Nuclear and Hadron Physics with Electromagnetic Probes

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Norfolk, VA USA

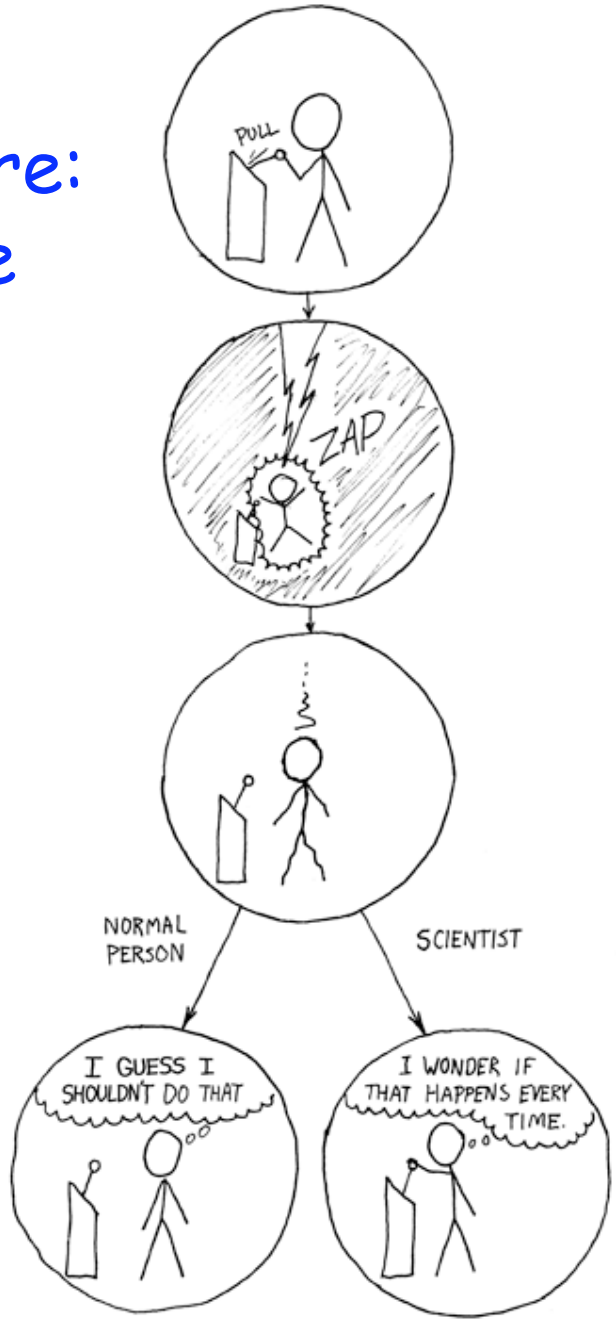
Lecture 1
UK Nuclear Physics Summer School
Newcastle 2007

Course Outline

- Lecture 1: Introduction to EM probes. Electron and photon interactions with nuclei. Beams and detectors. Elastic electron-nucleus scattering.
- Lecture 2: Nuclear Physics with electrons and photons. Charge, matter and momentum distributions. Single nucleons and correlated nucleon pairs.
- Lecture 3: Nucleon physics with electrons and photons. Proton and neutron structure, quark distributions
- Lecture 4: From nucleons to quarks. Nucleon modification in nuclei, exotic hadrons

Looking for patterns in nature: the difference

(xkcd.com)



Normal person:
"I guess I shouldn't do that"

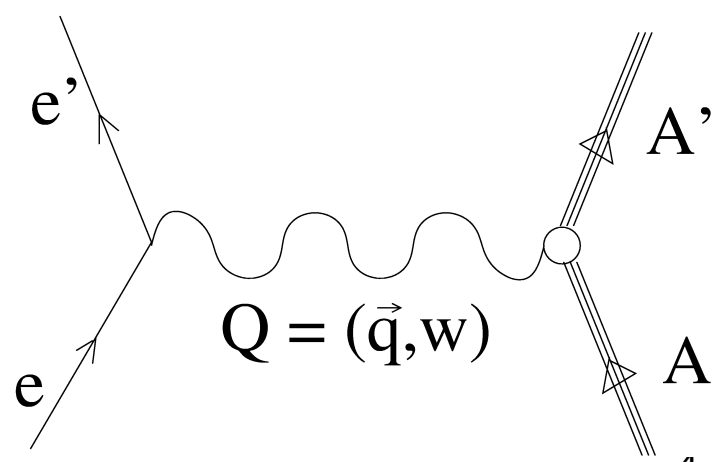
Scientist:
"I wonder if that happens every time?"

Why use electrons and photons?

- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak (α = 1/137)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

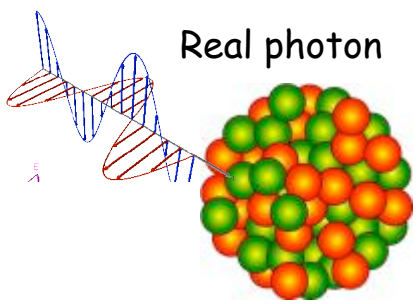
BUT:

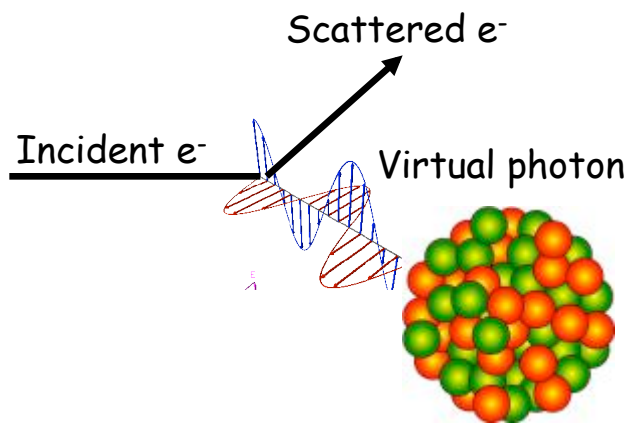
- Cross sections are small
- Electrons radiate
- Photons are hard to make



It's all photons!

- An electron interacts with a nucleus by exchanging a single* virtual photon.





Real photon:

Momentum q = energy [illegible]

Mass = $Q^2 = |\boldsymbol{q}|^2 -$ [illegible]$^2 = 0$

Virtual photon:

Momentum q [illegible] energy [illegible]

$Q^2 = -$ q[illegible]q[illegible] $= |\boldsymbol{q}|^2 -$ [illegible]^2 [illegible] 0

Virtual photon has mass!

([illegible] and [illegible] are both used for energy)

Electron beams need …

- High energy
  - $q \sim 2E \sin(\theta/2)$
  - $\lambda x < 0.2$ fm → $q > 1$ GeV/c
- High duty cycle (no large beam current variation)
  - Reduces accidental coincidences for multiparticle detection
  - Reduces detector rates, multiple hits, …
- High intensity (since cross sections are small)
- High resolution to separate nuclear levels
- High polarization (for spin asymmetry measurements)

Worldwide activity

E_{e-} = 2.8 GeV

E_{e-} = 27.5 GeV

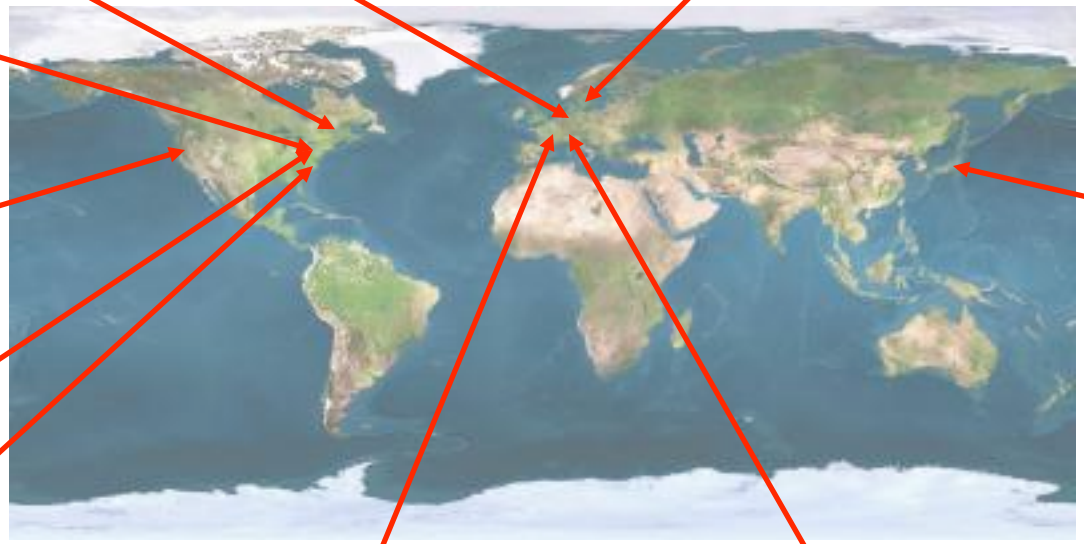
hermes

E_{e-} = 0.15GeV

MAX-lab

Jefferson Lab

E_{e-} = 6 GeV



SPring-8

E_{e-} = 8 GeV

Stanford Linear Accelerator Center

E_{e-} = 25 GeV

ODU

Duke University
Free Electron Laser Laboratory

H I γ S

E_{e-} = 1.2 GeV

ELSA

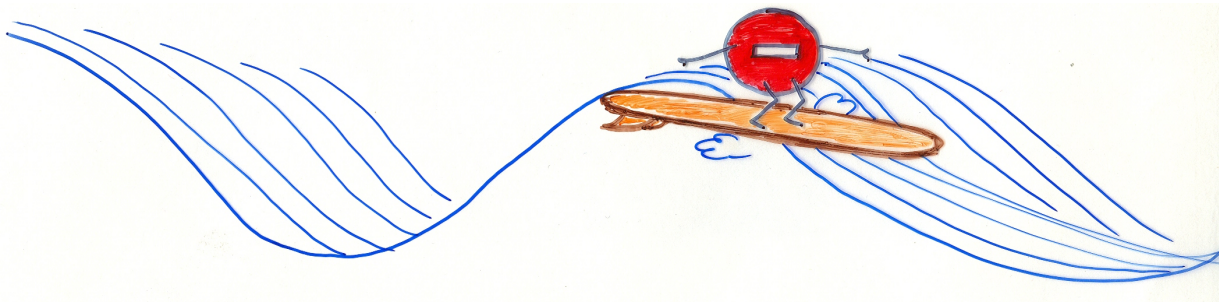
E_{e-} = 3.5 GeV

mami
MAINZER MIKROTRON

E_{e-} = 1.5 GeV

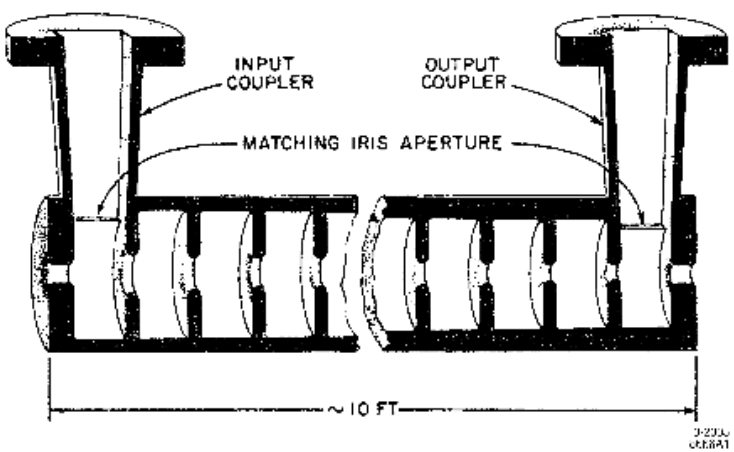
+ others… !

How to accelerate electrons:

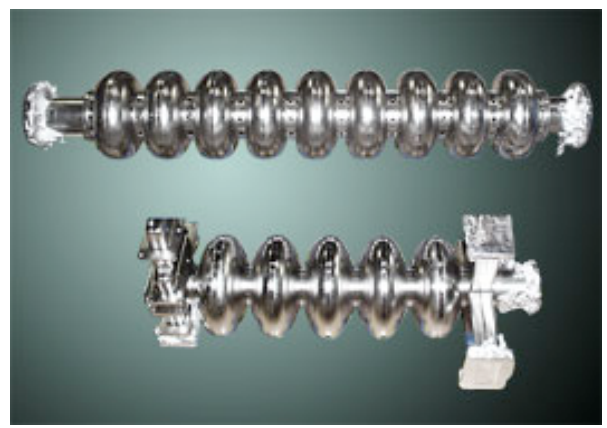


Surf the microwaves!

Accelerating cavity: disk loaded cylindrical wave guide
use TM_{01} mode to get a longitudinal electric field
match phase and velocity



DESY



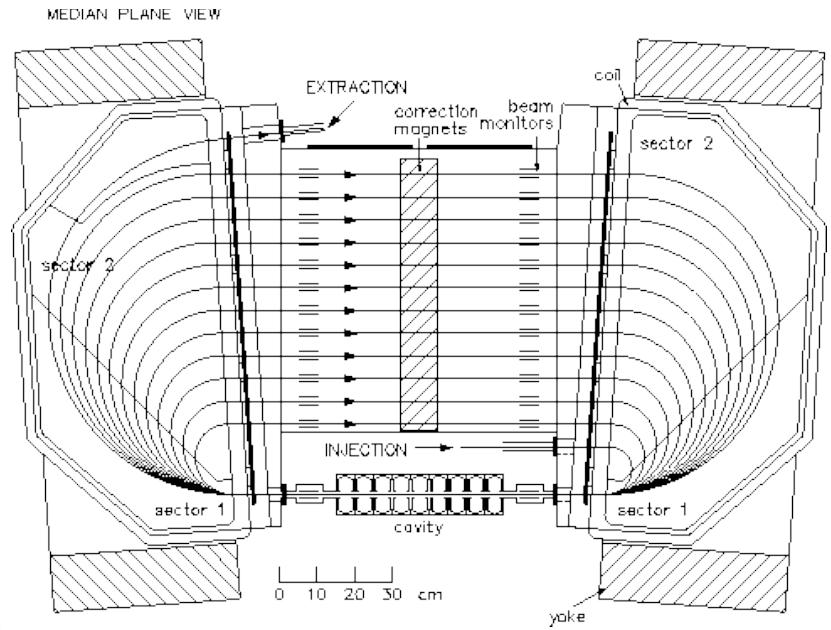
new

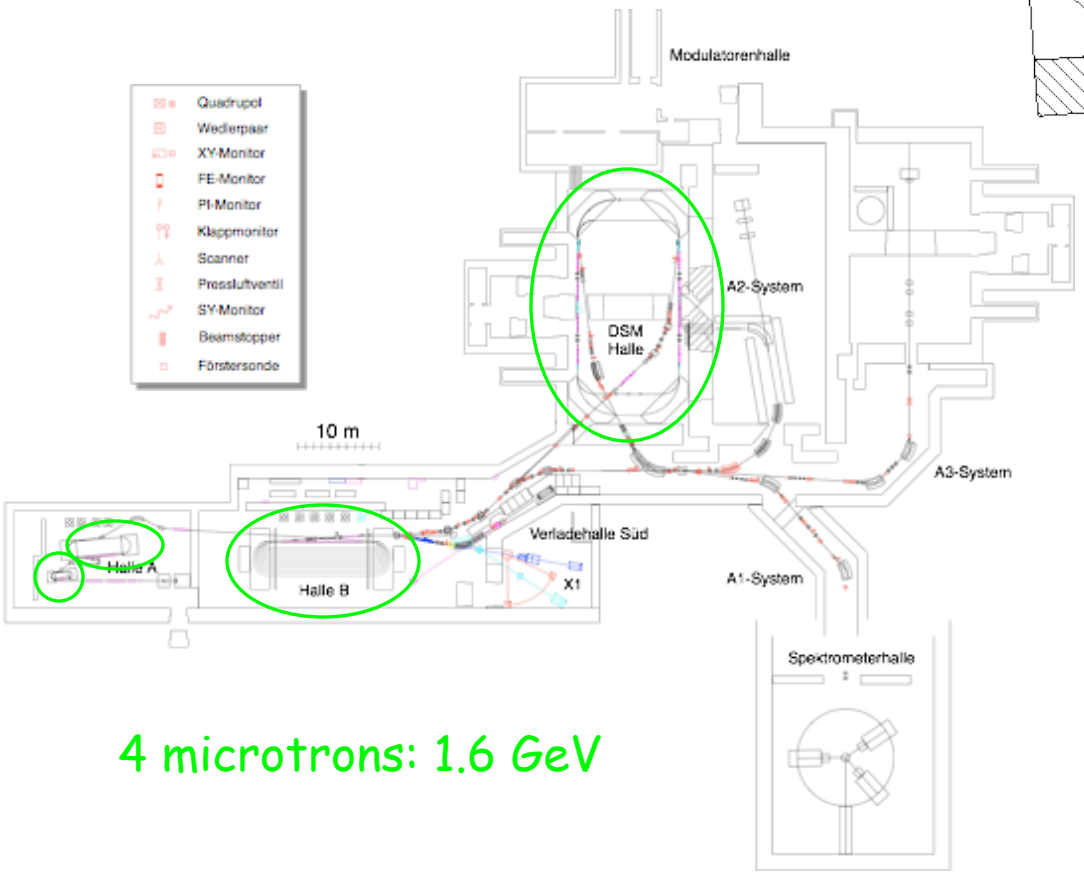
old

Jefferson Lab

Modern accelerators

- Race track microtrons (e.g.; Mainz)
  - Room temperature Cu cavities
  - Recirculate many times
  - Low gradient allows for CW

