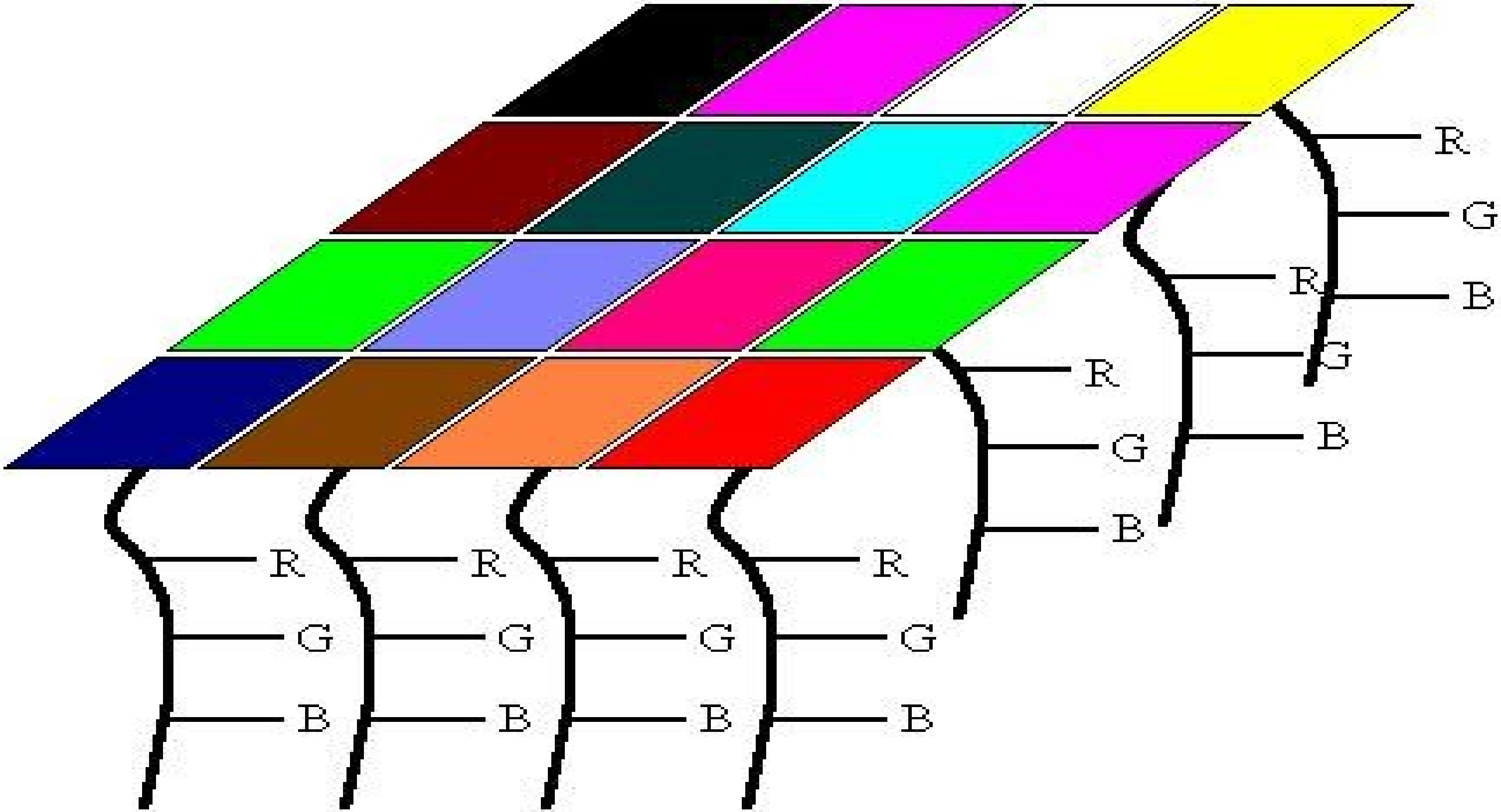
Use of Self Organizing Maps in Deep Inelastic Scattering

