

Phenomenology of *Transverse Momentum Dependent* Parton Distribution Functions

Umberto D'Alesio

Physics Department and INFN
University of Cagliari, Italy

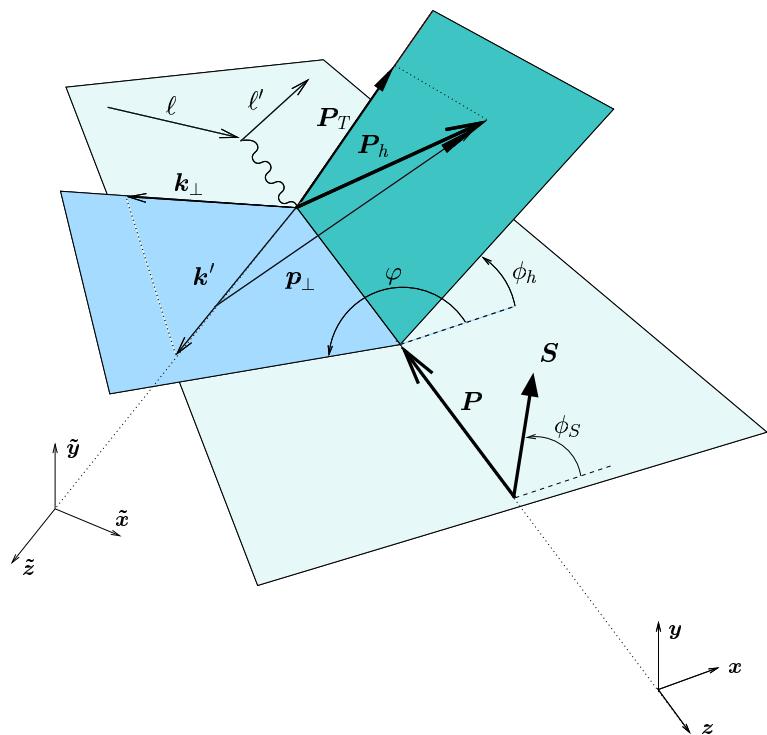
SIR2005, *International Workshop on*
Semi-Inclusive Reactions and 3D-Parton Distributions
May 18-20, 2005 Jefferson Lab, Newport News, Virginia, USA

[*M. Anselmino, M. Boglione, A. Kotzinian, E. Leader, S. Melis, F. Murgia, A. Prokudin*]

Outline

- Generalized pQCD approach with spin and \mathbf{k}_\perp -effects in distribution and fragmentation functions and elementary dynamics at LO;
- Cross sections and SSA in $\ell p \Rightarrow \ell' \pi X$:
Cahn effect (intrinsic motion), Sivers effect (intrinsic motion + spin), fit and comparison with data;
- SSA and the Sivers effect in $pp \rightarrow \pi X$: consistency (?);
Collins mechanism and the role of phases;
- SSA and the Sivers effect in $pp \rightarrow \ell^+ \ell^- X$: universality (?);
- Conclusions and outlook

