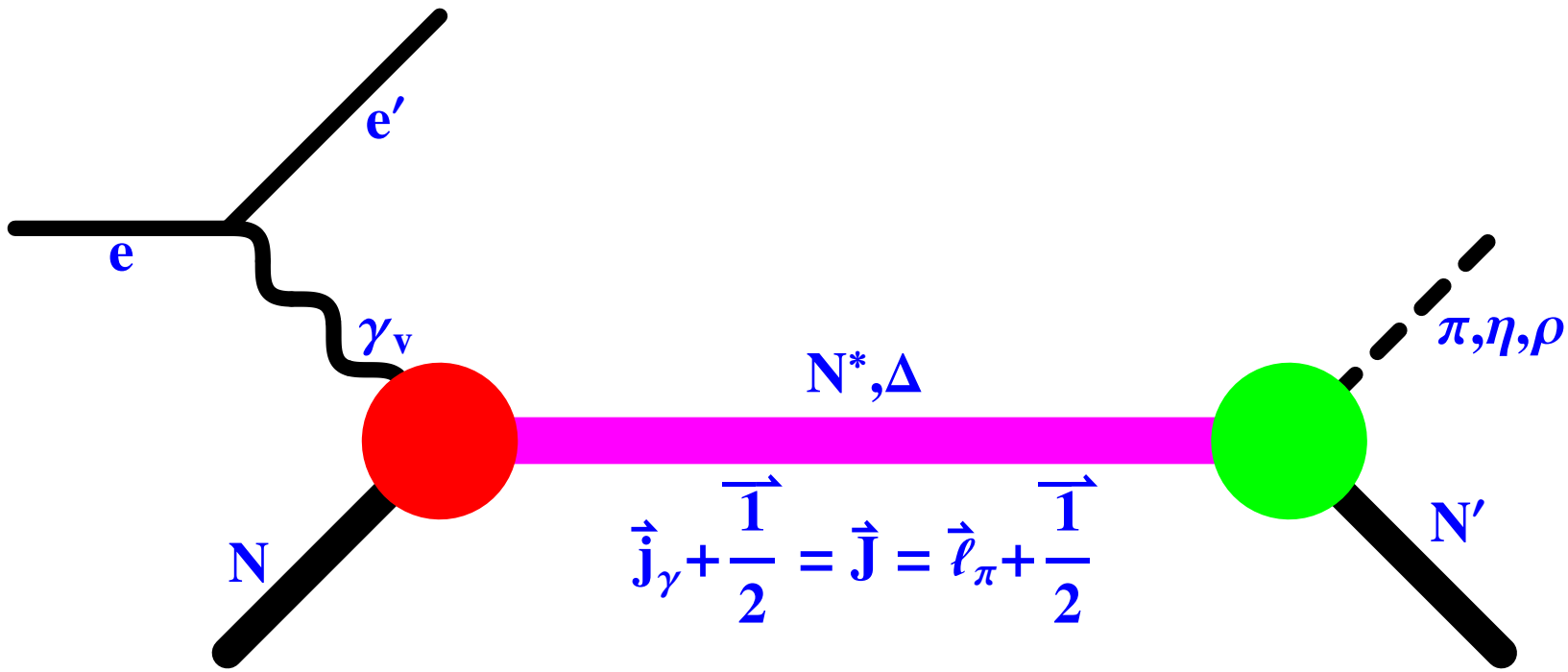
Response Functions for Delta Excitation at $Q^2 = 1$ (GeV/c)^2

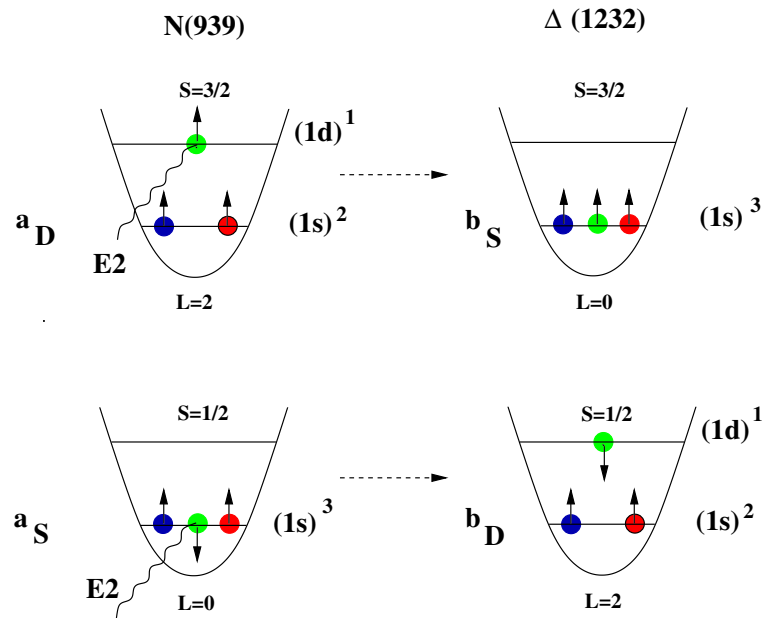
James J. Kelly
University of Maryland



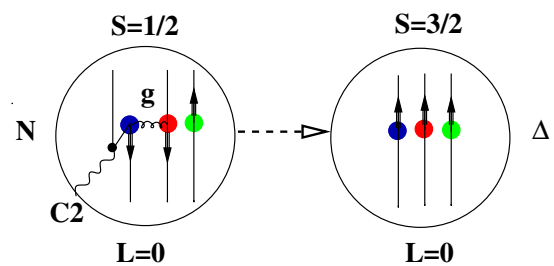
Origin of Quadrupole Amplitudes

configuration mixing: color hyperfine interaction gives $L = 2$ admixture

figure from Buchmann

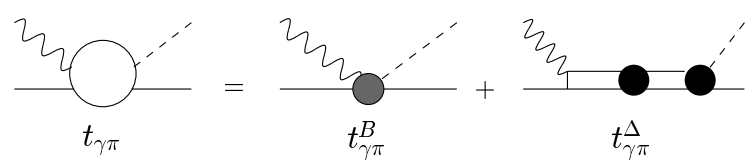


gluon or pion exchange currents with double spin flip figure from Buchmann



πN final state interactions

from Kamalov and Yang



Outline

Problem: resonance properties versus reaction dynamics
Method: polarization measurement of complex multipole amplitudes

- Introduction
- Review of low Q^2 data
- JLab experiment e91011
  - cross section and polarization analyses
  - Legendre and multipole analyses
  - quadrupole ratios
  - play with toy model
- Conclusions and outlook

Response Functions

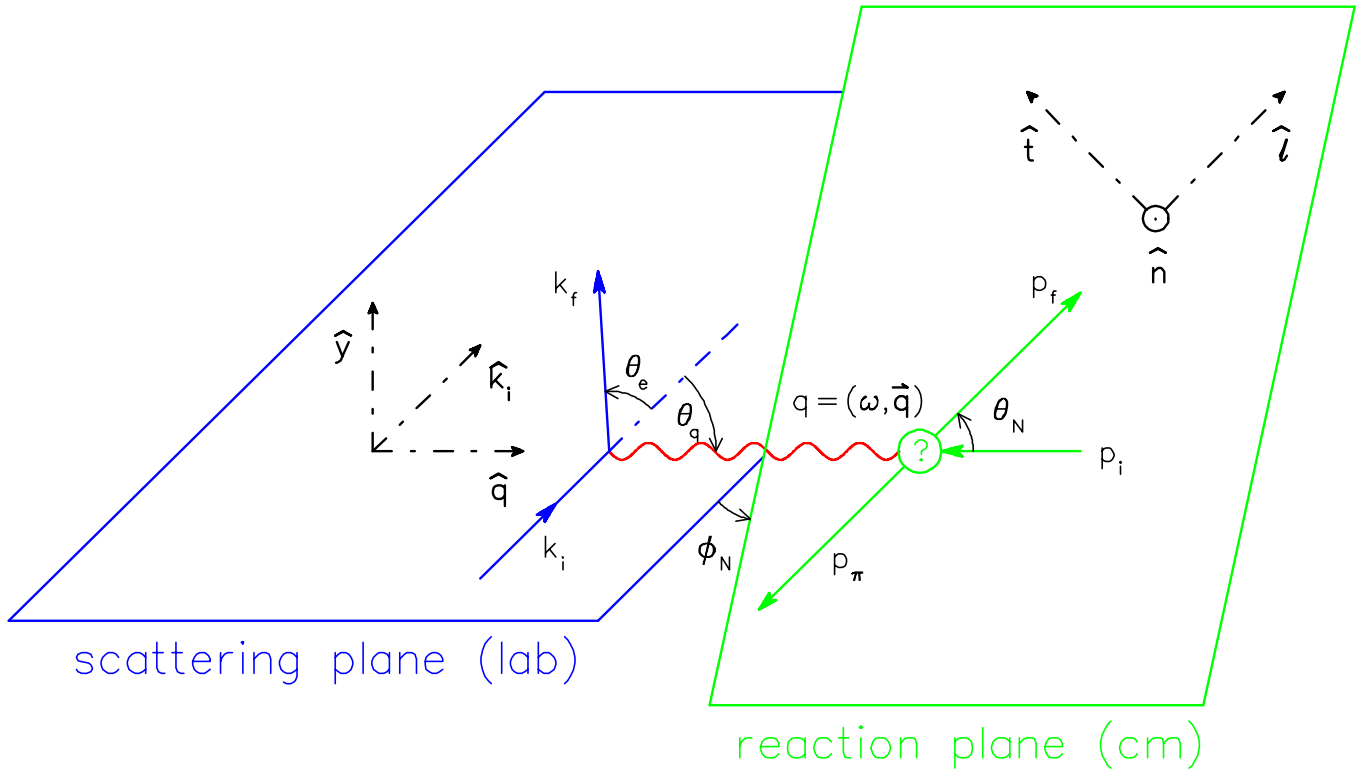
$$\frac{d^5\sigma}{dk_f d\Omega_e d\Omega_N} = \Gamma_\gamma \bar{\sigma} \left[1 + hA + \boldsymbol{S}\cdot(\boldsymbol{P} + h\boldsymbol{P}')\right]$$

$$\begin{aligned}
\bar{\sigma} &= \nu_0 \left[\nu_L \mathcal{W}_L + \nu_T \mathcal{W}_T + \nu_{LT}\mathcal{W}_{LT}\cos\phi + \nu_{TT}\mathcal{W}_{TT}\cos 2\phi\right] \\
P_n\,\bar{\sigma} &= \nu_0 \left[\nu_L \mathcal{W}_L^n + \nu_T \mathcal{W}_T^n + \nu_{LT}\mathcal{W}_{LT}^n\cos\phi + \nu_{TT}\mathcal{W}_{TT}^n\cos 2\phi\right] \\
A\,\bar{\sigma} &= \nu_0 \left[\nu'_{LT}\mathcal{W}'_{LT}\sin\phi\right] \\
P'_n\,\bar{\sigma} &= \nu_0 \left[\nu'_{LT}\mathcal{W}'^n_{LT}\sin\phi\right] \\
P_m\,\bar{\sigma} &= \nu_0 \left[\nu_{LT}\mathcal{W}_{LT}^m\sin\phi + \nu_{TT}\mathcal{W}_{TT}^m\sin 2\phi\right] \\
P'_m\,\bar{\sigma} &= \nu_0 \left[\nu'_{LT}\mathcal{W}'^m_{LT}\cos\phi + \nu'_{TT}\mathcal{W}'^m_{TT}\right] \qquad m \in \{\ell, t\}
\end{aligned}$$

Reduced responses: $\mathcal{W}_\eta = R_\eta \sin^{n_\eta}\theta$, $n_\eta \in \{0,1,2\}$

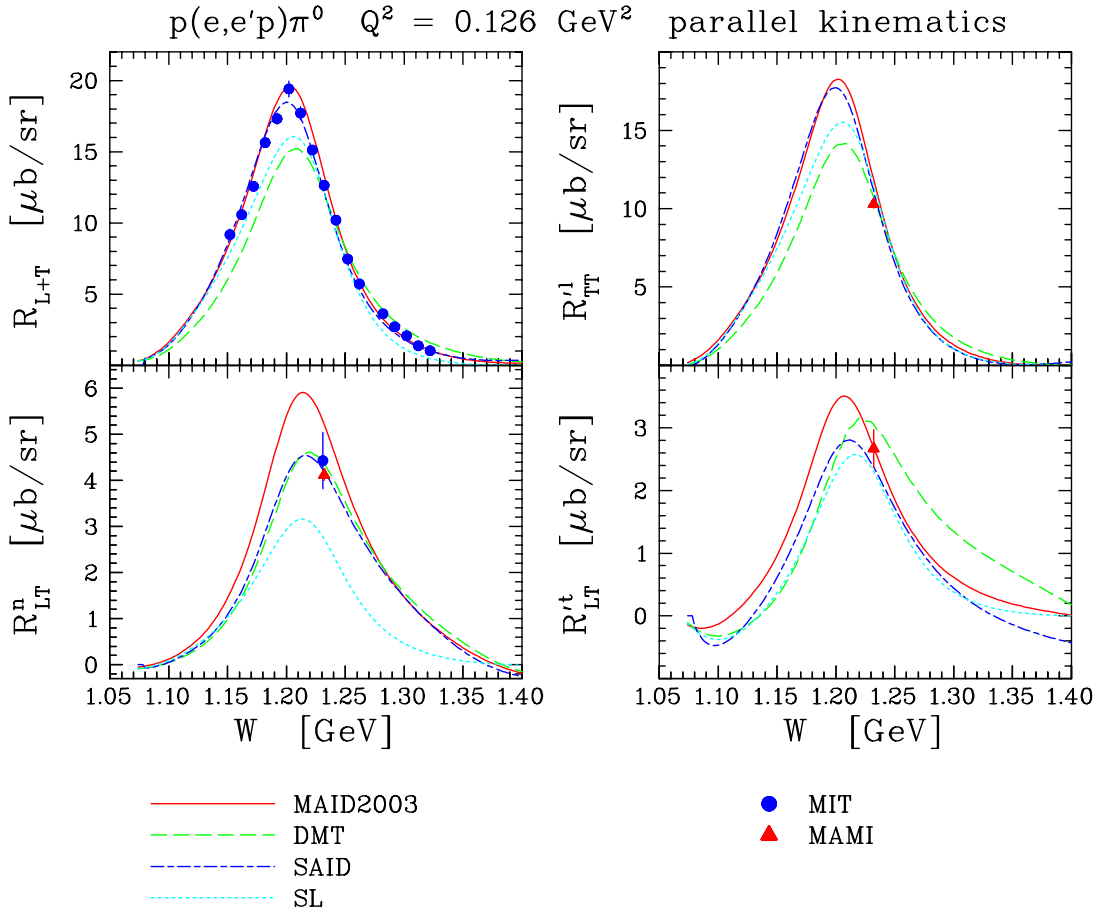
Legendre expansion: $R_\eta = \sum_k A_k^{(\eta)} P_k(x)$ $\qquad x = \cos\theta_\pi^{cm}$

Accessing Response Functions



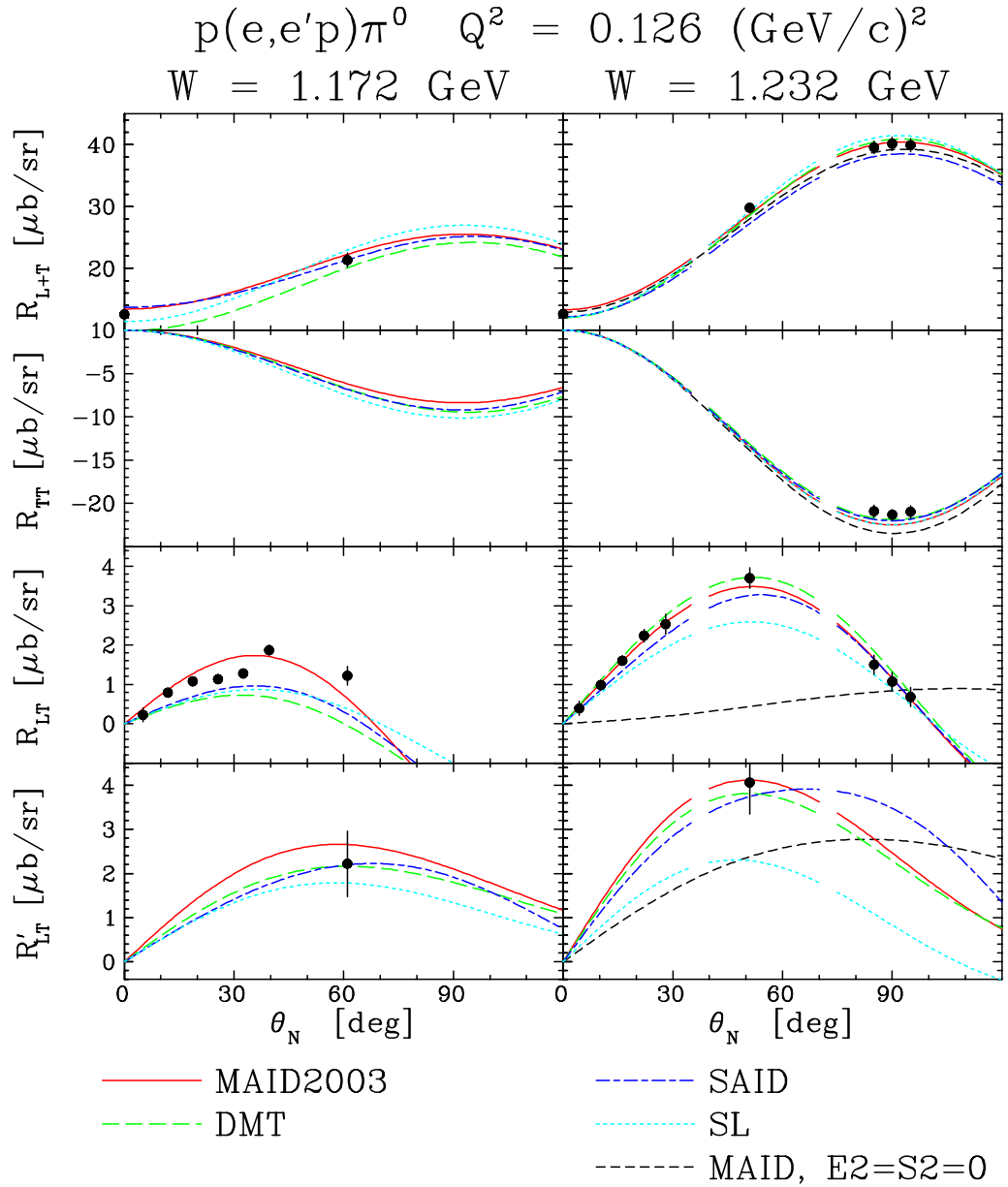
handle	real type	imaginary type
left/right	R_{LT}, $R'^{\ell,t}_{LT}$, $R'^{\ell,t}_{TT}$	R^n_{LT}
OOP	R_{TT}, R'^n_{LT}, R^n_{TT}	R'_{LT}, $R^{\ell,t}_{LT}$, $R^{\ell,t}_{TT}$
Rosenbluth	R_L, R_T $(\nu_T R_{L+T} = \nu_L R_L + \nu_T R_T)$	R^n_L, R^n_T

Parallel kinematics, $Q^2 \approx 0.126$ (GeV/c)^2



- Dynamical models fail on low-W side of Δ
- SL underestimates R^n_{LT}
- marginal failure of consistency relation

