



HUGS
2012

**27th Annual Hampton University
Graduate Studies Program**

June 4 - June 22, 2012

HUGS at Jefferson Lab Summer School is designed for graduate students, and focuses primarily on both experimental and theoretical topics of current interest in strong-interaction physics. The program is simultaneously intensive, friendly and casual, providing students many opportunities to interact with internationally renown lecturers and Jefferson Lab staff, as well as with other graduate students and visitors.

Program topics will include:

• Transverse Momentum of Partons	Maria-Elena Boglione (Turin U. & INFN Turin)
• Insights into the Spectrum of Hadrons from Lattice QCD	Jozef Dudek (ODU & JLab)
• Dualities and QCD	Josh Erlich (W&M)
• Meson Electroproduction and Imaging Studies	Tanja Horn (CUA)
• Flavor Structure and Electroweak Interactions	Paul Reimer (ANL)
• Nucleons in the Nucleus	Larry Weinstein (ODU)

**Application Deadline:
April 2, 2012**



www.jlab.org/HUGS

Transverse Momentum of partons

Mariaelena Boglione



UNIVERSITÀ
DEGLI STUDI
DI TORINO
ALMA UNIVERSITAS
TAURINENSIS



Lecture 3

Facts of life

This is a worm gear



These are worm gear distribution functions

$$h_{1L}^{q\perp}(x, \mathbf{k}_T^2)$$

$$g_{1T}^q(x, \mathbf{k}_T^2)$$

Alessandro Bacchetta

An elephant is not a mouse multiplied by a thousand

Larry Weinstein

Neohipparion (small horse)



(a)

Mastodon



(b)

This is a pretzel



This is pretzelosity

$$h_{1T}^{q\perp}(x, \mathbf{k}_T^2)$$

Matthias Burkhardt

We are physicists:
we are arrogant,
we act with prejudice, and
we make assumptions
based on our prejudice.
Whatever we say,
we say it with authority.

Paul Reimer

We will be talking about ...

- Deep Inelastic scattering
- Parton model

Lecture 1

Lecture 2

- Transverse momentum of partons
- 3D kinematics (i.e. kinematics with partonic transverse momentum)
- Integrated distribution and fragmentation functions (transversity and all that ...)
- Transverse Momentum Dependent distribution and fragmentation functions

- Unpolarized SIDIS cross section
- Phenomenology of TMD's: extracting Transverse Momentum Dependent distribution functions from experimental data:

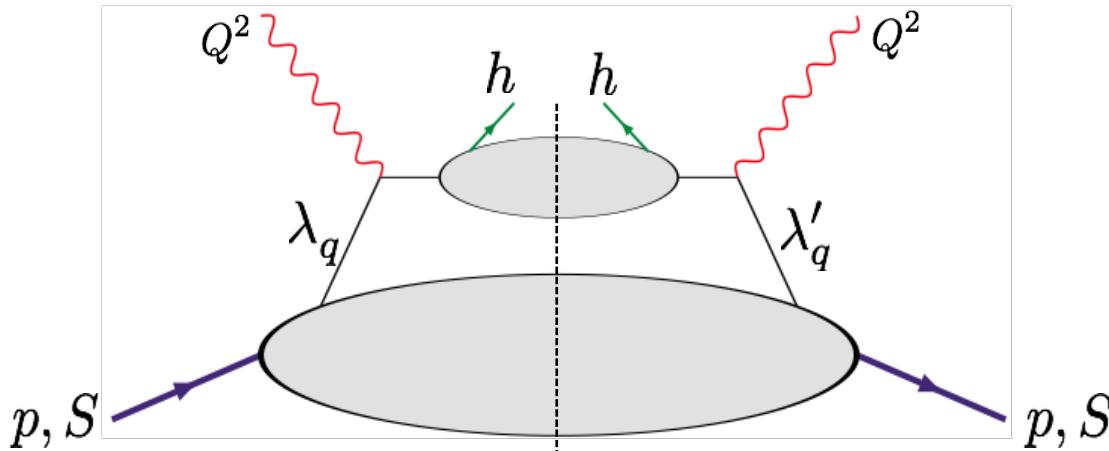
SIDIS

Lecture 3

- e+e- scattering
- Drell-Yan
- Scale dependence and TMD evolution scheme

Unpolarized SIDIS cross section

SIDIS FACTORIZATION



$$\frac{d\sigma^{\ell(S_\ell) + p(S) \rightarrow \ell' + h + X}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S}$$

$\rho_{\lambda_\ell, \lambda'_\ell}^{\ell, S_\ell} \otimes \rho_{\lambda_q, \lambda'_q}^{q/p, S} f_{q/p, S}(x, \mathbf{k}_\perp)$
TMD-PDF

Hard
Scattering

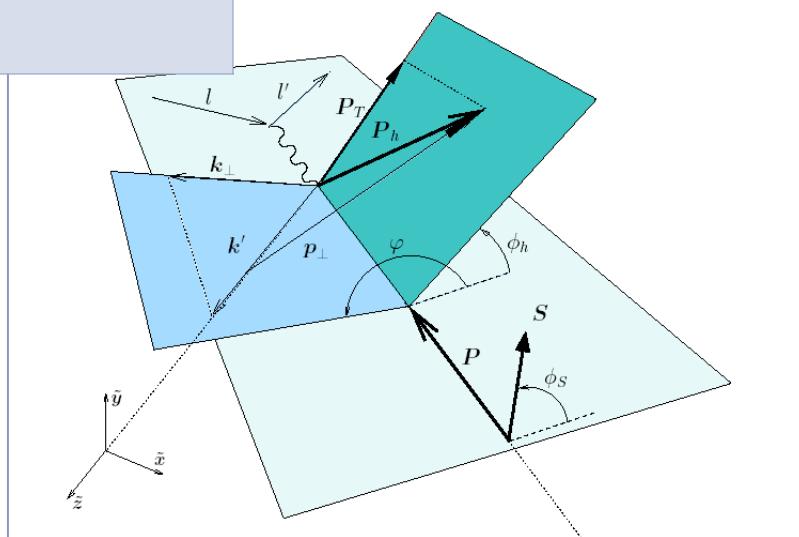
$\hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q} \hat{M}_{\lambda'_\ell, \lambda'_q; \lambda'_\ell, \lambda'_q}^* \otimes \hat{D}_{\lambda_q, \lambda'_q}^h(z, \mathbf{p}_\perp)$
TMD-FF

All three fundamental blocks contain phases.

The most general cross-section is obtained when all of them are kept into account.

TMD's in SIDIS

Trento
conventions



SIDIS in parton model
with intrinsic k_\perp

Factorization holds at large Q^2 ,
and

$$P_T \approx k_\perp \approx \Lambda_{QCD}$$

(Ji, Ma, Yuan)

$$x = x_B$$

$$z = z_h$$

$$P_T = z k_\perp + p_\perp$$

$$\mathcal{O}(k_\perp/Q)$$

Unpolarized Cross Section

$$d\sigma^{lp \rightarrow lhX} = \sum_q f_q(x, \mathbf{k}_\perp; Q^2) \otimes d\hat{\sigma}^{lq \rightarrow lq}(y, \mathbf{k}_\perp; Q^2) \otimes D_q^h(z, \mathbf{p}_\perp; Q^2)$$

$$d\hat{\sigma}^{\ell q \rightarrow \ell q} \propto \hat{s}^2 + \hat{u}^2 = \frac{Q^4}{y^2} \left[1 + (1-y)^2 - 4 \frac{k_\perp}{Q} (2-y) \sqrt{1-y} \cos \varphi \right]$$

$$\mathcal{O}(k_\perp/Q)$$

TMD's in SIDIS

$$\ell = E(1, \sin \theta, 0, \cos \theta) = (E, \ell)$$

$$\ell' = \ell - q$$

$$q = \frac{1}{2} \left(W - \frac{Q^2}{W}, 0, 0, W + \frac{Q^2}{W} \right)$$

$$P = P_0(1, 0, 0, -1),$$

$$k = \left(xP_0 + \frac{k_\perp^2}{4xP_0}, k_\perp, -xP_0 + \frac{k_\perp^2}{4xP_0} \right)$$

