Transverse Momentum of partons

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I am very grateful to:

- **Paul Reimer,**
  for breaking the ice with the first lecture of the week and
  for giving a great introduction to DIS processes

- **Alexei Prokudin, Alberto Accardi and Mark Schlegel,**
  for inspiration and more … (actually I have hacked many
  of their slides while putting together my introductory lecture)

- **All of my collaborators,**
  Alexei Prokudin and Stefano Melis in particular, for their fantastic work:
  most of the results I will present in these lectures have only been
  achieved thanks to their ability and dedication.

We will be talking about ...

- Deep Inelastic scattering
- Drell- Yan
- e^+e^- scattering processes
- Parton model
- Transverse momentum of partons
- 3D kinematics (i.e. kinematics with partonic transverse momentum)
- Collinear distribution and fragmentation functions (transversity and all that ...)
- Transverse Momentum Dependent distribution and fragmentation functions
- Phenomenology of TMD's: extracting Transverse Momentum Dependent distribution functions from experimental data (SIDIS, Drell-Yan, e+e- scattering)
- Scale dependence and TMD evolution scheme

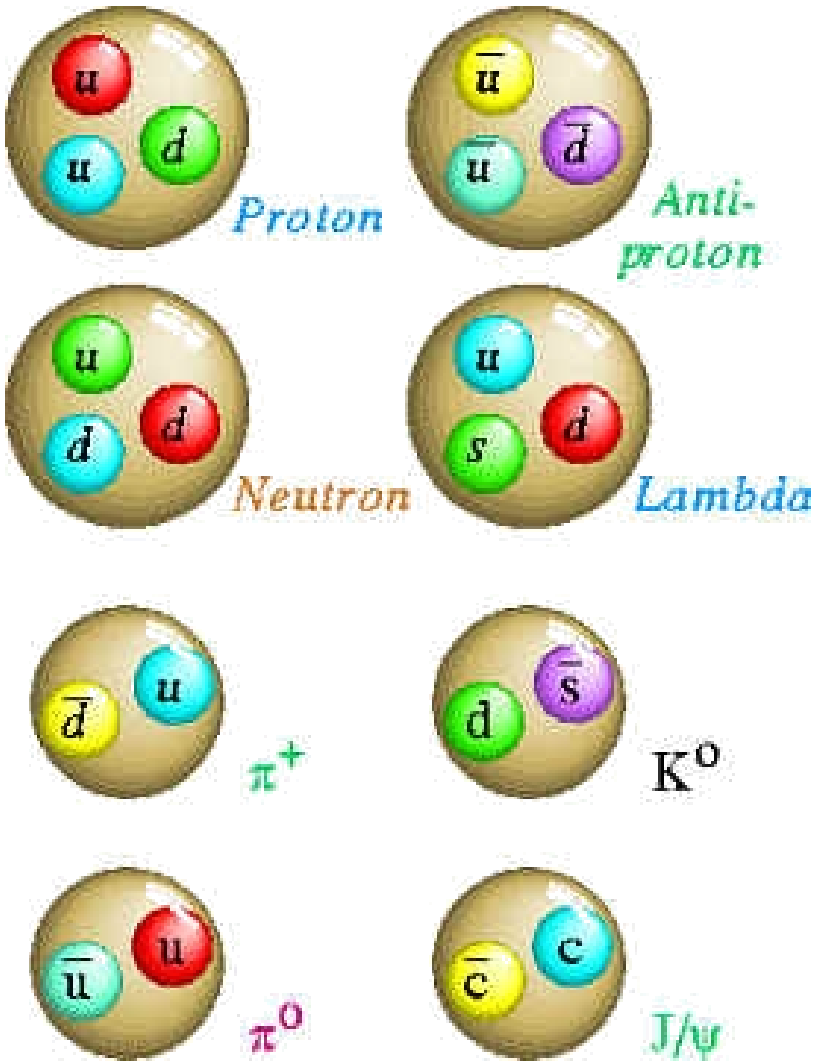
We will be talking about ...

- Deep Inelastic scattering
- Drell- Yan
- e^+e^- scattering processes
- Parton model

Lecture 1

- Transverse momentum of partons
- 3D kinematics (i.e. kinematics with partonic transverse momentum)
- Integrated distribution and fragmentation functions (transversity and all that ...)
- **T**ransverse **M**omentum **D**ependent distribution and fragmentation functions
- Phenomenology of TMD's: extracting **T**ransverse **M**omentum **D**ependent distribution functions from experimental data
- Scale dependence and TMD evolution scheme

Hadron Structure



- 6 flavors (and 3 colors):

  up, down, strange – light
  charm, bottom, top – heavy

  spin ½
  isospin (u = ½, d = – ½)
  strangeness (s = 1)

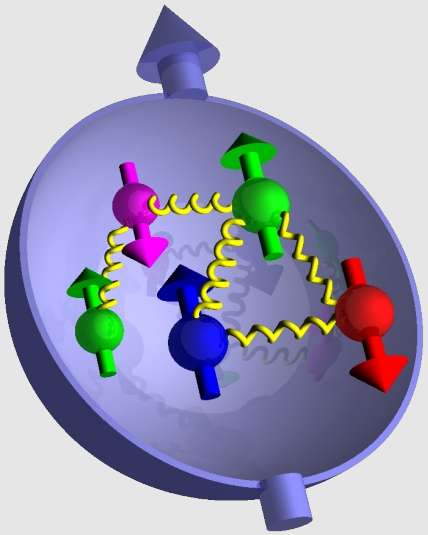
- confined in colorless hadrons
  - mesons – 2 quarks
  - baryons – 3 quarks

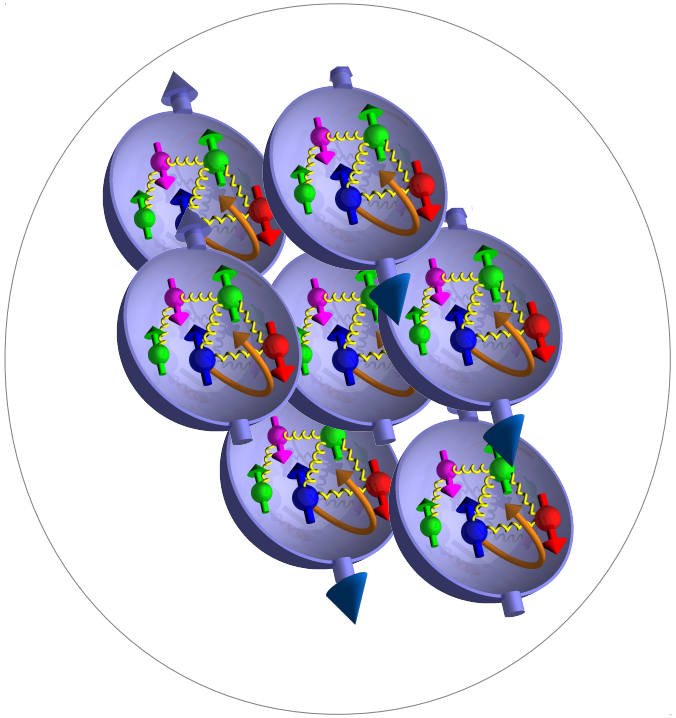
Thanks to Alberto Accardi for this slide ...

Hadron Structure

Nucleons are made of three (valence) quarks, spinning and orbiting around, kept together by a special glue, made of gluons.

Moreover, inside hadrons there is an arbitrary number of quark-antiquark pairs (made of sea quarks originated from gluons via $g \rightarrow q\bar{q}$ (themeselves radiated from quarks).





Nucleons and their elementary constituents, quarks and gluons, interact with each other inside nuclei.

How did we learn about that ?