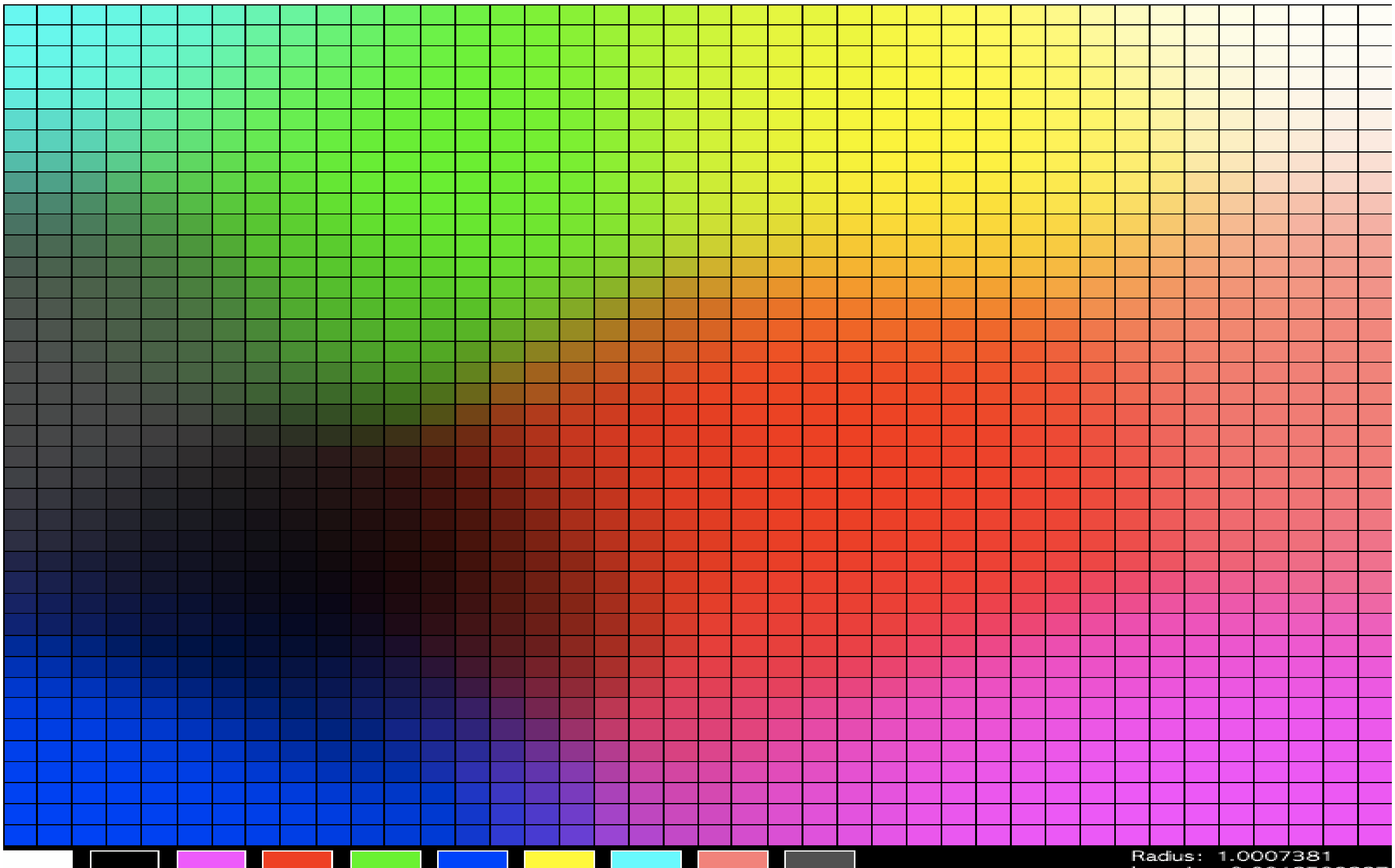
Self Organizing Maps

- Self Organizing Maps (SOM) are a type of neural network that uses *unsupervised learning*.
- Unsupervised learning refers to a type of neural network data evolution where an initial data set is molded to fit a final data set without the final data set being used as a reinforcement.
- A Self Organizing Map is a Map of Training Data on which an input set of training data, represented by an n dimensional vector, is placed.

SOM Color Example: a 4X4 Grid of Map Vectors That Comprise a Map



SOM Color Example After Self Organizing Process

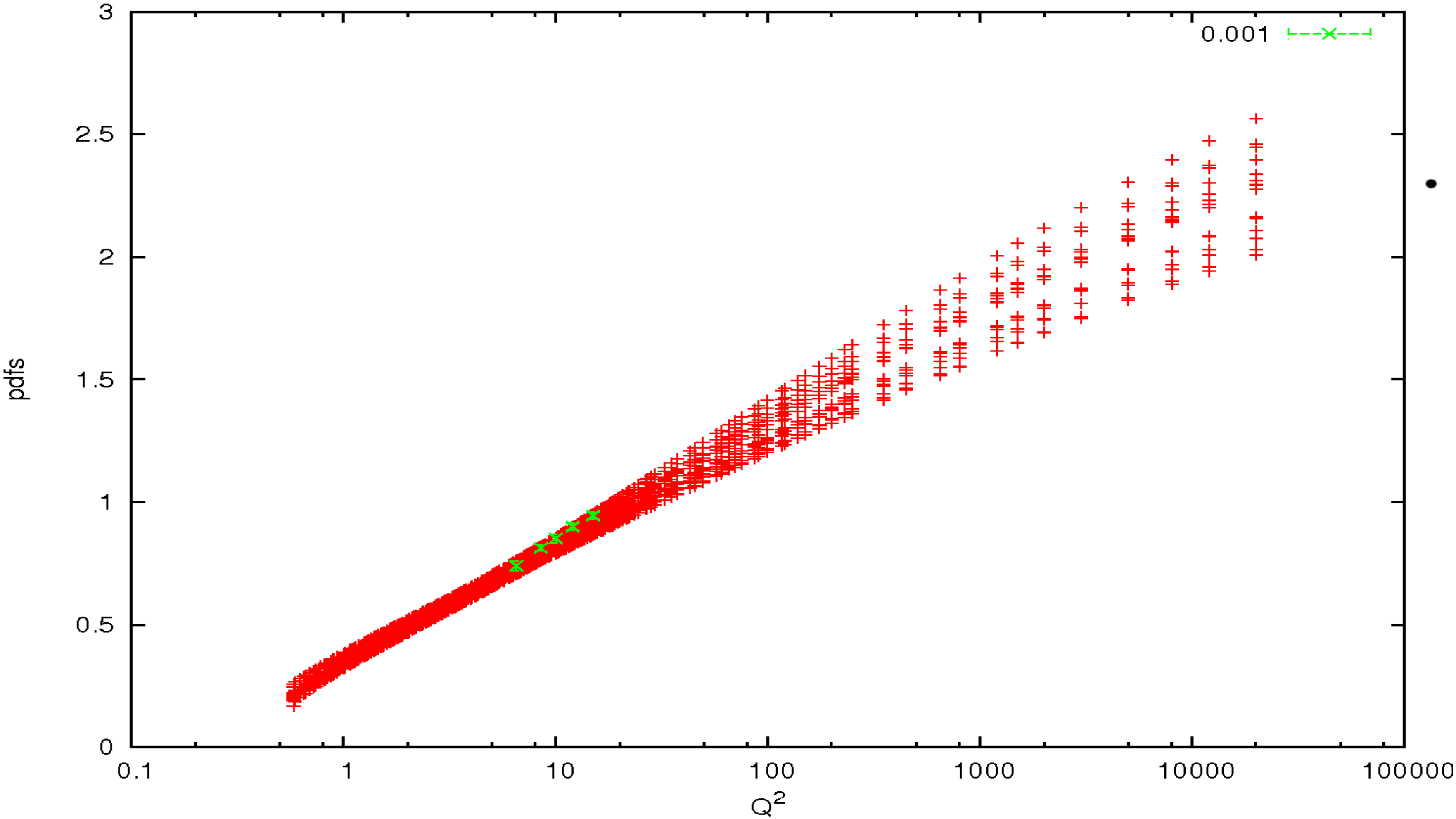


SOMPDF Methodology

- The SOM map training vectors formed from a parameterized set of Parton Distribution Functions, or PDFS, where the parameters are varied semi-randomly. The input training vector is also formed from a semi-randomly varied, parameterized PDF.
- The parameters for the PDF are "wiggled" for the map and the input vector.
- This way, after the map is trained, the $\chi 2$ for the input PDFs, relative to the experimental data, can be minimized using the genetic algorithm.

