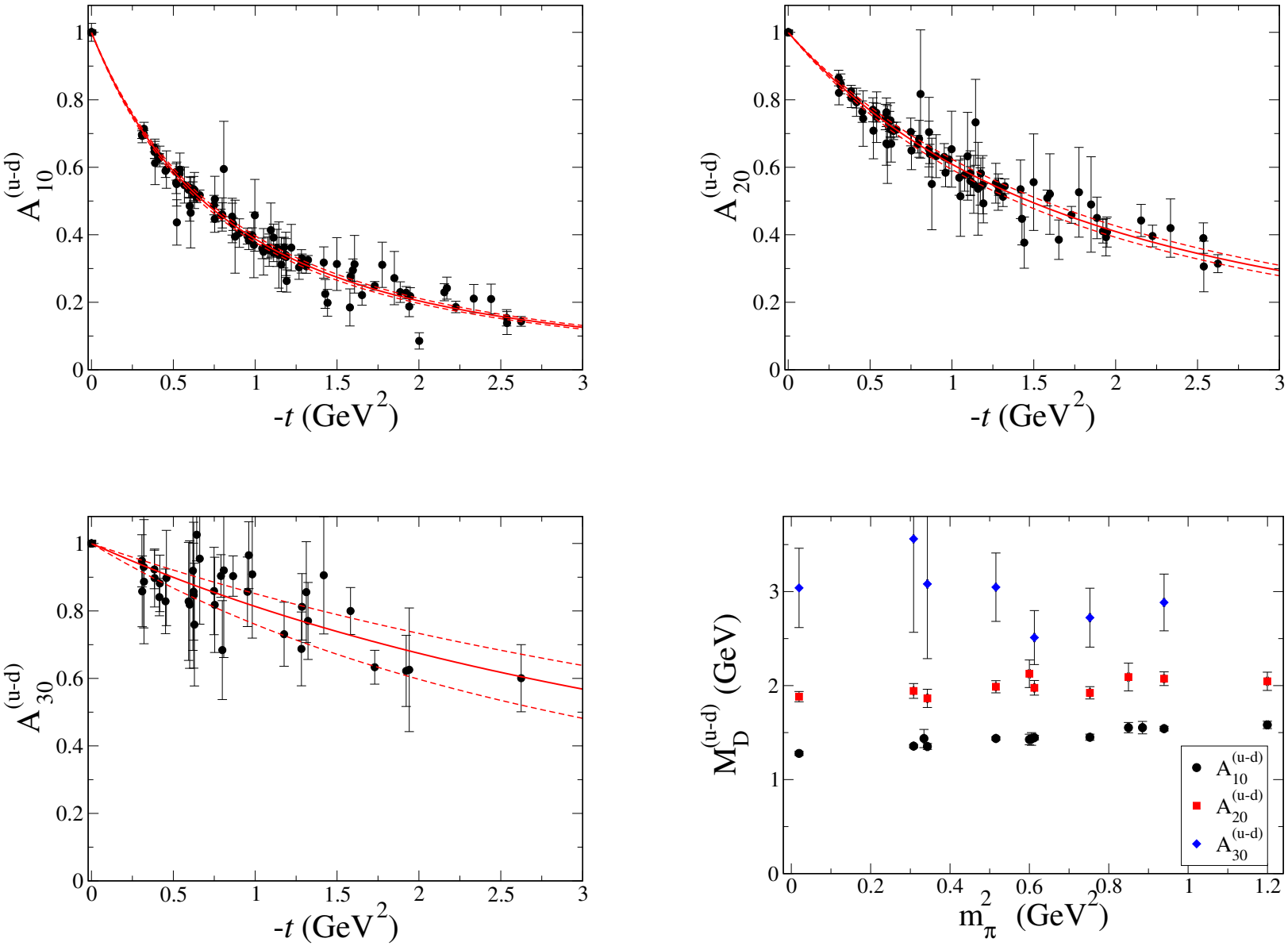


Impact Parameter Space

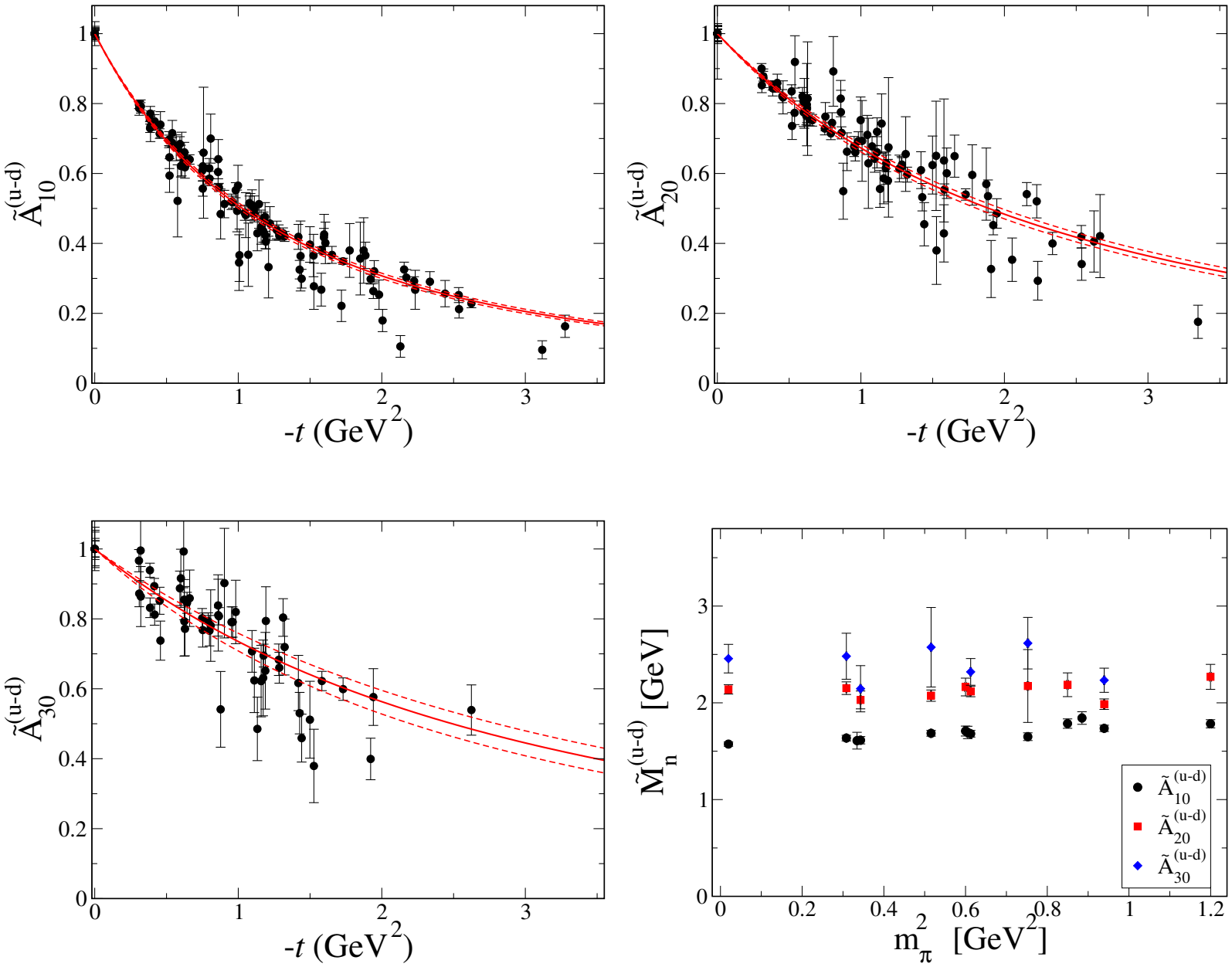
- Flattening of the GFFs $\Longrightarrow$
  quark distribution $\rightarrow \delta^2(\vec{b}_\perp)$ as $x \rightarrow 1$
- Form factors well described by a dipole
- Extrapolate dipole masses to chiral limit

$$A_{n0}(t) = \frac{A_{n0}(0)}{(1 - t/M_n^2)^2} \rightarrow \bar{A}_{n0}(t) = \frac{1}{(1 - t/(M_n^0 + \alpha_n m_\pi^2)^2)^2}$$