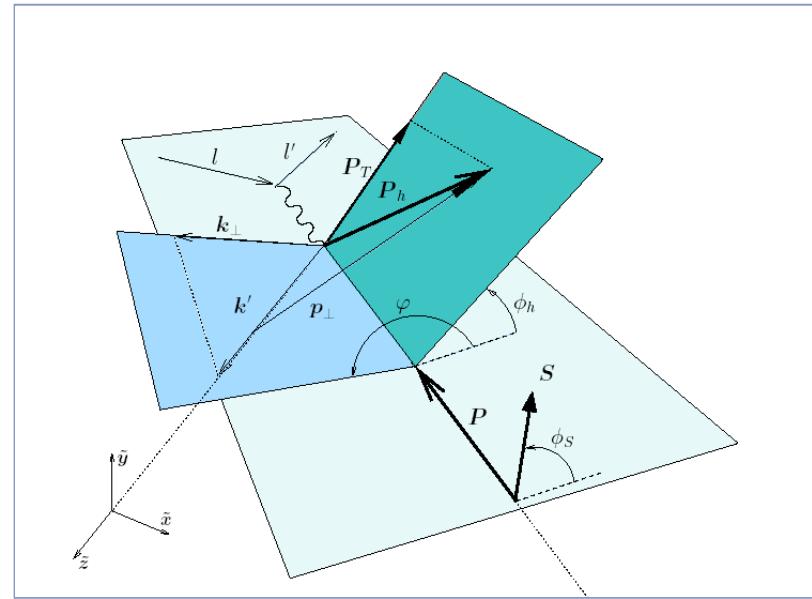
$$\hat{s} = xs - 2\ell \cdot k_\perp - k_\perp^2 \frac{x_B}{x} \left(1 - \frac{x_B s}{Q^2} \right)$$

$$\hat{t} = -Q^2$$

$$\hat{u} = -x \left(s - \frac{Q^2}{x_B} \right) + 2\ell \cdot k_\perp - k_\perp^2 \frac{x_B^2 s}{x Q^2}$$

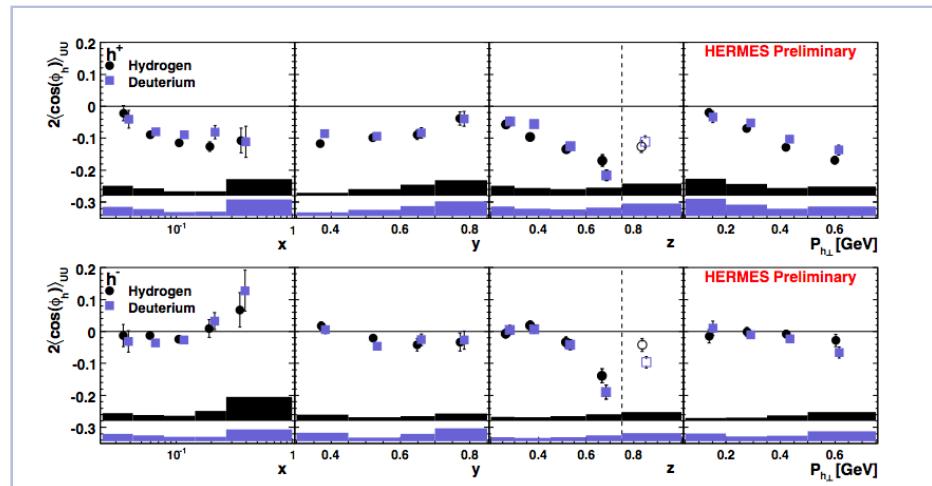
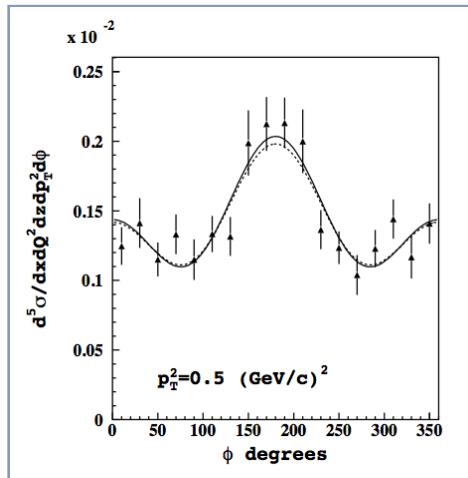
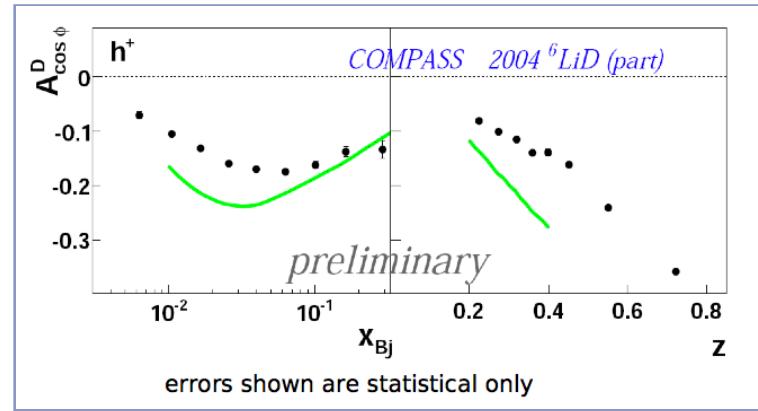
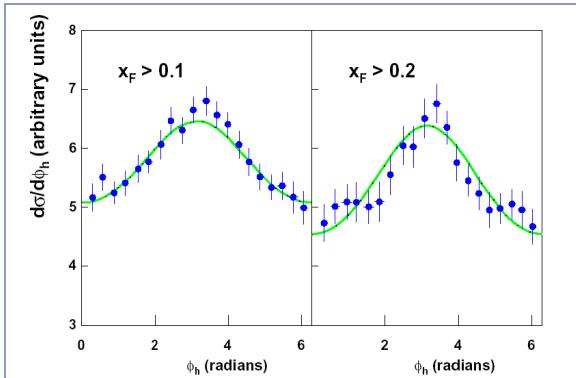
$$x = \frac{1}{2} x_B \left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}} \right)$$

$$\begin{aligned} \hat{s}^2 &= \frac{Q^4}{y^2} \left[1 - 4 \frac{k_\perp}{Q} \sqrt{1-y} \cos \varphi \right] + \mathcal{O} \left(\frac{k_\perp^2}{Q^2} \right) \\ \hat{u}^2 &= \frac{Q^4}{y^2} (1-y)^2 \left[1 - 4 \frac{k_\perp}{Q} \frac{\cos \varphi}{\sqrt{1-y}} \right] + \mathcal{O} \left(\frac{k_\perp^2}{Q^2} \right) \end{aligned}$$



TMD in unpolarized SIDIS \rightarrow Cahn Effect

Azimuthal dependence induced by quark intrinsic motion



Parametrization of the Unpolarized TMD

Assume a simple, factorized form for the TMD distribution and fragmentation functions, with a gaussian dependence on the intrinsic transverse momentum

$$f_q(x, k_\perp) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-k_\perp^2 / \langle k_\perp^2 \rangle}$$

x and z
dependence
from world data
fits

Normalized Gaussian
(one free parameter)

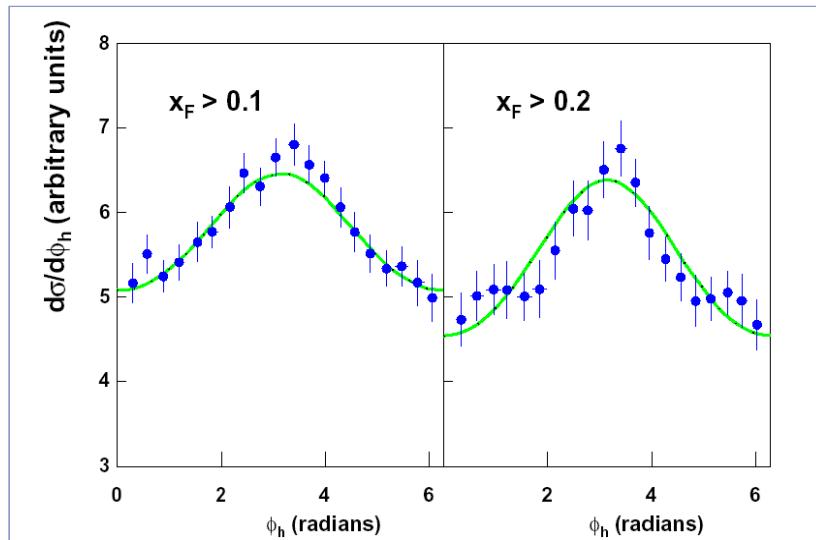
$$D_q^h(z, p_\perp) = D_q^h(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-p_\perp^2 / \langle p_\perp^2 \rangle}$$

TMD in unpolarized SIDIS → Cahn Effect

Determine the value of the two free parameters by fitting experimental data

$$\langle k_\perp^2 \rangle = 0.28 \text{ (GeV)}^2 \quad \langle p_\perp^2 \rangle = 0.25 \text{ (GeV)}^2 \quad (\text{Gaussian widths})$$

M.Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, Phys. Rev. D71:074006,2005.