Probing the nucleon structure

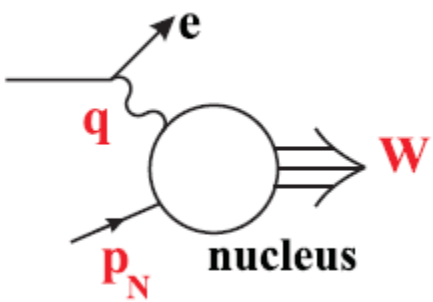
↓

Electron - Nucleus Scattering
Electron - Proton Scattering
Deep Inelastic Scattering

↪ DIS provided first direct evidence for the existence of quarks inside nucleons (Nobel Prize 1990: Freedman, Kendall, Taylor)

Electron - Nucleus Scattering

e⁻-nucleus scattering is an ideal probe of the structure of a composite object



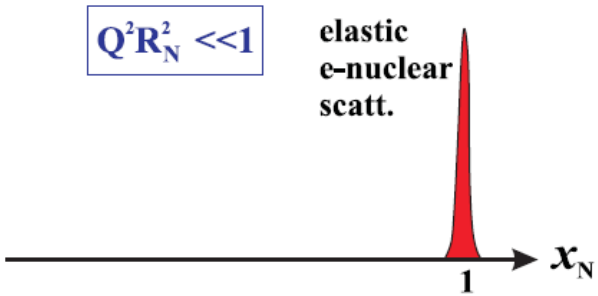
$$\nu = E - E'$$

$$Q^2 \equiv -q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$W^2 = (p_N + q)^2 = M_N^2 + 2p_N \cdot q + q^2$$

$\lambda \sim 1/Q$

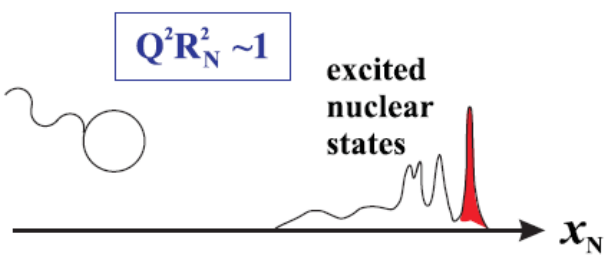
Increasing the energy of e, the wavelength of the photon decreases



$\Lambda \gg R_N$

The photon sees the nucleus as a point and scatters elastically, with $W=M_N$

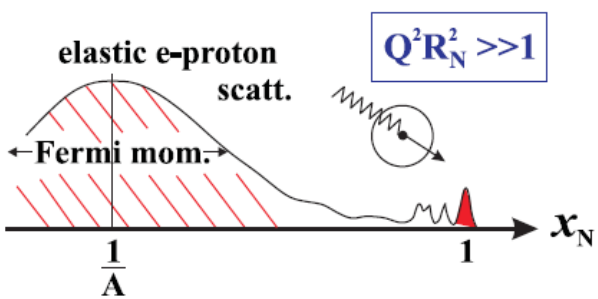
$$x_N \equiv \frac{Q^2}{2p_N \cdot q} = \left(\frac{Q^2}{2M_N\nu}\right)_{\text{lab.}} = 1$$



$\Lambda \approx R_N$

The photon is energetic enough to excite the nucleus

Now $W > M_N$ and $x_N < 1$

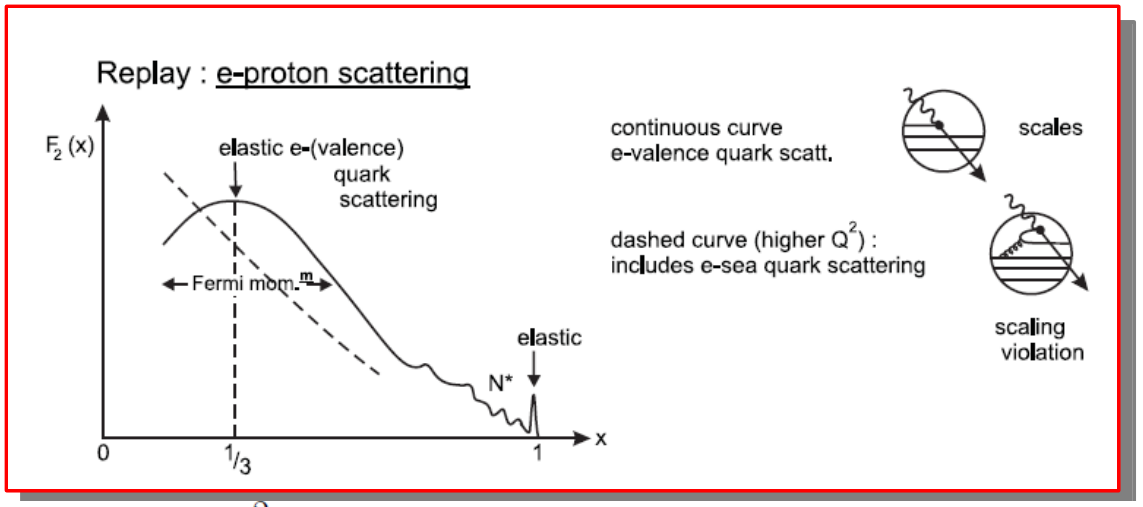


$\Lambda \ll R_N$

The photon probes deep in the nucleus, which breaks up → The photon sees the nucleons in the nucleus

$$x_N = \frac{M}{M_N}\left(\frac{Q^2}{2M\nu}\right)_{\text{lab.}} = \frac{1}{A}$$

