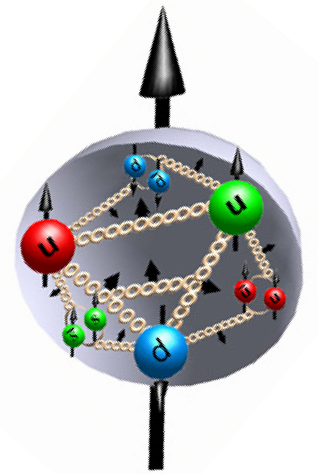
Nucleon Spin Decomposition



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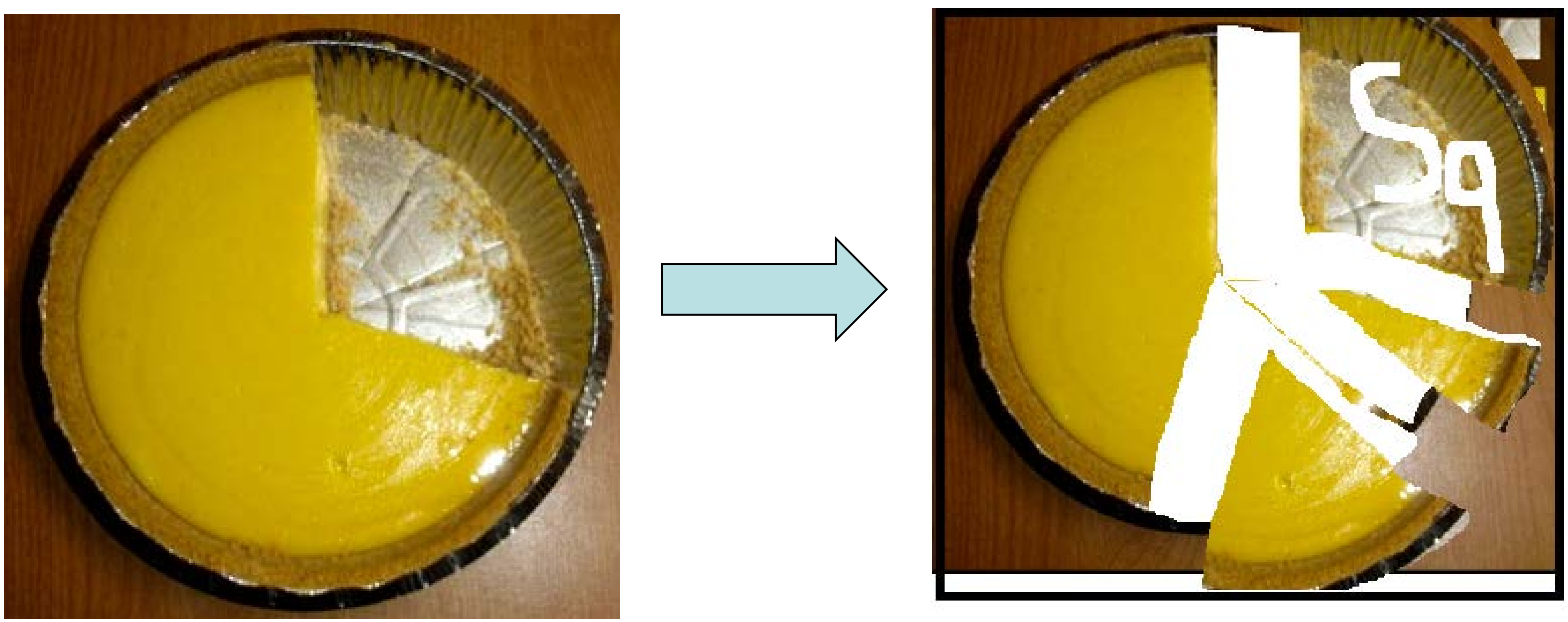
HUGS 2012 , Jlab.