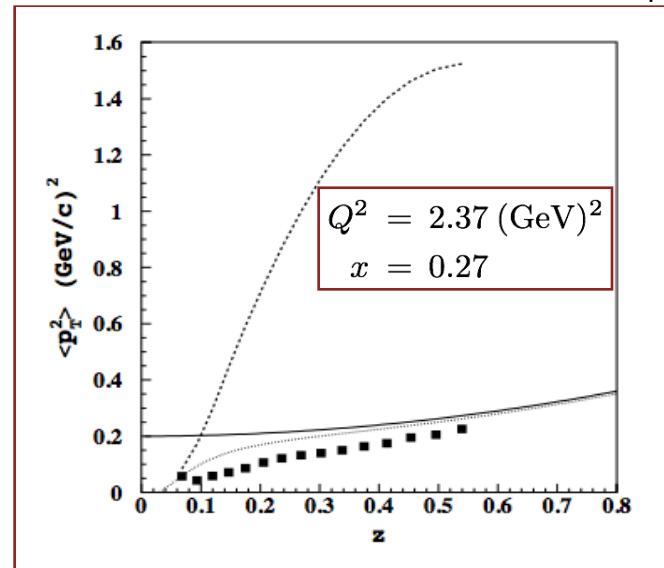


A $\cos\phi$ dependence is also generated by Boer-Mulders⊗Collins term, via a kinematical effect in $d\Delta\hat{\sigma}$, not included in this fit.

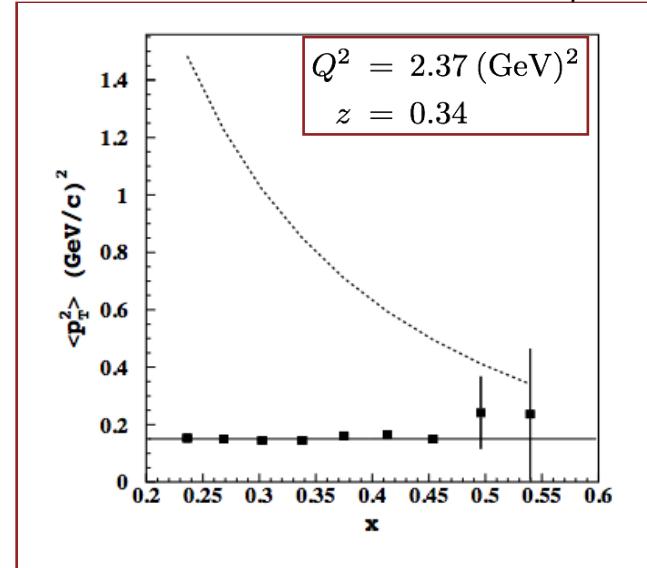
At $\mathcal{O}(k_\perp^2/Q^2)$ further dependence on $\cos(2\phi)$ is generated

P_T dependence of data in agreement with a Gaussian k_\perp dependence of unpolarized TMDs

CLAS data, arXiv:0809.1153 [hep-ex]



CLAS data, arXiv:0809.1153 [hep-ex]



Hint of a z -dependence
at small z values

No hint of x dependence
in the explored region

solid line → { Gaussian TMD's with $\langle k_\perp^2 \rangle = 0.25$ $\langle p_\perp^2 \rangle = 0.20$
 $\langle P_T^2 \rangle = z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle$ $\mathcal{O}(k_\perp/Q)$

Polarized SIDIS cross section

General SIDIS cross section

$$\begin{aligned}
& \frac{d\sigma^{\ell(S_\ell)p(S) \rightarrow \ell' h X}}{dx_B dQ^2 dz_h d^2\mathbf{p}_\perp d\phi_S} = \sum_{q_i} \int d^2\mathbf{k}_\perp \tilde{J} \frac{4e_q^2 e^4}{y^2} \left\{ \right. \\
& \frac{1}{2} f_{q_i/p} [1 + (1-y)^2] D_{q_f}^h + \frac{1}{2} \Delta f_{s_y/p}^{q_i} \frac{P_T}{p_\perp} (1-y) \left[\cos(\phi_h + \phi_\perp) - z_h \frac{k_\perp}{P_T} \cos 2\phi_\perp \right] \Delta^N D_{q_f}^h \\
& - 2(2-y) \sqrt{1-y} \frac{k_\perp}{Q} \left[f_{q_i/p} \cos \phi_\perp D_{q_f}^h + \frac{1}{2} \Delta f_{s_y/p}^{q_i} \frac{P_T}{p_\perp} \left(\cos \phi_h - z_h \frac{k_\perp}{P_T} \cos \phi_\perp \right) \Delta^N D_{q_f}^h \right] \\
& + \frac{1}{2} S_L \left[-\frac{P_T}{p_\perp} (1-y) \Delta f_{s_x/S_L}^{q_i} \left(\sin(\phi_h + \phi_\perp) - z_h \frac{k_\perp}{P_T} \sin 2\phi_\perp \right) \Delta^N D_{q_f}^h \right. \\
& + 2(2-y) \sqrt{1-y} \frac{k_\perp}{Q} \frac{P_T}{p_\perp} \Delta f_{s_x/S_L}^{q_i} \left(\sin \phi_h - z_h \frac{k_\perp}{P_T} \sin \phi_\perp \right) \Delta^N D_{q_f}^h \\
& + P_z^l \left([1 - (1-y)^2] \Delta f_{s_z/S_L}^{q_i} D_{q_f}^h - 4y \sqrt{1-y} \frac{k_\perp}{Q} \Delta f_{s_z/S_L}^{q_i} \cos \phi_\perp D_{q_f}^h \right) \\
& + \frac{1}{2} S_T \left[-\frac{1}{2} [1 + (1-y)^2] \Delta^N f_{q_i/S_T} \sin(\phi_S - \phi_\perp) D_{q_f}^h \right. \\
& + P_z^l [1 - (1-y)^2] \Delta f_{s_z/S_T}^{q_i} \cos(\phi_S - \phi_\perp) D_{q_f}^h - P_z^l 4y \sqrt{1-y} \frac{k_\perp}{Q} \Delta f_{s_z/S_T}^{q_i} \cos(\phi_S - \phi_\perp) \cos \phi_\perp D_{q_f}^h \\
& - \frac{P_T}{p_\perp} (1-y) \Delta^- f_{s_y/S_T}^{q_i} \sin(\phi_S - \phi_\perp) \left(\cos(\phi_h + \phi_\perp) - z_h \frac{k_\perp}{P_T} \cos 2\phi_\perp \right) \Delta^N D_{q_f}^h \\
& - \frac{P_T}{p_\perp} (1-y) \Delta f_{s_x/S_T}^{q_i} \cos(\phi_S - \phi_\perp) \left(\sin(\phi_h + \phi_\perp) - z_h \frac{k_\perp}{P_T} \sin 2\phi_\perp \right) \Delta^N D_{q_f}^h \\
& + \frac{P_T}{p_\perp} 2(2-y) \sqrt{1-y} \frac{k_\perp}{Q} \Delta^- f_{s_y/S_T}^{q_i} \sin(\phi_S - \phi_\perp) \left(\cos \phi_h - z_h \frac{k_\perp}{P_T} \cos \phi_\perp \right) \Delta^N D_{q_f}^h \\
& + \frac{P_T}{p_\perp} 2(2-y) \sqrt{1-y} \frac{k_\perp}{Q} \Delta f_{s_x/S_T}^{q_i} \cos(\phi_S - \phi_\perp) \left(\sin \phi_h - z_h \frac{k_\perp}{P_T} \sin \phi_\perp \right) \Delta^N D_{q_f}^h \\
& \left. + 2(2-y) \sqrt{1-y} \frac{k_\perp}{Q} \Delta^N f_{q_i/S_T} \sin(\phi_S - \phi_\perp) \cos \phi_\perp D_{q_f}^h \right\} .
\end{aligned}$$