SIDIS: $\ell p \rightarrow \ell' h X$

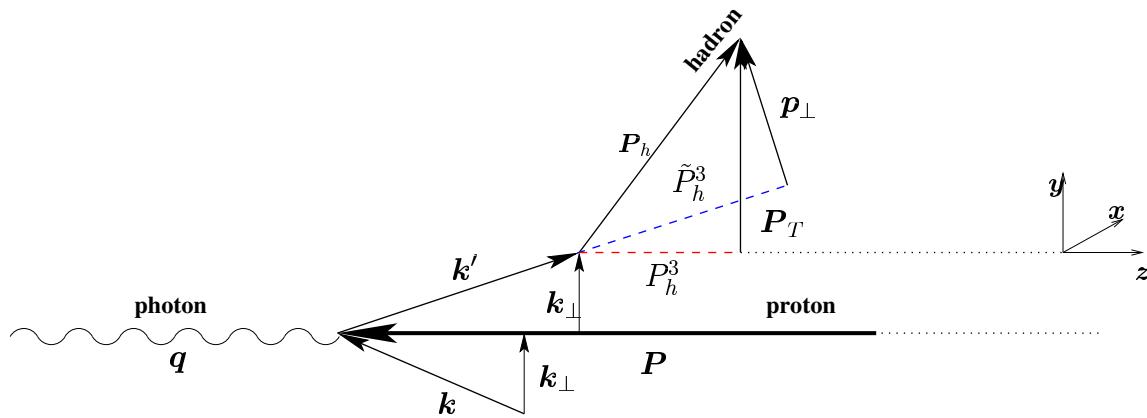


$$d\sigma^{\ell p \rightarrow \ell' h X} = \\ \sum_q f_{q/p} \otimes d\sigma^{\ell q \rightarrow \ell' q} \otimes D_q^h$$

\otimes means x and k_\perp convolutions

$P_T \simeq \Lambda_{QCD} \simeq k_\perp$: factorization
 X. Ji, J.-P. Ma and F. Yuan,
 PLB597 (04), PRD71 (05)

Three dimensional kinematics of the SIDIS process $\ell p \rightarrow \ell' h X$.



$$\frac{d^5 \sigma^{\ell p \rightarrow \ell' h X}}{dx_B \, dQ^2 \, dz_h \, d^2 \mathbf{P}_T} \simeq \sum_q e_q^2 \int d^2 \mathbf{k}_{\perp} \, f_q(x_B, k_{\perp}) \, \frac{2\pi\alpha^2}{x_B^2 s^2} \, \frac{\hat{s}^2 + \hat{u}^2}{Q^4} \, D_q^h(z_h, p_{\perp})$$

where $[\mathbf{k}_{\perp} = k_{\perp}(\cos \varphi, \sin \varphi, 0)]$

$$\hat{s}^2 + \hat{u}^2 \simeq \frac{Q^4}{y^2} \left(1 + (1-y)^2 - 4 \frac{k_{\perp}}{Q} (2-y) \sqrt{1-y} \cos \varphi \right) + \mathcal{O}(k_{\perp}^2/Q^2)$$

- $f_q(x, k_{\perp}) = f_q(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$ $D_q^h(z, p_{\perp}) = D_q^h(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-p_{\perp}^2 / \langle p_{\perp}^2 \rangle}$

$$\frac{d^5 \sigma^{\ell p \rightarrow \ell' h X}}{dx_B \, dQ^2 \, dz_h \, d^2 \mathbf{P}_T} \simeq \sum_q \frac{2\alpha^2 e_q^2}{Q^4} f_q(\textcolor{violet}{x}_{\textcolor{blue}{B}}) D_q^h(\textcolor{violet}{z}_h) \left[1 + (1-y)^2 - 4 \frac{(2-y)\sqrt{1-y} \langle k_{\perp}^2 \rangle z_h P_T}{\langle P_T^2 \rangle Q} \cos \phi_h \right] \frac{1}{\langle P_T^2 \rangle} e^{-P_T^2/\langle P_T^2 \rangle}$$

