Transverse Momentum of partons

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Lecture 4

We will be talking about ...

- Deep Inelastic scattering
- Parton model
- Transverse momentum of partons

Lecture 1

Lecture 2

- 3D kinematics (i.e. kinematics with partonic transverse momentum)
- Integrated distribution and fragmentation functions (transversity and all that ...)

- Transverse Momentum Dependent distribution and fragmentation functions
- Unpolarized SIDIS cross section
- Phenomenology of TMD's: extracting the Sivers distribution function from SIDIS experimental data.

Lecture 3

- Simultaneous extraction of transversity and Collins functions from SIDIS and e^+e^- scattering experimental data.

  Overview of Drell-Yan processes

Lecture 4

- Scale dependence and TMD evolution scheme

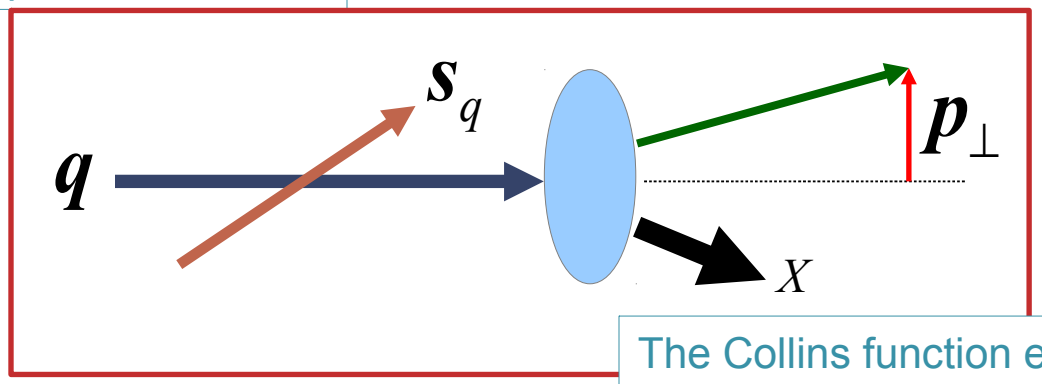
The Collins Fragmentation Function

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h} H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron

The Collins function is chirally odd

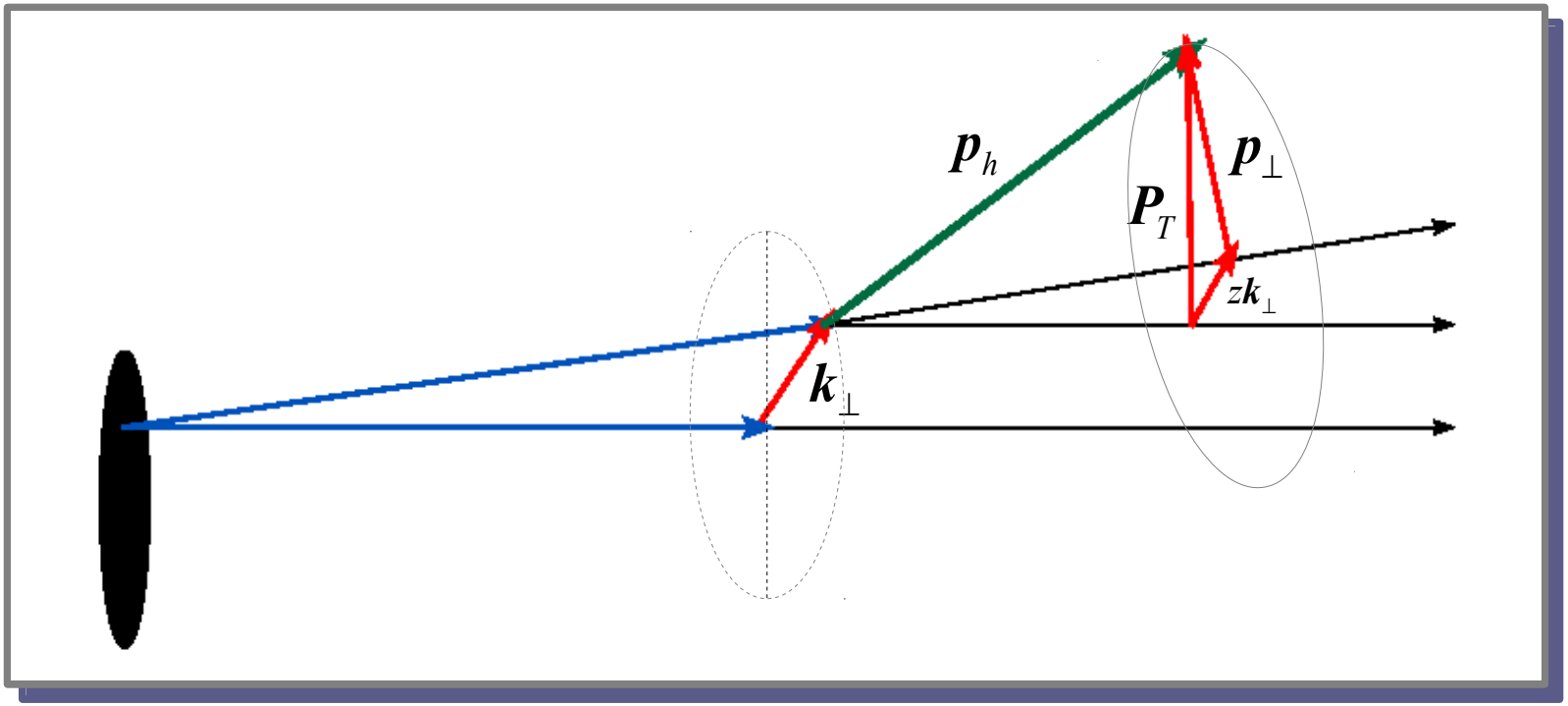


The Collins function embeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron

The Fragmentation Process

Little portrait of φ_q^h

φ_q^h is the azimuthal angle of the detected hadron h around the direction of the parent fragmenting quark q.
Technically it is the azimuthal angle of $p_\perp$ in the helicity frame of q.



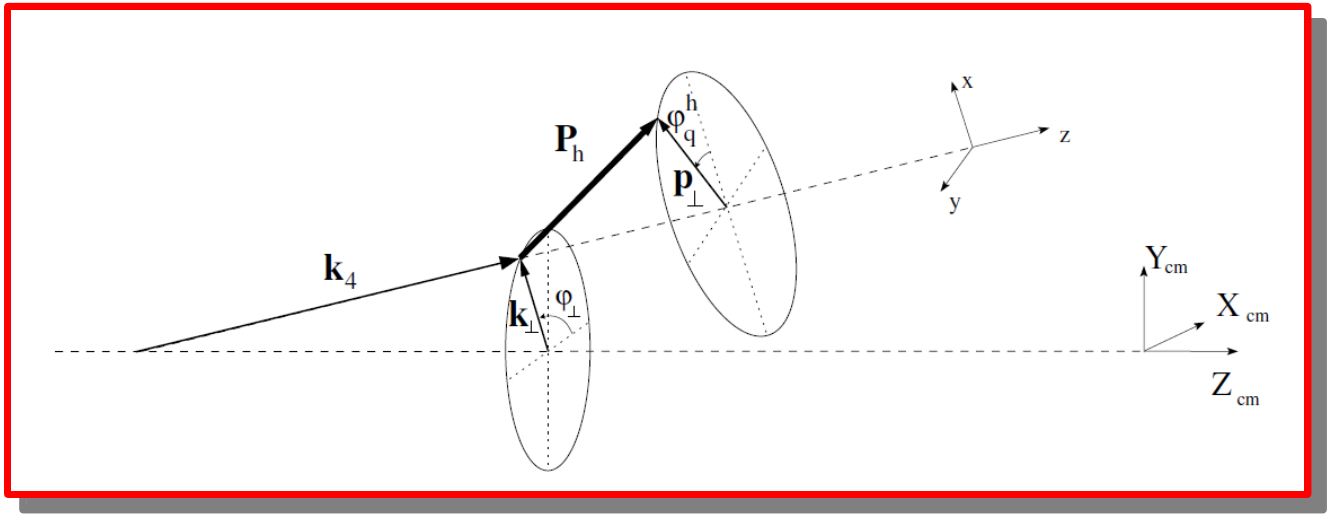
The Fragmentation Process

Little portrait of φ_q^h

φ_q^h is the azimuthal angle of the detected hadron h around the direction of the parent fragmenting quark q.
Technically it is the azimuthal angle of $p_\perp$ in the helicity frame of q.