General SIDIS cross section

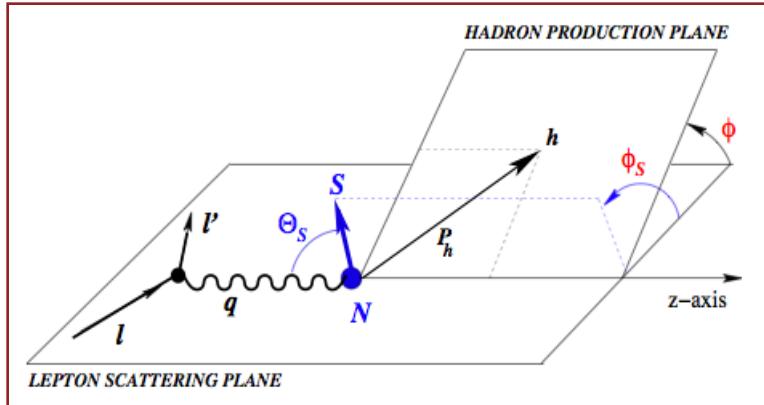
$$\frac{d\sigma^{\ell(S_\ell)p(S) \rightarrow \ell'hX}}{dx_B dQ^2 dz_h d^2\mathbf{p}_\perp d\phi_S} =$$

The F structure functions contain all the TMD's

$$\begin{aligned}
 & \frac{4e^2}{y^2} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y} \cos \phi_h F_{UU}^{\cos \phi_h} + (1-y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right\} \\
 & + S_L \left[(1-y) \sin 2\phi_h F_{UL}^{\sin 2\phi_h} + (2-y)\sqrt{1-y} \sin \phi_h F_{UL}^{\sin \phi_h} \right] \\
 & + S_L P_z^l \left[\frac{1 - (1-y)^2}{2} F_{LL} + y\sqrt{1-y} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
 & + S_T \left[\frac{1 + (1-y)^2}{2} \sin(\phi_h - \phi_S) F_{UT}^{\sin(\phi_h - \phi_S)} \right. \\
 & \quad + (1-y) \left(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right) \\
 & \quad \left. + (2-y)\sqrt{1-y} \left(\sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right) \right] \\
 & + S_T P_z^l \left[\frac{1 - (1-y)^2}{2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 & \quad \left. + y\sqrt{1-y} \left(\cos \phi_S F_{LT}^{\cos \phi_S} + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right) \right]
 \end{aligned}$$

- ❖ Studying Sivers, Collins and other mechanisms is complicated by the fact that all these effects mix and overlap in the polarized SIDIS cross section and azimuthal asymmetries !
- ❖ Way out : appropriate azimuthal moments of the asymmetries !

TMDs in polarized SIDIS



$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a$$

$$F_{LT}^{\cos(\phi-\phi_s)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a$$

$$F_{UT}^{\sin(\phi-\phi_s)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even
TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a}$$

$$F_{UT}^{\sin(\phi+\phi_s)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

$$F_{UT}^{\sin(3\phi-\phi_s)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

chiral-odd
TMDs

$$\frac{1}{Q} \cos \phi F_{UU}^{\cos \phi} \sim f_1^q \otimes D_1^q \otimes d\hat{\sigma} + (h_1^{q\perp} \otimes H_1^{q\perp} \otimes d\Delta\hat{\sigma})$$

Cahn kinematical
effects

TMD's in SIDIS one by one ...

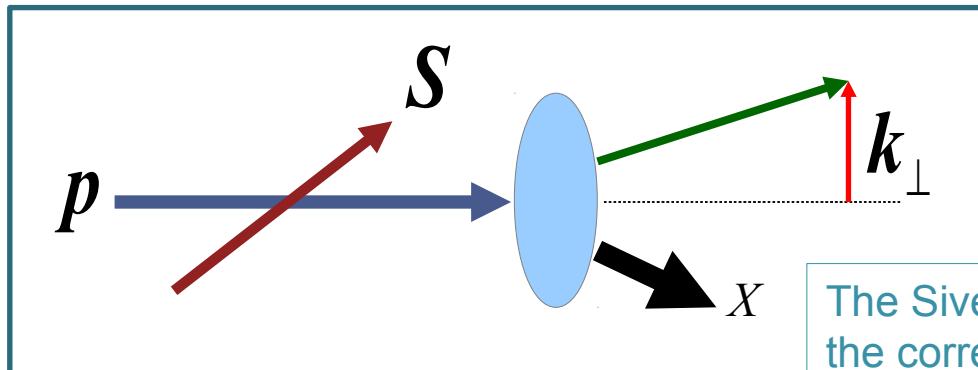
The Sivers Distribution Function

$$f_{q/p,S}(x, k_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

$$= f_{q/p}(x, k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp) S \cdot (\hat{p} \times \hat{k}_\perp)$$

The Sivers function is T-odd

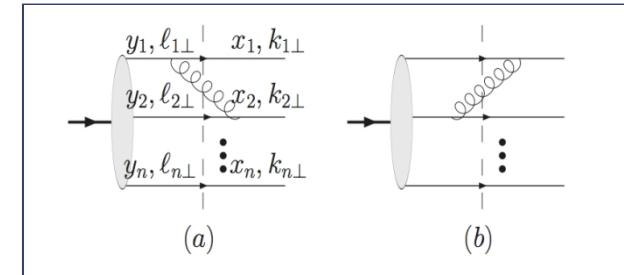
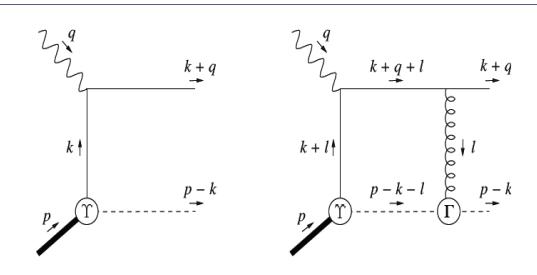


The Sivers function embeds the correlation between the proton spin and the quark transverse momentum

The Sivers Distribution Function

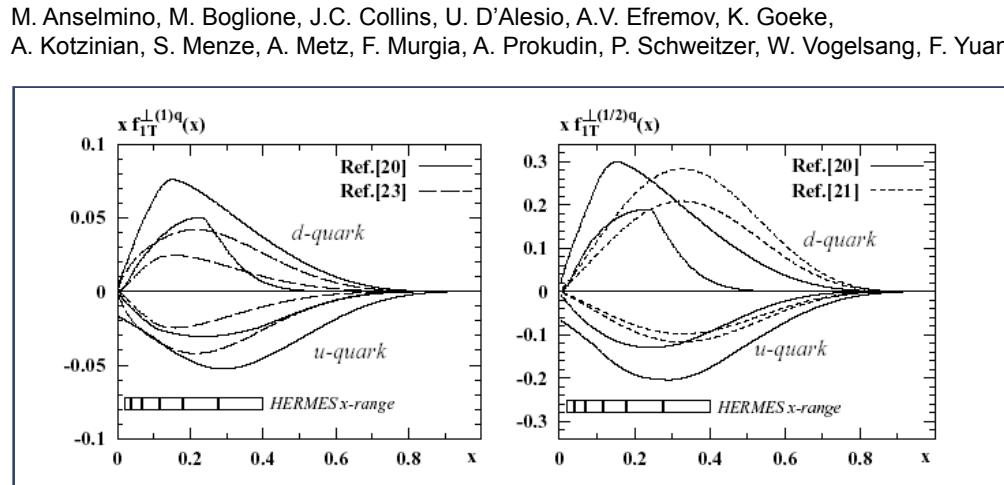
Models →

Bacchetta, Gamberg, Goldstain, Mukherjee, Metz, Amrath, Shaefer, Yang, Brodsky, Schmidt, Hwang, Scopetta, Courtoy, Frattini, Vento, Radici, Pasquini, Yuan ...