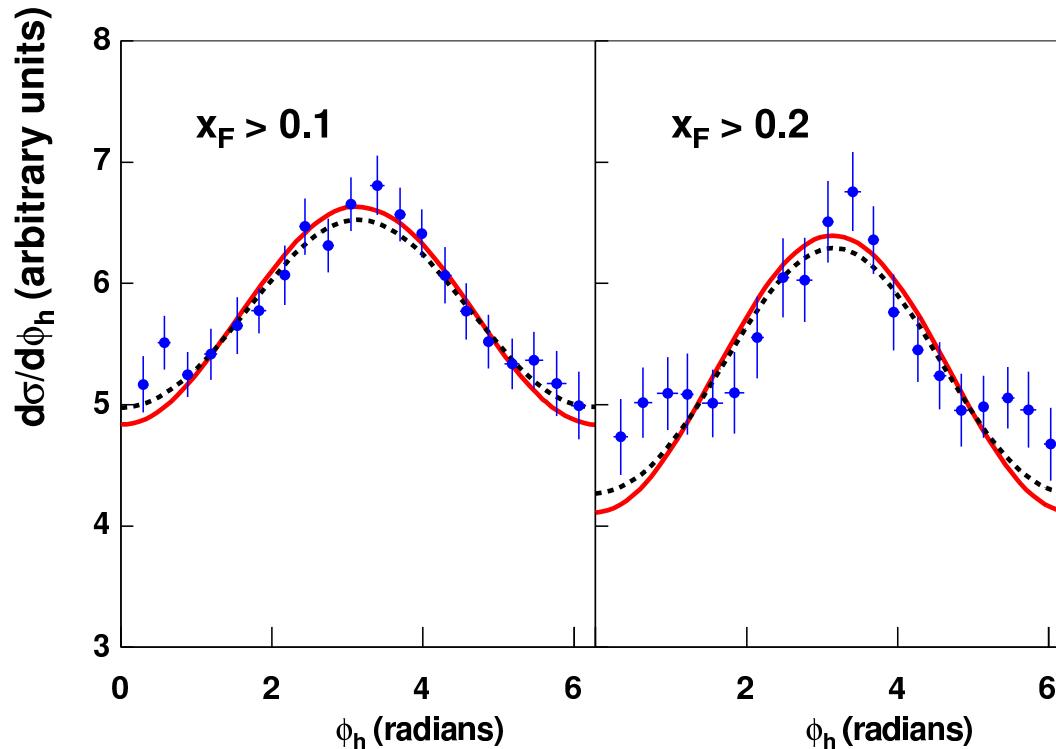
where

$$\langle P_T^2 \rangle = \langle p_{\perp}^2 \rangle + z_h^2 \langle k_{\perp}^2 \rangle$$

- $\cos \phi_h$ dependence in UNPOL SIDIS: **Cahn effect** (1978);
- tool to extract $\langle p_{\perp}^2 \rangle$ and $\langle k_{\perp}^2 \rangle$;
- $x_B - z_h$ factorization preserved only at $\mathcal{O}(k_{\perp}/Q)$.

Best choice (constant values) to fit data [M. Anselmino *et al.* PRD71 (05)]:

$$\langle k_{\perp}^2 \rangle = 0.25 \text{ (GeV/c)}^2 \quad \langle p_{\perp}^2 \rangle = 0.20 \text{ (GeV/c)}^2 .$$



Fits to EMC data for $d\sigma/d\phi_h$: exact kinematics (dashed line), $\mathcal{O}(k_\perp/Q)$ order solid (red) line.

$P_T > 0.2$ GeV/ c $y < 0.8$ $Q^2 > 4$ (GeV/ c) 2 EMC, [ZPC34 (87)]

Azimuthal SSA in $\ell p \rightarrow \ell' \pi X$: A_{UT} and the Sivers effect

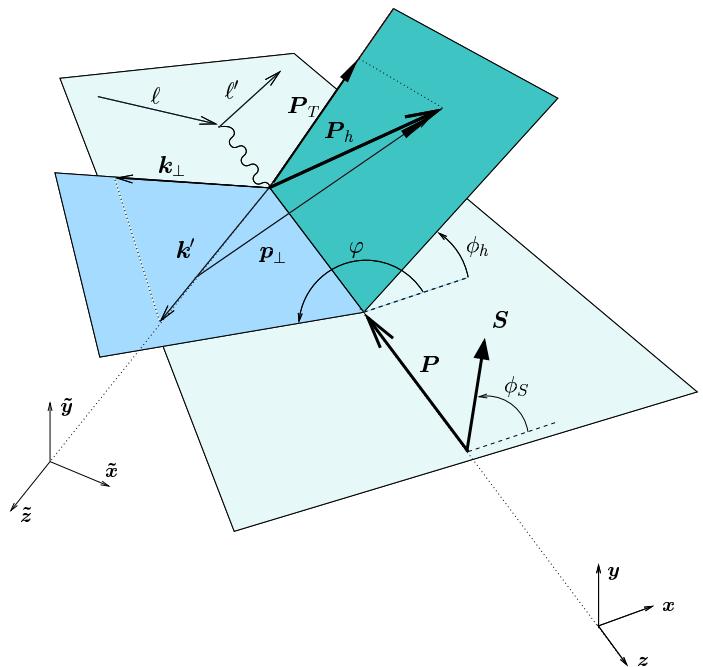
In a transversely polarized hadron ($\mathbf{S}_T \cdot \hat{\mathbf{p}} = 0$) the number density of unpolarized partons is given by

