the excited spectrum of QCD

the spectrum of excited hadrons

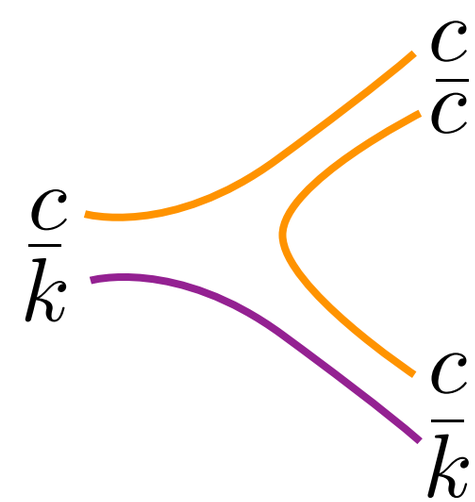
let's begin with a **convenient fiction** :

imagine that QCD were such that there was a spectrum of **stable** excited hadrons

e.g. suppose we set up QCD with just two degenerate flavours of quark with mass roughly that of the charm quark

$$m_c = m_k \sim 1.5\,\mathrm{GeV}$$

then we'd expect a spectrum of $c\bar{k}$ hadron states starting at about 3 GeV that are stable up to about 6 GeV

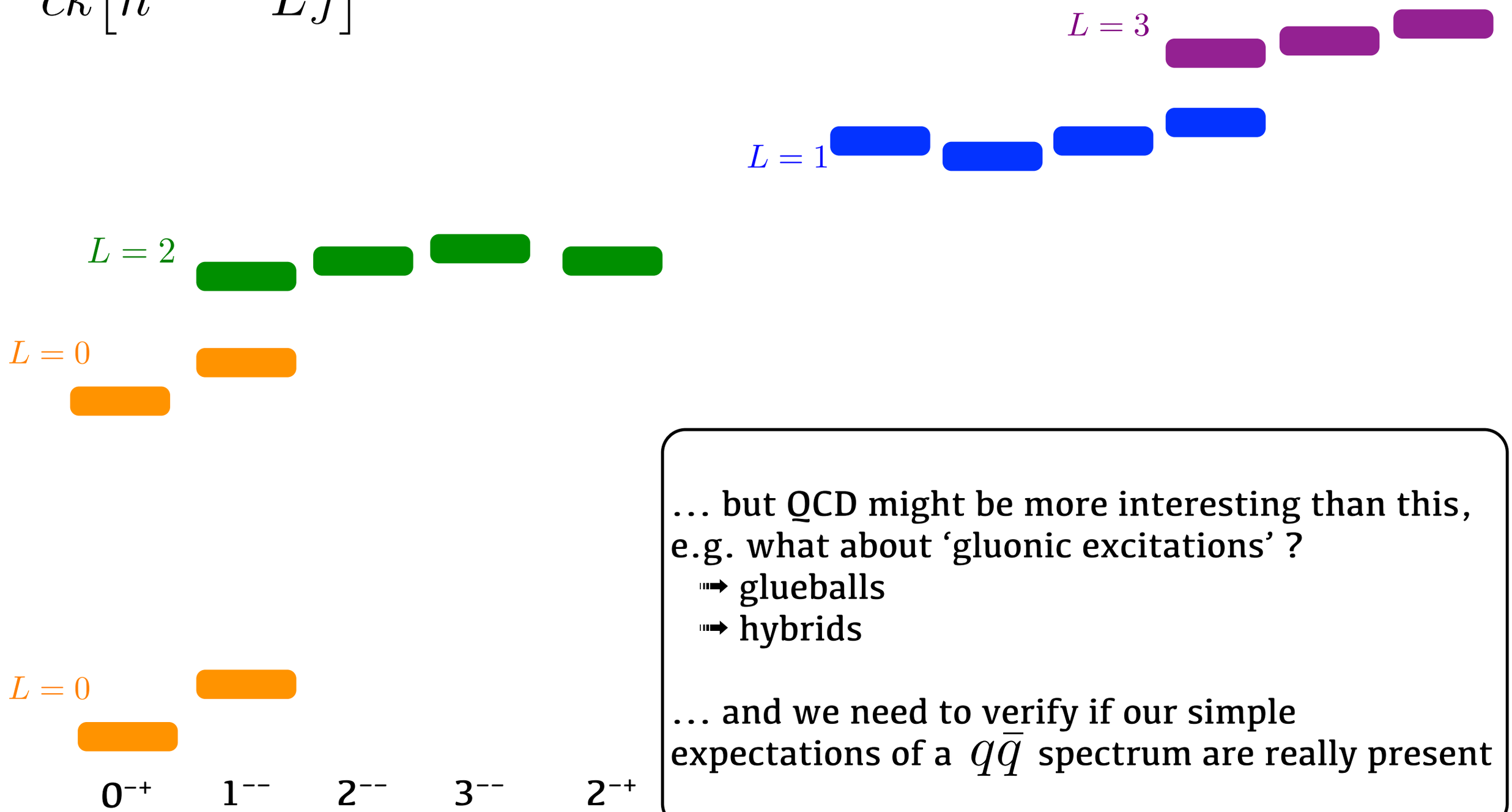


except perhaps if glueballs are important ?

the spectrum of excited hadrons

might expect something like the non-relativistic quark model

$$c\bar{k}\left[n\,^{2S+1}L_J\right]$$



… but QCD might be more interesting than this, e.g. what about 'gluonic excitations' ?

- ➟ glueballs
- ➟ hybrids

… and we need to verify if our simple expectations of a $q\bar{q}$ spectrum are really present

the spectrum of excited hadrons

we'd like to map out the spectrum of states in each J^{PC}

need interpolating fields that transform like the desired J^{PC}

e.g. local fermion bilinears

$$\bar{\psi}\gamma_5\psi \sim 0^{-+}$$
$$\bar{\psi}\psi \sim 0^{++}$$
$$\bar{\psi}\gamma_i\psi \sim 1^{--}$$
$$\bar{\psi}\gamma_5\gamma_i\psi \sim 1^{++}$$
$$\epsilon_{ijk}\bar{\psi}\gamma_j\gamma_k\psi \sim 1^{+-}$$

... very limited in J^{PC} coverage

one possible extension: include gauge-covariant derivatives

$$\overleftrightarrow{D}_i = \overleftarrow{D}_i - \overrightarrow{D}_i = \overleftarrow{\partial}_i - \overrightarrow{\partial}_i - 2igA_i$$

e.g. $\bar{\psi}\overleftrightarrow{D}_i\psi \sim 1^{--}$

$$\bar{\psi}\gamma_i\overleftrightarrow{D}_j\psi \sim ? \qquad \begin{matrix} i = 1\ldots3 \\ j = 1\ldots3 \end{matrix}$$

9 elements

operator is reducible

the spectrum of excited hadrons

$\bar{\psi}\gamma_i \overleftrightarrow{D}_j \psi \sim ?$ $\quad i=1\ldots3$, $j=1\ldots3$ **9 elements** operator is reducible

very easy to build a scheme where the operators are irreducible:

$$\gamma_m \equiv \sum_i \epsilon_i(m)\gamma_i \qquad \vec{\epsilon}(m=\pm) = \mp\tfrac{1}{\sqrt{2}}\big[1,\pm i,0\big]$$

$$\overleftrightarrow{D}_m \equiv \sum_i \epsilon_i(m)\overleftrightarrow{D}_i \qquad \vec{\epsilon}(m=0) = \big[0,0,1\big]$$

spin-1 circular basis

$$\Longrightarrow \big\langle 1m_1;1m_2\big|JM\big\rangle\, \bar{\psi}\gamma_{m_1}\overleftrightarrow{D}_{m_2}\psi \sim J^{++}$$

with J=0,1,2

Hadron Spectrum Collaboration has used up to three derivatives:

$$\big\langle 1m_1;j_{234}m_{234}\big|JM\big\rangle$$
$$\big\langle 1m_3;j_{24}m_{24}\big|j_{234}m_{234}\big\rangle$$
$$\big\langle 1m_2;1m_4\big|j_{24}m_{24}\big\rangle$$
$$\bar{\psi}\gamma_{m_1}\overleftrightarrow{D}_{m_2}\overleftrightarrow{D}_{m_3}\overleftrightarrow{D}_{m_4}\psi$$

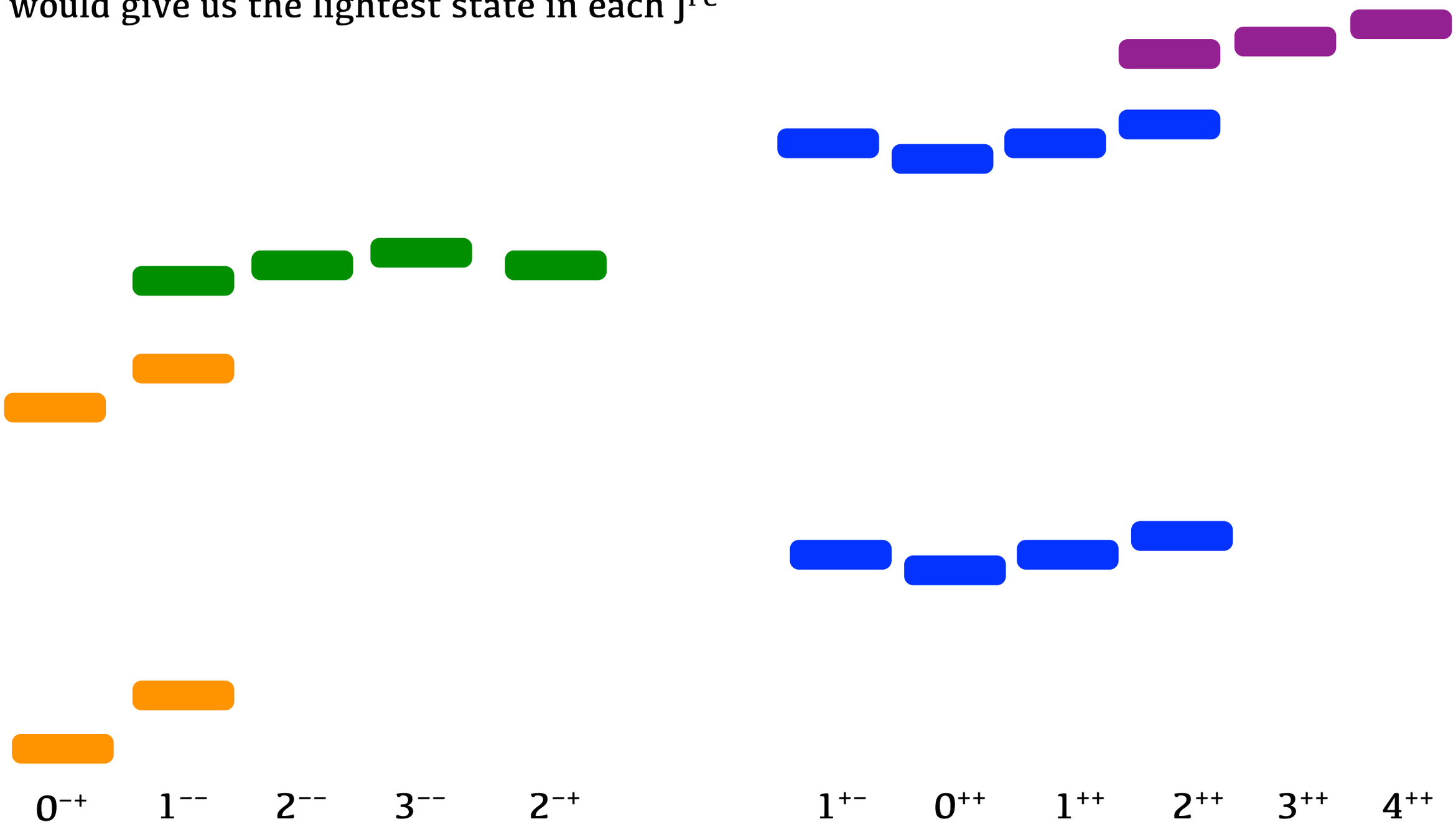
can build a big basis this way covering all J ≤ 4

PRL103 262001 (2009)
PRD82 034508 (2010)

the spectrum of excited hadrons

so we could compute correlators for each J^{PC} and look at effective masses at large t

would give us the lightest state in each J^{PC}



we want more than this . . .

the spectrum of excited hadrons

we need to be able to extract **excited** states

$$C(t) = \sum_{\mathfrak{n}} A_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t}$$

a weighted sum of exponentials
- just do a fit to the time-dependence ?

(fit variables : A_0, A_1 … , E_0, E_1 …)

this is a very bad way to approach this problem

➟ suppose two states are (nearly) degenerate
- fit won't be able to tell if there are two states or one !

➟ how do we determine how many states to include in the fit
- if we decrease t_{min} to use more of the data, need more states ?

fortunately there is a very powerful method available …

variational approach

suppose we have multiple operators for a given J^{PC} $\quad \mathcal{O}_1,\ \mathcal{O}_2,\ \mathcal{O}_3 \ldots$

e.g. $J^{PC} = 1^{--}$

$$\bar{\psi}\gamma_m\psi$$
$$\bar{\psi}\overleftrightarrow{D}_m\psi$$
$$\langle 1m_1; 1m_2|1m\rangle\bar{\psi}\gamma_5\overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}\psi$$
$$\langle 1m_1; 2m_D|1m\rangle\langle 1m_2; 1m_3|2m_D\rangle\bar{\psi}\gamma_{m_1}\overleftrightarrow{D}_{m_2}\overleftrightarrow{D}_{m_3}\psi$$
$$\vdots$$

compute a **matrix of correlation functions**

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j(0)|0\rangle$$
$$= \sum_{\mathfrak{n}} Z_i^{(\mathfrak{n})} Z_j^{(\mathfrak{n})} e^{-E_{\mathfrak{n}} t} \qquad Z_i^{(\mathfrak{n})} = \langle \mathfrak{n}|\mathcal{O}_i(0)|0\rangle$$

solve the 'generalised eigenvalue problem' : $C(t)v^{(\mathfrak{n})} = \lambda_{\mathfrak{n}}(t)C(t_0)v^{(\mathfrak{n})}$

eigenvalues, 'principal correlators' $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

eigenvectors are 'orthogonal' $v^{(\mathfrak{m})\dagger}C(t_0)v^{(\mathfrak{n})} = \delta_{\mathfrak{m},\mathfrak{n}}$

variational approach

the interpretation is relatively simple

the eigenvectors indicate the optimal linear combination of $\mathcal{O}_i$ to interpolate $|\mathfrak{n}\rangle$

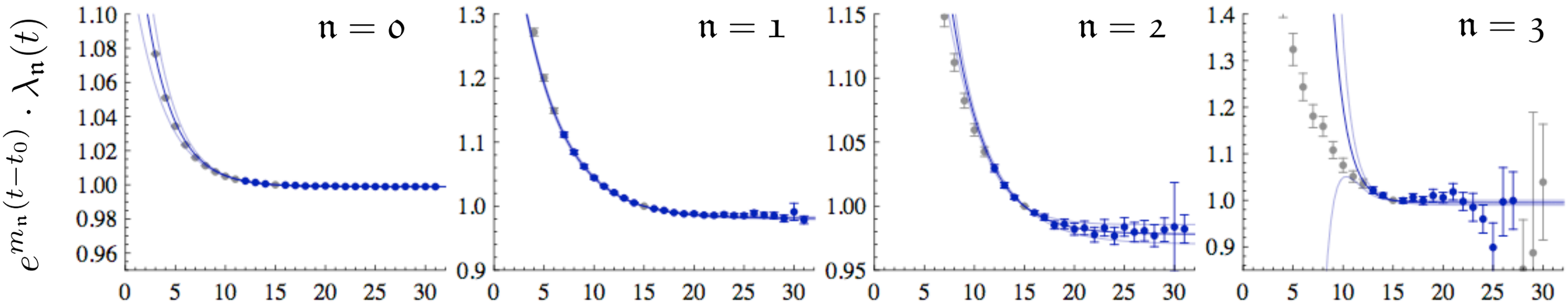
$$\Omega_{\mathfrak{n}} = \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i \qquad\qquad \langle \mathfrak{m}|\Omega_{\mathfrak{n}}|0\rangle \approx \delta_{\mathfrak{m},\mathfrak{n}}$$

degenerate states are easy to deal with - they might have $E_{\mathfrak{m}} = E_{\mathfrak{n}}$

- but they have orthogonal $v^{(\mathfrak{m})}, v^{(\mathfrak{n})}$

variational approach

principal correlators $t_0 = 15$



fit each with

$$\lambda(t) = (1 - A)e^{-m(t-t_0)} + Ae^{-m'(t-t_0)}$$

with m,m',A as parameters

(throw away: m', A)

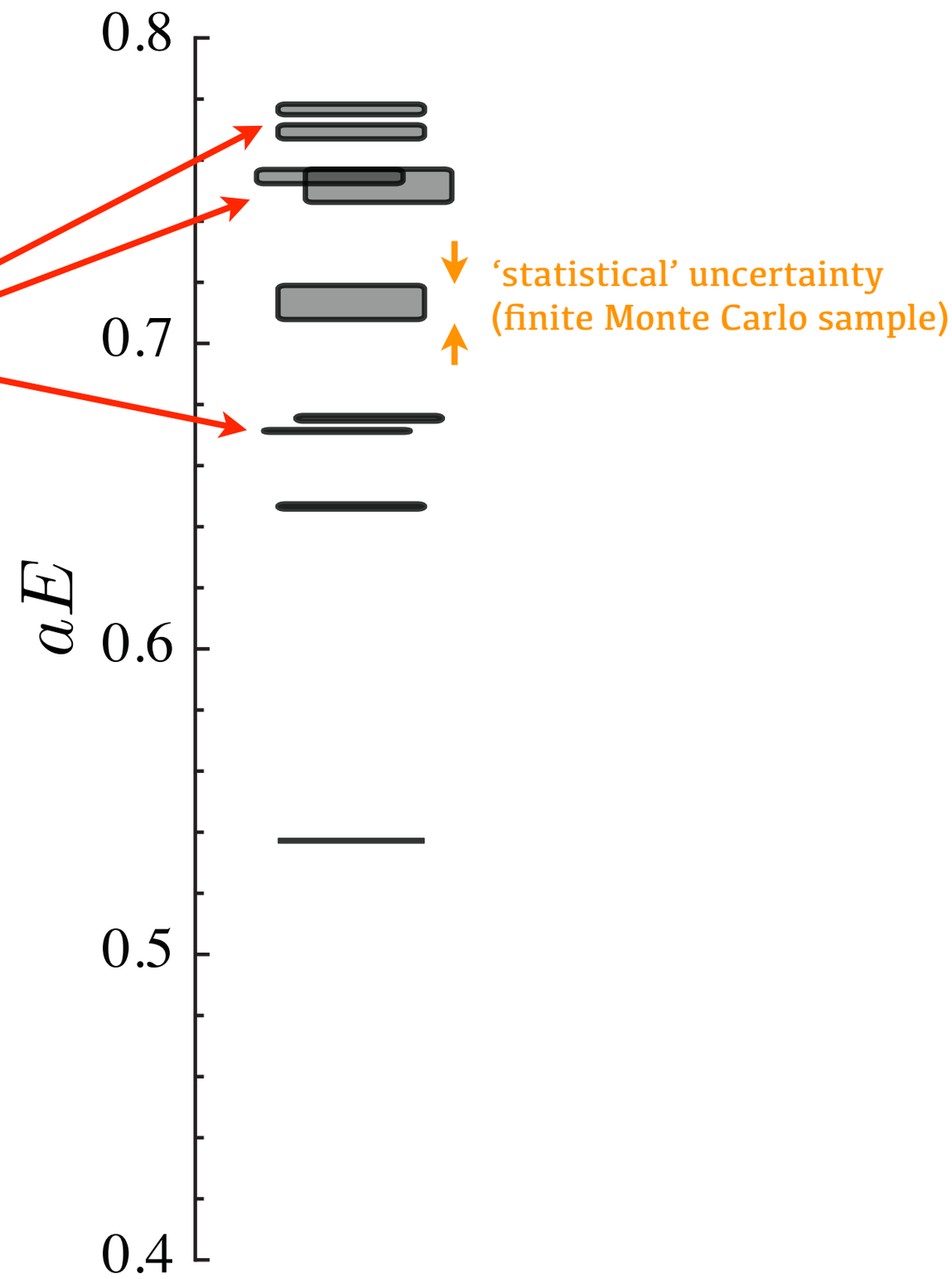
a real example - T_1^{--} in charmonium

superimposed J=1,3,4 spectra

26 operators

variational analysis of
26×26 matrix of correlators

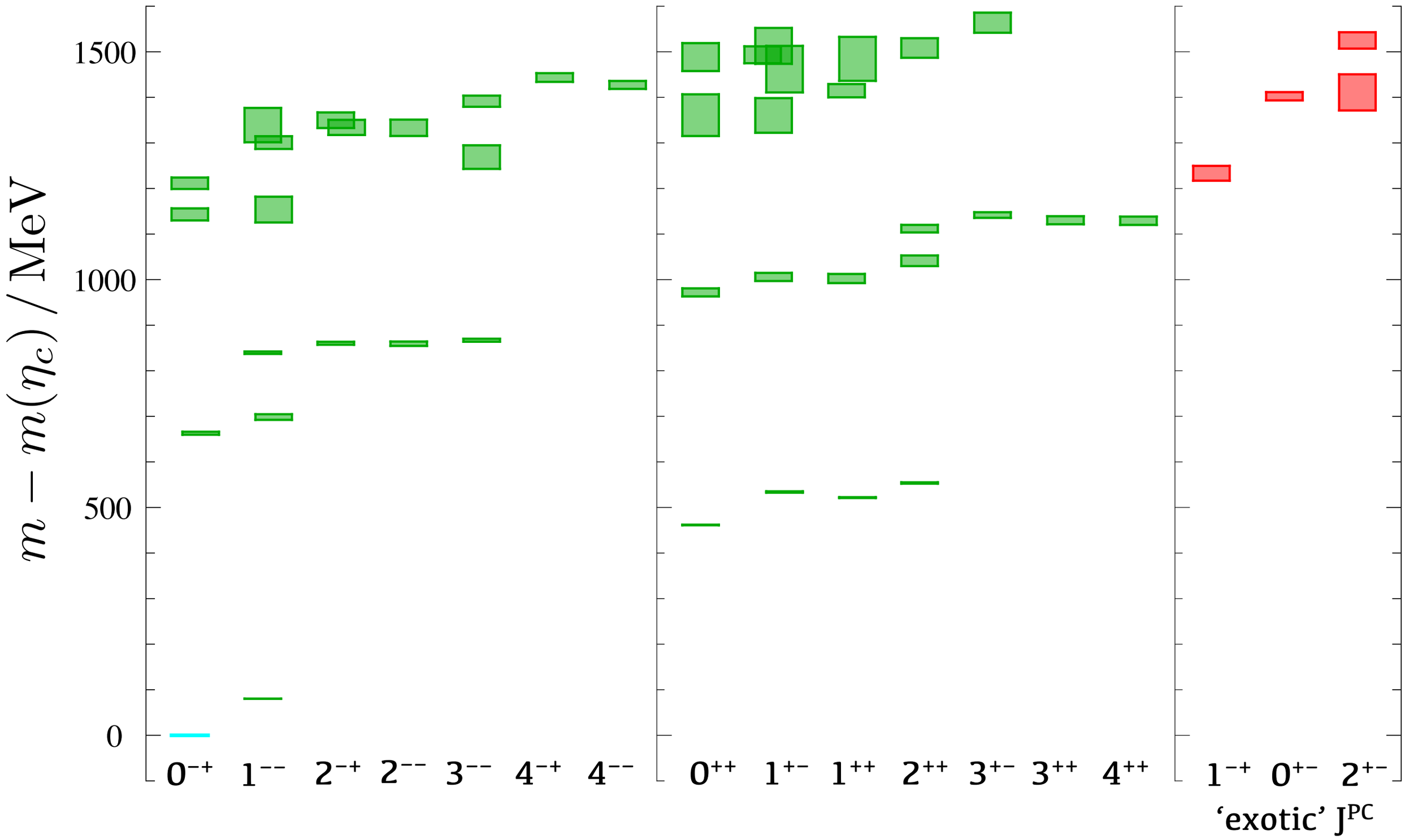
multiple approximate degeneracies



the charmonium spectrum from a lattice QCD calc

perform variational analysis in each quantum number

Hadron Spectrum Collaboration
arXiv:1204.5425



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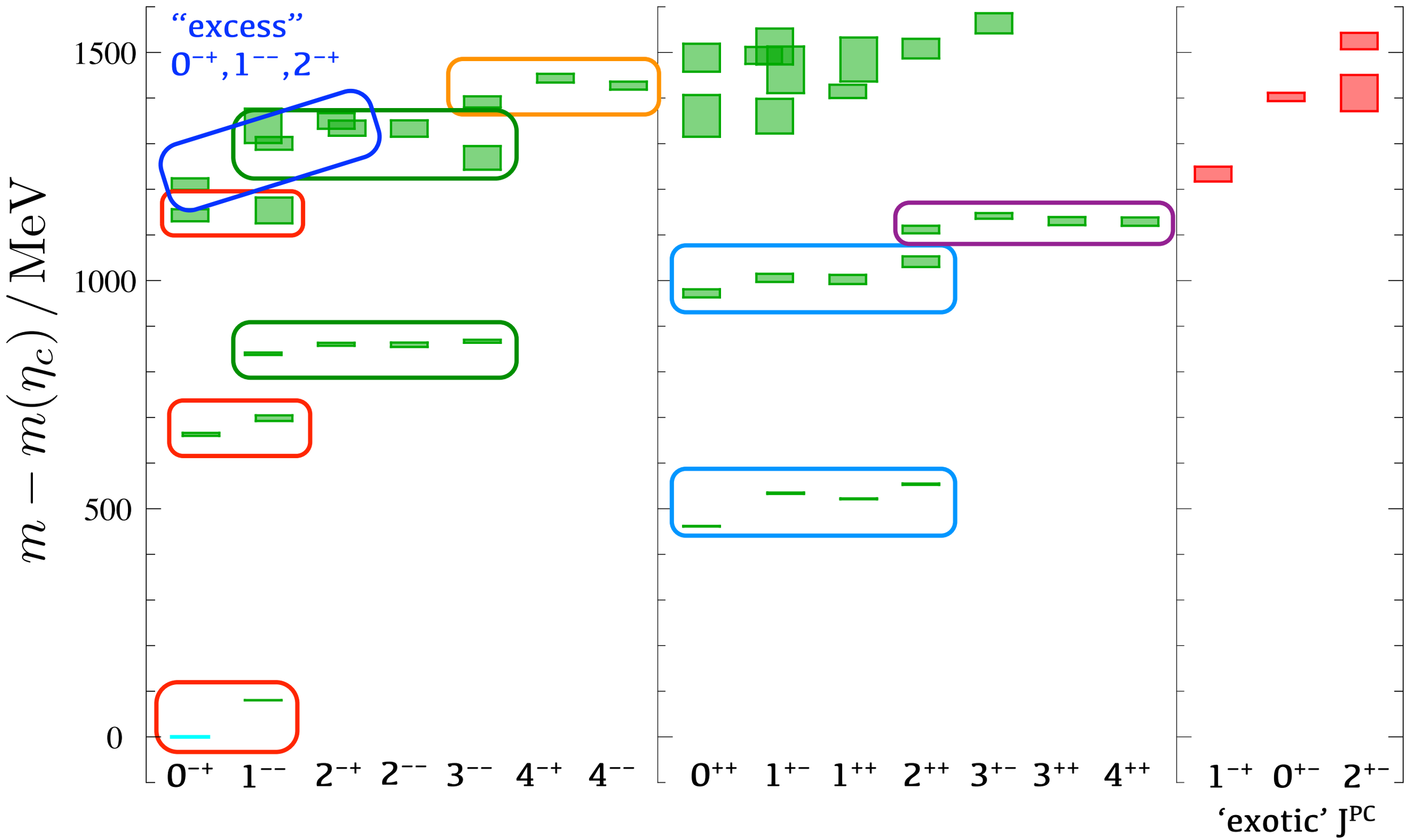