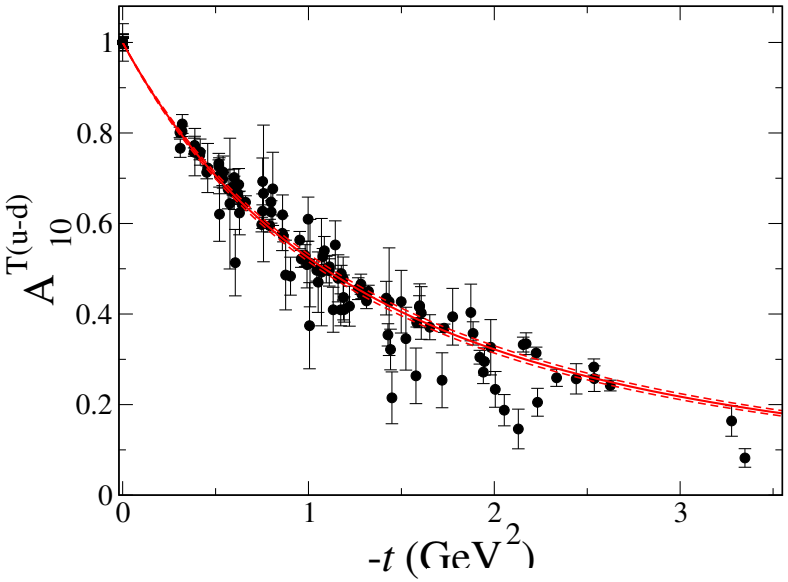
Described By A Dipole

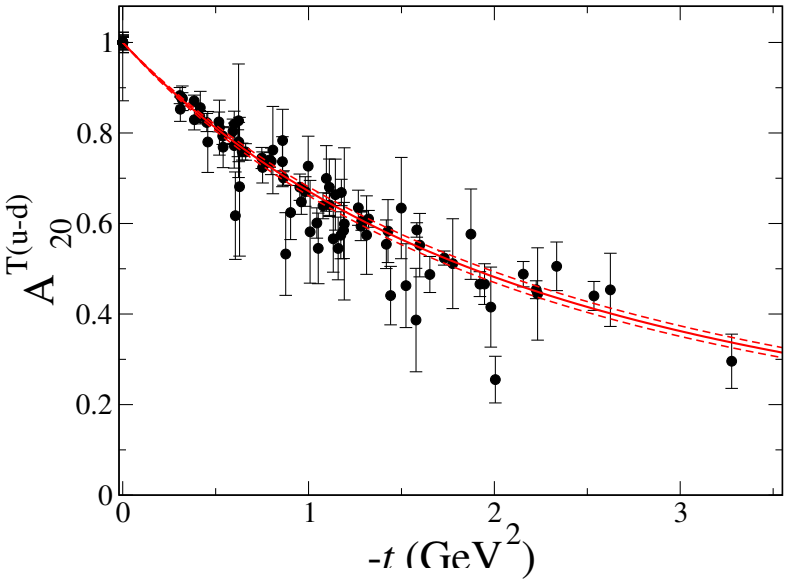


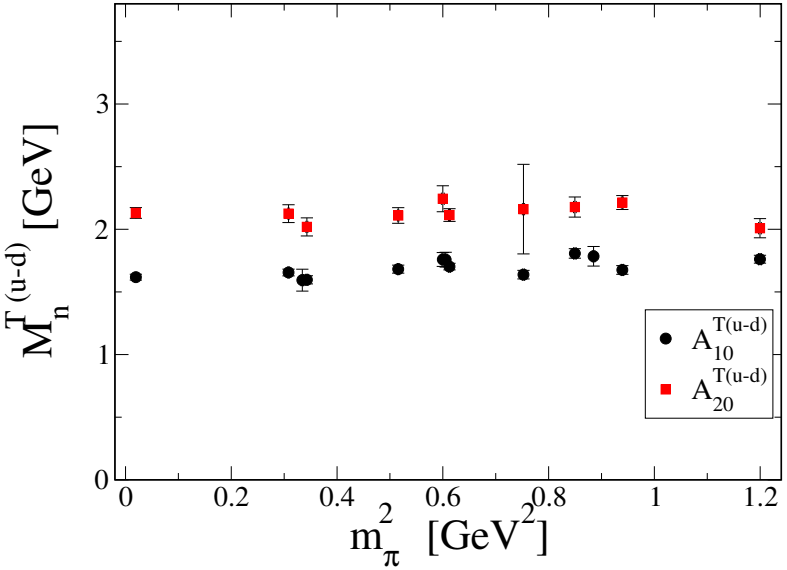
Polarised


$\tilde{A}_{10}^{(u-d)}$
$\tilde{A}_{20}^{(u-d)}$
$\tilde{A}_{30}^{(u-d)}$
-t (GeV^2)
$\tilde{M}_n^{(u-d)}$ [GeV]
m_π^2 [GeV^2]
$\tilde{A}_{10}^{(u-d)}$
$\tilde{A}_{20}^{(u-d)}$
$\tilde{A}_{30}^{(u-d)}$