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

$\Delta^N f_{q/p^\uparrow}(x, k_\perp)$: Sivers function ($\simeq f_{1T}^\perp$)

$$d\sigma^{\ell p^\uparrow \rightarrow \ell' \pi X} \simeq \sum_q f_{q/p^\uparrow} \otimes d\sigma^{\ell q \rightarrow \ell' q'} \otimes D_q^h + \dots \quad [k_\perp - \text{factorization}]$$

by proper weighting $A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \Rightarrow$



$$\mathbf{S}_T \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) = \sin(\varphi - \phi_S)$$

weight: $\sin(\phi_h - \phi_S)$

$$A_{UT}^{\sin(\phi_h - \phi_S)} =$$

$$\sum_q \int d\phi_S d\phi_h d^2 \mathbf{k}_\perp \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp) \sin(\phi_h - \phi_S)$$

$$\sum_q \int d\phi_S d\phi_h d^2 \mathbf{k}_\perp f_q(x, k_\perp) \frac{d\hat{\sigma}^{\ell q \rightarrow \ell q}}{dQ^2} D_q^h(z, p_\perp)$$

Parameterization of the Sivers function

For each light quark flavour $q = u, d, \bar{u}, \bar{d}$ we adopt the factorized form

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 2 \mathcal{N}_q(x) h(k_\perp) f_{q/p}(x, k_\perp)$$

where

$$\mathcal{N}_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

$$h(k_\perp) = \frac{2k_\perp M}{k_\perp^2 + M^2}$$

alternative k_\perp depend. : $h'(k_\perp) = \sqrt{2e} \frac{k_\perp}{M'} e^{-k_\perp^2/M'^2}$

13 parameters

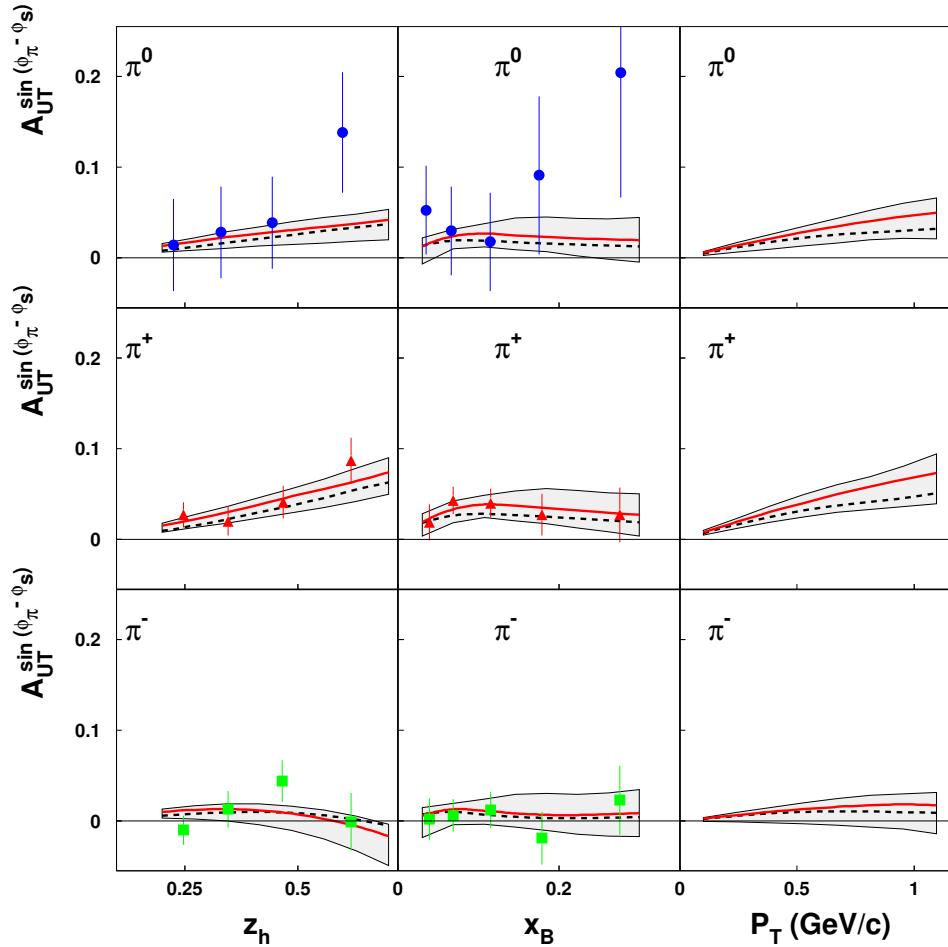
$N_{u_v} = 0.42 \pm 0.18$	$N_{d_v} = -1.0 \pm 1.8$
$a_{u_v} = 0.0 \pm 3.3$	$a_{d_v} = 1.1 \pm 1.2$
$b_{u_v} = 2.6 \pm 1.8$	$b_{d_v} = 5.0 \pm 3.6$
$N_{\bar{u}} = 1.0 \pm 1.9$	$N_{\bar{d}} = -1.0 \pm 1.9$
$a_{\bar{u}} = 0.52 \pm 0.43$	$a_{\bar{d}} = 0.0 \pm 4.5$
$b_{\bar{u}} = 0.0 \pm 3.1$	$b_{\bar{d}} = 0.0 \pm 2.8$