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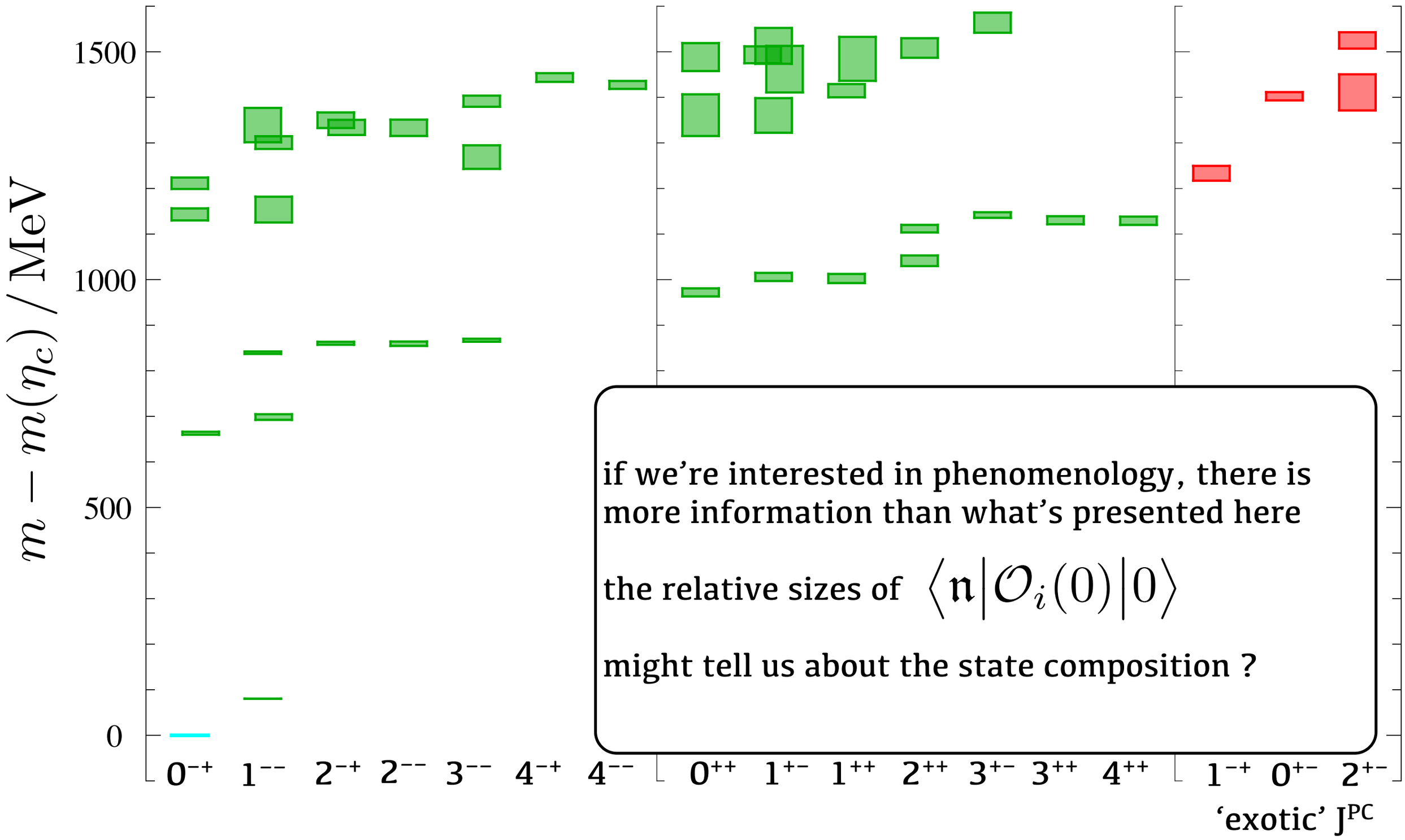


if we're interested in phenomenology, there is more information than what's presented here

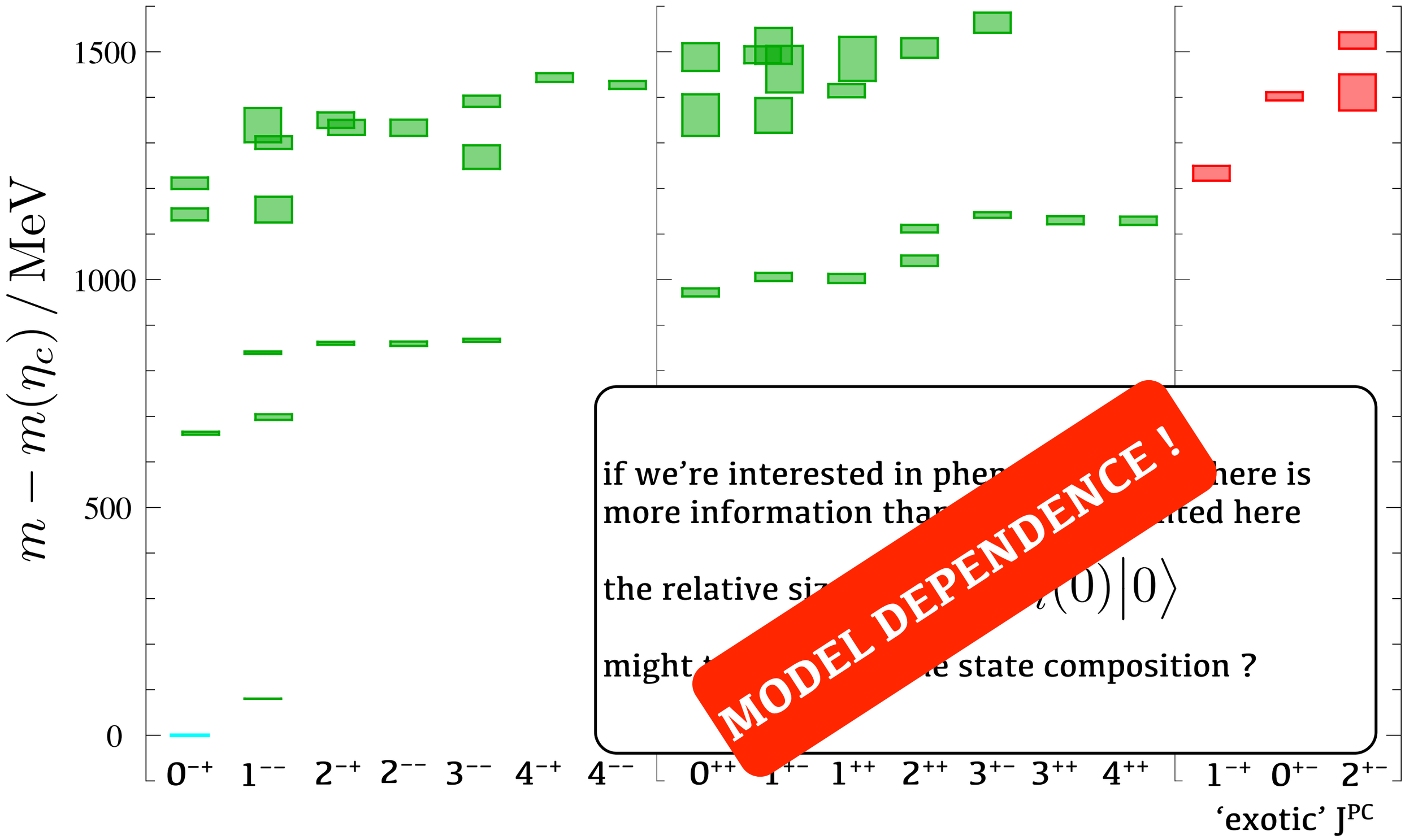
the relative sizes of $\langle \mathfrak{n} | \mathcal{O}_i(0) | 0 \rangle$

might tell us about the state composition ?

the charmonium spectrum from a lattice QCD calc

perform variational analysis in each quantum number

Hadron Spectrum Collaboration
arXiv:1204.5425



if we're interested in phen here is
more information tha ted here

the relative siz $(0)|0\rangle$

might t e state composition ?

MODEL DEPENDENCE !

back to the operators ...

e.g. $J^{PC}=1^{--}$ consider a model-interpretation

$$\bar{\psi}\gamma_m \tfrac{1}{2}(1-\gamma_0)\psi$$

spin-structure: $$\psi \sim \begin{bmatrix} 1 \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \end{bmatrix} \chi$$

$$\tfrac{1}{2}(1-\gamma_0)\psi \sim \begin{bmatrix} 1 \\ 0 \end{bmatrix} \chi$$

upper component projector

$$\bar{\psi}\gamma_m \tfrac{1}{2}(1-\gamma_0)\psi \sim \phi^\dagger \sigma_m \chi$$

$^3S_1$

MODEL DEPENDENCE !

back to the operators ...

e.g. $J^{PC}=1^{--}$ consider a model-interpretation

$$\langle 1m_1; 2m_2|1m\rangle\ \bar{\psi}\gamma_{m_1} D^{[2]}_{J=2,m_2}\,\tfrac{1}{2}(1-\gamma_0)\psi$$

$$D^{[2]}_{J,m} \equiv \langle 1m_1; 1m_2|Jm\rangle\, \overleftrightarrow{D}_{m_1}\, \overleftrightarrow{D}_{m_2}$$

without gauge-fields: $D^{[2]}_{J=2,m} \to Y^m_2(\overleftrightarrow{\partial})$

$$\sim \langle 1m_1; 2m_2|1m\rangle\ \cdot \phi^\dagger \sigma_{m_1}\chi\ \cdot Y^{m_2}_2(\vec{q})$$

$^3D_1$

$q\bar{q}$ relative momentum

back to the operators ...

e.g. $J^{PC}=1^{--}$

$$\bar{\psi}\gamma_5 D^{[2]}_{J=1,m}\,\tfrac{1}{2}(1-\gamma_0)\psi$$

$$D^{[2]}_{J=1,m} \equiv \langle 1m_1;1m_2|1m\rangle \overleftrightarrow{D}_{m_1}\overleftrightarrow{D}_{m_2}$$

antisymmetric CGC

without gauge-fields: $D^{[2]}_{J=1,m} \to 0$

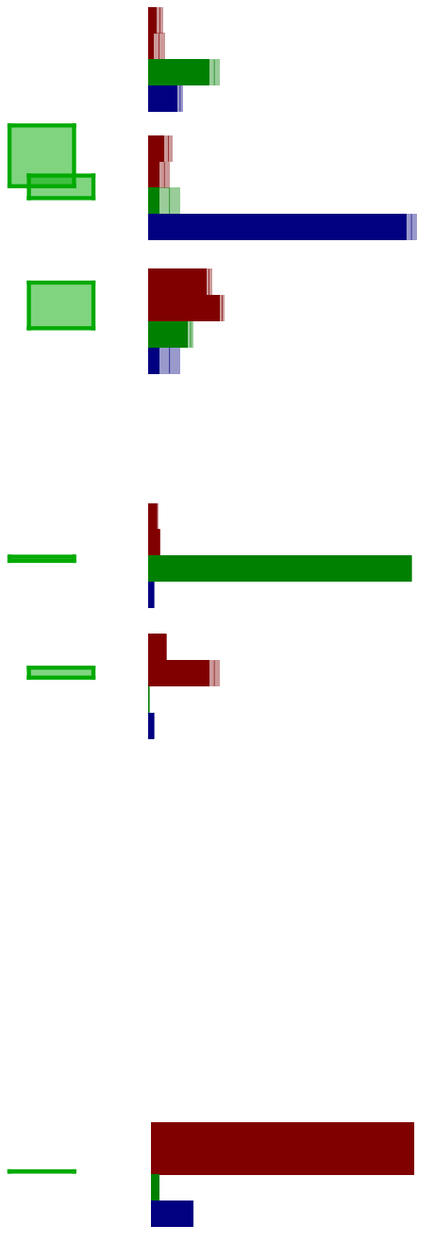
with gauge-fields $D^{[2]}_{J=1,m} \propto [D_i, D_j] \propto F_{ij}$ chromomagnetic part of field-strength tensor

$$\underbrace{\bar{\psi}\gamma_5 t^a \psi}_{q\bar{q}_{\mathbf{8}}(^1S_0)}\, B^a_m$$

$^1\mathrm{hyb}_1$

operator overlaps

e.g. $J^{PC}=1^{--}$



$^3D_1$

$^1\mathrm{hyb}_1$

$^3S_1$?

$^3D_1$

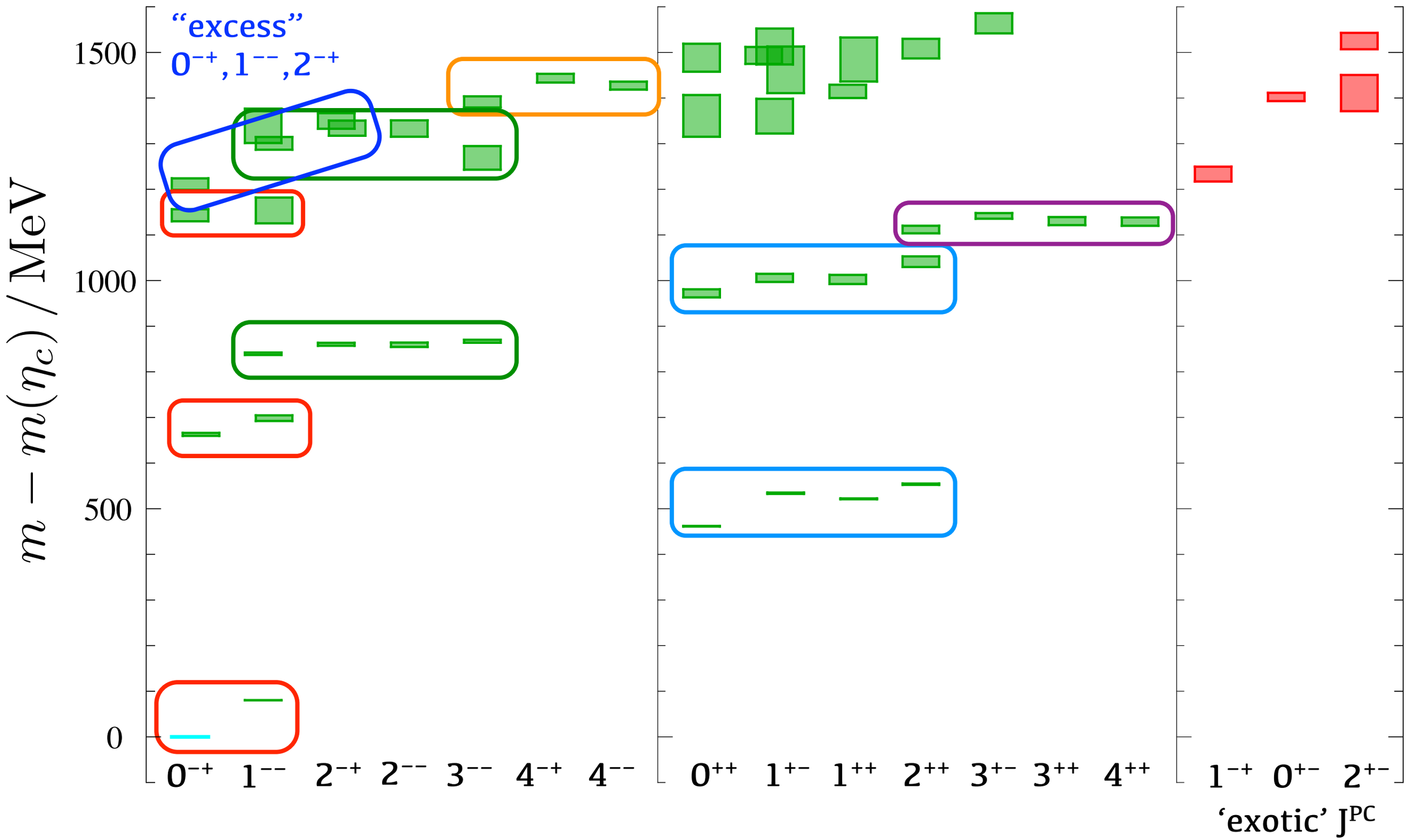
$^3S_1$

$^3S_1$

the charmonium spectrum from a lattice QCD calc

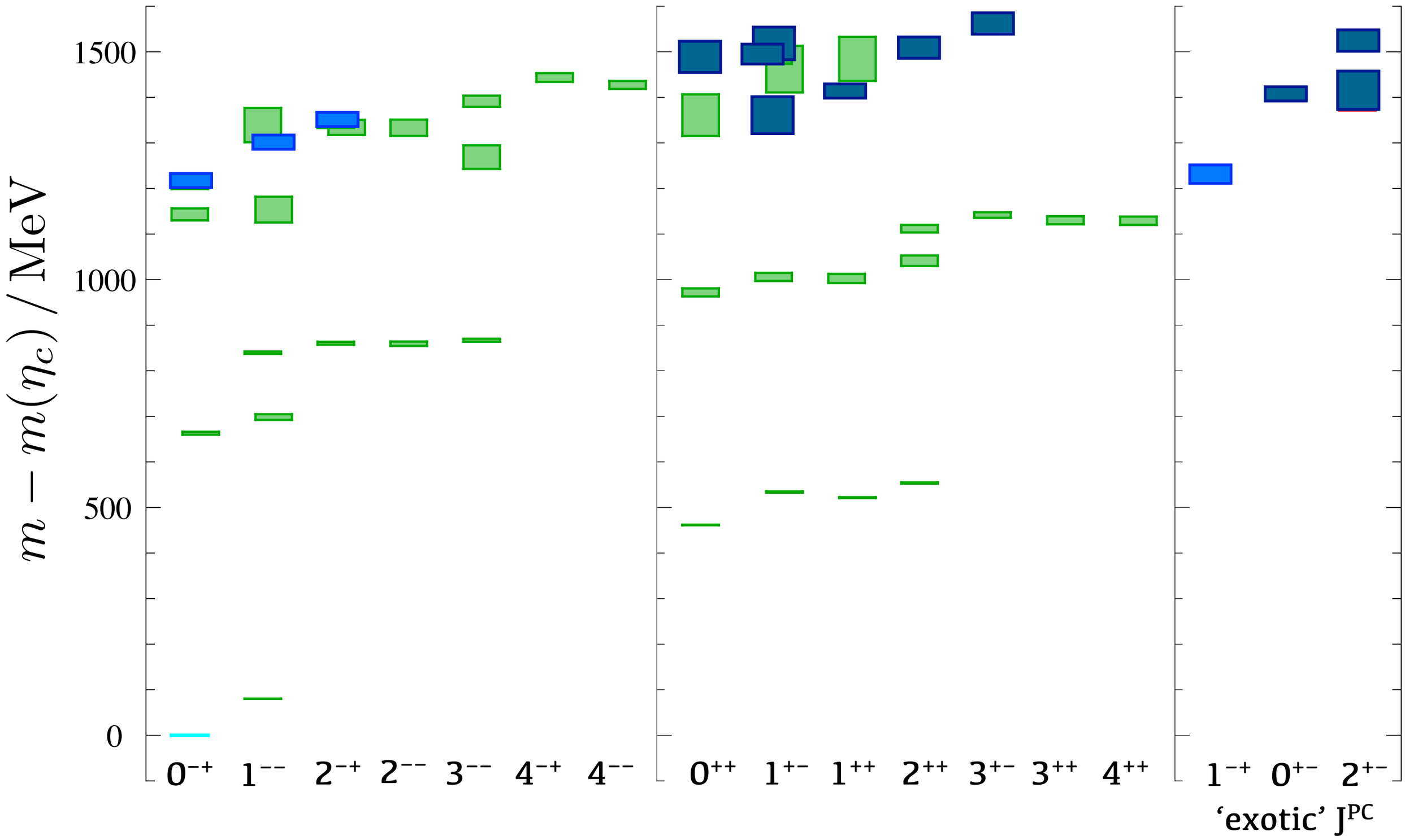
perform variational analysis in each quantum number

Hadron Spectrum Collaboration
arXiv:1204.5425



the charmonium spectrum from a lattice QCD calc

can isolate dominant hybrid character across the spectrum



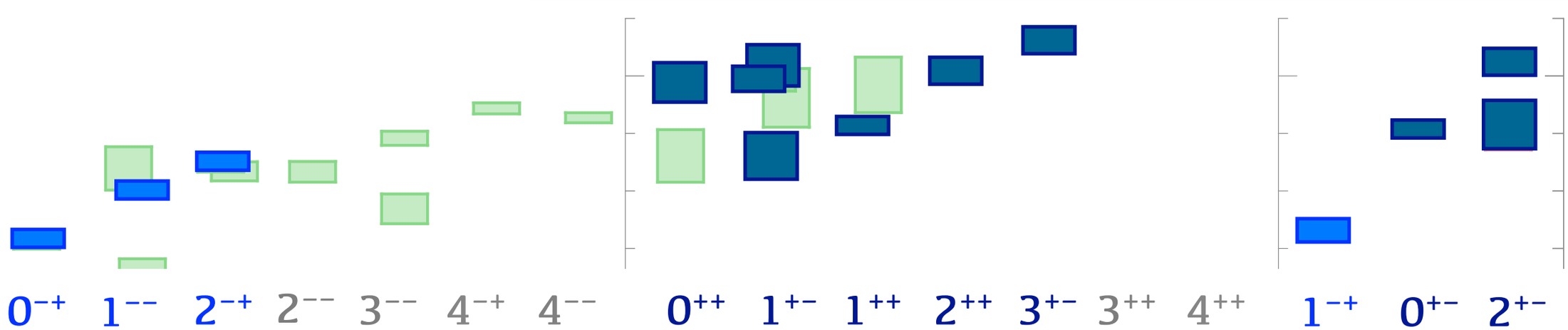
hybrid mesons

a phenomenology of hybrid mesons based upon QCD calculations

a chromomagnetic field configuration is lowest excitation

$$q\bar{q}_{\mathbf{8}}(^1S_0)B_{\mathbf{8}} \sim 0^{-+} \otimes 1^{+-} = 1^{--}$$
$$q\bar{q}_{\mathbf{8}}(^3S_1)B_{\mathbf{8}} \sim 1^{--} \otimes 1^{+-} = (0,1,2)^{-+}$$

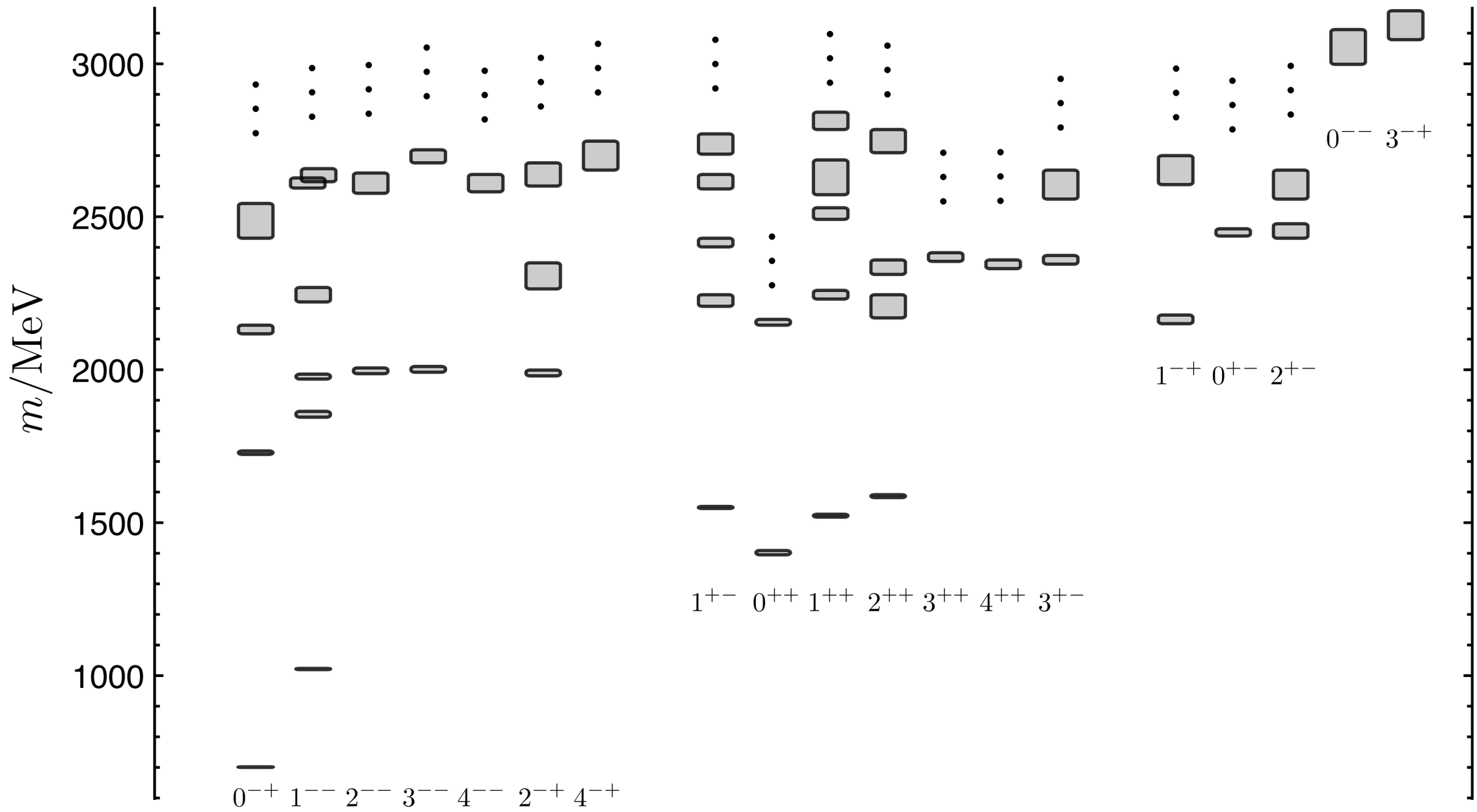
$$q\bar{q}_{\mathbf{8}}(^1P_1)B_{\mathbf{8}} \sim 1^{+-} \otimes 1^{+-} = (0,1,2)^{++}$$
$$q\bar{q}_{\mathbf{8}}(^3P_0)B_{\mathbf{8}} \sim 0^{++} \otimes 1^{+-} = 1^{+-}$$
$$q\bar{q}_{\mathbf{8}}(^3P_1)B_{\mathbf{8}} \sim 1^{++} \otimes 1^{+-} = (0,1,2)^{+-}$$
$$q\bar{q}_{\mathbf{8}}(^3P_2)B_{\mathbf{8}} \sim 2^{++} \otimes 1^{+-} = (1,2,3)^{+-}$$



lighter quarks - isovector mesons

three flavours of quark - all at the strange quark mass

m("π") ~ 700 MeV



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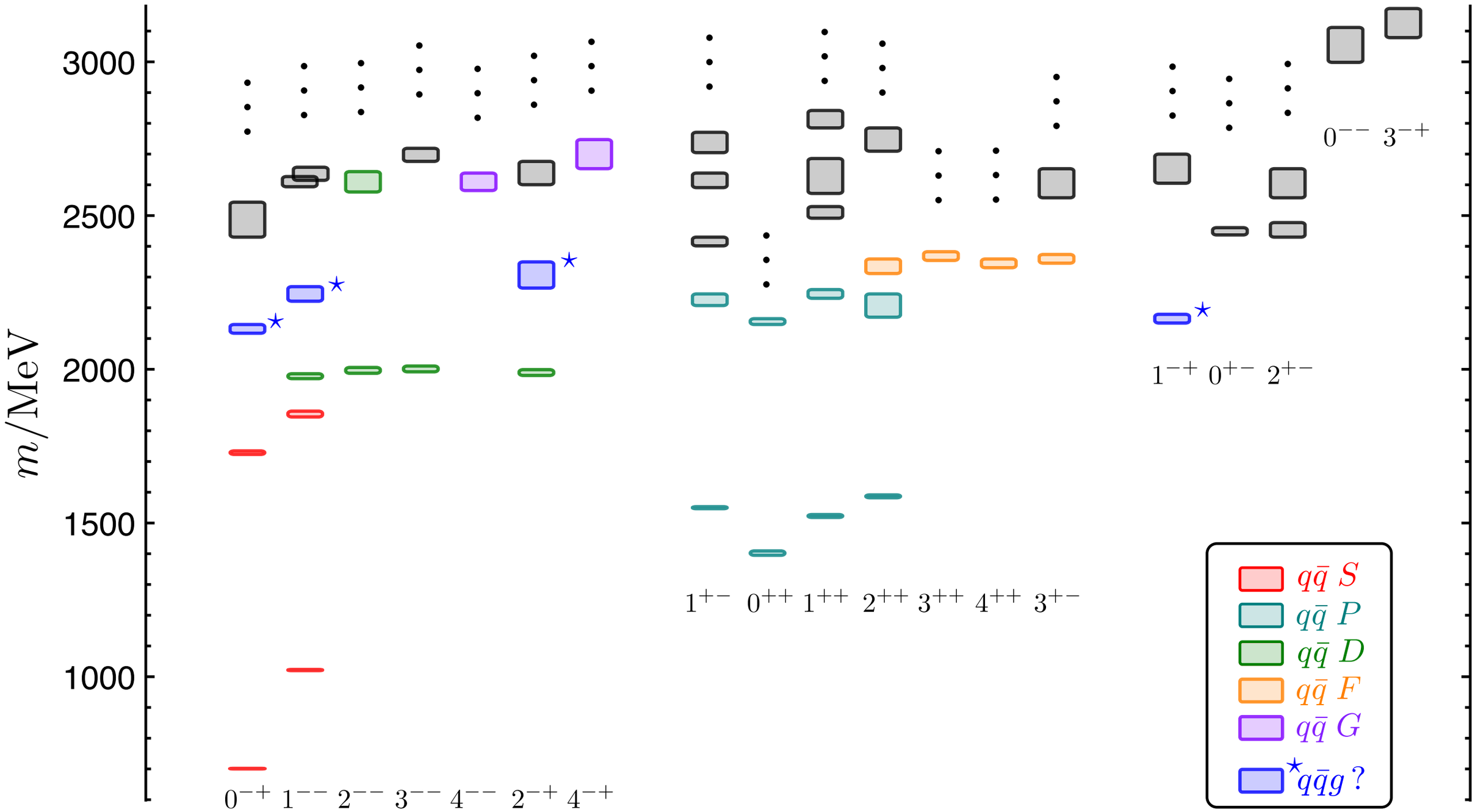
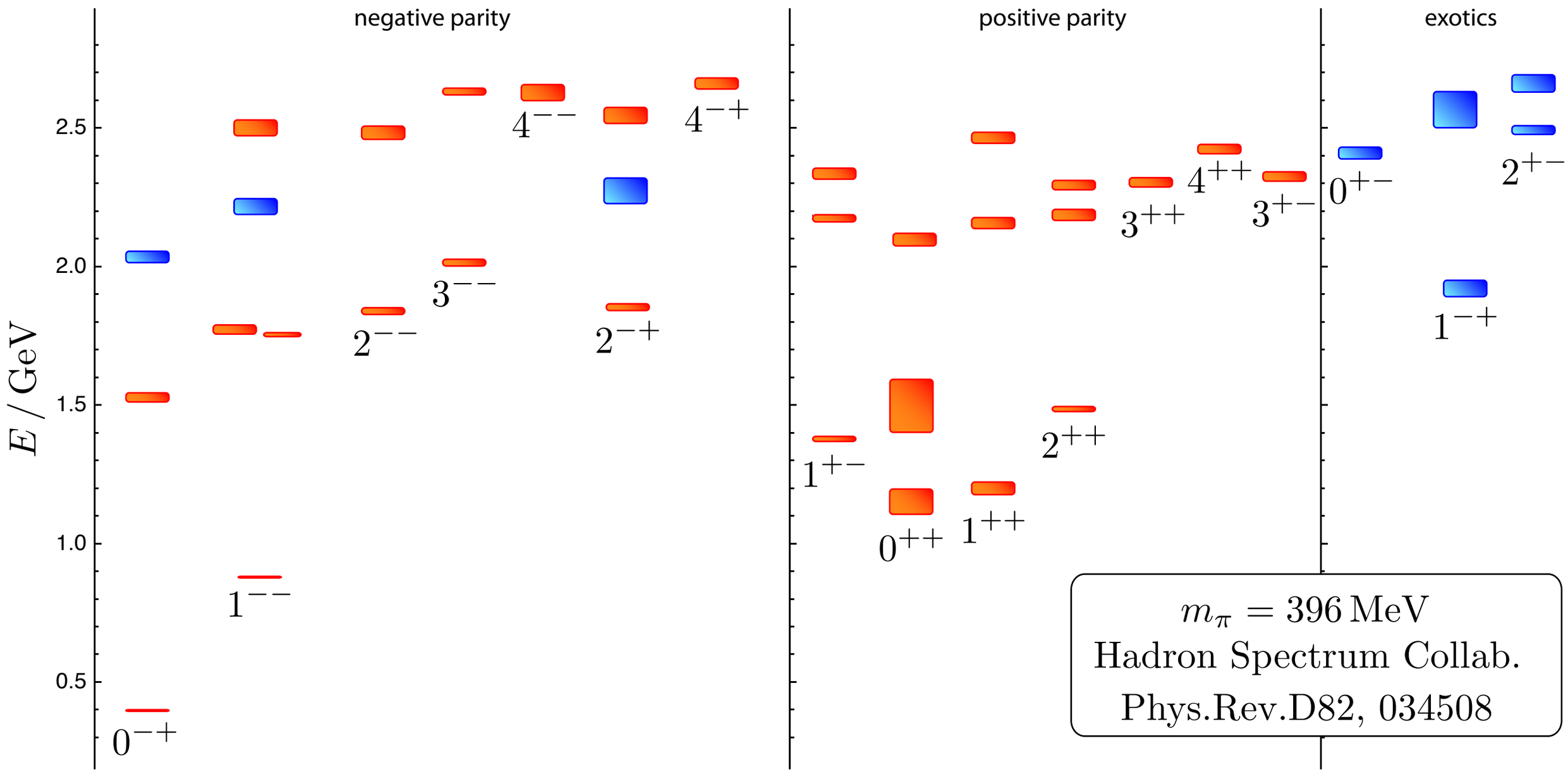
interpretations based on operator overlaps