$$\cos\varphi_q^h = \hat{p}_\perp \cdot \hat{x}$$
$$\sin\varphi_q^h = \hat{p}_\perp \cdot \hat{y}$$

$$\cos\varphi_q^h = \frac{P_T}{p_\perp}\left[\cos(\phi_h - \phi_\perp) - z_h \frac{k_\perp}{P_T}\right]$$
$$\sin\varphi_q^h = \frac{P_T}{p_\perp}\sin(\phi_h - \phi_\perp)$$



Transversity and Collins Functions

- The Collins function is chirally odd, so it must appear in physical processes convoluted with another chirally odd function.
- In SIDIS it appears , for instance, in combination with either transversity or
- Boer-Mulders distribution functions.
- The transversity distribution function contains basic information on the spin structure of the nucleons.
- Being related to the expectation value of a chiral odd operator, transversity appears in physical processes which require a quark helicity flip; therefore it cannot be measured in usual DIS.
- Transversity function can be measured in single or double polarized Drell-Yan → planned experiments in polarized pp at PAX
- At present, the only chance of gathering information on the transversity distribution function is SIDIS, where it appears associated to the Collins fragmentation function.
- DOUBLE PUZZLE: we cannot determine the transversity distribution function if we do not know the Collins fragmentation function.

Transversity and Collins Functions

OUR STARTEGY :

A global fit of HERMES and COMPASS SIDIS single spin asymmetries combined with BELLE data on $e^+e^- \rightarrow h_1 h_2 X$ will allow us to determine both the transversity distribution and the Collins fragmentation functions of u and d quarks.

Collins Effect in SIDIS

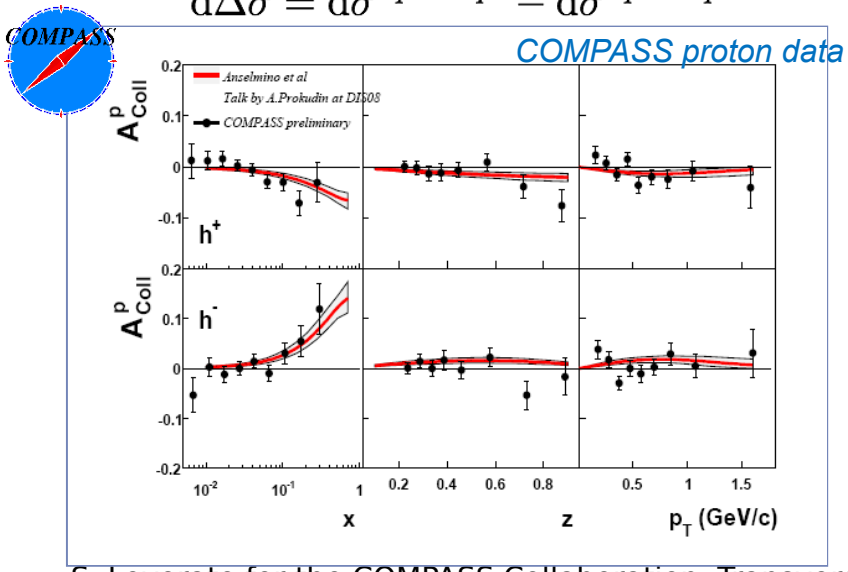
$F_{UT}^{\sin(\phi+\phi_S)}$

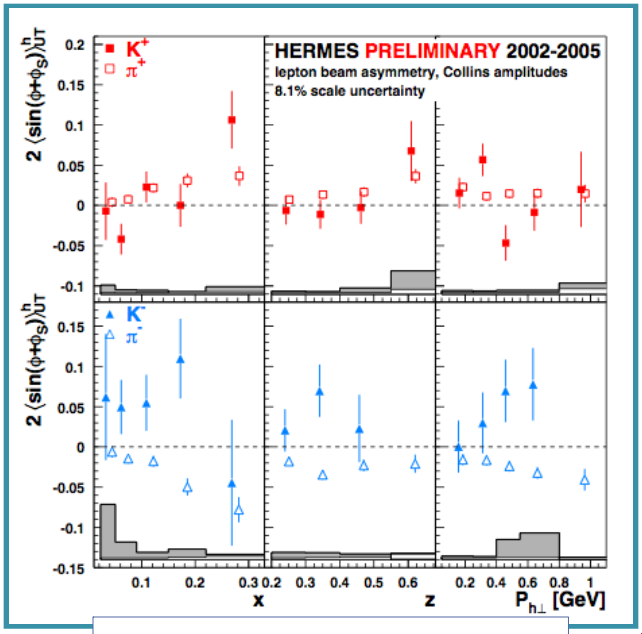
$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \sum_q h_{1q}(x, k_\perp) \otimes d\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

The Collins effect in SIDIS couples to transversity

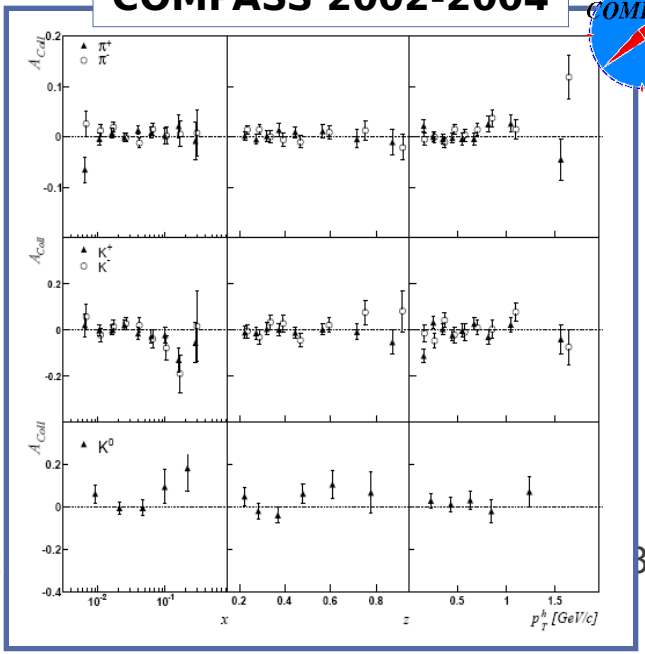
$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2 \frac{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\, \sin(\phi+\phi_S)}{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

$$d\Delta\hat{\sigma} = d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\uparrow} - d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\downarrow}$$





COMPASS 2002-2004

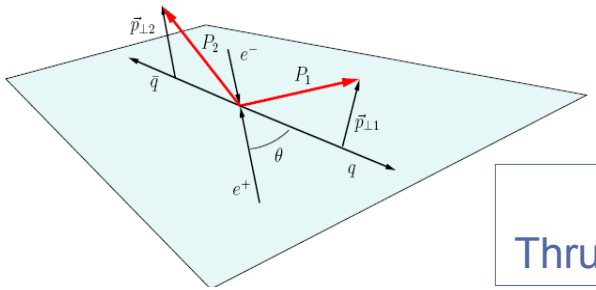


S. Levorato for the COMPASS Collaboration, Transversity 2008

Simultaneous determination of Transversity and Collins functions

We need to determine two convoluted unknown functions

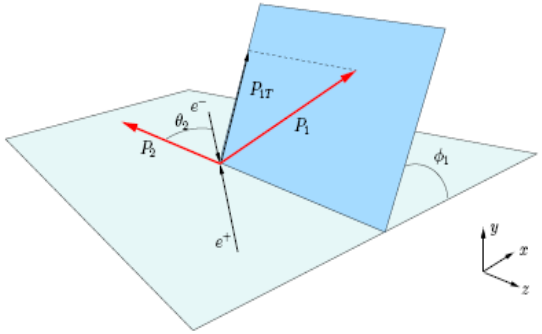
- Fix one of the two functions according to some theoretical model and use SIDIS data to determine the other (see for example Efremov, Goeke, Schweitzer)
- Perform a simultaneous fit of SIDIS and e+e-→h1h2X BELLE data.