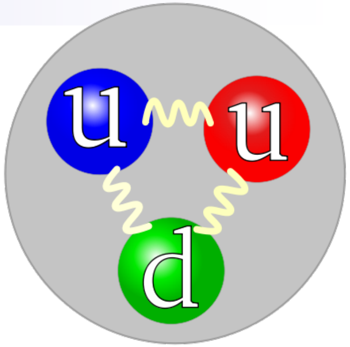
Outline

- ❑ Motivation
- ❑ Introduction
- ❑ Different results suggested / Latest Development
- ❑ summary of results



Motivation

Spin of Nucleon (proton):



- Before 1980's: quarks carry all of the nucleon spin
- Suggestion by European muon collaboration (EMC):
  - Spin carried by quarks insufficient to justify total spin of nucleon

- ➢ SMC at CERN, HERMES at DESY, COMPASS, Jlab etc. confirmed the original discovery of EMC
- ➢ only ~ 30% of nucleon spin is by quark spin.
- ➢ where remaining ~70% comes from ?
- ➢ How is nucleon spin is carried by its constituents ?

Spin crisis in nuclear physics !!!!

missing angular momentum !!!!

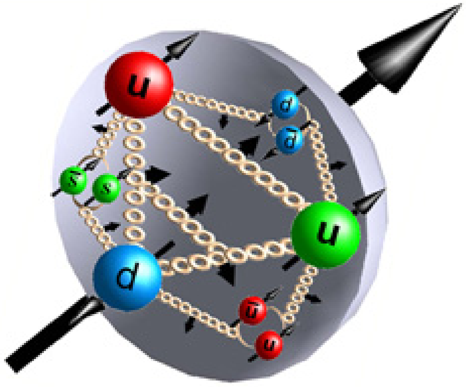
Introduction

Factors contributing for remaining 70% of nucleon spin:

- Quark Orbital Angular Momentum (OAM)
- from Quantum Chromodynamics (QCD):

(exchange of gluon to bind quarks inside the nucleon)

  - Gluon spin
  - Gluon OAM



Spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Sigma_q + \Sigma_g + L_q + L_g .$$

- Quark spin- from polarized Deep Inelastic Collision (DIS)
- Gluon spin –measured by many experiments
- Quark and gluon OAM- Generalized Parton Distribution (GPD) through Deeply Virtual Compton Scattering (DVCS) in Jlab

Introduction

Summary of current status of nucleon spin:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^Q + \Delta g + \text{Orbital Angular Momenta ?}$$

$\Delta\Sigma^Q$: fairly precisely determined ! ($\sim 1/3$)

Δg : likely to be small , but large uncertainties

$\Downarrow$

What carries the remaining 2 / 3 of nucleon spin ?

quark OAM ? gluon spin ? gluon OAM ?

Results Suggested for Nucleon Spin

Decomposition of Nucleon Spin

Problem: **Big problem : Orbital Angular Momenta !!!!!**

To decompose J (total angular momentum) into contributions from different constituents

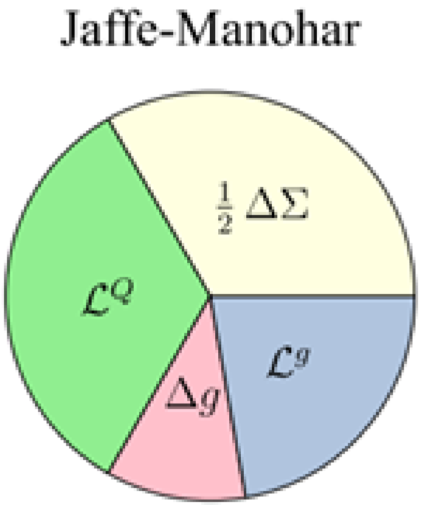
- changing gauge may also shift angular momentum between various degree of freedom
- Decomposition depends on gauge and quantization scheme
- Not necessarily be unique- like culture
- What is the precise (QCD) definition of each terms of the decomposition
- How we extract each terms by means of direct measurement?