Example of PDF envelope for x = 0.001

Red Dots = PDF Envelopes, Green Dots = Experimental data



Genetic Algorithm

- For a NXN map of PDFs, the training process was repeated for a given number of iterations.
- The training vectors with the best values of $\boldsymbol{\chi}^2$ were determined for each iteration.
- The best-matching training vectors were used to generate the next set of training vectors for the subsequent iteration.

PDFs and Their Purpose

- The cross section for deep inelastic scattering of a lepton off of a proton can be written in terms of a hard scattering cross section and $q_j(x, Q^2)$.
- - $q_j(x, Q^2)$ refers to the Parton Distribution Functions

  -Q^2 is the square of the momentum of the virtual photon exchanged

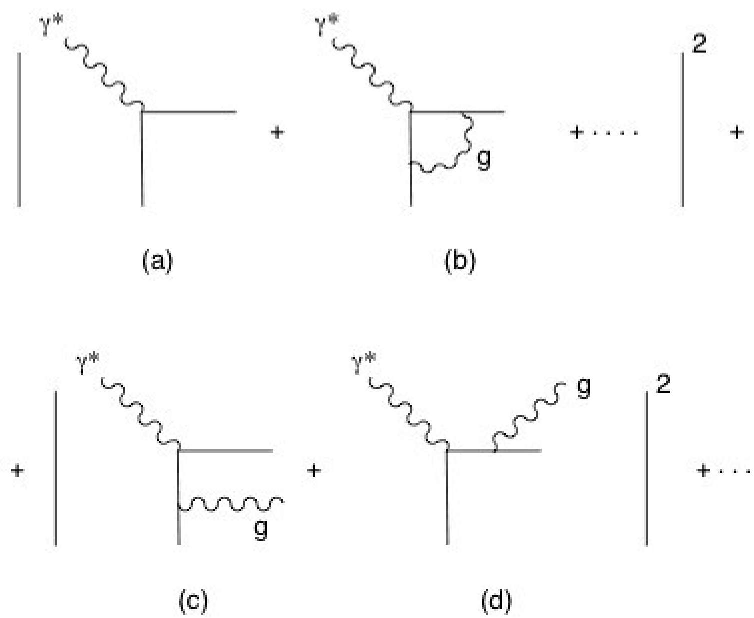
  -x is the fraction of the proton momentum contained in the parton momentum

  At the moment, direct info about these parton distribution functions can only be obtained through experiment

Constraining the PDFs

- The parton distributions used are the PDFs for the 3 lightest quarks, up, down and strange, and the gluons.
- For these PDFs, momentum conservation yields $\int_0^1 dx\, x\left(u + \bar{u} + d + \bar{d} + s + \bar{s} + g\right) = 1$
- Then there are the sum rules for the up, down and strange quarks:
$\int_0^1 dx\,(u - \bar{u}) = 2$, $\int_0^1 dx\left(d - \bar{d}\right) = 1$ and
$\int_0^1 dx\,(s - \bar{s}) = 0$

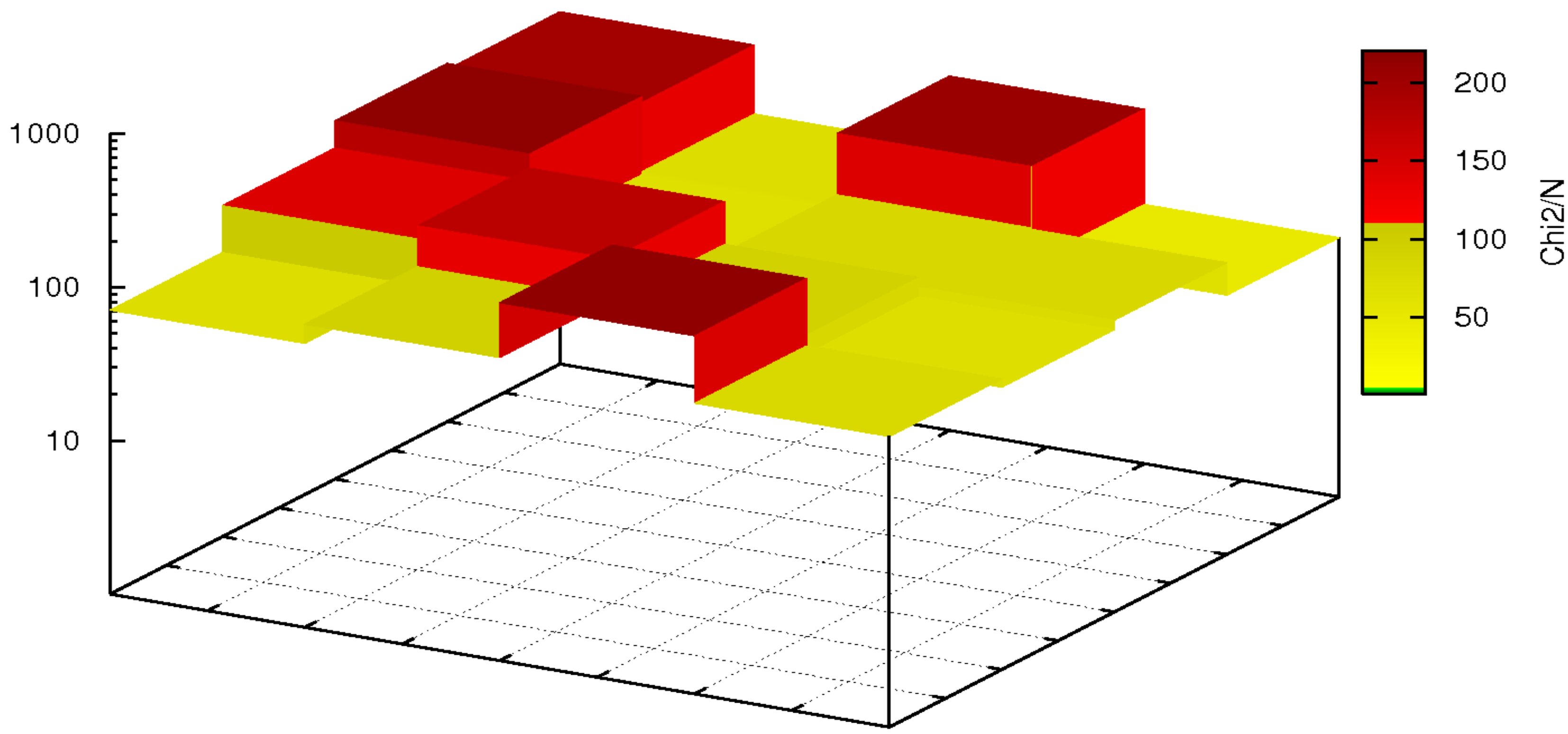
Momentum Evolution for the PDFs – Third Constraint