Electron - Proton Scattering

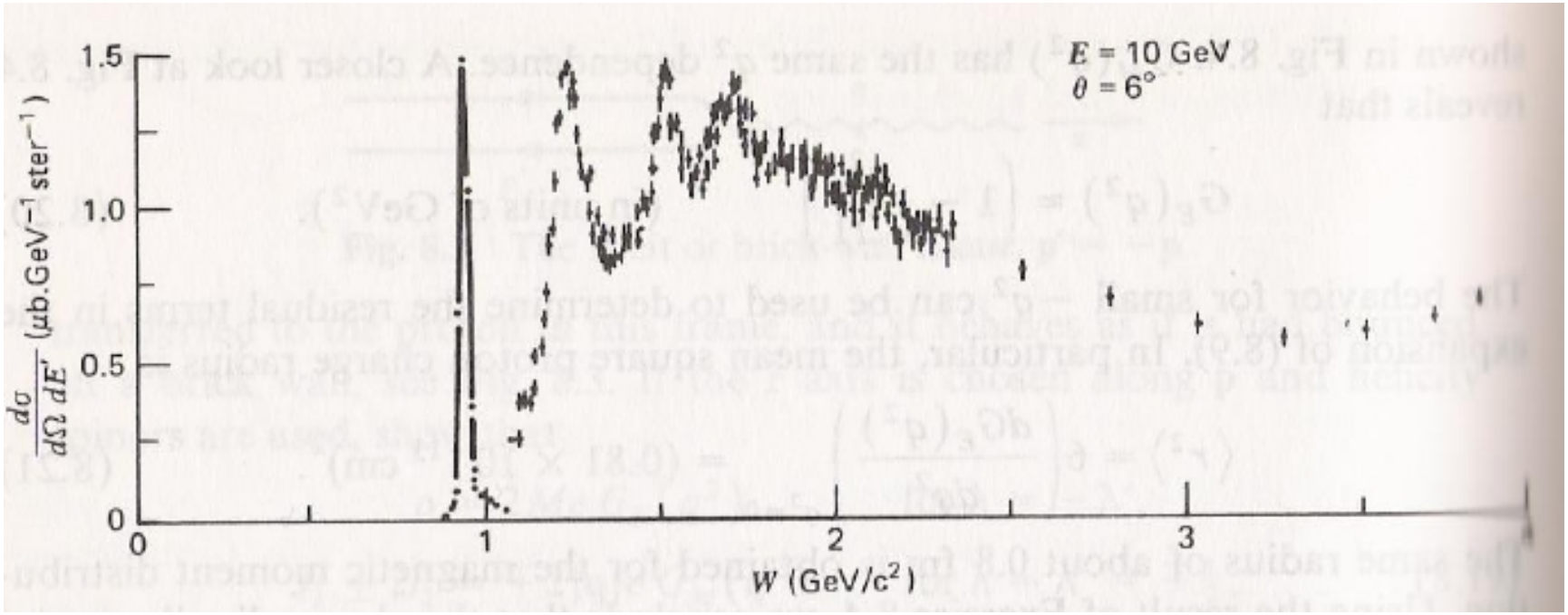


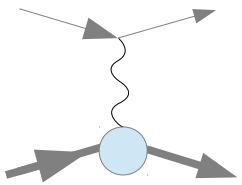
Let us increase Q^2 even further. Suppose that protons are made up of three point-like quarks, then high-energy electron–proton scattering will simply be a replay of electron–nucleus scattering one layer of substructure down. We have an analogous sequence of diagrams to those shown in Fig. 1, but with R_N replaced by the proton radius R. Also the scattering probabilities should now be plotted in terms of

$$x = \left(\frac{Q^2}{2p \cdot q} \right)$$

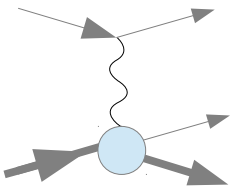
where p is the 4-momentum of the proton. The continuous curve in Fig. 2 is the analogue of the lowest plot in Fig. 1. It shows the elastic eq-scattering peak Fermi-smeared about $x = 1/3$, together with traces of the elastic ep peak at $x \sim 1$. If there were no further substructure, this curve would persist as Q^2 increases. We would have (Bjorken) scaling; the scattering depends only on the ratio $x = Q^2/2p \cdot q$, and not on the two variables, Q^2 and $p \cdot q$, individually. x is known as the Bjorken scaling variable.

Electron - Proton Scattering



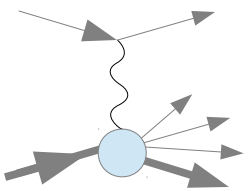


Elastic:
W=M
(no energy to produce any other particle)



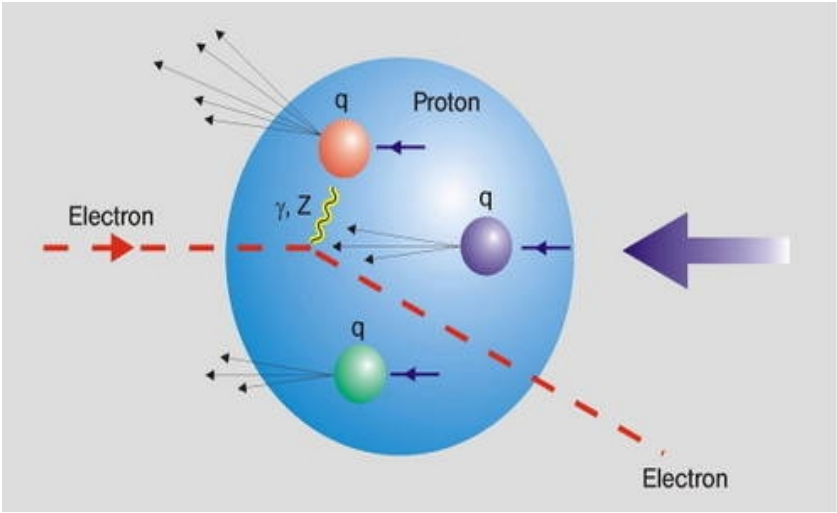
Inelastic (resonance region)

(W for only a few particles)



Deep inelastic scattering
(large W, more particles)

Deep Inelastic Scattering



Suppose the photon probes a quark carrying a fraction ξ of the proton momentum.
For massless quarks we find:

$$(\xi p + q)^2 = m_q^2 \simeq 0$$

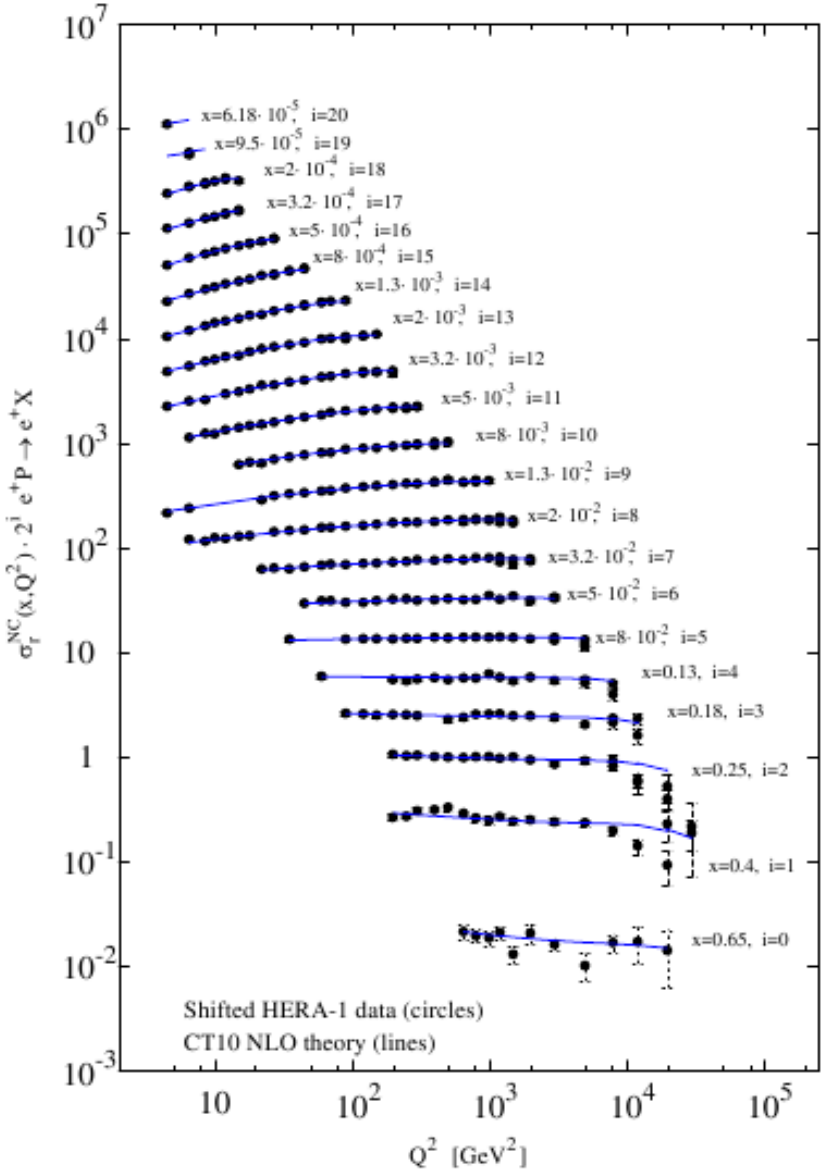
$$\xi \simeq Q^2/2p \cdot q = x$$

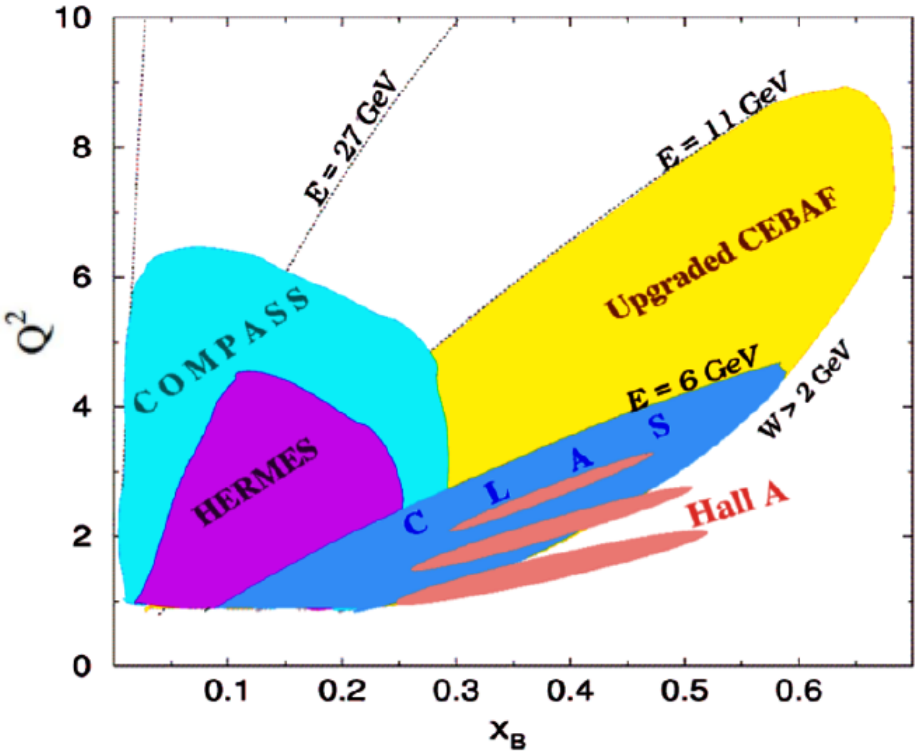
As Q^2 increases, more quarks are resolved which have to share the parent momentum. Each carries a smaller fraction ξ= x, and we have scaling violation, proportional to $\alpha_s \log(Q^2/\mu^2)$.

Scaling

Scaling violation

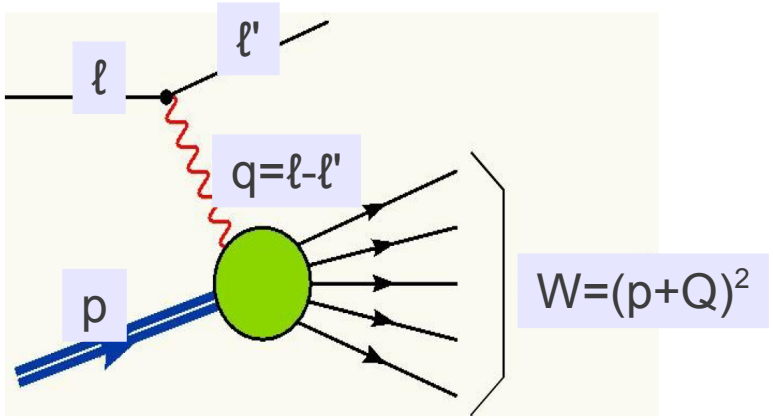
Deep Inelastic Scattering





Deep Inelastic Scattering

Inclusive lepton-hadron scattering:



$$\lambda = e, \mu, \nu, \ldots$$

$$N = p, n, \ldots$$

$$q^\mu = \ell^\mu - \ell'^\mu$$ Virtual photon momentum

$$p_X^\mu = \sum_{i \in X} p_i^\mu$$ Hadronic final state momentum

- Notation: $p^2 = p_\mu p^\mu \quad p \cdot q = p_\mu q^\mu$
- Masses: $p^2 = M^2 \quad \ell^2 = m_\lambda^2$

The photon is virtual: $q^2 < 0$

Neglect compared to Q^2 (MeV vs. GeV)

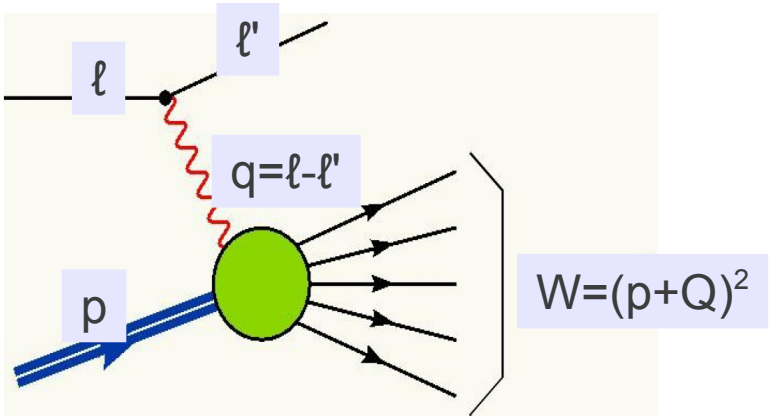
$$q^2 = (l - l')^2 = m_\lambda^2 + m_\lambda^2 - 2\ell \cdot \ell'$$

$$= -2EE' + |\vec{\ell}|\,|\vec{\ell'}| \cos\theta = -2EE'(1 - \cos\theta) \leq 0$$

$$m_\lambda^2 = 0 \Rightarrow |\vec{\ell}| = E$$

Deep Inelastic Scattering

Inclusive lepton-hadron scattering:



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Lorentz invariants

$$Q^2 = -q^2$$ virtuality

$$x_B = \frac{-q^2}{2p \cdot q}$$ Bjorken x

$$\nu = \frac{p \cdot q}{\sqrt{p^2}}$$ beam energy loss *

$$y = \frac{p \cdot q}{p \cdot \ell}$$ inelasticity *

$$W^2 = (p+q)^2$$ (final state) invariant mass

$$s = (p+\ell)^2$$ center-of-mass energy

Deep Inelastic Scattering Kinematics and Cross Section

$$d^3\sigma_{DIS} = \frac{1}{flux}\ \frac{d^3\ell'}{(2\pi)^3\, 2\,\ell'_0} \sum_x |M(e\,p \to e\,X)|^2 \qquad flux = 2\,s$$

$$i\,M(e\,p \to e\,X) = -i\,e\ \ \bar{u}(\ell')\gamma^\mu u(\ell)\ \left(\frac{-i}{q^2}\right)(ie)\ \langle X|J^\mu(q)|p\rangle$$

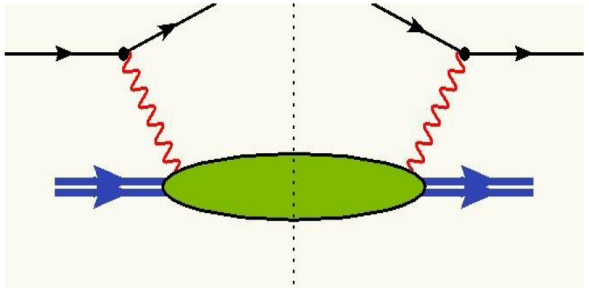
$$= \frac{-e^2}{q^2}\ \bar{u}(\ell')\gamma^\mu u(\ell)\ \int d^4x\, e^{iqx} \langle X|J^\mu(x)|p\rangle$$

where $J^\mu(q) = \int d^4x\, e^{iqx} J^\mu(x)$

- e.m. current
- local operator
- always conserved

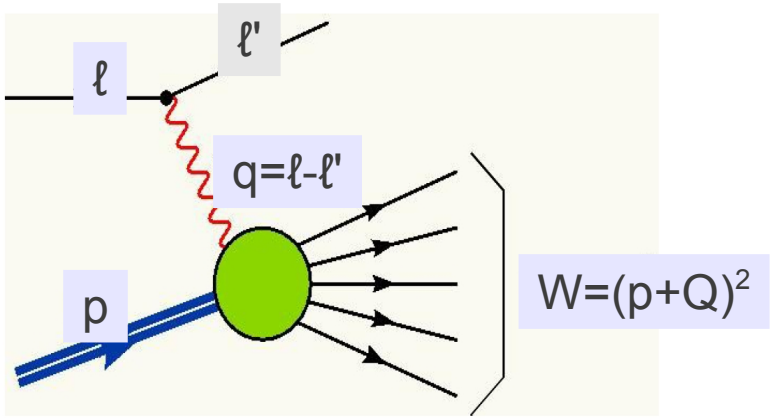
$$\frac{d^2\sigma}{dx\,dy} = \frac{\alpha^2}{Q^2}\ y\ \overbrace{L^{\mu\nu}}^{\text{Lepton tensor}}\ \underbrace{H_{\mu\nu}}_{\text{Hadron tensor}}$$

$$x = \frac{Q^2}{2\,p\cdot q} \qquad y = \frac{p\cdot q}{p\cdot \ell}$$



Deep Inelastic Scattering Kinematics and Cross Section

$$L_{\mu\nu} = 2[\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\, \ell \cdot \ell']$$