lighter quarks - isovector mesons

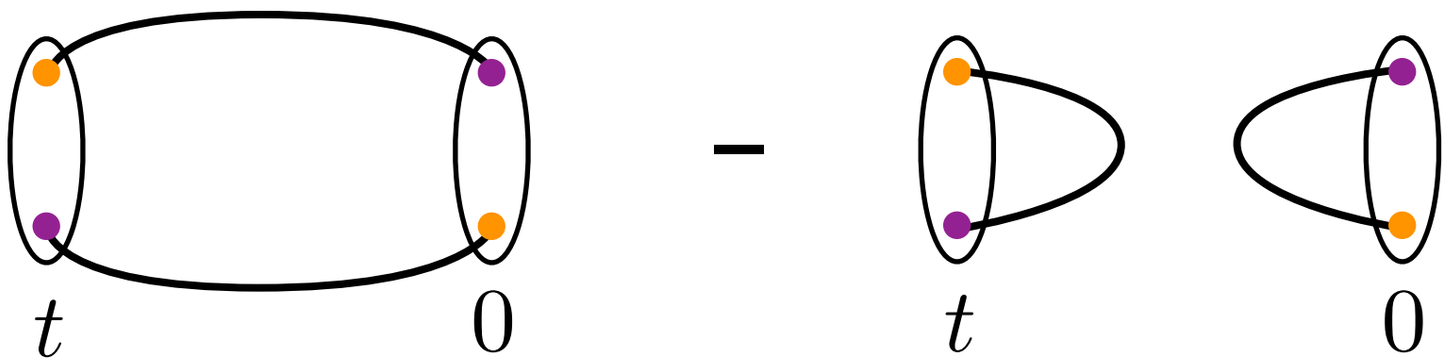
three flavours of quark
- degenerate up/down quarks
- correct strange quark mass

m(π) ~ 400 MeV



isoscalar mesons

difference w.r.t. isovector mesons is addition of ‘disconnected’ diagrams



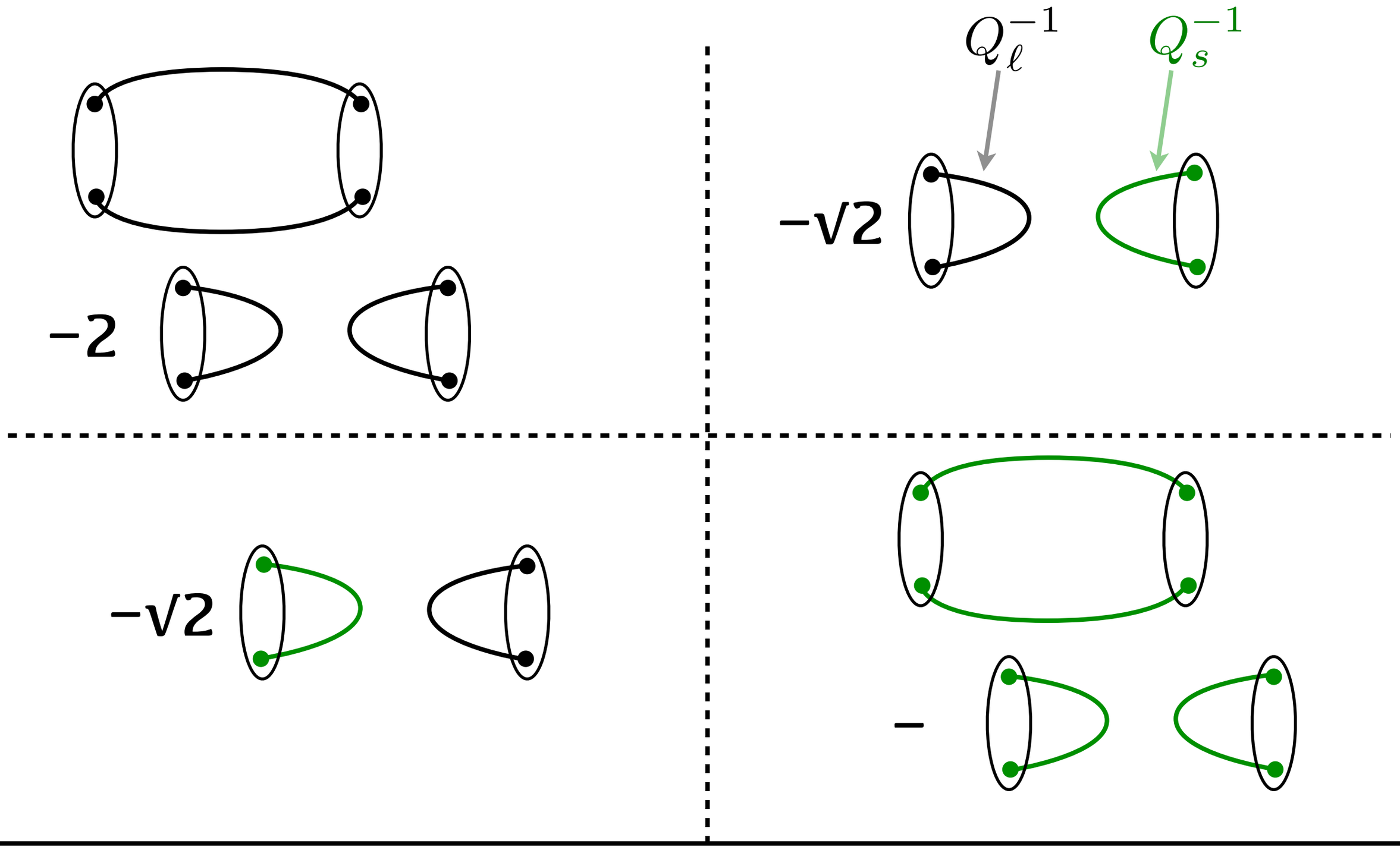
$$\bar{\psi}\mathbf{\Gamma}'\psi(t)\cdot\bar{\psi}\mathbf{\Gamma}\psi(0) \quad - \quad \bar{\psi}\mathbf{\Gamma}'\psi(t)\cdot\bar{\psi}\mathbf{\Gamma}\psi(0)$$

$$\mathrm{tr}\left[Q^{-1}_{t,0}\mathbf{\Gamma}Q^{-1}_{0,t}\mathbf{\Gamma}'\right] \qquad \mathrm{tr}\left[Q^{-1}_{t,t}\mathbf{\Gamma}'\right]\,\mathrm{tr}\left[Q^{-1}_{0,0}\mathbf{\Gamma}\right]$$

challenging using
‘traditional’ methods

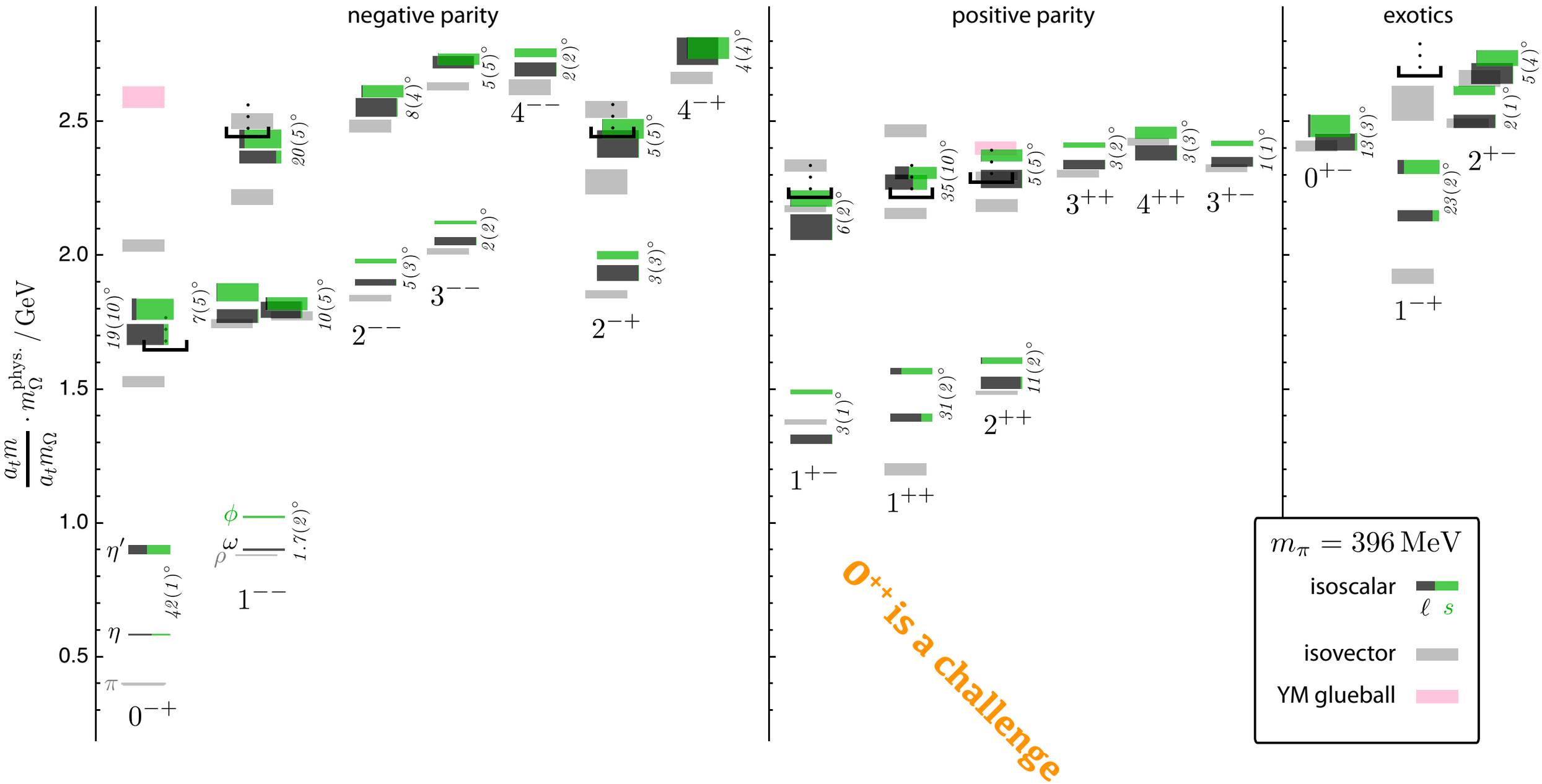
isoscalar mesons

hidden 'light' and hidden 'strange' can mix $\frac{1}{\sqrt{2}}\left(u\bar{u}+d\bar{d}\right)$ $s\bar{s}$



isoscalar mesons

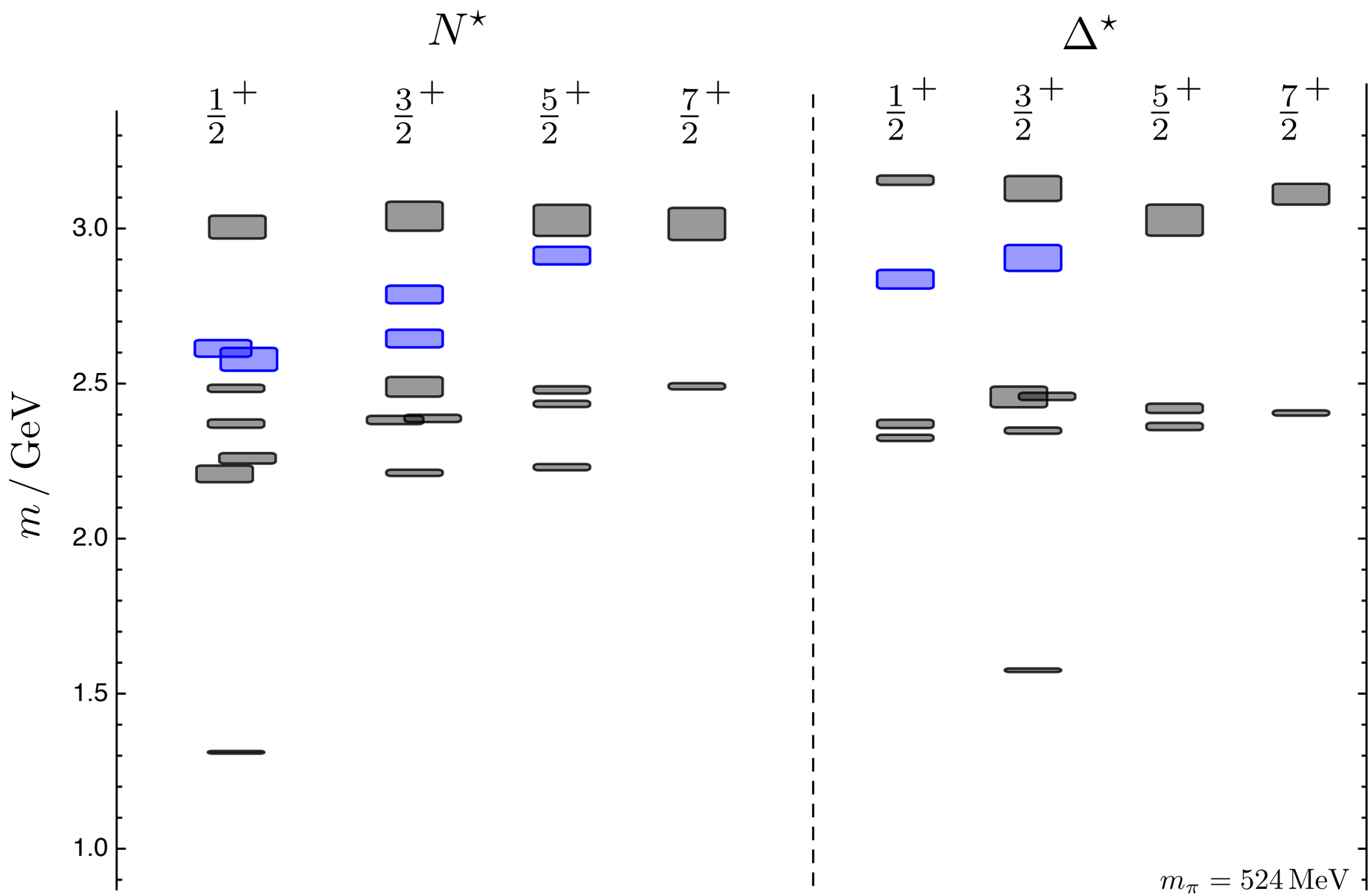
Hadron Spectrum Collaboration
PRD83 111502 (2011)



baryons

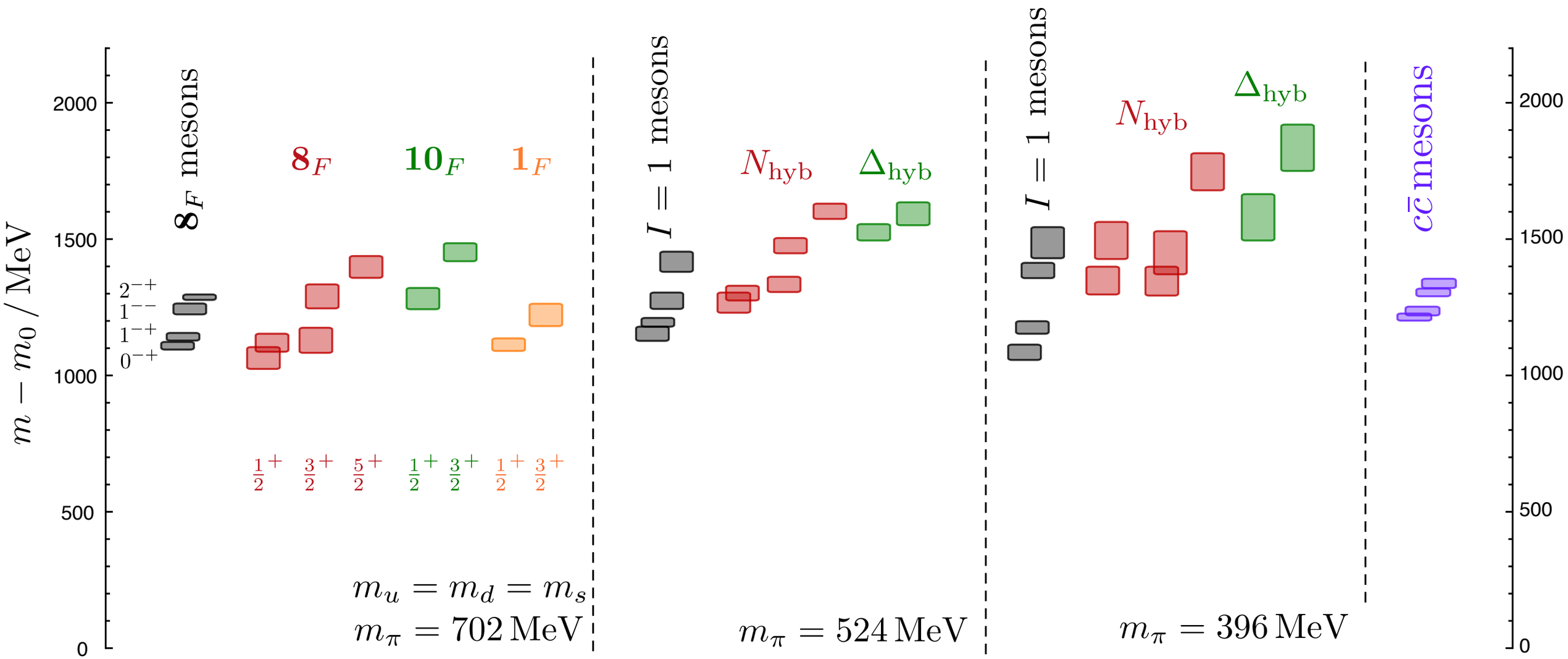
analogous large basis of operators for baryons - three quark fields respecting permutation (anti-)symmetry

Hadron Spectrum Collaboration
PRD84 074508 (2011)
PRD85 054016 (2012)



hybrids (mesons and baryons)

approximately remove the 'quark mass' contribution to the hybrid mass



light hybrid mesons - m_ρ
light hybrid baryon - m_N
charmonium hybrids - m_{η_c}

chromomagnetic gluonic excitation scale ~ 1.3-1.4 GeV ?

a **model dependent interpretation** of lattice QCD calculations

the resonance spectrum of QCD

or,

"where are you hiding the scattering amplitudes?"

real QCD

real QCD has very few stable particles : $\pi,\ K,\ N,\ \Sigma,\ \Lambda,\ \Xi,\ \Omega$

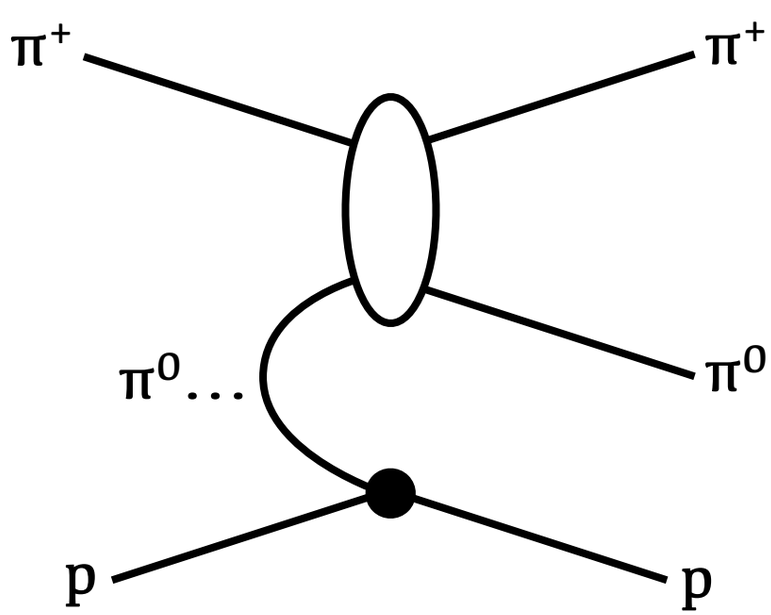
'states' like ρ , Δ … are **resonances**

asymptotic states of the theory include multi-pion states

e.g. in the CM frame $\int d\hat{p}\ Y_L^m(\hat{p})\ \left|\pi(\vec{p})\pi(-\vec{p})\right\rangle$

with $\left|\vec{p}\right|$ varying continuously up from zero

e.g. ππ scattering in isospin-1



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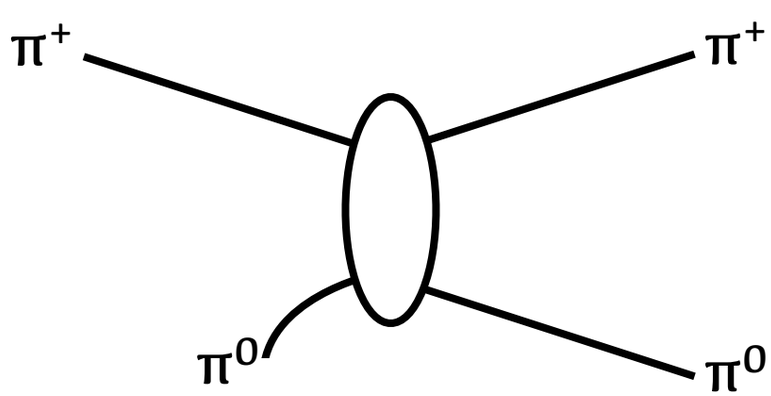
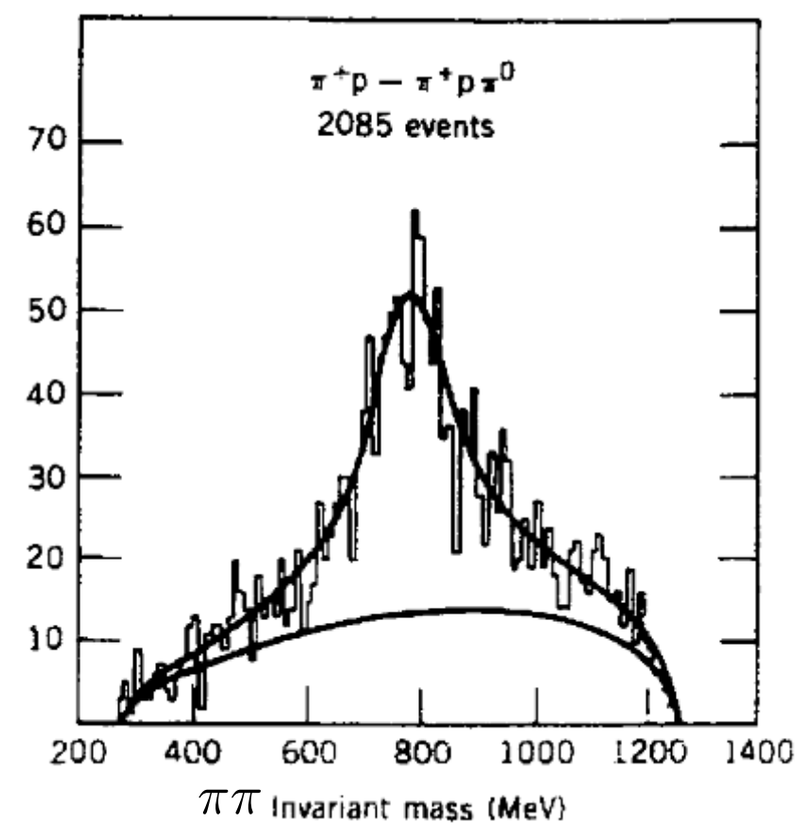


for small t



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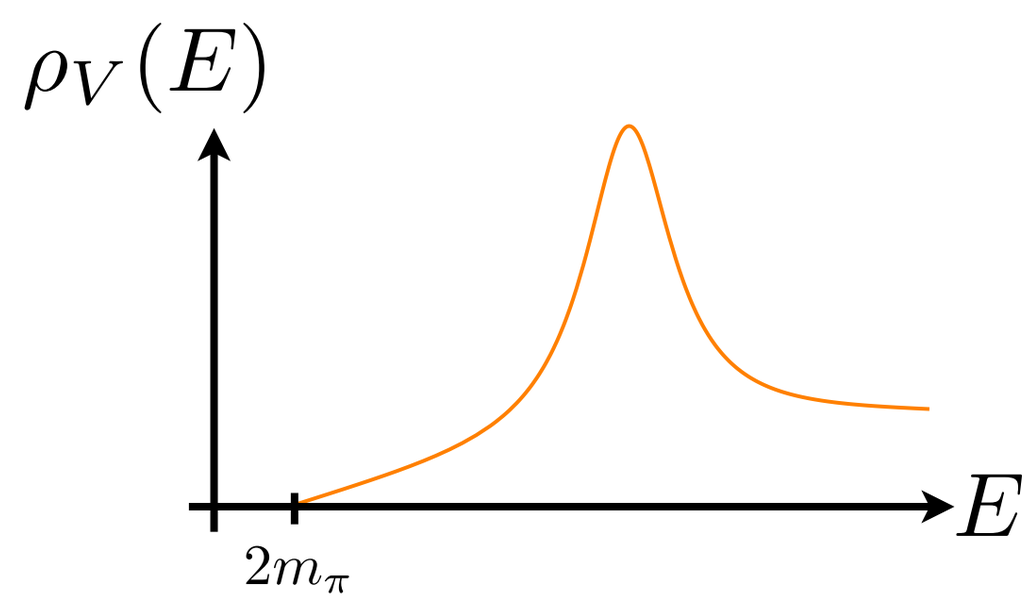
with $\left|\vec{p}\right|$ varying continuously up from zero

within the field-theory we have correlators

e.g. the (Euclidean) vector correlator

$$C_V(t) = \int dE\ e^{-Et}\ \rho_V(E)$$

spectral function



the spectrum is continuous !