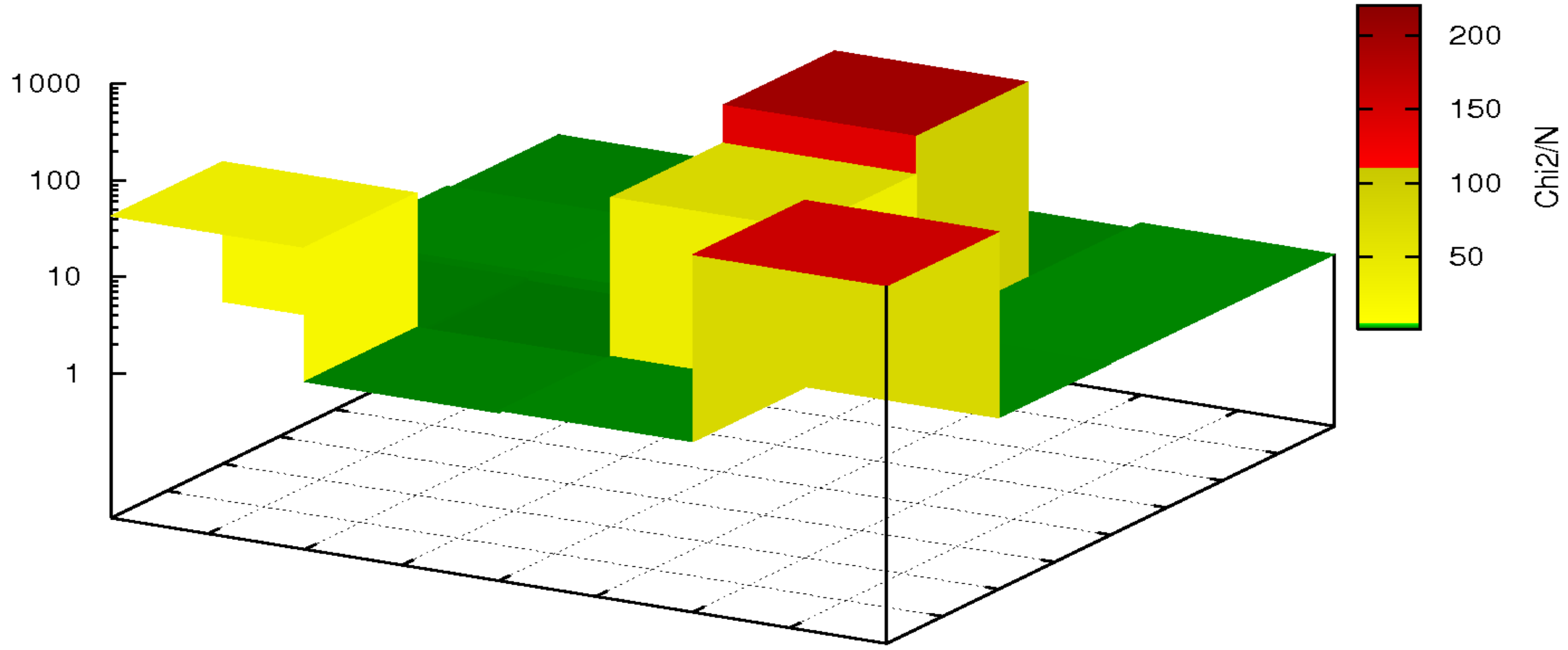
The integrals of the PDFs needed to be equal to the evolution factors, which are functions of In (q2).

The third condition was that the PDFs had to be normalized according to the momentum transfer evolution