BELLE
Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$
$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1+\cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

BELLE
Hadronic plane method



$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1\, z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1+\cos^2\theta_2} \cos(2\,\phi_1) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Parametrization of unpolarized distribution and fragmentation functions

Unpolarized distributions

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

x and z dependence from world data fits

Gaussian $k_\perp$ and $p_\perp$ dependences

Gaussian widths

$\langle k_\perp^2 \rangle = 0.25\ \mathrm{GeV}^2$

$\langle p_\perp^2 \rangle = 0.20\ \mathrm{GeV}^2$

Parametrization of transversity and Collins functions

Transversity Distribution Function

$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x)+\Delta q(x)\right]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

$$\mathcal{N}_q^T(x) = N_q^T\, x^\alpha(1-x)^\beta\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta}$$

$$\mathcal{N}_q^C(z) = N_q^C\, z^\gamma(1-z)^\delta\,\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma\delta^\delta},$$

x and z dependence from world data fits

Gaussian k and p dependences

Collins Fragmentation Function

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\,D_{h/q}(z)\,h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

$$h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M}\,e^{-p_\perp^2/M^2}$$

Free parameters for simultaneous determination of Transversity and Collins functions

- N_u^T (varying between -1 and +1)
- N_d^T (varying between -1 and +1)
- N_{fav}^C (varying between -1 and +1)
- N_{unf}^C (varying between -1 and +1)

9 parameters fit

- M^2
- α
- β
- γ
- δ

Simultaneous determination of Transversity and Collins functions

- As the two different sets of Belle data are based on a different analysis of the same experimental events, they are strongly correlated.

- Therefore, we treat them separately:

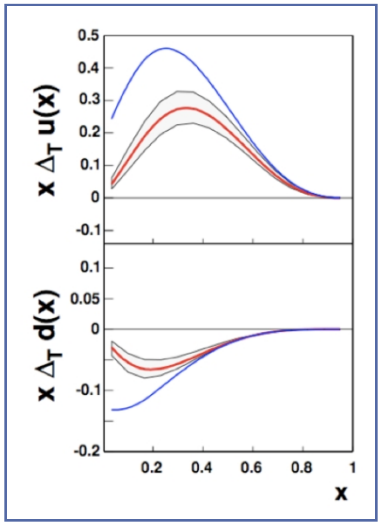
  FIT I → HERMES + COMPASS + BELLE_cos($\varphi_1+\varphi_2$)

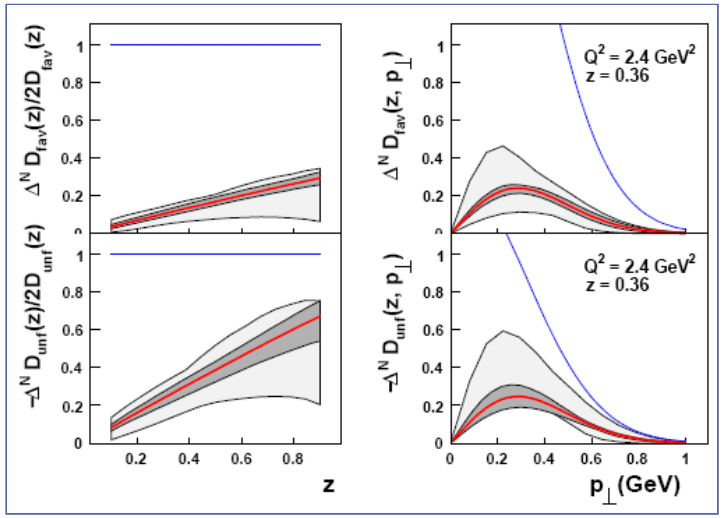
  FIT II → HERMES + COMPASS + BELLE_cos($2\varphi_0$)

- By using 2006-BELLE data, Fit I and Fit II are perfectly compatible within the errors; the asymmetries they generate are hardly distinguishable

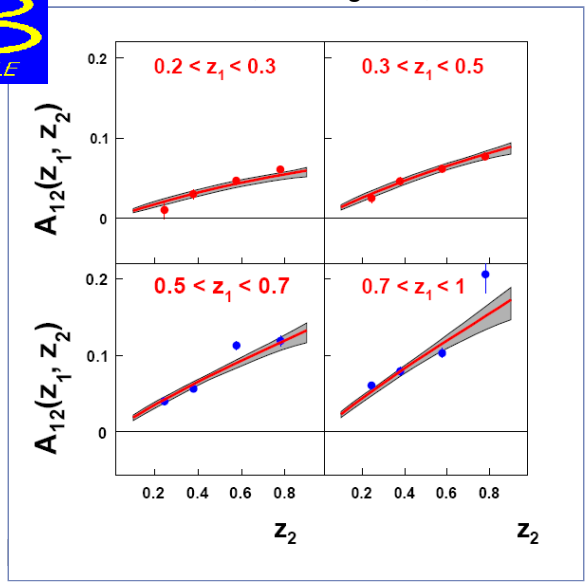
- 2008-BELLE data have fantasticly small error bars, but checking their consistency is more problematic. Here we show the results we obtain from FIT I (HERMES + COMPASS + BELLE_cos($\varphi_1+\varphi_2$)