real QCD

real QC s : π, K, N, Σ, Λ, Ξ, Ω

'states'

asym clude multi-pion states

$Y_L^m(\hat{p}) \left|\pi(\vec{p})\pi(-\vec{p})\right\rangle$

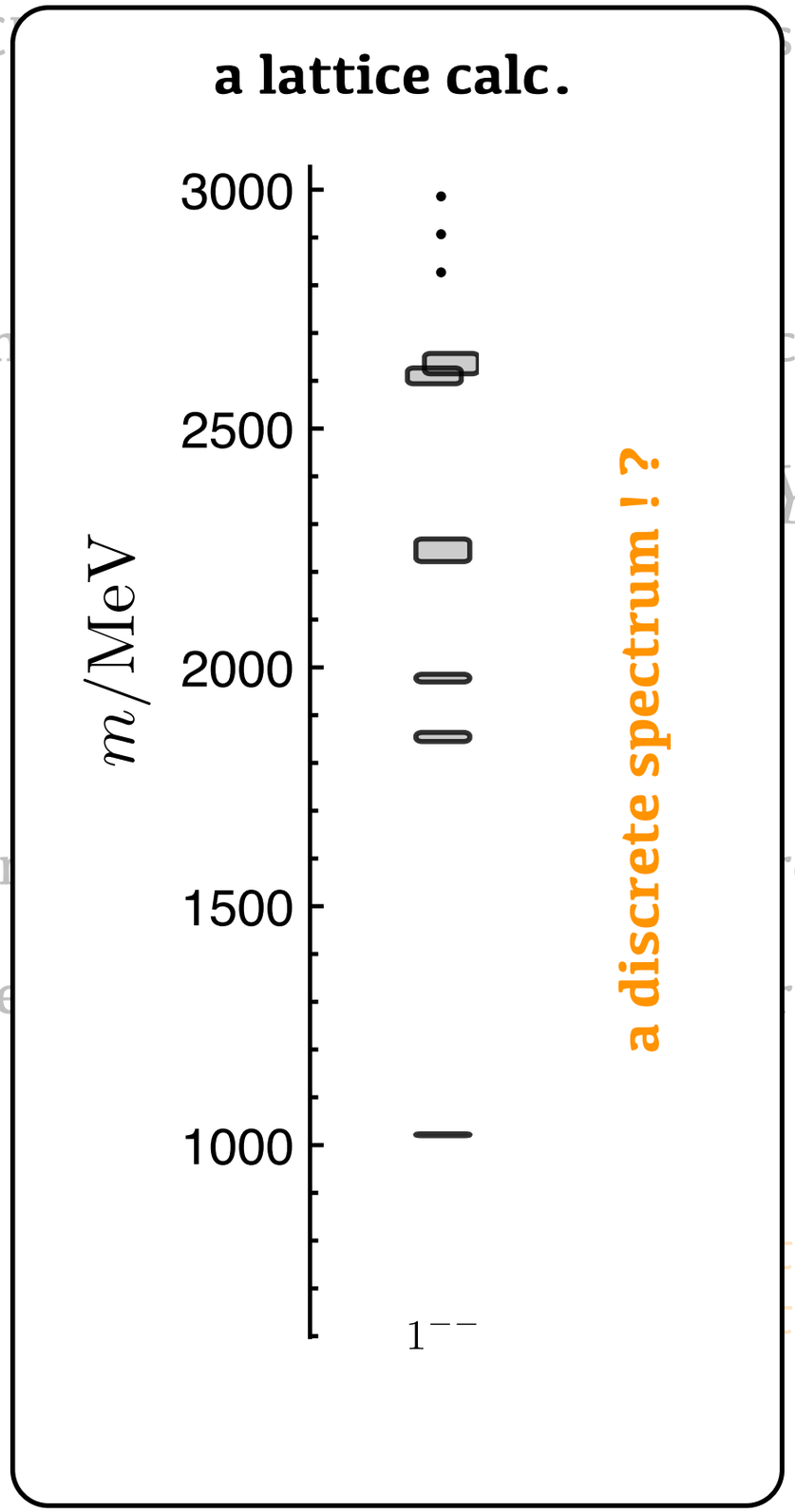
with $\left|\vec{p}\right|$ varying continuously up from zero

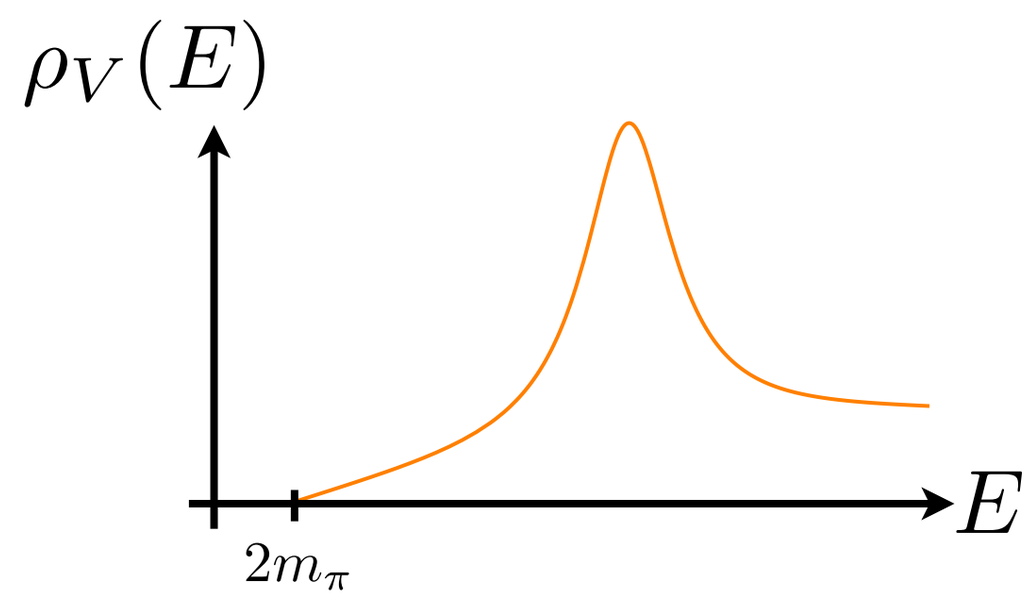
within relators

e relator

tral

tion

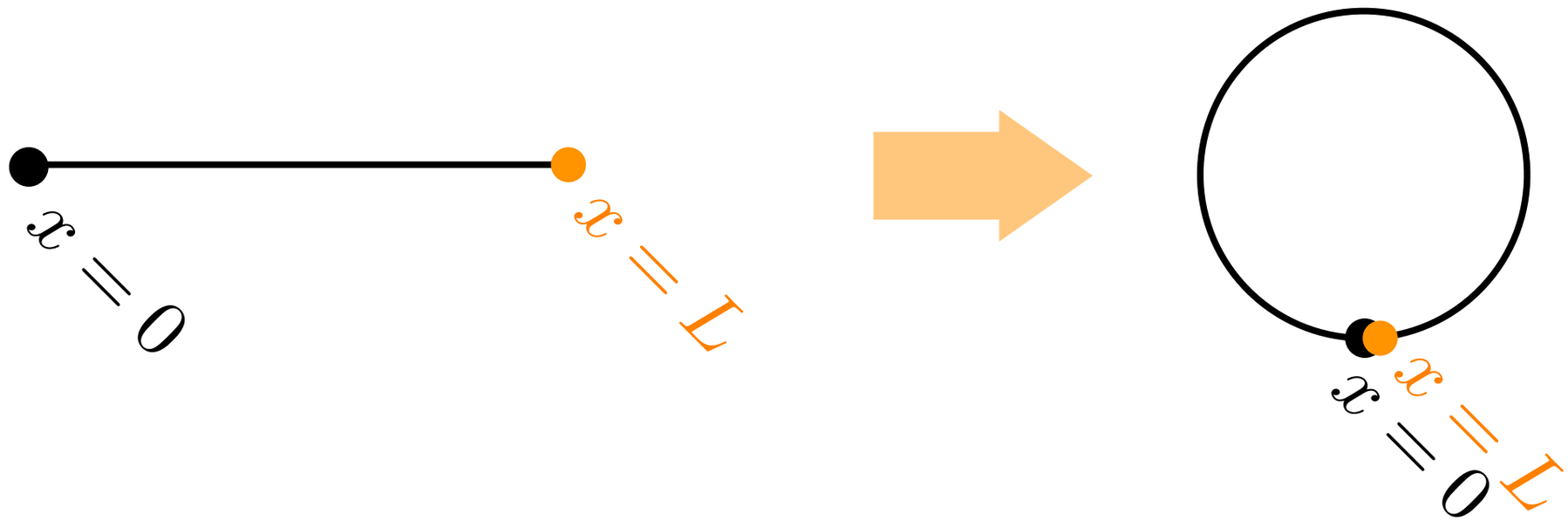




the spectrum is continuous !

field theory in a finite volume

consider the case of one space dimension with a periodic boundary condition



this will make the allowed momenta of a free particle discrete :

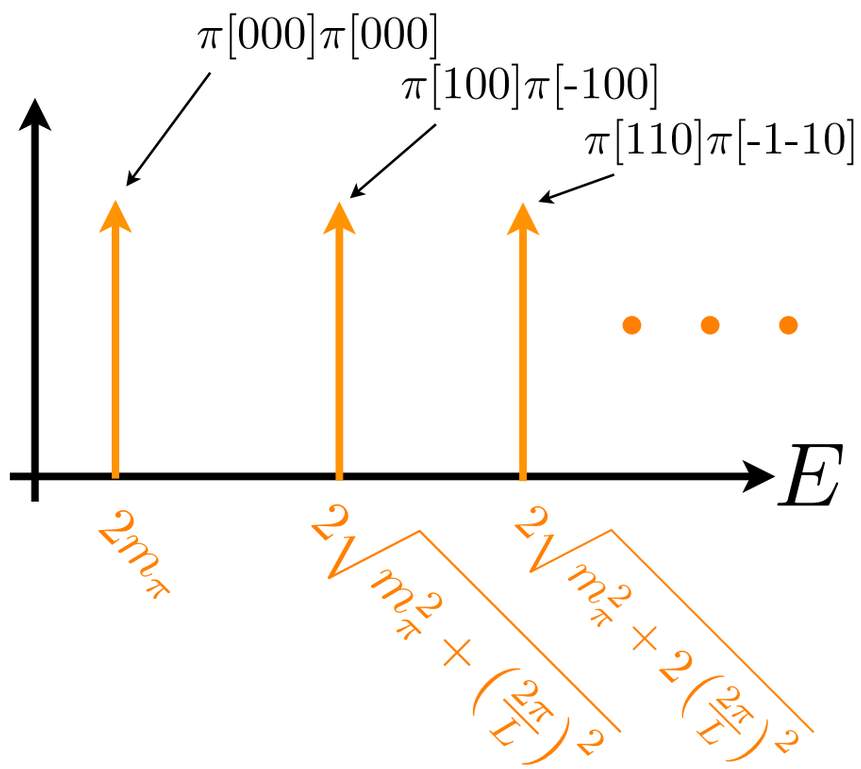
$$\psi_p(x) = e^{ipx} \quad \text{free particle}$$

$$\psi_p(x) = \psi_p(x + L) \quad \text{periodic boundary condition}$$

$$e^{ipL} = 1 \implies \boxed{p = \frac{2\pi}{L} n} \quad \text{for integer } n$$

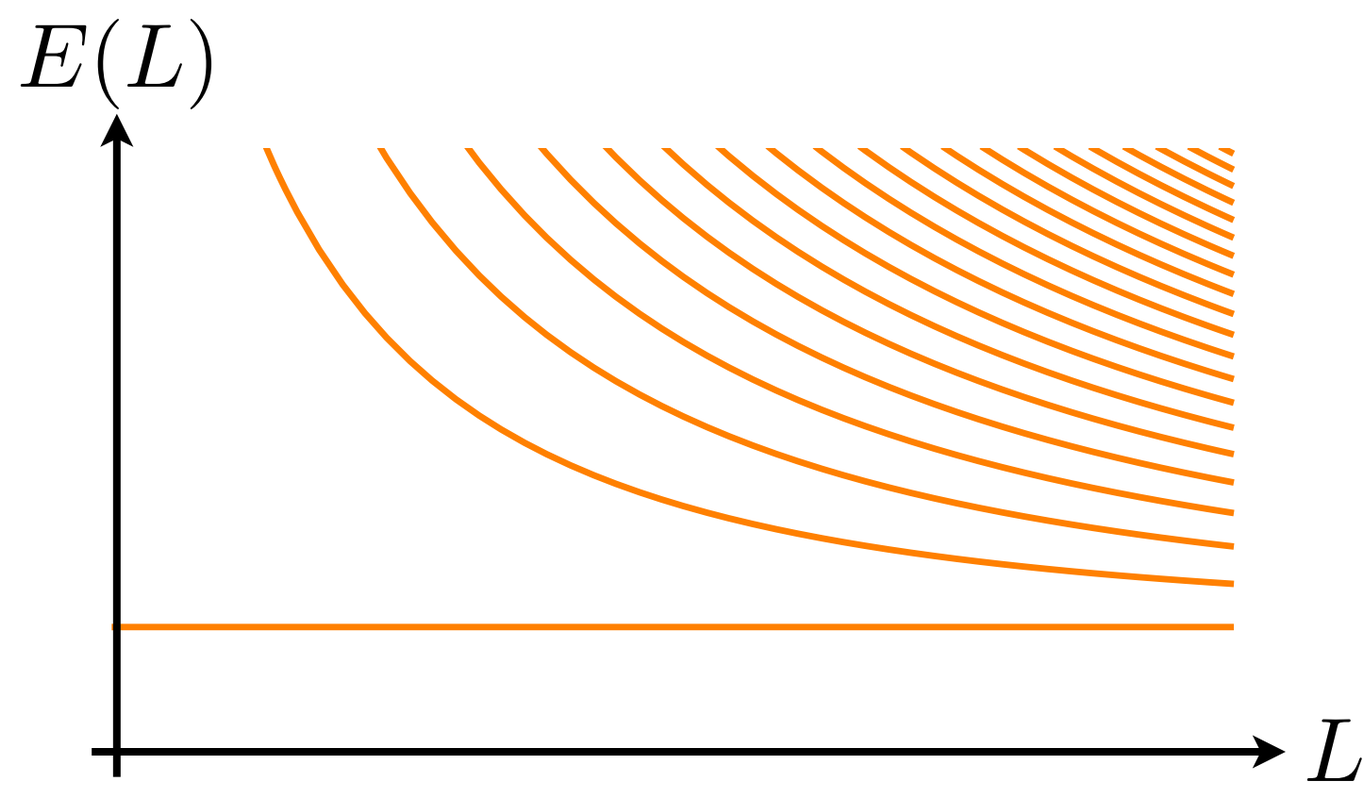
non-interacting two-particle states in a finite volume

in a three-dimensional cubic box $\vec{p} = \frac{2\pi}{L}\left[n_x, n_y, n_z\right]$



non-interacting
spectral function

the spectrum is discrete and volume-dependent



two-particle states in a finite volume

but hadrons do interact, and sometimes strongly

e.g. $\pi\pi \to \rho \to \pi\pi$

$\pi N \to \Delta \to \pi N$

what is the impact of these interactions on the finite-volume spectrum ?

two-particle states in a finite volume

e.g. non-rel quantum mechanics in one-dimension

two spinless bosons separated by x

interacting through a potential $V(x)$

in center-of-momentum frame

scattering solutions of

$$-\frac{1}{m}\frac{d\psi}{dx^2} + V(x)\psi = E\psi$$

$$E = \frac{k^2}{m} > 0$$

$$\psi(x \to \infty) = N \cos\left[k|x| + \delta(k)\right]$$

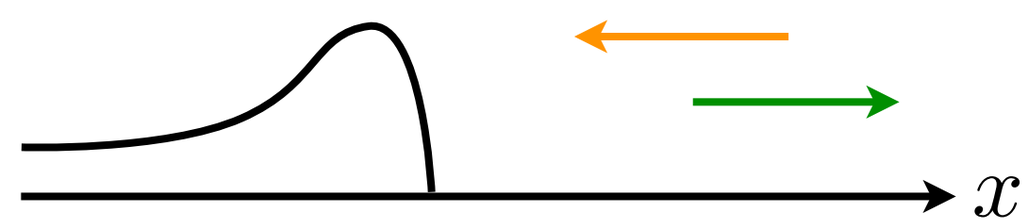
scattering
'phase-shift'

scattering phase-shift contains
all the information about elastic scattering

$$x > 0 \qquad \underbrace{\left(\frac{N}{2}e^{-i\delta}\right)}_{\text{normalisation}} \left(e^{-ikx} + e^{2i\delta(k)} e^{ikx}\right)$$

wave incoming
from x→+∞

reflected wave



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$$\psi(x \to \infty) = N \cos\left[k|x| + \delta(k)\right]$$

finite length world $-L/2 < x < L/2$ with periodic b.c.

wavefunction and derivative continuous at boundary $\frac{1}{\psi}\frac{d\psi}{dx}\Big|_{x=-L/2} = \frac{1}{\psi}\frac{d\psi}{dx}\Big|_{x=+L/2}$

$$-k\tan\left[\frac{kL}{2} + \delta(k)\right] = k\tan\left[\frac{kL}{2} + \delta(k)\right]$$

$$\tan\left[\frac{kL}{2} + \delta(k)\right] = 0$$

$$kL + 2\delta(k) = 0 \mod 2\pi$$

two-particle states in a finite volume

e.g. quantum mechanics in one-dimension

$$kL + 2\delta(k) = 0 \mod 2\pi$$

discrete & **volume-dependent** spectrum of scattering states

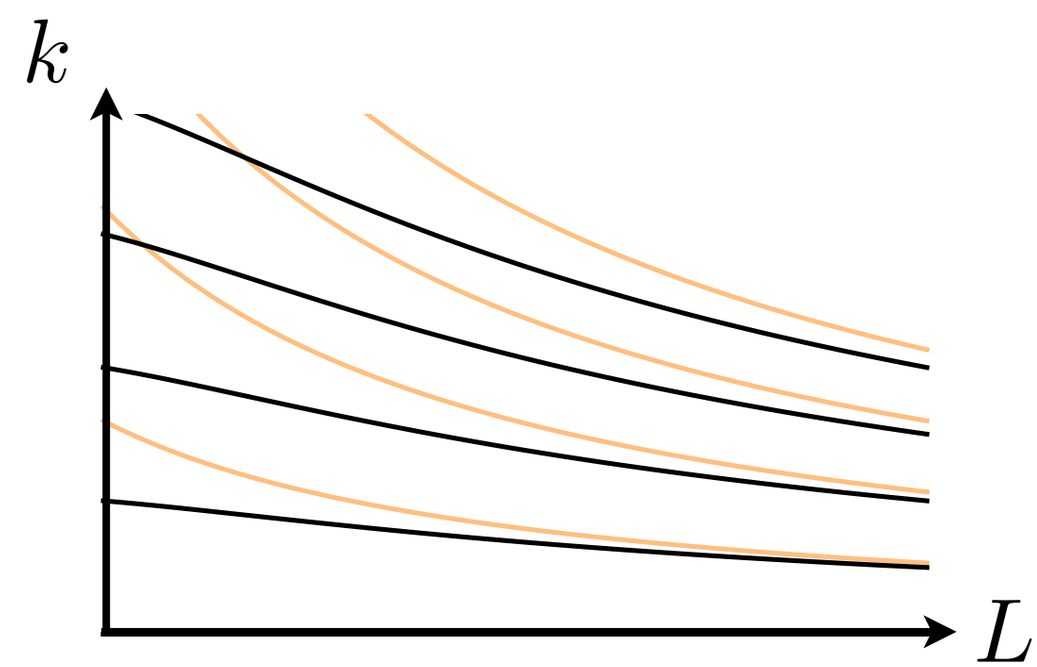
$$E = \frac{k^2}{m} \qquad k = \frac{2\pi}{L}\left(n - \frac{\delta(k)}{\pi L}\right)$$

non-interacting momentum

shift due to interaction

e.g. a weak attraction

$$\delta(k) = a\,k$$



two-particle states in a finite volume

the analogous expression for two-particle elastic scattering
in a finite cubic volume has been derived by Lüscher

somewhat complicated by the lack of full rotational symmetry (a cube)

for our purposes, we'll pretend that it's as simple as

$$\delta_\ell(E) = f_\ell(E, L)$$

phase-shift in partial wave ℓ
at scattering energy E

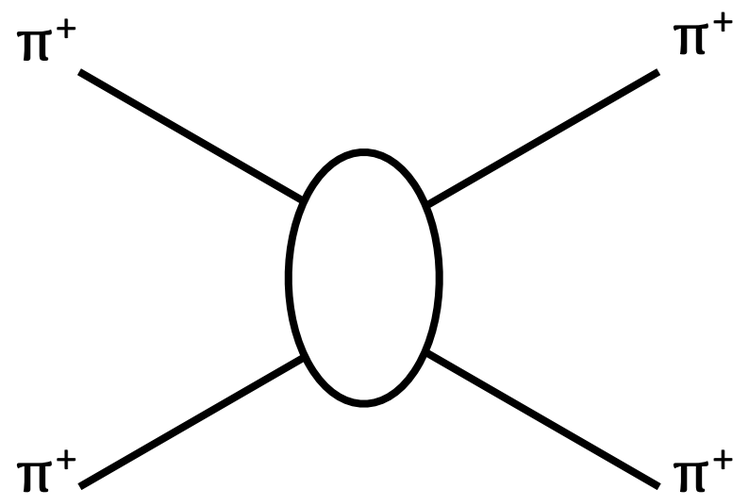
known function of
energy and box length

so we 'measure' $E_{\mathfrak{n}}$ on one or more volumes

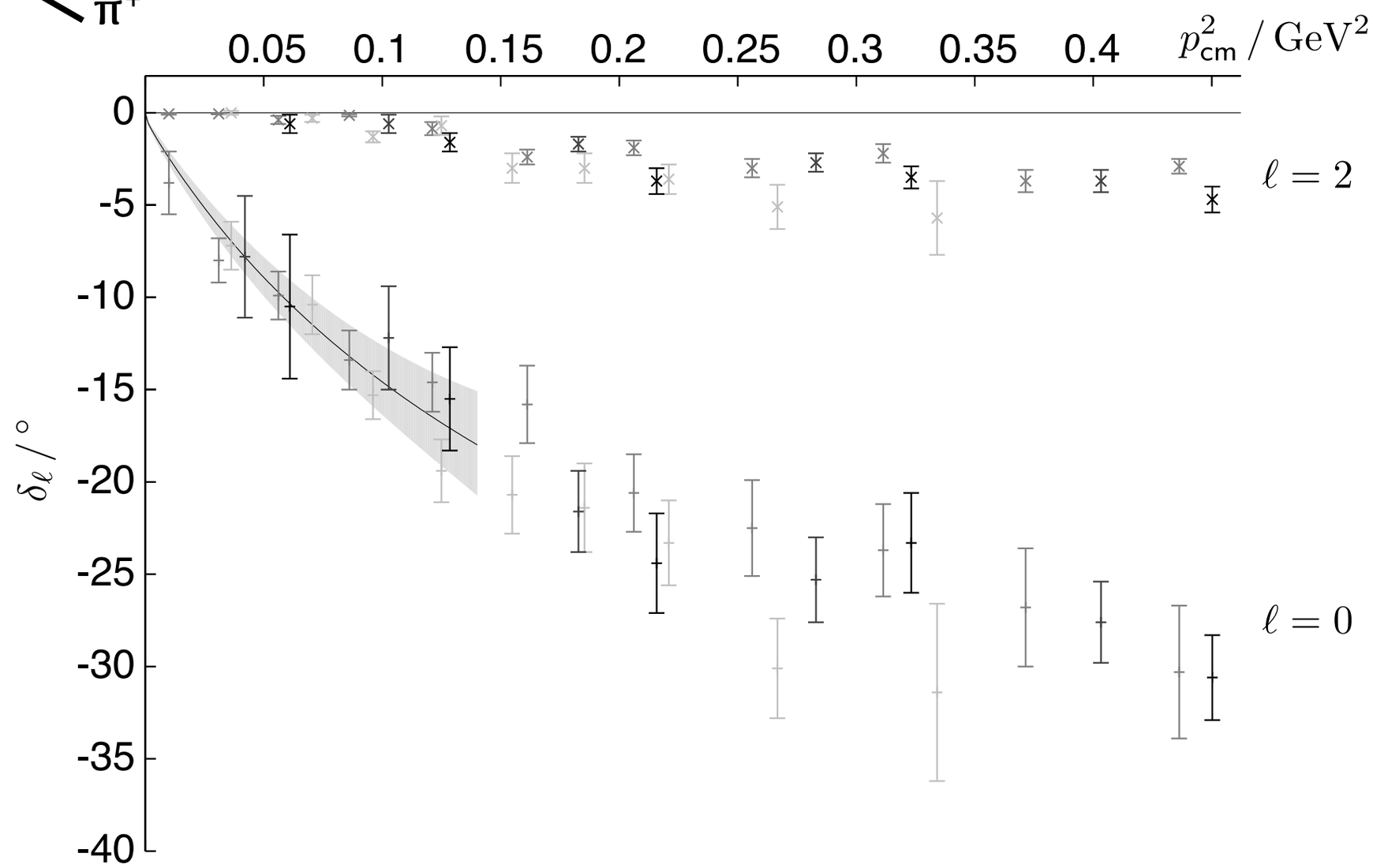
and plug into the formula to determine δ at discrete energies

ππ isospin-2 scattering

e.g.



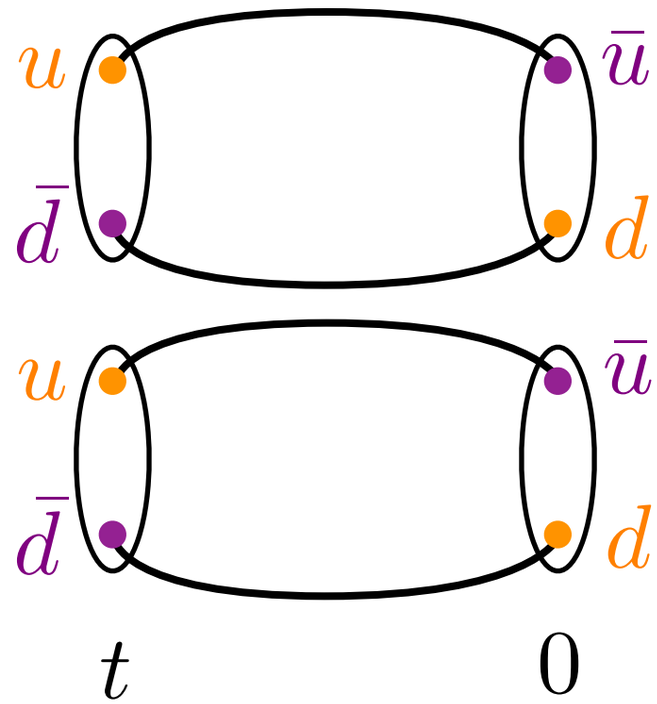
empirically: weak & repulsive scattering

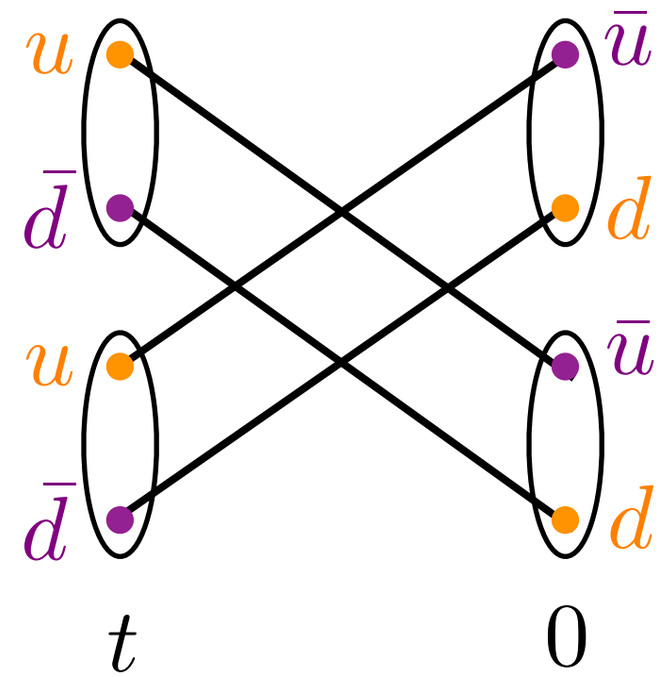


ππ isospin-2 scattering - field theory calculation

$$C(t) = \langle 0 | \mathcal{O}^\dagger_{\pi^+}(t) \mathcal{O}^\dagger_{\pi^+}(t) \cdot \mathcal{O}_{\pi^+}(0) \mathcal{O}_{\pi^+}(0) | 0 \rangle$$

no quark-annihilation
in these correlators





variational in a basis
of operators :