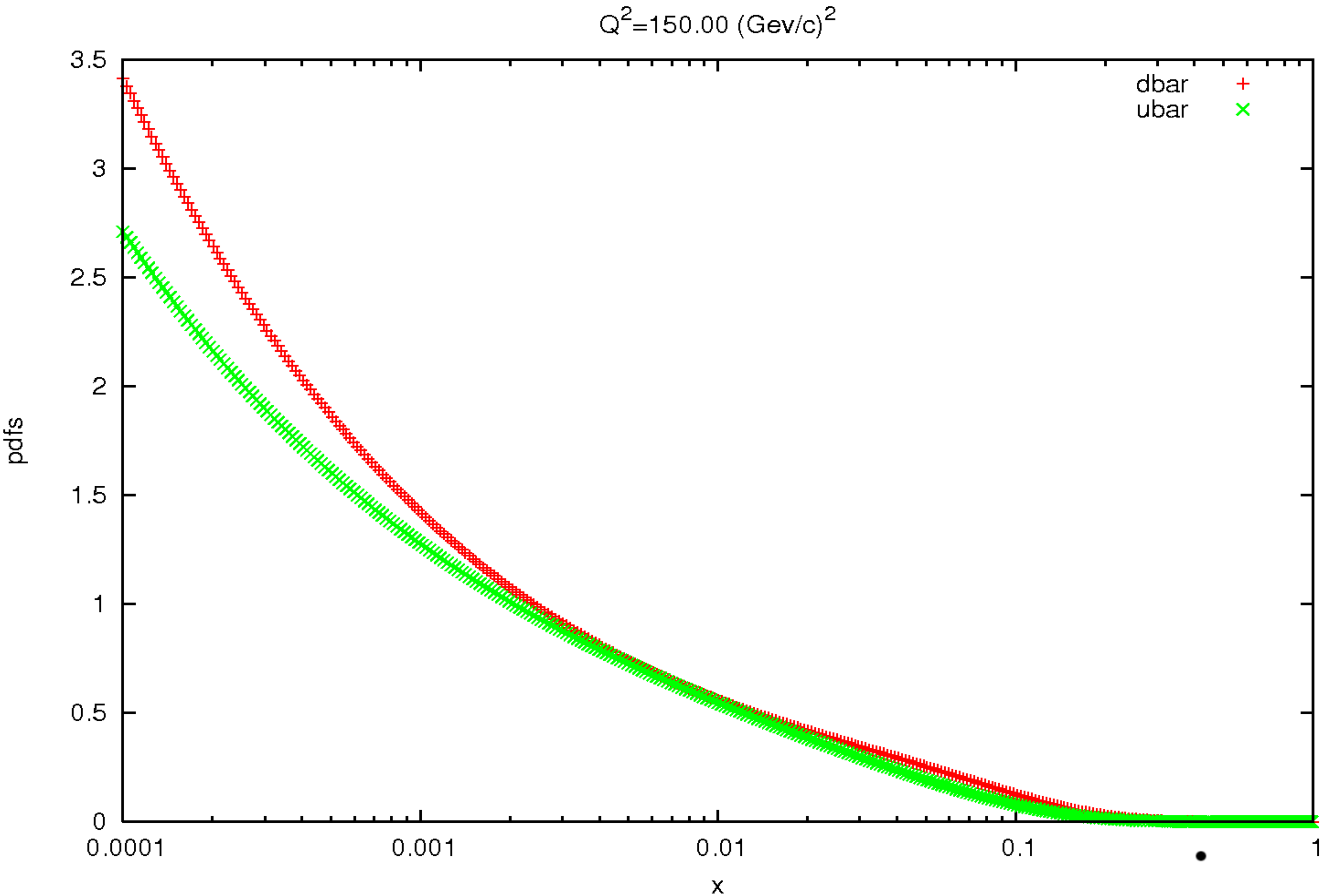
Map of χ2 for the SOMPDF Initial Iteration



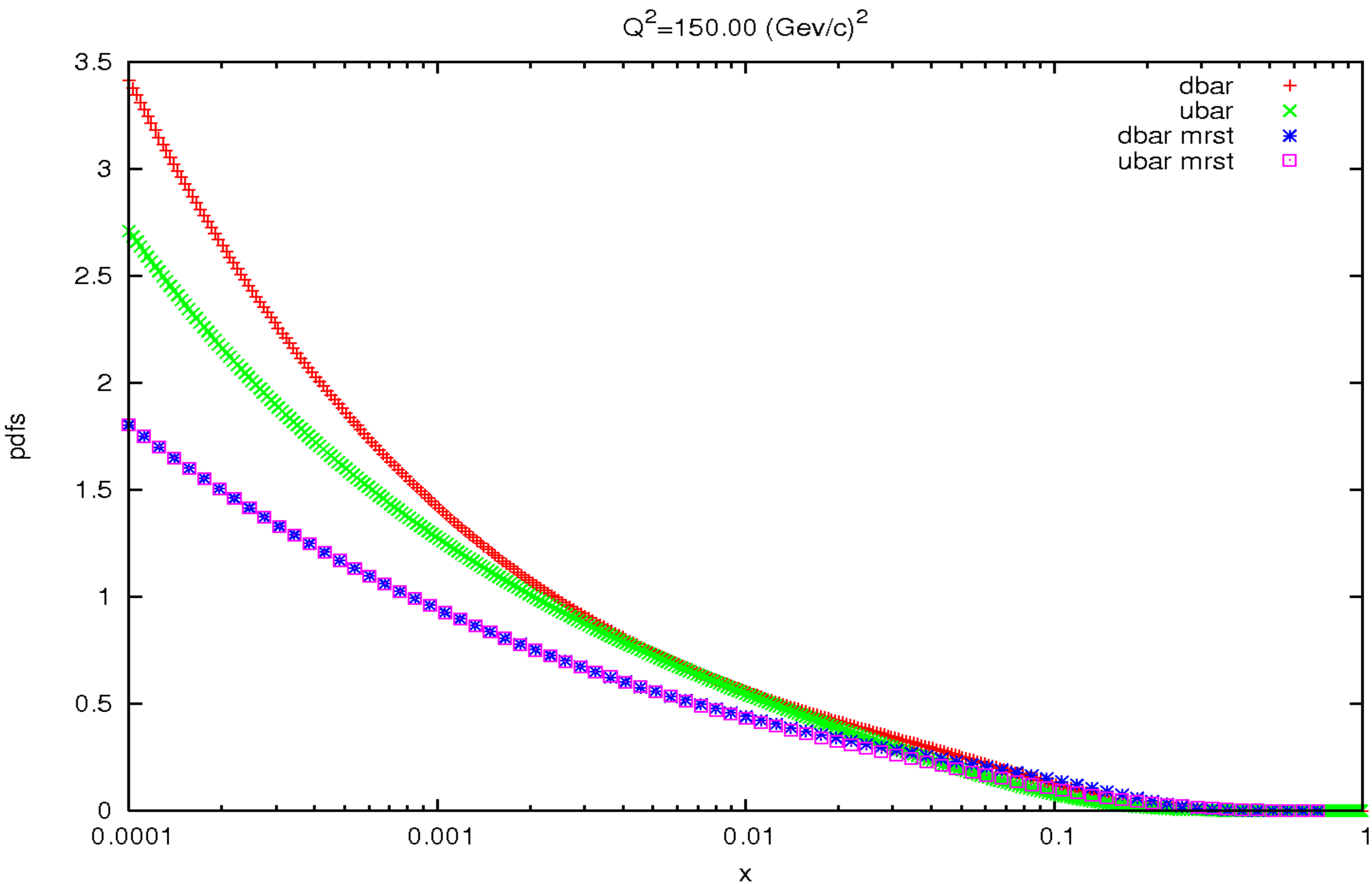
Map of χ2 for the SOMPDF Final Iteration