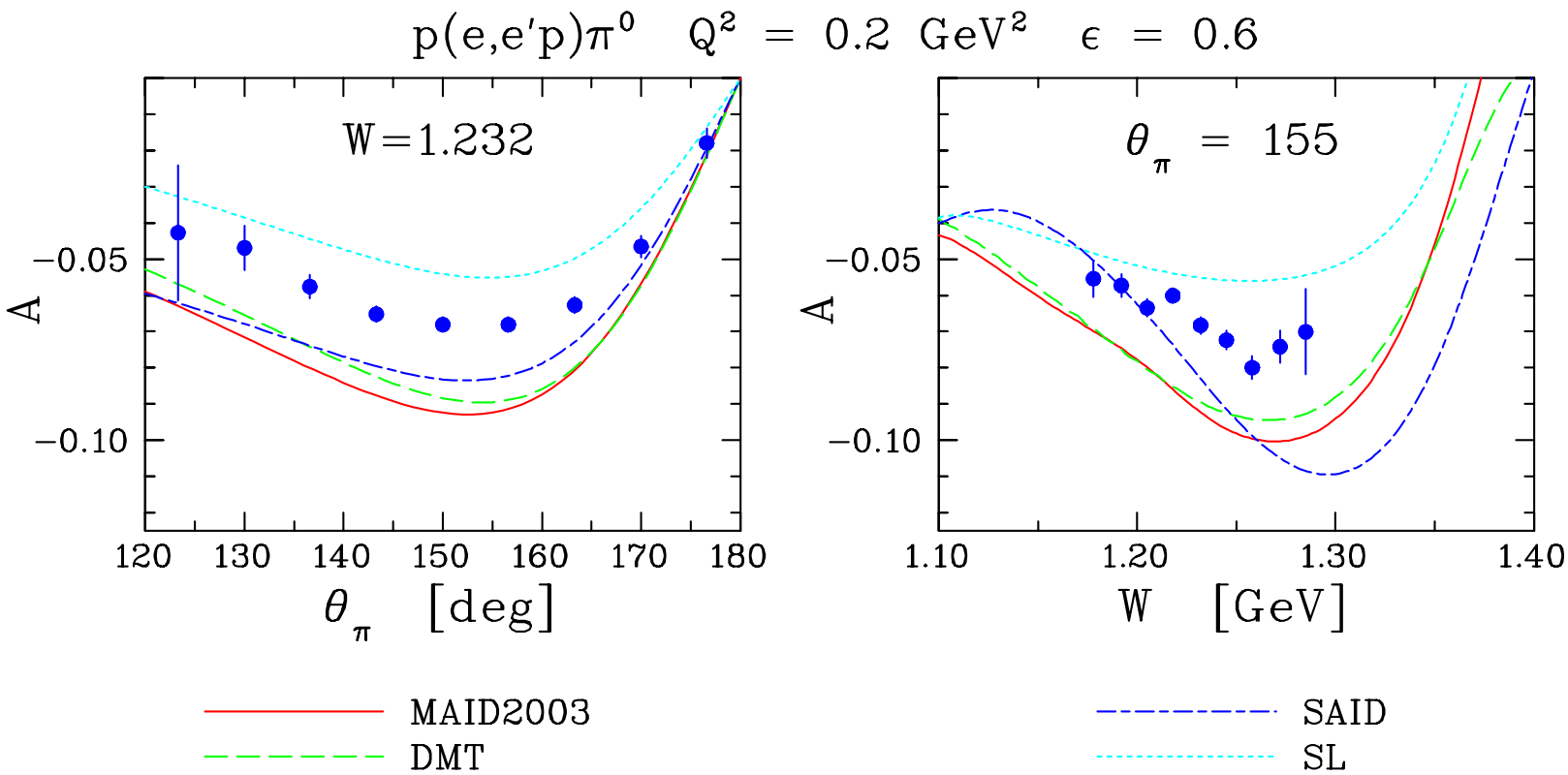
MIT-OOPS, $Q^2 \approx 0.126$ (GeV/c)^2



- The most microscopic model (SL) has the most difficulty
- No dramatic problems in MAID, DMT, or SAID here
- R_{LT} clearly requires significant S_{1+}
- Limited sensitivity to E_{1+}

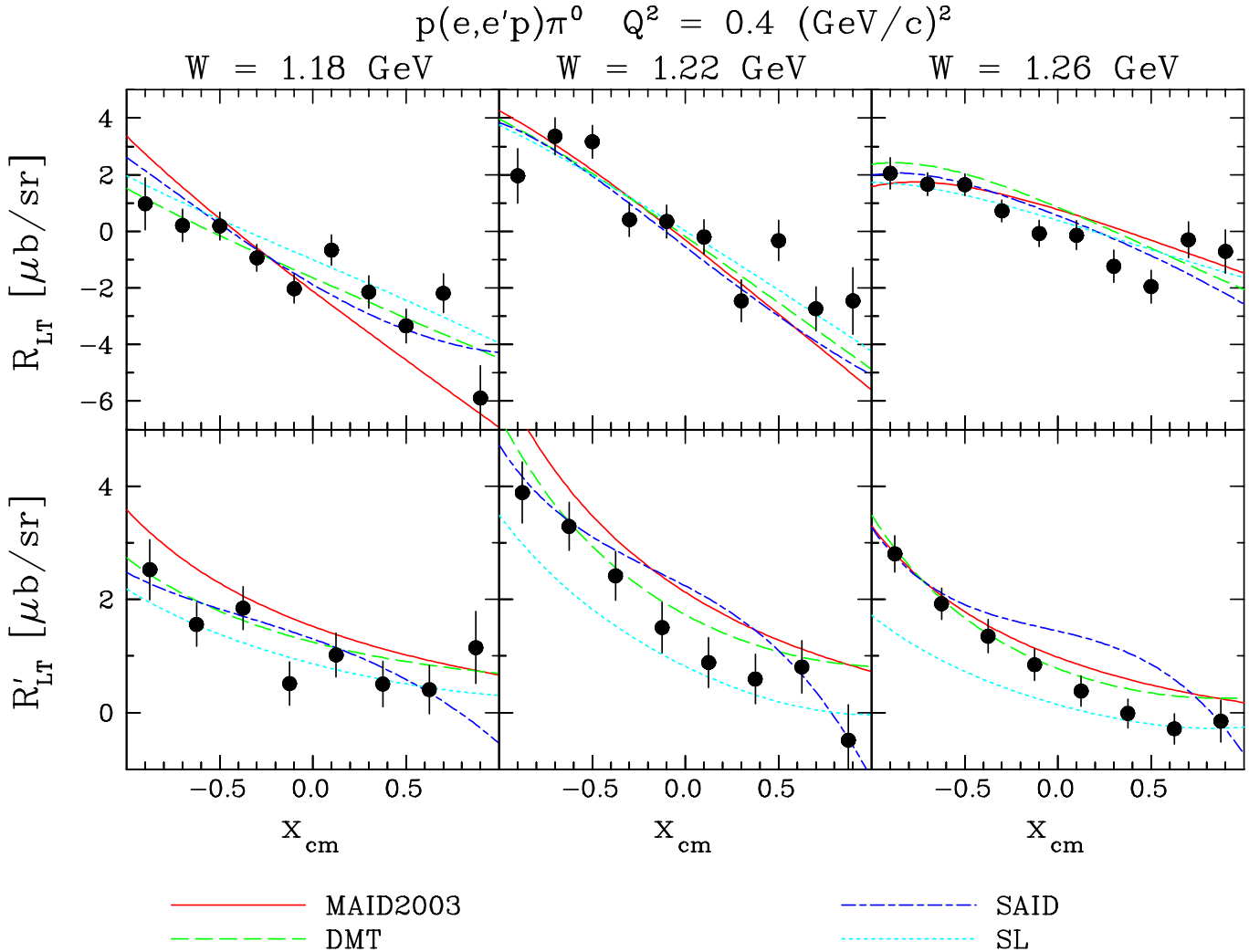
Beam analyzing power $Q^2 \approx 0.2$ (GeV/c)^2

Bartsch *et al.* at MAMI using spectrometer tilted out of plane



- Sensitive to nonresonant contributions and FSI.
- Considerable variation among models.

R_{LT}, R'_{LT}, $Q^2 \approx 0.4$ (GeV/c)^2; Joo *et al.* at CLAS



- Terms beyond sp truncation $\Longrightarrow$ deviation from linearity.
- Larger variations and nonlinearities for R'_{LT} than R_{LT}.

JLab e91011: recoil polarization in $p(\vec{e}, e'\vec{p})\pi^0$

- spokespersons: Kelly, Sarty, Frullani, Lourie (emeritus)
- Z. Chai: cross section (MIT, 2003)
- R. Roché: recoil polarization (FSU, 2003)
- O. Gayou: final polarization analysis (MIT postdoc)
- M. Jones: analysis guru
- Hall A collaboration: cast of thousands

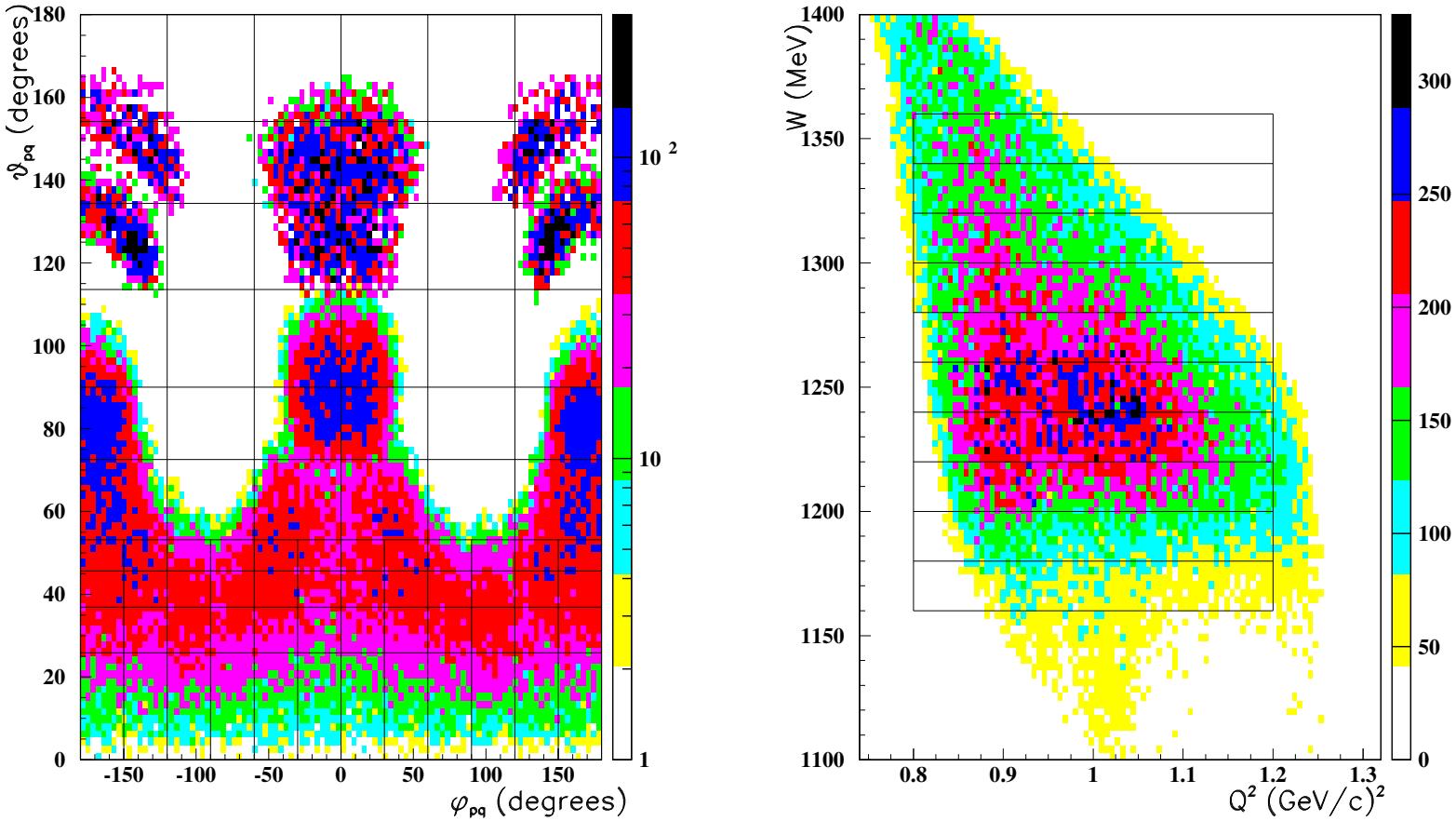
Nominal kinematics:

$1.17 \leq W \leq 1.35$ GeV
$Q^2 = 1.0 \pm 0.2$ (GeV/c)^2
$E_0 = 4.53$ GeV, $\epsilon \approx 0.95$
$-1 \leq x_{\text{cm}} \leq +1$

Beam properties:

current up to 110 μA
polarization $\approx 72\%$
duration: 36 days, 180 C

Kinematic coverage



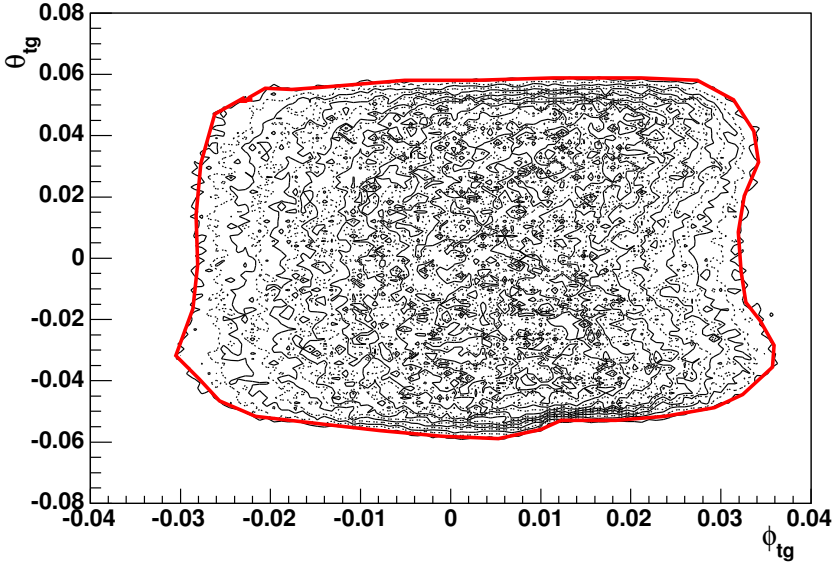
- Kinematic focusing $\Longrightarrow$ considerable OOP coverage.
- need to compensate for variation in $\overline{Q^2}$
- sweet spot: $1.21 \leq W \leq 1.29$

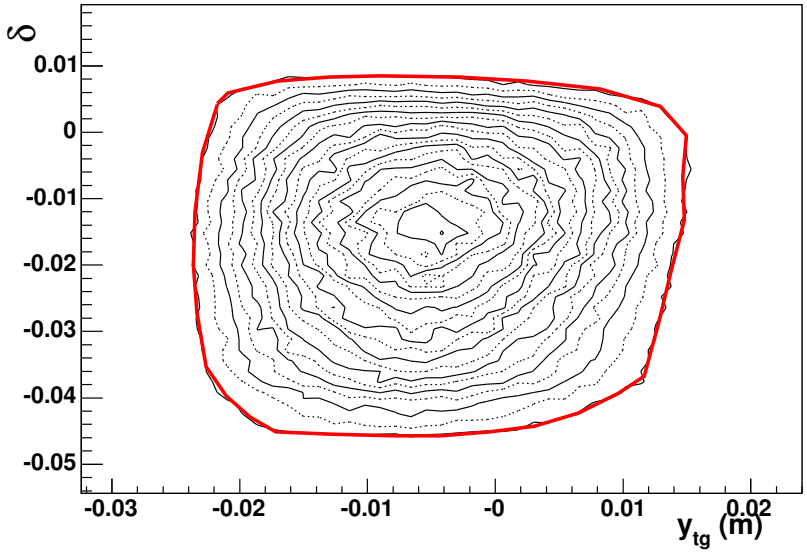
Acceptance Function

(Z. Chai)

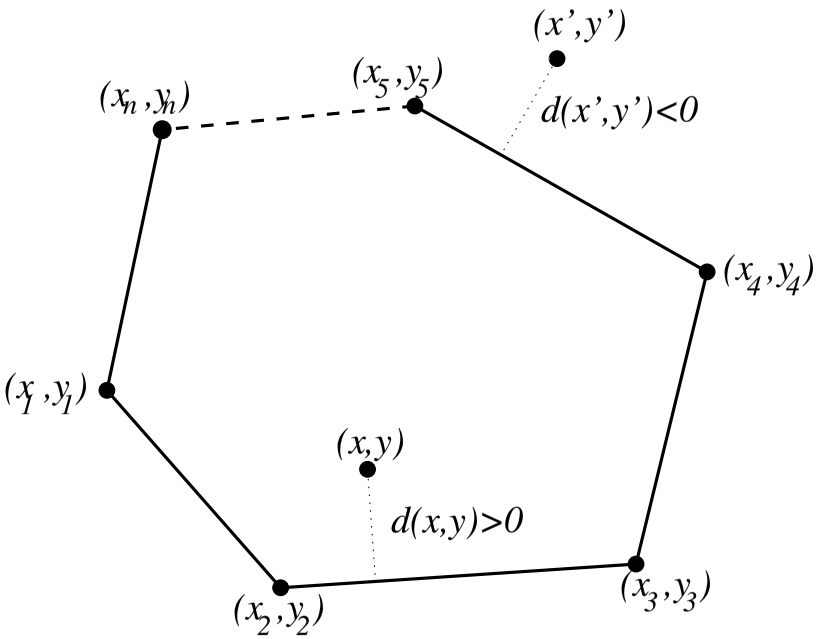
- Measure $(x, y, \theta\phi)_{fp}$ at focal plane $\Longrightarrow$ reconstruct $(\delta, y, \theta, \phi)_{tgt}$ at target.
- Some trajectories in 4-dimensional volume of target variables strike apertures or miss detectors.
- Use coincidence data to define acceptance function R.
  - advantage: suppress accidentals
  - disadvantage: depends upon kinematics
- Two-dimensional projections
  - 4 variables $\Longrightarrow$ 6 two-dimensional projections for each spectrometer
  - draw polygonal boundaries, scale to equal area