Best values of the parameters of the Sivers functions (fit to HERMES data).

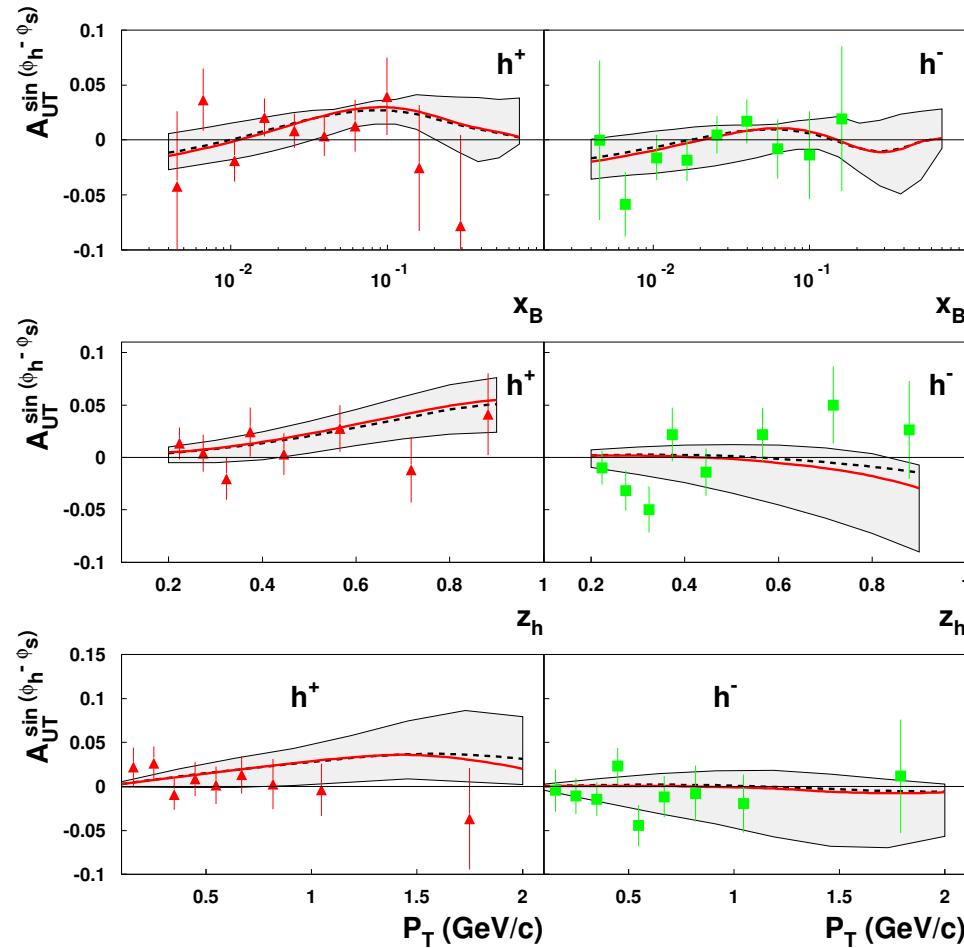
$$M^2 [M'^2] = 0.25 [0.36] (\text{GeV}/c)^2$$