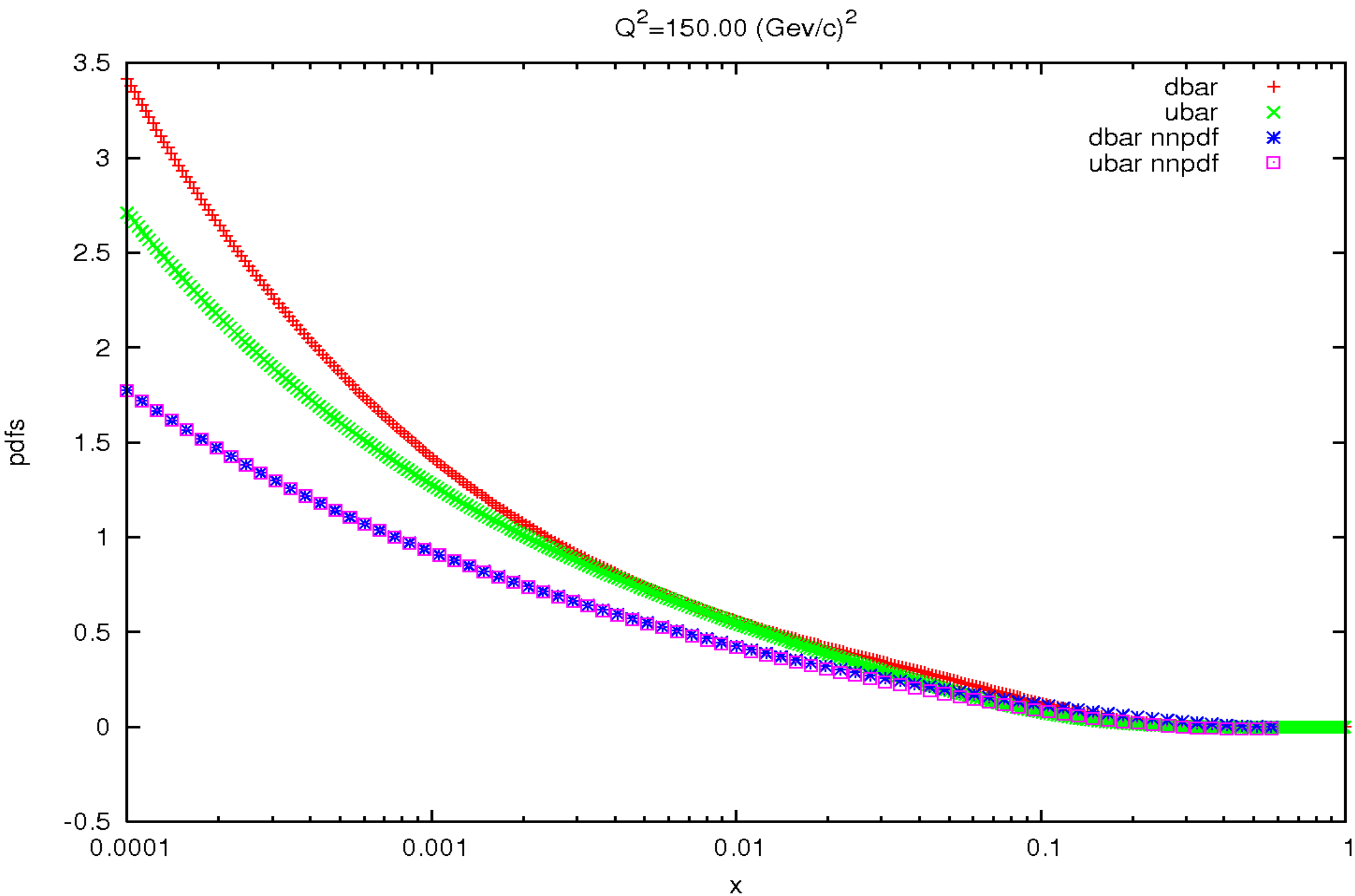
Ub and Db Plots for Proton Data, χ2 = 1.92



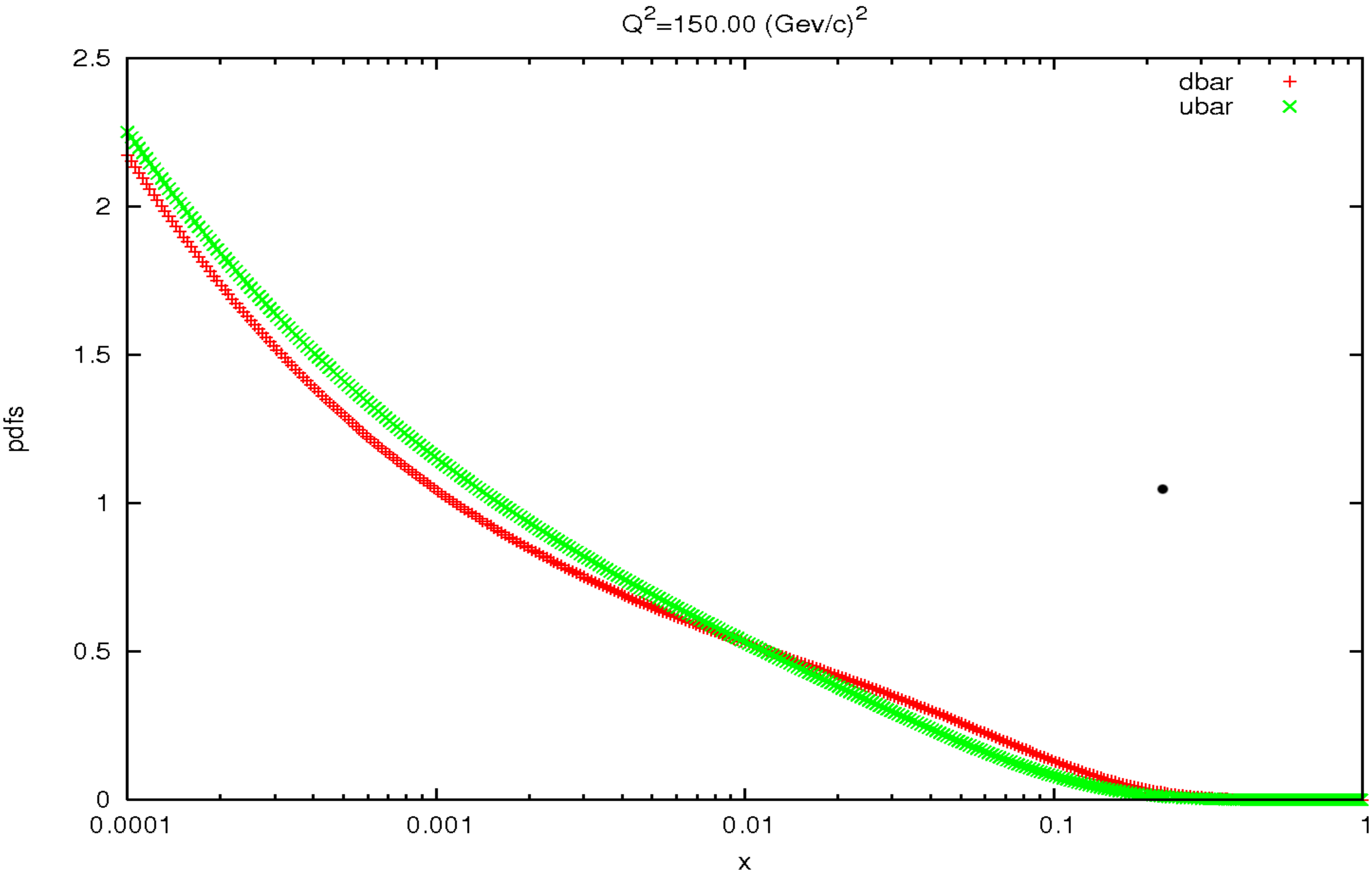
Comparison of Ub and Db Plots for Proton Data with MRST Experimental Data, χ2 = 1.92