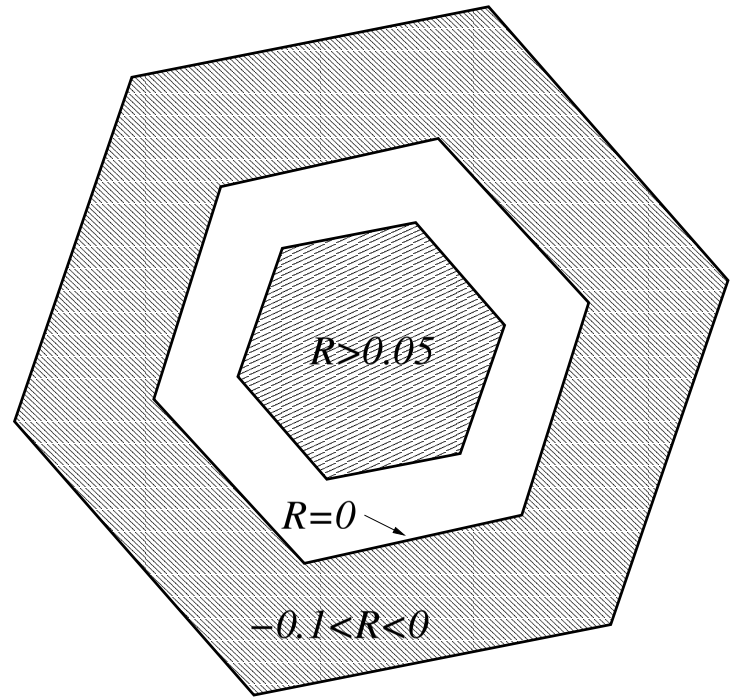
Sample projections





distance from border



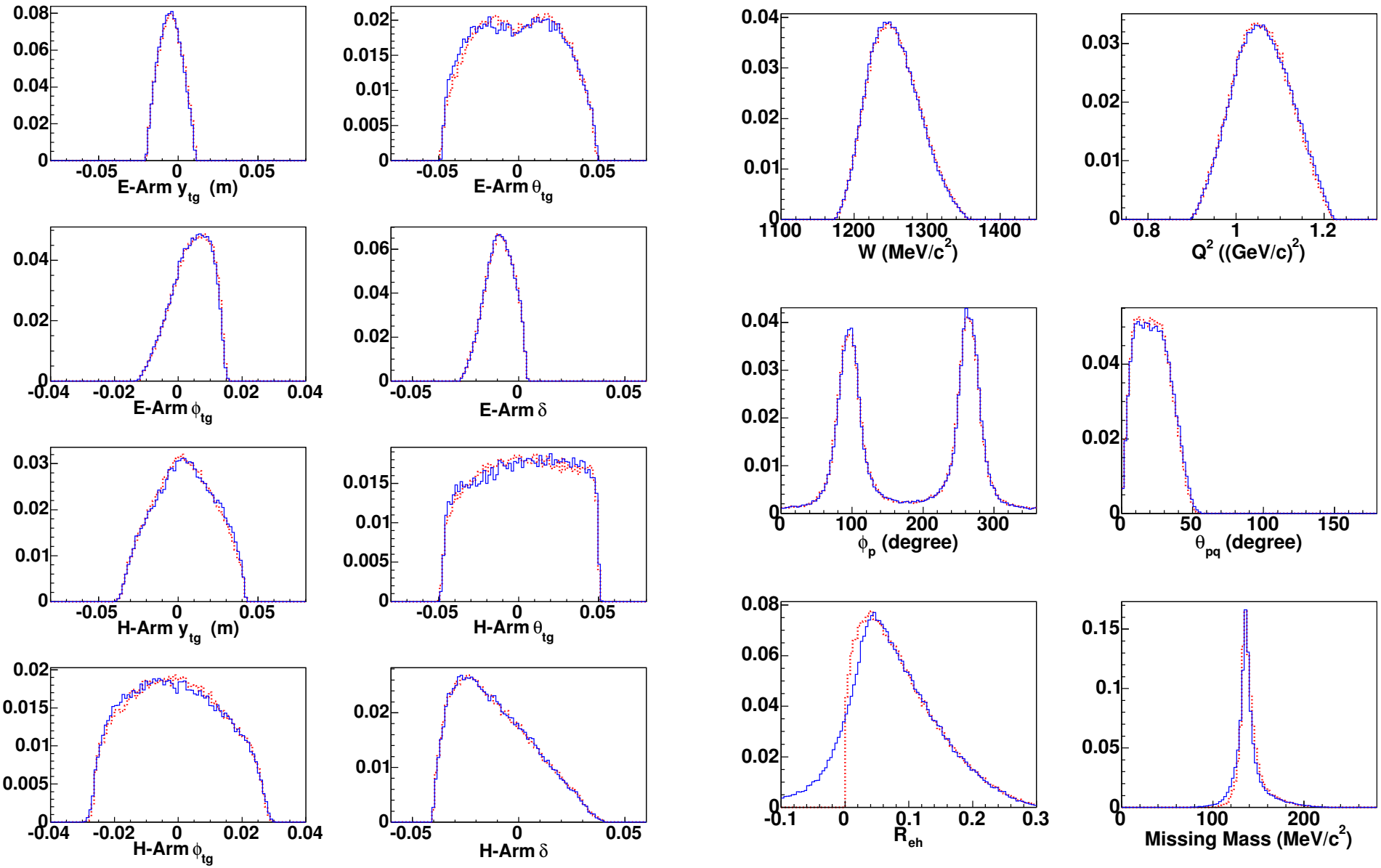


- Define R_i as distance to nearest segment; determine whether inside or outside
- For each spectrometer, define $R = \pm \min R_i$ as positive for all R_i inside, negative if any outside.
- Expect constant detection probability for $R > R_{cut}$ sufficiently far from any edge.

Simulation

- `EPIPROD` event generator in `MCEEP` using
  - MAID2000 cross section weighting
  - R-function acceptance
  - additional smearing at FP
  - radiative corrections
- Choose R_{cut} on data/theory plateau
- Good reproduction of kinematic distributions
  - angle offset determined by missing mass is consistent with survey

Acceptance Simulation



Cross section calculation

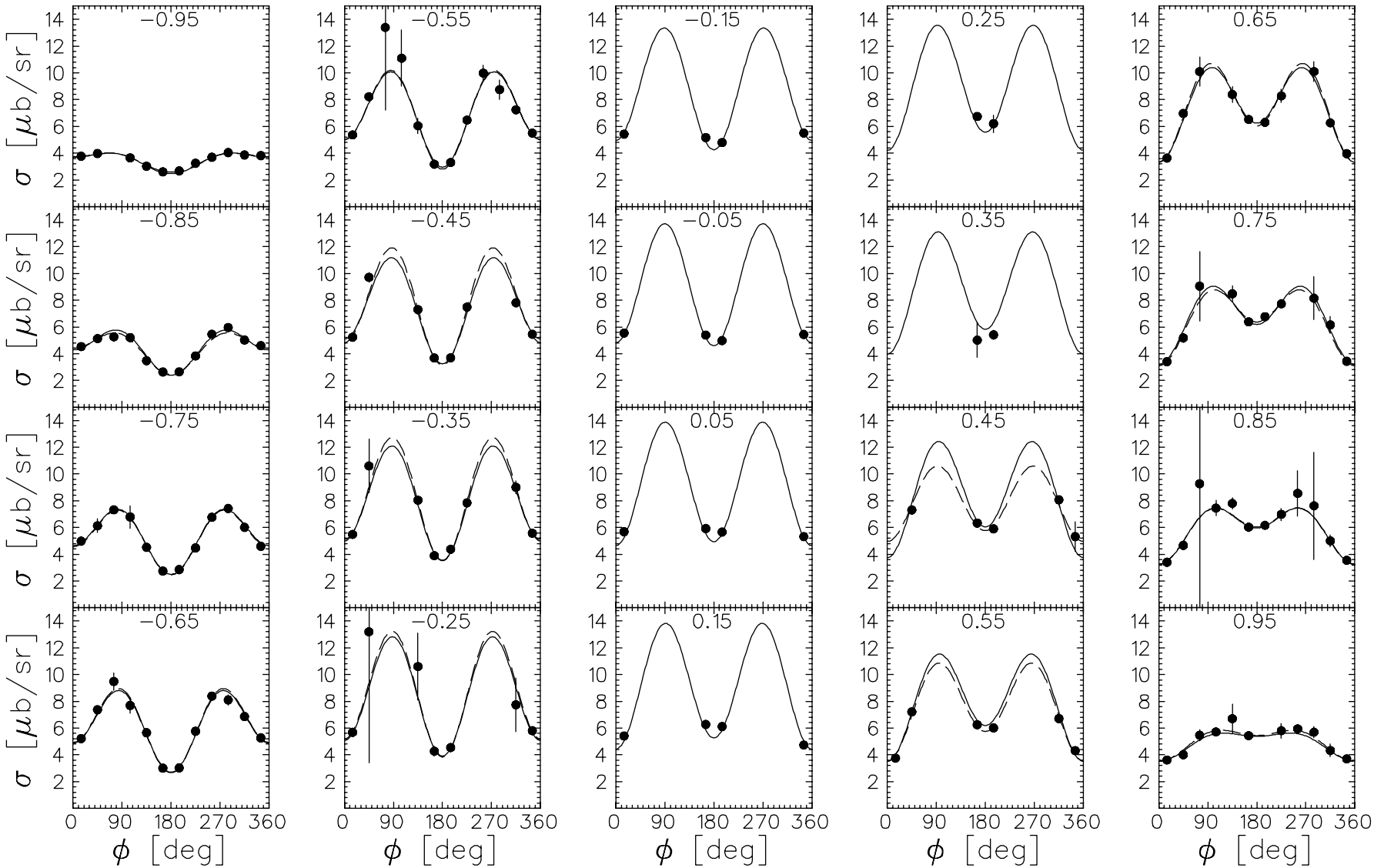
Simulation corrects for acceptance averaging, such that

$$\bar{\sigma} = \frac{Y}{Y_{\text{MC}}} \bar{\sigma}_{\text{model}} \frac{f_{\text{CDT}} f_{\text{EDT}} f_{\text{abs}}}{\epsilon_{\text{trigger}} \epsilon_{\text{track}}}$$

gives cross section for *nominal kinematics* at bin center.

- Y = experimental yield corrected for background
- Y_{MC} = simulated yield for model $\bar{\sigma}_{\text{model}}$
- f_{CDT} = computer dead time (~ 1.1)
- f_{EDT} = electronic dead time (~ 1.1 @ 1 MHz)
- f_{abs} = absorption in target & windows (~ 1.01)
- $\epsilon_{\text{trigger}}$ = trigger efficiency ($\sim 97\%$)
- ϵ_{track} = tracking efficiency (0.85 – 1 per arm)

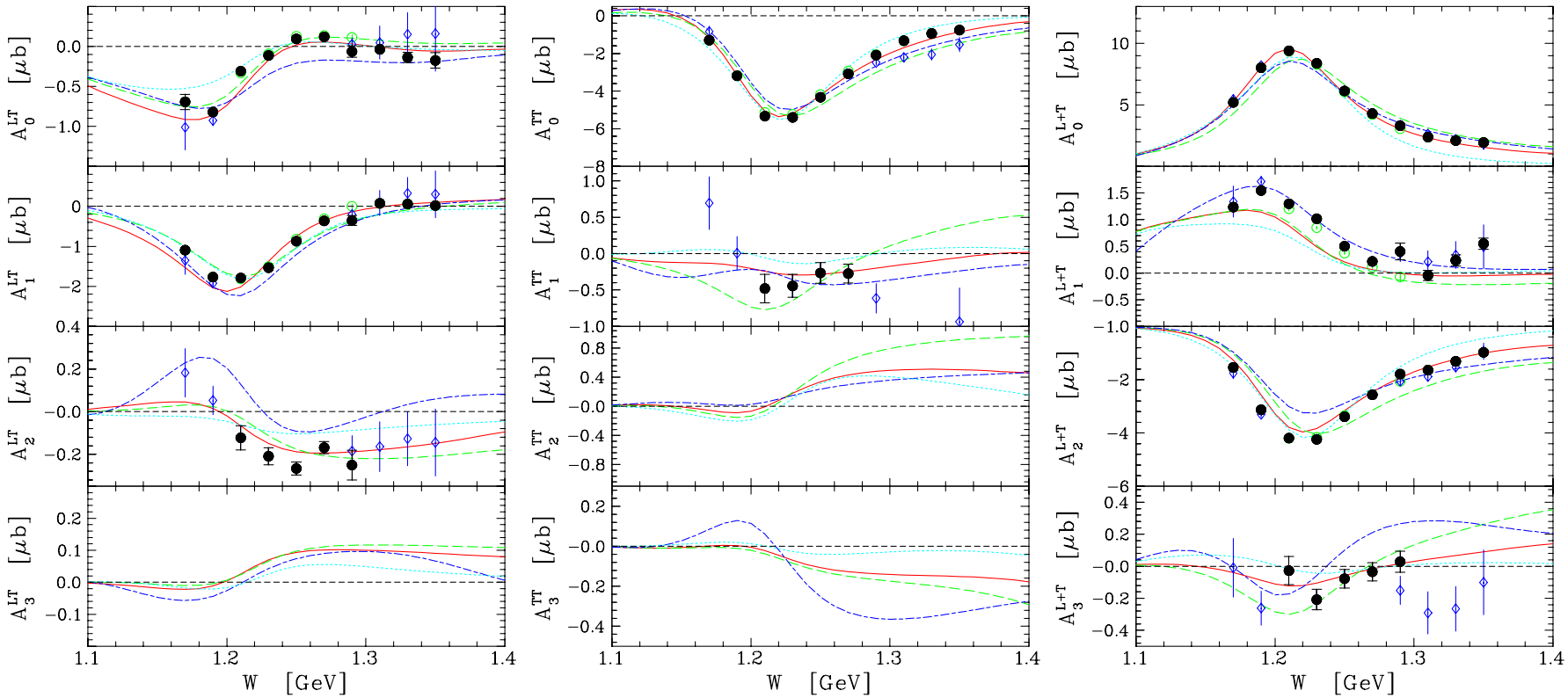
Differential cross section



dashed: extract responses

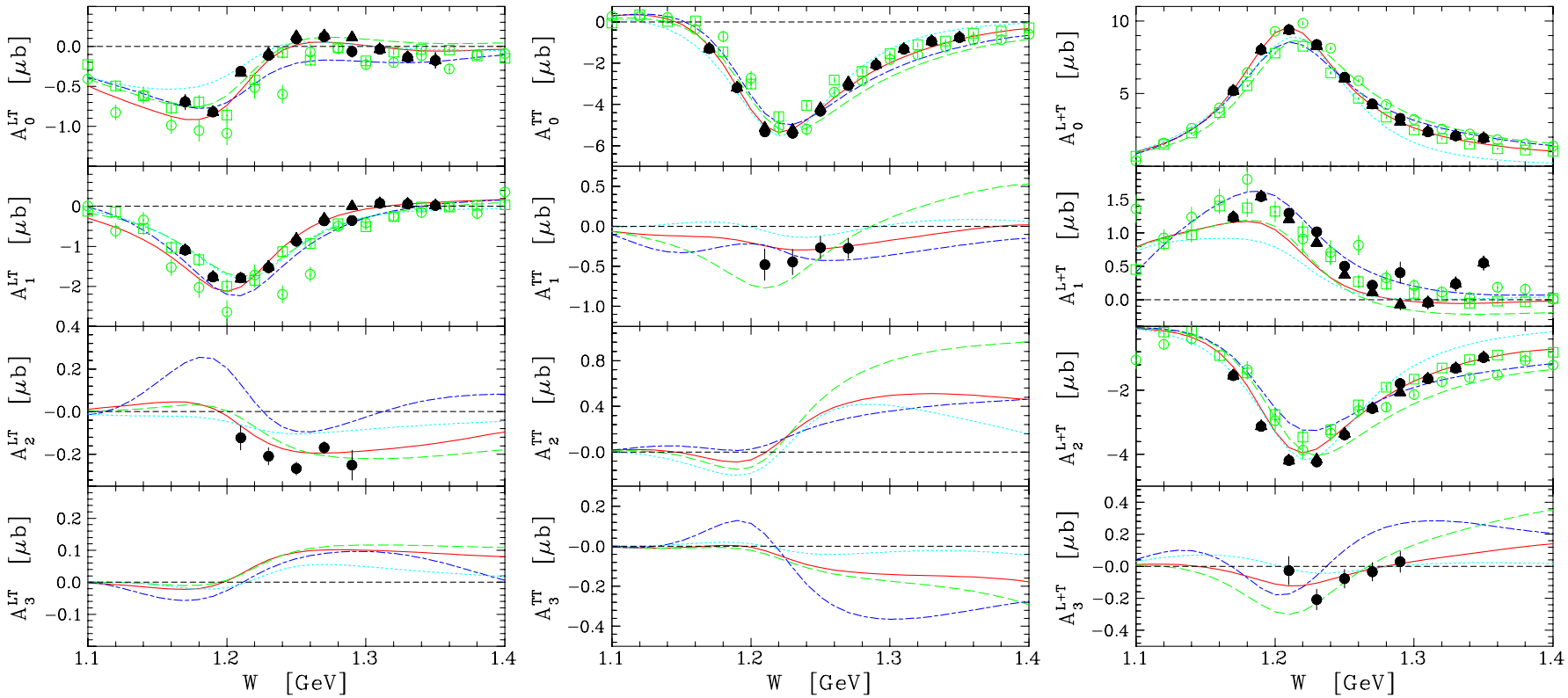
solid: Legendre

Legendre for cross section



- Reasonable agreement with models.
- Evidence for terms beyond sp truncation.
- Unanticipated structure in A_1^{L+T} for $W > 1.3$ GeV?

Comparison with CLAS



- Use dipole form factor to scale CLAS data for $Q^2 = 0.90, 1.15$ to $Q^2 = 1$ (GeV/c)^2.
- Generally good agreement with CLAS.
- Better statistics but less kinematic coverage.

Traditional quadrupole analysis

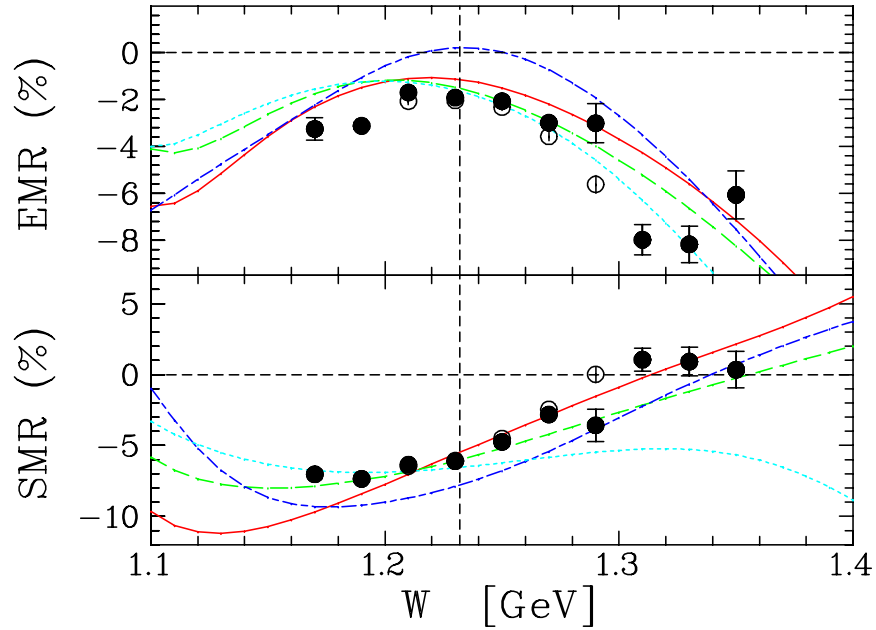
Assume:

- sp truncation
- M_{1+} dominance

$$\tilde{R}_{EM} = \frac{3A_2^{L+T} - 2A_0^{TT}}{12A_0^{L+T}}$$

$$\tilde{R}_{SM} = \frac{A_1^{LT}}{3A_0^{L+T}}$$

Same formulas, theory and data.



method	SMR, %	EMR, %	χ^2_ν	$\chi^2_N(\sigma)$
sp	-6.07 ± 0.11	-2.04 ± 0.13	1.7	1.6
$sp+$	-6.11 ± 0.11	-1.92 ± 0.14	1.5	1.4
CLAS [a]	-7.4 ± 0.4	-1.8 ± 0.4		

[a] Average of $Q^2 = 0.9$ (GeV/c)^2 for $E_i = 1.645$ and 2.445 GeV.