[M. Anselmino *et al.* PRD71 (05)]

- consistent with the analysis by A.V. Efremov *et al.* hep-ph/0412353



Fit to HERMES data (PRL94 [05]) on $A_{UT}^{\sin(\phi_\pi - \phi_S)}$ for $\ell p^\uparrow \rightarrow \ell' \pi X$. Exact kinematics (dashed line), $\mathcal{O}(k_\perp/Q)$ order (solid bold line) and theoretical uncertainty.



Our estimates vs. COMPASS preliminary data (05) on $A_{UT}^{\sin(\phi_h - \phi_S)}$ for $\ell D \rightarrow \ell' h^\pm X$.

Single Spin Asymmetries in $A^\dagger B \rightarrow \pi X$: helicity formalism and k_\perp

- Configuration ($AB \equiv pp$):

AB center of mass frame - A along $+Z$ -axis, π in the XZ plane with p_T parallel to $+X$ -axis. $S_A = \uparrow$ or \downarrow along Y -axis, $S_B = 0$.

- k_\perp 's imply: non-planar $ab \rightarrow cd$ scattering; different \perp directions.

Parton helicity amplitudes in pp c.o.m frame in terms of hel. amplitudes in partonic c.o.m. frame.

$$\text{boost + rotations: } \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b} = \hat{M}_{\lambda_c, \lambda_d; \lambda_a, \lambda_b}^0(\hat{s}, \hat{t}) e^{i \lambda_m \xi_m} e^{i(\lambda_a - \lambda_b) \phi_c''}$$

Notice: various combinations of “T-odd” effects in (un)polarized cross sections but

by including proper phases + numerical (8-dim., VEGAS) integration:

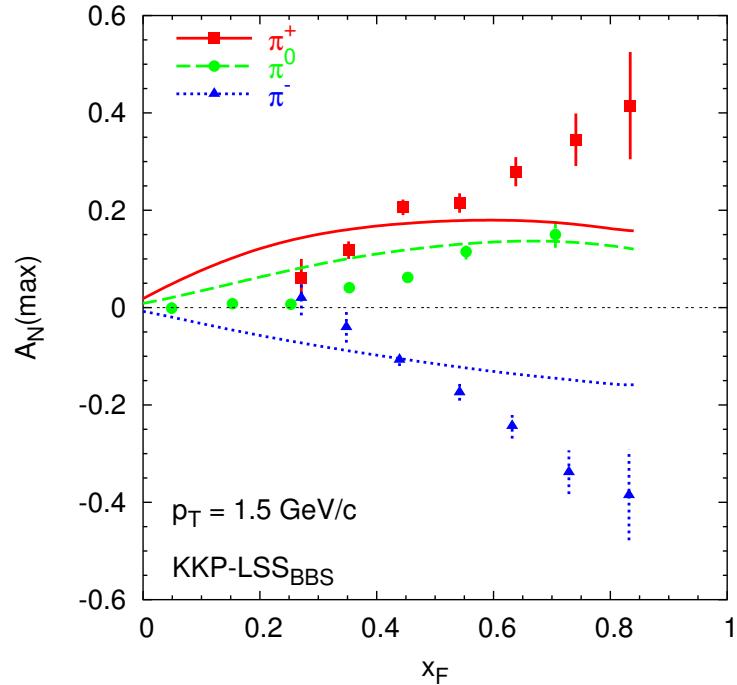
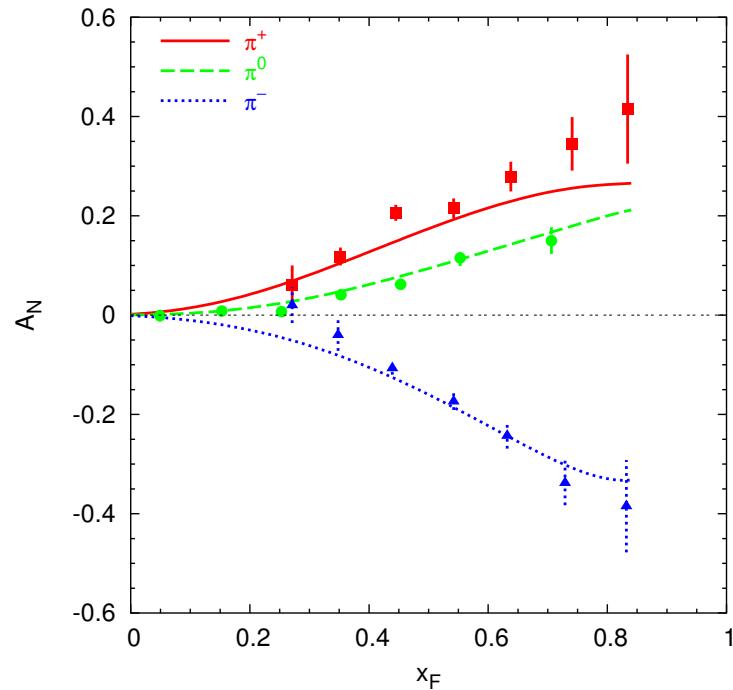
$$d\sigma^{\text{unp}} \simeq f_{a/p} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c}$$