Comparison of Ub and Db Plots for Proton Data with NNPDF Experimental Data, χ2 = 1.92



Ub and Db Plots for Deuteron Data, χ2 = 1.92



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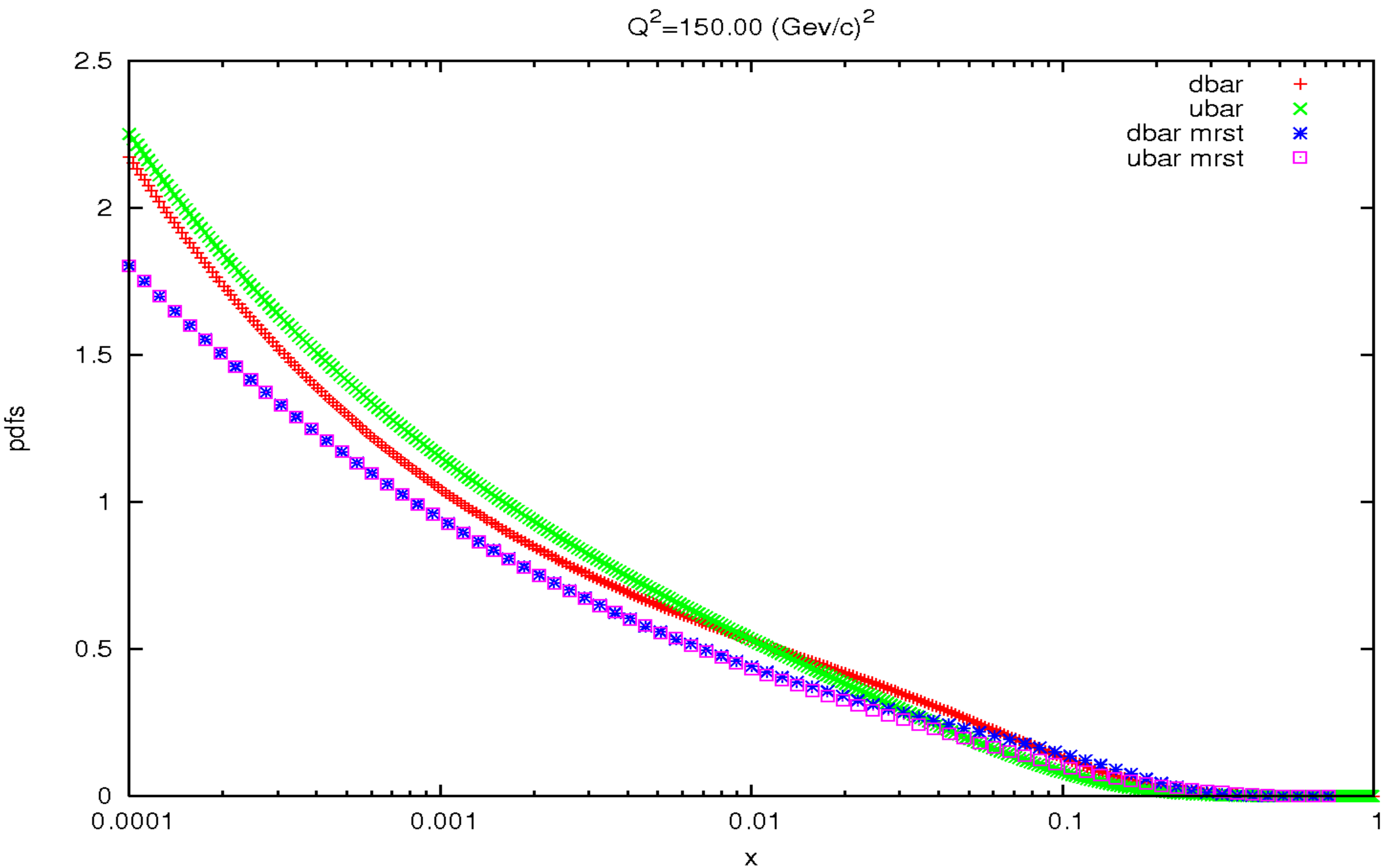