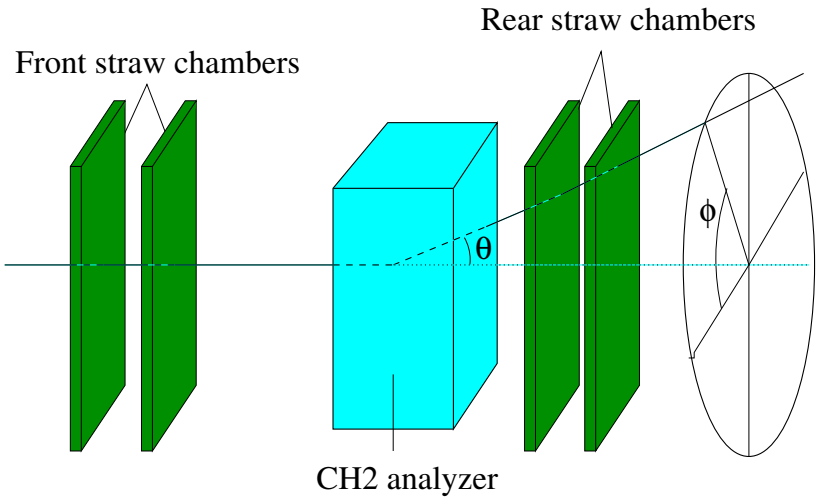
Reliability of Legendre method

Project Legendre coefficients from truncated multipole expansions and compute transitional EMR,SMR estimators.

model	$R_{SM}^{(p\pi^0)}$	$\tilde{R}_{SM}^{(p\pi^0)}$ $\ell_\pi \leq 1$	$\tilde{R}_{SM}^{(p\pi^0)}$ $\ell_\pi \leq 5$	$R_{EM}^{(p\pi^0)}$	$\tilde{R}_{EM}^{(p\pi^0)}$ $\ell_\pi \leq 1$	$\tilde{R}_{EM}^{(p\pi^0)}$ $\ell_\pi \leq 5$
Legendre			-6.11			-1.92
MAID2003	-6.73	-6.37	-5.63	-1.65	-0.57	-1.12
DMT	-7.21	-6.77	-6.10	-1.77	-0.70	-1.47
SAID	-8.71	-7.78	-7.88	$+0.17$	$+1.96$	$+0.22$
SL	-6.59	-6.69	-6.58	-2.29	-1.29	-1.58

Neither sp truncation nor M_{1+} dominance supported by models! Need more complete data for multipole fit.

Maximum-likelihood method for responses



$S_{\alpha\beta}$ spin transport matrix

A analyzing power

ξ false aymmetry

$\bar{\sigma}$ from Legendre fit

$$L = \prod_{\text{events}} \frac{1}{2\pi}\left(1 + \xi - \varepsilon_x \sin\phi_{\text{fpp}} + \varepsilon_y \cos\phi_{\text{fpp}}\right)$$

$$\varepsilon_\alpha = \xi_\alpha + A(\theta_{\text{fpp}}) \sum_\beta S_{\alpha\beta}\Pi_\beta$$

$$\bar{\sigma}\Pi_\beta = \sum_\eta \nu_{\beta\eta}(P_e, \epsilon, \theta, \phi) R_\eta$$

Maximal use of data. No binning in ϕ or ϕ_{fpp}.

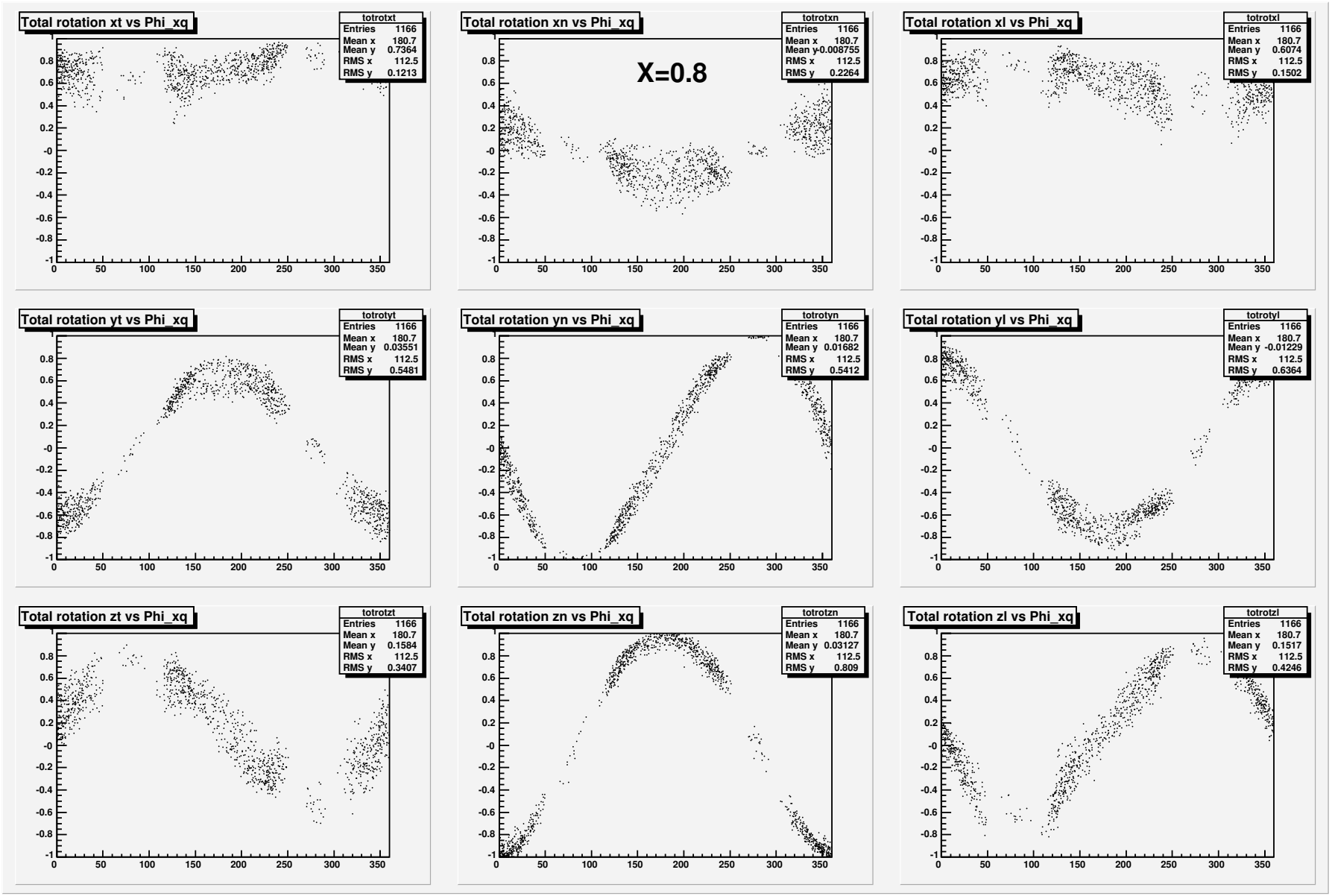
Spin rotation

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}_{\text{FPP}} = R_{\text{fpp}}\, C\, R_{\text{spectrometer}}\, R_{\text{hall}}\, R_W \begin{pmatrix} \Pi_t \\ \Pi_n \\ \Pi_\ell \end{pmatrix}_{\pi N cm}$$

- R_W: Wigner rotation from $cm \rightarrow lab$
- R_{hall}: to hall basis
- $R_{\text{spectrometer}}$: to the spectrometer basis
- C: COSY transport through spectrometer
- R_{fpp}: to the local FPP coordinate system

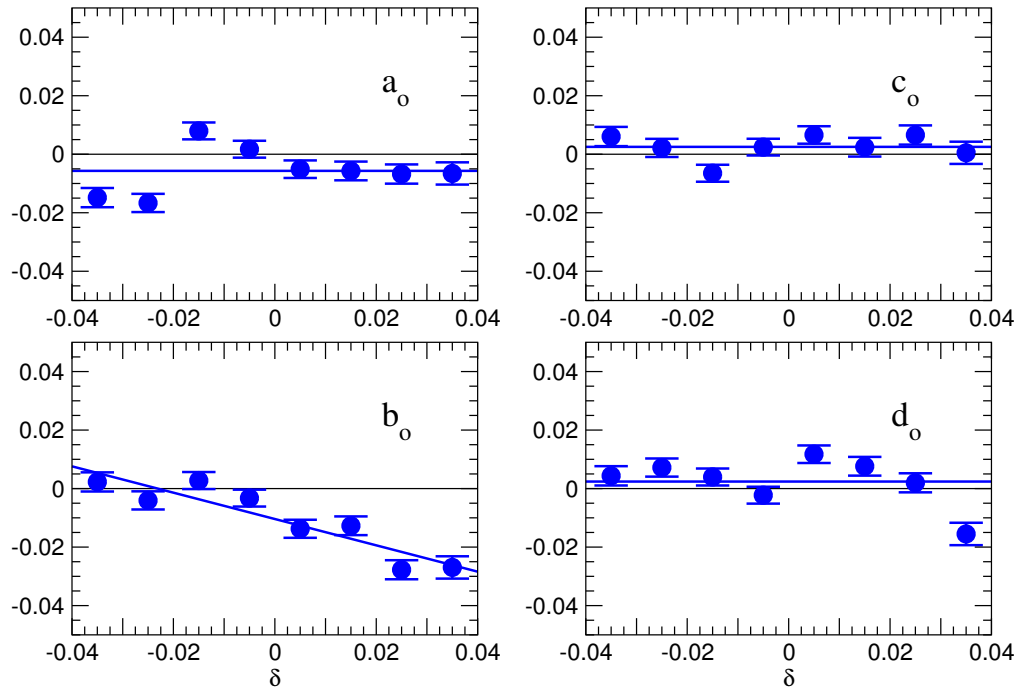
Evaluated event-by-event.

Spin rotation matrix



False asymmetry

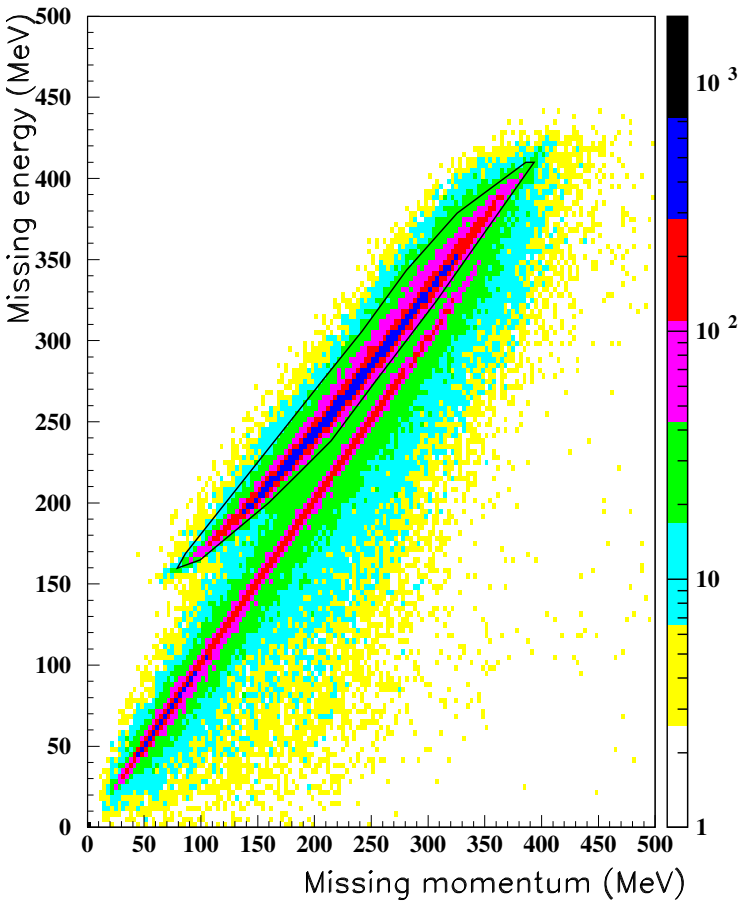
$$\xi = a_0 \sin\phi_{\text{fpp}} + b_0 \cos\phi_{\text{fpp}} + c_0 \sin 2\phi_{\text{fpp}} + d_0 \cos 2\phi_{\text{fpp}}$$



- calibrate assuming helicity-independent polarization vanishes for elastic scattering
- depends primarily on δ, not p
- small effects with little systematic uncertainty

Elastic subtraction

$$\varepsilon_\alpha = A_y(\theta_{\rm fpp}) \sum_\beta (f_1 S^{(1)}_{\alpha\beta} T^{(1)}_\beta + f_2 S^{(2)}_{\alpha\beta} T^{(2)}_\beta)$$



- elastic contribution, f_2, visible only for $\theta_{pq} = -50°, -90°$
- compute elastic polarization, $T^{(2)}$, using form factors
- effect on $N \rightarrow \Delta$ responses minimal

Bin centering for responses

Acceptance-averaged $(\overline{W}, \overline{Q^2})$ depend upon $\overline{x}$; desired quantities (multipoles or Legendre coefficients) do not.

- increases scatter wrt to fit
- could artificially enhance higher partial waves
- angular dependence of $\overline{Q^2}$ most important for this experiment

Additive method:

$$R(W, Q^2, \bar{x}, \bar{\epsilon}) = R(\overline{W}, \overline{Q^2}, \bar{x}, \bar{\epsilon}) - \frac{\partial R}{\partial W}(\overline{W} - W) - \frac{\partial R}{\partial Q^2}(\overline{Q^2} - Q^2)$$

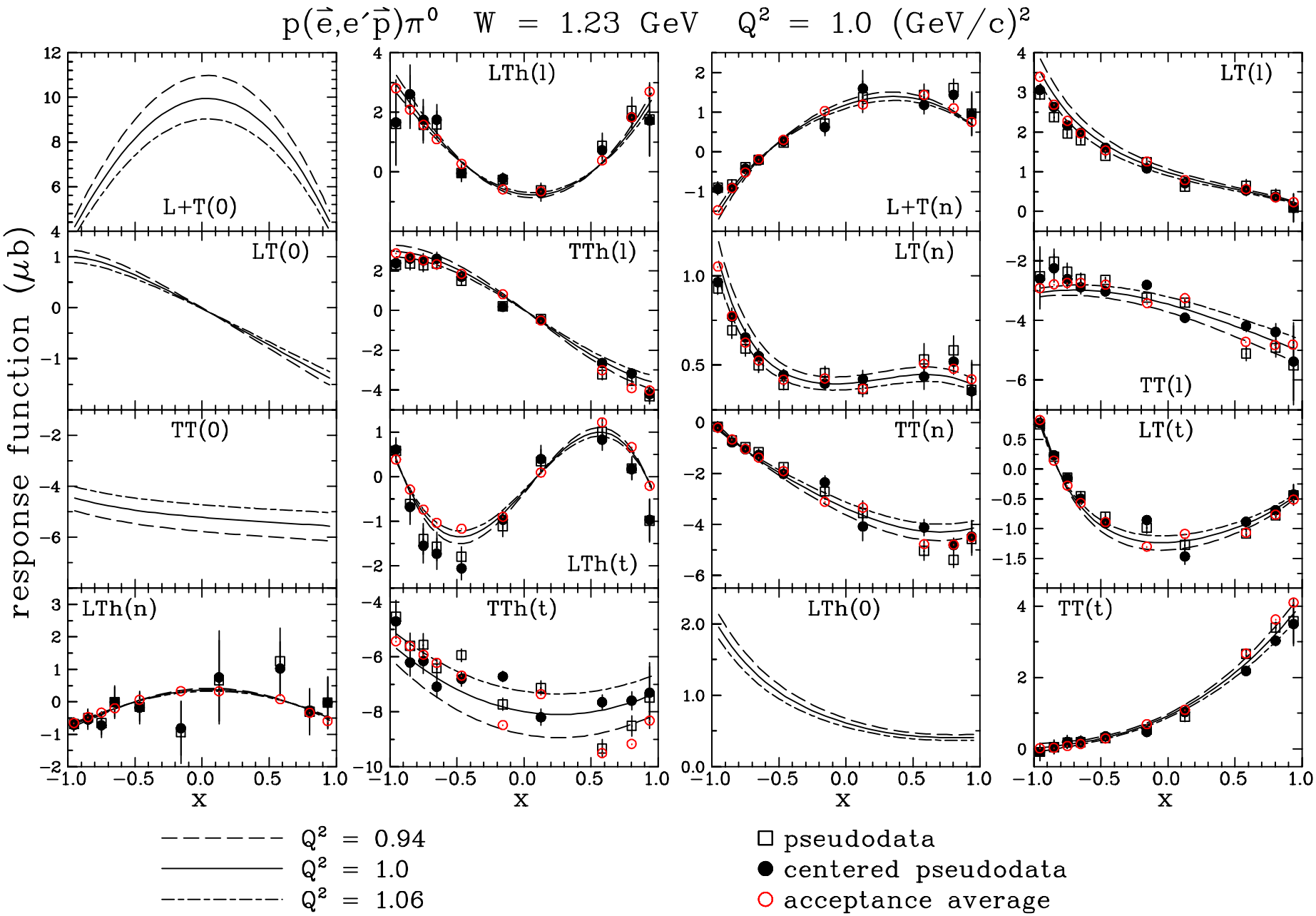
Multiplicative method:

$$R(W, Q^2, \bar{x}) = R(\overline{W}, \overline{Q^2}, \bar{x}) \left(G(Q^2)/G(\overline{Q^2}) \right)^2$$

Pseudodata test method

- use data to sample kinematical distributions and acceptance accurately; accumulate acceptance-averaged kinematics
- use model to compute polarization at target for each event; accumulate acceptance-averaged response functions and polarization
- transport polarization to FPP, use observed θ_{fpp}
- use rejection method to sample ϕ_{fpp} randomly with distribution based upon model polarization
- apply maximum likelihood method to ϕ_{fpp} pseudodata
- compare $max(L)$ responses with model at both nominal and acceptance-averaged kinematics