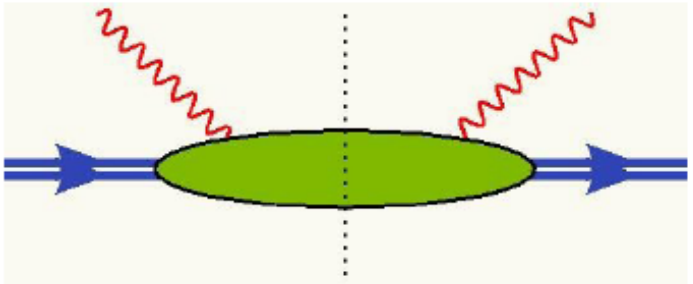


Write the photon-proton vertex in the most general possible way, without making any assumption on the nature of strong interactions

$$H_{\mu\nu} = \frac{1}{2} \sum_{spin} \sum_X [\langle X | J^\mu(0) | p \rangle]^\star \; [\langle X | J^\nu(0) | p \rangle] \; (2\pi)^4 \; \delta(q - p + P_X) \; =$$

$$= \frac{1}{2} \sum_{spin} \int d^4 x \, e^{iqx} \; \langle p | J^\mu(x) J^\nu(0) | p \rangle$$

Deep Inelastic Scattering
Hadronic Tensor



$$\langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle$$

Sum over *all* hadronic final states:

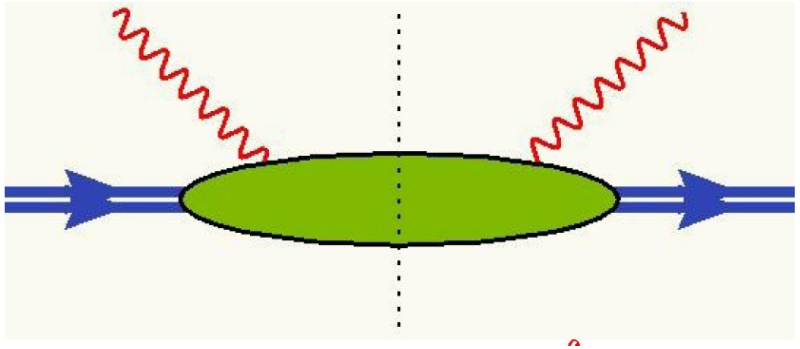
$$H^{\mu\nu}(P,q) = \sum\!\!\!\!\!\!\int_X \langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle(2\pi)^4\delta^{(4)}(P+q-P_X)$$

$$= \int d^4x e^{iq\cdot x}\langle P,S|J^\mu(x)\sum\!\!\!\!\!\!\int_X |X\rangle\langle X|J^\nu(0)|P,S\rangle$$

$$= \int d^4x\, e^{iq\cdot x}\langle P,S|[J^\mu(x)\,,\ J^\nu(0)]|P,S\rangle$$

Deep inelastic Scattering
General properties of the hadronic tensor

- Hermiticity of the current $J^\mu(x)$: $H^{\mu\nu}(p,q) = H^{\nu\mu\,*}(p,q)$
- Covariance : $H^{\mu\nu}$ is a rank two Lorentz tensor constructed with p^μ and q^μ
- Current conservation : $\partial_\mu J^\mu(x) = 0 \;\rightarrow\; q_\mu\, H^{\mu\nu}(p,q) = q_\nu\, H^{\mu\nu}(p,q) = 0$



$$\frac{1}{4\pi} H_{\mu\nu}(p,q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) \overbrace{F_1(x,Q^2)}^{\text{Structure function}}$$

$$+ \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) \underbrace{F_2(x,Q^2)}_{\text{Structure function}}$$

Deep Inelastic Scattering
Parton Model

$$\frac{d^2\sigma_{DIS}}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2}\left[y\,F_1(x,Q^2) + \frac{1-y}{xy}\,F_2(x,Q^2)\right]$$

Having made no assumptions on the nature of strong interactions

Now compare it with point-like particle cross section

$$\frac{d^2\sigma_{e\mu}}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2}\left[y\,\frac{x}{2}\delta(1-x) + \frac{1-y}{xy}\,\delta(1-x)\right]$$

This dependence is typical of spin 1/2

$$\begin{cases} F_1(x,Q^2) = \frac{x}{2}\delta(1-x) \\ F_2(x,Q^2) = \delta(1-x) \end{cases}$$

Independent of Q^2 → Bjorken scaling

In other words, for a point-like object the structure functions have the form of a delta function which forces x=1

Deep Inelastic Scattering
Parton Model

Bjorken scaling

In 1969 Bjorken proposed that observing a proton with high Q^2 (i.e. short λ) photon, in the limit

$Q^2 \rightarrow \infty$

$\nu \rightarrow \infty$

but x_B finite, one should find a behaviour similar to that of point-like particle scattering, with F_1 and F_2 depending only on the ratio $x_B = Q^2 /(2\, q\, p)$ and not on Q^2 and ν separately.

This picture is however frame dependent – needs a good choice:

- **Rest frame**: assume finite energy of interactions, *i.e.*, finite interaction times
- **Infinite momentum frame** along *z* direction: times are dilated, interaction is slower and slower, until as $p_z \rightarrow \infty$ they appear to not interact at all – this is the right frame for our intuitive picture (and for precise realization in field theory)

Parton Model

- When observed by high Q^2 photons, protons appear as a collection of loosely bound constituents, called PARTONS
  In the interaction, The photon gets scattered by one of these partons

- Parton scatterings do not interfere among each other: scattering off a proton is the incoherent sum over all the individual partonic scatterings

- Each parton in a fast moving proton is characterized by a **probability distribution** in longitudinal momentum:

  $f_q^p(\xi)$ is the probability of finding parton q inside the proton p carrying a fraction between ξ and $\xi + d\xi$ of the proton momentum

- Therefore the total cross section, within the parton model, can be written

$$\sigma(e(\ell) + p(P) \rightarrow e(\ell') + X) =$$
$$= \sum_q e_q^2 \int_0^1 d\xi\, f_q(\xi)\, \sigma(e(\ell) + q(\xi P) \rightarrow e(\ell') + q(p'))$$

Or simply $$\boxed{d\sigma_{DIS} = \sum_q e_q^2 \int_0^1 d\xi\, f_q^p(\xi)\, \hat{\sigma}_{partonic}}$$

Parton Model

This means replacing

p → ξ P , i.e. x → x / ξ and dx → d x / ξ

In the formula for point-like particle scattering.