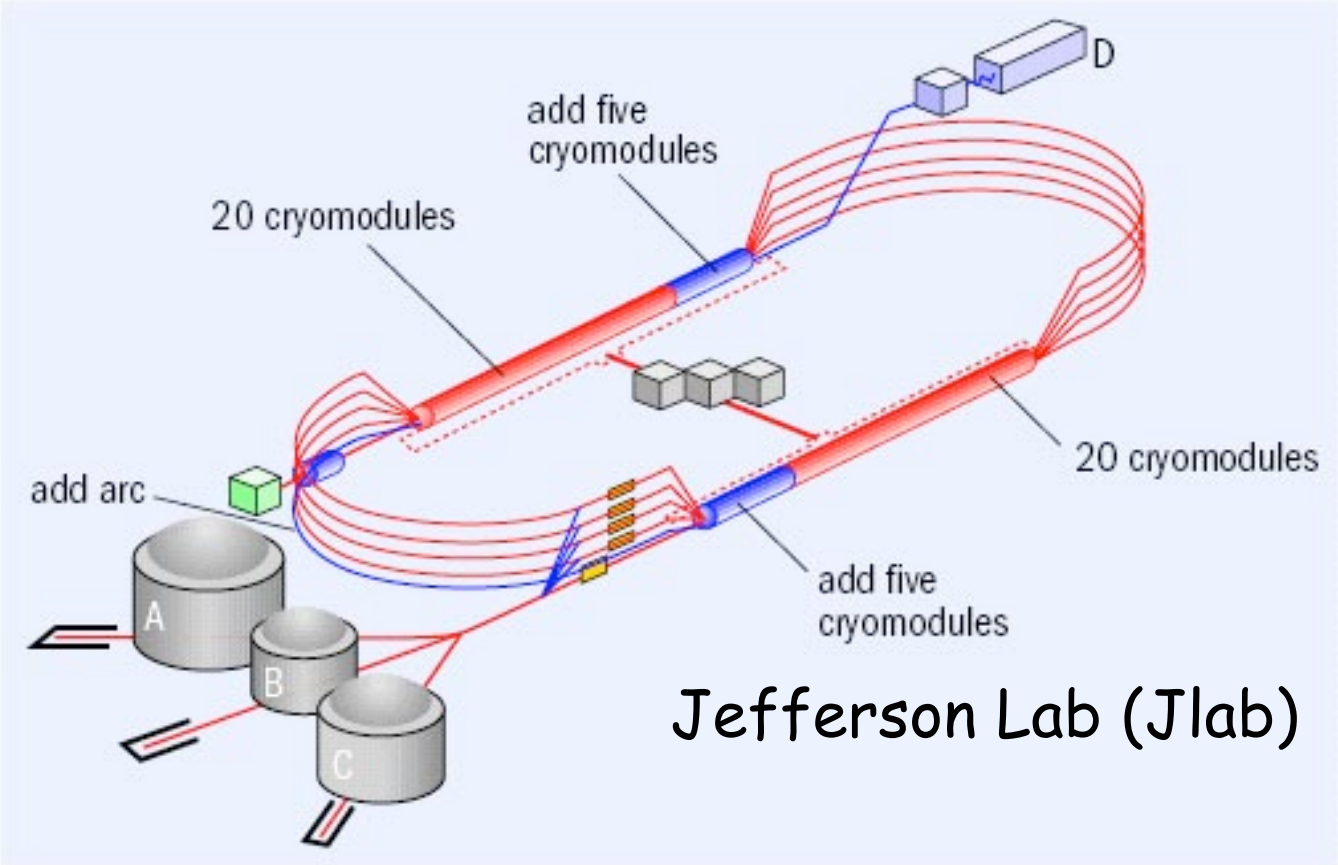




4 microtrons: 1.6 GeV

Modern accelerators

- Superconducting Linear Accelerators (LINACs)
  - 2K niobium cavities, very low resistive losses
  - Recirculate few times
  - High gradient (5-50 MeV/m)
  - CW

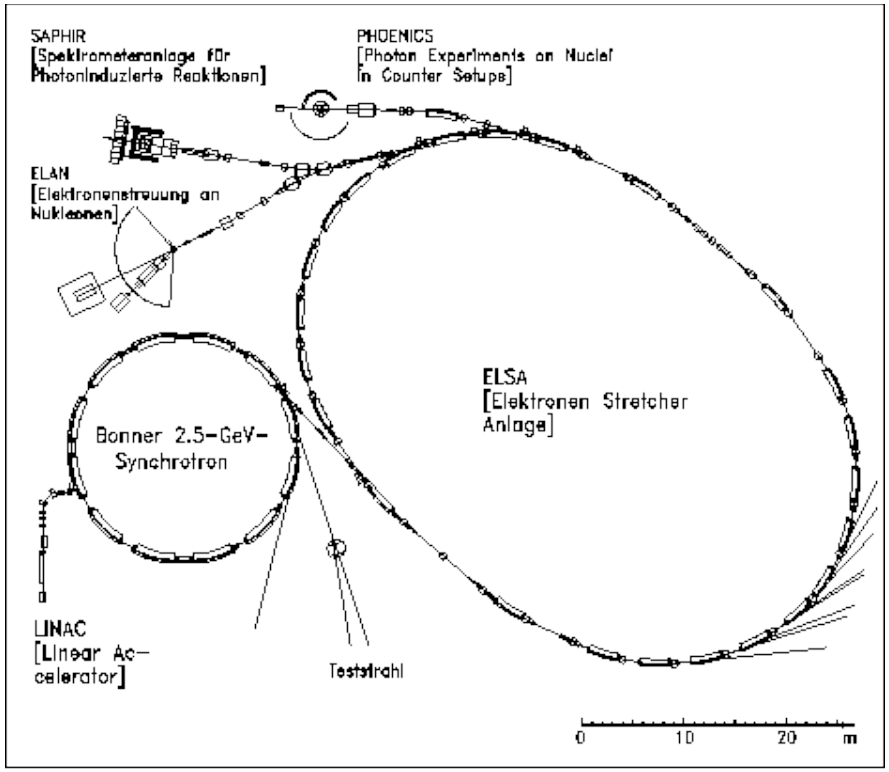


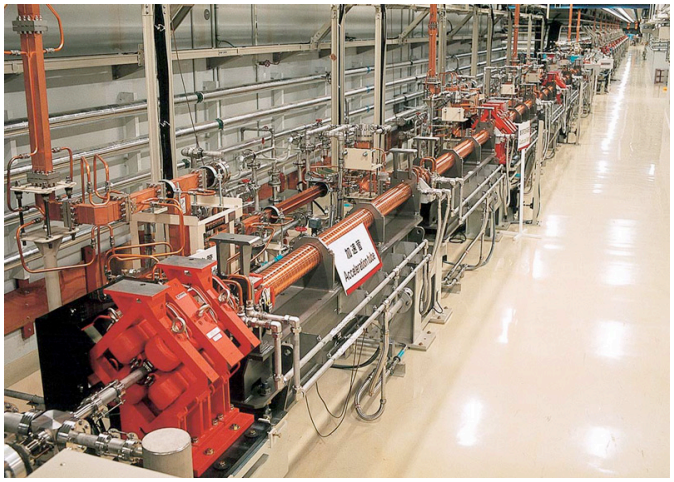
Jefferson Lab (Jlab)

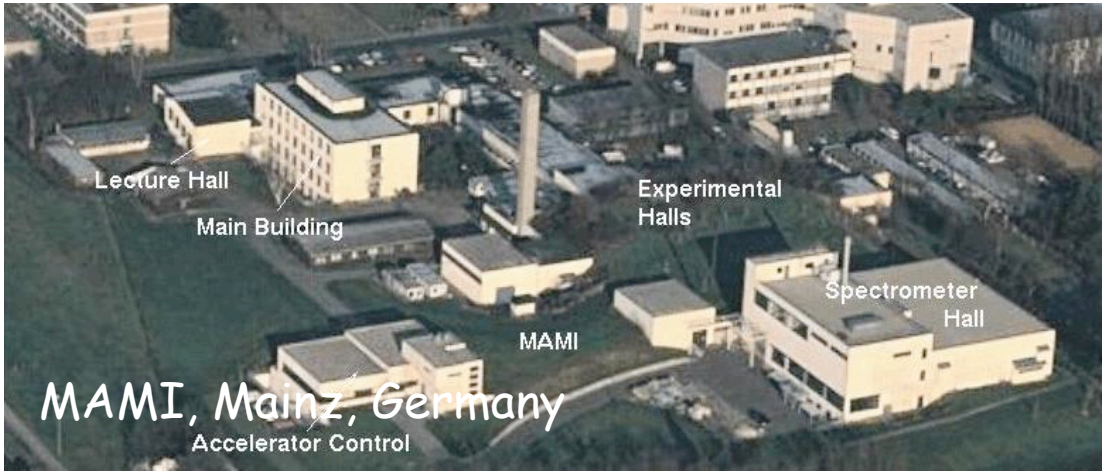
CEBAF at Jefferson Lab

Modern accelerators

- Linac plus stretcher ring
  (SPring8, Bonn, Frascati)
  - Electron beam generated in pulsed linac
  - Beam stored in stretcher ring and extracted slowly
  - Many rings built for synchrotron radiation studies
  - Beam typically less stable, noisier, and less focused







MAMI, Mainz, Germany

Jefferson Lab, USA

Postcard collection

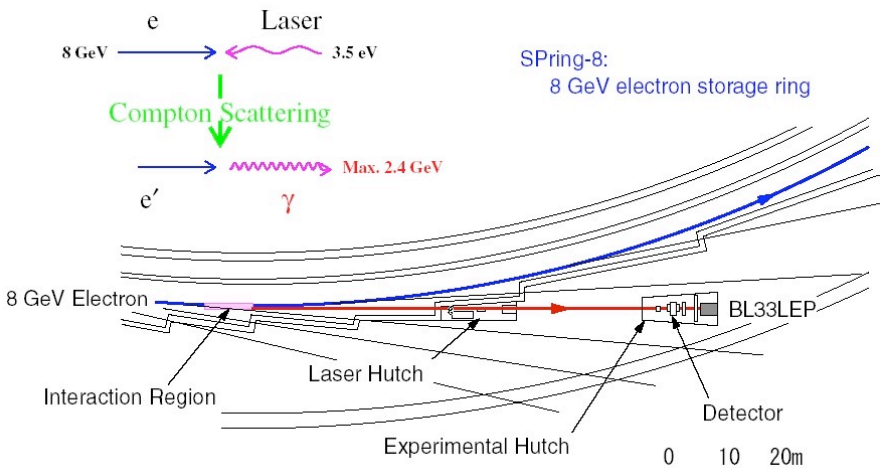
Spring 8, Japan

Hermes, DESY, Germany

Real photon beams from electron beams

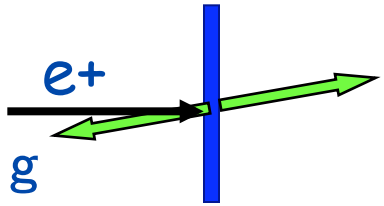
- **Laser back-scattering**

(LEGS, FRASCATI, DUKE, Spring8)
E_g smaller than beam energy
Facilities up to 4 GeV



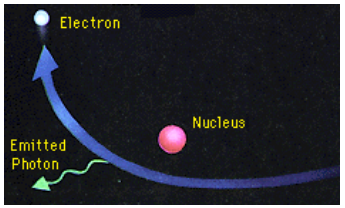
- **Positron annihilation in flight**

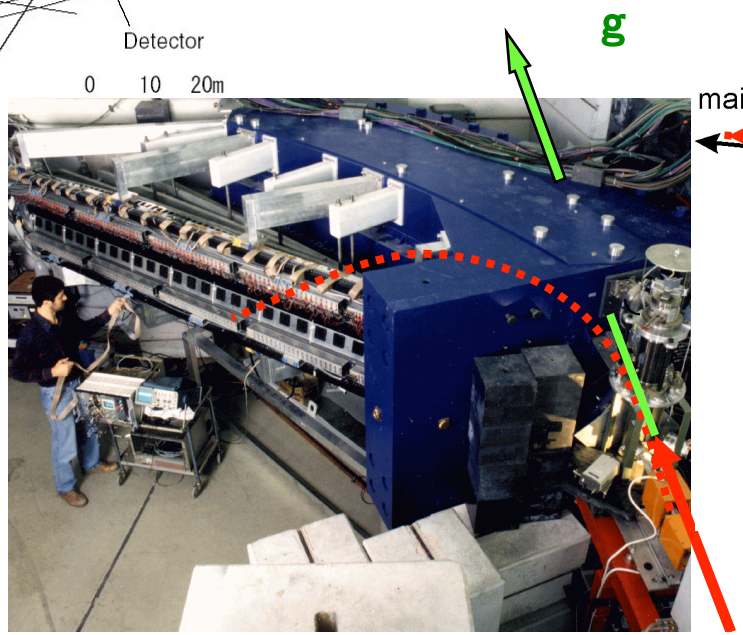
Only successful up to ~50 MeV
Bremsstrahlung background also present

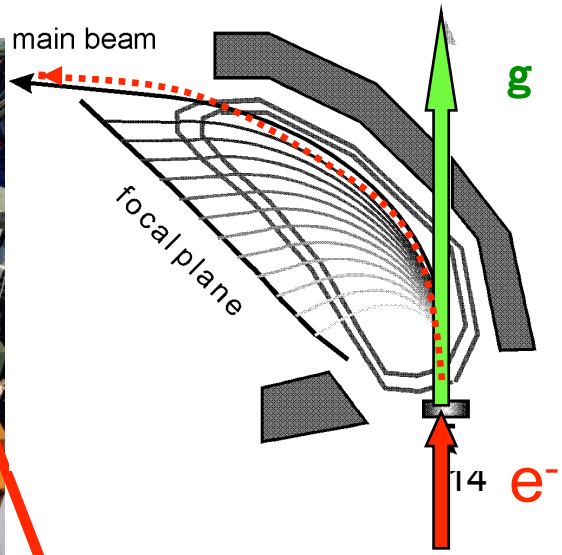


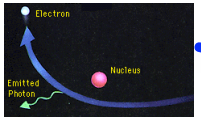
- **Tagged Bremsstrahlung**

(MAMI, Bonn, Lund, Jlab)
Eg up to ~beam energy
Facilities up to 6 GeV
(12 GeV planned at JLab)

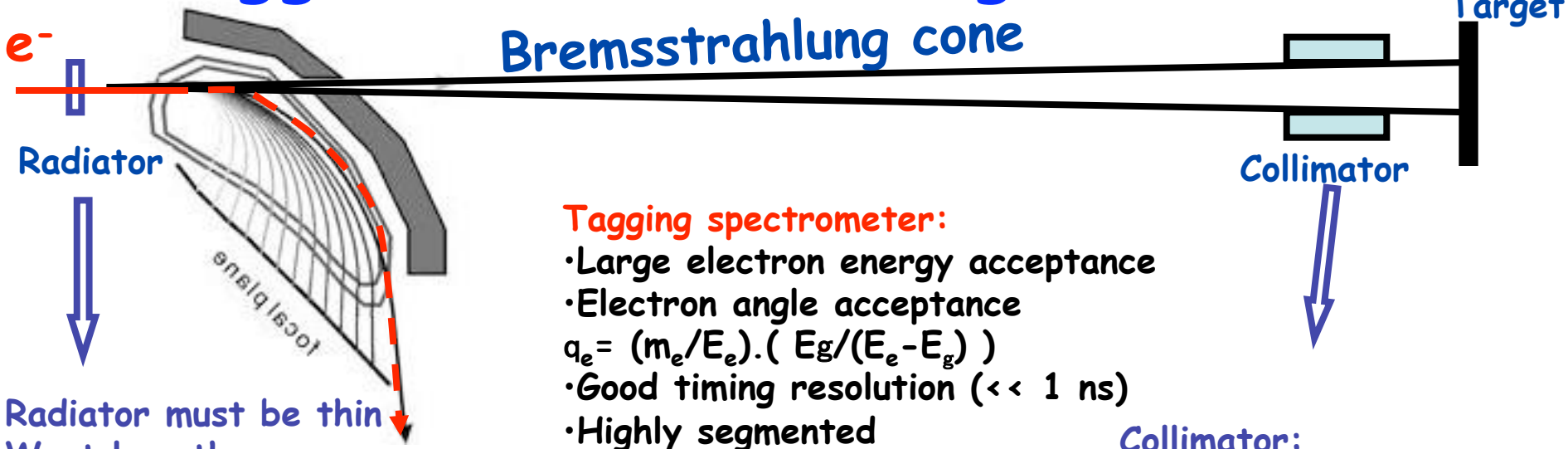






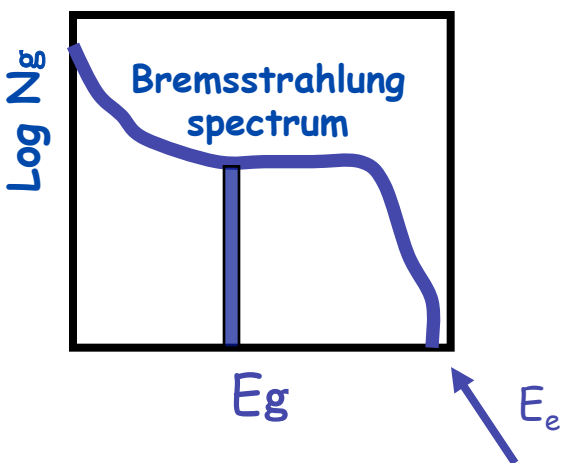


Tagged Bremsstrahlung



Radiator must be thin
Want less than one photon per electron

Keep radiator to target distance small to keep beamspot small



Tagging spectrometer:

- Large electron energy acceptance
- Electron angle acceptance

$q_e = (m_e/E_e).(Eg/(E_e-E_g))$

- Good timing resolution (<< 1 ns)
- Highly segmented
  - Good energy resolution
  - High rate capability

Reals/Randoms ~ $D/R_e t$

D = Duty factor of beam
R_e = mean electron rate
t = resolving time

Collimator:

- Smaller beam spot
- Enhance polarisation (see later)

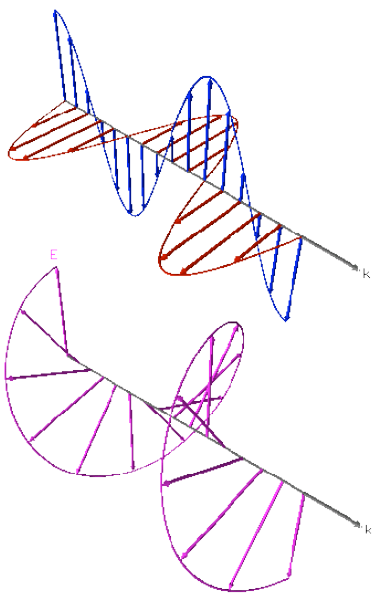
Polar angle of cone containing 50% of g

$Q_c = m_e/E_e$

e.g. MAMI – 855 MeV
Q_c = 0.6 mrad

Polarisation in real photon beams

Linear Polarisation



Circular Polarisation

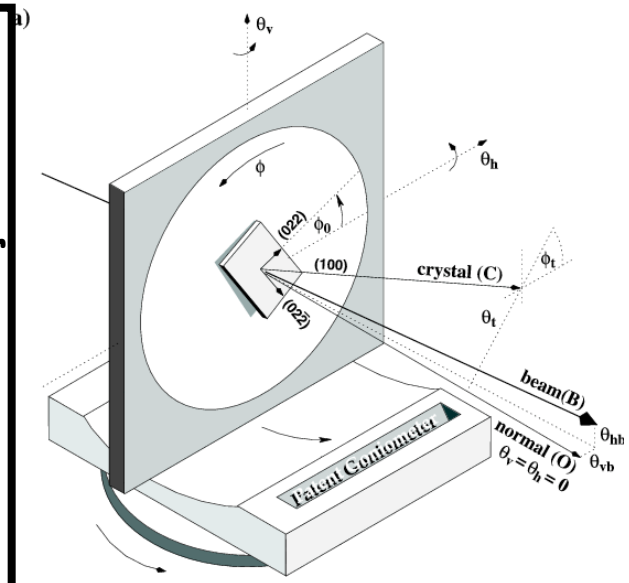
Bremsstrahlung facilities:

Linear polarisation:

- Coherent Bremsstrahlung
- Crystalline (thin diamond) radiator
- Orient diamond to give polarised photons in certain photon energy ranges

Circular polarisation:

- Polarised electrons and amorphous radiator.