Simultaneous determination of Transversity and Collins functions

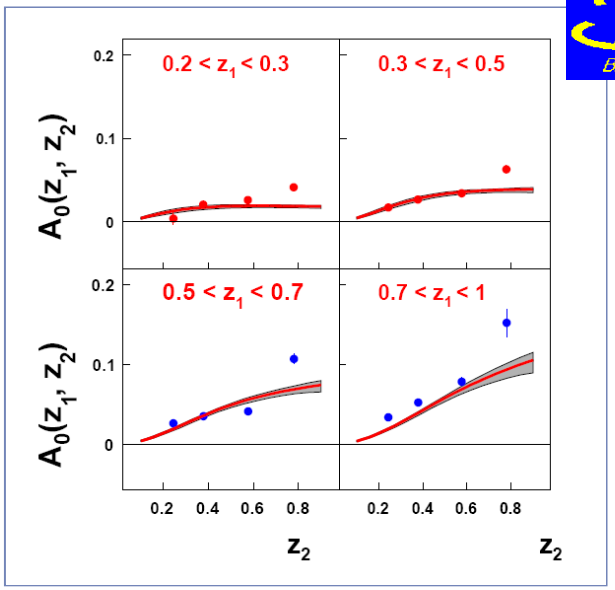




M.Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk

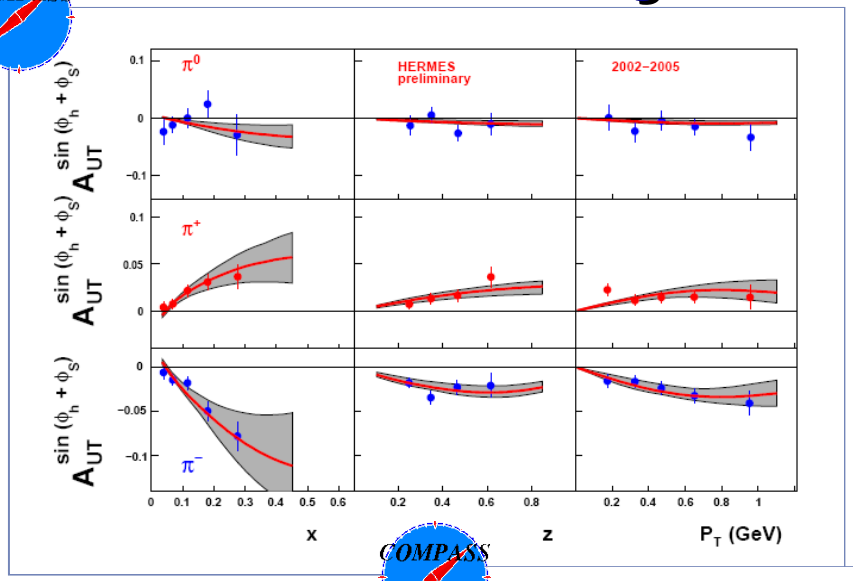


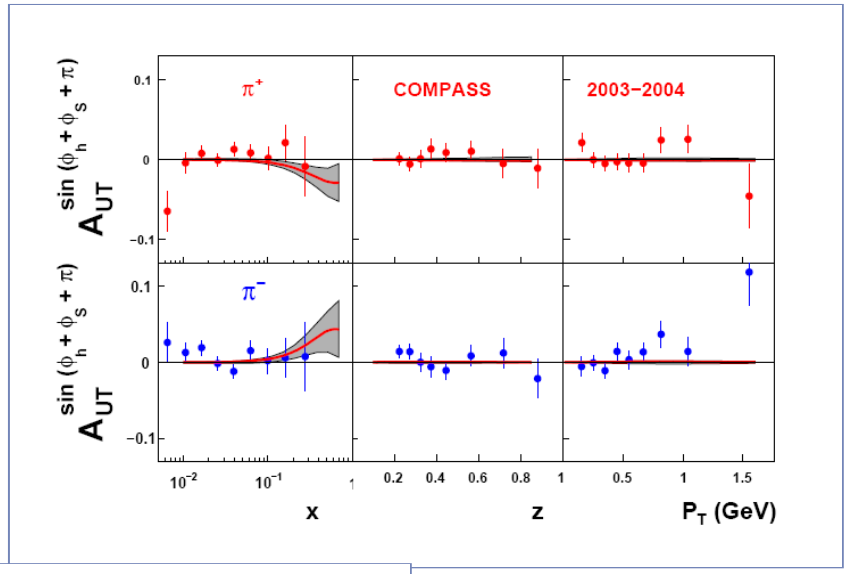
R. Saidl et al., Phys. Rev. D 78 (2008) 032011

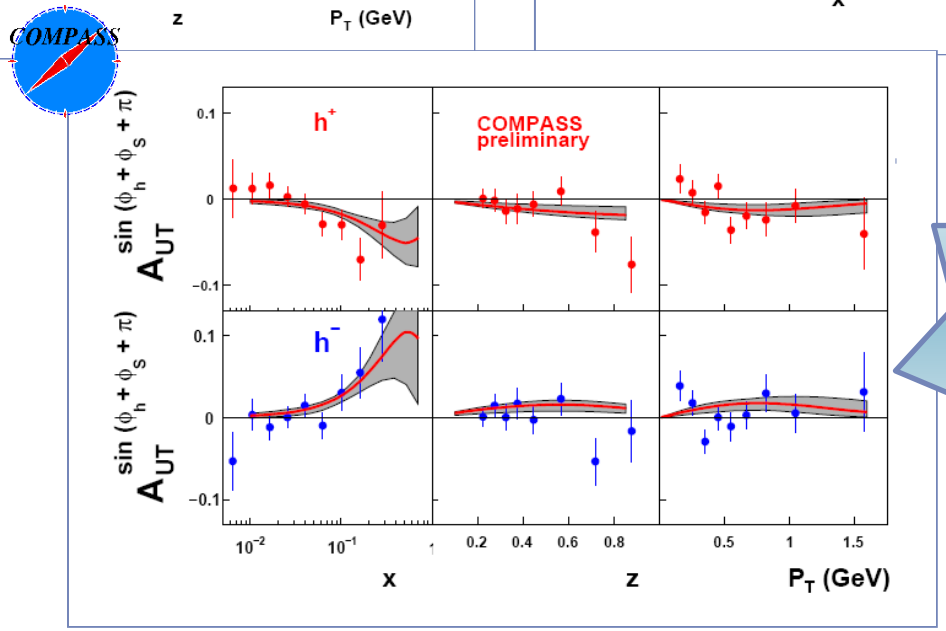


R. Saidl et al., Phys. Rev. D 78 (2008) 032011

Simultaneous determination of Transversity and Collins functions







Predictions

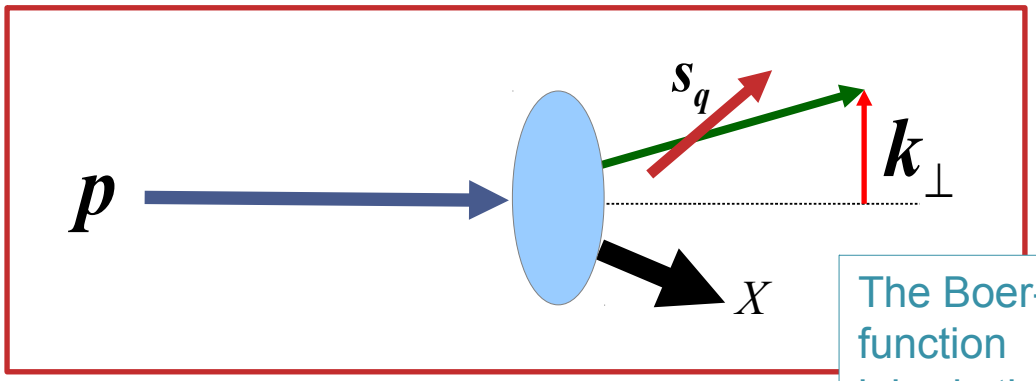
The Boer-Mulders Distribution Function

$$f_{q,s_q/p}(x,\boldsymbol{k}_\perp) = \frac{1}{2} f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta f_{q^\uparrow/p}(x,k_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

$$= \frac{1}{2} f_{q/p}(x,k_\perp) - \frac{1}{2}\frac{k_\perp}{M} h_1^{\perp q}(x,k_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)$$

The Boer-Mulders function is related to the probability of finding a polarized quark inside an unpolarized proton

The Boer Mulders function is chirally odd



The Boer-Mulders function inbeds the correlation between the quark spin and its transverse momentum

Extraction of the Boer-Mulders Function

V. Barone, S. Melis and A. Prokudin, ArXiv:0912.5194 (to be published in Phys. Rev. D)

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

$A = \propto f_1 \otimes D_1$ is the usual azimuthal-independent contribution

$B \propto \frac{1}{Q}(f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp)$ subleading - Cahn + Boer-Mulders effect

$C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ +...other unknown twist-4 terms?

Extraction of the Boer-Mulders Function

- Simple parametrization of the Boer-Mulders functions:

$$h_1^{\perp q}(x, k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x, k_\perp)$$ for valence quarks

$$h_1^{\perp q}(x, k_\perp) = -|f_{1T}^{\perp q}(x, k_\perp)|$$ for sea quarks

2 parameter fit

Sivers function as determined by Anselmino et al, ***Eur.Phys.J.A39:89-100,2009.***

Boer-Mulders function $h_1^\perp(x, k_\perp)$ describes distribution of transversely polarised quarks in an unpolarised hadron. $\cos(2\phi_h)$ asymmetry is generated in SIDIS by convolution with Collins FF.