Pseudodata results



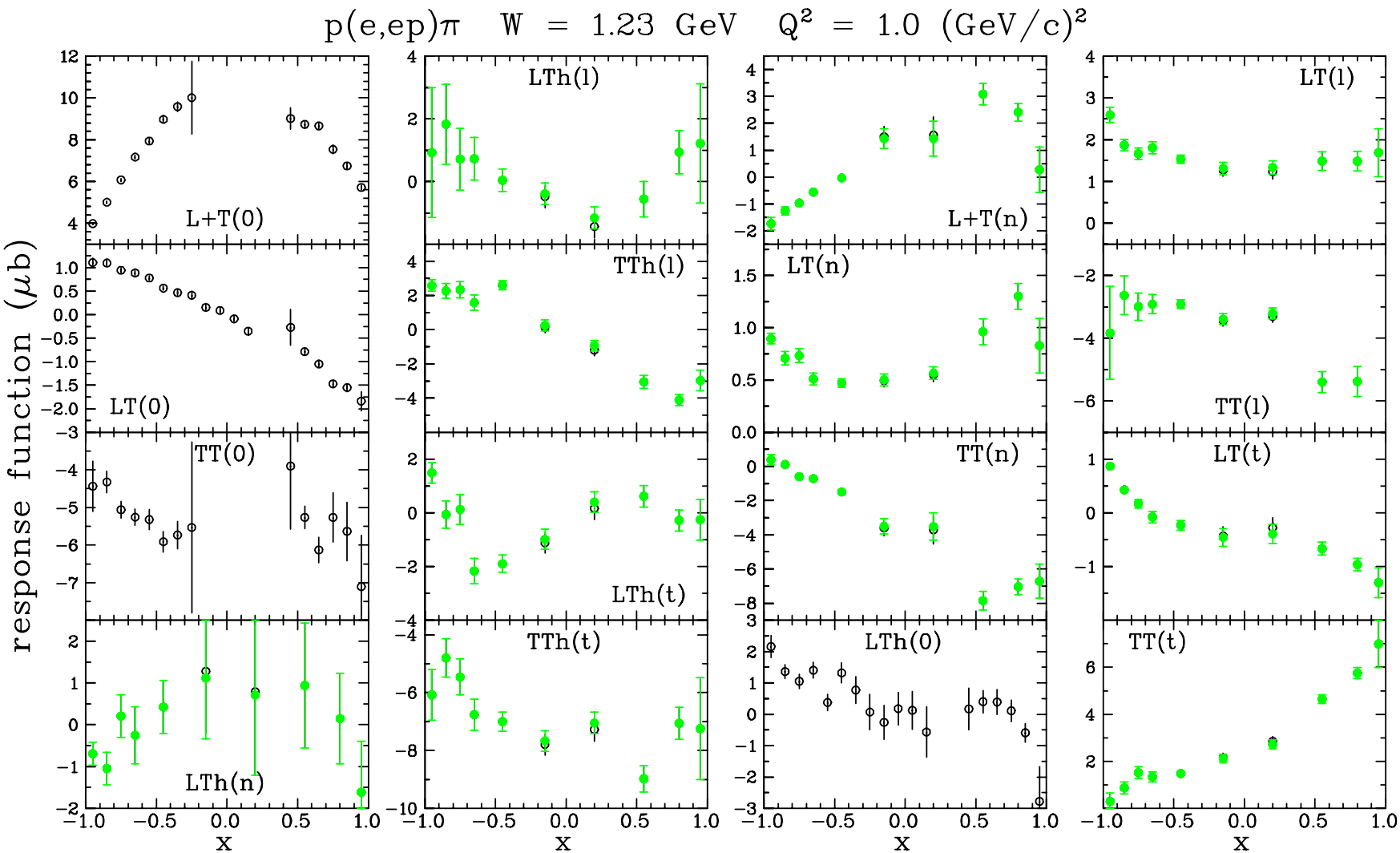
Pseudodata conclusions

- analysis more reliable for responses than polarizations — dispersion in spin-transport causes problems for binning wrt ϕ
- fluctuations in pseudodata correlated with differences between nominal and acceptance-averaged kinematics
- $(\overline{Q^2} - Q^2)$ more important than $(\overline{W} - W) \Longrightarrow$ multiplicative bin centering sufficient
- little sensitivity to model for bin-centering form factor
- method fundamentally sound

Systematic uncertainties in responses

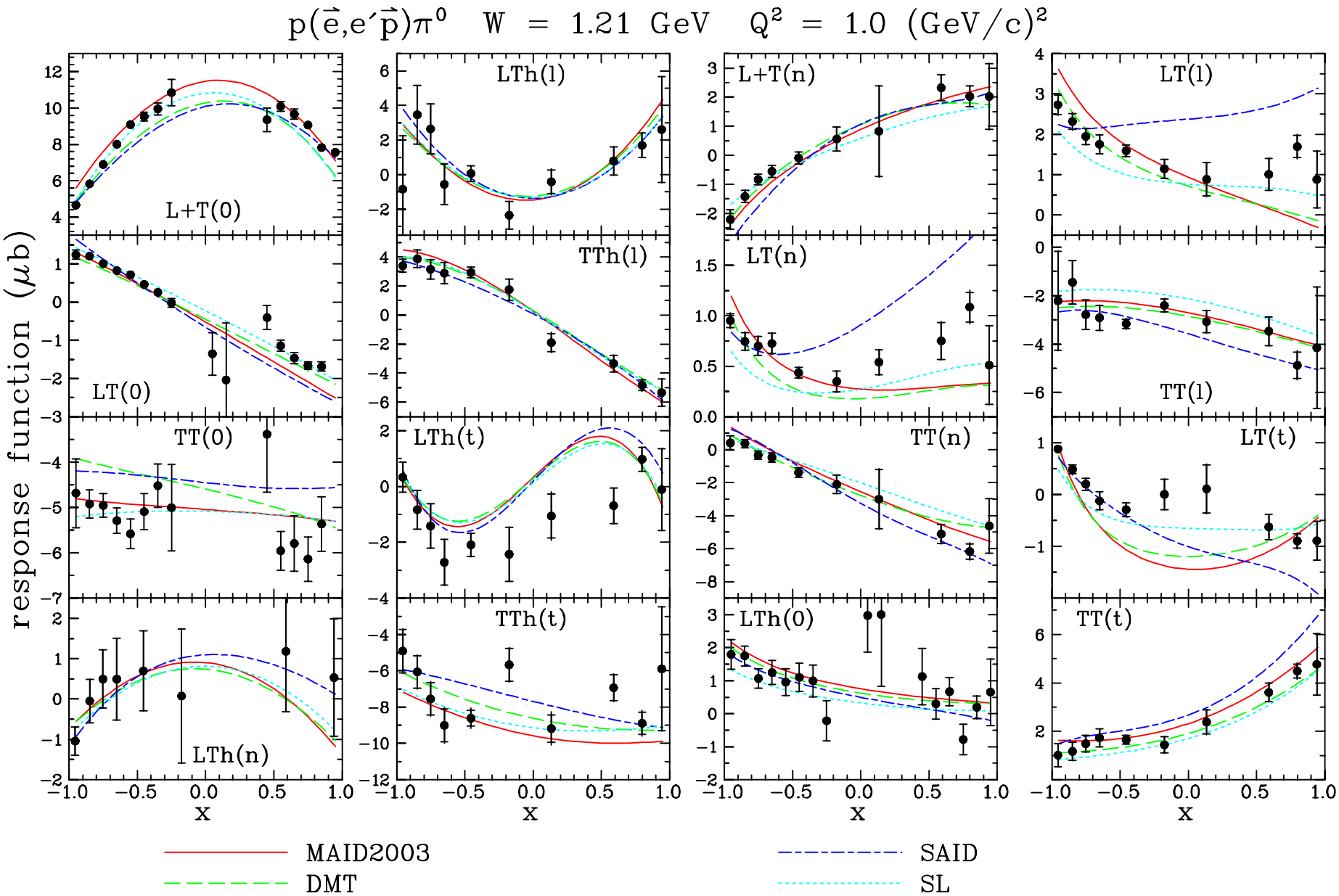
- normalization uncertainties
  - cross section, $\pm 3\%$
  - beam polarization, $\delta P_e / P_e \approx \pm 1.4\%$
  - analyzing power, $\delta A_y / A_y \approx \pm 2\%$
- estimated by replay or fit with perturbation
  - elastic contamination, $\delta f_{el} / f_{el} \sim 1$
  - false asymmetry, $\delta \xi / \xi \approx 0.1$
  - spin rotation, 6 parameters for Pentchev geometrical approximation
  - bin centering: compare dipole vs. Sato-Lee form factor

Pentchev versus COSY

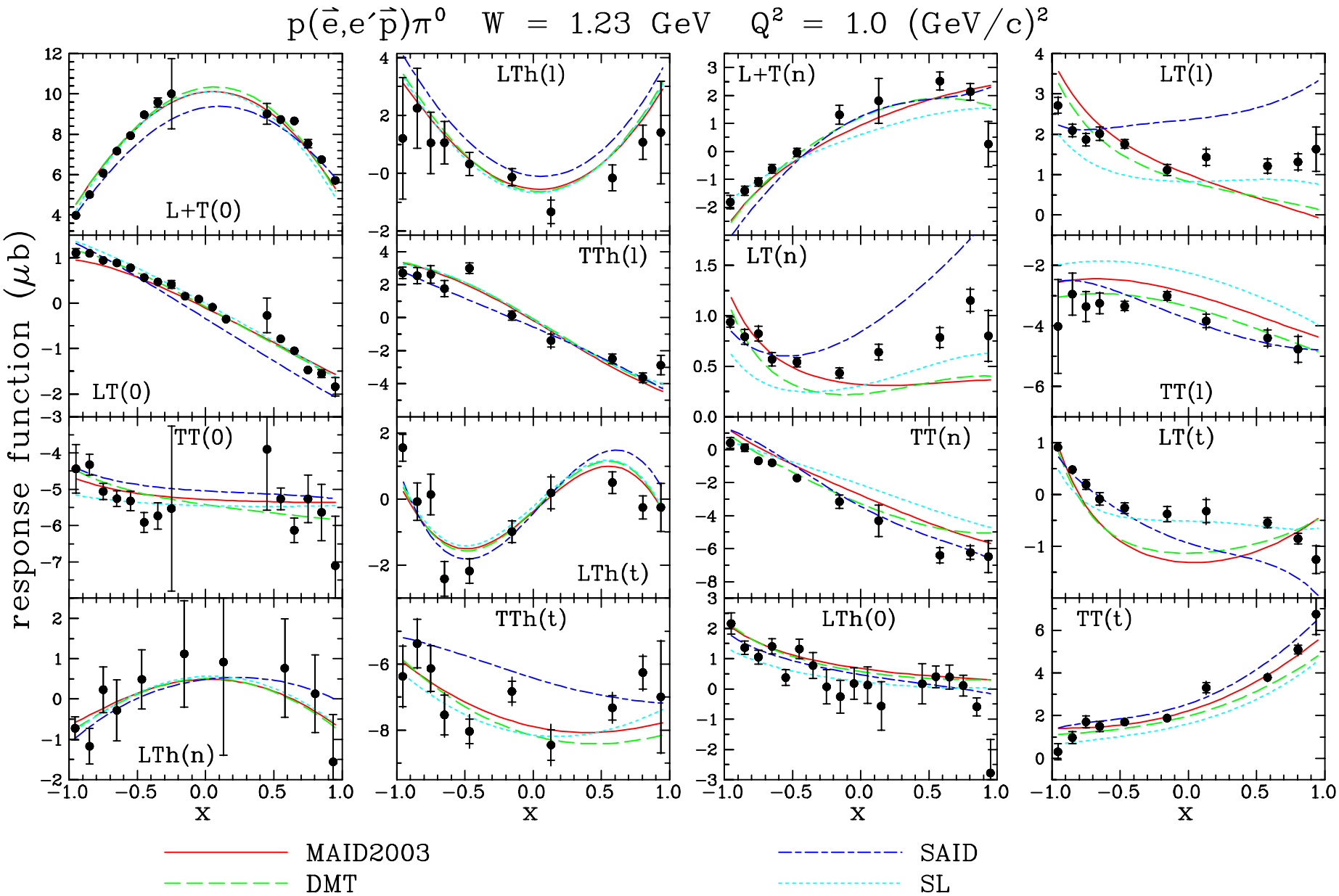


Pentchev geometrical model very similar to COSY; suitable for analysis of systematic uncertainties due to spin rotation