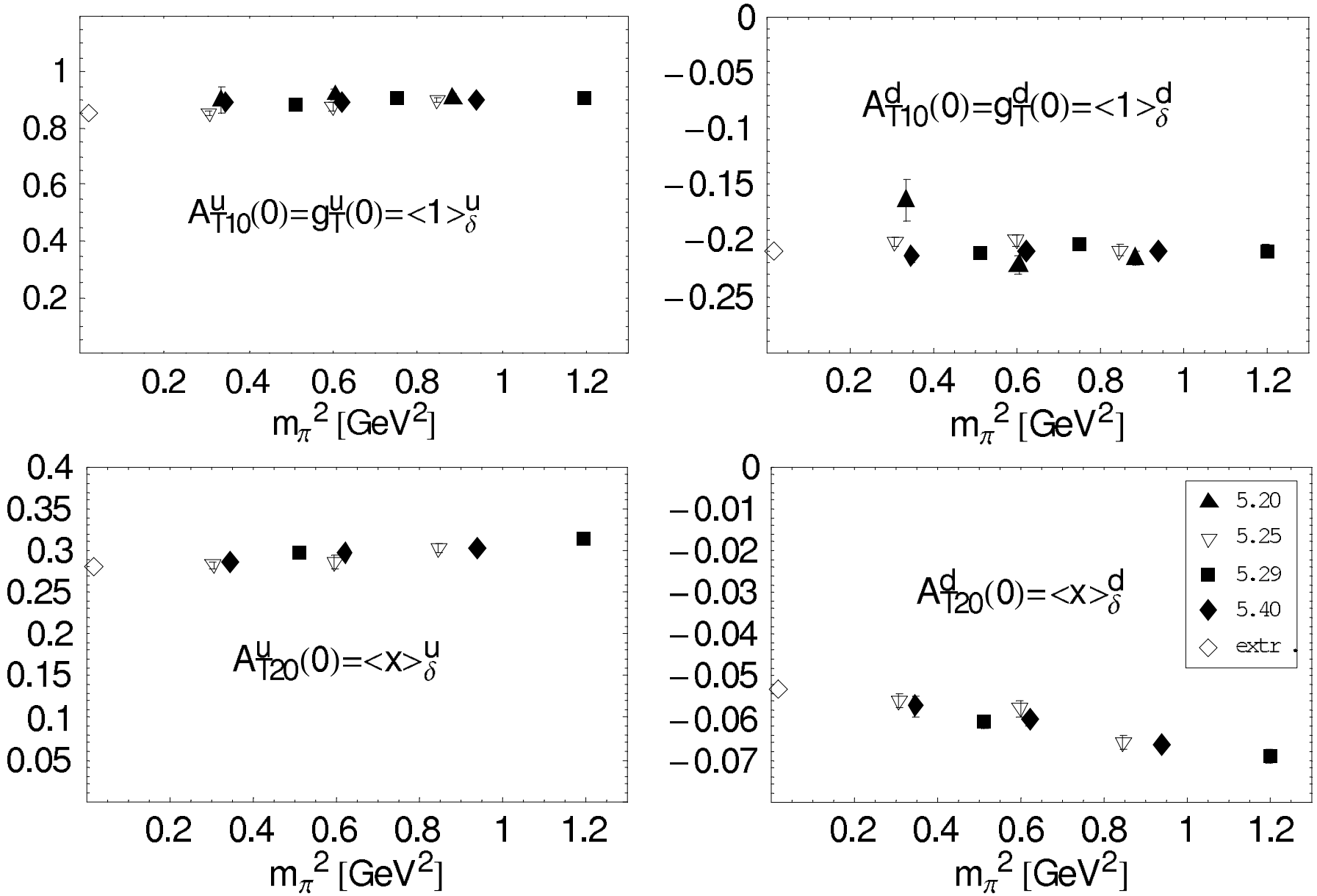
Generalised Transversity





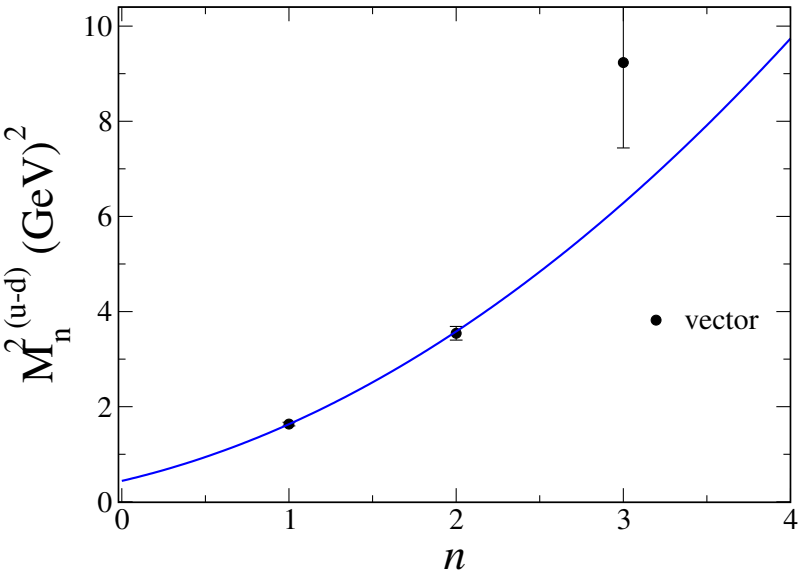