Therefore the parton model cross section becomes

$$\frac{d^2\sigma_{DIS}}{dx\,dy} = \sum_q e_q^2 \int_0^1 d\xi \; f_q(\xi) \; \frac{4\pi\alpha^2}{Q^2} \; \underbrace{\delta\left(1-\frac{x}{\xi}\right)} \; \left(\frac{1}{\xi}\right) \; \left(\frac{1-y}{xy}\xi - \frac{xy}{2\xi}\right)$$

$$\boxed{\xi\,\delta(\xi - x) \;\rightarrow\; \xi = x \;\; !!!}$$

$$\boxed{\frac{d^2\sigma_{DIS}}{dx\,dy} = \frac{2\pi\alpha^2 s}{Q^4} \sum_q e_q^2 \; x \; f_q(x) \; [1+(1-y)^2]}$$

PARTON MODEL DIS CROSS SECTION

Parton Model

Comparing the parton model DIS cross section

$$\frac{d^2\sigma_{DIS}}{dx\,dy} = \frac{2\pi\alpha^2 s}{Q^4} \sum_q e_q^2 \; x \; f_q(x) \; [1+(1-y)^2]$$

with the DIS cross section obtained only on general principles

$$\frac{d^2\sigma_{DIS}}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2} \left[y \; F_1(x,Q^2) + \frac{1-y}{xy} \; F_2(x,Q^2) \right]$$

We find a match only if

$$F_2(x,Q^2) = 2x F_1(x,Q^2)$$

CALLAN GROSS RELATION
(partons have spin 1/2)

$$F_2(x,Q^2) \rightarrow F_2(x) = \sum_q e_q^2 \; x \; f_q^p(x)$$

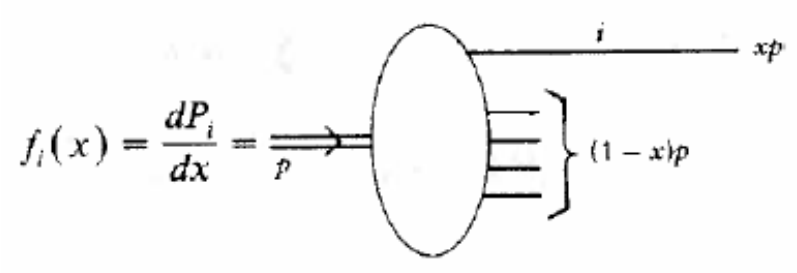
Bjorken limit, scaling

Parton Model interpretation
of the Structure Functions

Parton Model

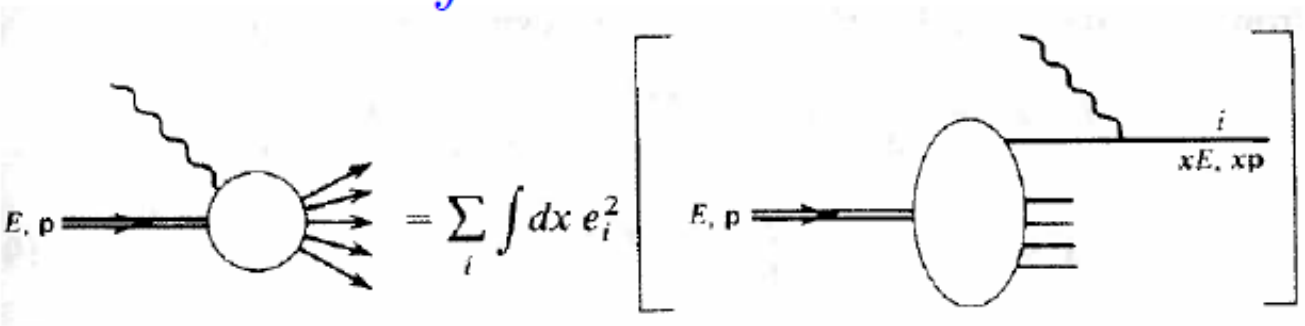
Define a parton distribution

$f_i(x)dx =$ Probability of finding a parton *i* with momentum Fraction between *x* and *x+dx*



For a DIS process (as an example)

$$\sigma(p,q) = \int dx\, f_i(x)\hat{\sigma}_i(xp,q)$$



Parton Model

What can we expect in general?

- There are gluons, expect also "sea" quark anti-quark pairs
- **Proton charge: +1**, but *u*: +2/3, *d*: -1/3, *s*: -1/3, *g*: 0

$$1 = \frac{2}{3}\int_0^1 dx\left[u(x) - \bar{u}(x)\right] - \frac{1}{3}\int_0^1 dx\left[d(x) - \bar{d}(x)\right] - \frac{1}{3}\int_0^1 dx\left[s(x) - \bar{s}(x)\right]$$

- **Proton isospin: 1/2**, but *u*: +1/2, *d*: -1/2, *s*: 0, *g*: 0

$$\frac{1}{2} = \frac{1}{2}\int_0^1 dx\left[u(x) - \bar{u}(x)\right] - \frac{1}{2}\int_0^1 dx\left[d(x) - \bar{d}(x)\right]$$

- **Proton strangeness: 0**, but *u*: 0, *d*: 0, *s*: 1, *g*: 0

$$0 = \int_0^1 dx\left[s(x) - \bar{s}(x)\right]$$

Parton Model

Parton sum rules (for the proton):

$$2 = \int_0^1 dx \big[\underbrace{u(x) - \bar{u}(x)}_{= u_V(x)}\big]$$

$$1 = \int_0^1 dx \big[\underbrace{d(x) - \bar{d}(x)}_{= d_V(x)}\big]$$

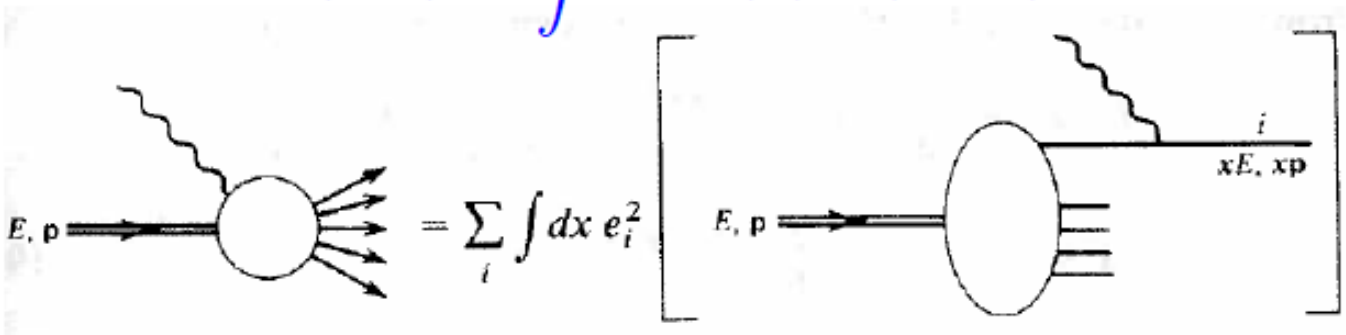
$$0 = \int_0^1 dx \big[s(x) - \bar{s}(x)\big]$$

Parton Model

Caveats:

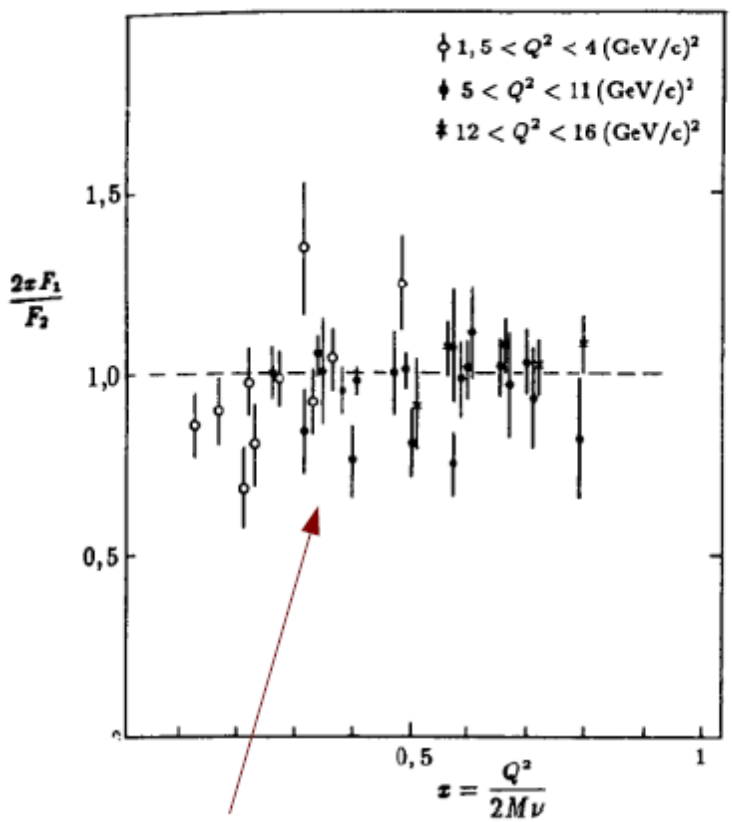
- Tacit probabilistic assumption: we are multiplying probabilities rather than amplitudes

$$\sigma(p,q) = \int dx\, f_i(x)\hat{\sigma}_i(xp,q)$$



  - justified by time dilation / smallness of photon wavelength arguments
  - Can be broken by soft (long wavelength) initial state interactions between the proton and quark lines
- Likewise, we are assuming the same parton distribution applies to other process: "universality"
- The non-trivial proof in QCD is called "QCD factorization theorem"

Parton Model



Charged partons have spn ½ (quarks!)

log(Q^2) scaling violations

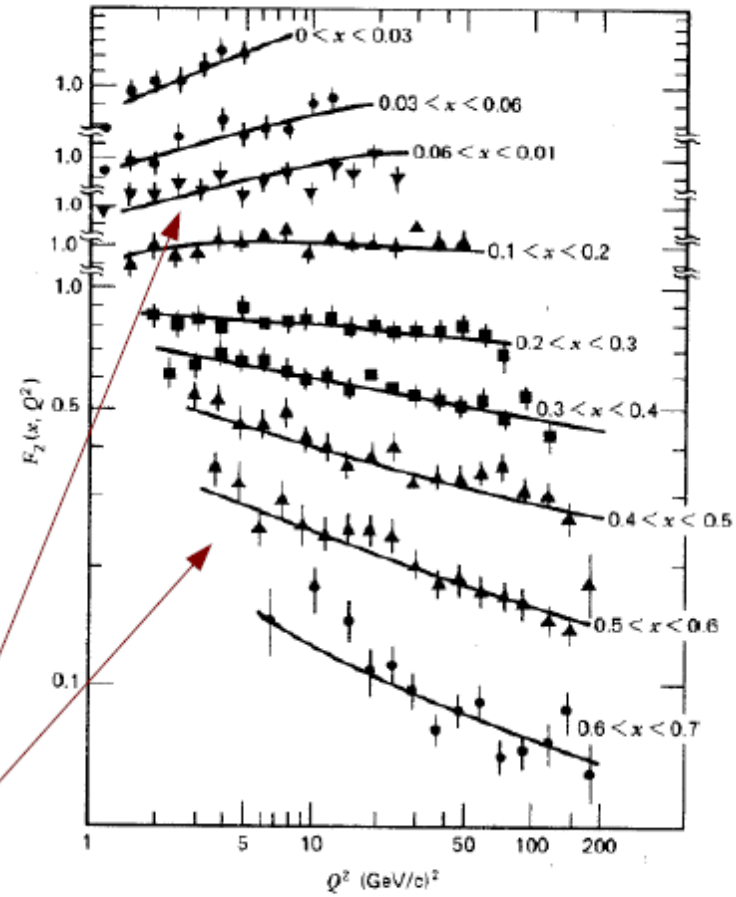
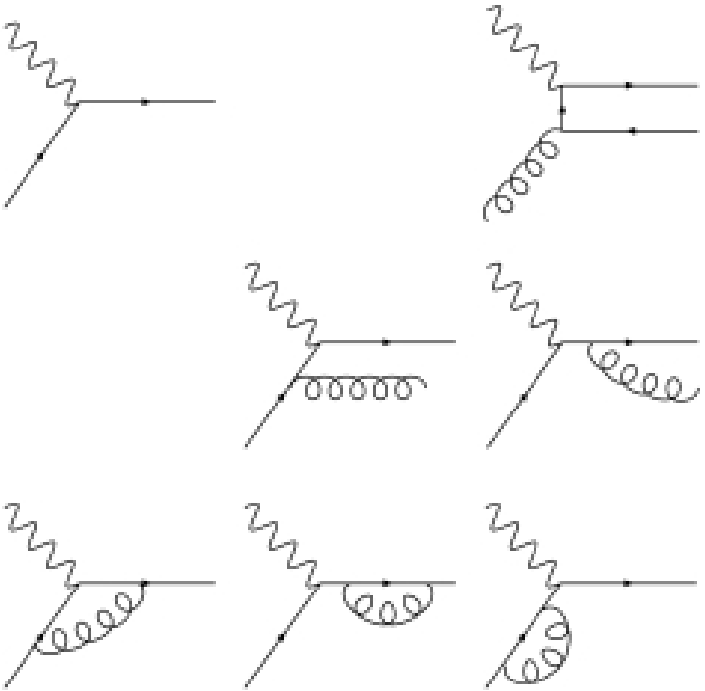


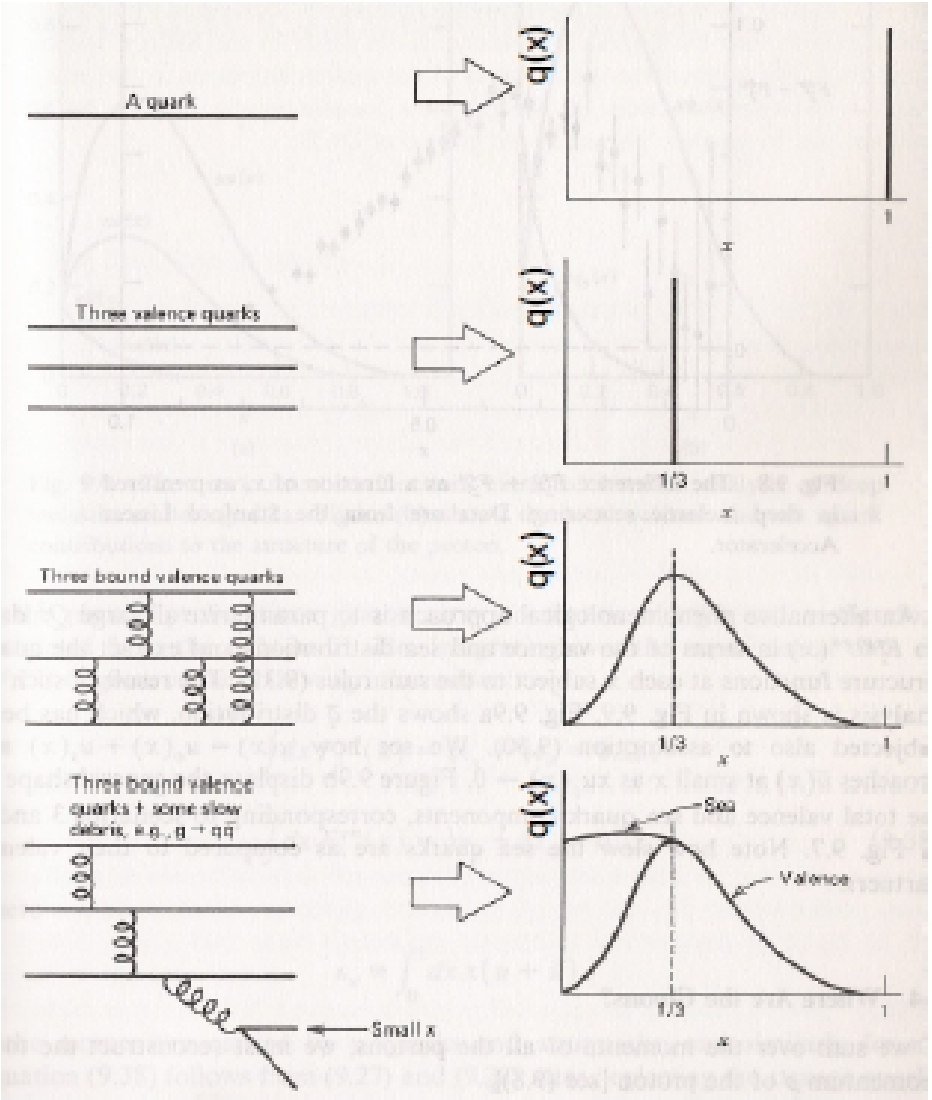
Fig. 10.10 Deviations from scaling. With increasing Q^2, the structure function $F_2(x, Q^2)$ increases at small x and decreases at large x. The data are from the CDHS counter experiment at CERN.