"ππ" of various
relative momenta

e.g. $\vec{P} = [000]$

$\pi_{[000]}\pi_{[000]}$

$\pi_{[100]}\pi_{[-100]}$

$\pi_{[110]}\pi_{[-1-10]}$

$\pi_{[111]}\pi_{[-1-1-1]}$

$\vdots$

ππ isospin-2 scattering

Hadron Spectrum Collaboration
arXiv:1203.6041

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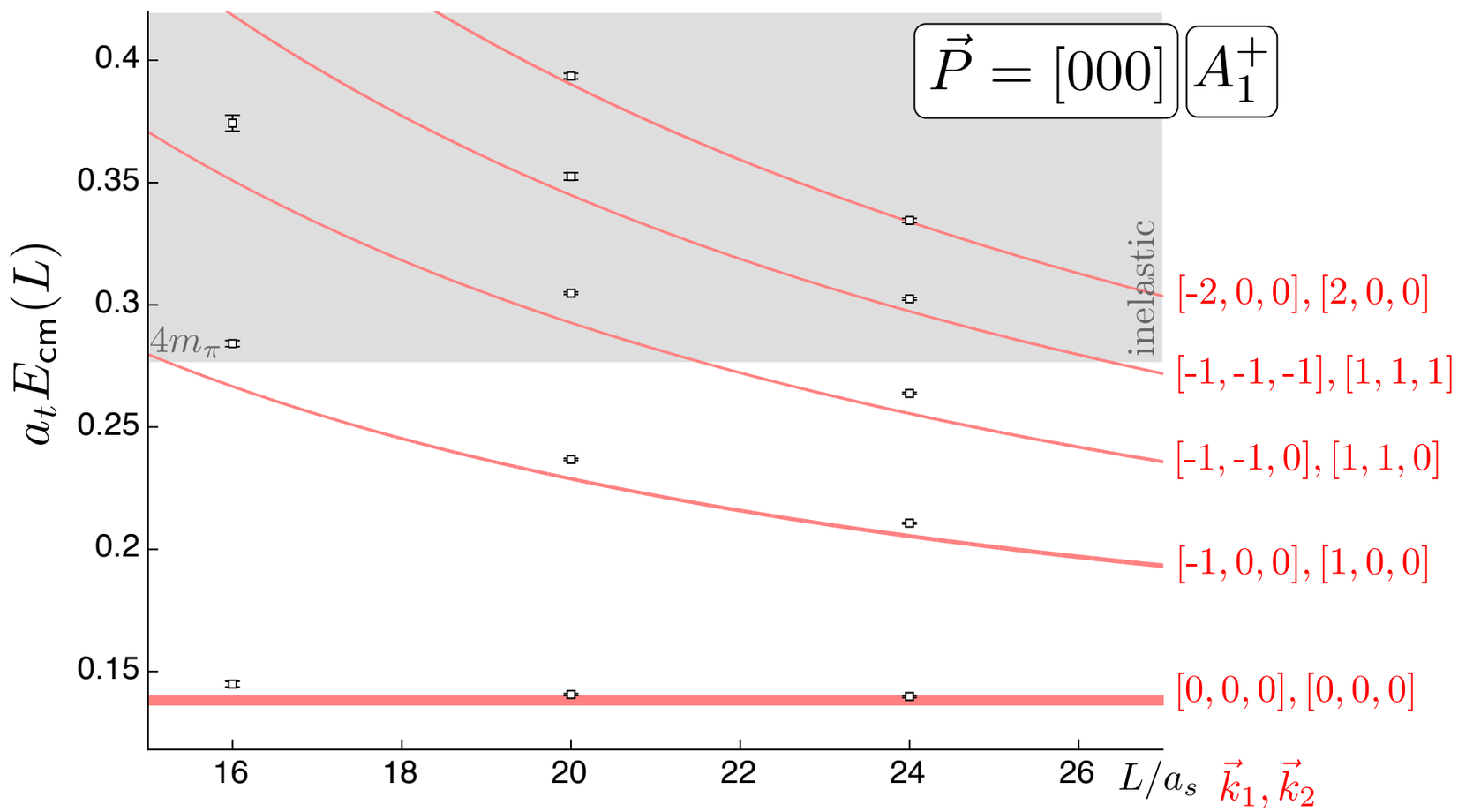
$\pi_{[111]}\pi_{[-1-1-1]}$

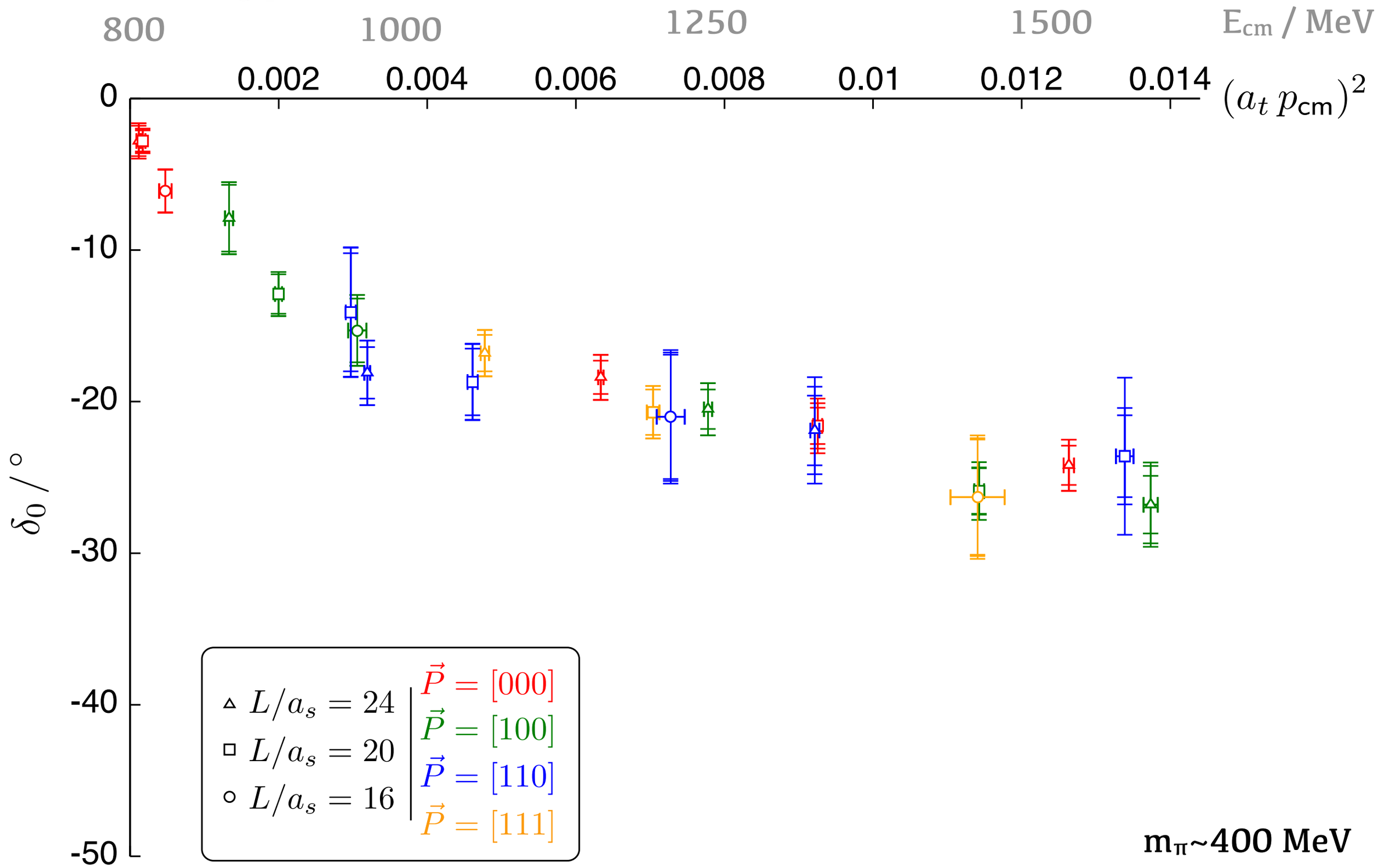
$\vdots$

extracted energies shifted upward slightly from non-interacting pion pairs



ππ isospin-2 scattering

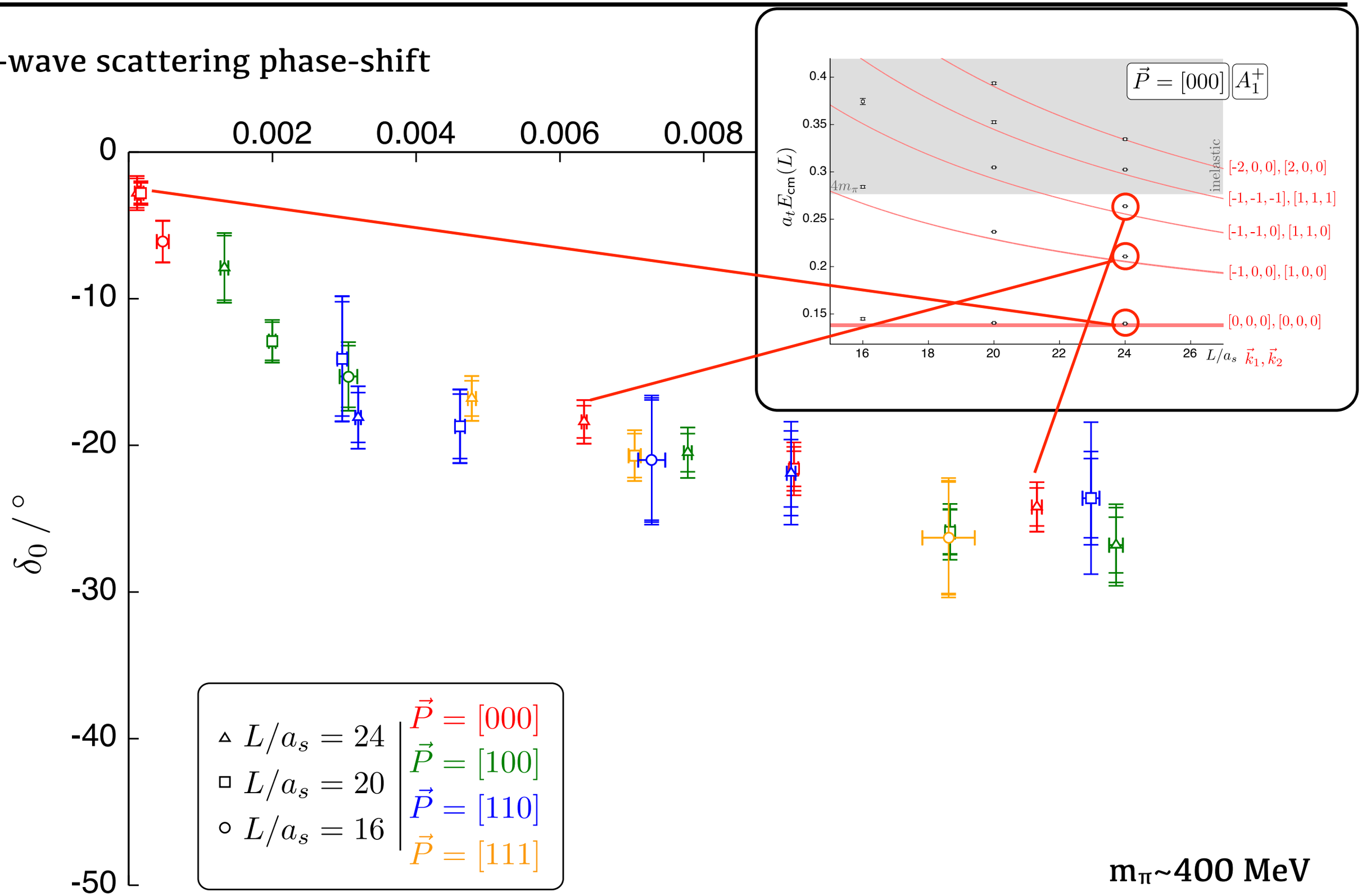
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

S-wave scattering phase-shift



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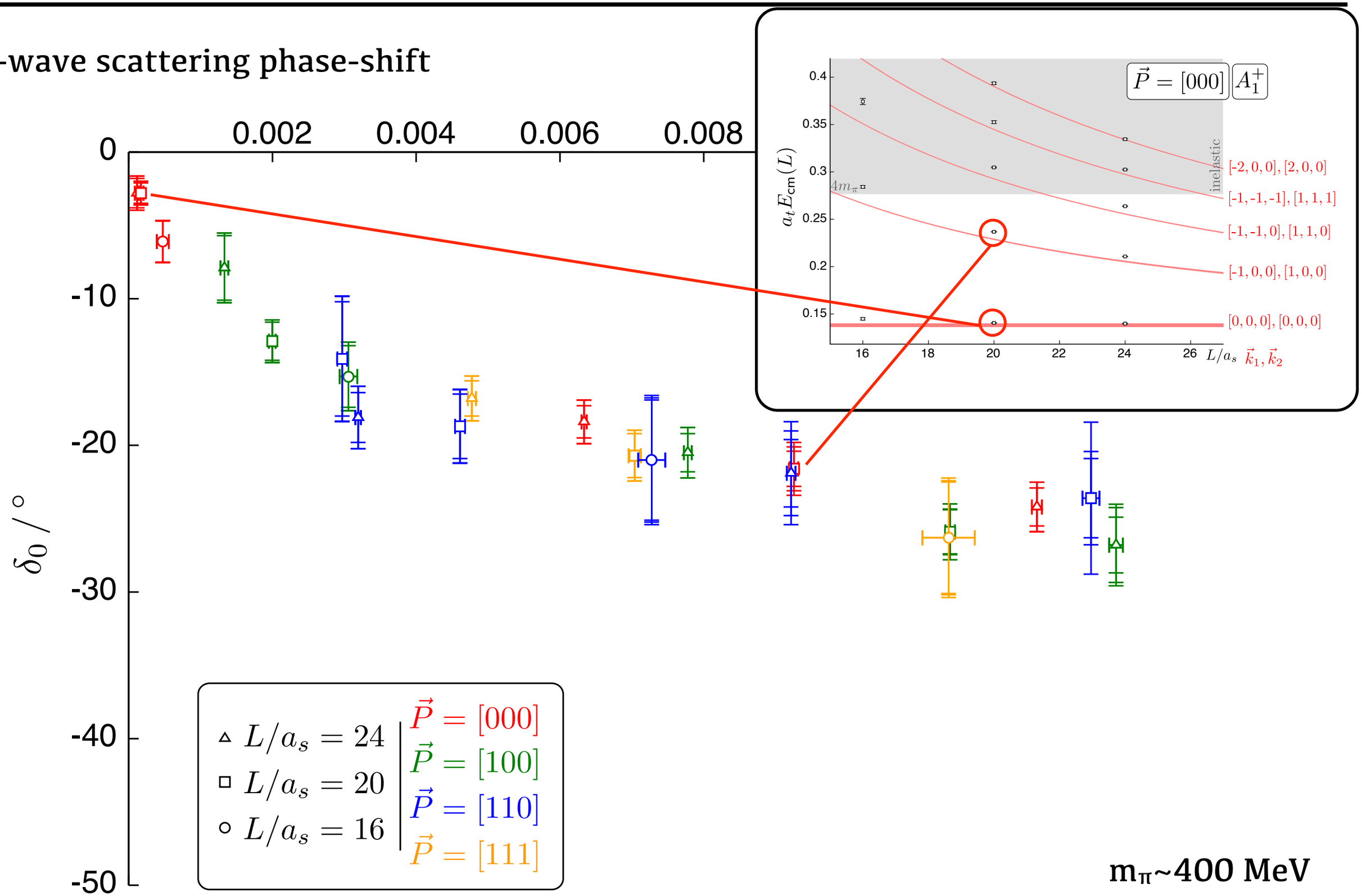

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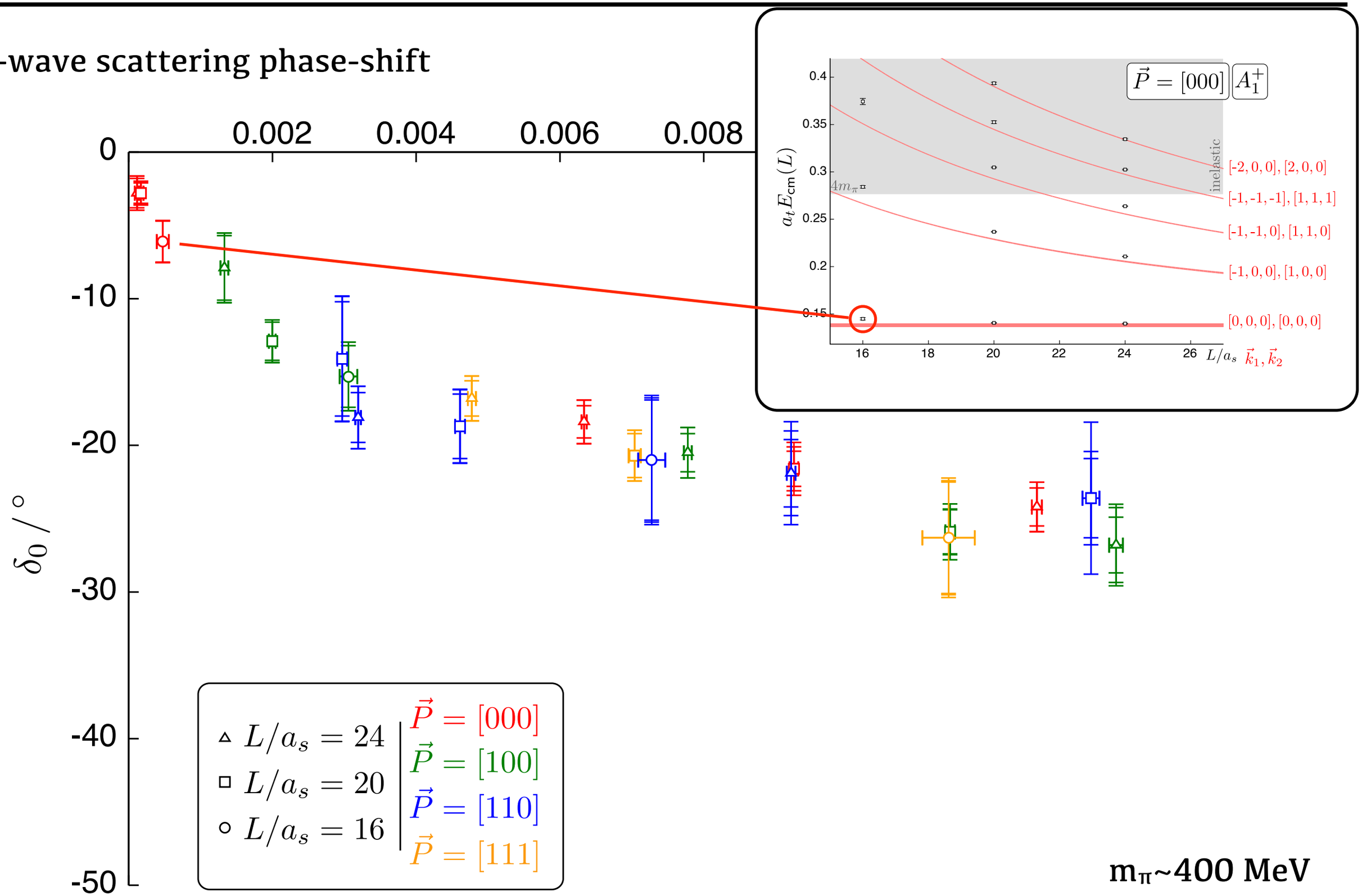
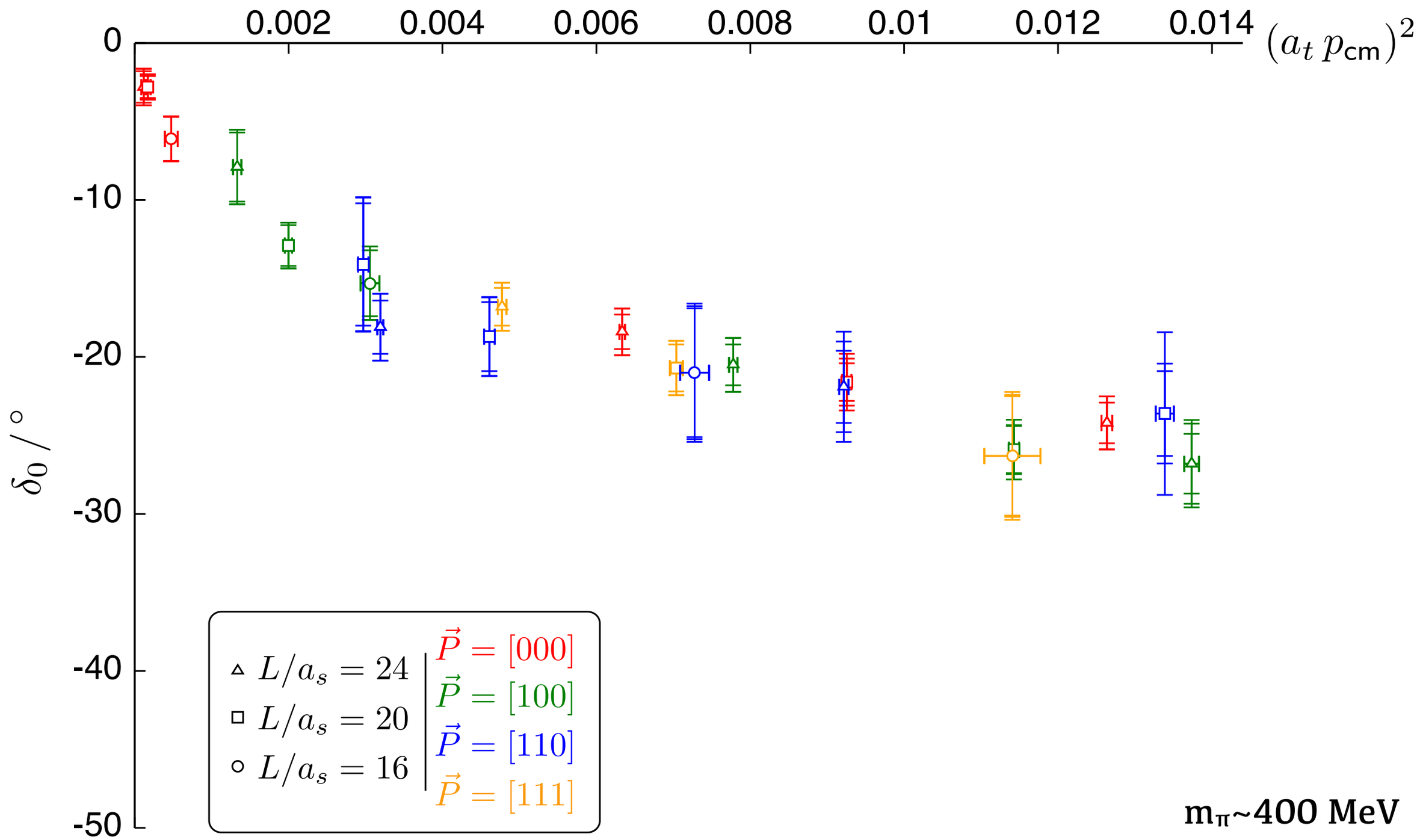


m_π~400 MeV

ππ isospin-2 scattering

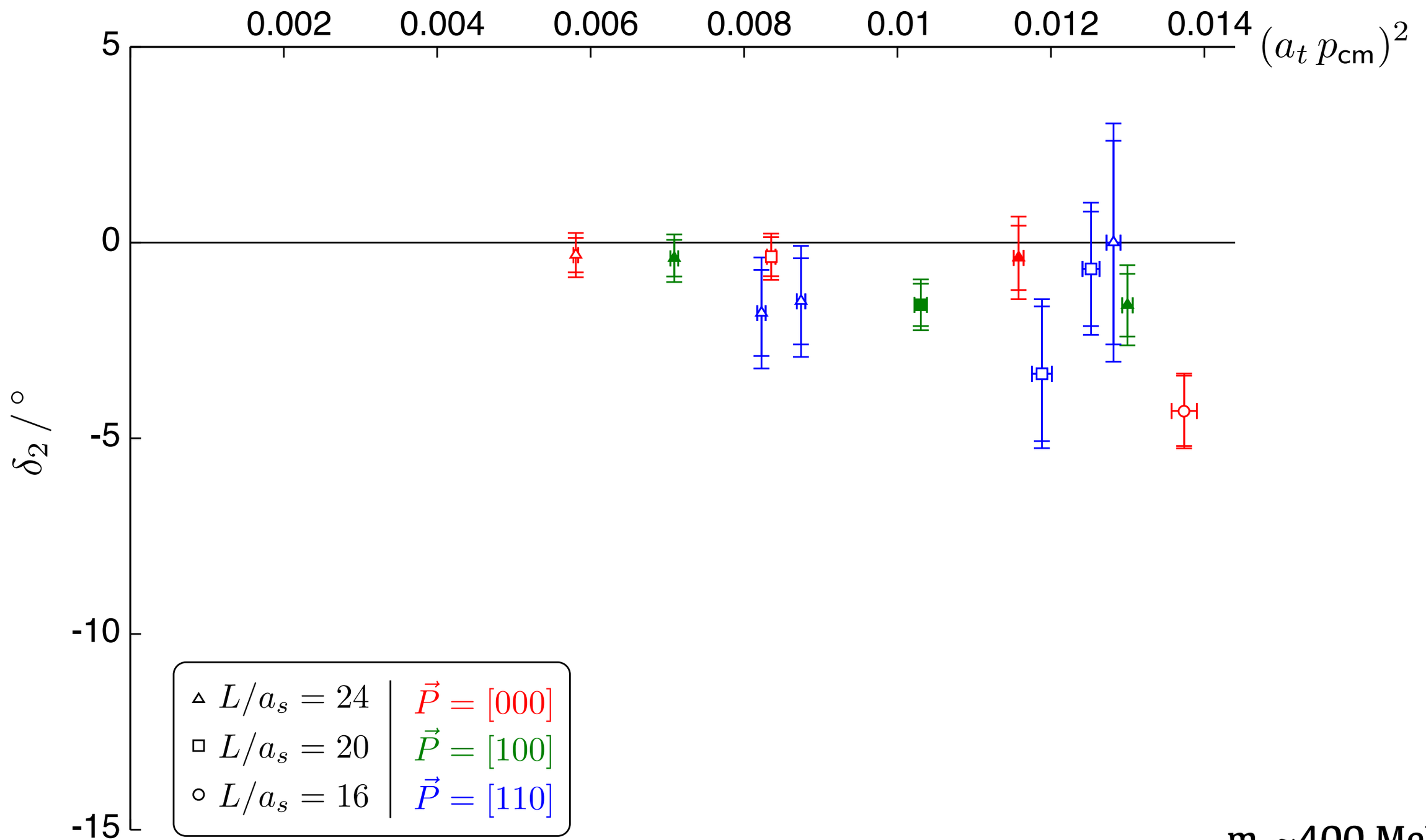
S-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

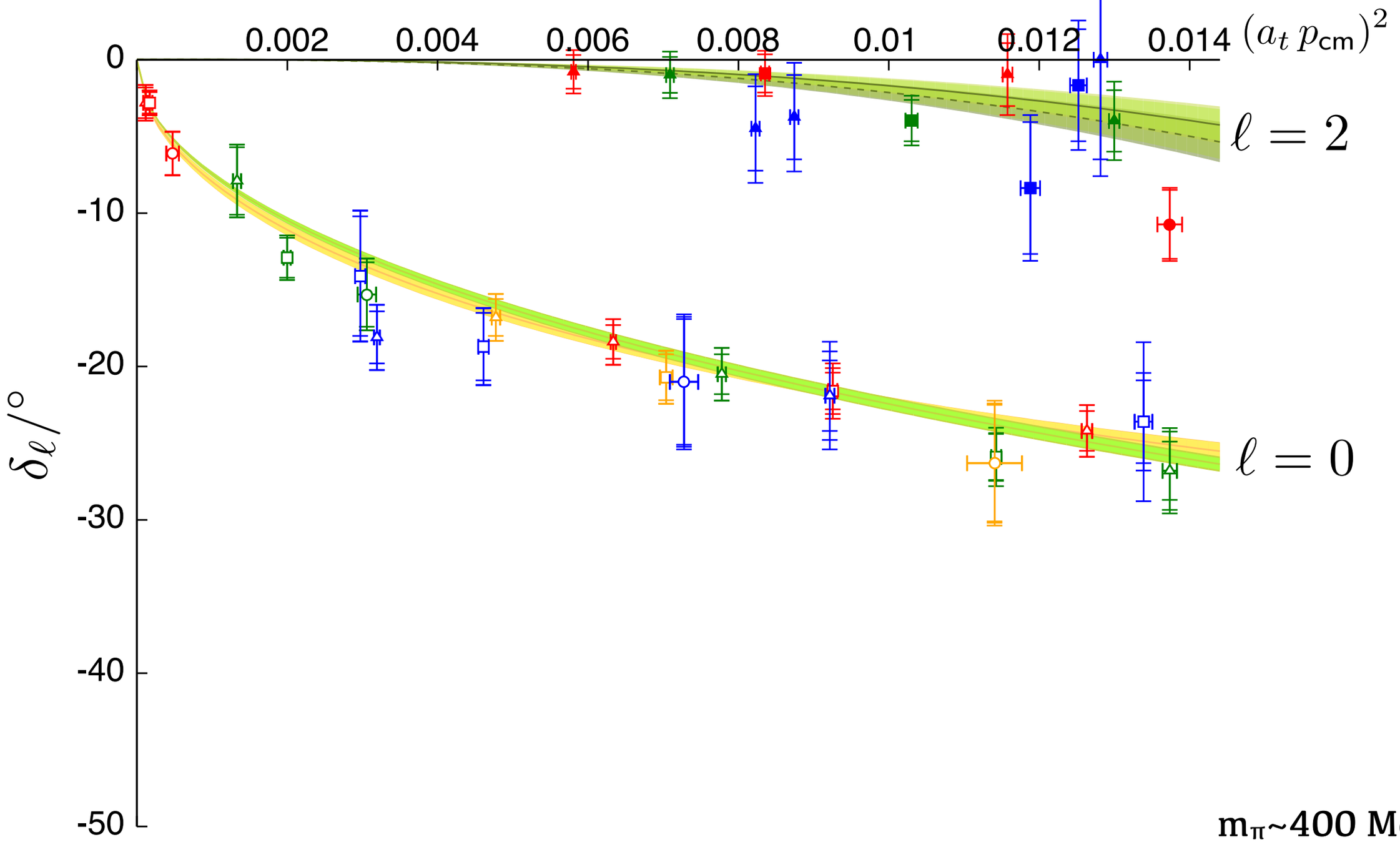
D-wave scattering phase-shift



m_π~400 MeV

ππ isospin-2 scattering

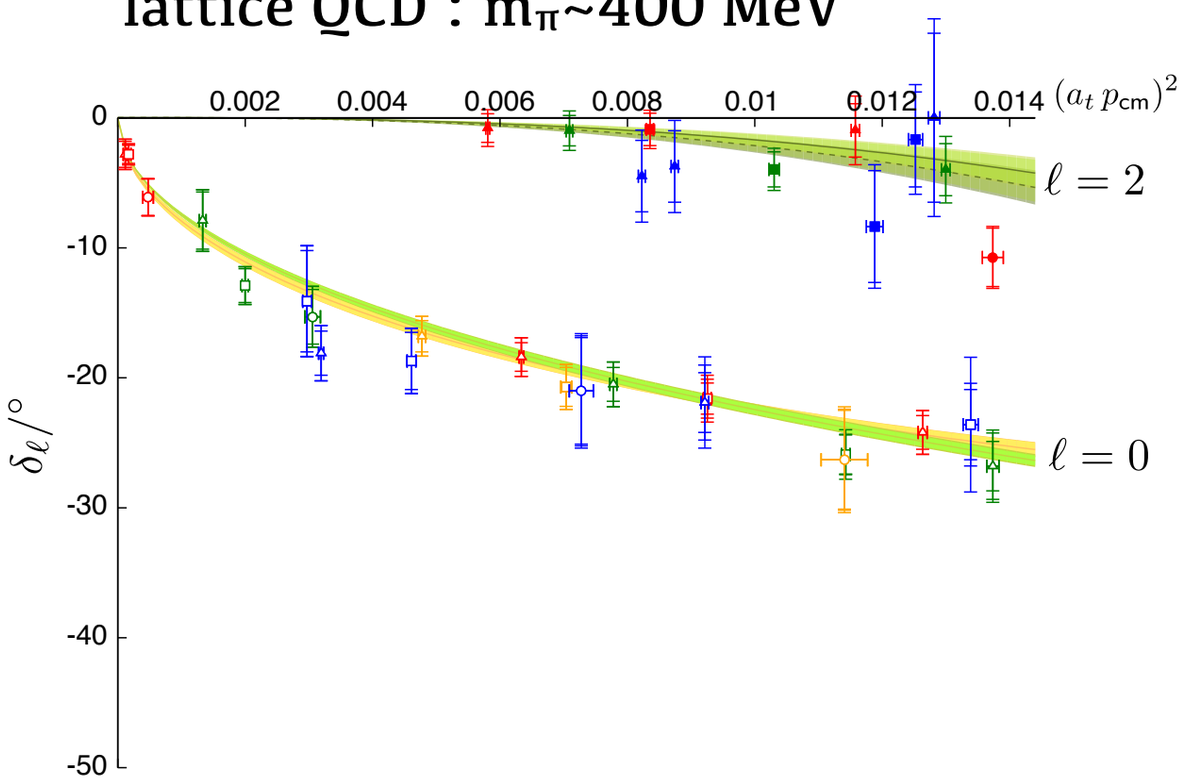
summary (with effective range fits) $$k^{2\ell+1}\cot\delta_\ell = \frac{1}{a_\ell} + \frac{1}{2}r_\ell k^2 + \ldots$$