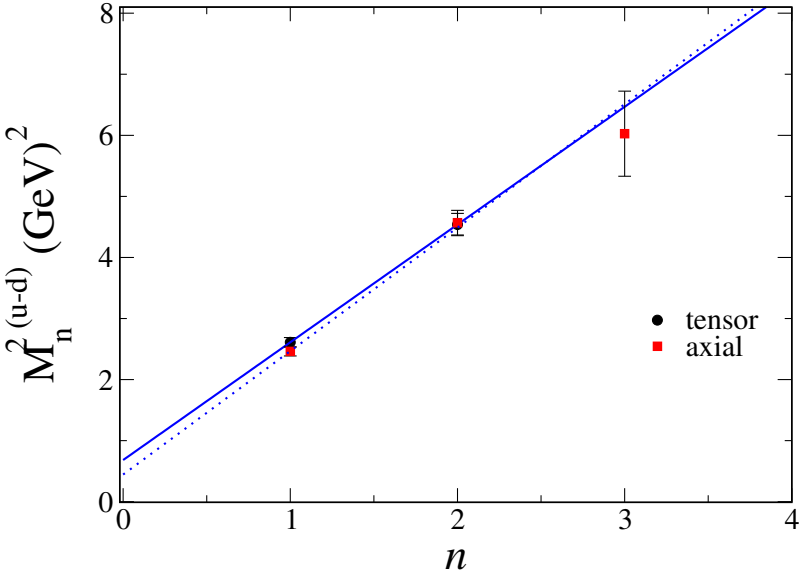
Generalised Transversity, ($\overline{\mathrm{MS}}$ 4 GeV^2)



Regge Trajectory?