Determining
the Sivers
function

Fits →



$$f_{1T}^{\perp(1)q} = \int d^2 k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^{\perp q}(x, k_\perp)$$

$$f_{1T}^{\perp(1/2)q}(x) = \int d^2 k_\perp \frac{k_\perp}{M} f_{1T}^{\perp q}(x, k_\perp)$$

The first and 1/2-transverse moments of the **Sivers quark distribution functions**.

The fits were constrained mainly (or solely) by the preliminary HERMES data in the indicated x-range.

The curves indicate the 1- σ regions of the various parameterizations.

Sivers effect in SIDIS single spin asymmetry

$$A_{\text{UT}} = \frac{d\sigma^{\ell p^\uparrow \rightarrow \ell' h X} - d\sigma^{\ell p^\downarrow \rightarrow \ell' h X}}{d\sigma^{\ell p^\uparrow \rightarrow \ell' h X} + d\sigma^{\ell p^\downarrow \rightarrow \ell' h X}}$$

$$A_{\text{UT}}^{\sin(\varphi_h - \varphi_s)} = \frac{\sum_q e_q^2 \int d\varphi_s d\varphi_h d^2 \vec{k}_\perp \Delta^N f_{q/p^\uparrow}(x, \vec{k}_\perp) \frac{d\sigma}{dQ^2} D_{h/q}(z, \vec{p}_\perp) \sin(\varphi - \varphi_s) \sin(\varphi_h - \varphi_s)}{\sum_q e_q^2 \int d\varphi_s d\varphi_h d^2 \vec{k}_\perp f_{q/p}(x, \vec{k}_\perp) \frac{d\sigma}{dQ^2} D_{h/q}(z, \vec{p}_\perp)}$$

Two soft mechanisms at work in SIDIS:

- **Distribution fn.** → probability to find quark q carrying a light-cone fraction x of the parent proton momentum, an intrinsic transverse momentum \vec{k}_\perp , at scale Q2.
- **Fragmentation fn.** → describes the hadronization of the struck quark into the final, detected hadron.

Both mechanisms play an important role in determining total cross section and spin asymmetries

Parametrization of the Sivers distribution function

$$h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M} e^{-k_{\perp}^2/M^2}$$

$$\Delta^N f_{q/p}^{\uparrow}(x, k_{\perp}) = 2 N_q(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$$

$$N_q(x) = N_q x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

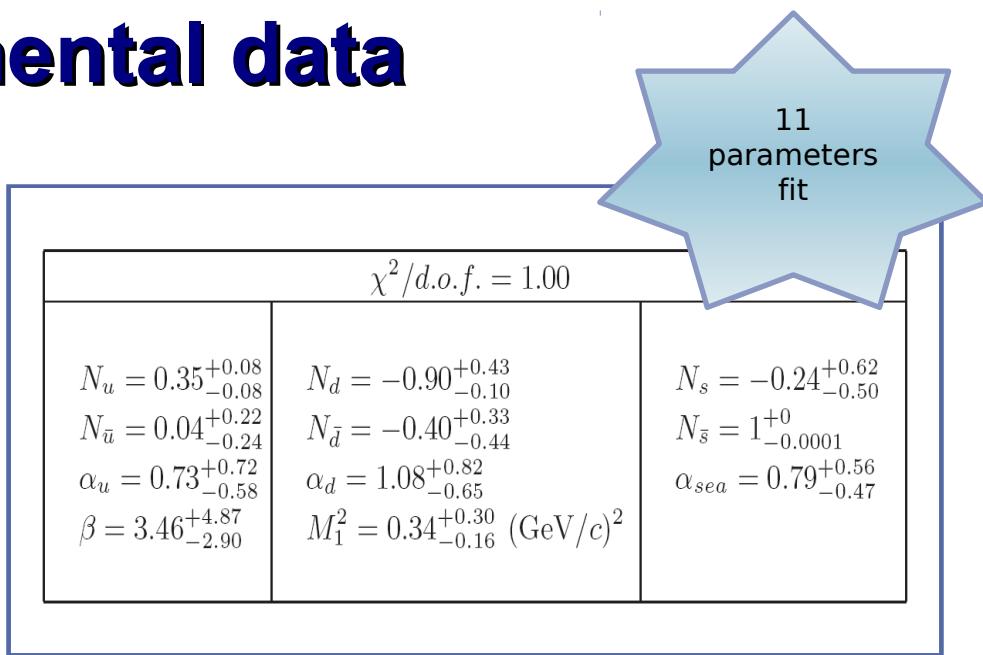
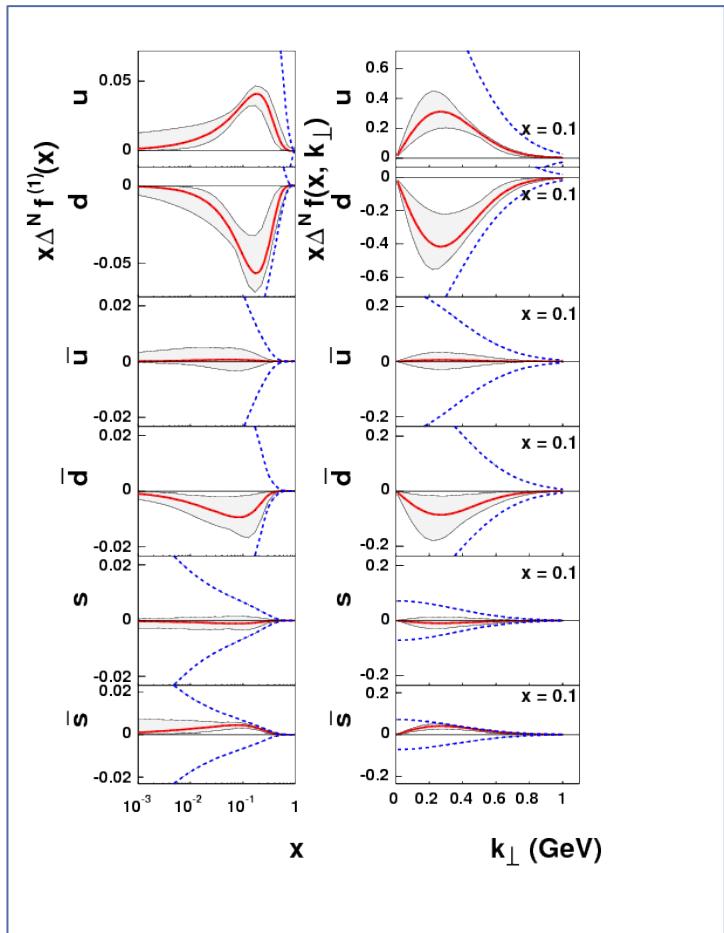
with $-1 \leq N_q \leq 1$

$$f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

$|N_q(x)|$ and $|h(k_{\perp})|$ are smaller than 1 for any x and for any k_{\perp}

Positivity Bound $\frac{|\Delta^N f_{q/p}^{\uparrow}(x, k_{\perp})|}{2 f_{q/p}(x, k_{\perp})} \leq 1$ automatically satisfied

Extraction of Sivers Function from SIDIS experimental data



✓ Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0$ $f_{1T}^{\perp u} < 0$

- $\Delta^N f_{d/p^\uparrow} < 0$ $f_{1T}^{\perp d} > 0$

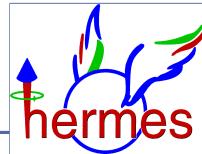
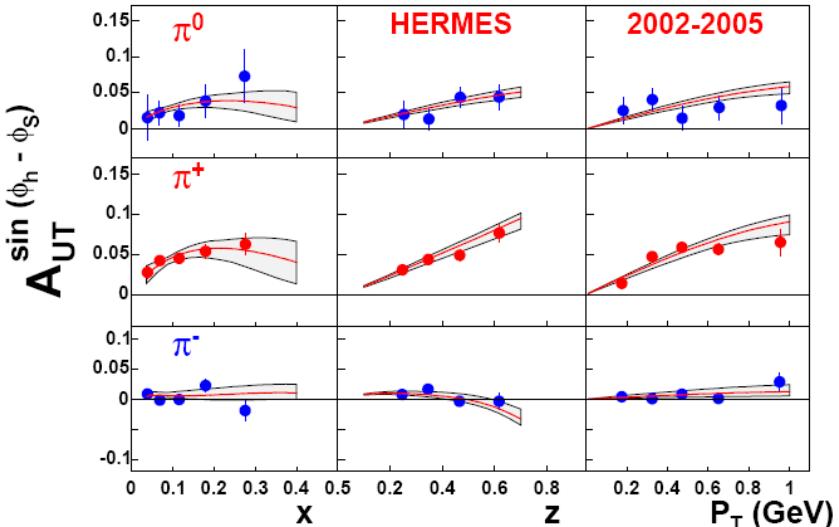
✓ Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0$ $f_{1T}^{\perp \bar{s}} < 0$