Some Decomposers of Nucleon Spin :

- Jaffe Manohar Decompostion
- Ji Decomposition
- Chen, sun and Leader et. all
- Decomposition by wakamatsu
- OAM in QED / QCD
- Chen, sun et. all decomposition
- some latest updates

Results Suggested for Nucleon Spin

Basic Gauge Principle: Observables must be gauge invariant



JM Decomposition of Nucleon Spin :

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q \mathscr{L}_q^z + \frac{1}{2}\Delta G + \mathscr{L}_g^z$$

$$M^{+xy} = \frac{1}{2}\sum_q q_+^\dagger \gamma_5 q_+ + \sum_q q_+^\dagger (\vec{r} \times i\vec{\partial})^z q_+ + \varepsilon^{+-ij} TrF^{+i}A^j + 2TrF^{+j}(\vec{r} \times i\vec{\partial})^z A^j$$

where $q_+ = \frac{1}{2}\gamma^-\gamma^+ q$ is the dynamical component of the quark field operators and A^+

in light cone gauge = 0

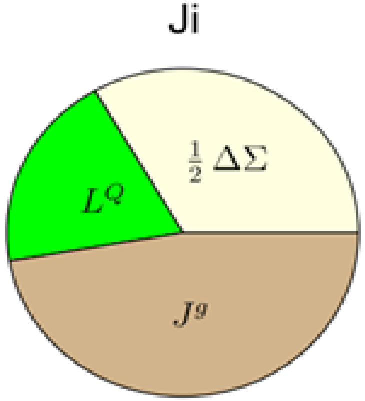
- not gauge invariant
- Quark spin -polarized DIS
- Δg- from polarized DIS
- OAM on light-like hypersurface

Pizza quattro stagioni

R.L. Jaffe and A. Manohar, NPB 337, 1990 ~ 500 citations

Results Suggested for Nucleon Spin

Ji Decomposition of Nucleon Spin :



Pizza tre stagioni

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q^z + J_g^z$$

$$M^{0xy} = \frac{1}{2}\sum_q q^\dagger \Sigma^z q + \sum_q q^\dagger (\vec{r} \times i\vec{D})^z q + [\vec{r} \times (\vec{E} \times \vec{B})]^z$$

where $i\vec{D} = i\vec{\partial} - g\vec{A}$.

- Each term is separately gauge invariant
- OAM on space –like hypersurface
- $\mathbf{J}_q$ accessible through GPDs (X. Ji 1997)

$$J_q^z = \frac{1}{2}\Delta q + L_q^z = \frac{1}{2}\int_0^1 dx x [H_q(x,0,0) + E_q(x,0,0)].$$

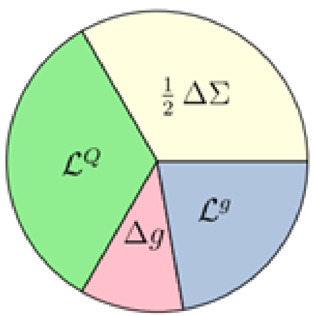
- DVCS to probe $\mathbf{J}_q = \mathbf{S}_q + \mathbf{L}_q$
- No further decomposition of J^g
- i.e. no identification of gluon spin/OAM

X. Ji PRL 78, 1997

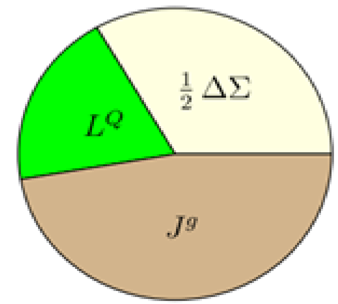
Results Suggested for Nucleon Spin

Compare : Ji and JM Decomposition

Jaffe-Manohar



Ji



$$\mathcal{L}^Q \neq L^Q$$

$$L_Q(\mathrm{JM}) \sim \psi^\dagger \boldsymbol{x} \times \boldsymbol{p}\, \psi \qquad\qquad L_Q(\mathrm{Ji}) \sim \psi^\dagger \boldsymbol{x} \times (\boldsymbol{p} - g\,\boldsymbol{A})\, \psi$$

canonical OAM | **dynamical OAM**

($\boldsymbol{p}$: **canonical momentum**) | ($\boldsymbol{p} - g\,\boldsymbol{A}$: **dynamical momentum**)

not gauge invariant ! | gauge invariant !

$$\Delta g + \mathcal{L}^g \neq J^g$$

no sense to mix them ► **J_q -** Δg has no connection to OAM

Important question: how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

Results Suggested for Nucleon Spin

OAM in Scalar Diquark Model by MB andBC:

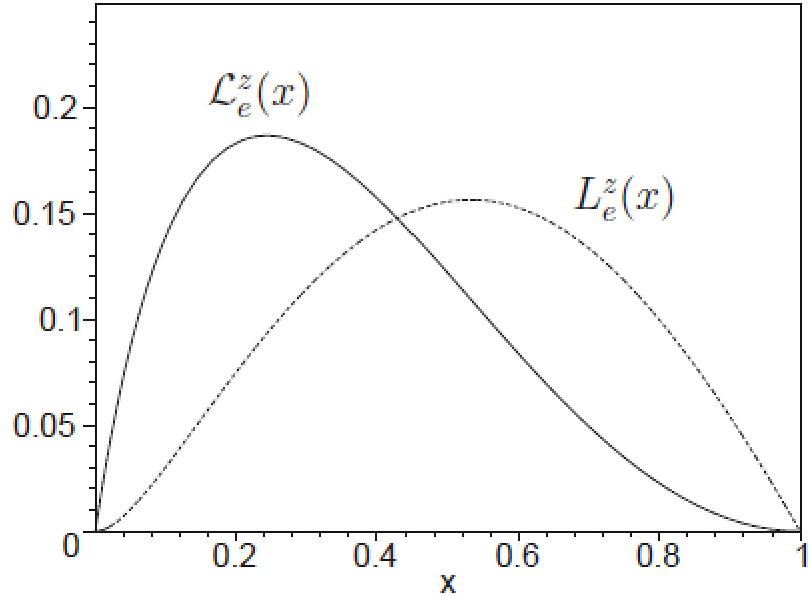
- Toy model for nucleon- nucleon splits into **quark and scalar 'diquark'**

Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$

$$J_q^z = \frac{1}{2}\Delta q + L_q^z = \frac{1}{2}\int_0^1 dx x[H_q(x,0,0) + E_q(x,0,0)].$$

Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$

$$\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$$



MB and BC PRD 79, 2009

Results Suggested for Nucleon Spin

Orbital Angular Momentum in Quantum electrodynamics(QED):
(Angular Momentum Decomposition in QED)

OAM of e^- according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^\uparrow_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^\uparrow_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$

e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\,[q(x)+E(x,0,0)] - \frac{1}{2}\Delta q$

$$\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$$

Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and defining $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields

$\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$; α/4π is small.

Similar calculation in QCD for quark and gluon:

Applying these results to a (massive) quark with $J^z = +\frac{1}{2}$ yields to $\mathcal{O}(\alpha_s)$

$$\mathcal{L}_q^z - L_q^z = \frac{\alpha_s}{3\pi},$$

i.e., for $\alpha_s \approx 0.5$ about 10% of the spin budget for this quark.

Coined new term "Vector potential "to contribute to nucleon spin

MB and BC PRD 79, 2009

Results Suggested for Nucleon Spin

Important: so far, quest for gauge invariant decomposition of J_g

Gauge Invariant Decomposition by Chen, Sun et. all:

basic idea

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

which is a kind of generalization of the decomposition of photon field into the transverse and longitudinal components in QED :

$$\boldsymbol{A}_{phys} \Leftrightarrow \boldsymbol{A}_\perp, \qquad \boldsymbol{A}_{pure} \Leftrightarrow \boldsymbol{A}_{||}$$

$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys}$ with $\nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$

$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L'_q\right) + S'_g + L'_g$ with Δq as in JM/Ji

$$L'_q = \int d^3x \, \langle P,S| \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}_{pure}\right)^3 q(\vec{x}) \, |P,S\rangle$$

$$S'_g = \int d^3x \, \langle P,S| \left(\vec{E} \times \vec{A}_{phys}\right)^3 |P,S\rangle$$

$$L'_g = \int d^3x \, \langle P,S| \, E^i \left(\vec{x} \times \vec{\nabla}\right)^3 A^i_{phys} \, |P,S\rangle$$

Chen Sun Goldman PRL 103, 2009

Results Suggested for Nucleon Spin

$$\vec{J}_{\rm QCD} = \int d^3x\psi^\dagger\frac{1}{2}\vec{\Sigma}\psi + \int d^3x\vec{x}\times\psi^\dagger\frac{1}{i}\vec{D}\psi + \int d^3x\vec{E}\times\hat{\vec{A}} + \int d^3x\vec{x}\times E_i\vec{D}\hat{A}_i$$
$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g.$$

- S'_g very small , but large uncertainties
- Reduces Jaffe-Manohar decomposition in a Gauge

$$\boldsymbol{A}_{pure} = 0, \quad \boldsymbol{A} = \boldsymbol{A}_{phys}$$

- **OAM is canonical OAM operator**
- S (gluon spin in coulomb Gauge) = (5/9) Δg(light cone gauge).
- Suggests that under proper identification, gluon spin to nucleon spin may be drastically smaller than conventional wisdom .