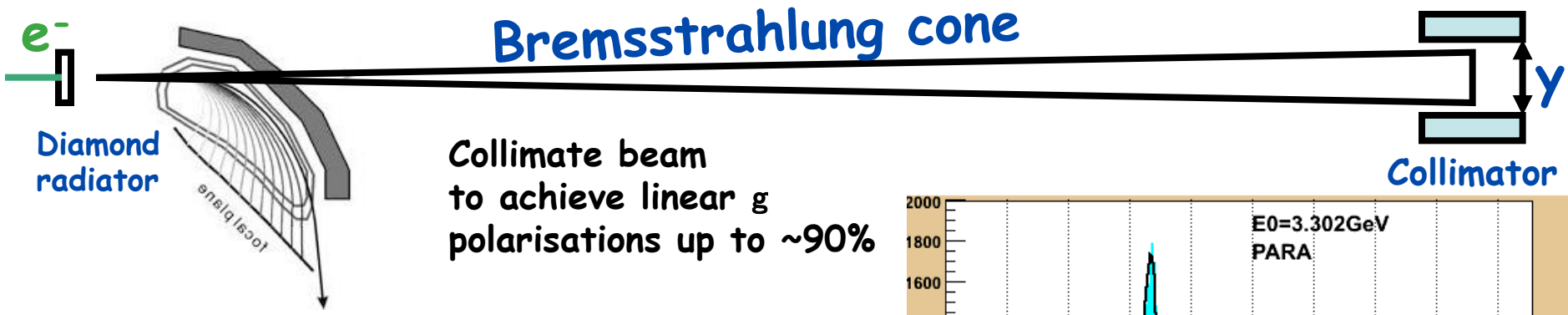


Laser backscattering:

Linear polarisation:

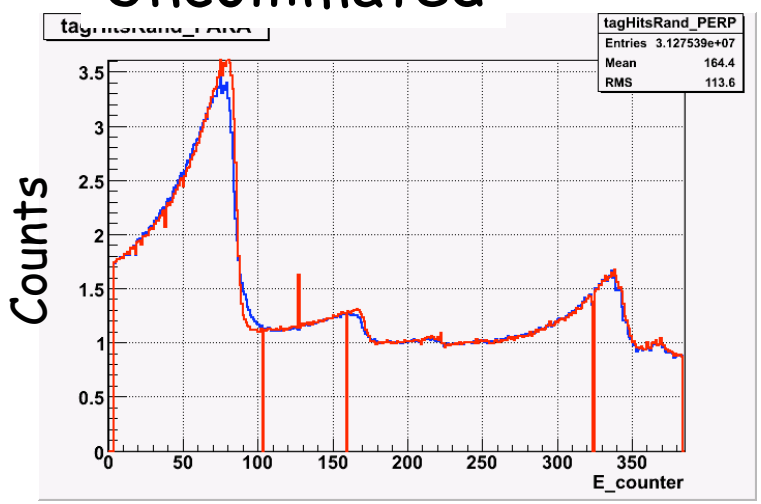
- Linearly polarised laser light backscattering from electrons.
- High degrees of polarisation (~100%) achievable

Coherent bremsstrahlung

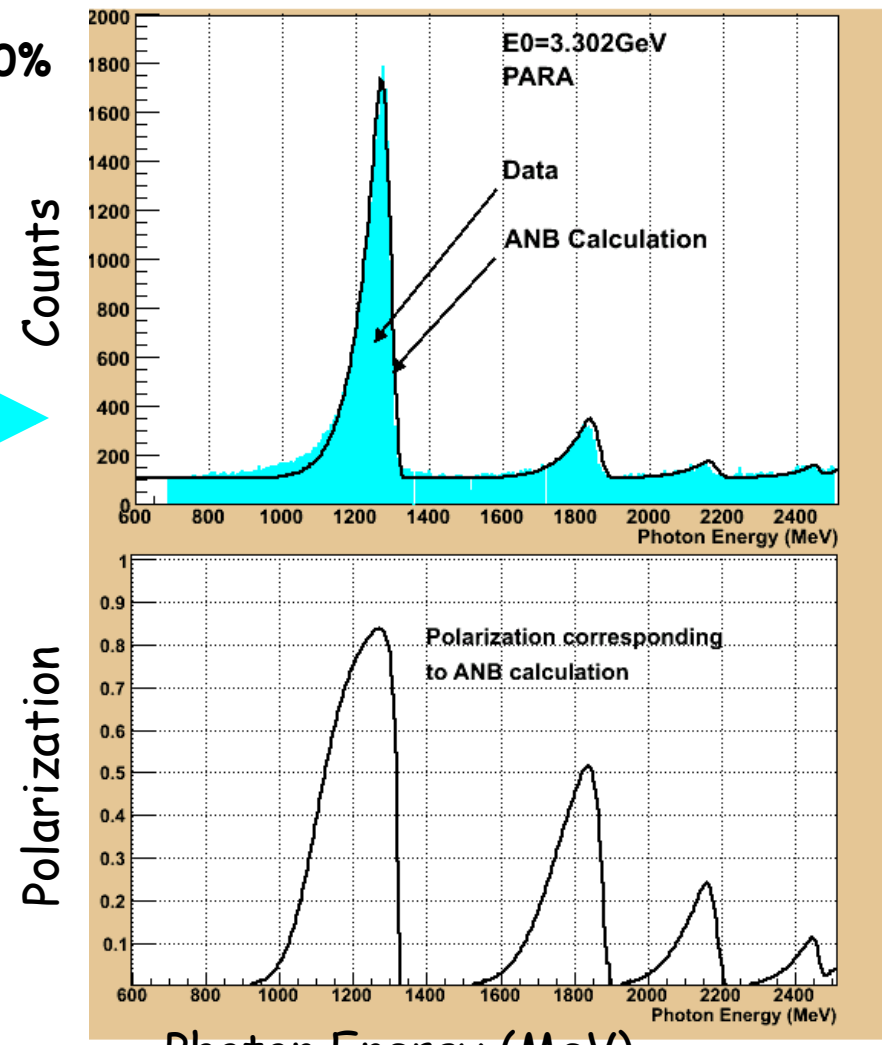


Collimate beam to achieve linear g polarisations up to ~90%

Uncollimated



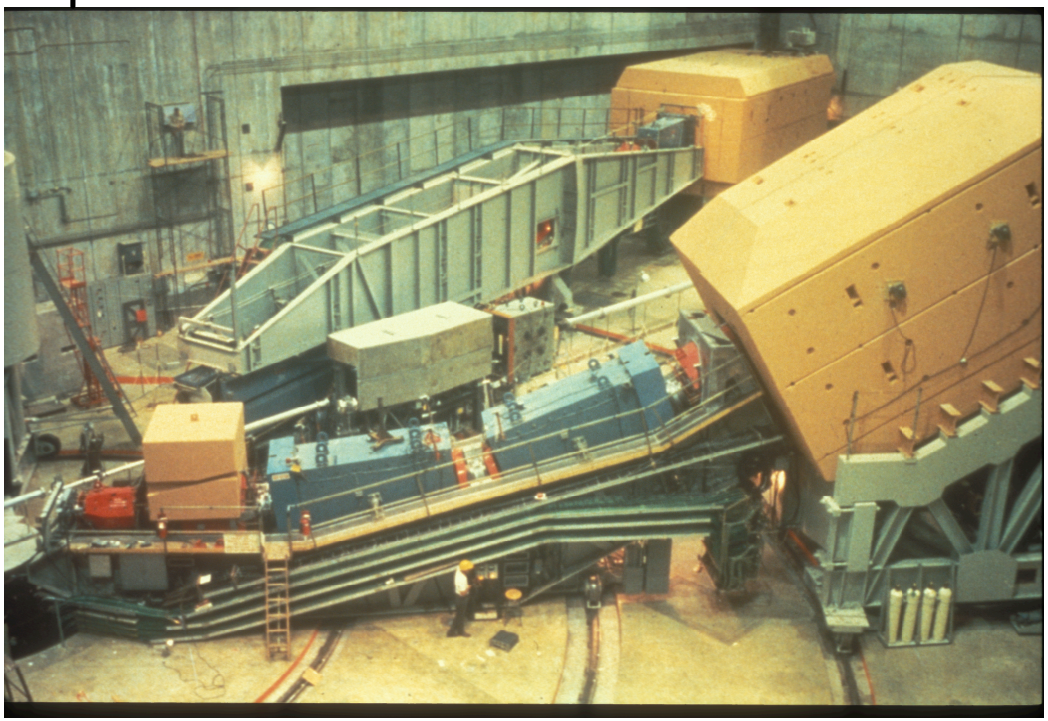
Photon Energy (a.u.)



Photon Energy (MeV)

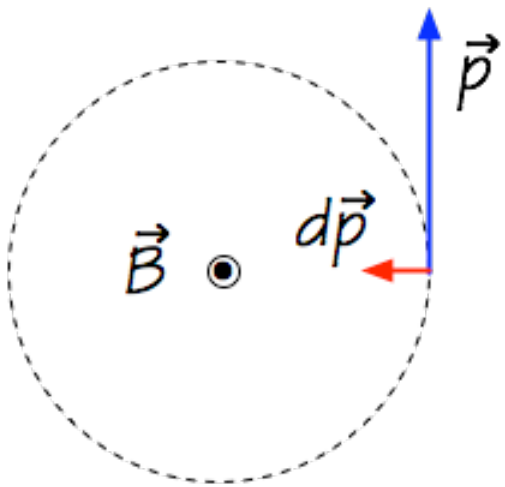
Spectrometers/Detectors

- 'standard' magnetic spectrometers
- Internal target spectrometers
- Large acceptance spectrometers
- Nonmagnetic spectrometers



SLAC

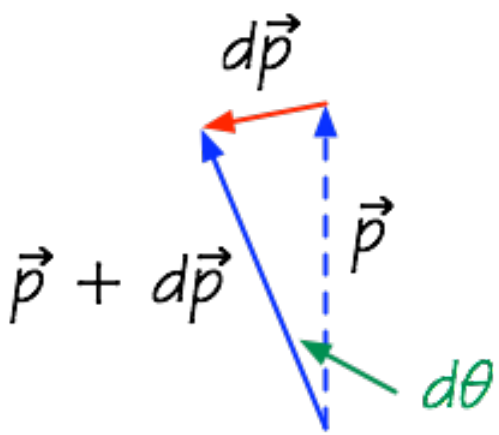
Deflection of electrons in magnetic field



Lorentz Force

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{d\vec{p}}{dt}$$

$$d\vec{p} = q(\vec{v}dt) \times \vec{B} = qd\vec{l} \times \vec{B}$$



$$d\theta = \frac{dp}{p} = \frac{q}{p}Bdl$$

$$\Rightarrow \Delta\theta = \frac{q}{p}\int Bdl$$

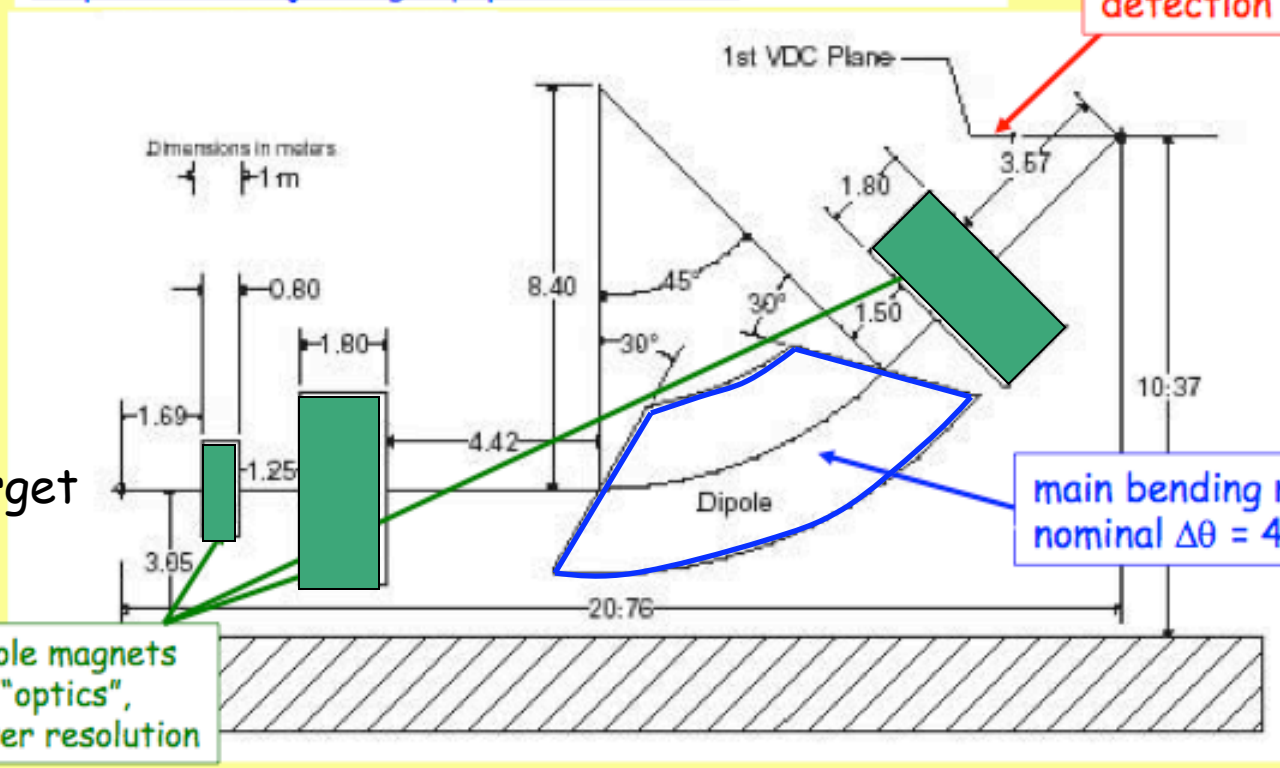
Deflection $\Delta\theta$, even if B field is not uniform

Deflection in magnetic field measures the momentum

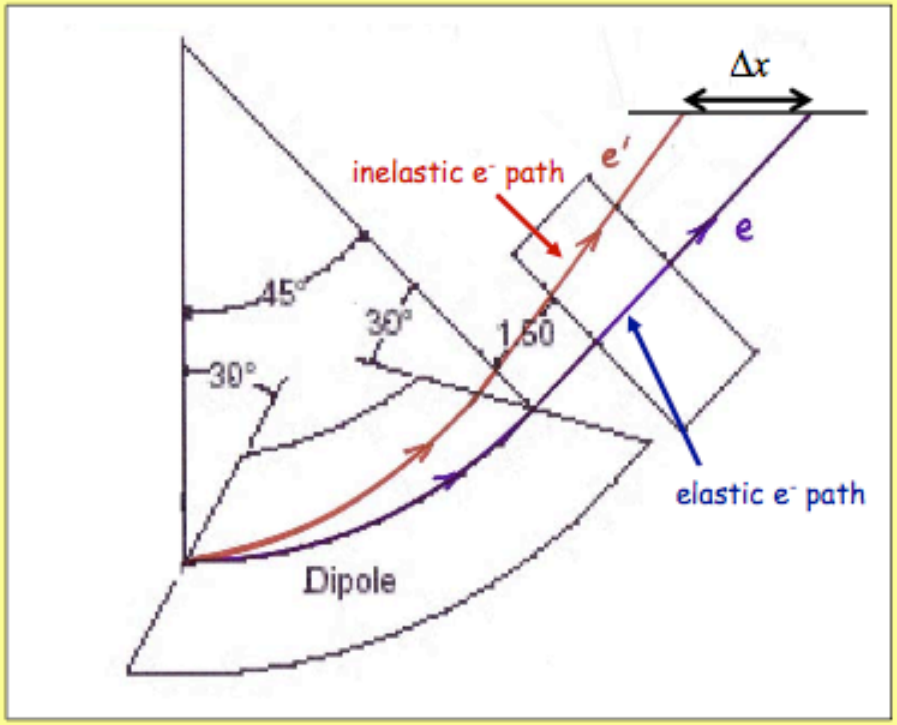
$$\Delta\theta = \frac{e}{p}\int B\, d\ell = \frac{const.}{p} \quad \text{for central trajectories}$$

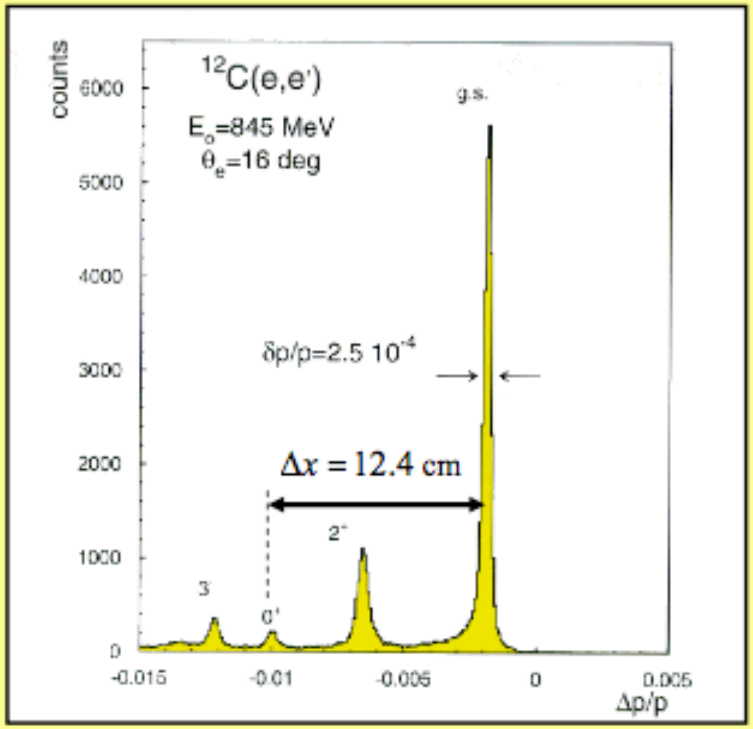
Hall A High Resolution Spectrometer (HRS) at JLab:

http://hallaweb.jlab.org/equipment/HRS.htm



Magnets and resolution





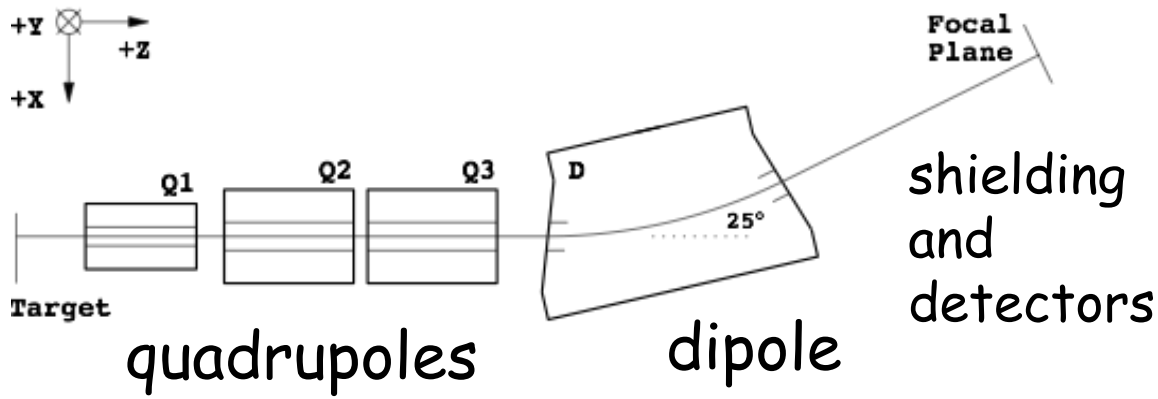
Dispersion: D = 12.4 cm/ %

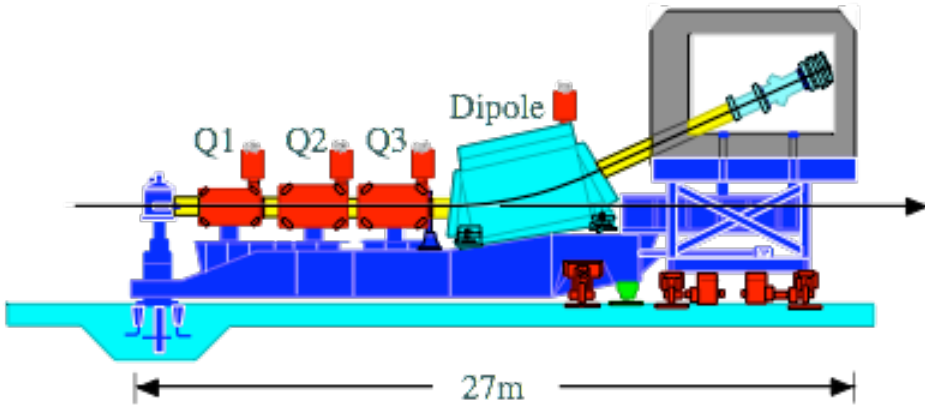
→ A 1% shift from the central momentum corresponds to a deflection at the focal plane of 12.4 cm **more** than the elastic peak.