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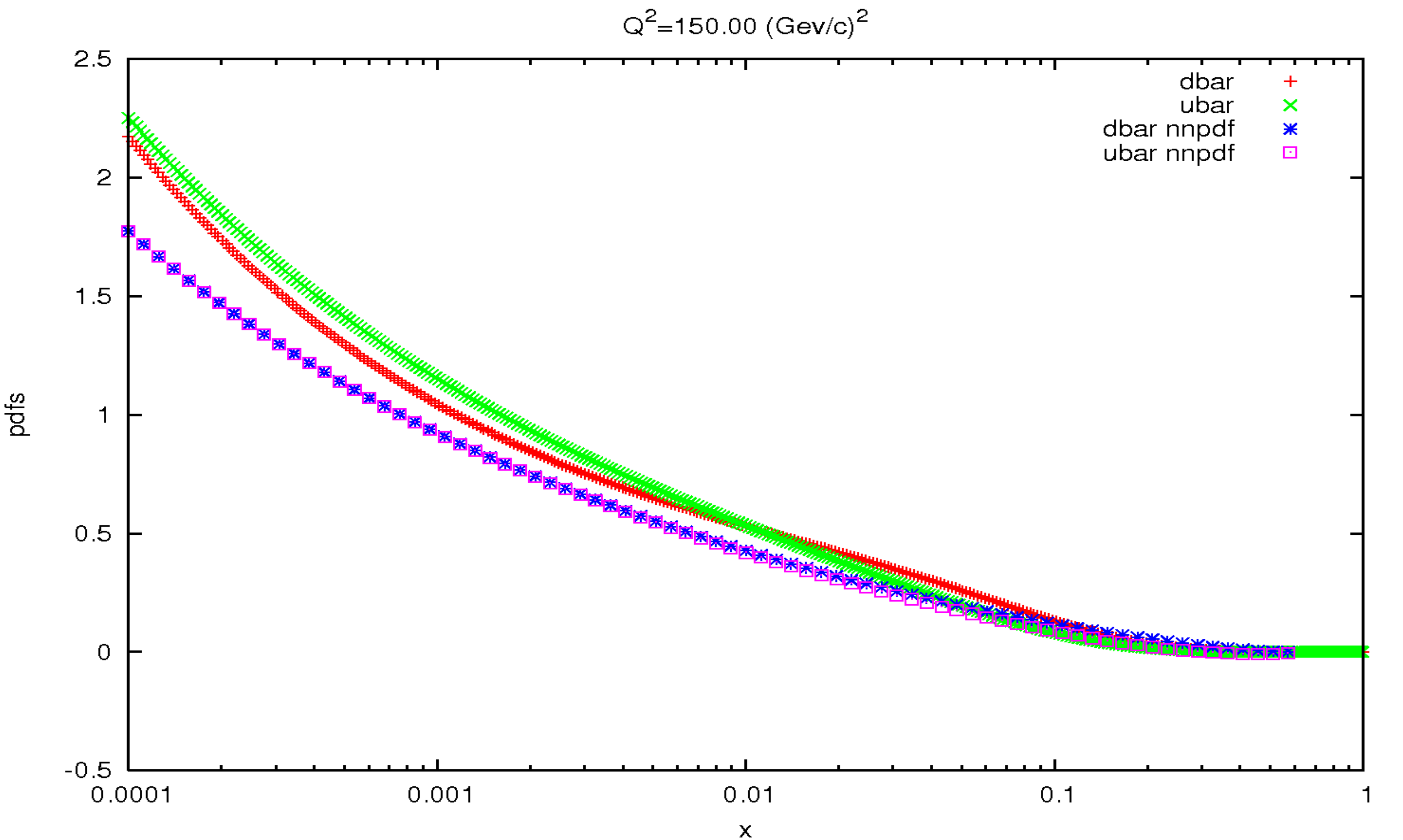
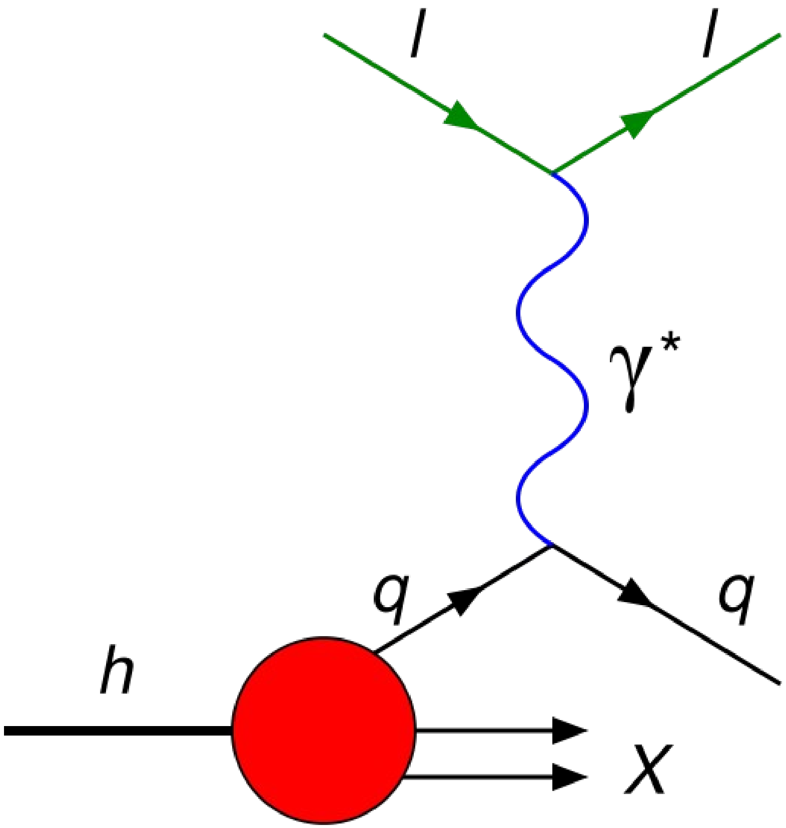


Background on DIS



- Deep Inelastic Scattering is used to probe the structure of hadrons-including the proton.
- In this setup, an electron, muon or neutrino scatters off of a proton and exchanges a virtual photon with momentum *q* with a parton (quark) in the proton.
- This leads to a series of partons in the proton carrying fractions of the proton's initial momentum

The Need for Neural Networks

- Neural Networks are organized systems of data used to train and evolve a given data set to more closely match a known data set. Here they are used to train PDF data.
- The parameterized PDFs vary significantly from group to group.
- In order to fit them with the experimental values for the PDF we attempt to evolve the PDFs from an initial set of conditions using neural networks.
- Then we can analyze how well the PDFs that different groups have created can fit the experimental data.
- The neural network we attempt to use is the Self Organizing Map.