$$\begin{aligned} d\sigma^\uparrow - d\sigma^\downarrow &\simeq \Delta^N f_{a/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c} && \text{“Sivers effect”} \\ &+ h_{1a} \otimes f_{b/p} \otimes d\Delta\hat{\sigma} \otimes \Delta^N D_{\pi/c^\uparrow} && \text{“Collins effect”} \end{aligned}$$

Notice: the explicit phases entering the convolution are

$$\Delta^N f_{a/p^\uparrow} \otimes f_{b/p} \otimes d\hat{\sigma} \otimes D_{\pi/c} \rightarrow \cos \phi_a$$

$$h_{1a} \otimes f_{b/p} \otimes d\Delta\hat{\sigma} \otimes \Delta^N D_{\pi/c^\uparrow} \rightarrow \cos[\phi_a + \phi_c'' - \xi_a - \tilde{\xi}_a + \xi_c + \tilde{\xi}_c + \phi_\pi^H]$$



A_N at $E = 200$ GeV vs. x_F at $p_T = 1.5$ GeV/c. Data are from [E704] PLB261-264 (1991).

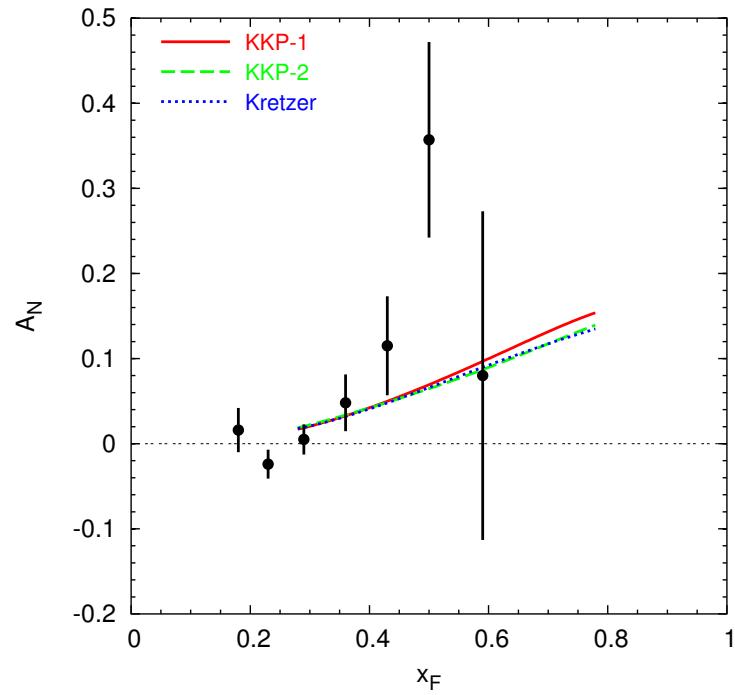
Left plot is with Sivers effect (valence-like). Right plot is with Collins effect (full saturated).