JLab Hall C HMS

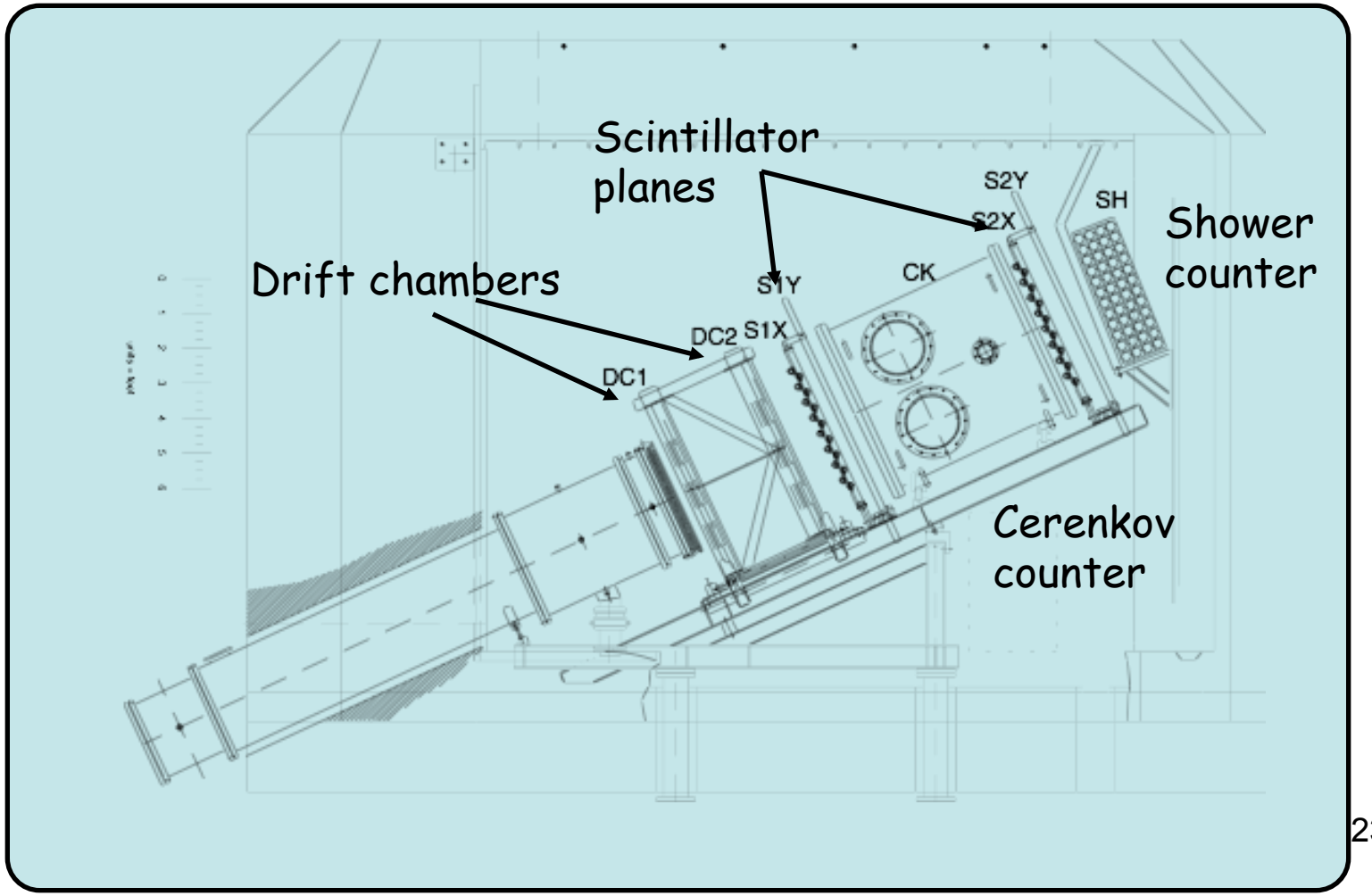
Standard magnetic spectrometers:

- Dipole disperses particles
- Quads improve
  - angular acceptance
  - resolution
- Small aperture
  - High luminosity
- Large B fields
  - Good resolution



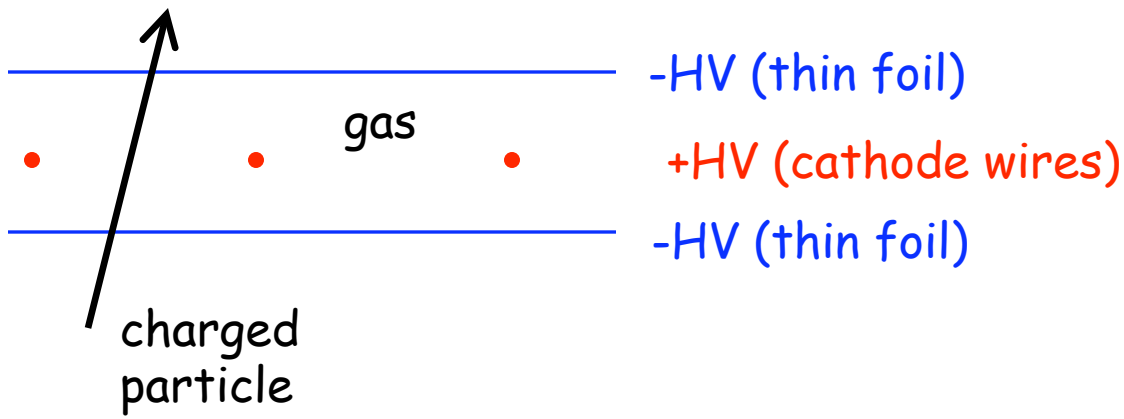


HMS detectors



Detectors I

Wire chambers measure position (and angle)



1. Charged particle passes through wire chamber and knocks out electrons from the gas.
2. Electrons drift in the E field to the cathode wire, colliding with gas molecules
3. Close to the wire, the mean free path times the electric field is large enough to ionize the gas molecules. Avalanche!
4. Read the signal on the cathode wire

Detectors II

Scintillators: time (~100 ps) and energy measurement

- Typically a doped plastic or crystal (eg: Ge, NaI, BaF_2)
- Charged particle passes through scintillator (or neutral particle interacts) and excites atomic electrons. These de-excite and emit light.
- Minimum energy loss (when $\beta\gamma \approx 1$) is $dE/dx = 2$ MeV/(gm/cm^2)

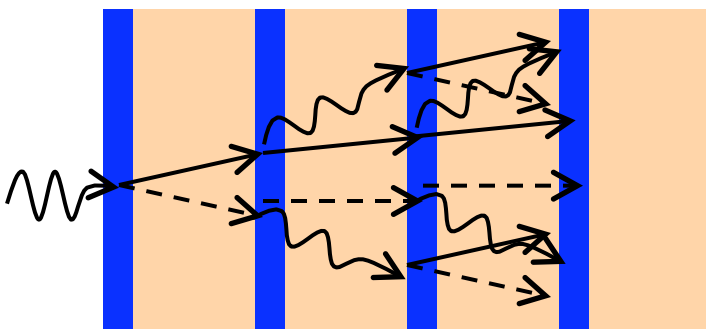
Cerenkov counter: threshold velocity measurement.

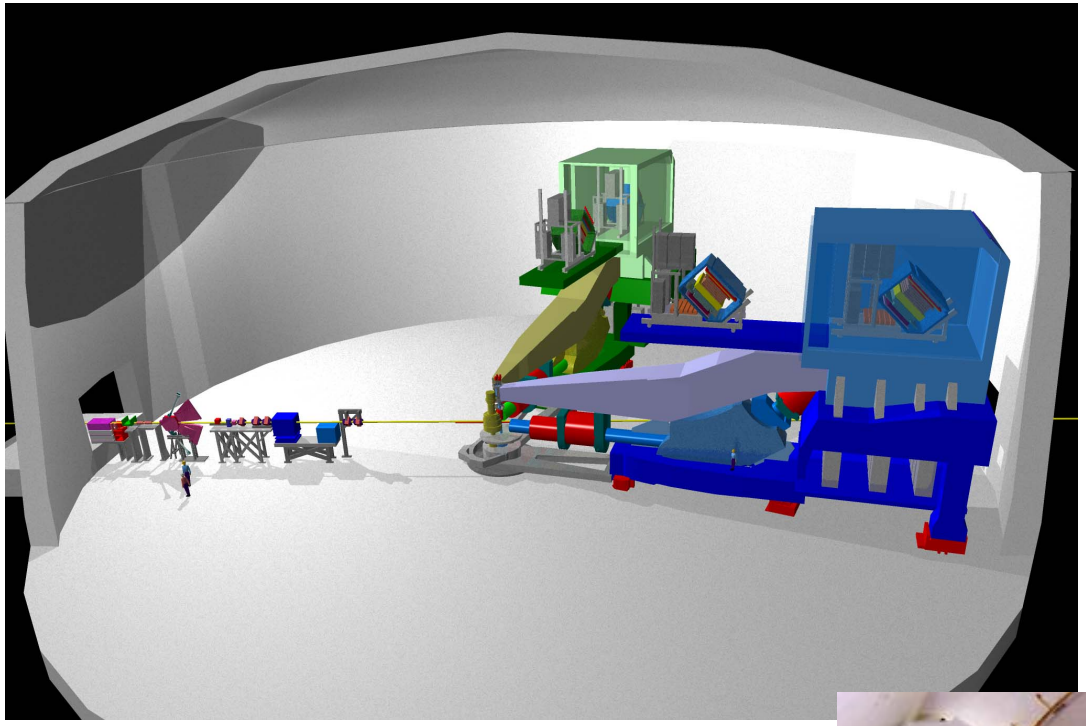
- Typically an empty box with smoke (ie: a gas) and mirrors
- Local light speed is $v = 1/n < c$
- Particles travelling faster than v will emit Cerenkov light (an electromagnetic 'sonic boom')
- The opening angle of the cerenkov cone is related to the particle's velocity

Detectors III

Electromagnetic shower counters: measure energy (also time) and detect neutral particles.

- Electrons and photons passing through material shower
- After one radiation length of material on average:
  - Electrons emit a bremstrahlung photon
  - Photons convert to an electron/positron pair
- After 10 radiation lengths, one e^- or γ is now ~1000 particles
- Simple design: alternating layers of lead (R_L = 6 mm) and scintillant
  - Particles shower in the lead
  - Charged particles deposit energy in the scintillant

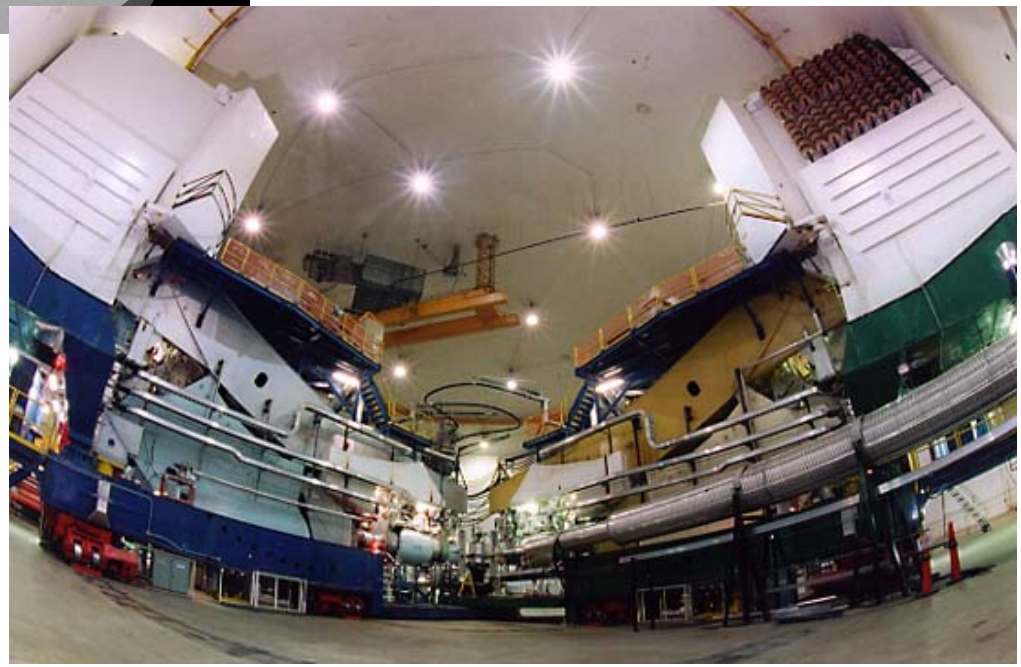




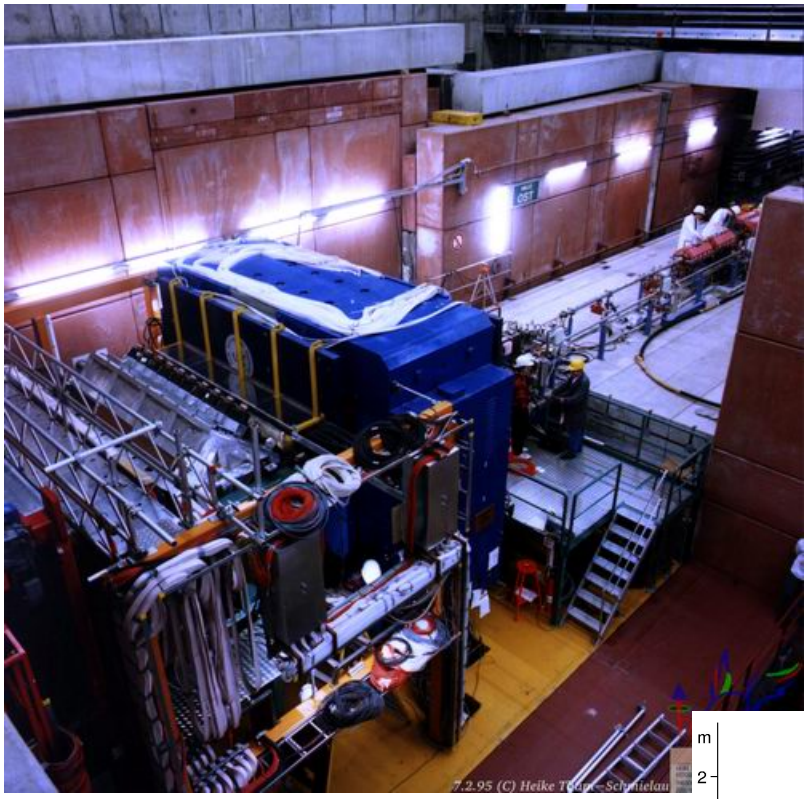
JLab Hall A

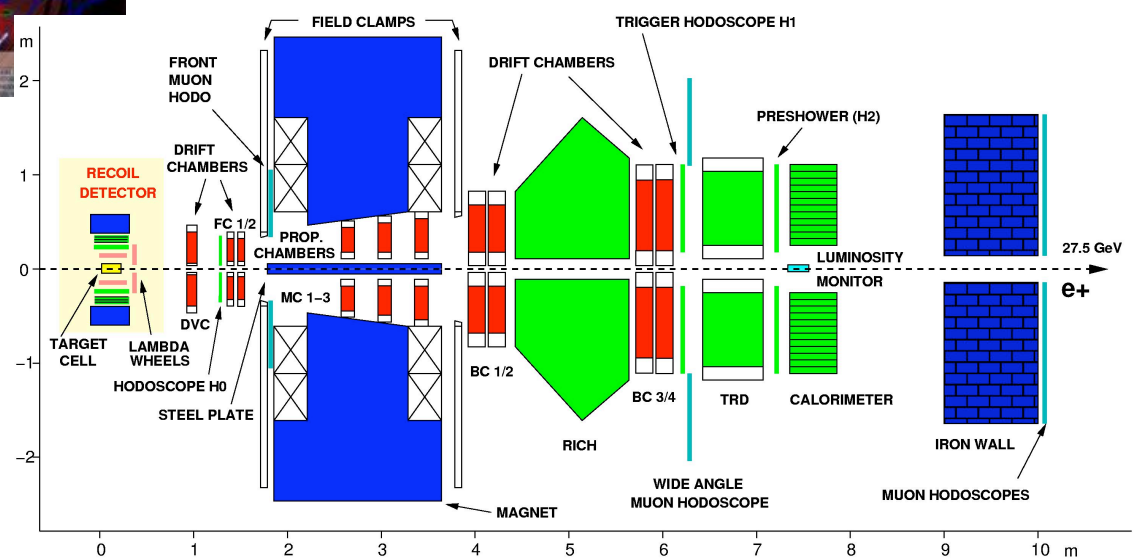
High Resolution Spectrometers (HRS)

Momentum acceptance	+/- 4.5%
Momentum resolution	10^{-4}
Solid angle	6 msr
Angular resolution	
vertical	2 mr
horizontal	0.6 mr