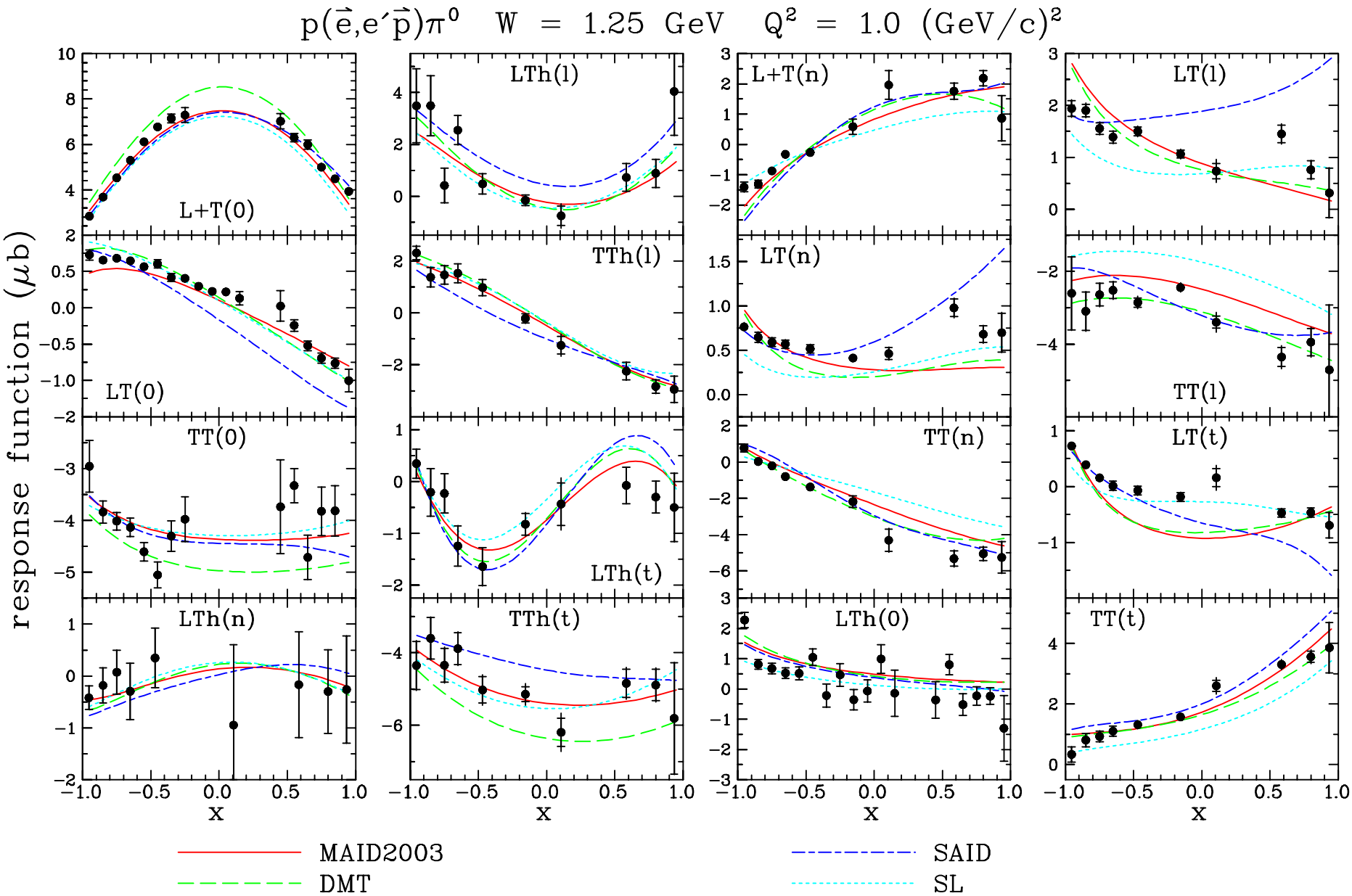
Model responses, W=1.21



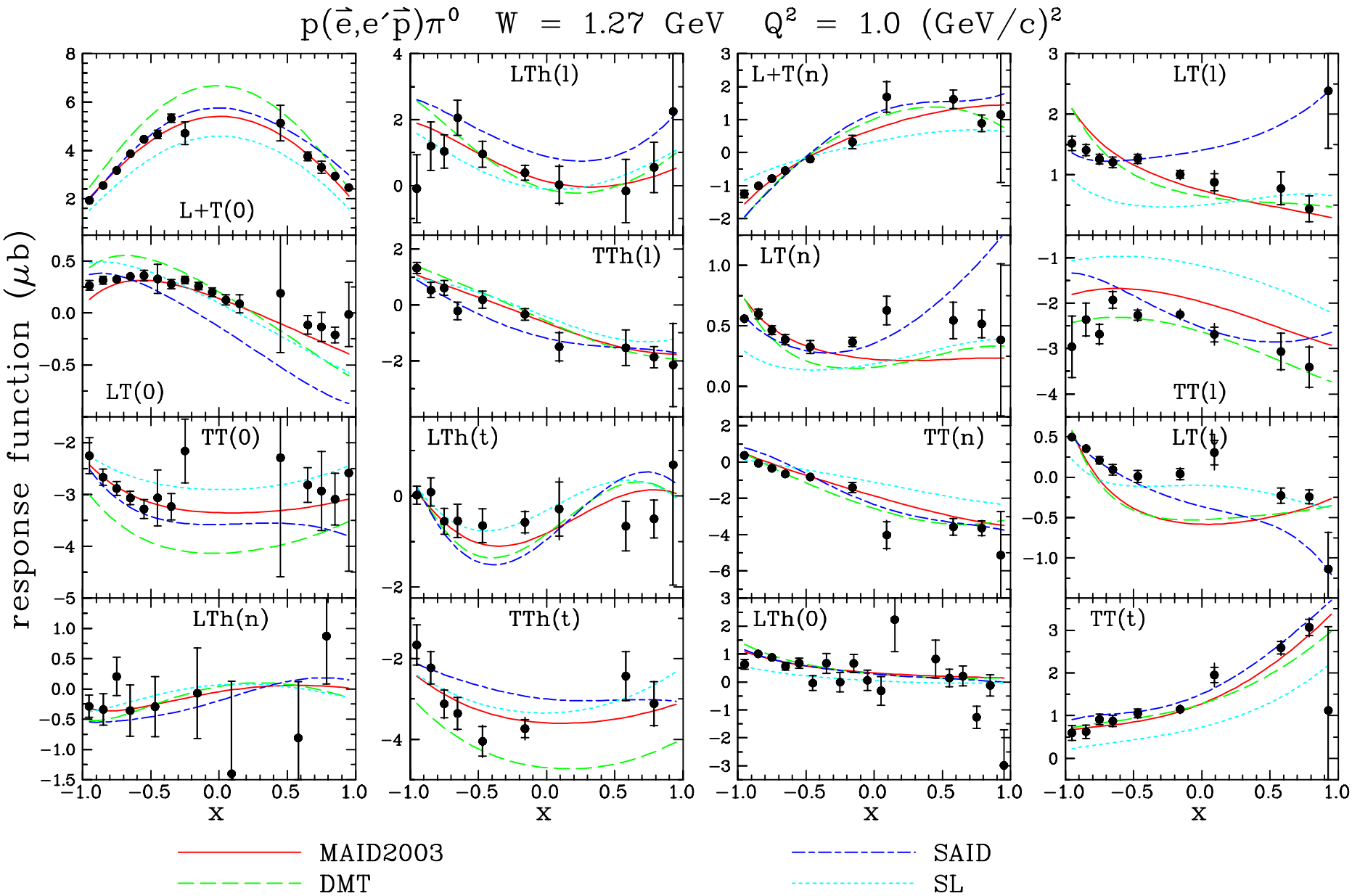
Model responses, W=1.23



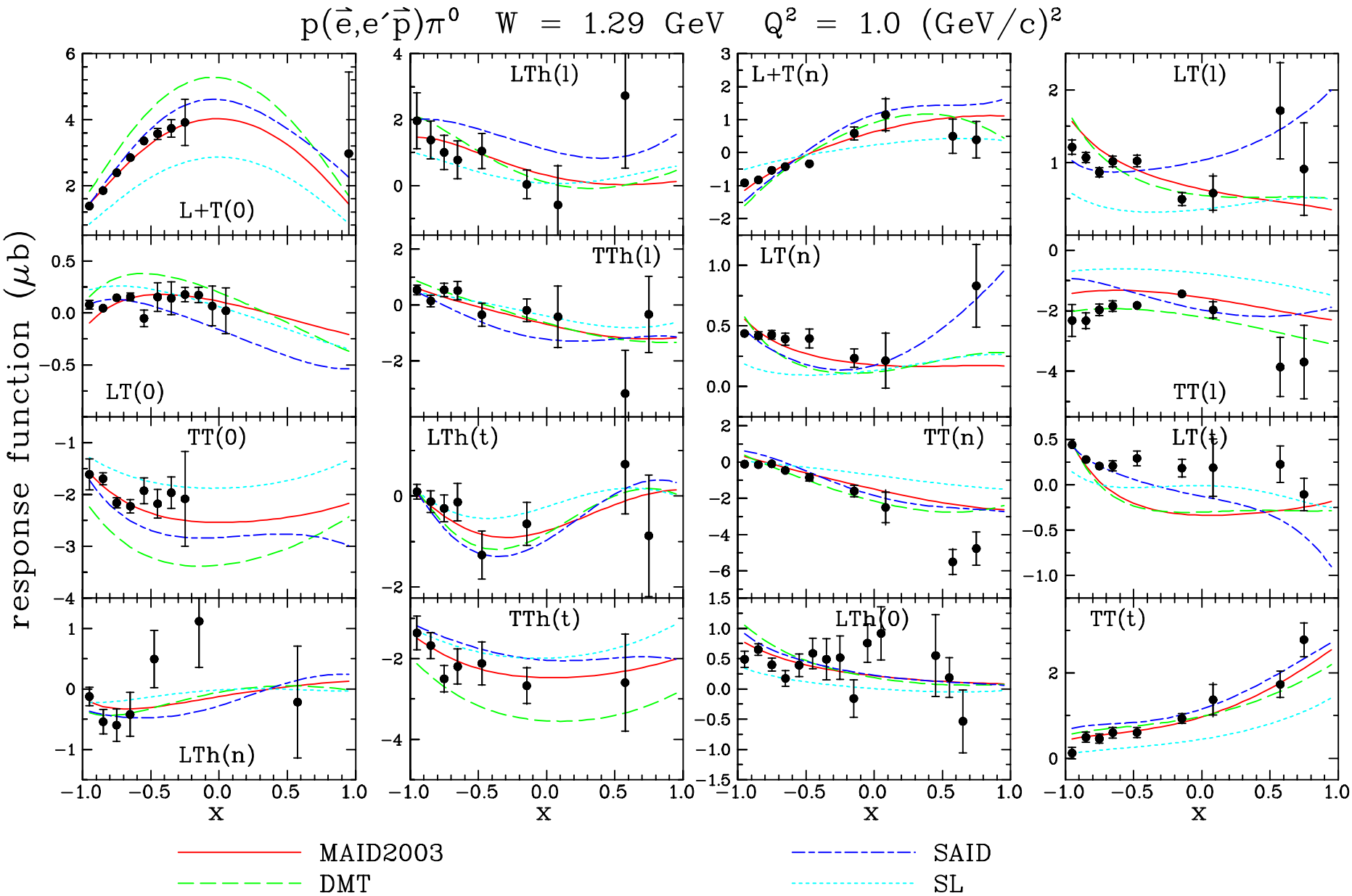
Model responses, W=1.25



Model responses, W=1.27



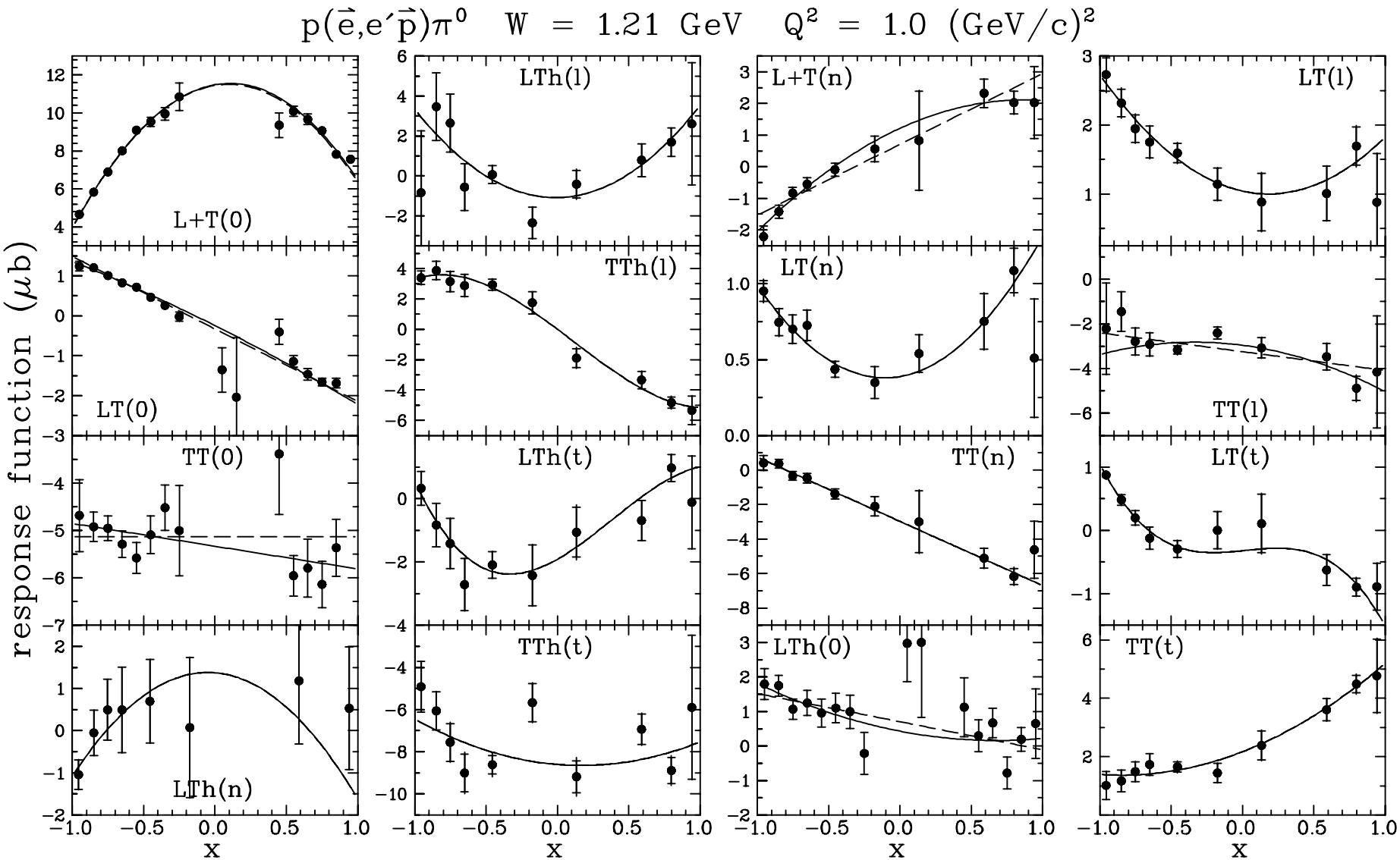
Model responses, W=1.29



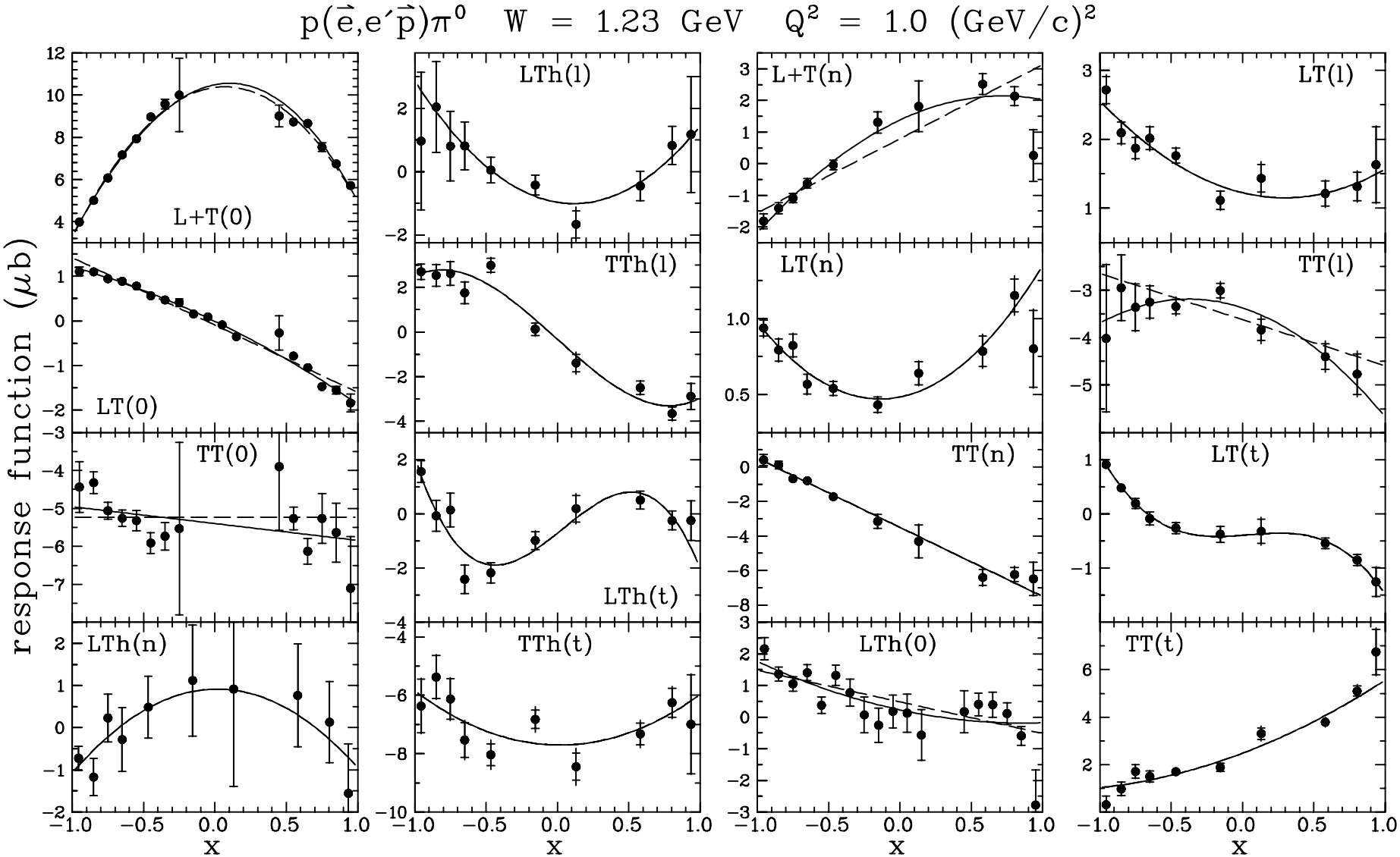
Summary of responses

- angular distributions for 14 responses and 2 Rosenbluth combinations:
  - 12 observed for first time
  - columns 1-2, depending upon $\mathrm{Re}H_iH_j^*$, show relatively small model variations
  - columns 3-4, depending upon $\mathrm{Im}H_iH_j^*$, show rather large model variations
  - MAID generally most successful of these models
- 10 W bins, responses scaled to $Q^2 = 1$ using dipole form factor
- systematic errors rarely visible

Legendre analysis, W=1.21

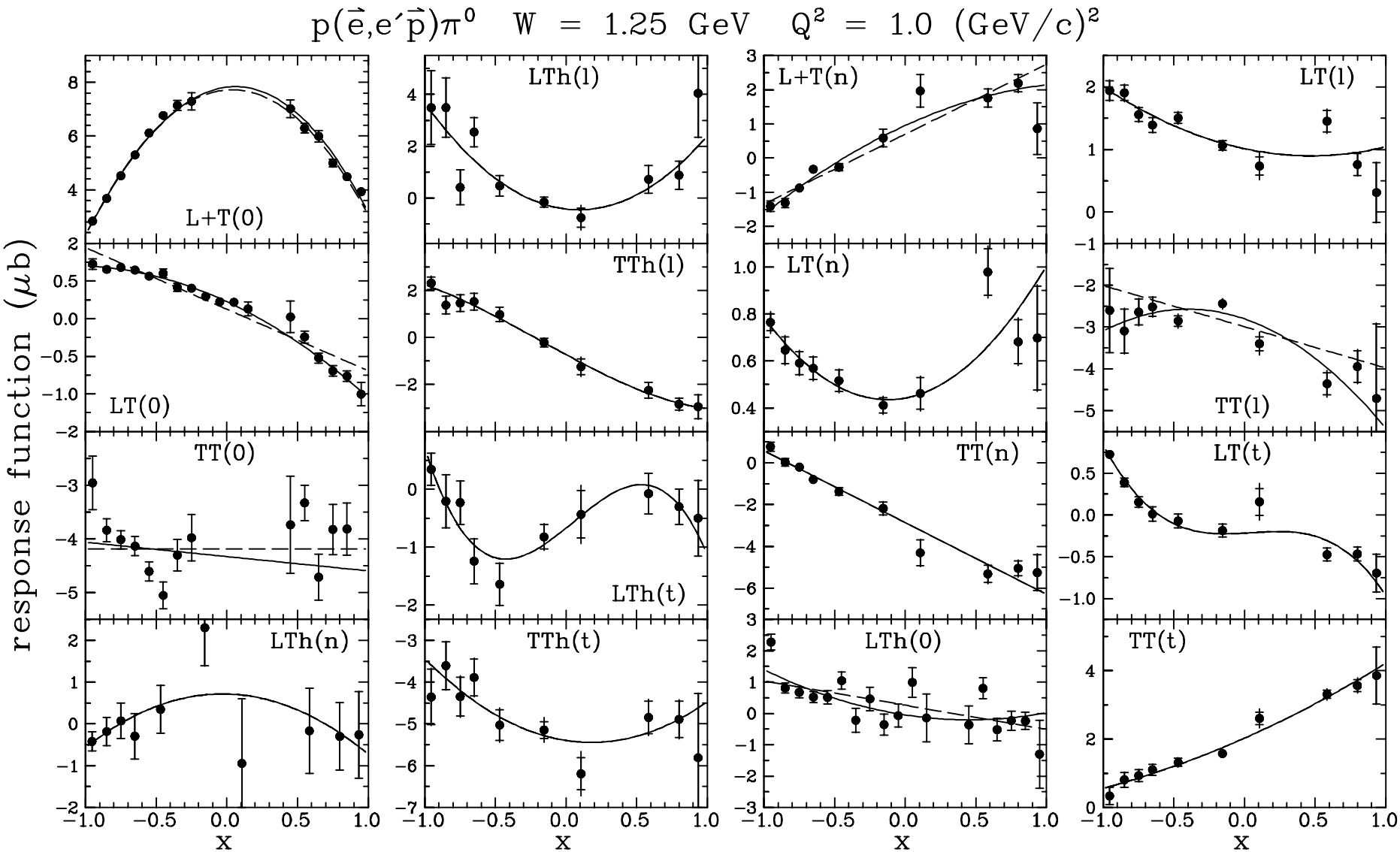


Legendre analysis, W=1.23

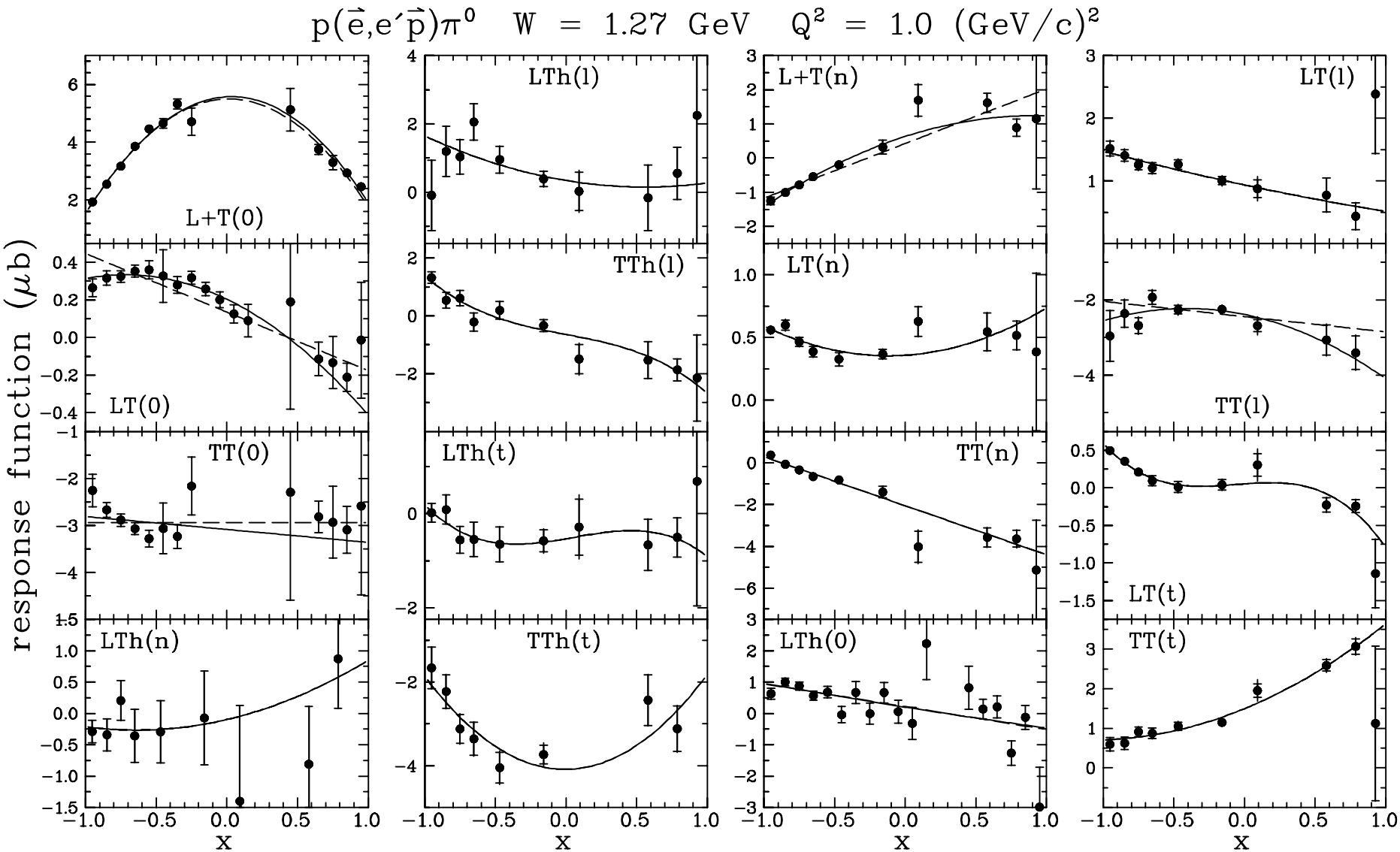


Need terms beyond sp truncation.