Where do gluons come about ?

We know that quarks are not free: they interact *via* gluons

- gluon coupling: α_s
- Appear at Next-to-Leading Order (NLO)

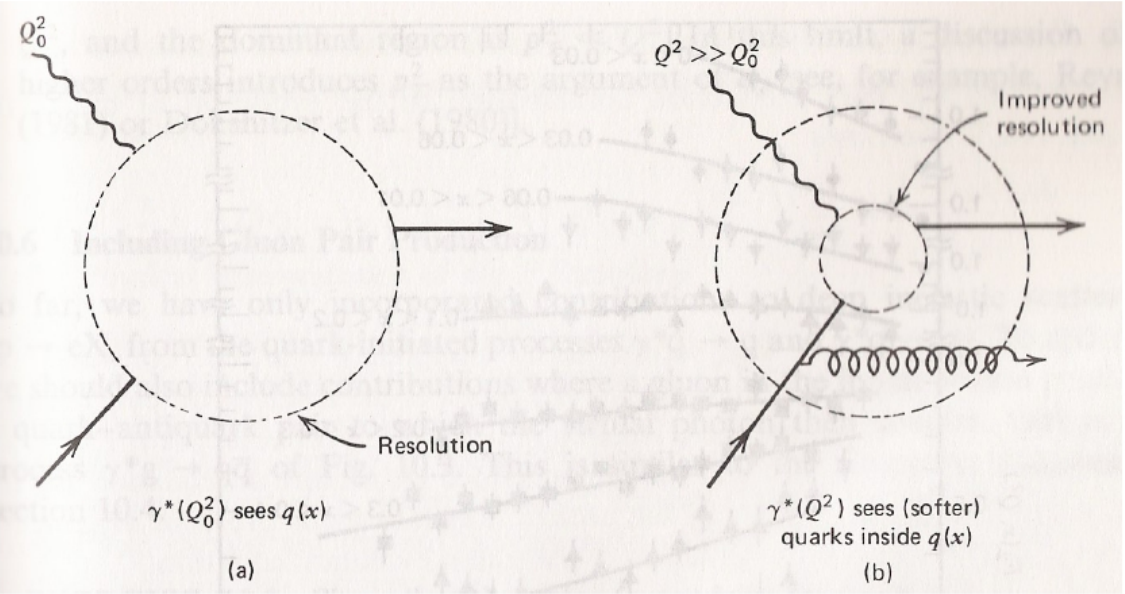




QCD Evolution Equations

QCD evolution – DGLAP equations

$$\frac{d\, f_\ell(x,\mu)}{d\log\mu} = \frac{\alpha_S(\mu)}{2\pi}\sum_m f_m(z,\mu)\int_x^1 \frac{dz}{z} P_{\ell m}\left(\frac{x}{z}\right)$$



- PDF are non-perturbative and need to be experimentally measured (↪ global fits, lectures 3-4)
- However, if they are known at Q_0, they can be calculated at any Q

QCD Improved Parton Model

To invoke asymptotic freedom, and keep perturbation theory under control (LO vs. NLO vs. NNLO, ...) choose

$$\mu = \mu_R = Q$$

At LO one obtains the "QCD improved" parton model

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 q(x_B, Q^2)$$

Now depends on Q !

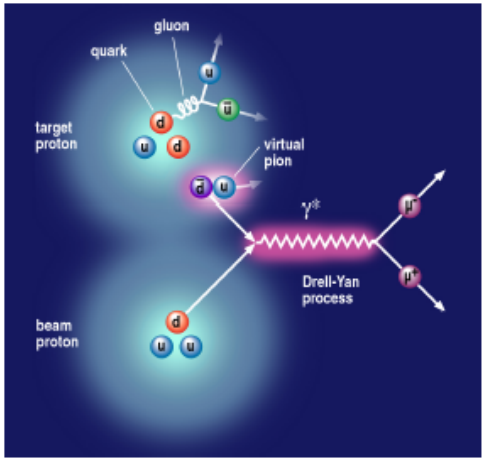
– Bjorken scaling is broken, but only logarithmically, by DGLAP evolution, and the running of α_s. Agrees with data.

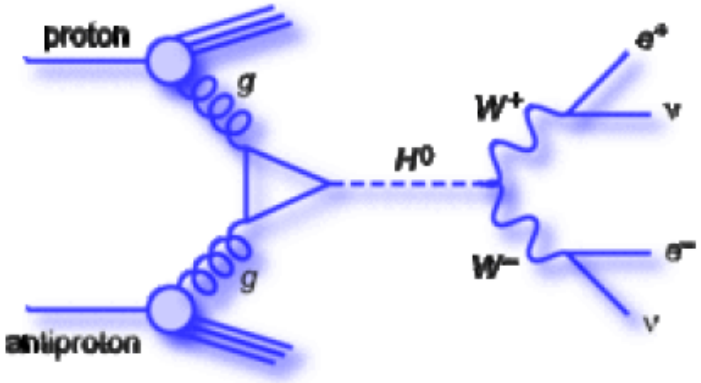
Factorization and Universality

Factorization and universality are 2 indivisible facets of the QCD factorization theorem

The factorization theorem also insures that the PDFs obtained from DIS experiments can be used to calculate other processes ("universality" of the PDFs)

- *e.g.*, Drell-Yan lepton-pair production, or Higgs production





- In fact, factorization and universality need to be proved process-by-process, but hold for a large number of these (some seen in Lecture 3)

Summary

- The understanding of the structure of a proton at short distances is the key ingredient to be able to predict cross sections of hadronic processes

- The binding forces responsible for the quark confinement are due to the exchange of soft gluons (if the gluons were hard, the recoil would tend to brak the proton apart, rather then keep it together), which occur on a rather long time scale ($1/m_p$ or longer).

- When we investigate the proton with a highly virtual photon, the interaction between photon and quark should take place in a much shorter typical time (1/Q). When the photon gets inside the proton and sees the quark, the struck quark has no time to negotiate a coherent response with the other quarks, because the time scale to “talk” to its pals would be too large compared to the time scale of the photon interaction. Therefore, the quark has no option but to interact with the photon as if it were a free particle.

- To good approximation, i.e. at LO, the highly virtual photon sees a static snapshot of the inner guts of the proton.

- The inner dynamics of strong interactions (gluons) become evident only at higher orders, with QCD evolution and scaling violations.