$$N_u = +0.40 \quad a_u = 2.0 \quad b_u = 0.3$$

$$N_d = -0.90 \quad a_d = 2.0 \quad b_d = 0.2$$

\simeq CONSISTENCY with SIDIS

Transversity funct. and
Collins funct. full saturated



Predictions of $A_N(pp \rightarrow \pi^0 X)$ in terms of Sivers effect alone [U.D. and F. Murgia PRD70 (05)] at $\sqrt{s} = 200$ GeV and $\eta = 3.8$ vs. x_F . Data are from [STAR] PRL92 (2004).

Consistency between Sivers functions from $p^\uparrow p \rightarrow \pi X$ and $\ell p^\uparrow \rightarrow \ell' \pi X$

- $\ell p \rightarrow \ell' \pi X$ (HERMES): moderately small $x_B(x) \Rightarrow$ explore “sea region” and No gluons at LO;
- $pp \rightarrow \pi X$ (E704, STAR), large A_N at large $x_F \Rightarrow$ constraint to the valence behaviour of TMD pdf’s. [U.D. and F. Murgia PRD70 (04)]
gluon-sea cancelation at work at $x_F \simeq 0$?
- Interplay of Collins effect ?
New analysis [M.Anselmino et al. PRD71 (05)] with full microscopic dynamics \Rightarrow suppression of the Collins effect;
- Theory: “universality” of TMD’s $|_{pp \rightarrow hX}$ (if any) \Rightarrow UNKNOWN

Access to the Sivers effect and universality: SSA in $p^\uparrow p \rightarrow \ell^+ \ell^- X$ processes

[M. Anselmino, U.D., F. Murgia, PRD67 (03)]

Differential cross sections in the variables: M^2, y, \mathbf{q}_T [$q_T^2 \ll M^2$]

Angular distribution of the lepton pair production plane: integrated over.

$$d\sigma^\uparrow - d\sigma^\downarrow \simeq \Delta^N f_{q/A^\uparrow}(x_a, \mathbf{k}_{\perp a}) \otimes \hat{f}_{\bar{q}/B}(x_b, \mathbf{k}_{\perp b}) d\hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-}$$

Other mechanisms for SSA in Drell-Yan processes:

$$\sum_q h_1(x_a, \mathbf{k}_{\perp a}) \otimes \Delta^N f_{\bar{q}^\uparrow/B}(x_b, \mathbf{k}_{\perp b}) \otimes d\Delta\hat{\sigma}^{q\bar{q} \rightarrow \ell^+ \ell^-} \quad [\text{D. Boer PRD60 (99)}]$$

$$d\Delta\hat{\sigma} = d\hat{\sigma}^{\uparrow\uparrow} - d\hat{\sigma}^{\uparrow\downarrow} \simeq \cos 2\phi \quad [\phi = \widehat{P_A N}_{\ell^+\ell^-}] \quad \int d\phi \Rightarrow 0$$

Universality [J. Collins PLB536 (02)]:

$$\Delta^N f_{q/A^\uparrow}(x, k_\perp)|_{DY} = -\Delta^N f_{q/A^\uparrow}(x, k_\perp)|_{DIS} ??? \dots$$

Conclusions and outlook

- Phenomenology of the Sivers effect in different processes and kinematical configurations:
 1. $\ell p \rightarrow \ell' \pi X$ (HERMES and COMPASS A_{UT} data - fit)
intrinsic motion and the Cahn effect
 2. $pp \rightarrow \pi X$ (E704 and STAR A_N data - fit) consistency w.r.t (1)
 3. $pp \rightarrow \ell^+ \ell^- X$ direct access and universality
- Generalized pQCD approach including k_\perp effects within the helicity formalism to DSA, SSA and unpol. cross sections for $pp \rightarrow hX$;
- Azimuthal spin asymmetries in SIDIS: role of Collins effect (detailed k_\perp approach in progress);
- ep^\uparrow -data in different kinematical regions and $p^\uparrow p$ -data at large energies and at large p_T , with transversely polarized targets, essential for a deep and consistent understanding of spin and k_\perp - dependent distr.