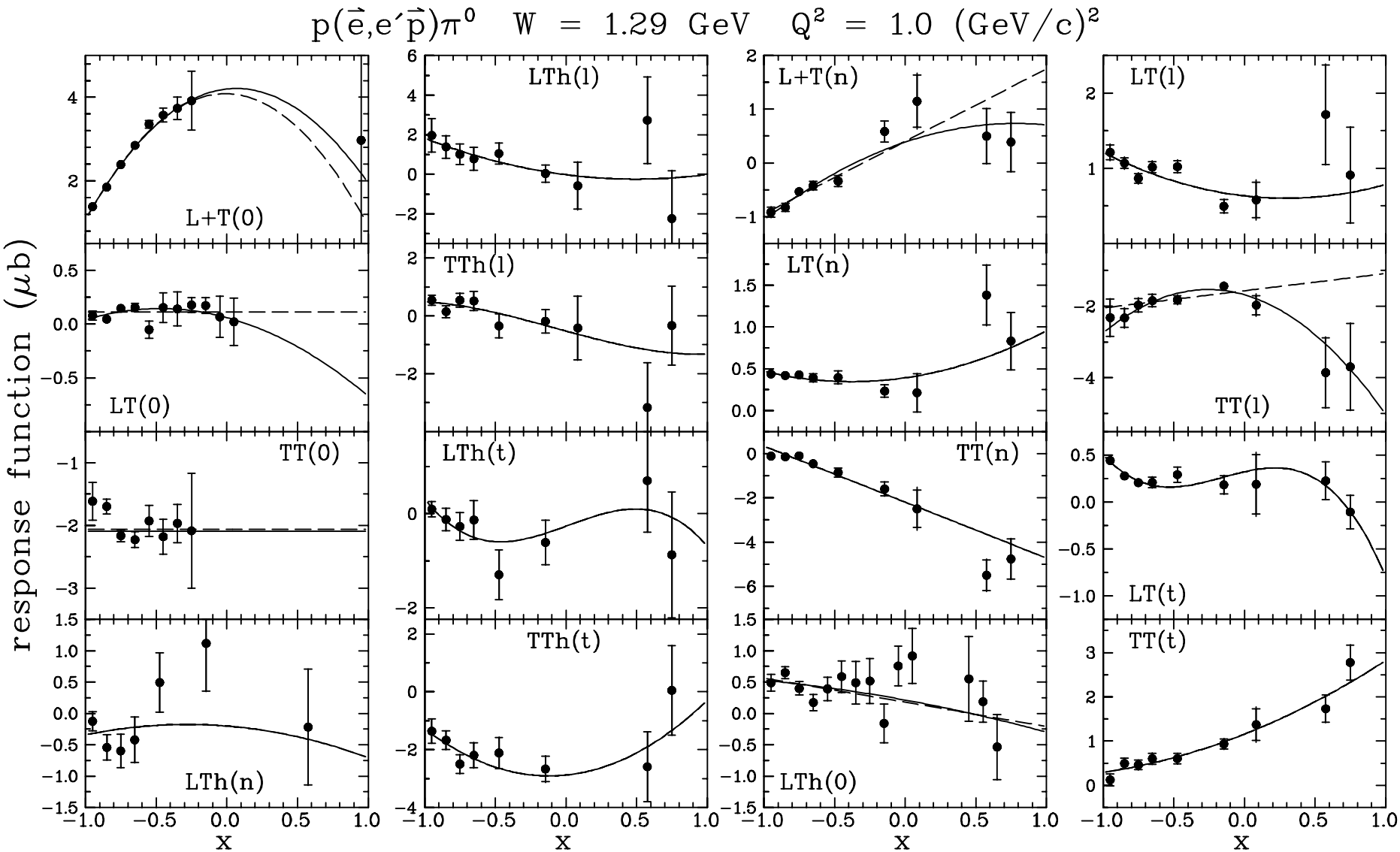
Legendre analysis, W=1.25



Legendre analysis, W=1.27



Legendre analysis, W=1.29



Multipole Analysis

Represent each amplitude (real or imaginary part of $M_{\ell\pm}$, $E_{\ell\pm}$, $S_{\ell\pm}$) as

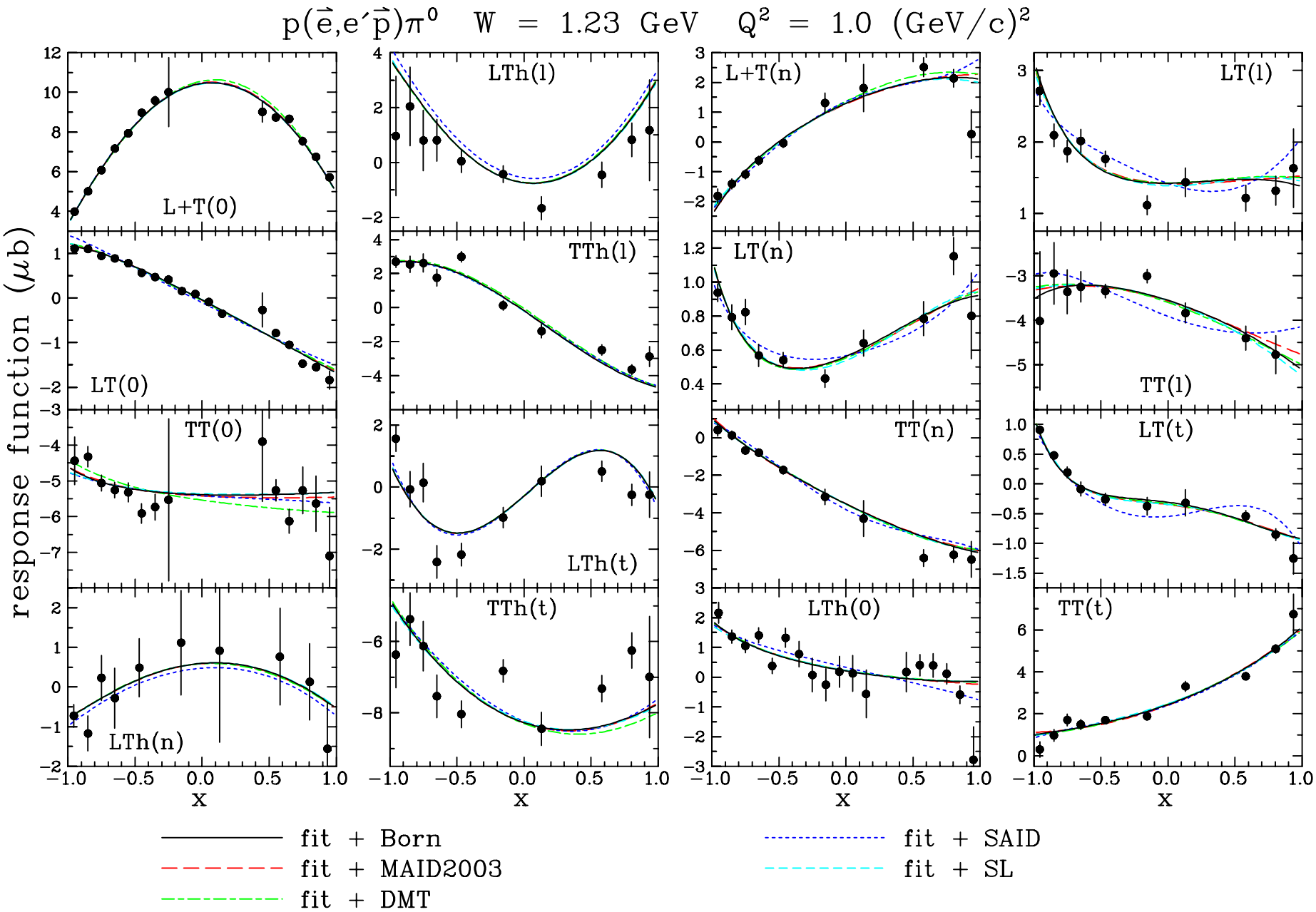
$$A(W,Q^2) = A^{(0)}(W,Q^2) + \Delta A(W,Q^2)$$

where $A^{(0)}$ from *baseline model* and ΔA is *fitted correction*.

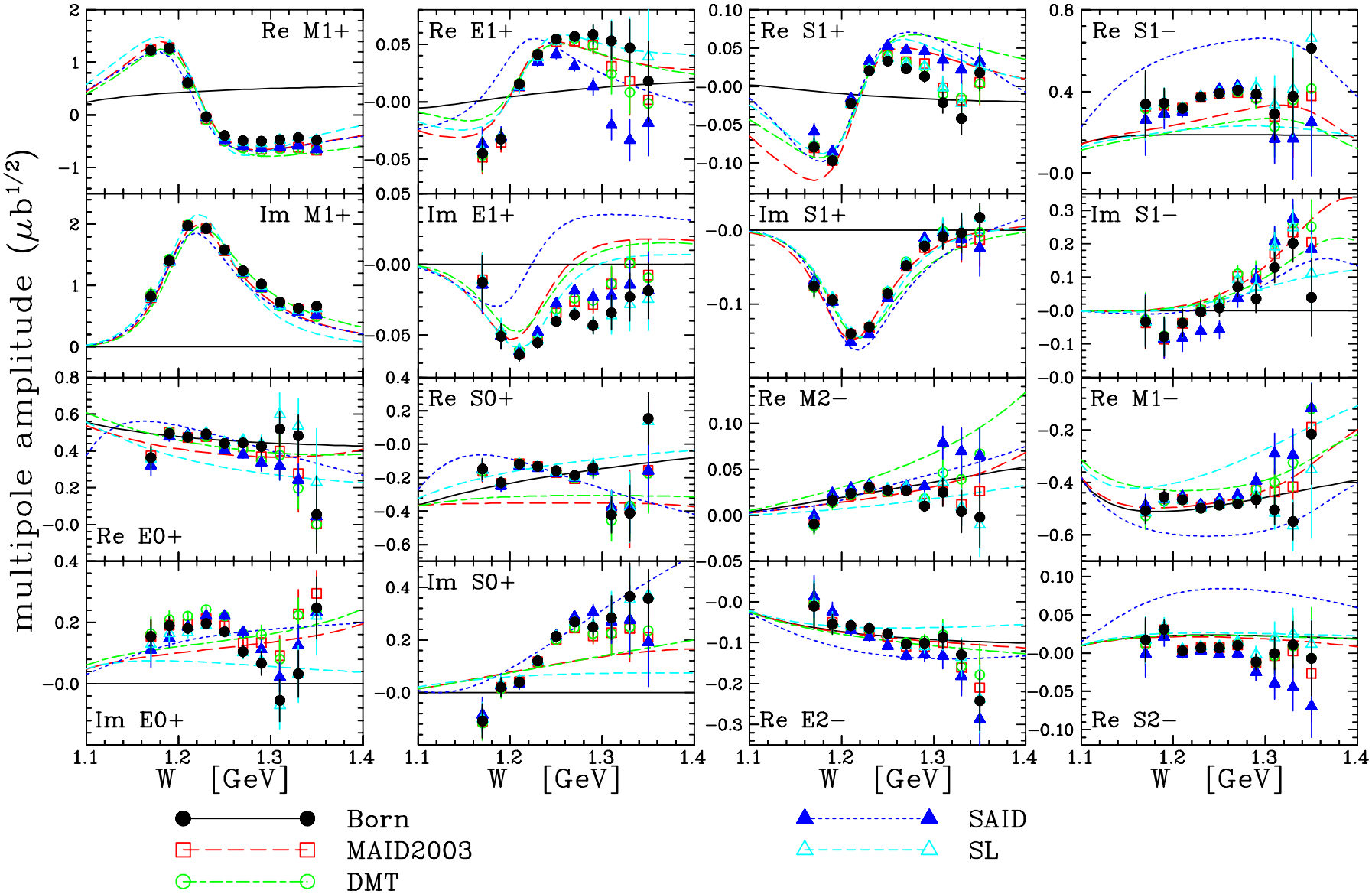
- fit all σ, R data for specified (W,Q^2) simultaneously
- vary appropriate subset of low partial waves
- higher multipoles from baseline model
- relatively little sensitive to choice of baseline model
- enforces symmetries and positivity constraints that are ignored by Legendre analysis

Typically fit 14 angular distributions using only 14-17 parameters.