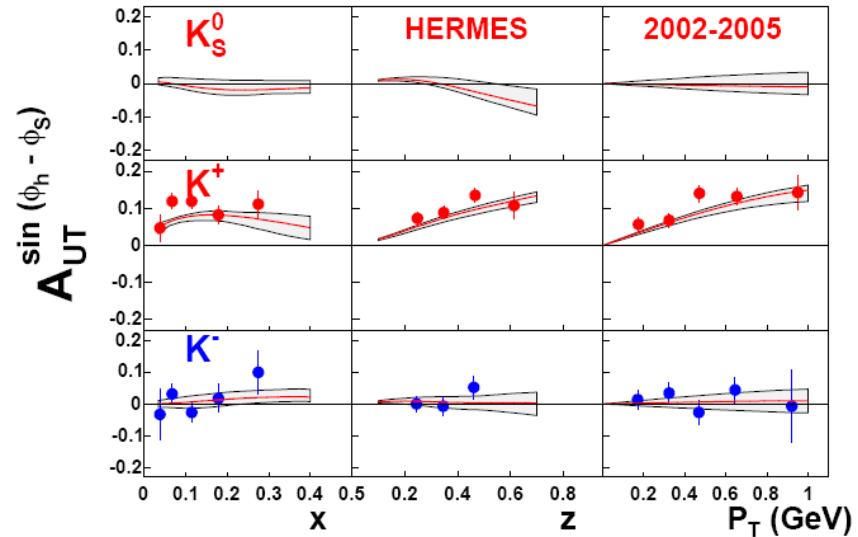
M.Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk,
Eur.Phys.J.A39:89-100,2009.

Extraction of Sivers Function from SIDIS experimental data

π^+ production at HERMES



K^+ production at HERMES



M.Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk,
Eur.Phys.J.A39:89-100,2009.

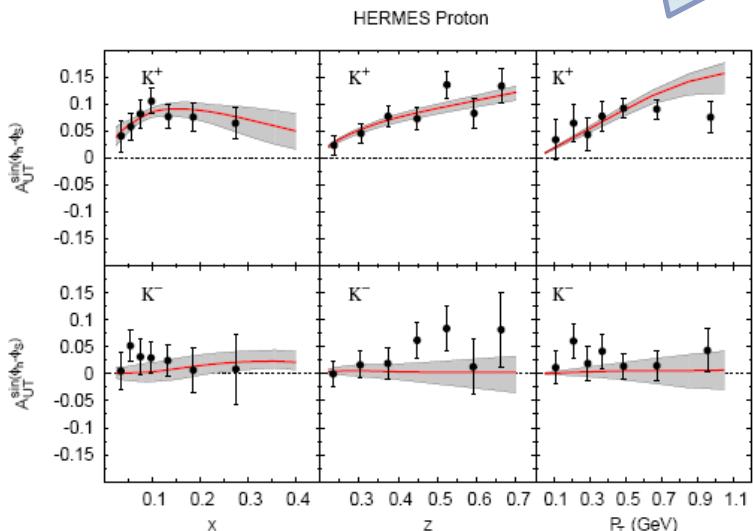
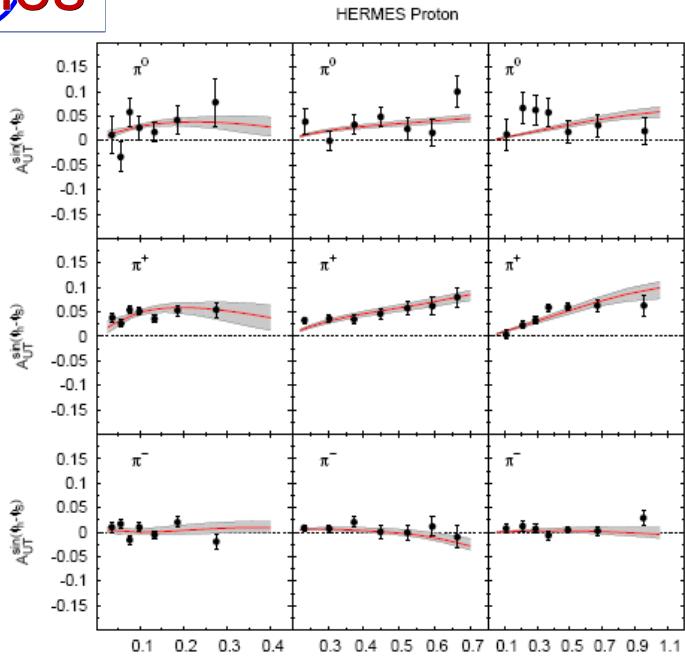
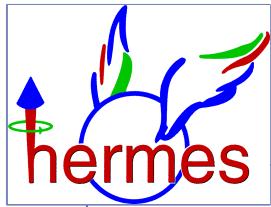
$$D_d^{K_S^0} = D_{\bar{d}}^{K_S^0} = \frac{1}{2} [D_u^{K^+} + D_{sea}^{K^+}]$$

$$D_{\bar{s}}^{K_S^0} = D_s^{K_S^0} = \frac{1}{2} [D_{\bar{s}}^{K^+} + D_{sea}^{K^+}]$$

$$D_u^{K_S^0} = D_{\bar{u}}^{K_S^0} = D_{sea}^{K^+} \equiv D_d^{K^+} = D_{\bar{u}}^{K^+} = D_s^{K^+} = D_{\bar{d}}^{K^+}$$

K^0 fragmentation functions

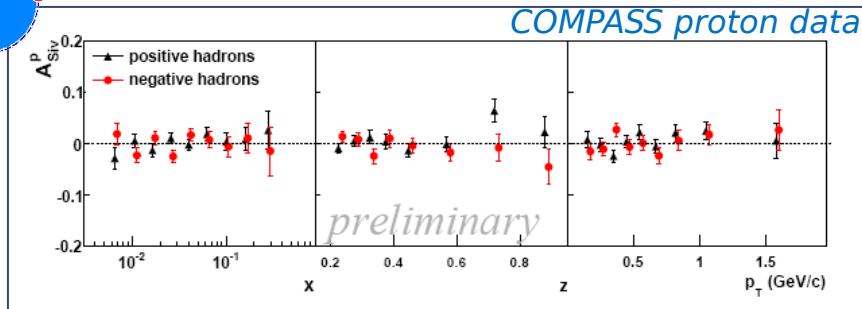
Extraction of Sivers Function from SIDIS experimental data



New
data
(2009)
old
fit !

HERMES Collaboration,
Phys.Rev.Lett.103:152002,2009.