- Large-N_c predictions $h_1^{\perp u} \simeq h_1^{\perp d}$
- Burkardt's approach $h_1^\perp$ and $f_{1T}^\perp$ are connected to GPD's. $h_1^{\perp u,d} \simeq \frac{\mathcal{K}_T{}^{u,d}}{\mathcal{K}^{u,d}} f_{1T}^{\perp u,d}$, $h_1^{\perp u,d} < 0$

  arXiv:0705.1573 [hep-ph], Phys.Rev.**D 72**,094020
- Lattice QCD result: $\frac{\mathcal{K}_T{}^u}{\mathcal{K}^u} \simeq \frac{3}{1.67}$, $\frac{\mathcal{K}_T{}^d}{\mathcal{K}^d} \simeq \frac{1.9}{2.03}$

  hep-lat/0612032

A. Prokudin, talk at Workshop on Transverse Spin Physics, Beijing (2008)

Extraction of the Boer-Mulders Function

FIT I (C term)

- ✓ GRV98 PDF
- ✓ DSS FF
- ✓ Gaussians: $\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$

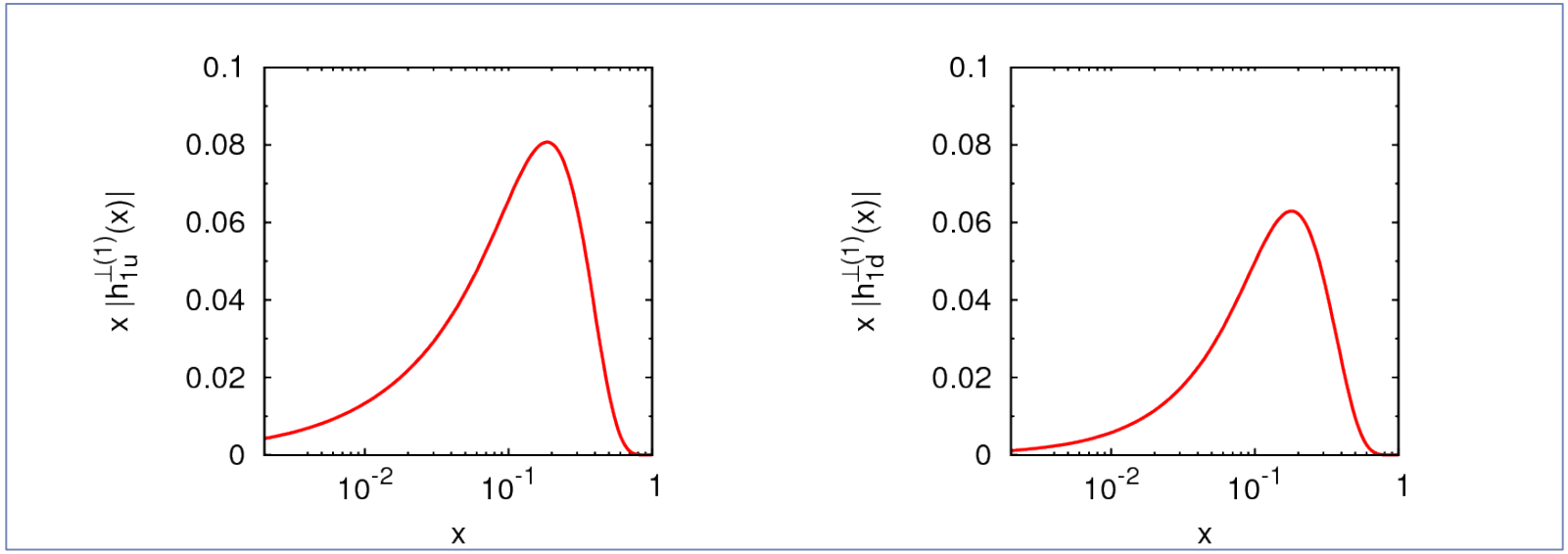
  $\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$

  (from Cahn-cosφ effect)
- ✓ Simulated evolution (unp.)

Experimental data

- ➢ HERMES proton and deuteron target (x,z,PT) charged hadrons
- ➢ COMPASS deuteron target (x,z,PT) charged hadrons

Extraction of the Boer-Mulders Function



FIT I

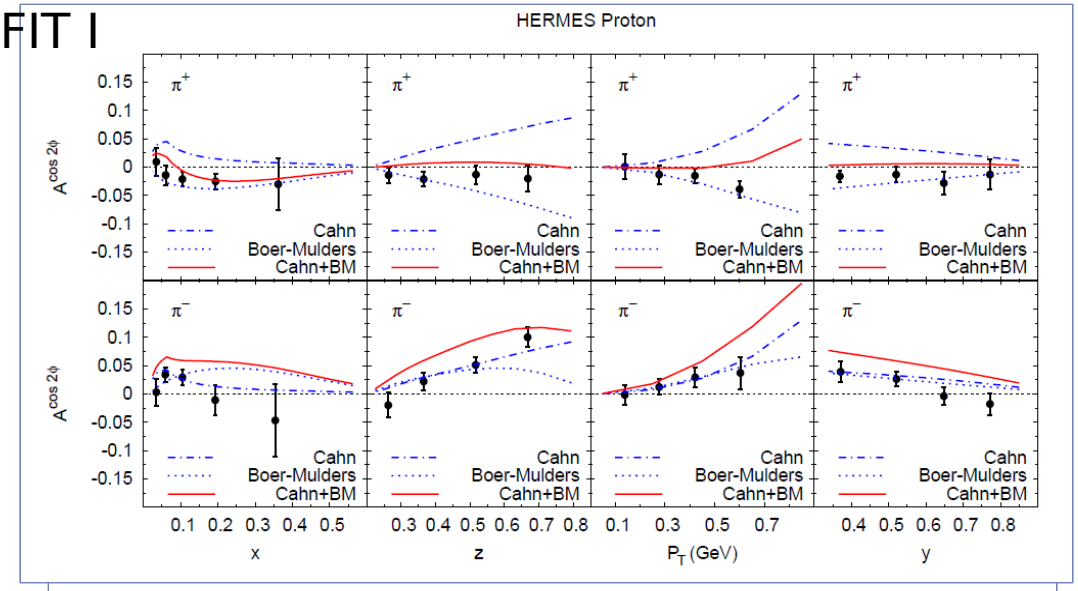
- $\chi^2/d.o.f. = 3.73$
- $\lambda_u = 2.0 \pm 0.1$
- $\lambda_d = -1.11^{+0.00}_{-0.02}$

Bound saturated

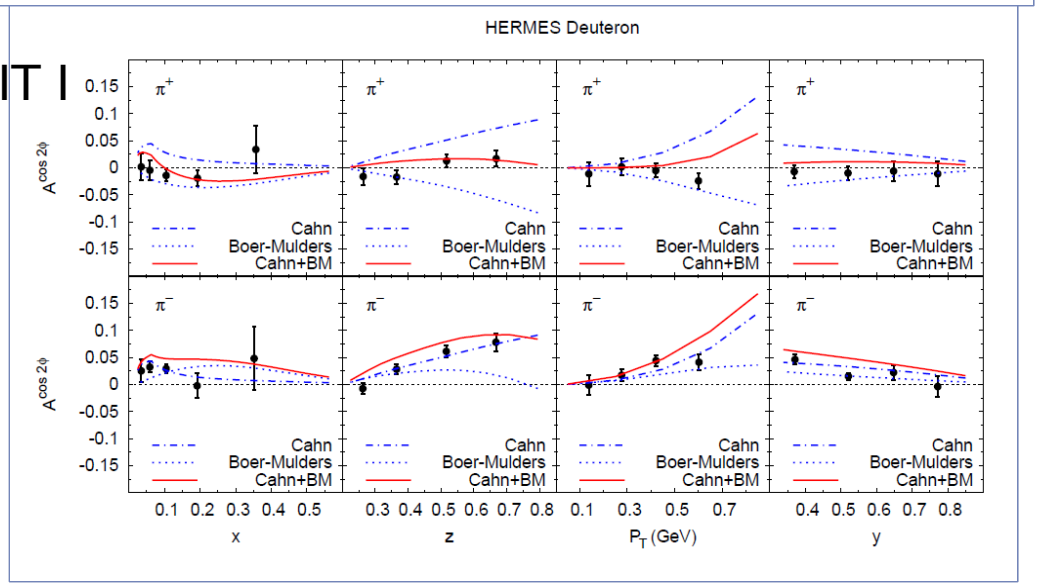
$\Rightarrow$ $h_1^{\perp d}$ and $h_1^{\perp u}$ both negative