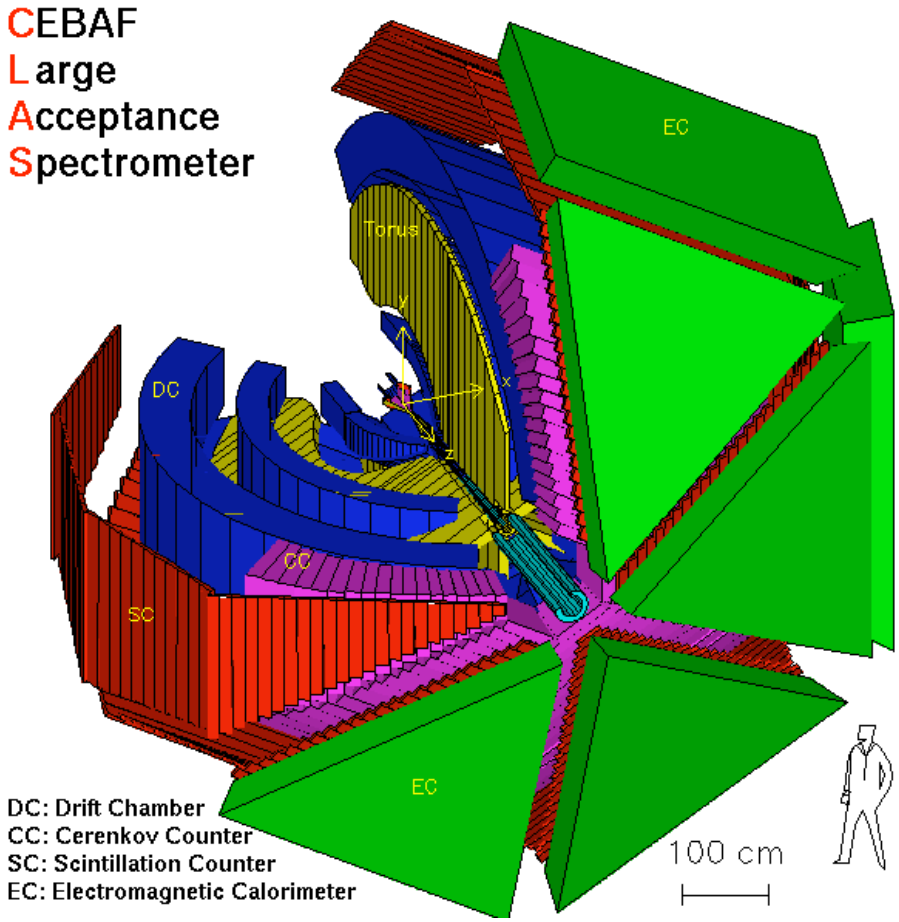


Internal target spectrometer: Hermes at DESY





JLab CLAS


CEBAF
Large
Acceptance
Spectrometer
EC
Torus
DC
CC
SC
EC
100 cm
DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter

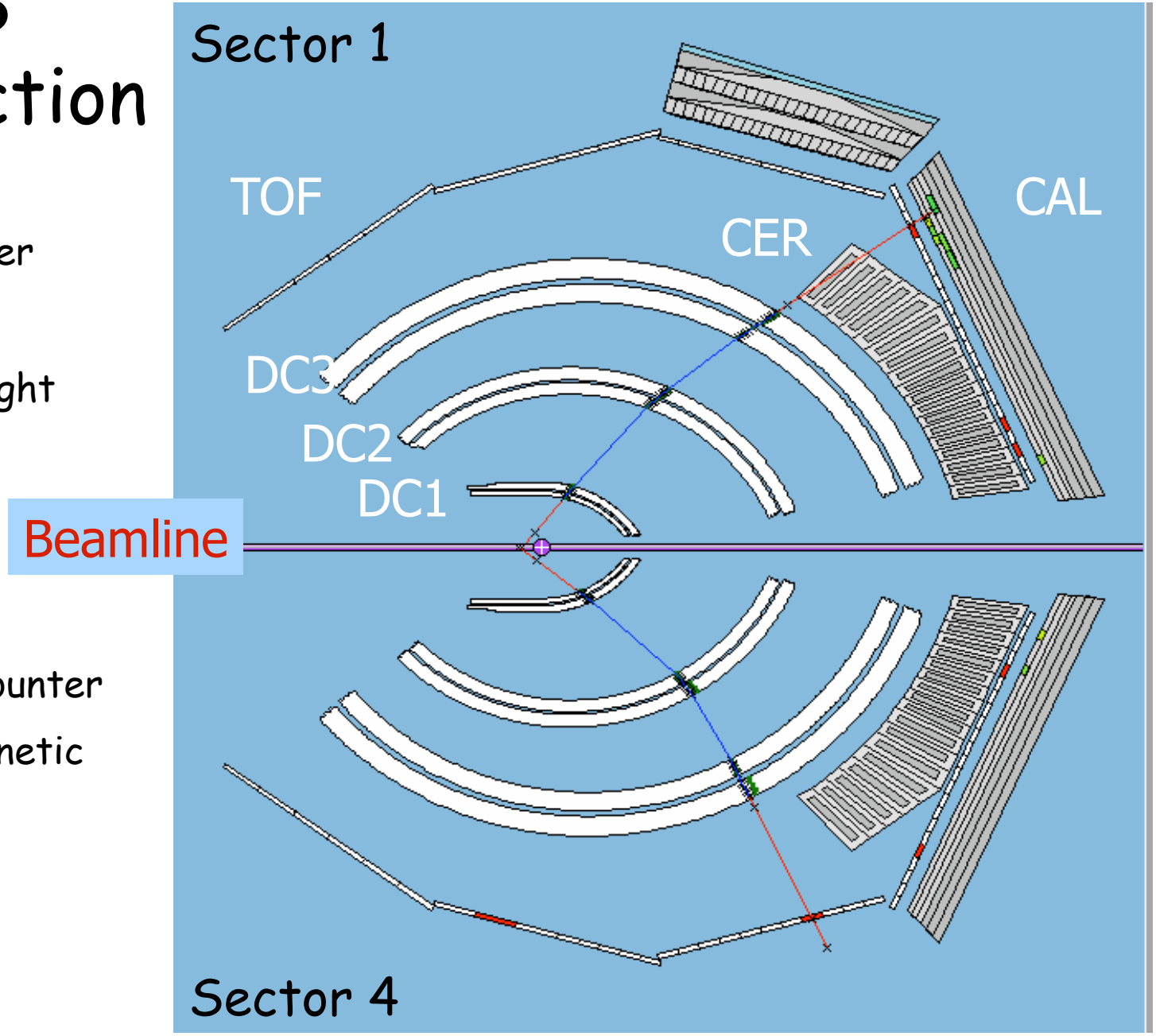
CLAS cross section

DC = Drift Chamber (wire chamber)

TOF = Time of Flight (scintillator)

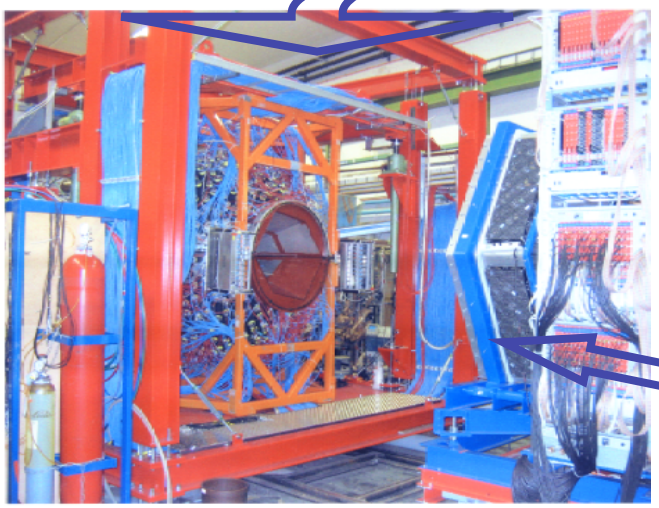
CER = Cerenkov counter

CAL = Electromagnetic calorimeter

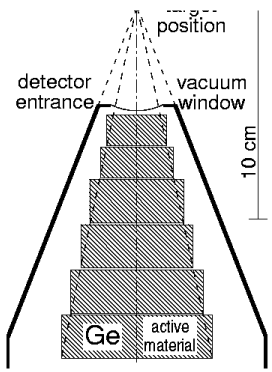


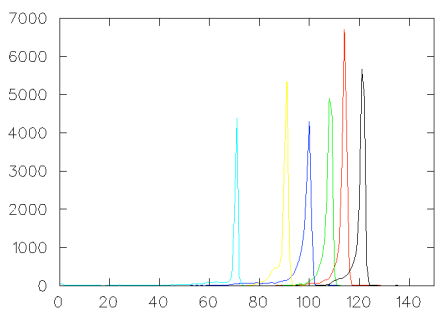
Associated Particle detectors

- detect neutral particles (primarily photons)

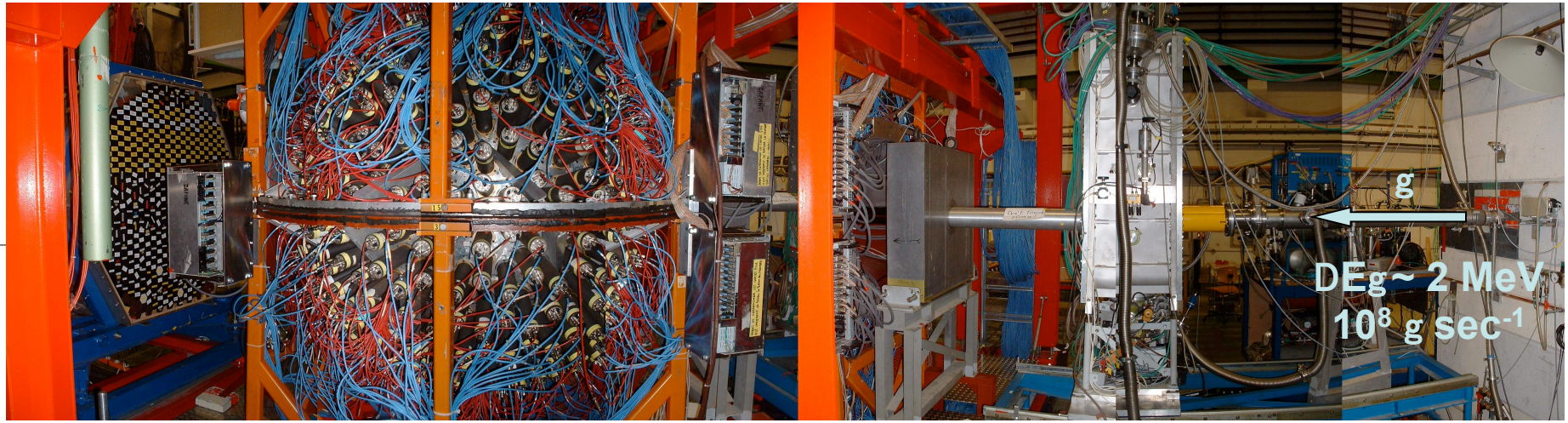


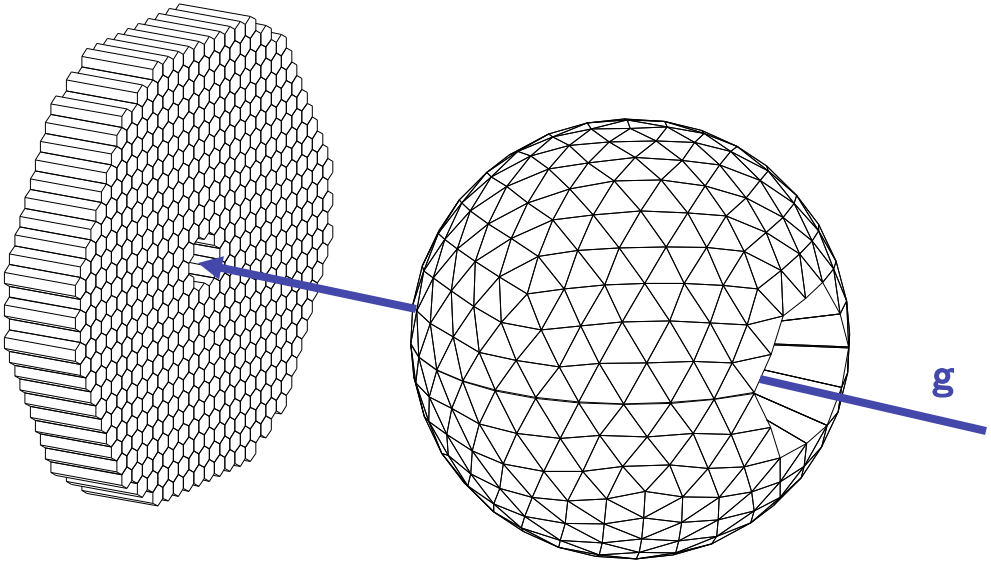
- Crystal Ball at MAMI
  672 element NaI detector
- Determines energies and momenta of high energy photons
- Scottish groups built sub-detector to also allow charged particle detection
- TAPS BaF_2 array covers forward angles





- Also use Germanium detectors as high resolution proton/pion Detectors e.g. Edinburgh Ge6 array



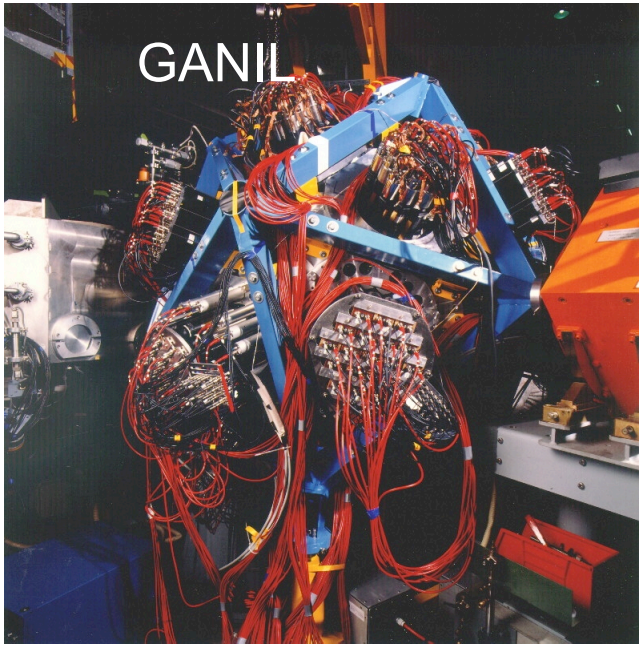


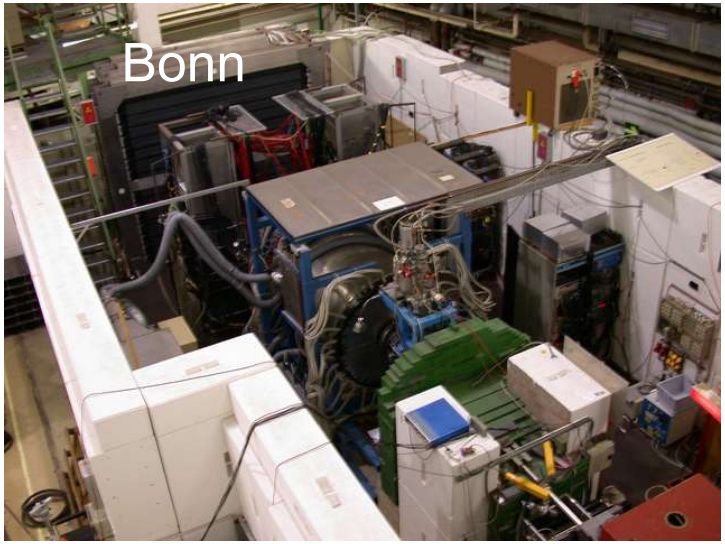
TAPS
528 BaF_2 crystals

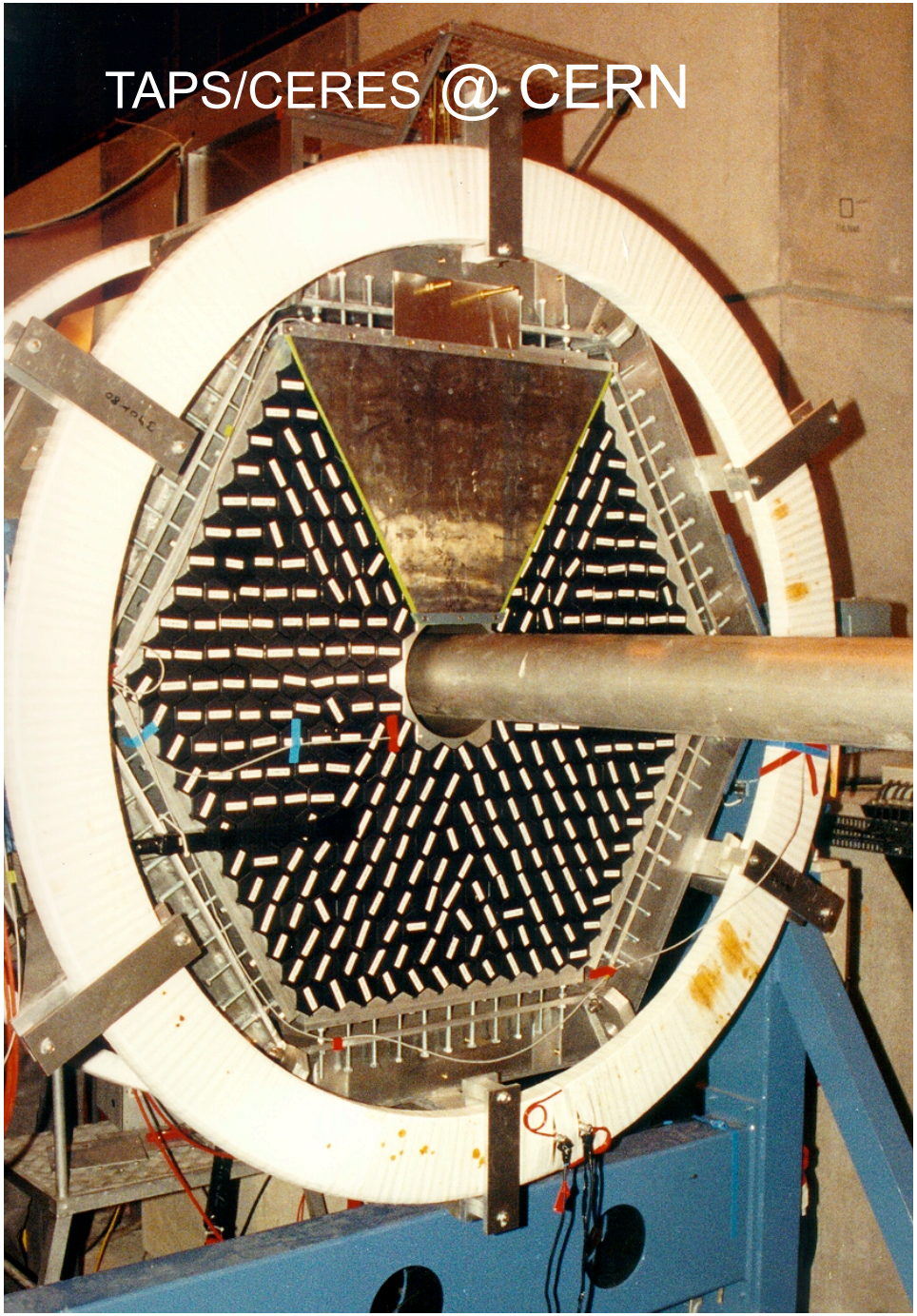
Crystal Ball
672 NaI crystals

Crystal Ball and TAPS at MAMI

"Travelling Around" Photon Spectrometer

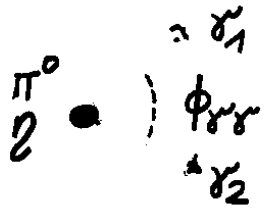






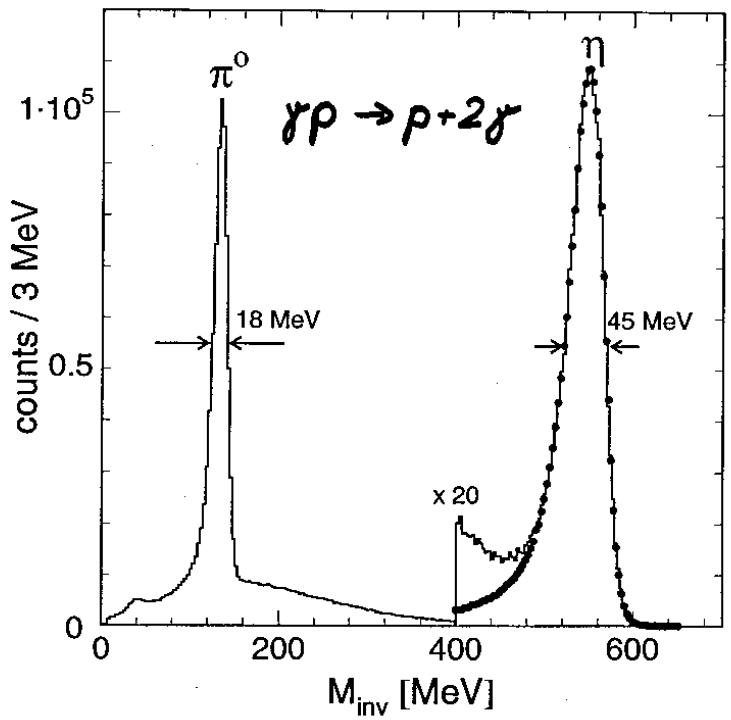
What do we use photon detectors for?