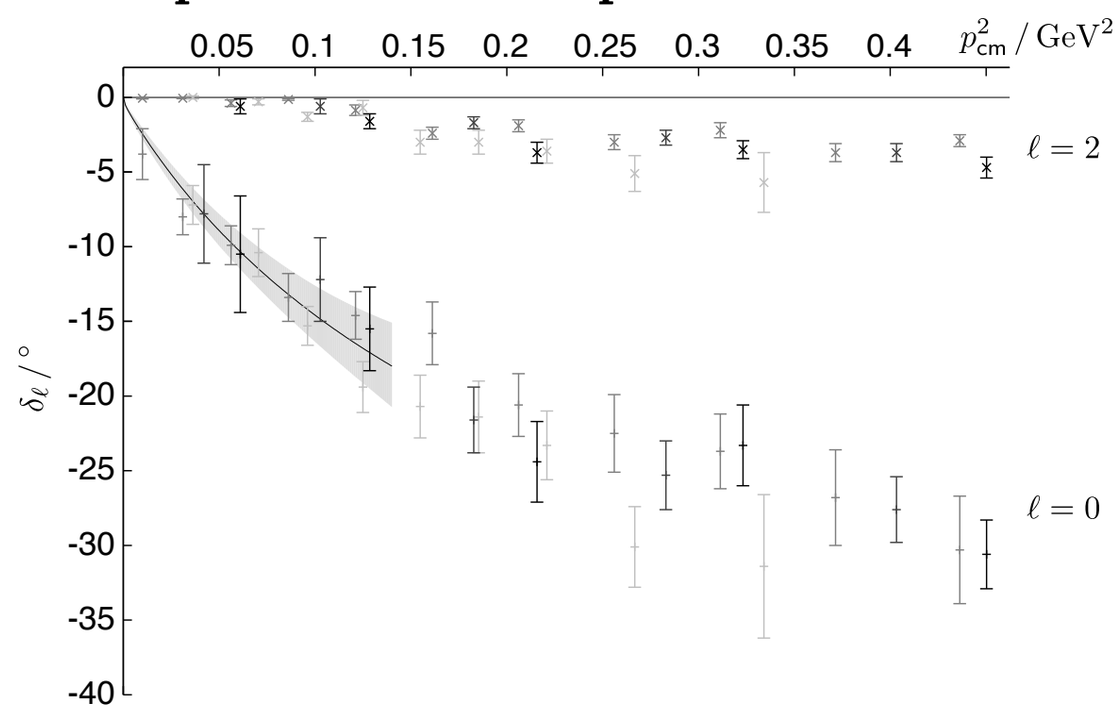
m_π~400 MeV

ππ isospin-2 scattering

lattice QCD : m_π~400 MeV



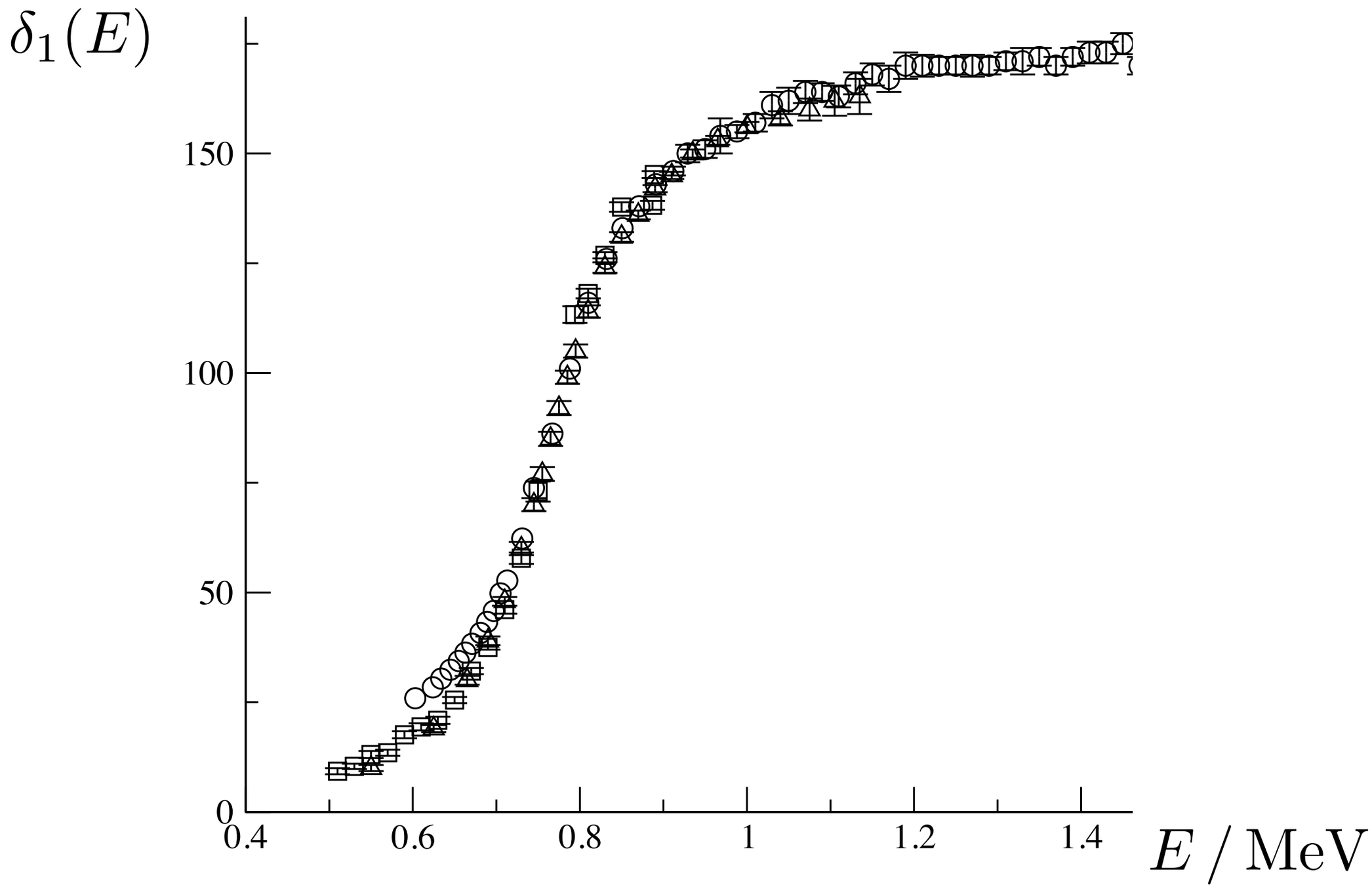
experimental compendium



ππ isospin-1 scattering

more interesting scattering channels are those featuring resonances

e.g. the ρ in ππ



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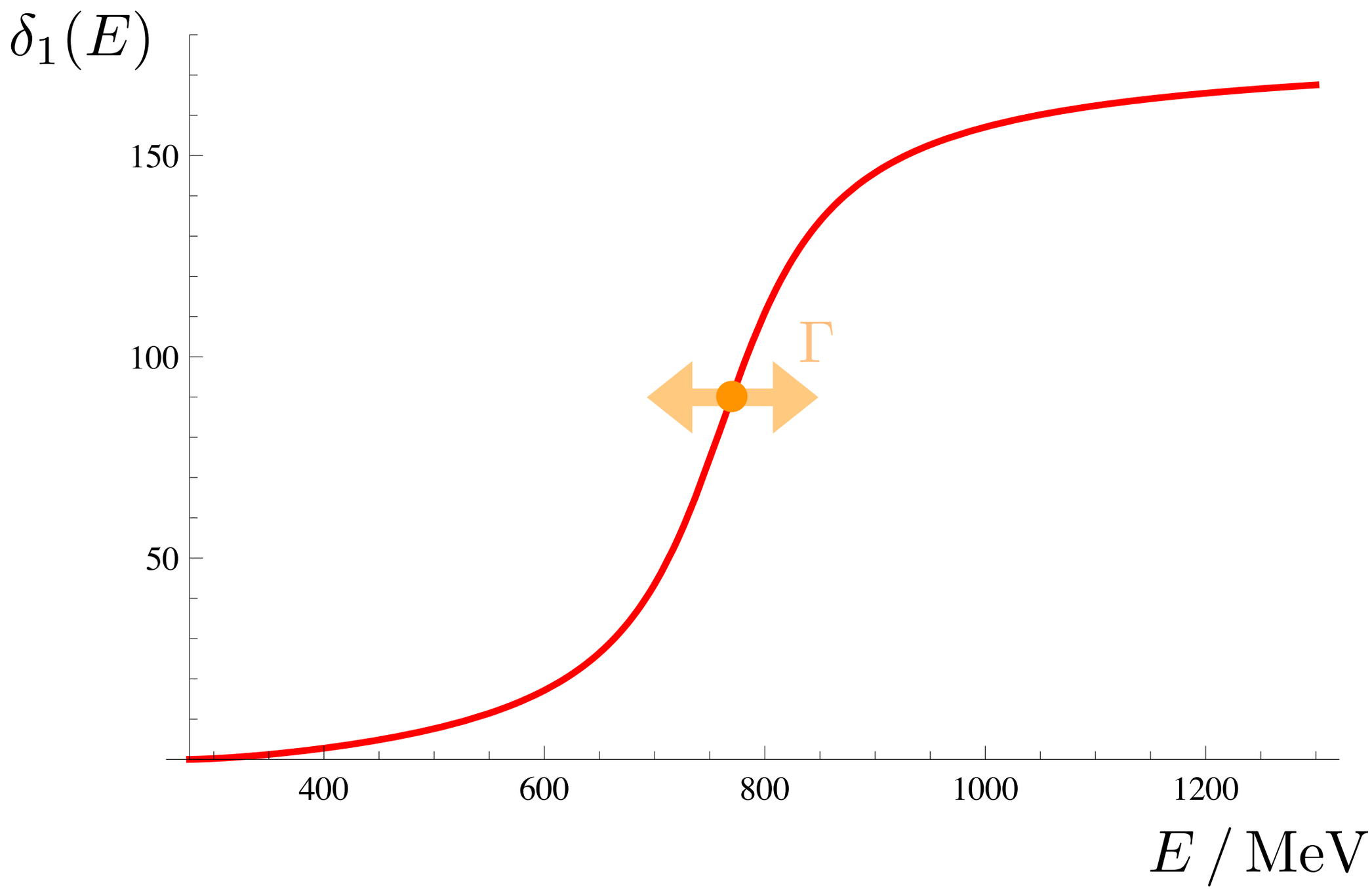

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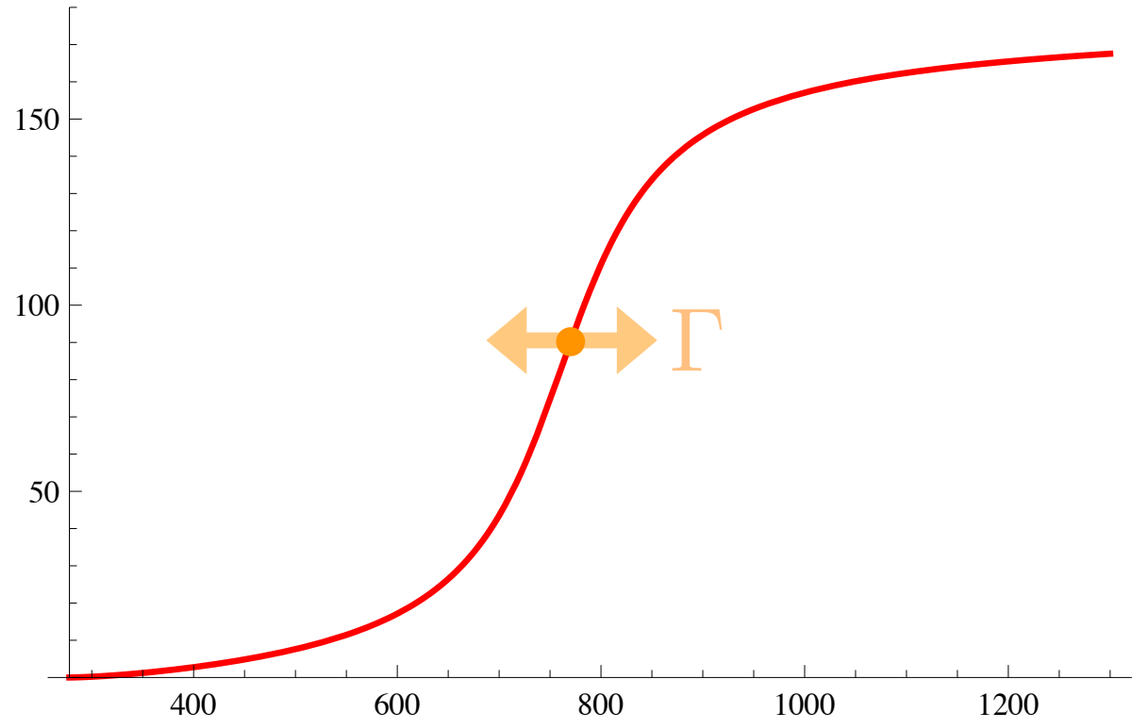
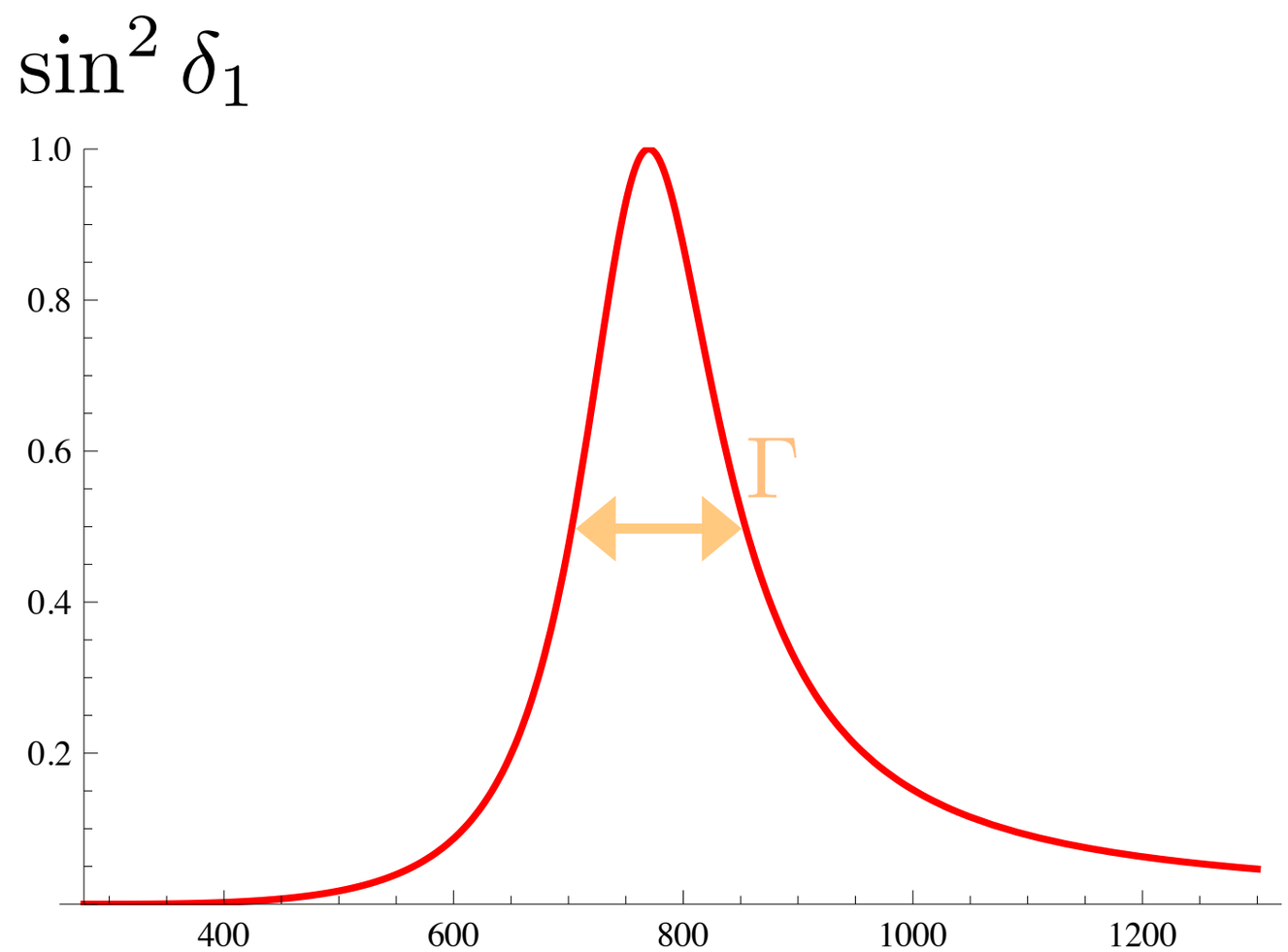
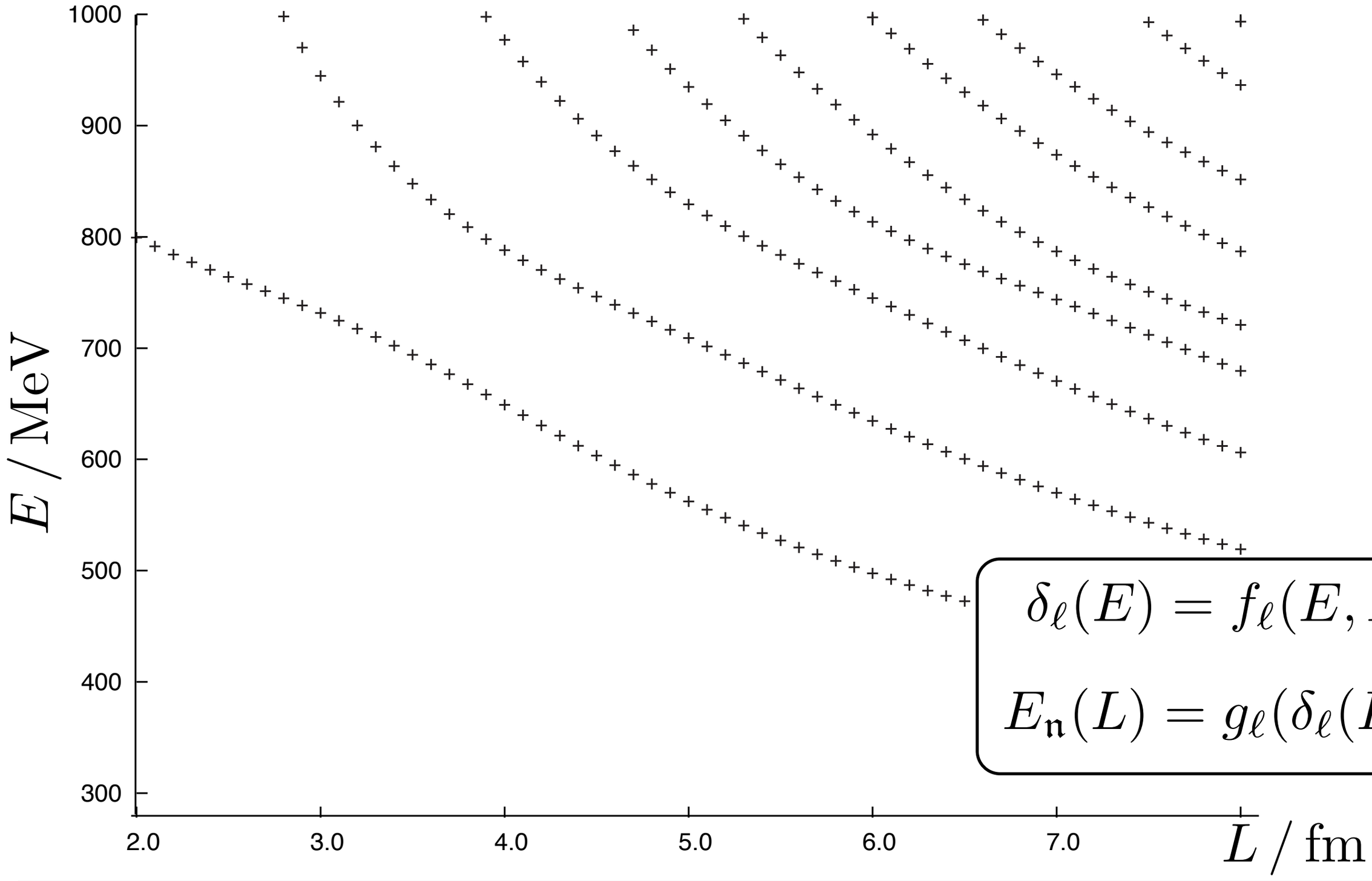




ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance

m_π=139 MeV
m_ρ=770 MeV
Γ_ρ=150 MeV

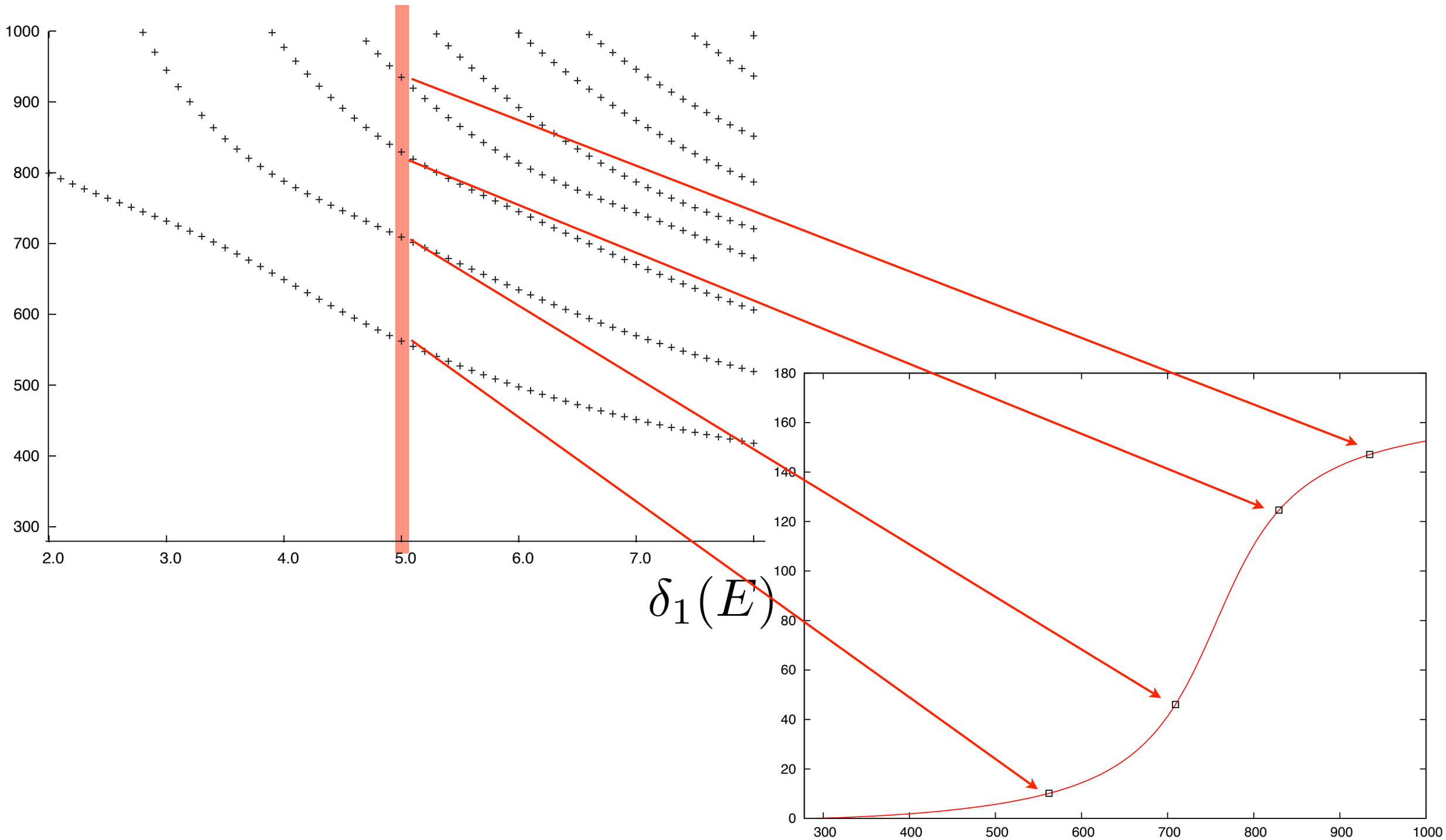


$$\delta_\ell(E) = f_\ell(E, L)$$

$$E_{\mathfrak{n}}(L) = g_\ell(\delta_\ell(E), L)$$

ππ isospin-1 scattering

a strongly volume-dependent spectrum



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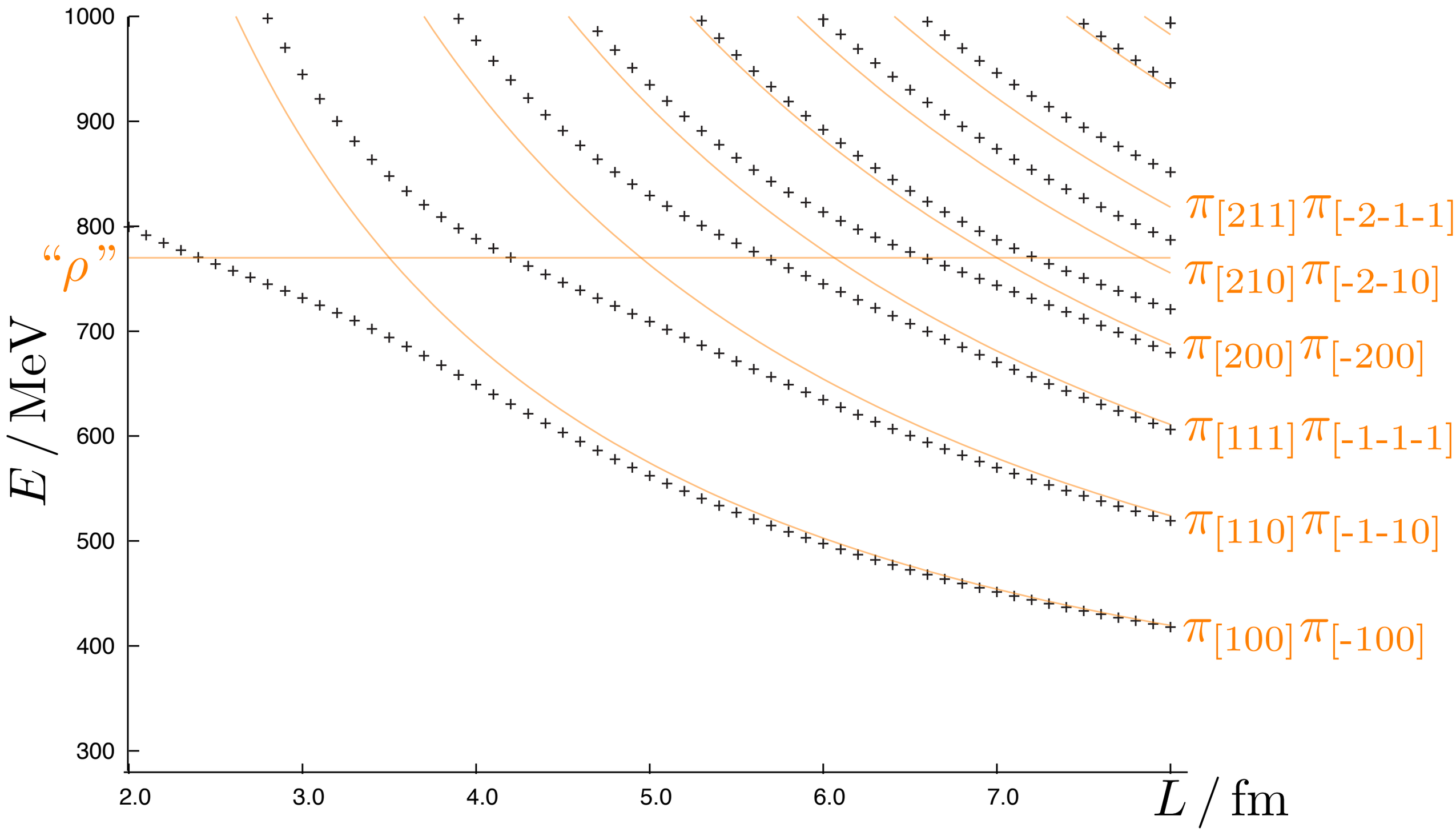