Extraction of the Boer-Mulders Function

FIT I

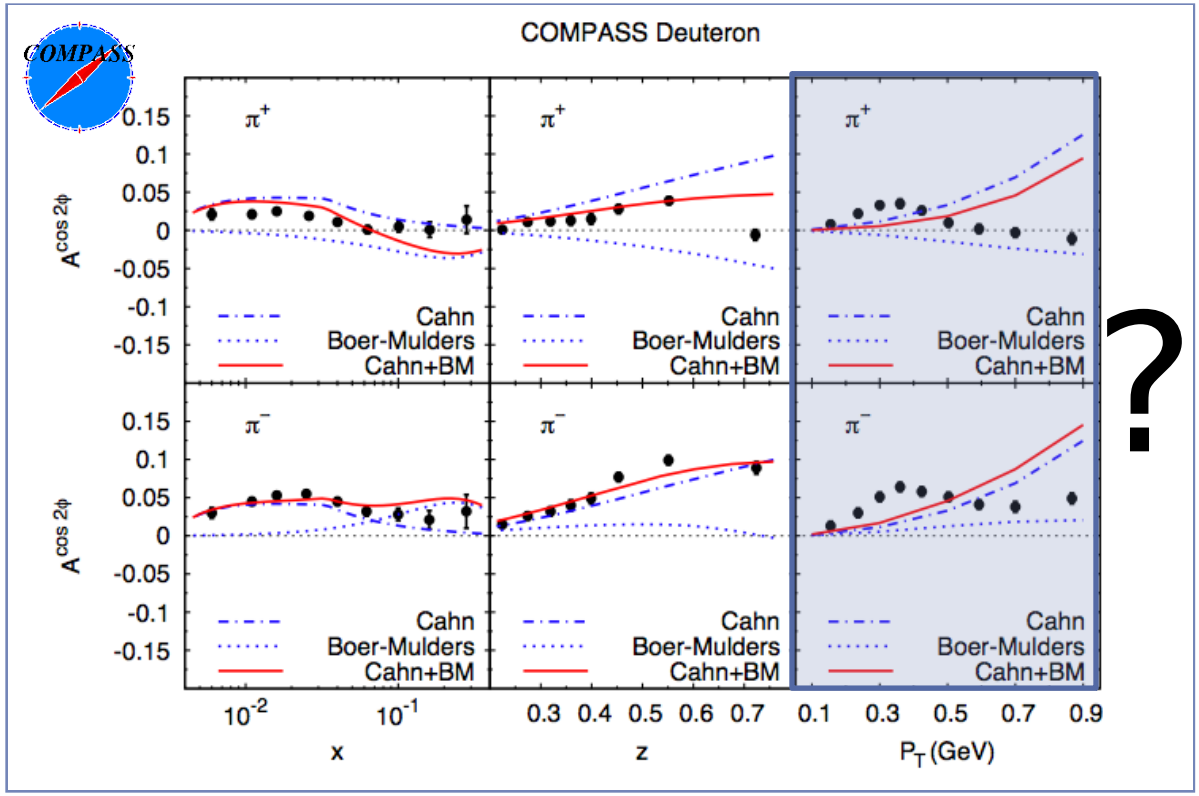


FIT I



- ✓ Cahn effect (Twist-4) comparable to BM effect
- ✓ Same sign of Cahn contribution for positive and negative pions
- ✓ BM contribution opposite in sign for positive and negative pions

Extraction of the Boer-Mulders Function



?

Extraction of the Boer-Mulders Function

FIT II

COMPASS

$\langle k_\perp^2 \rangle = 0.25\ (\mathrm{GeV}/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$

~ EMC

HERMES

$\langle k_\perp^2 \rangle = 0.18\ (\mathrm{GeV}/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (\mathrm{GeV}/c)^2$

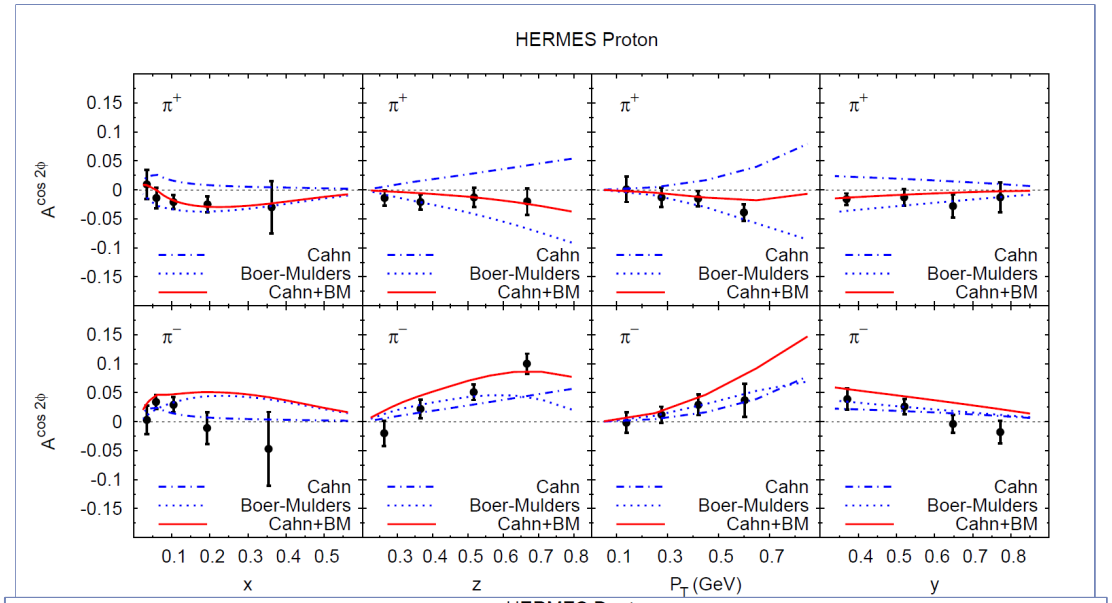
~ HERMES MC

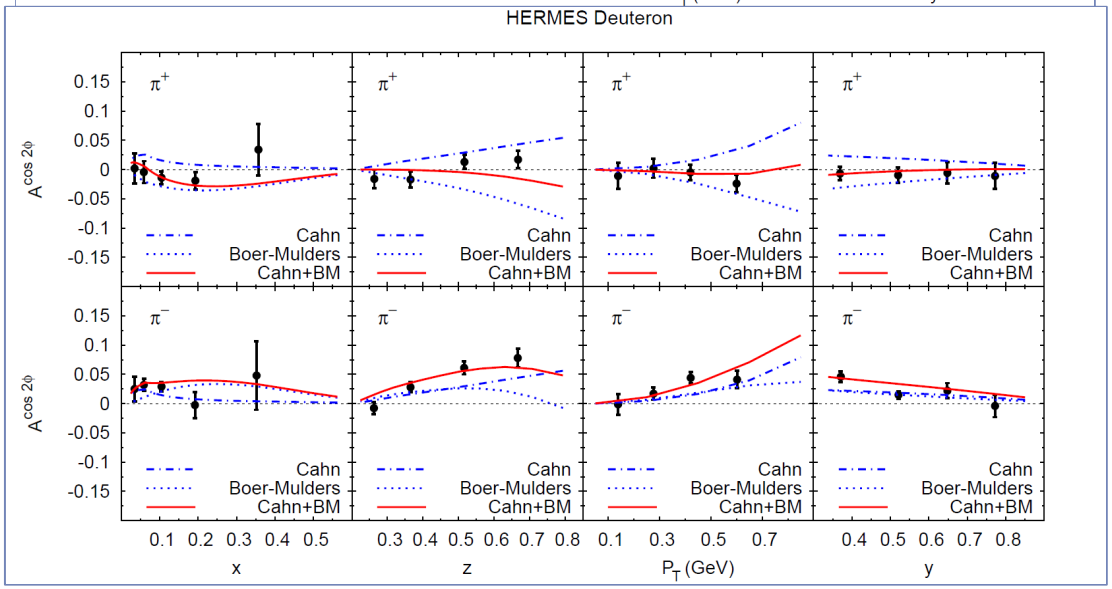
FIT II

- $\chi^2/d.o.f. = 2.41$
- $\lambda_u = 2.1 \pm 0.1$
- $\lambda_d = -1.11^{+0.00}_{-0.02}$