Primarily used to reconstruct neutral mesons which decay to photons
Eg. $p^0 \rightarrow 2g$, $h \rightarrow 2g$



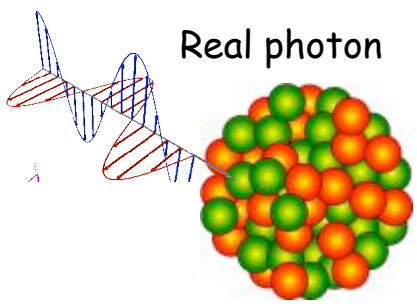
$$M_{inv} = \sqrt{(E_{\gamma_1} + E_{\gamma_2})^2 - (P_{\gamma_1} + P_{\gamma_2})^2} =$$

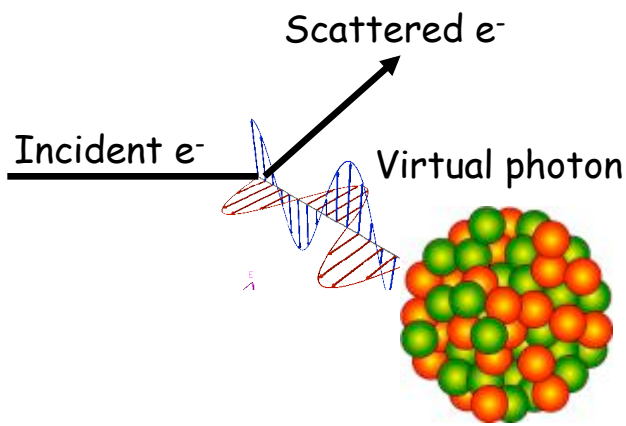
$$= \sqrt{2 E_{\gamma_1} E_{\gamma_2} (1 - \cos(\phi_{\gamma\gamma}))}$$



It's all photons!

- An electron interacts with a nucleus by exchanging a single* virtual photon.





Real photon:

Momentum q = energy [illegible]

Mass = $Q^2 = |q|^2 -$ [illegible]$^2 = 0$

Virtual photon:

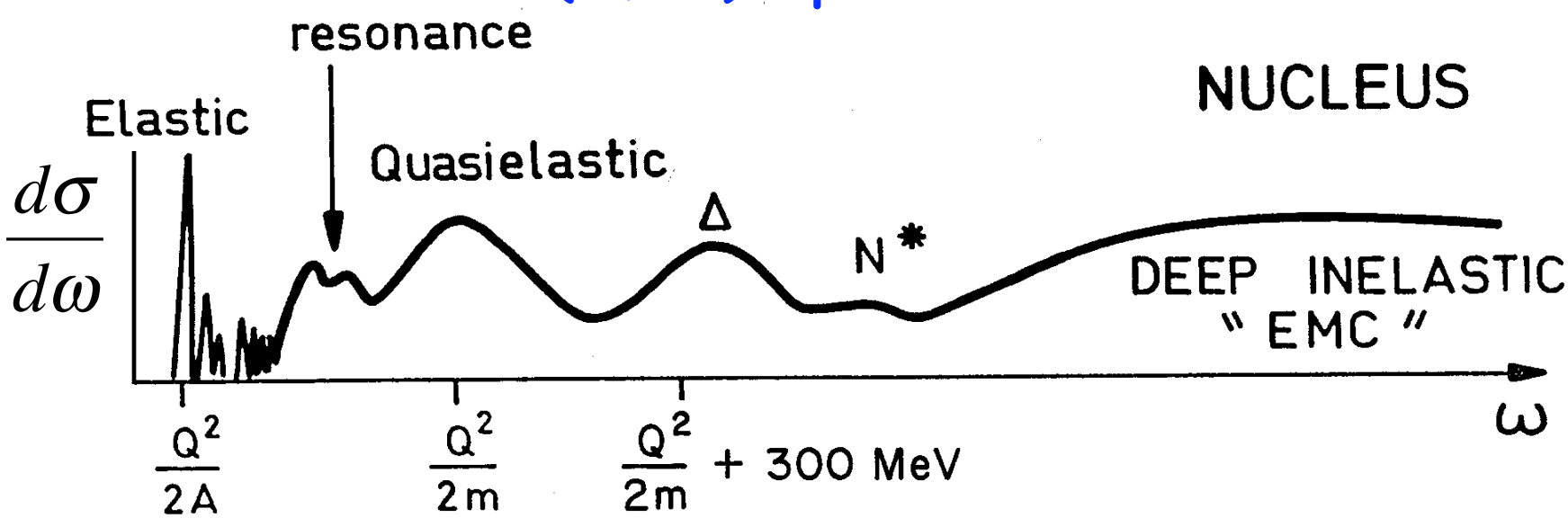
Momentum q [illegible] energy [illegible]

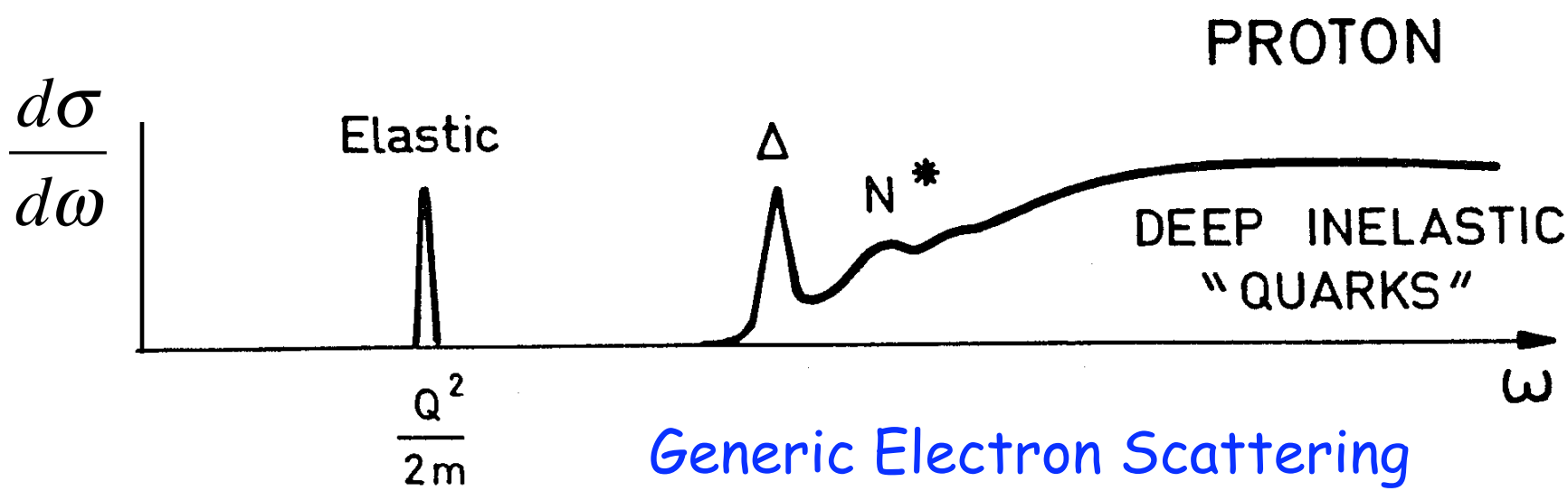
$Q^2 = -$ q[illegible]q[illegible] = $|q|^2 -$ [illegible]^2 [illegible] 0

Virtual photon has mass!

([illegible] and [illegible] are both used for energy)

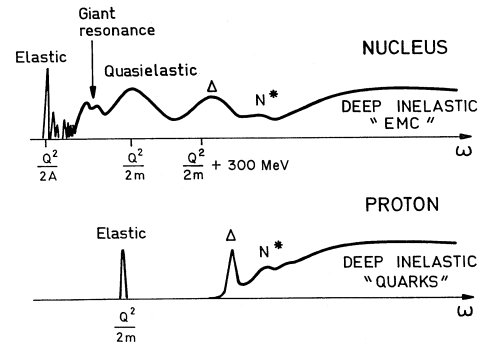
(e,e') spectrum





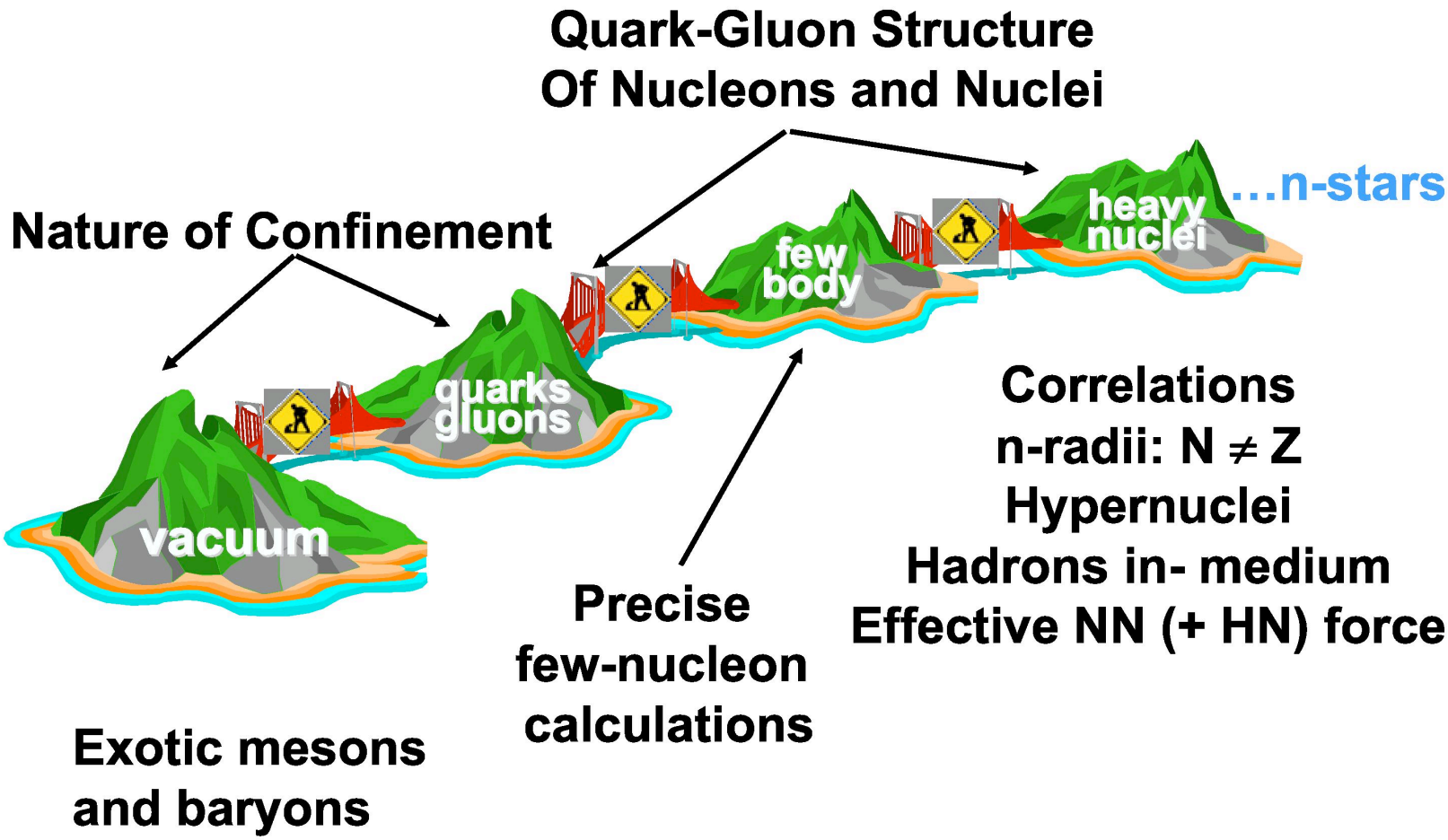
Generic Electron Scattering
at fixed momentum transfer

Experimental goals:



- Elastic scattering
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF (e.g. T_{20})
- Quasielastic scattering
  - Shell structure
  - Momentum distributions, occupancies
  - Medium modification of the nucleon
  - Nuclear transparency and color transparency
- (deep) inelastic scattering
  - Baryon resonance excitation (Δ, N*, Y, exotics?)
  - Nucleon structure
    - Quark and gluon distributions
  - Meson production (W, W, W, W, K, exotics?)

Program central to all of nuclear science



Energy vs length

Select spatial resolution and excitation energy independently

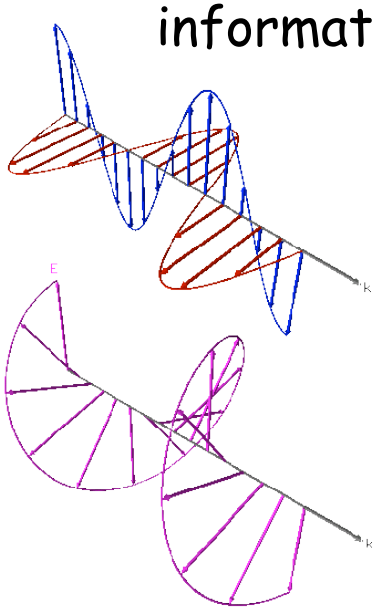
- Photon energy ω determines excitation energy
- Photon momentum q determines spatial resolution ω ω hbar/q

Three cases:

- Low q
  - Photon wavelength ω larger than the nucleon size (R_p)
- Medium q: 0.2 < q < 1 GeV/c
  - ω ~ R_p
  - Nucleons resolvable
- High q: q > 1 GeV/c
  - ω < R_p
  - Nucleon structure resolvable

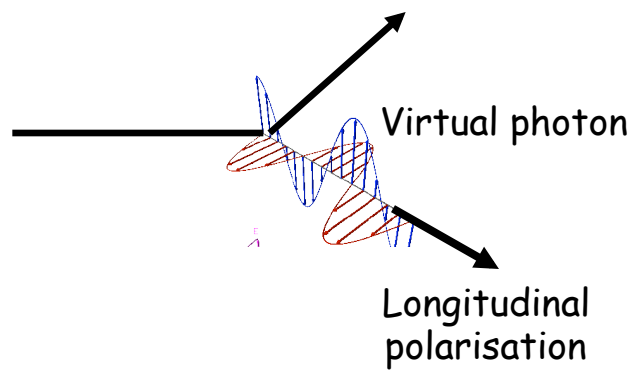
Types of photon Polarisation

- Both real and virtual photons can have polarisation
  - Real photons: transverse pol only
  - Virtual photons: longitudinal or transverse
- Determining azimuthal distribution of reaction products around these polarisation directions gives powerful information (see later)



Linear polarisation
(Electric field vector oscillates in a plane)

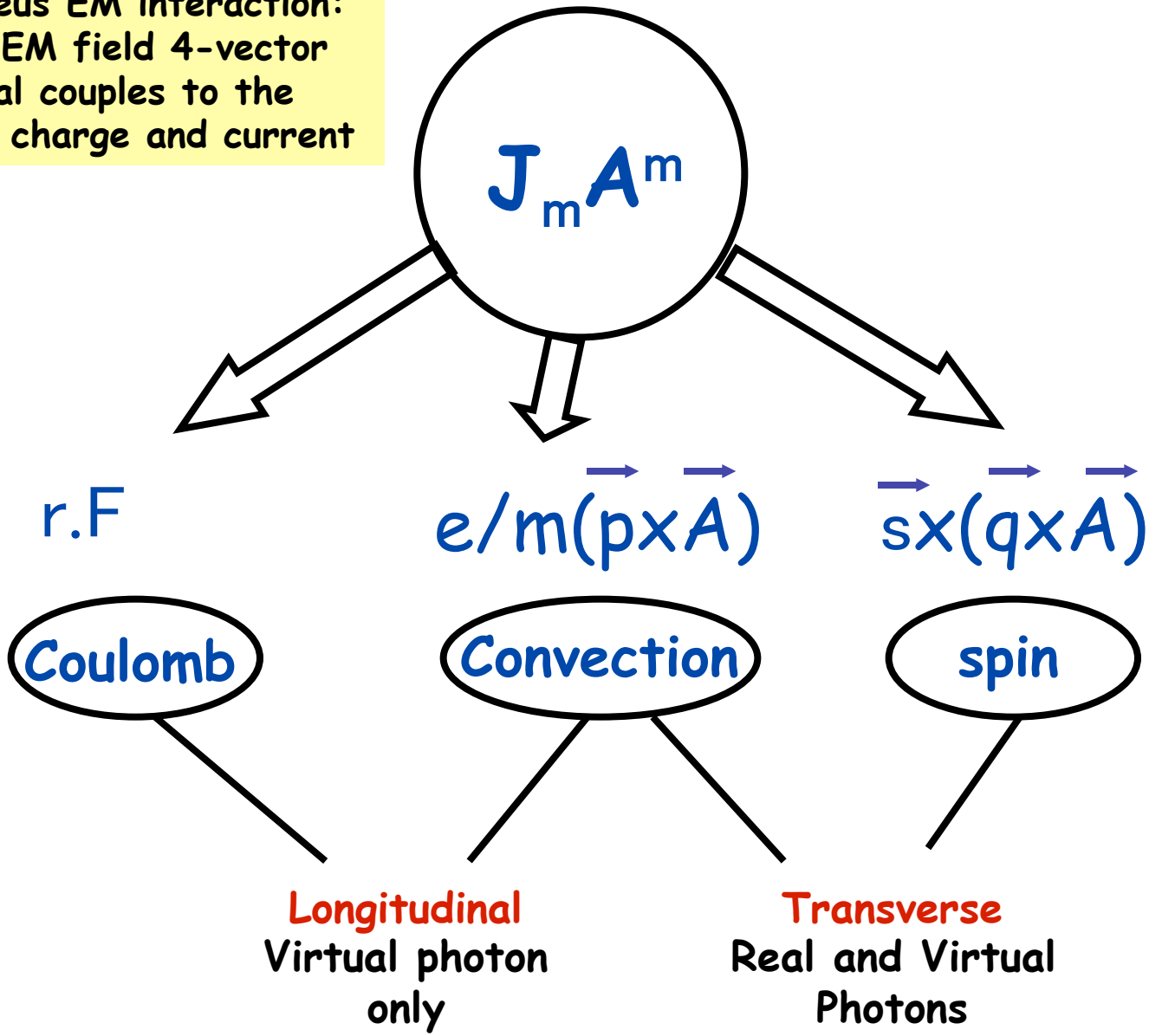
Circular polarisation
(Electric field rotates clockwise or anticlockwise)