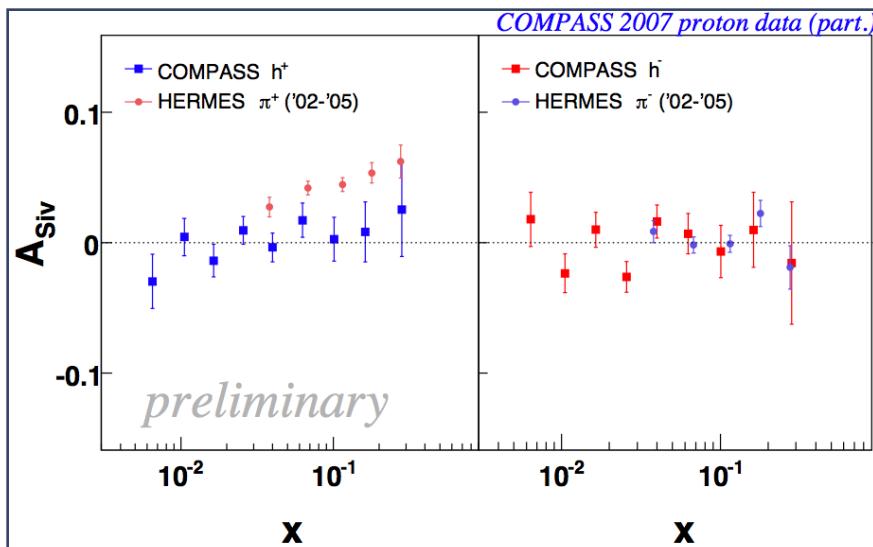
The Sivers Distribution Function



S. Levorato for the COMPASS Collaboration,
Transversity 2008

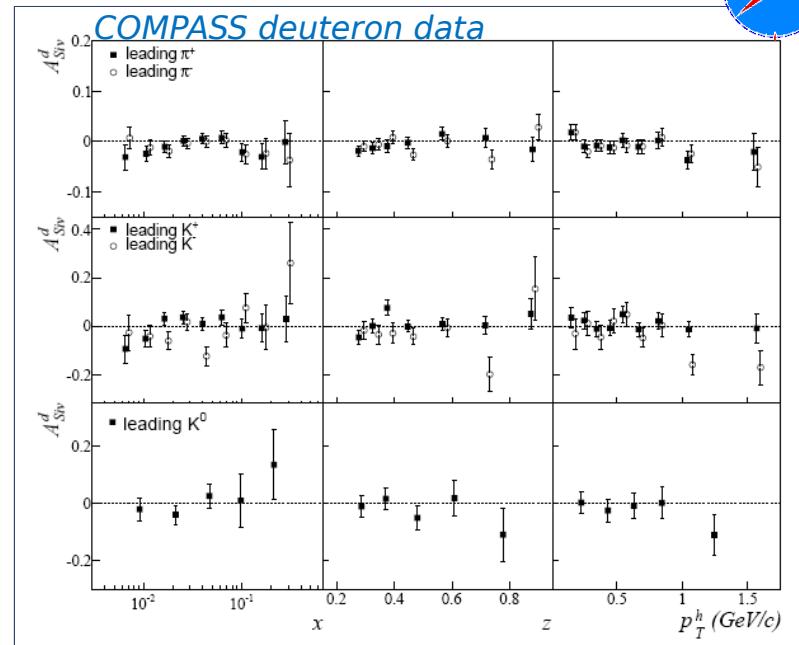


COMPASS vs HERMES, problems?



TMDs - Lecture 3 Courtesy of F.
Bradamante

M. Boglione - HUGS 2012



COMPASS Collaboration, Phys. Lett. B673: 127-135, 2009

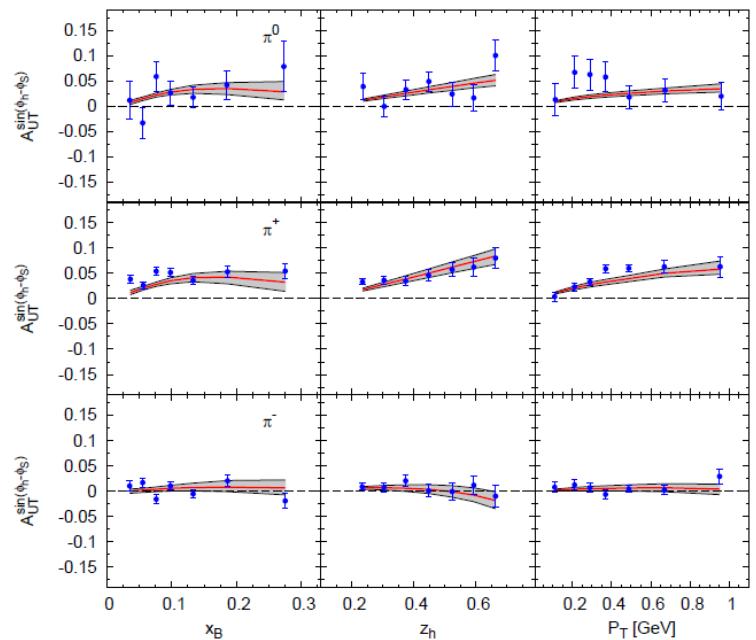
*Until recently,
COMPASS found
Sivers = 0.
Only HERMES data
revealed Sivers effect.
New COMPASS Sivers
data provide
further check!*

The Sivers Distribution Function

New data, new fit



HERMES PROTON - DGLAP

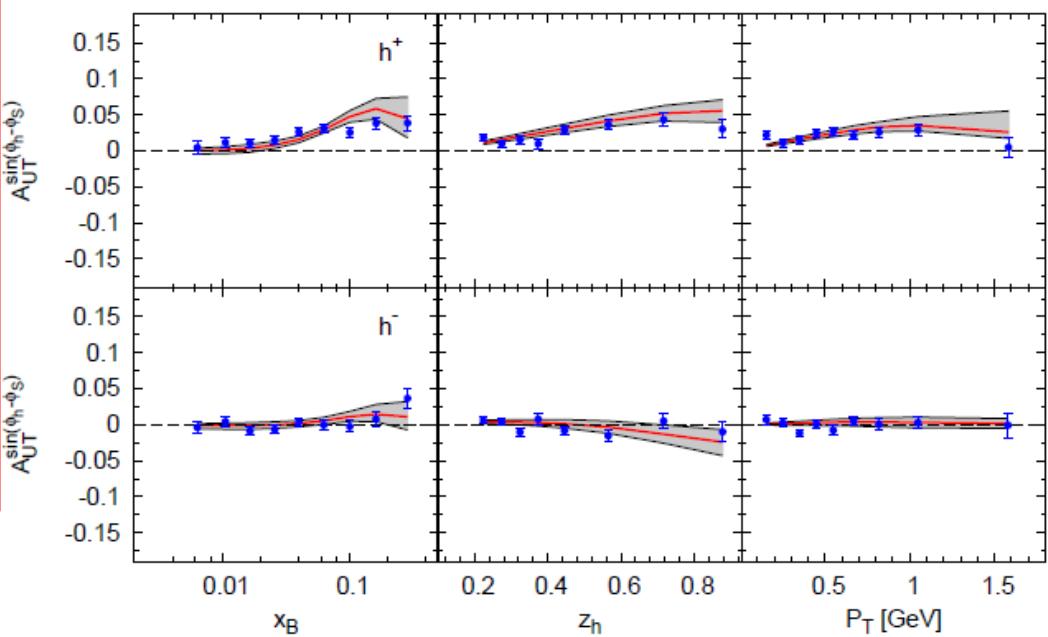


A. Airapetian et al, HERMES Collaboration,
Phys. Rev. Lett. 103, 152002 (2009)

M. Anselmino, M. Boglione, S. Melis,
ArXiv:1204.1239
Accepted for publication in Phys. Rev. D.



COMPASS PROTON - DGLAP

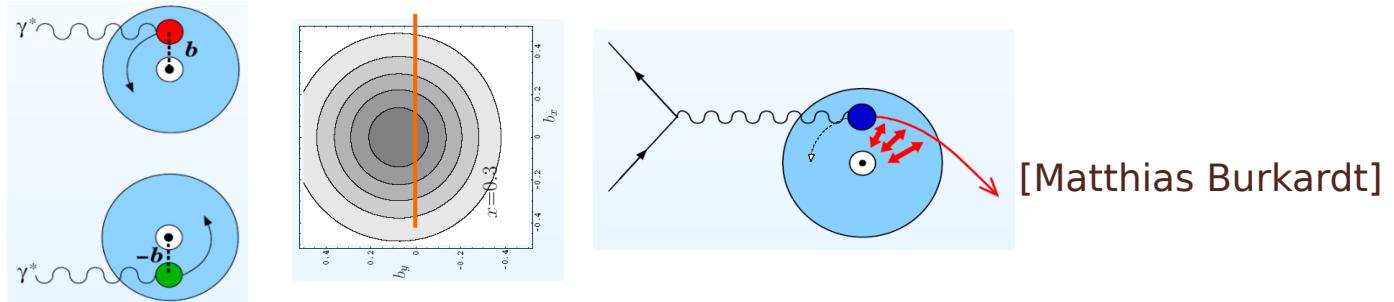


F. Bradamante et al, COMPASS Collaboration, arXiv:1111.0869

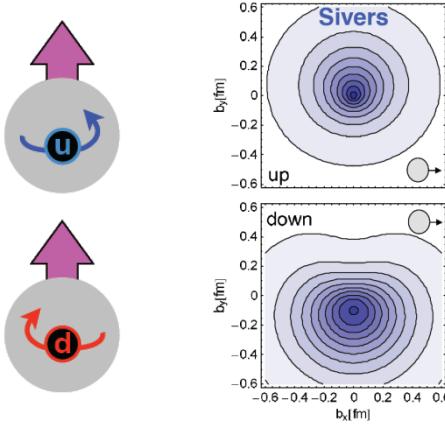
The Sivers Distribution Function

Spin-orbit correlations

A non-zero Sivers function requires non-zero orbital angular momentum !