Better description of HERMES but the BM is unchanged

Extraction of the Boer-Mulders Function

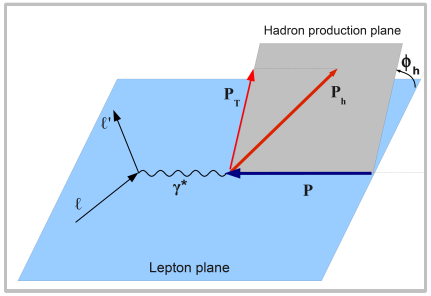




V. Barone,
S. Melis,
A. Prokudin
ArXiv: 0912.5194

Boer-Mulders Distribution Function in unpolarized SIDIS

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dy\, dz_h\, d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\, s y^2}\left\{\frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}\right\}$$



Azim. indep. unp. contrib.

$$F_{UU} \propto f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp)\,,$$

Cahn effect **Boer-Mulders effect**

$$F_{UU}^{\cos\phi_h} \propto \frac{k_\perp}{Q}\left[\, f_{q/p}(x,k_\perp) \otimes D_{h/q}(z,p_\perp) + \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)\right]$$

$\cos\phi_h$

Boer-Mulders effect

$$F_{UU}^{\cos 2\phi_h} \propto \Delta f_{q^\uparrow/p}(x,k_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z,p_\perp)\,.$$

$\cos 2\phi_h$

Intrinsic partonic motion in unpolarized SIDIS $\langle \cos \varphi_h \rangle$

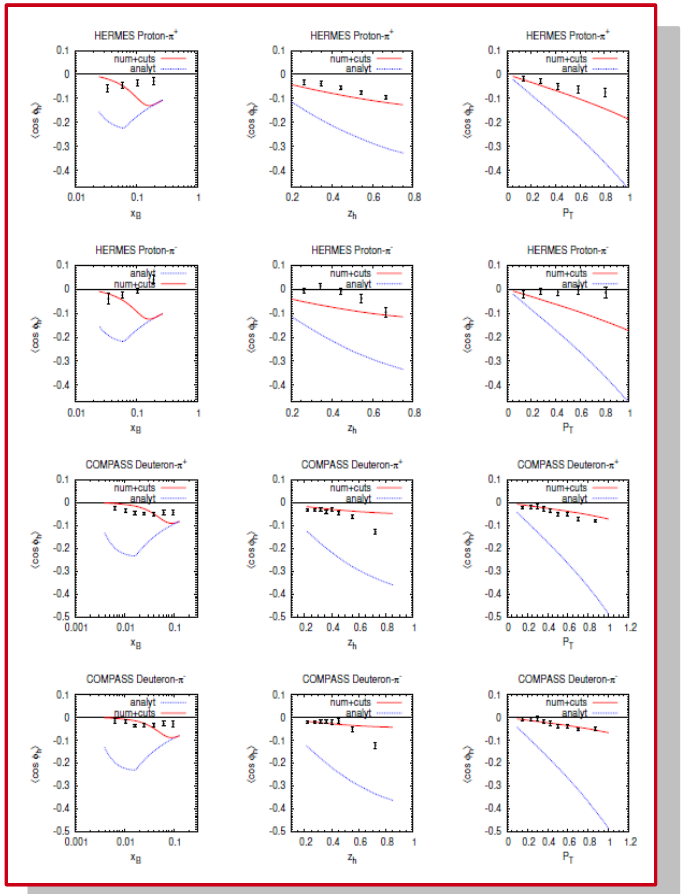
- The integration over $k_\perp$ from zero to infinity is a crude assumption.
- The parton model provides kinematical limits on the transverse momentum size.

By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

$$k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2$$

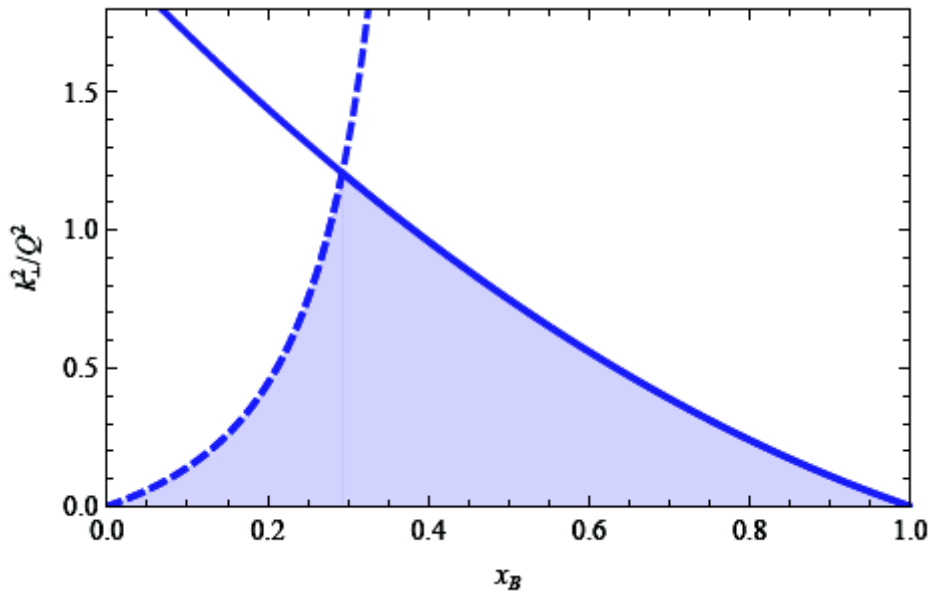
By requiring the parton not to move backward with respect to its parent hadron, we find

$$\boldsymbol{k}_\perp^2 \leq \frac{x_B(1 - x_B)}{(2x_B - 1)^2} Q^2$$



M. Boglione, S. Melis, A. Prokudin, arXiv;1106.6177
To appear in Phys. Rev. D

Intrinsic partonic motion in unpolarized SIDIS $\langle \cos \varphi_h \rangle$



The last three TMD's ...

what about the last 3 TMDs? any relation with the others?

$$g_{1T}^{\perp(1)a}(x) \simeq x \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y)$$

$$h_{1L}^{\perp(1)a}(x) \simeq -x^2 \int_x^1 \frac{\mathrm{d}y}{y^2}\, h_1^a(y)$$

$$h_{1T}^{\perp(1)a}(x) \simeq g_1^a(x) - h_1^a(x)$$

neglecting twist-3 contributions

similar to the Wandzura-Wilczek relation

$$g_T^a(x) \simeq \int_x^1 \frac{\mathrm{d}y}{y}\, g_1^a(y)$$

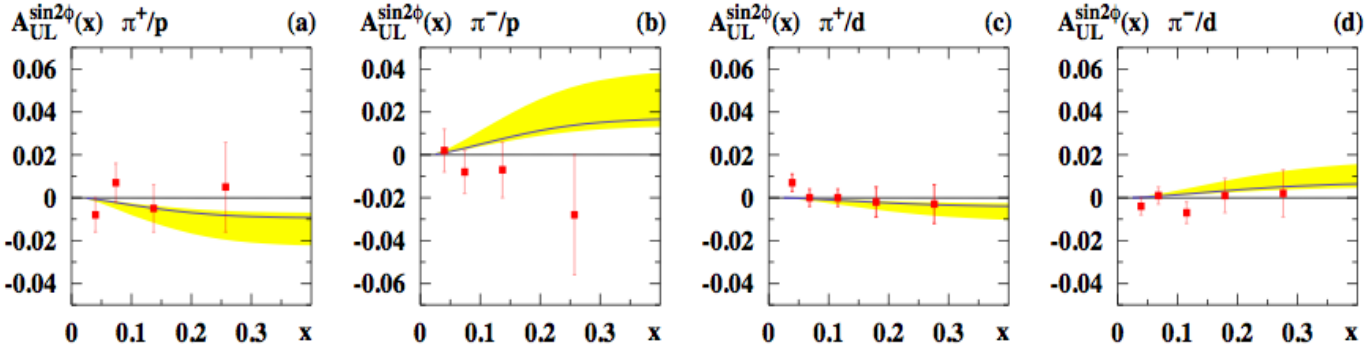
supported by experiment

$$g_{1T}^{\perp(1)a}(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{k_\perp^2}{2m_N^2}\, g_{1T}^{\perp a}(x, k_\perp^2)$$