Virtual photons can also have a longitudinal polarisation component - related to coulomb (charge) scattering

Real and virtual photon coupling

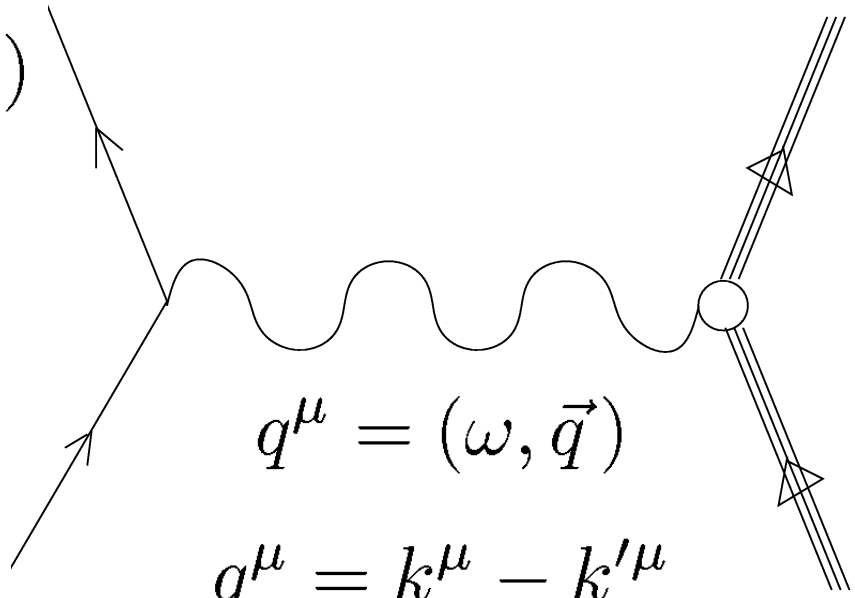
Photon-nucleus EM interaction:
Photon EM field 4-vector potential couples to the nuclear charge and current



Inclusive electron scattering (e,e')

$k'^{\mu} = (E', \vec{k}\,')$

$p'^{\mu} = (E_p, \vec{p}_p)$



Lab frame kinematics

$q^{\mu} = (\omega, \vec{q})$

$k^{\mu} = (E, \vec{k}\,)$

$q^{\mu} = k^{\mu} - k'^{\mu}$

$p^{\mu} = (M, \vec{0}\,)$

Invariants:

$$p^{\mu} p_{\mu} = M^2$$

$$p_{\mu} q^{\mu} = M\omega$$

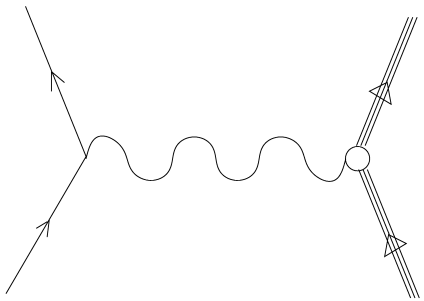
$$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2$$

$$W^2 = (q^{\mu} + p^{\mu})^2 = p'_{\mu} p'^{\mu}$$

Formalism

- Inclusive scattering:
  - measure scattering angle θ_e and energy E'_e ($\omega = E_e - E'_e$) and the cross section $d\sigma/d\omega d\Omega$
- One photon exchange:

$$\mathcal{M}_n = \frac{e^2}{q^2}\bar{u}(k')\gamma_\mu u(k) < n|J_\mu(0)|p, S >$$



$$\begin{aligned} d\sigma &= \frac{1}{4M\vec{k}}\sum_n |\mathcal{M}_n|^2(2\pi)^4\delta^4(p+q-p')\frac{d^3k'}{(2\pi)^3 2E'} \\ &= \frac{|\vec{k}'|}{ME}\frac{\alpha^2}{Q^4}L^{\mu\nu}H_{\mu\nu}d\omega d\Omega \end{aligned}$$

$L^{\mu\nu}$ and $H_{\mu\nu}$ are the lepton and hadron tensors

More Formalism

Lepton tensor known (QED):

$$
\begin{aligned}
L_{\mu\nu} &= \sum_{s'} (\bar{u}(k',s')\gamma_\mu u(k,s))^* (\bar{u}(k',s')\gamma_\nu u(k,s)) \\
&= 2(k_\mu k'_\nu + k_\nu k'_\mu) + q^2 g_{\mu\nu} + 2im_l \epsilon_{\mu\nu\alpha\beta} q^\alpha s_l^\beta
\end{aligned}
$$

Spin dependent

Hadron tensor unknown:

$$
\begin{aligned}
H_{\mu\nu} &= \frac{1}{4\pi} \sum_n < p,S|\hat{J}_\mu(0)|n >< n|\hat{J}_\nu(0)|p,S > (2\pi)^4 \delta^4(p+q-p') \\
&= \frac{1}{4\pi} \int d^4\xi \exp(iq\cdot\xi) < p,S|[\hat{J}_\mu(\xi), \hat{J}_\nu(0)]\,|p,S >
\end{aligned}
$$

Nuclear current operators

Two approaches:

1. Calculate $J_{\square}$ from models or
2. Create most general hadron tensor from tensors (bilinear Lorentz vectors) $A^i_{\square\square} = a_{\square} b_{\square}$ and Lorentz scalers $S_i = a_{\square} b^{\square}$: $H_{\square\square} = \square A^i_{\square\square} F^i(S_1, S_2, \ldots)$

Most general hadron tensor:

EM current conservation $\boxed{\text{W}}$ $q^\mu H_{\mu\nu} = H_{\mu\nu} q^\nu = 0$

parity conservation $\boxed{\text{W}}$ $H_{\mu\nu}(p,q) = H_{\mu\nu}(\tilde{p},\tilde{q})$

$\tilde{p} = (p_0, -\vec{p}), \tilde{q} = (q_0, -\vec{q})$

Time reversal symmetry $\boxed{\text{W}}$ $H_{\mu\nu}(p,q) = H^*_{\mu\nu}(\tilde{p},\tilde{q})$

Available independent four vectors for (e,e'):

- target momentum p$\boxed{\text{W}}$
- photon momentum q$\boxed{\text{W}}$
- target polarization S$\boxed{\text{W}}$

$$H^{nospin}_{\mu\nu}(p,q) = F_1\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + F_2\frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)$$

$$H^{spin}_{\mu\nu}(p,q) = \frac{M}{p\cdot q}\left[\epsilon_{\mu\nu\alpha\beta}q^\alpha S^\beta(g_1+g_2) - \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta g_2\right]$$

Spin dependent

Four independent structure functions

- Spin **independent**: $F_1(Q^2,x)$ and $F_2(Q^2,x)$
- Spin **dependent**: $g_1(Q^2,x)$ and $g_2(Q^2,x)$

(arguments also written as (Q^2,$\boxed{\text{W}}$) or (Q^2,$\boxed{\text{W}}$))

$$x = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2M\omega}$$

Elastic cross section ($p'^2 = m^2$)

Recoil factor Form factors

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left\{\left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\kappa^2 F_2^2(Q^2)\right] + \frac{Q^2}{2M^2}[F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right\}$$

$$= \sigma_M \frac{E'}{E}\left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau \tan^2\frac{\theta}{2} G_M^2(Q^2)\right]$$

$$= \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4} R_L(Q^2) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right) R_T(Q^2)\right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4E^2 \sin^4\left(\frac{\theta_e}{2}\right)}$$

F_1, F_2: Dirac and Pauli form factors

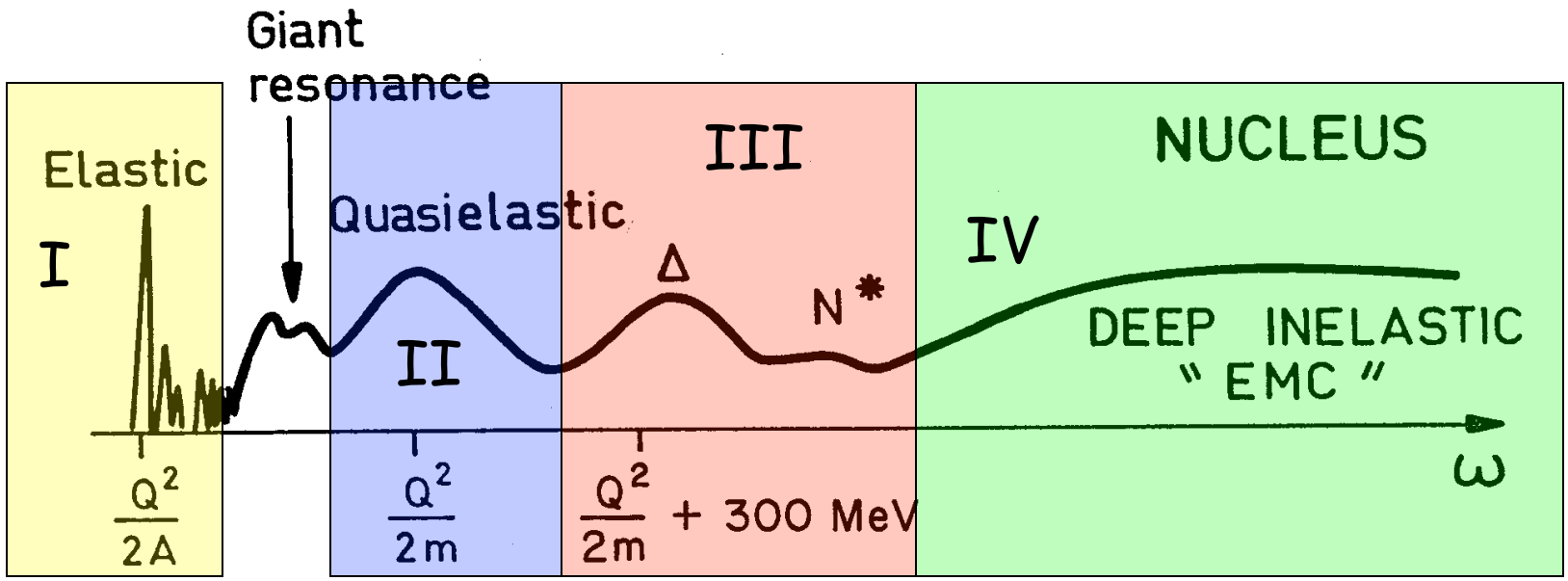
G_E, G_M: Sachs form factors (electric and magnetic)

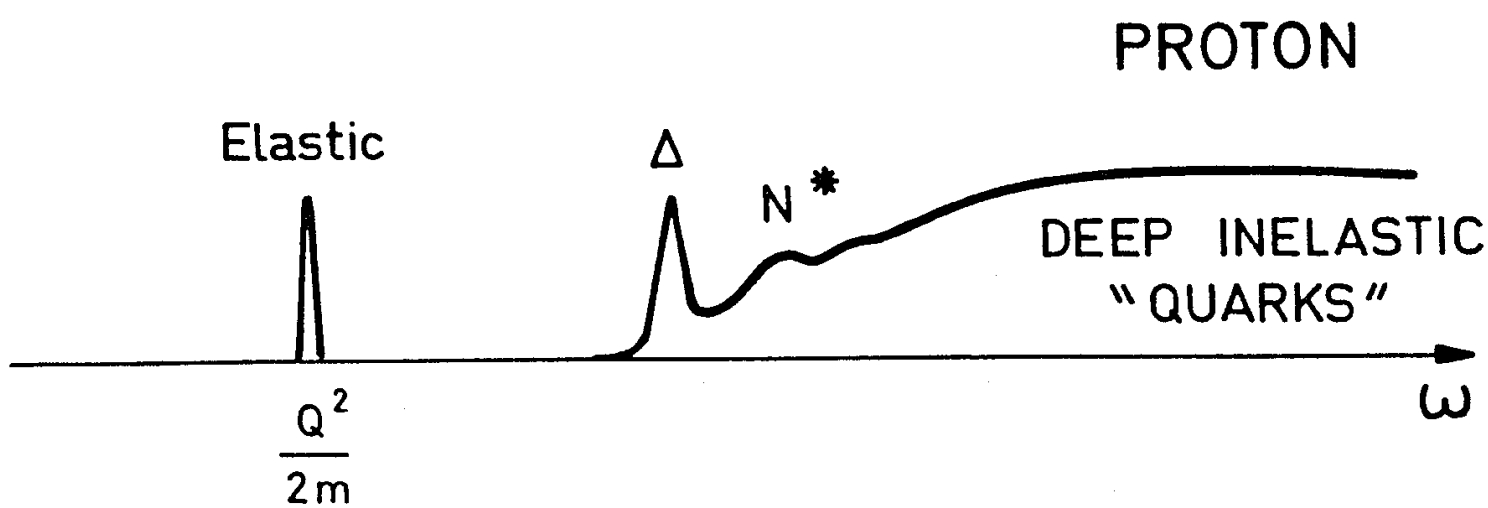
$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$ $t = Q^2/4M^2$

$G_M(Q^2) = F_1(Q^2) + \tau F_2(Q^2)$ k = anomalous magnetic moment

R_L, R_T: Longitudinal and transverse response fn

Electron-nucleus interactions

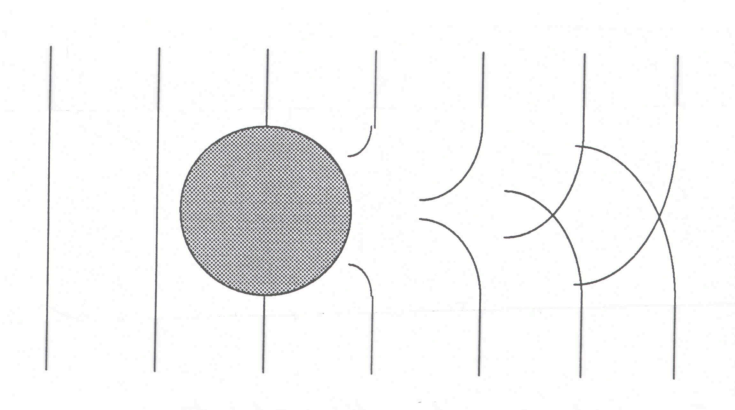




Electrons as Waves

Scattering process is quantum mechanical

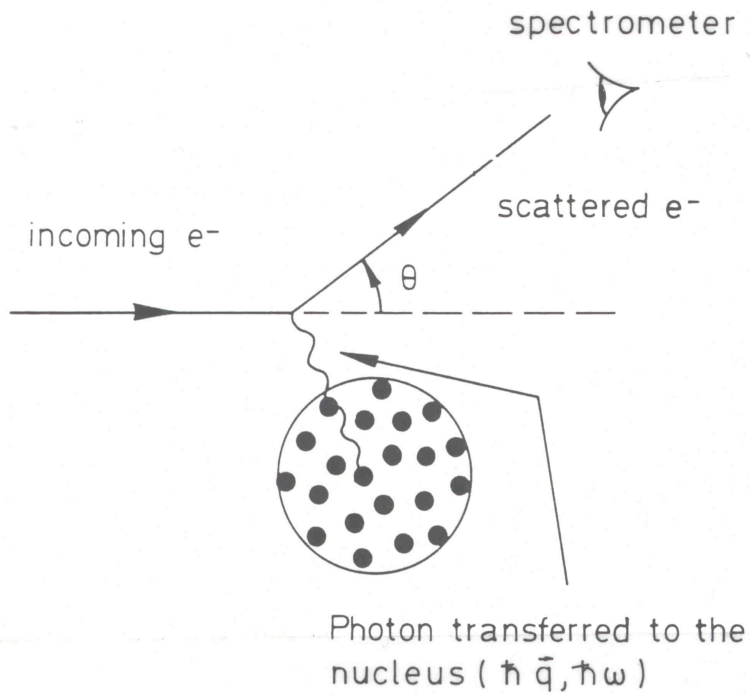
De broglie wavelength: $\lambda = \frac{h}{p}$

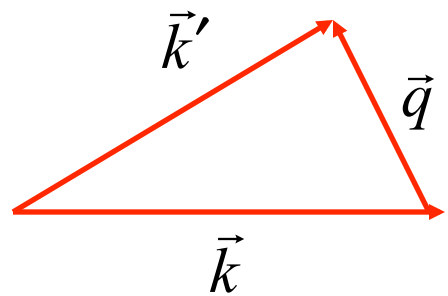


Electron energy: $E_e \approx pc$

resolving "scale": $\lambda = \frac{2\pi(197 MeV \cdot fm)}{E_e}$

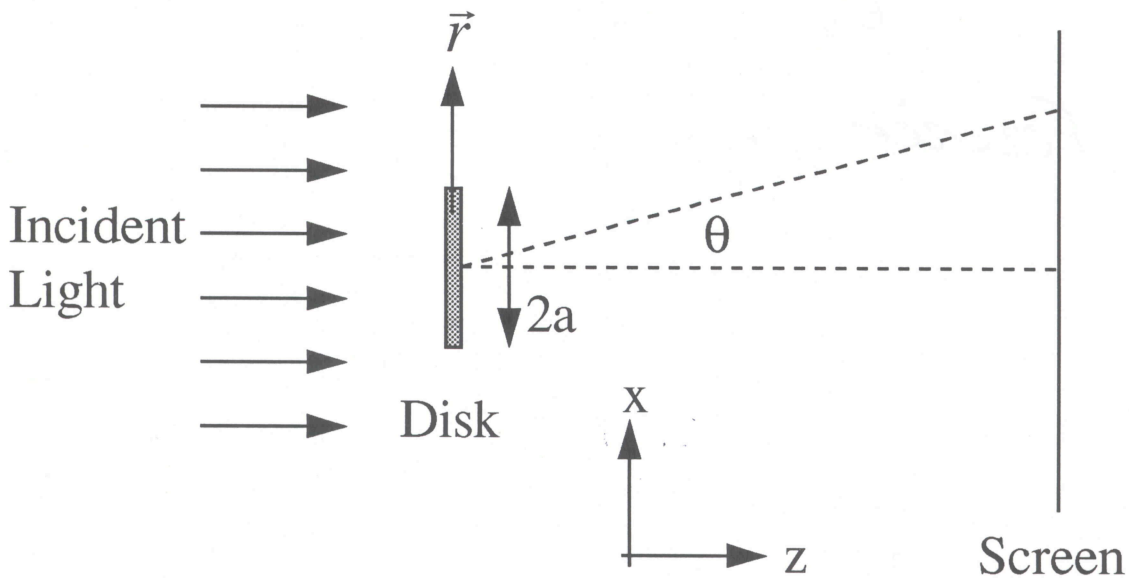
Analogy between elastic electron scattering and diffraction





Simple analogy for elastic electron scattering….