Chen et al.'s papers arose **quite a controversy** on the **feasibility** of complete decomposition of nucleon spin.

- X. Ji, Phys. Rev. Lett. 104 (2010) 039101 : 106 (2011) 259101.
- S. C. Tiwari, arXiv:0807.0699.
- X. S. Chen et al., arXiv:0807.3083 ; arXiv:0812.4336 ; arXiv:0911.0248.
- Y. M. Cho et al., arXiv:1010.4336 ; arXiv:1102.1130.
- X. S. Chen et al., Phys.Rev. D83 (2011) 071901.
- E. Leader, Phys. Rev. D83 (2011) 096012.
- Y. Hatta, Phys. Rev. D84 (2011) 041701R.

Chen, Sun Goldman February 2011

Results Suggested for Nucleon Spin

Gauge Invariant Decomposition by Wakamatsu:

- Tried to give a clear picture of complete decomposition of nucleon spin:

$$J_{QCD} = \boldsymbol{S}^q + \boldsymbol{L}^q + \boldsymbol{S}^g + \boldsymbol{L}^g$$

M. W., Phys. Rev. D81 (2010) 114010.
M. W., Phys. Rev. D83 (2011) 014012.
M. W., Phys. Rev. D84 (2011) 037501.

$$\boldsymbol{S}^q = \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x$$

$$\boldsymbol{L}^q = \int \psi \, \boldsymbol{x} \times (\boldsymbol{p} - g \, \boldsymbol{A}) \, \psi \, d^3x$$

$$\boldsymbol{S}^g = \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x$$

$$\boldsymbol{L}^g = \int E^{aj} \, (\boldsymbol{x} \times \nabla) \, A^{aj}_{phys} \, d^3x + \boxed{\int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x}$$

•Vector potential found in 'MB and BC'

"potential angular momentum"

- Quark part is common to Ji decomposition
- Quark and gluon intrinsic spin part common to "Chen et. all" decomposition

quark OAM extracted from combined analysis of GPD and polarized PDFs is **dynamical quark OAM**, not **the canonical OAM** as predicted by Chen et. all

A crucial difference with the Chen decomp. appears in the orbital parts

$$\boldsymbol{L}^q + \boldsymbol{L}^g = \boldsymbol{L}'^q + \boldsymbol{L}'^g$$

$$\boldsymbol{L}^g - \boldsymbol{L}'^g = - (\boldsymbol{L}^q - \boldsymbol{L}'^q) = \int \rho^a \, (\boldsymbol{x} \times \boldsymbol{A}^a_{phys}) \, d^3x$$

Results Suggested for Nucleon Spin

QED framework also supports his work:
(Physical meaning of potential angular momentum)

It represents angular momentum associates with the longitudinal part of electric field generated by color-charged quarks !

Next:

Introduced the covariant generalization for all decomposition:

(1) It is useful to find relations to high-energy DIS observables.

(2) It is essential to prove Lorentz frame-independence of the decomposition.

$$\Delta q = \int_{-1}^{1} \Delta q(x)\, dx, \qquad \Delta g = \int_{-1}^{1} \Delta g(x)\, dx.$$