for a recent model of all twist-2 TMDs see Bacchetta et al., arXiv:0807.0323

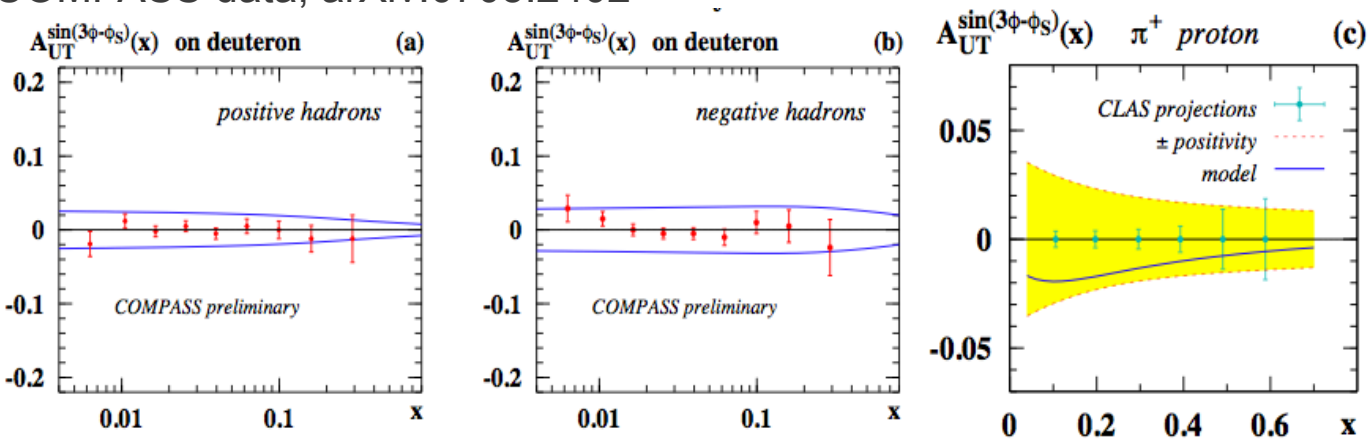
The last three TMD's ...

HERMES data, PRL 84 (2000) 4047; PL B562 (2003) 182



$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2\, h_{1L}^{\perp a} \otimes H_1^{\perp a}$$

COMPASS data, arXiv:0705.2402



$$F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2\, h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

H. Avakian., A.V. Efremov, P. Schweitzer, F. Yuan
→ Bag Model predictions arXiv:0805.3355 [hep-ph]

Pretzelosity

What do we know about it?

- in transversely polarized nucleon: measure of quark polarization $\perp$ quark p_T
  Piet Mulders, Rick Tangerman 1995

- tells us deviation of nucleon shape from sphere
  Gerry Miller 2007 ('non-sphericity', 'pretzelosity')
  Matthias Burkardt 2007 ('pretzel', 'peanut', 'baggle')

- pretzelosity-relation in bag: $g_1^q(x, p_T) - h_1^q(x, p_T) = h_{1T}^{\perp(1)q}(x, p_T)$
  Avakian, Efremov, PS, Yuan 2008

- also in spectator, light-cone constituent and covariant parton model
  Jakob et al 1997, Pasquini et al 2008, Efremov et al 2008

Spin-orbit correlations

Light-cone SU(6) quark-diquark model, Ma, Schmidt (1998)

also direct calculation: $h_{1T}^{\perp(1)q}(x) = -L^q(x)$

$\rightarrow$

$(-1)\, h_{1T}^{\perp(1)q}(x)\mathrm{d}x$ = contribution of quark with $x \in [x, x + dx]$ to light-cone angular momentum

i.e. transverse moment of pretzelosity = direct measure of L^q !

Universality of Sivers and Collins functions

- The Collins fragmentation function is universal (no initial/final state interactions, no effects induced by requiring color gauge invariance)

  J. Collins and A. Metz, Phys. Rev. Lett. 93,252001 (2004), F. Yuan, arXiv:0903.4680

- The Sivers distribution function (naively time reversal odd) is subject to initial/final state interaction – color gauge invariance requirements induce color factors (process dependence).

  C. J. Bomhof, P. J. Mulders,F. Pijlman, Eur. Phys. J. C 47, 147 (2006)

Example:

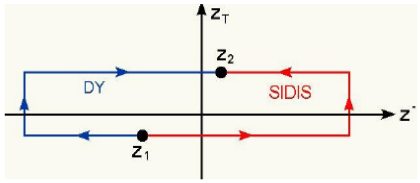
$$(\Delta^N f_{q/p^\uparrow})^{SIDIS} = -(\Delta^N f_{q/p^\uparrow})^{Drell-Yan}$$

- **If T-odd TMDs ≠ 0: Gauge Link not neglegible, physical effect:**

Initial / Final state interactions

Wilson line process-dependent:

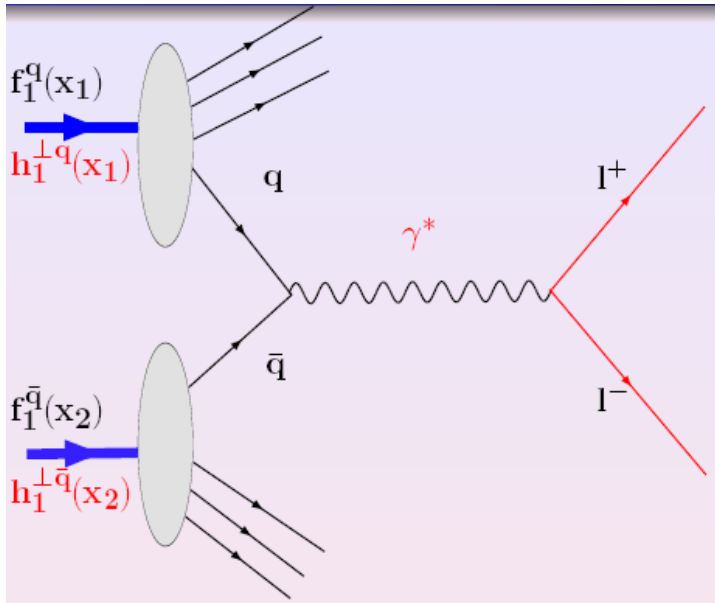
$$\mathcal{W}[z_1; z_2] = \mathcal{P}e^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$



Time reversal → **switches sign:**

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY} \qquad h_1^{\perp}\Big|_{DIS} = -h_1^{\perp}\Big|_{DY}$$

Drell-Yan Processes



$H_1 + H_2 \to l^+ + l^- + X$

Kinematical variables:

$M^2 \equiv q^2 = (p_q + p_{\bar{q}})^2$,

$y \equiv \frac{1}{2} \ln \frac{q_0 + q_L}{q_0 - q_L}$,

$\tau \equiv x_1 x_2 = \frac{M^2}{s}$

$f_1^q(x_1)$ – parton density function (PDF)

$$\sigma = \sum_q f_1^q(x_1, \mathbf{k}_\perp) \otimes \sigma^{q\bar{q} \to l^+ l^-} \otimes f_1^{\bar{q}}(x_2, \mathbf{p}_\perp) + (q \leftrightarrow \bar{q})$$

Angular dependence in the Collins-Soper frame

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda + 3}\left(1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi\right)$$

A. Prokudin, talk at Workshop on Transverse Spin Physics, Beijing (2008)