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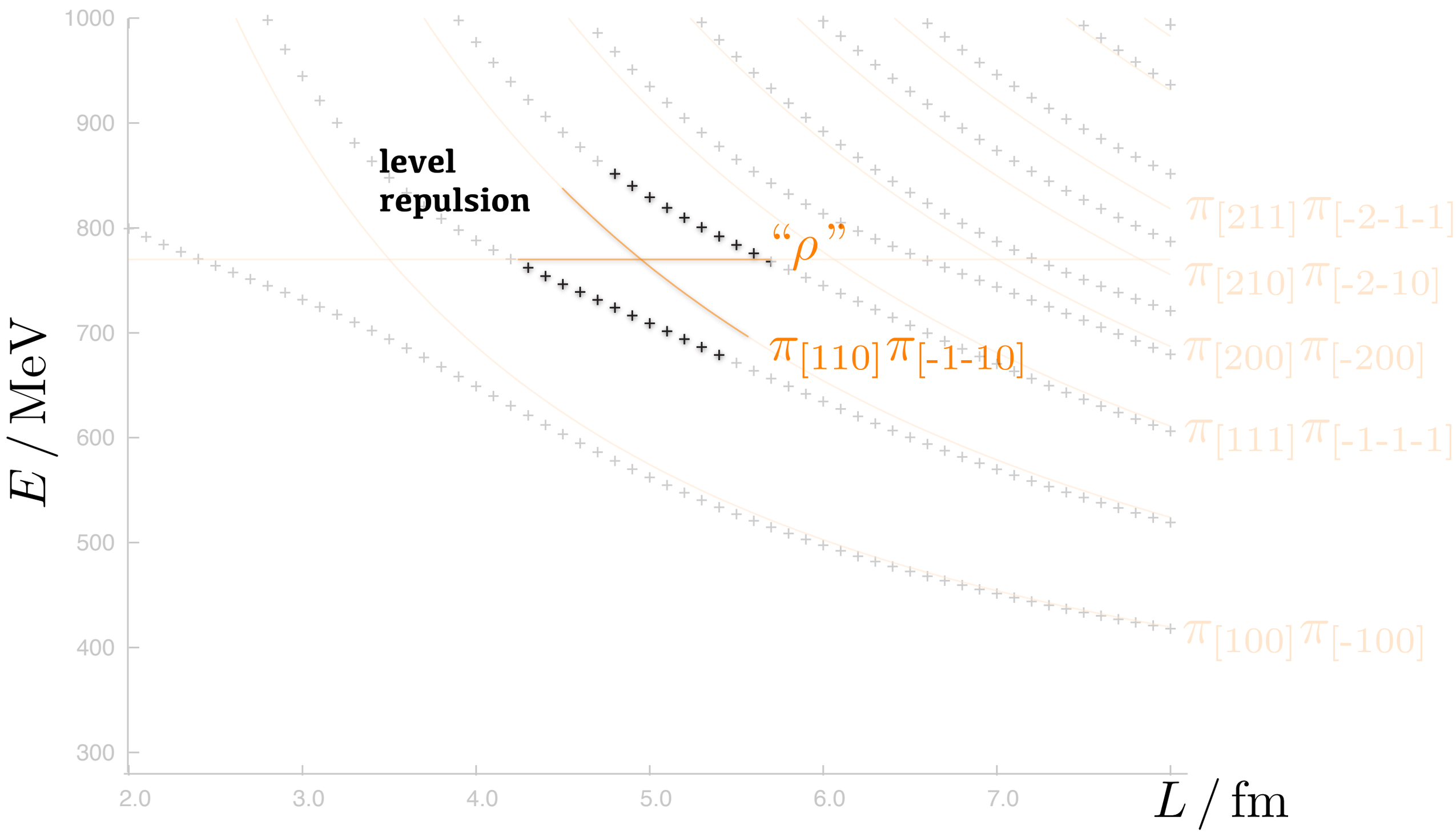

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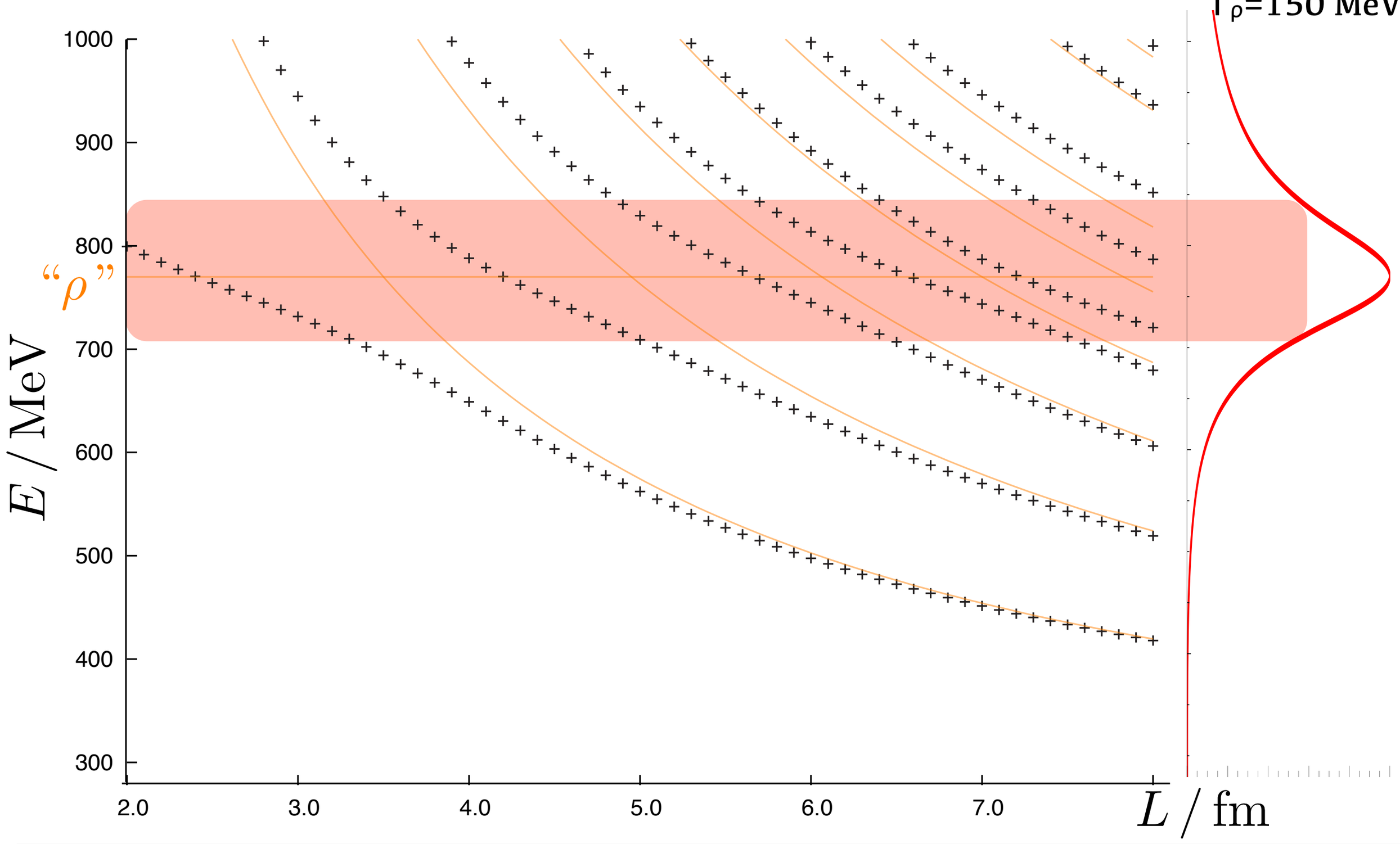

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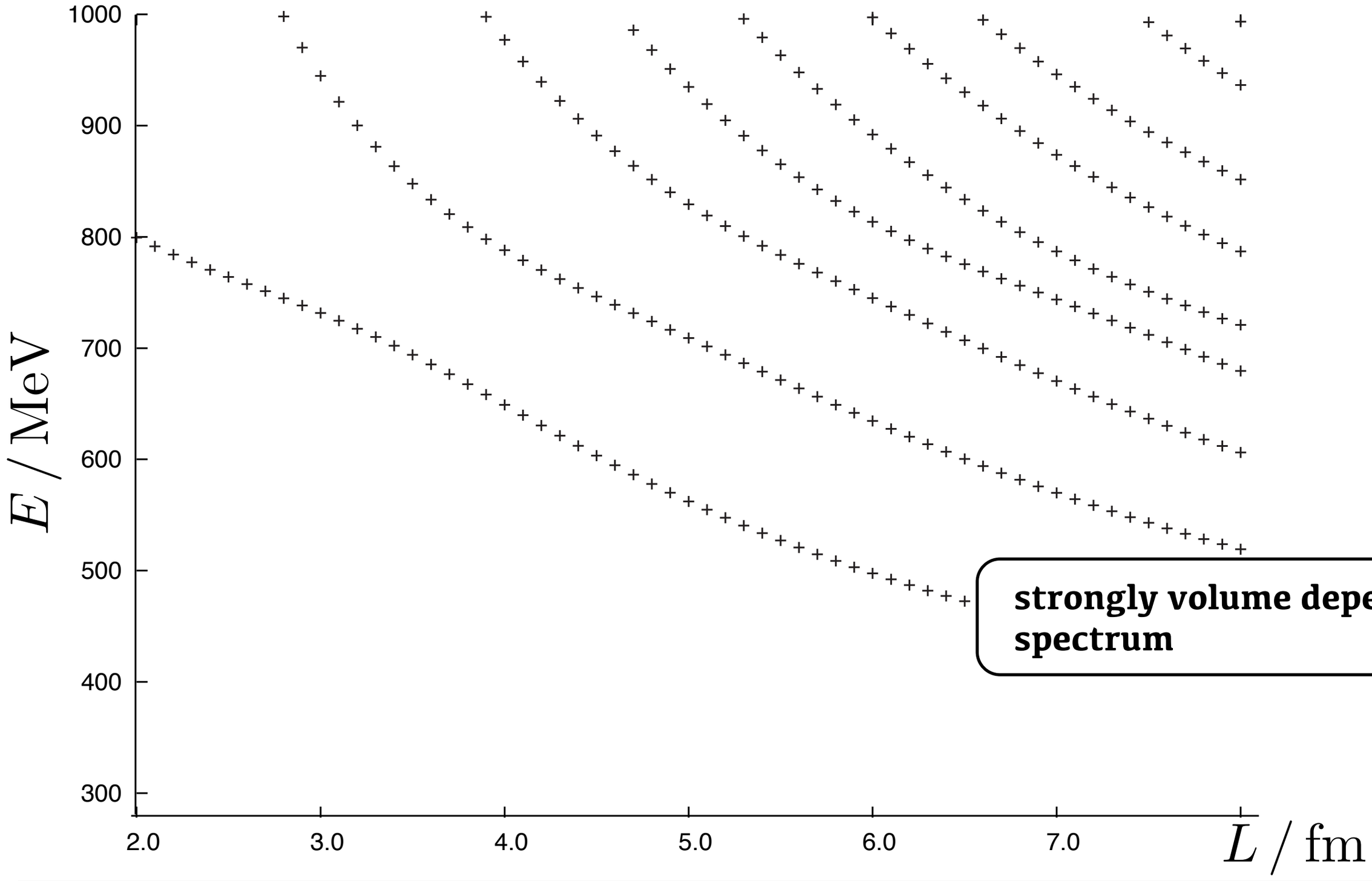
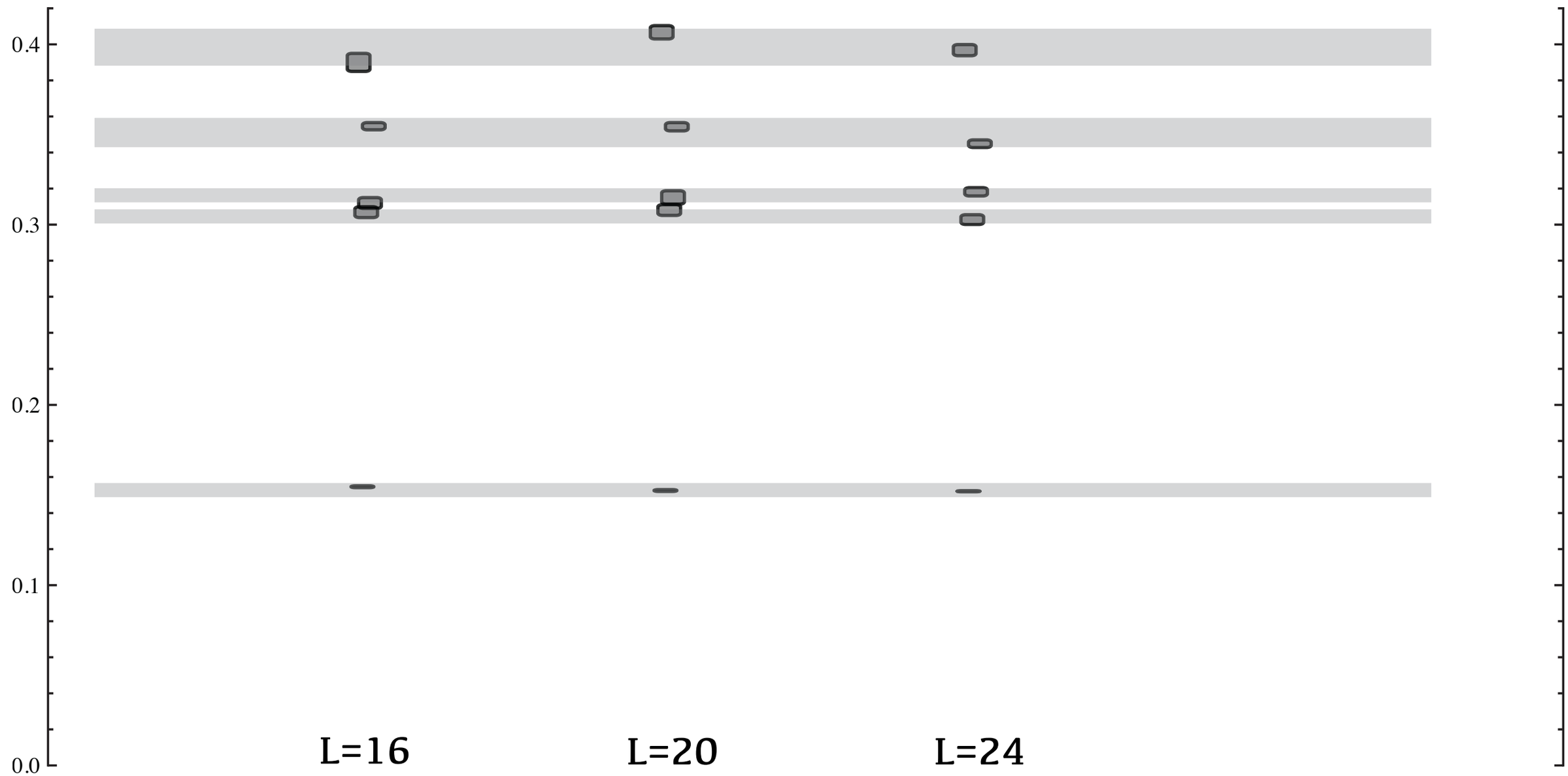


strongly volume dependent spectrum

the spectrum using 'local' operators - T_1^{--}

m_π~400 MeV

no systematic volume dependence observed in the spectrum



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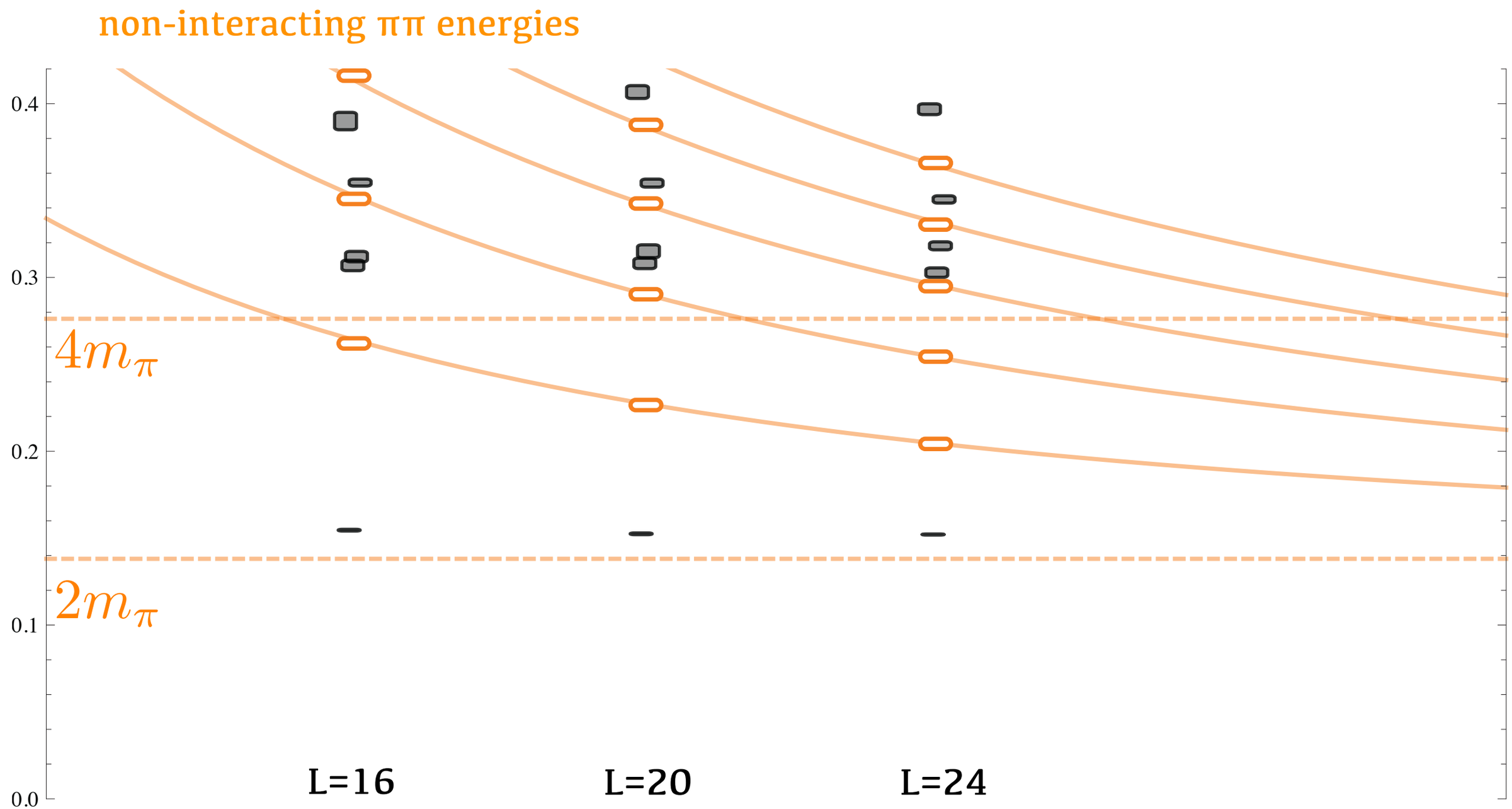

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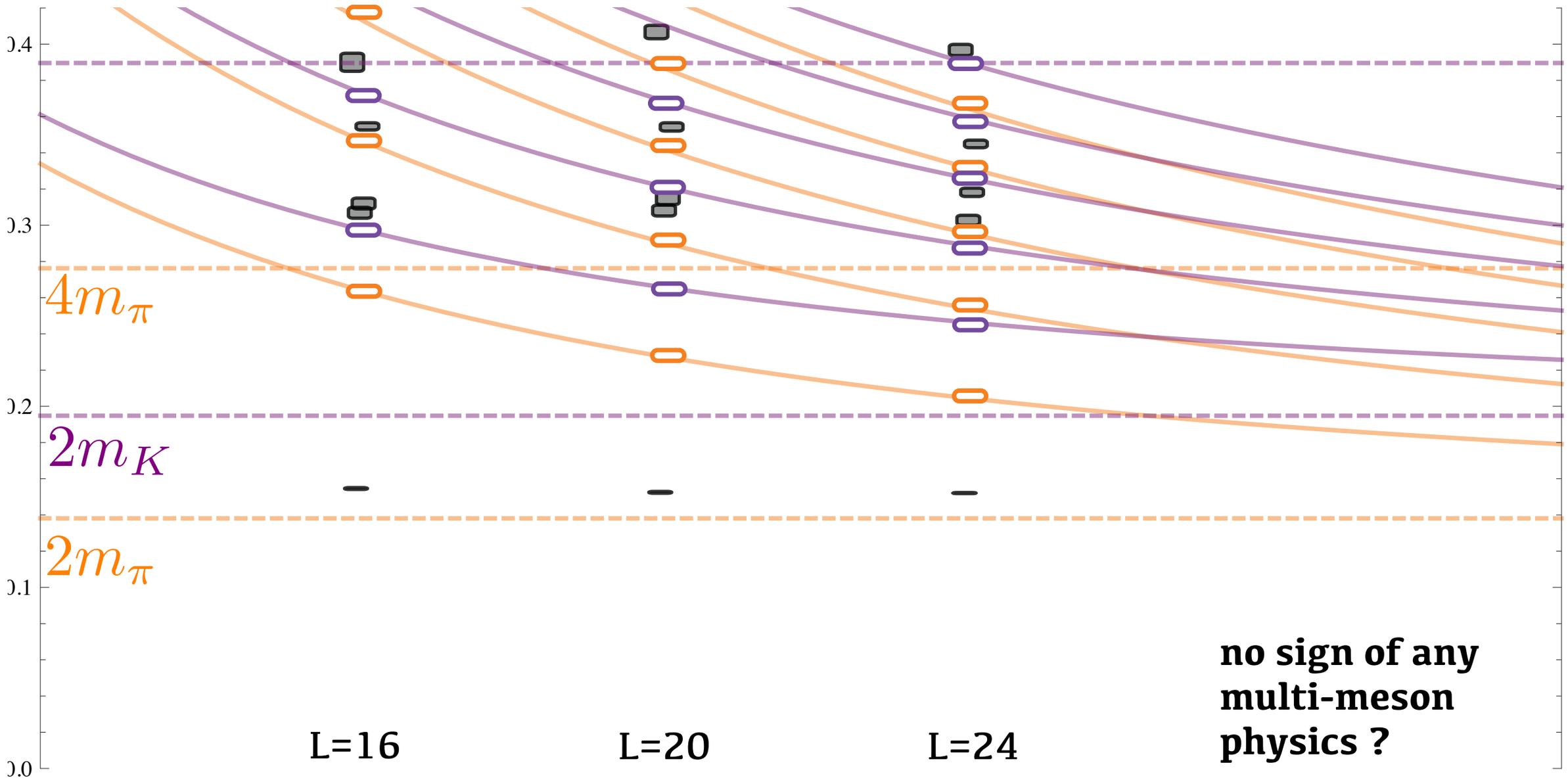


the spectrum using 'local' operators

we suspect that this effect can be understood

➟ hypothesise that energy eigenstates are superpositions of

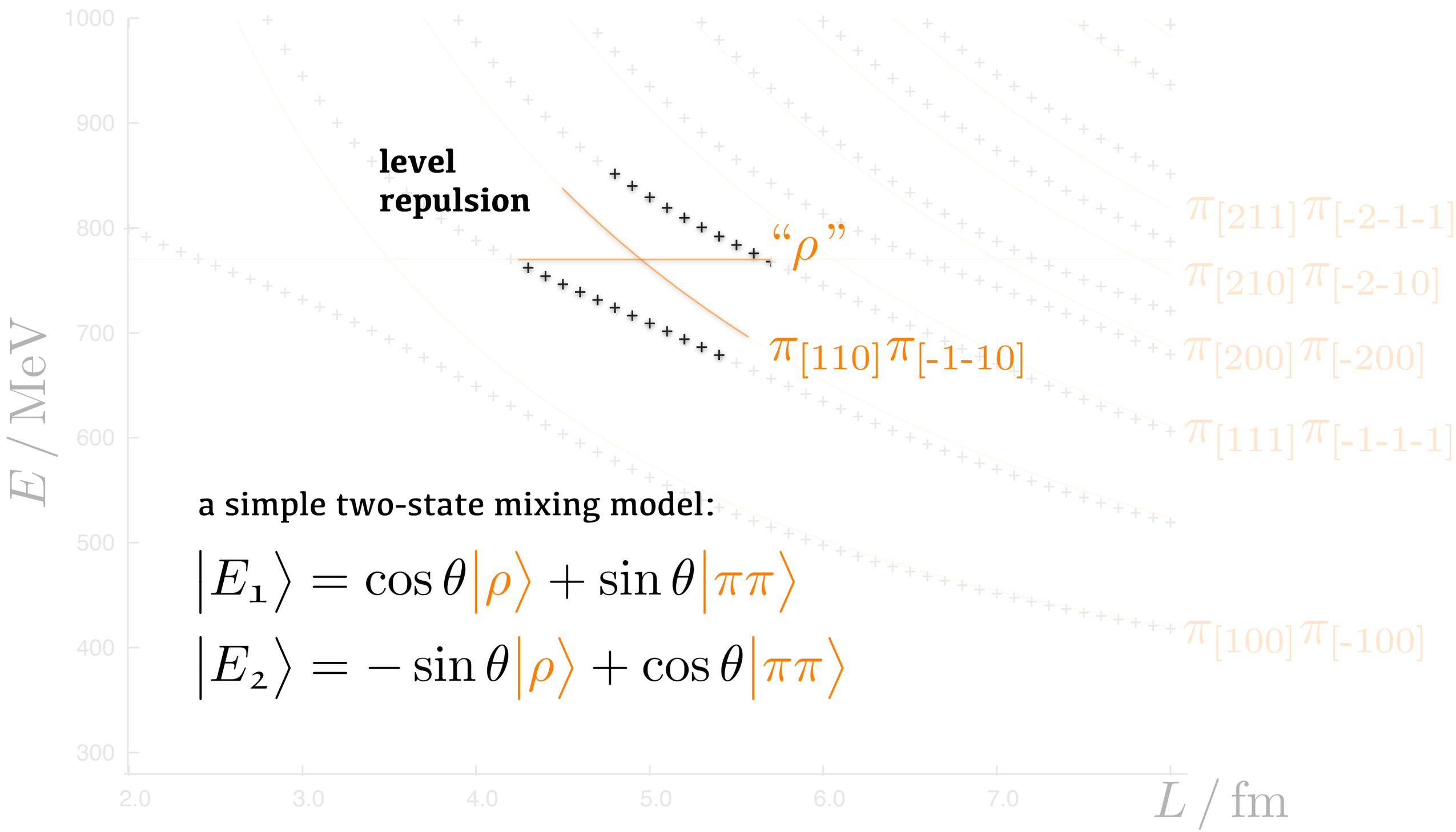
- ⋆ a $q\bar{q}$ state, call it $|\rho\rangle$
- ⋆ "non-interacting" $|\pi\pi\rangle$ basis states

➟ hypothesise that local $q\bar{q}$ operators have a suppressed overlap onto $|\pi\pi\rangle$

by at least a factor of $1/L^3$

ππ isospin-1 scattering

expected finite-volume spectrum given a ρ resonance



a simple two-state mixing model:

$$|E_1\rangle = \cos\theta|\rho\rangle + \sin\theta|\pi\pi\rangle$$

$$|E_2\rangle = -\sin\theta|\rho\rangle + \cos\theta|\pi\pi\rangle$$

the spectrum using 'local' operators

a simple two-state mixing model:

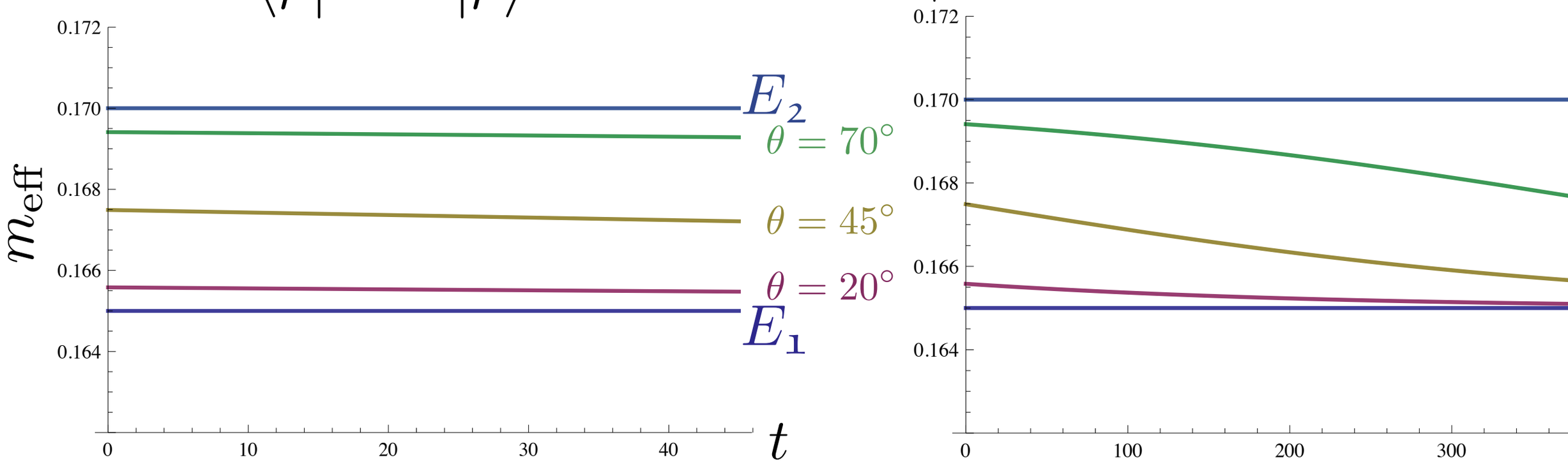
$$\left|E_1\right\rangle = \cos\theta\left|\rho\right\rangle + \sin\theta\left|\pi\pi\right\rangle$$
$$\left|E_2\right\rangle = -\sin\theta\left|\rho\right\rangle + \cos\theta\left|\pi\pi\right\rangle$$

if we only use operators which overlap well with $\left|\rho\right\rangle$ and not with $\left|\pi\pi\right\rangle$

then a variational solution won't be able to find the orthogonal combinations

the principal correlator will behave like

$$\left\langle\rho\left|e^{-Ht}\right|\rho\right\rangle = \cos^2\theta\, e^{-E_1 t} + \sin^2\theta\, e^{-E_2 t}$$



ππ isospin-1 scattering - lattice calculation

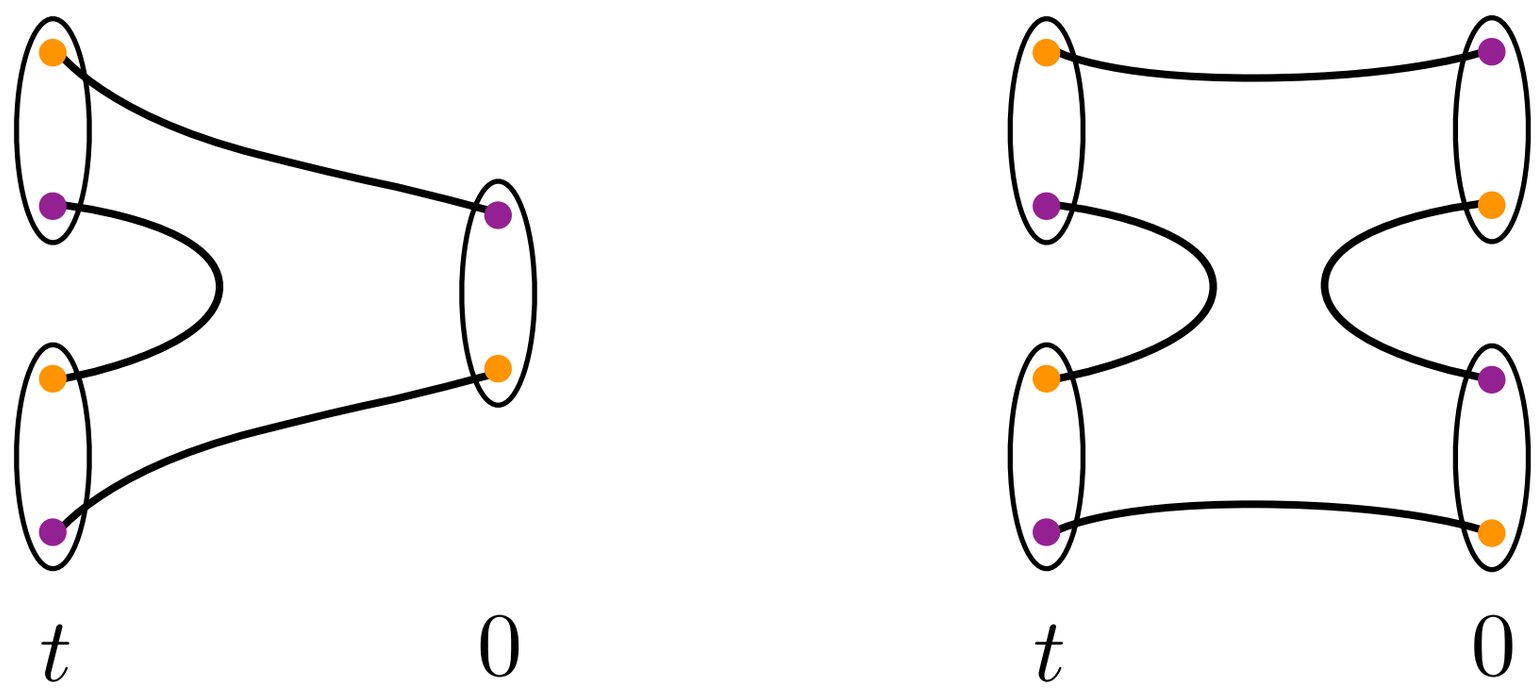
do it properly !

operator basis:

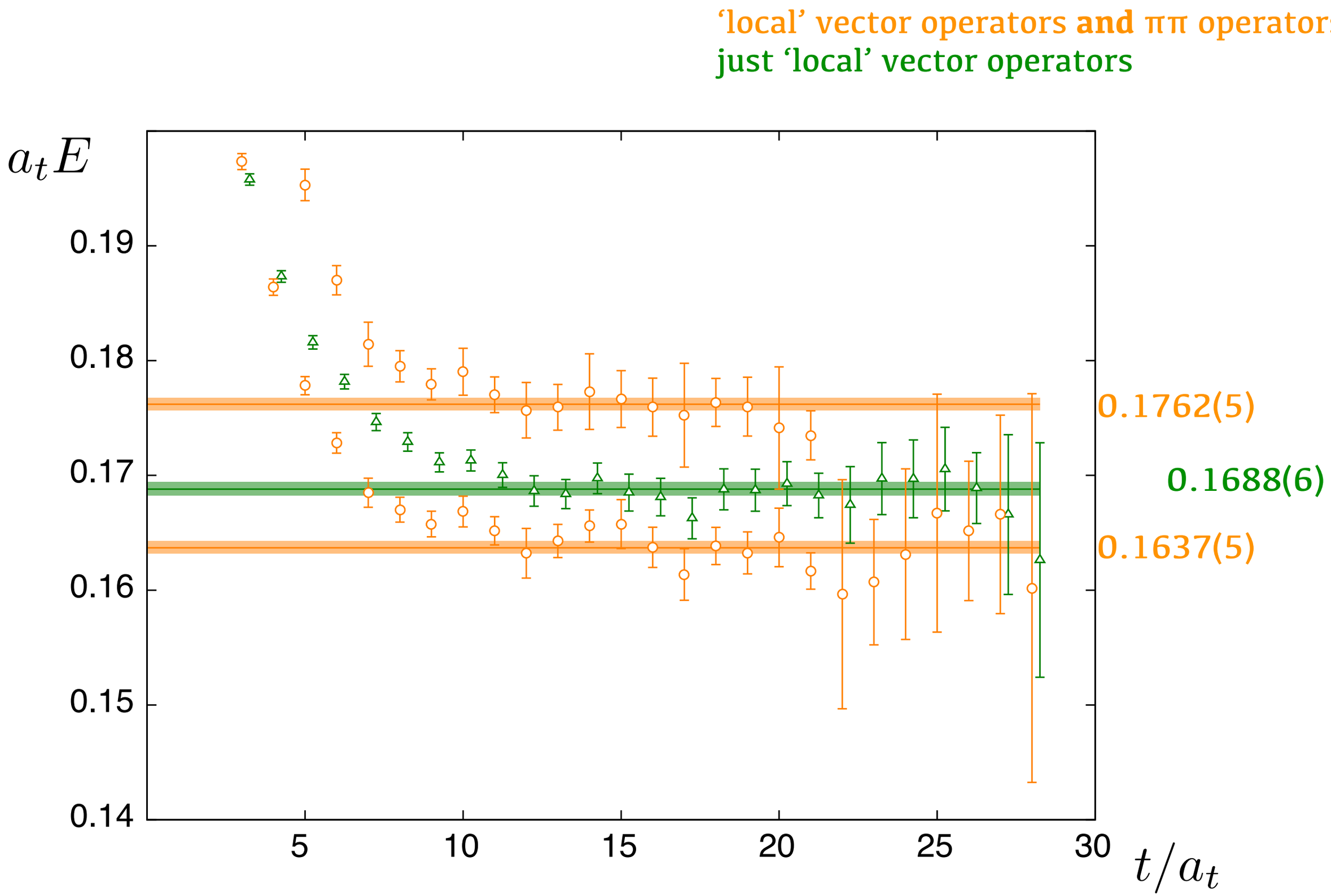
- ➟ usual big set of derivative-based fermion bilinears
- ➟ ππ-like operators of definite relative momentum

need quark annihilation diagrams

e.g.

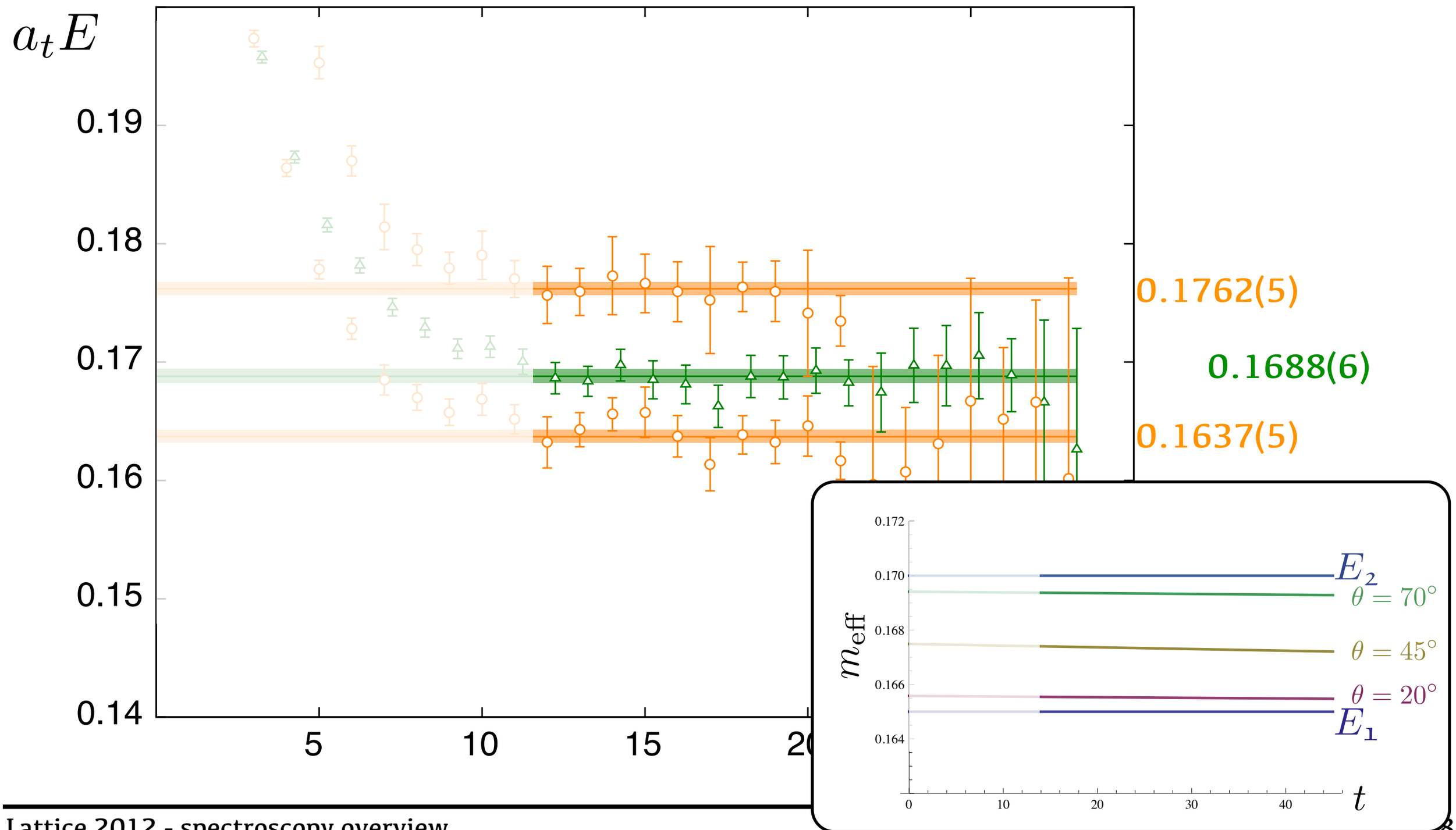


[100] A1 - with & without ππ operators



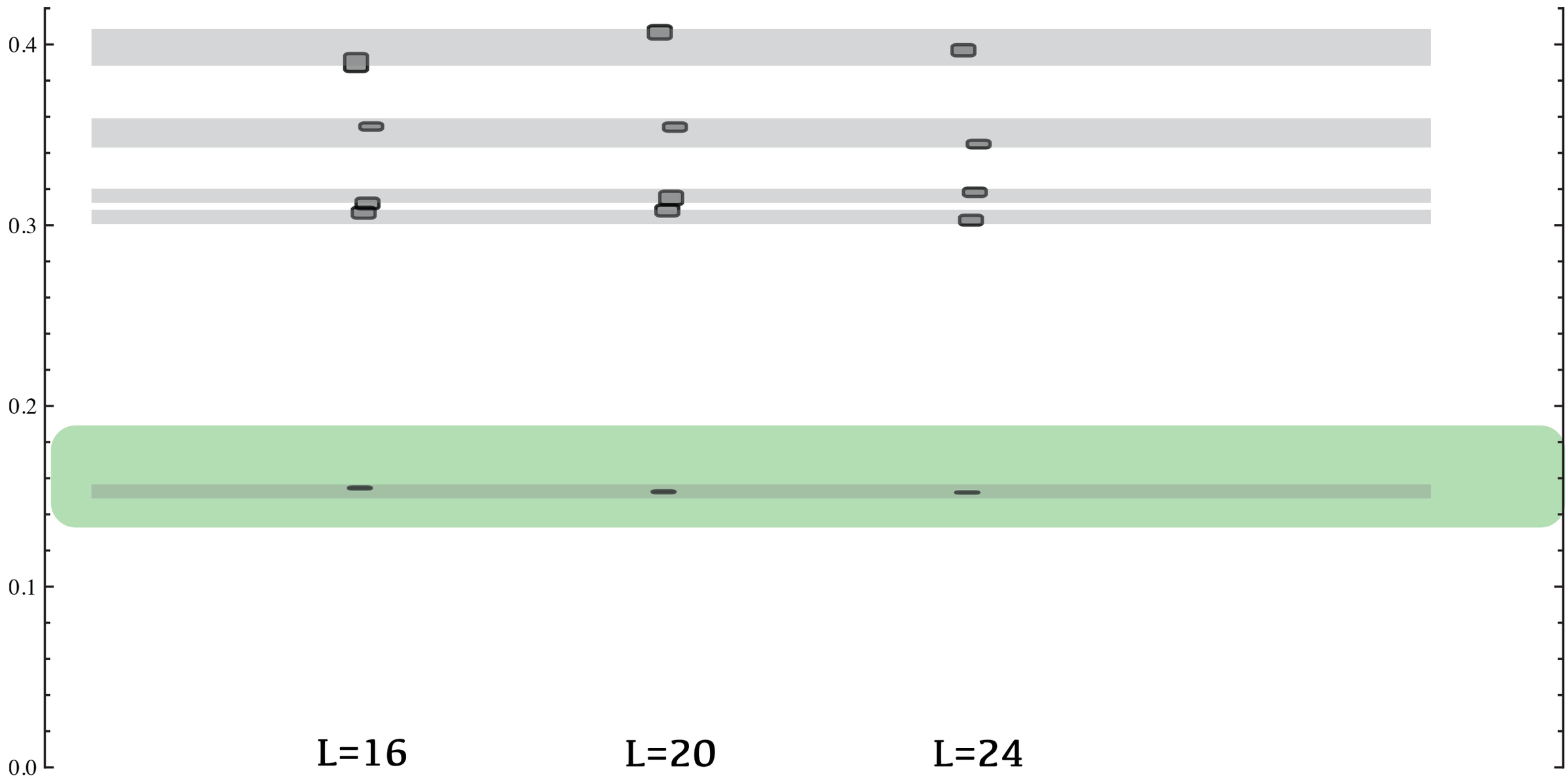
[100] A1 - with & without ππ operators

'local' vector operators **and** ππ operators
just 'local' vector operators



the spectrum using 'local' operators - T_1^{--}

m_π~400 MeV

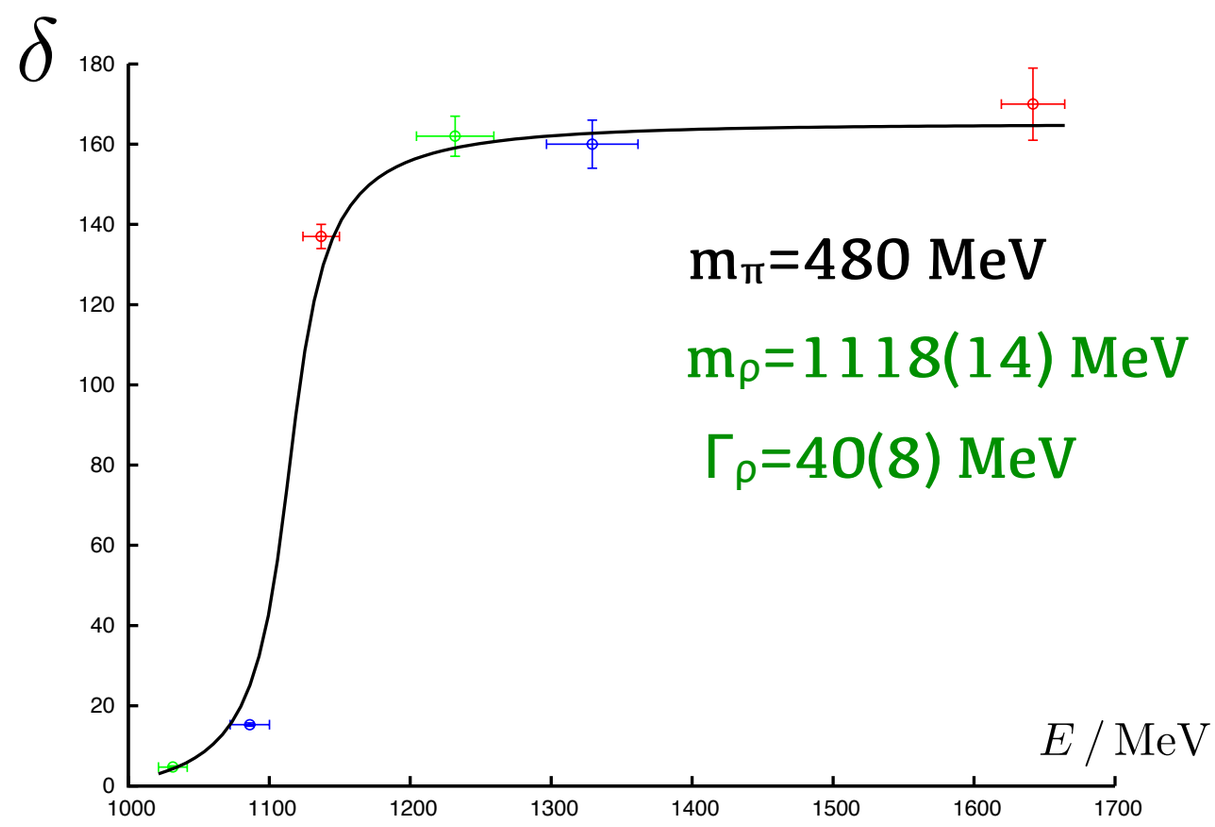


ππ isospin-1 scattering - pion mass dependence

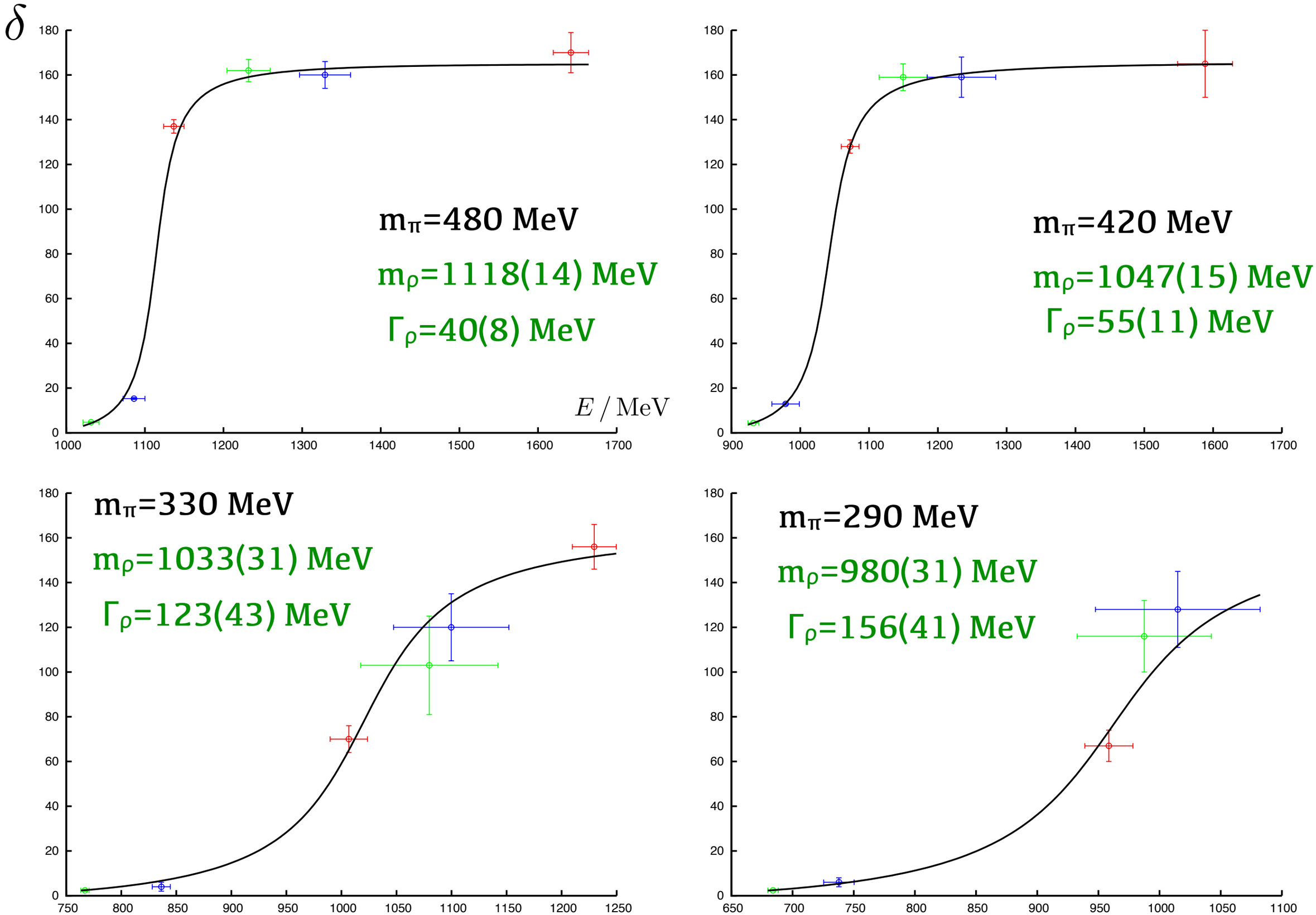
European Twisted Mass Collaboration
PRD83 (2011) 094505

- ➟ two-flavour calculation (no strange quarks)
- ➟ four different quark masses
- ➟ setting the lattice scale ?

➟ computed in relatively few frames



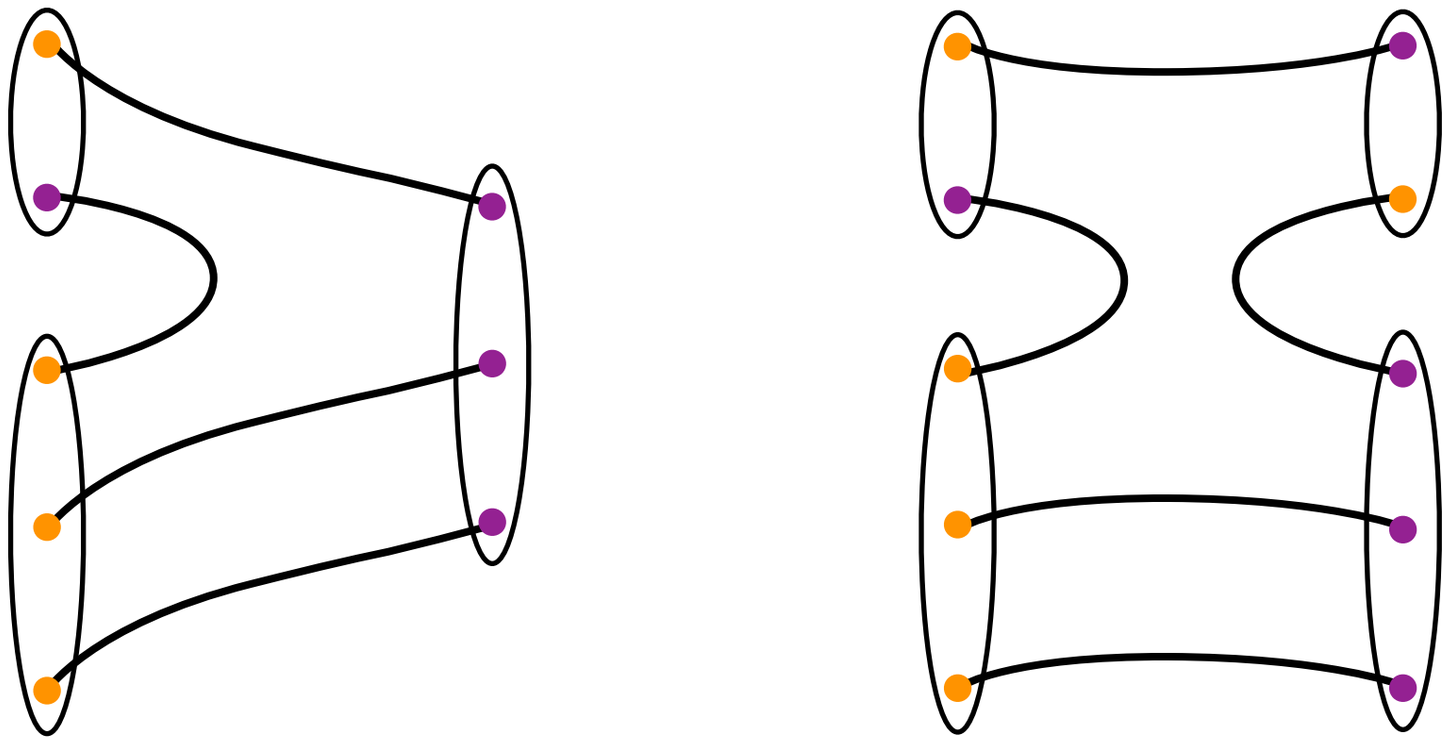
ππ isospin-1 scattering - pion mass dependence



forthcoming resonance calculations

computationally challenging:

➟ pion-nucleon elastic scattering (Δ resonance)



➟ lots of quark lines
➟ lots of matrix multiplication

… computer time

forthcoming resonance calculations

computationally challenging:

➟ pion-nucleon elastic scattering (Δ resonance)

requires untested formalism:

➟ meson-meson **inelastic** scattering (e.g a_0 in $\pi\eta$-$K\bar{K}$)

e.g. 2-channel inelastic scattering

$$\begin{bmatrix} \frac{4\pi}{k_1}\frac{\eta e^{2i\delta_1}-1}{2i} & \frac{4\pi}{\sqrt{k_1 k_2}}\frac{\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)}}{2} \\ \frac{4\pi}{\sqrt{k_1 k_2}}\frac{\sqrt{1-\eta^2}e^{i(\delta_1+\delta_2)}}{2} & \frac{4\pi}{k_2}\frac{\eta e^{2i\delta_2}-1}{2i} \end{bmatrix}$$

three real numbers at each scattering energy

finite-volume formalism

$$E_{\mathfrak{n}}(L) = f\left(\delta_1(E), \delta_2(E), \eta(E); L\right)$$

'measured' three unknowns

forthcoming resonance calculations

computationally challenging:

➟ pion-nucleon elastic scattering (Δ resonance)

requires untested formalism:

➟ meson-meson **inelastic** scattering (e.g a_0 in $\pi\eta$-$K\bar{K}$)

➟ three-meson decays, e.g. $\omega, a_1, a_2 \ldots \to \pi\pi\pi$

all the complications of building unitary, analytic scattering amplitudes present for experiment !

and to finish ...

thank you for your attention