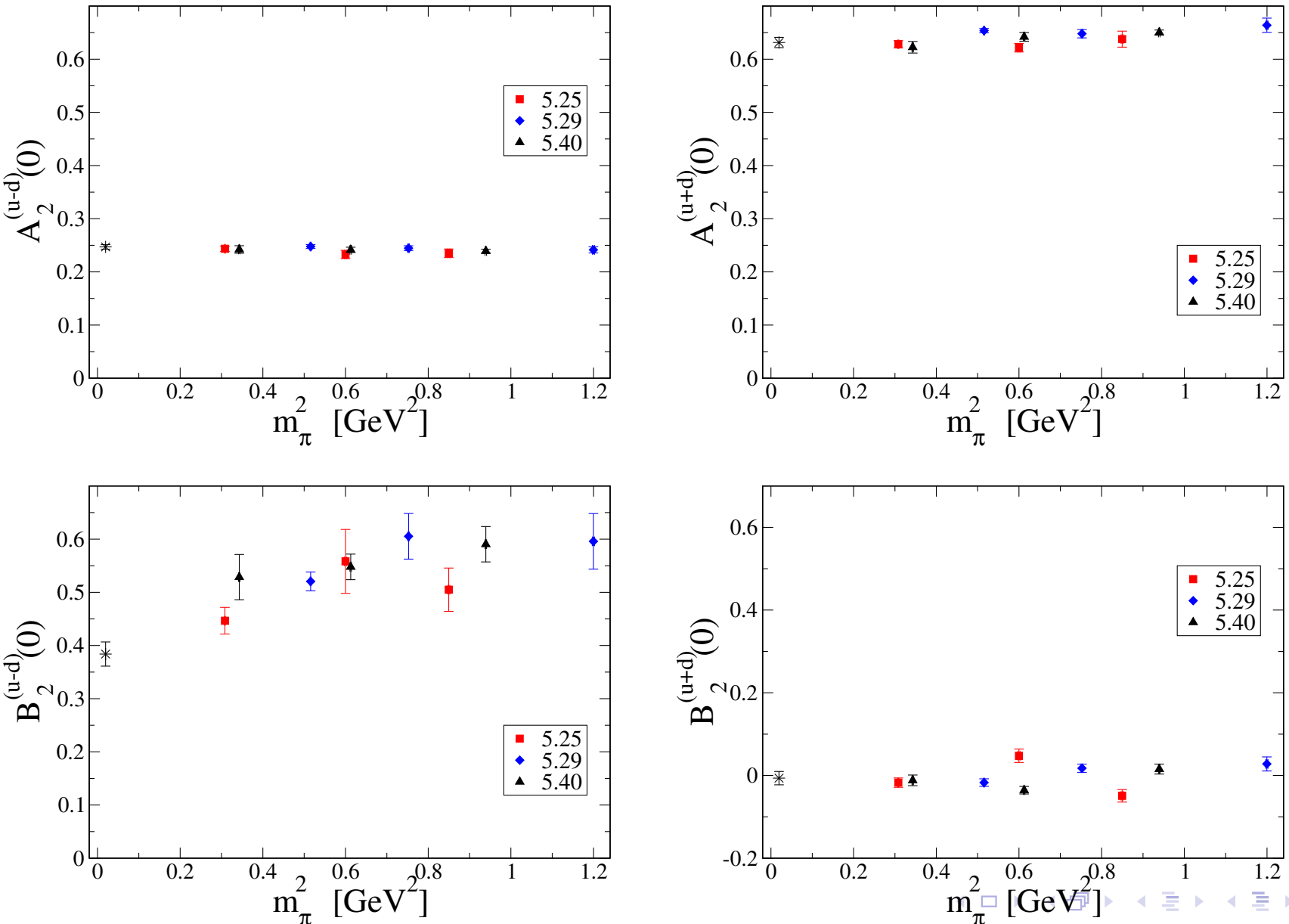
Regge: $\sqrt{M_n^2} = \alpha + n/\alpha'$ $\qquad$ $M_n^2 = \alpha + n/\alpha'$





$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1}\, H_q(x,0,t) = \frac{\langle x_q^{n-1} \rangle}{(1 - t/M_n^2)^2},$$

Angular Momentum $J^q = L^q + S^q = \frac{1}{2}(A_2^q + B_2^q)$, ($\overline{\mathrm{MS}}4$ GeV^2)



Conclusions and Outlook

- Lattice provides a useful tool for investigating (moments of) **GPDs**
  - "Flattening" of GFFs $A_{n0}(t)$ for increasing n
  - $\lim_{x\to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$
- Complete Current Analysis
  - Increase statistics for 2nd moment
  - Renormalisation
  - Finite volume effects
  - Chiral extrapolation
  - Partially Quenched
- Quark contribution to nucleon spin and angular momentum
- Compute $H_q(x, 0, t)$ and $\tilde{H}_q(x, 0, t)$ via inverse Mellin Transform