$$u_X(x, \mathbf{b}_\perp)$$



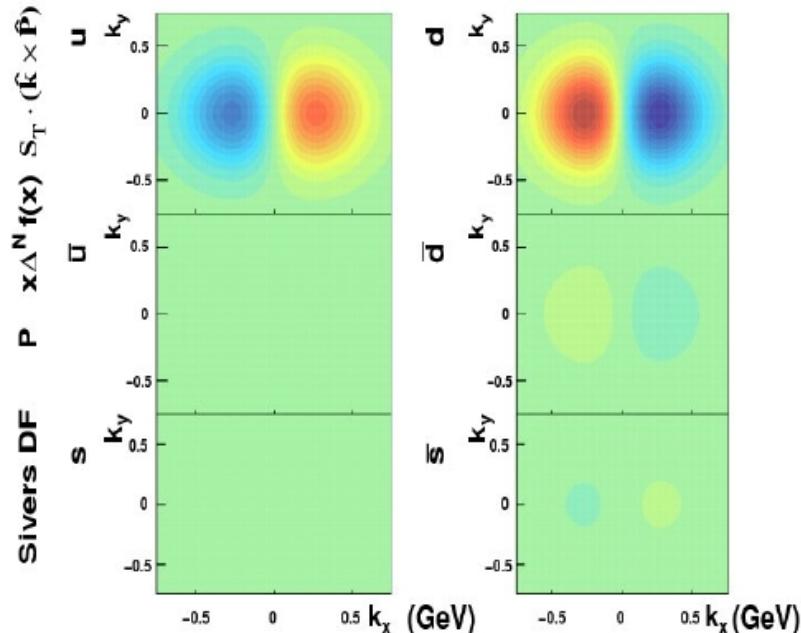
$$L_u > 0$$

Lattice [P. Haegler et al.]

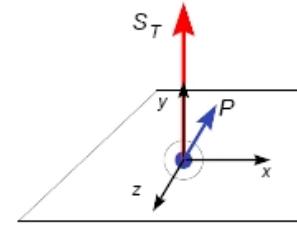
$$L_d < 0$$

3D view of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



Courtesy of Alexei Prokudin



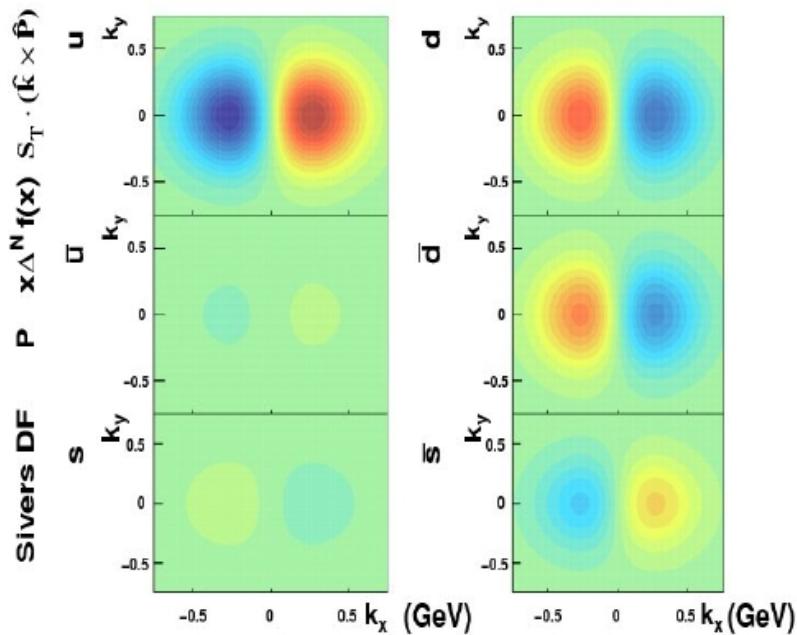
This is the three dimensional view of the proton as “seen” by the virtual photon.

Red color – more quarks. Blue color – less quarks. Distributions of quarks are not symmetrical and shifted due to final state interactions.

$$x = 0.2$$

3D view of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



Courtesy of Alexei Prokudin

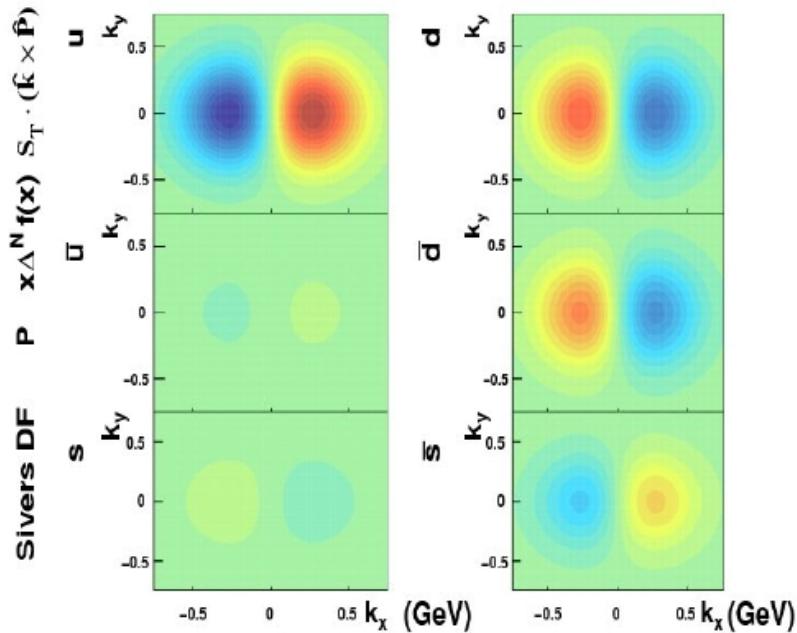
Sivers functions for *u*, *d* and *sea* quarks are extracted from **HERMES** and **COMPASS** data.

Red color – more quarks. Blue Color – less quarks. Sivers functions is a left – right asymmetry of quark distribution.
 $x = 0.01$

More information on sea quarks.
Please note that Sivers function is a dipole correlation of the type $S_T \cdot (\mathbf{k} \times \hat{\mathbf{P}})$ one example of those correlations that we almost discarded at the beginning of the seminar using symmetry reasoning.

3D view of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



Courtesy of Alexei Prokudin

Sivers functions for *u*, *d* and *sea* quarks are extracted from **HERMES** and **COMPASS** data.

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More information on sea quarks. Please note that Sivers function is a dipole correlation of the type $S_T \cdot (\mathbf{k} \times \hat{\mathbf{P}})$ one example of those correlations that we almost discarded at the beginning of the seminar using symmetry reasoning.

Summary

- We have seen how TMD distribution functions are parameterized and extracted from experimental data
- The SIDIS unpolarized cross section is already very interesting as it clearly shows evident signs of the presence of partonic intrinsic transverse momentum.
- The ideal processes to study TMDs are polarized cross sections and spin asymmetries, but they need a careful Fourier analysis to disentangle a large number of terms, each proportional to a different TMD or combination of TMDs.
- In this framework, the valence quark contribution to the Sivers function, which represents the density number of unpolarized quarks inside a transversely polarized proton, is extracted from data on the SIDIS Sivers azimuthal moment and is, at the moment, known fairly well.