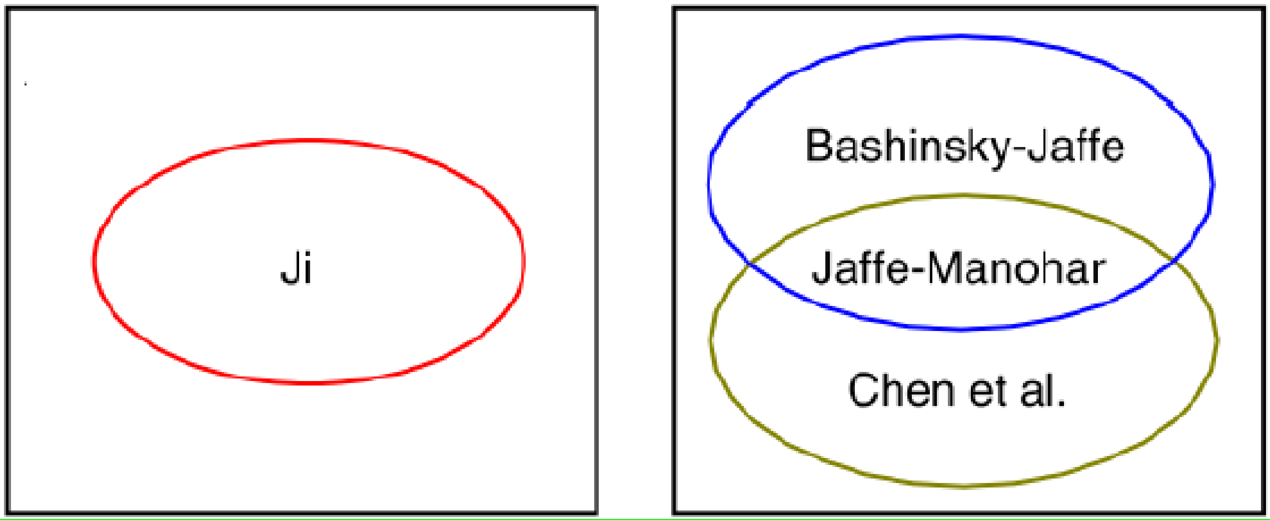
$$\begin{aligned} S_q &= \frac{1}{2}\Delta q, \\ L_q &= \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0) \right] - \frac{1}{2}\Delta q, \\ S_g &= \Delta g, \\ L_g &= \frac{1}{2}\left[A_{20}^g(0) + B_{20}^g(0) \right] - \Delta g. \end{aligned}$$

$$\begin{aligned} A_{20}^{q/g}(0) &= \int_{-1}^{1} x\, H^{q/g}(x,0,0)\, dx, \\ B_{20}^{q/g}(0) &= \int_{-1}^{1} x\, E^{q/g}(x,0,0)\, dx. \end{aligned}$$

Results Suggested for Nucleon Spin

we find two physically nonequivalent decompositions (I) and (II) .

Decomposition (I)   Decomposition (II)



The great advantage of decomposition (I) over (II) is:
Concrete connection between High Energy DIS(discussed by Wakamatsu)

Decomposition (II)

This decomposition reduces to any ones of Bashinsky-Jaffe, of Chen et al., and of Jaffe-Manohar, after an appropriate gauge-fixing in a suitable Lorentz frame, which reveals that **these 3 decompositions are all gauge-equivalent** !

These 3 are **physically equivalent** decompositions !

It was sometimes criticized that there are so many decompositions of nucleon spin.

$$
\begin{aligned}
\frac{1}{2} &= \frac{1}{2}\Sigma + L_Q + \Delta g + L_g \\
&= \frac{1}{2}\Sigma' + L'_Q + \Delta g' + L'_g \\
&= \frac{1}{2}\Sigma'' + L''_Q + \Delta g'' + L''_g \\
& \quad \vdots
\end{aligned}
$$

However, this is not true any more. One should recognize now that there are only two physically nonequivalent decompositions !

Decomposition (I)	Decomposition (II)
extension of **Ji's decomp.** including gluon part	nontrivial gauge-invariant extension of **Jaffe-Manohar's decomp.**
dynamical OAMs	**"canonical" OAMs**

Since both decompositions are gauge-invariant, there is a possibility that they both correspond to observables !

Goldman argued that the nucleon spin decomposition is frame-dependent !

- T. Goldman, arXiv:1110.2533.

This is generally true, but our interest here is the longitudinal spin sum rule.

♣ The longitudinal spin decomposition is certainly frame-independent !

Leader recently proposed a sum rule for transverse angular momentum.

- E. Leader, arXiv:1109.1230.

More you read more you get confused !!!!!!!

Thank you for your patience