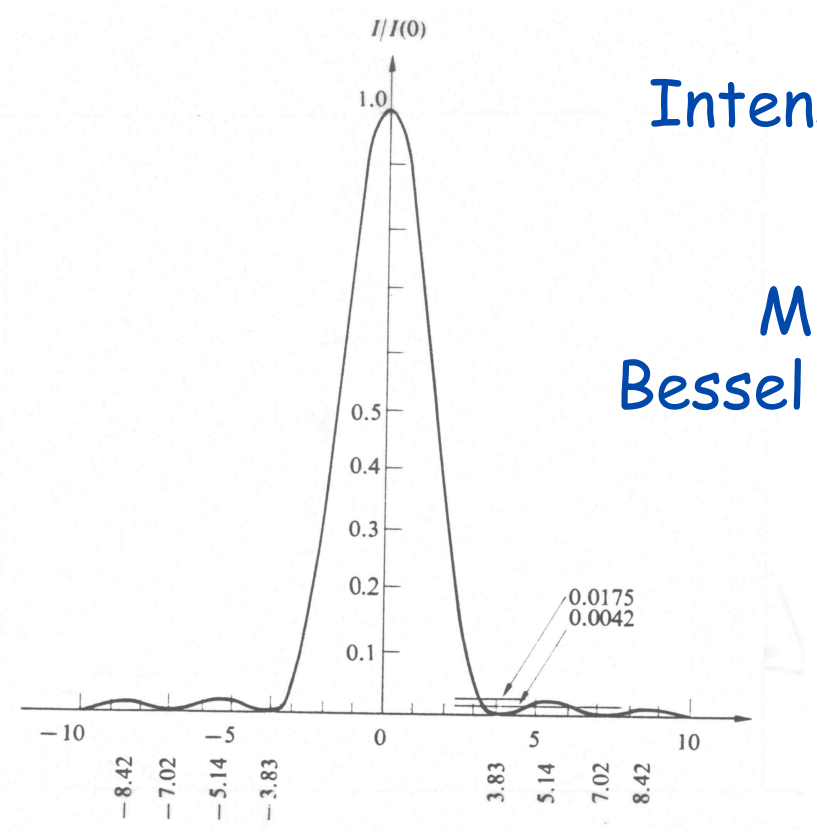
Classical Fraunhofer Diffraction



Amplitude of wave at screen:

$$\Phi \propto \int_0^a \int_0^{2\pi} \exp(ibr\cos\phi)\, r\, d\phi\, dr$$

Classical Fraunhofer Diffraction



Intensity: $\Phi^2 \propto \left(\frac{J_1((2\pi a/\lambda)\sin\theta)}{\sin\theta} \right)^2$

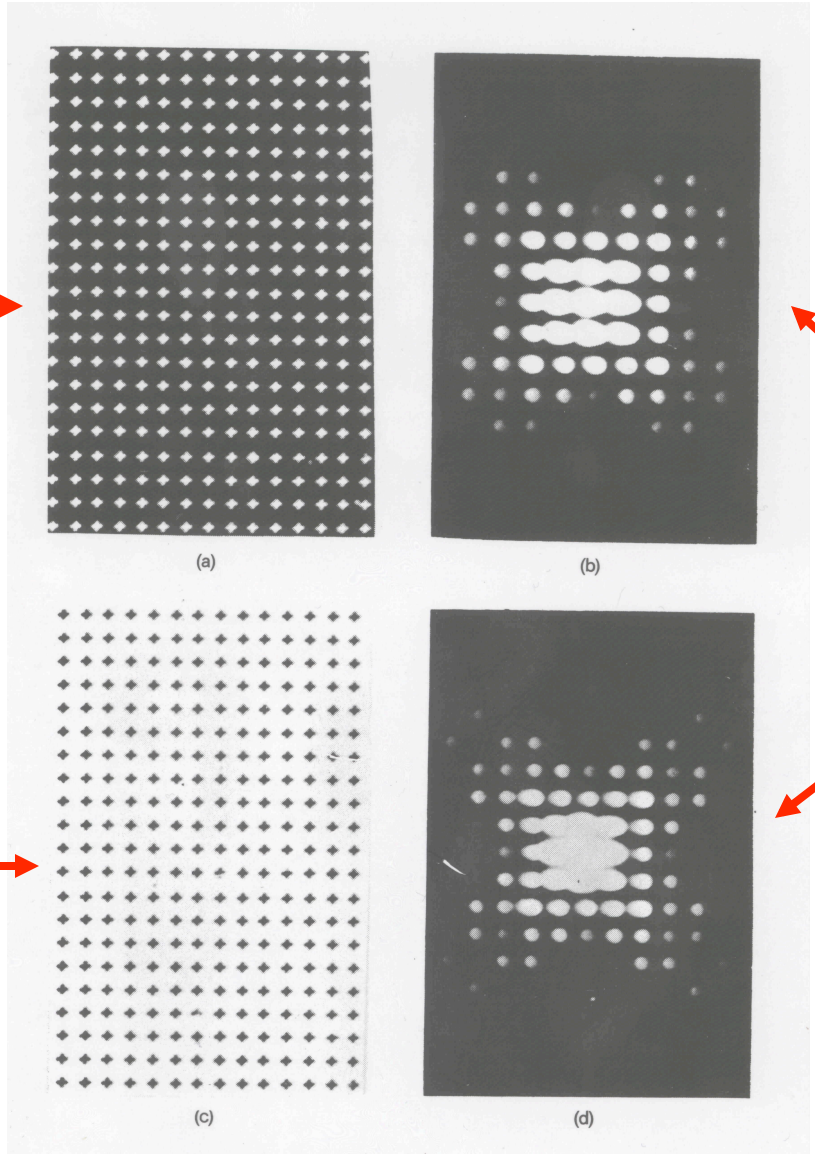
Minima occur at zeroes of Bessel function. 1st zero: x = 3.8317

...some algebra...

Hence $2a \approx \frac{1.22\lambda}{\sin\theta}$

Excursion: Babinet's principle

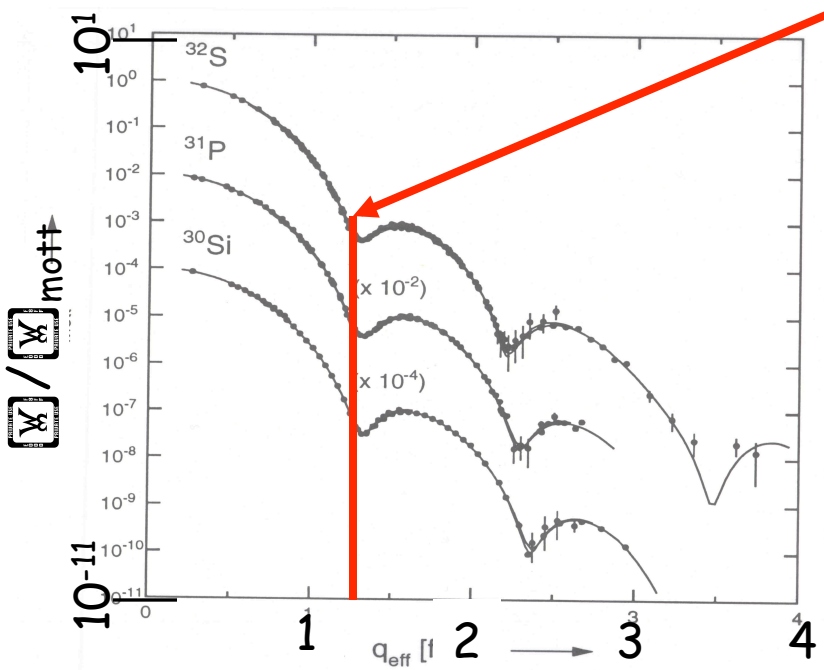
Screen with apertures



Complementary screen

Patterns appear the same

Example: $^{30}Si(e,e')$



q_{eff} (fm^{-1})

($1\ fm^{-1}$ = 197 MeV/c)

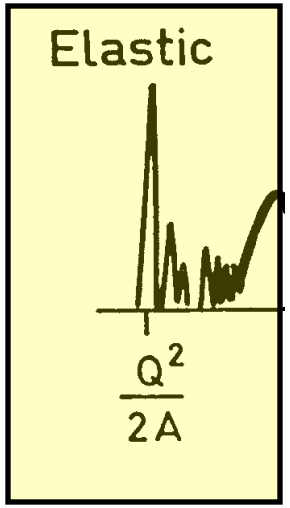
1st minimum = 1.3 fm^{-1}
=> q= 32.8^0

Electron energy = 454.3 MeV
=> l = 2.73 fm

Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm
(from fit to entire curve)

I. Elastic Electron Scattering from Nuclei (done formally)



Fermi's Golden Rule

$$\frac{d\sigma}{dQ} = \frac{2\pi}{\hbar} \left| M_{fi} \right|^2 D_f$$

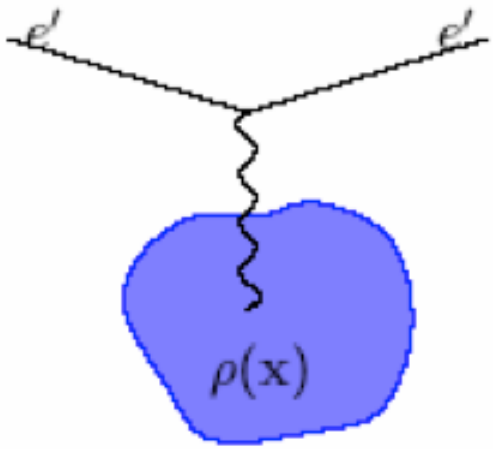
M_{fi}: scattering amplitude

D_f: density of the final states (or phase factor)

$$M_{fi} = \int \Psi_f^* V(x) \Psi_i d^3x$$

$$= \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x$$

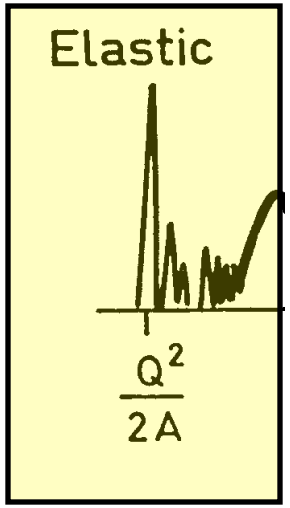
$$= \int e^{iq \cdot x} V(x) d^3x$$



Plane wave approximation for incoming and outgoing electrons

Born approximation (interact only once)

I. Elastic Electron Scattering from (spin-0) Nuclei



Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, ρ(x),

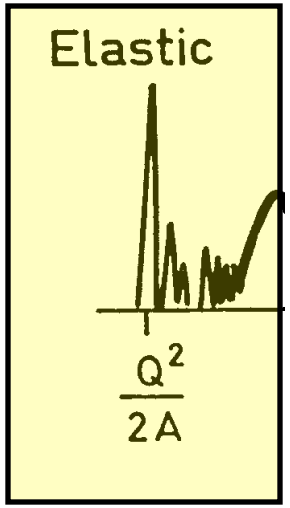
$$V(x) = -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{\rho(x')}{|x-x'|}d^3x'$$

$$\begin{aligned} M_{fi} &= -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iq\cdot x}\int \frac{\rho(x')}{|x-x'|}d^3x'd^3x \\ &= -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iqR}\left[\int \frac{e^{iq\cdot x'}\rho(x')}{|R|}d^3x'\right]d^3R \\ &= -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{e^{iqR}}{R}d^3R\int e^{iq\cdot x'}\rho(x')d^3x' \end{aligned}$$

$$F(q) = \int e^{iq\cdot x'}\rho(x')d^3x'$$

Charge form factor F(q)
is the Fourrier transform
of the charge distribution ρ(x)

I. Elastic Electron Scattering from (spin-0) Nuclei



Form factor and cross section

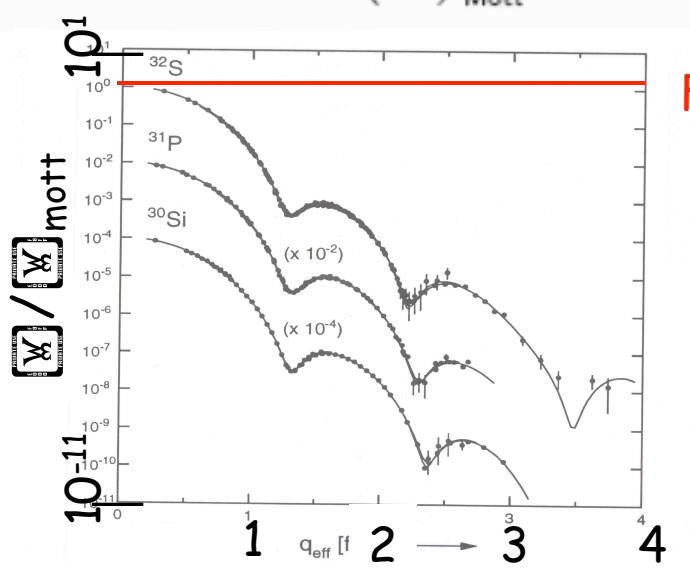
- For point-like particle, $\rho(x') = \delta(x')$ and $F(q) = 1 \rightarrow$ Rutherford-*like* scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}}$$

$\cos^2$ term only difference
From Rutherford formula
Arises from Dirac theory
For spin $\frac{1}{2}$ particle

- Scattering from a charge distribution

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(q)|^2$$

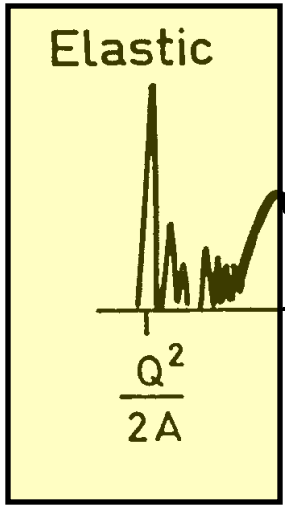


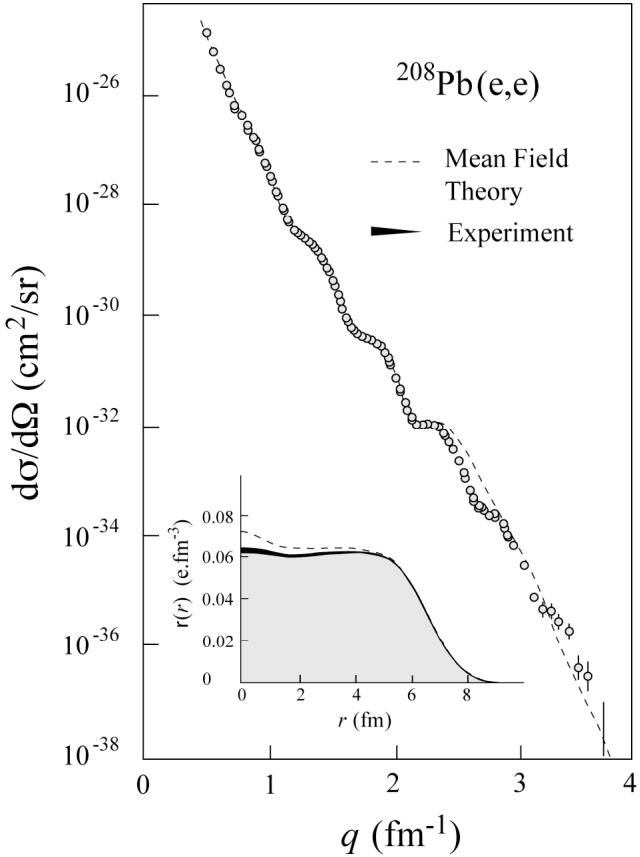
F=1 for point particle

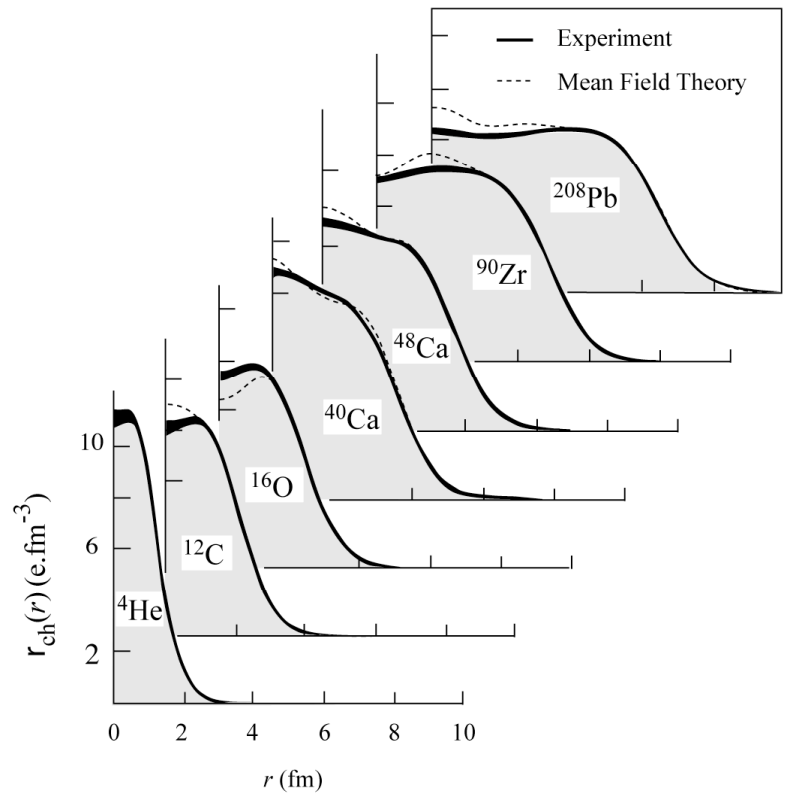
q_{eff} (fm^{-1})

1 fm^{-1} = 197 MeV/c

I. Elastic (e,e') Scattering ⇒ charge distributions







In '70s large data set was acquired on elastic electron scattering (mainly at Saclay) over large Q^2-range and for variety of nuclei

"Model-independent" analysis of these data provided accurate results on charge distribution for comparison with the best available theory: Mean-Field Density-Dependent Hartree-Fock

Summary

- Photons (real and virtual) are a precise probe of nuclear and nucleon structure
- Used in many labs around the world
  - Impressive accelerators
  - Big detectors
  - Lots of pretty pictures
- Beautiful measurements of the nuclear charge distribution