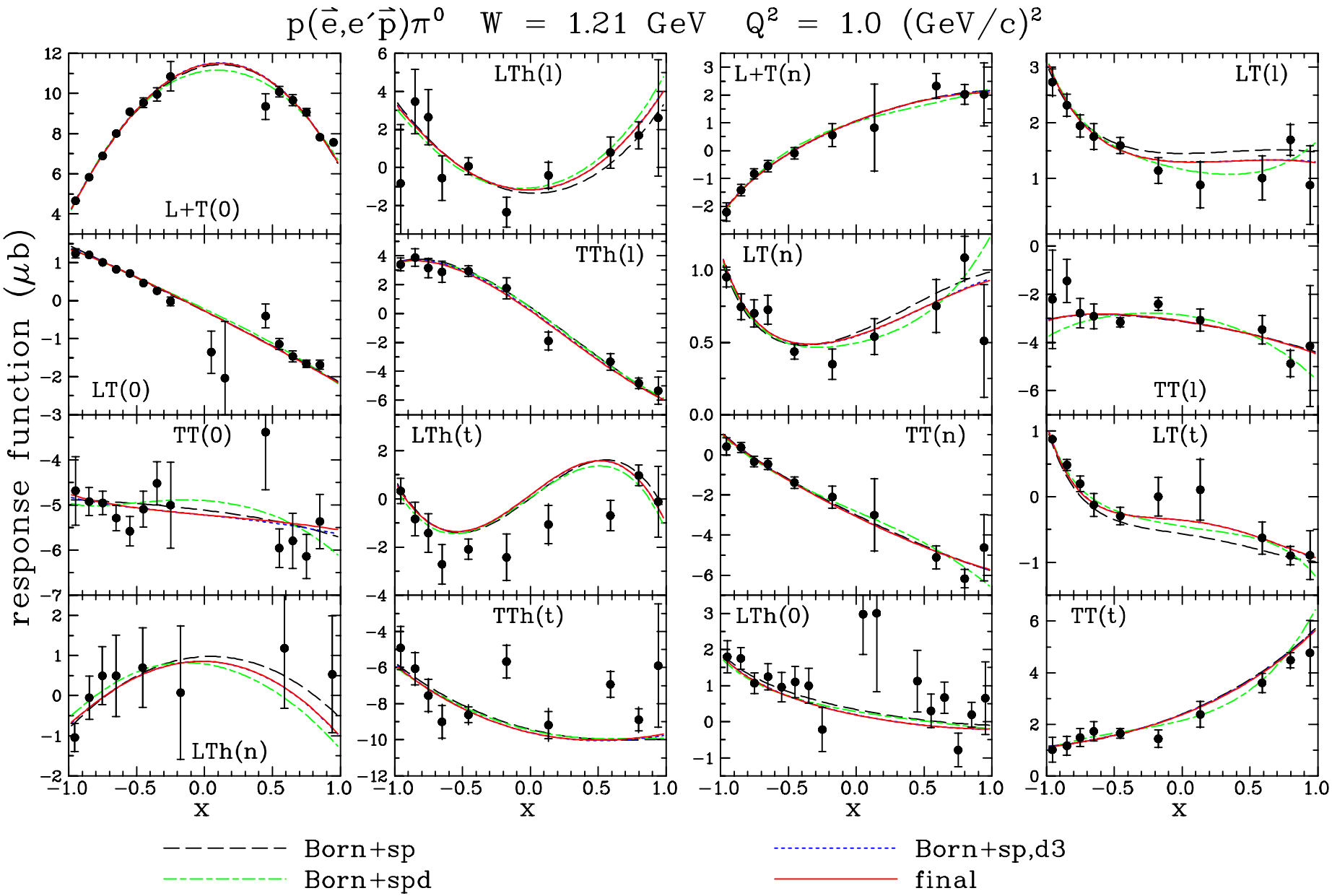
Baseline variation: fits



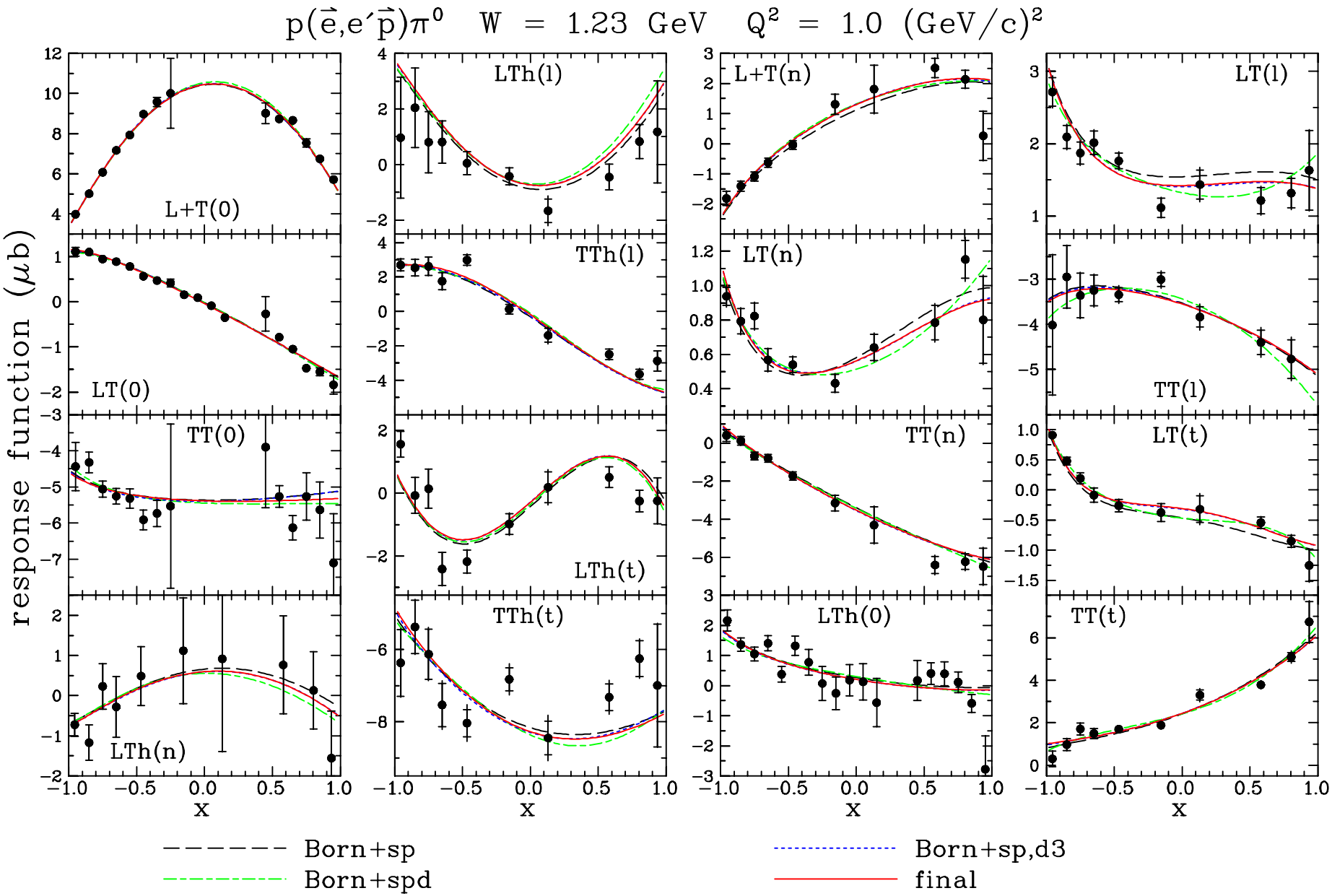
Baseline variation: multipoles



Multipole analysis, W=1.21

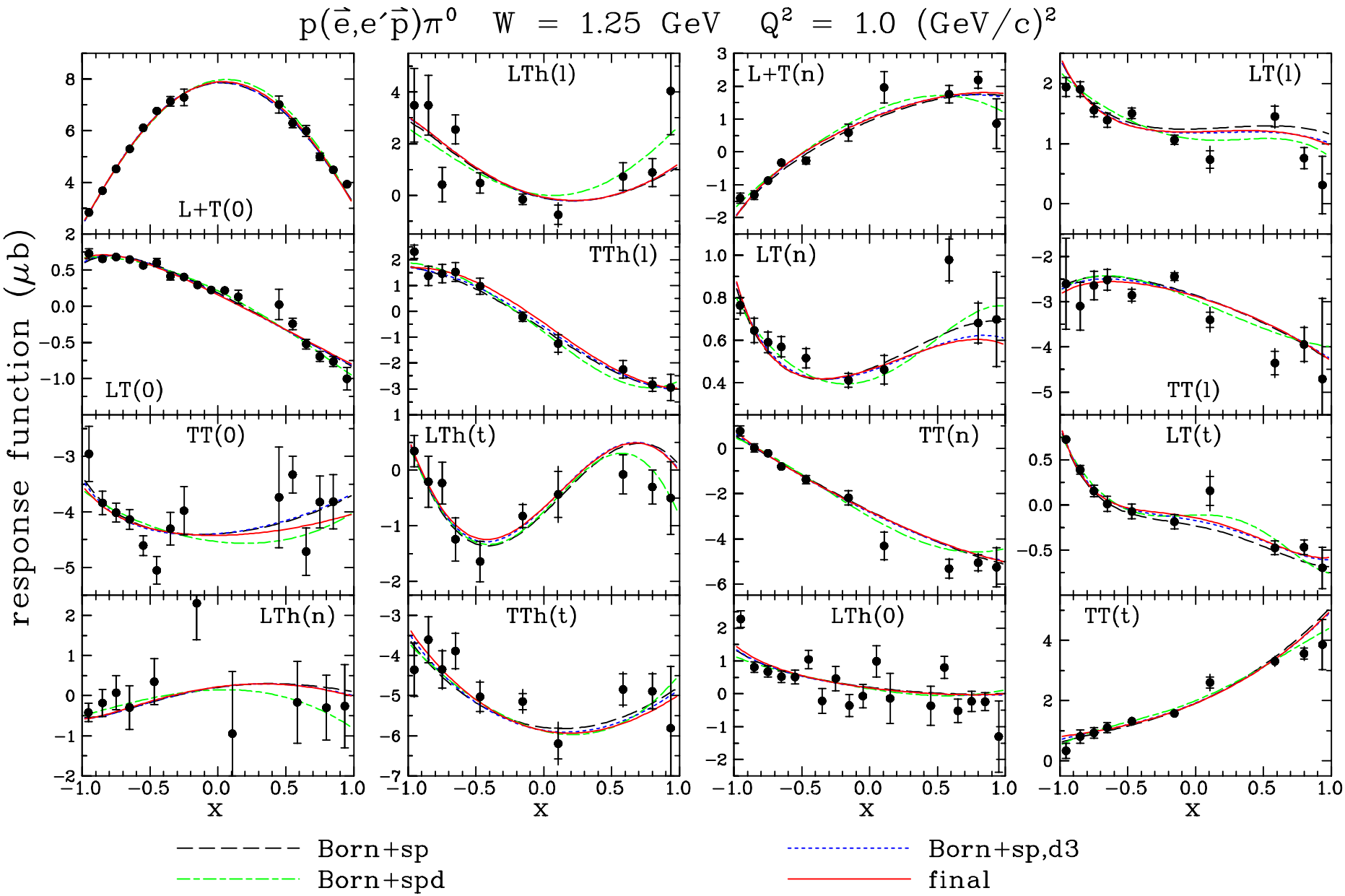


Multipole analysis, W=1.23

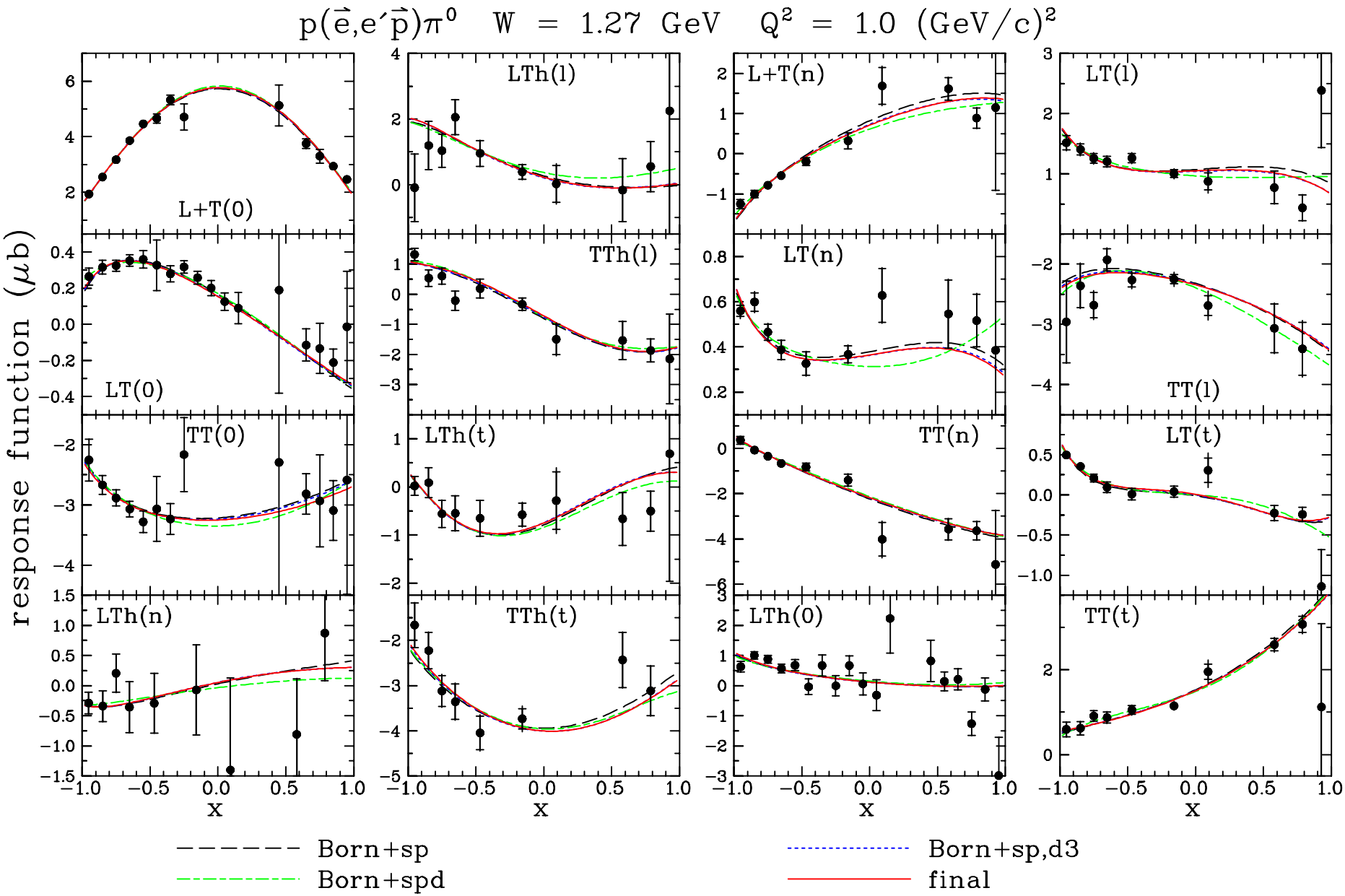


Largest improvements in I-type responses.

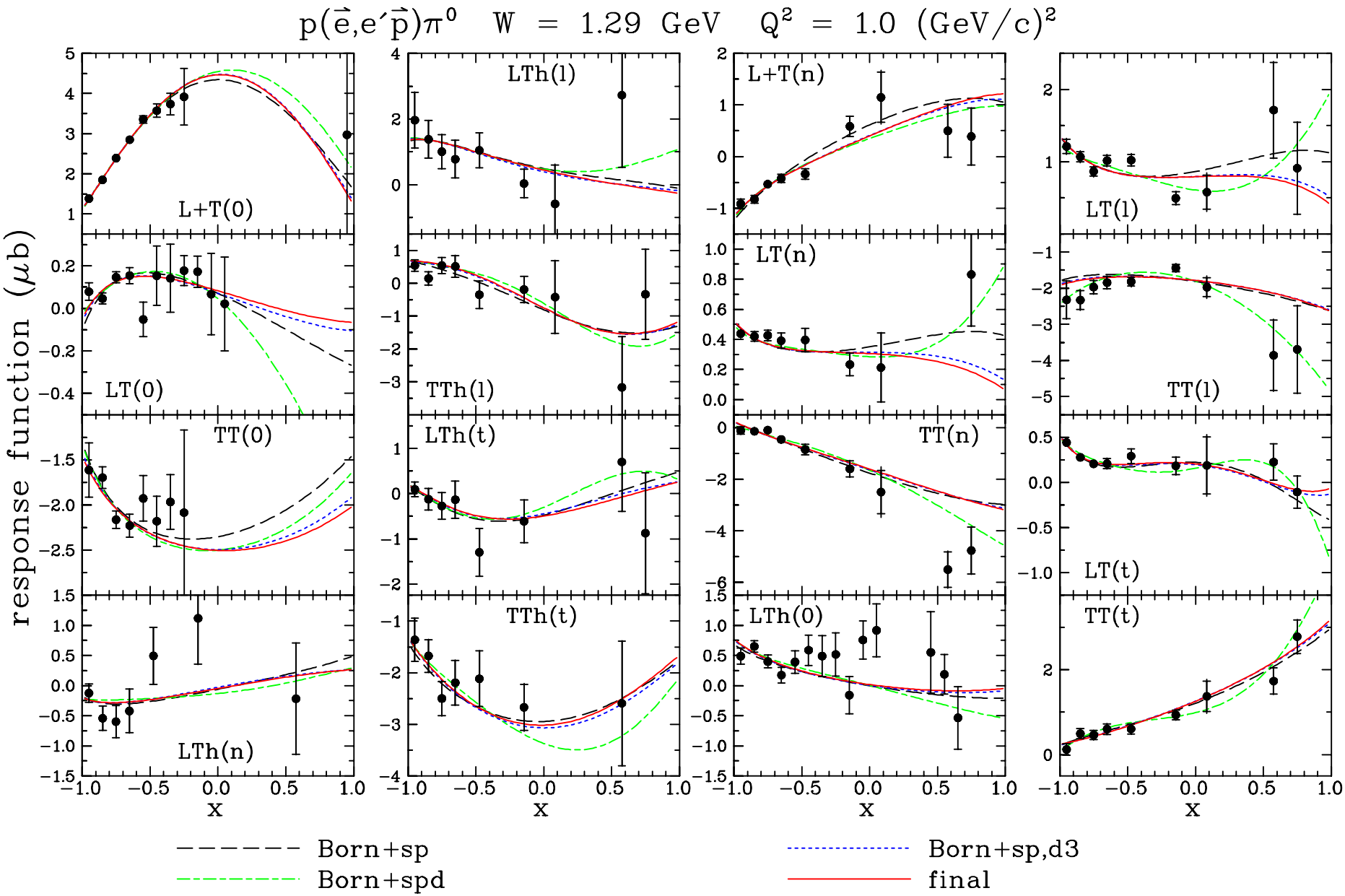
Multipole analysis, W=1.25



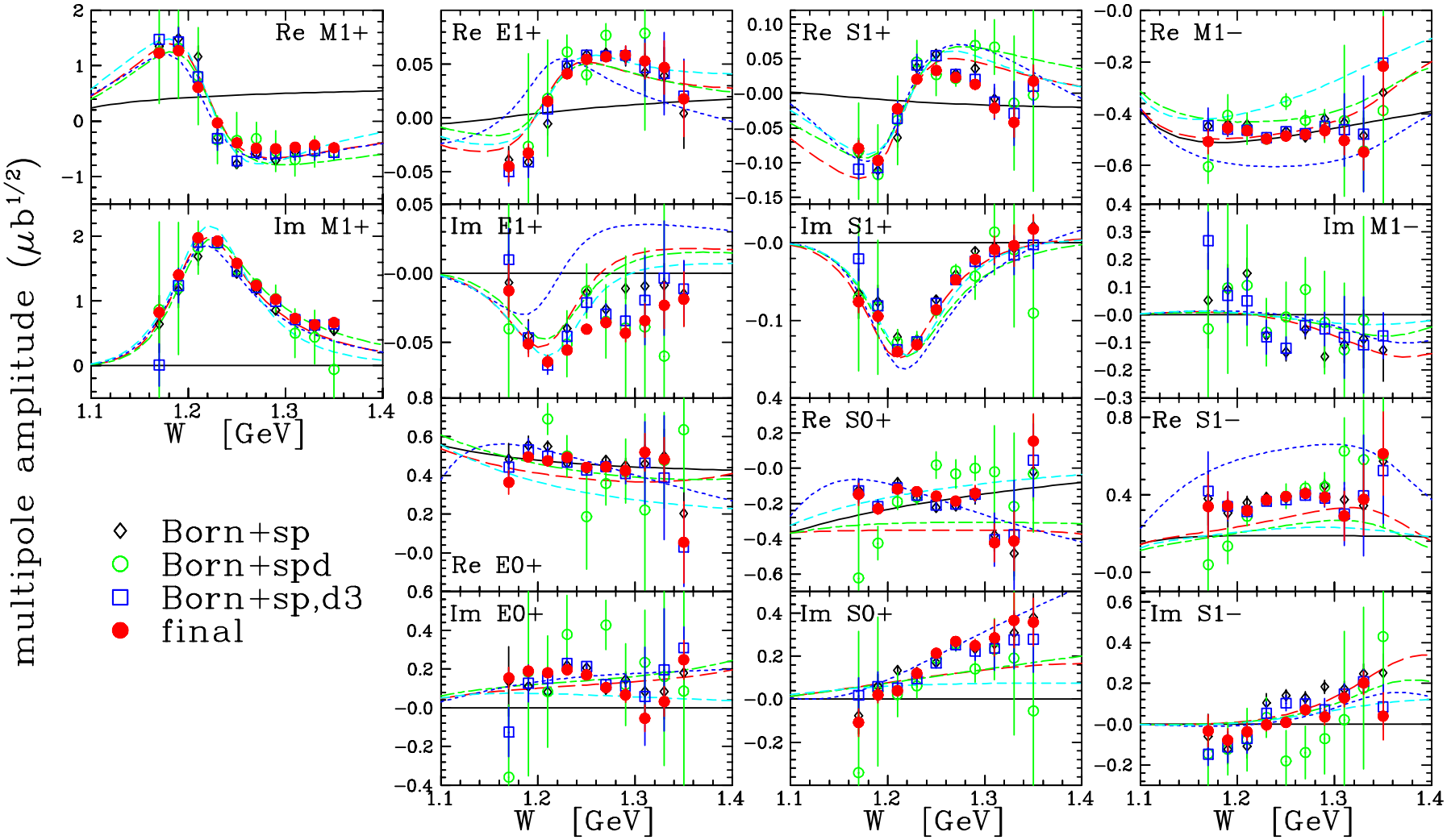
Multipole analysis, W=1.27



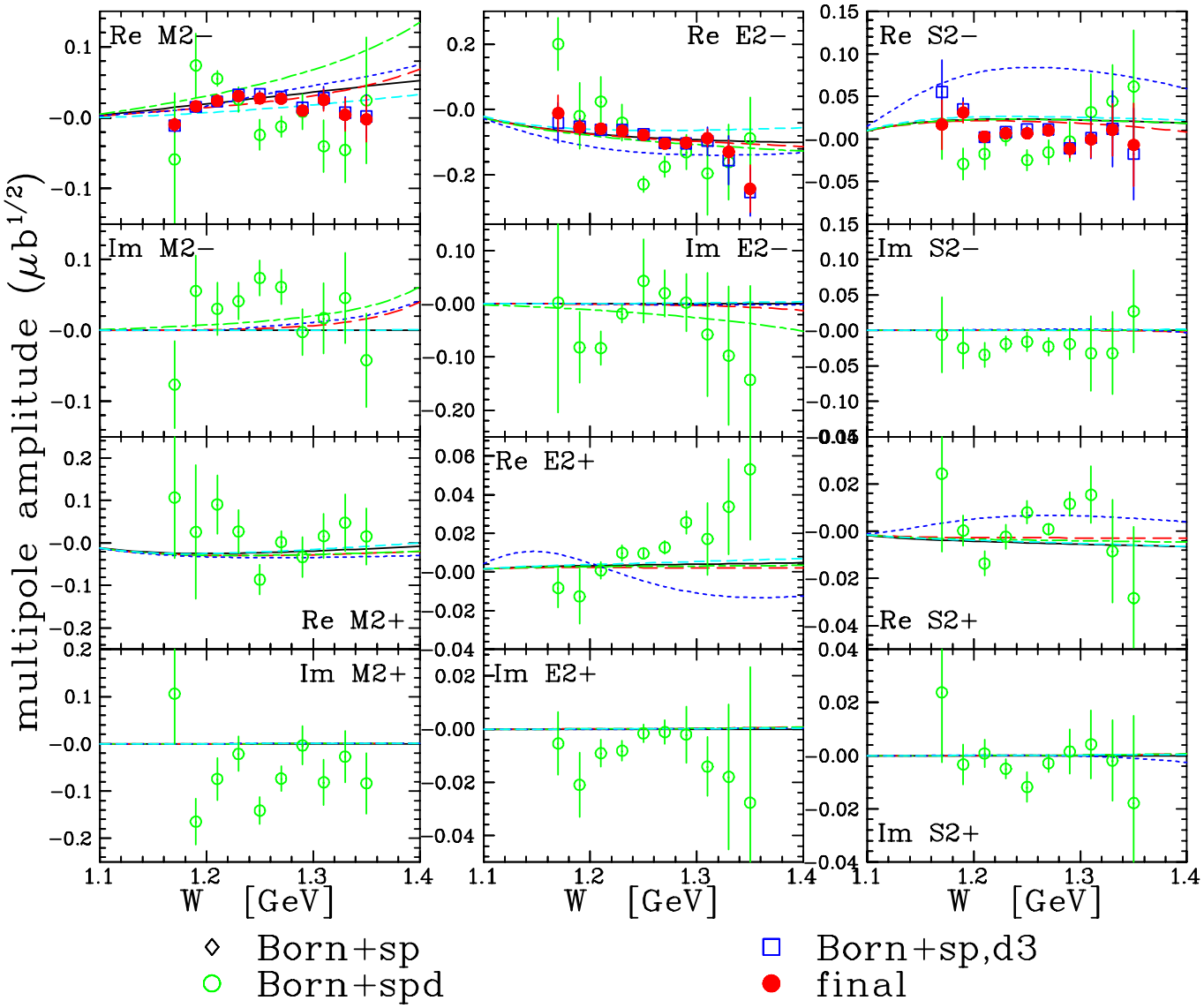
Multipole analysis, W=1.29



Truncation dependence: $\ell_\pi \leq 1$



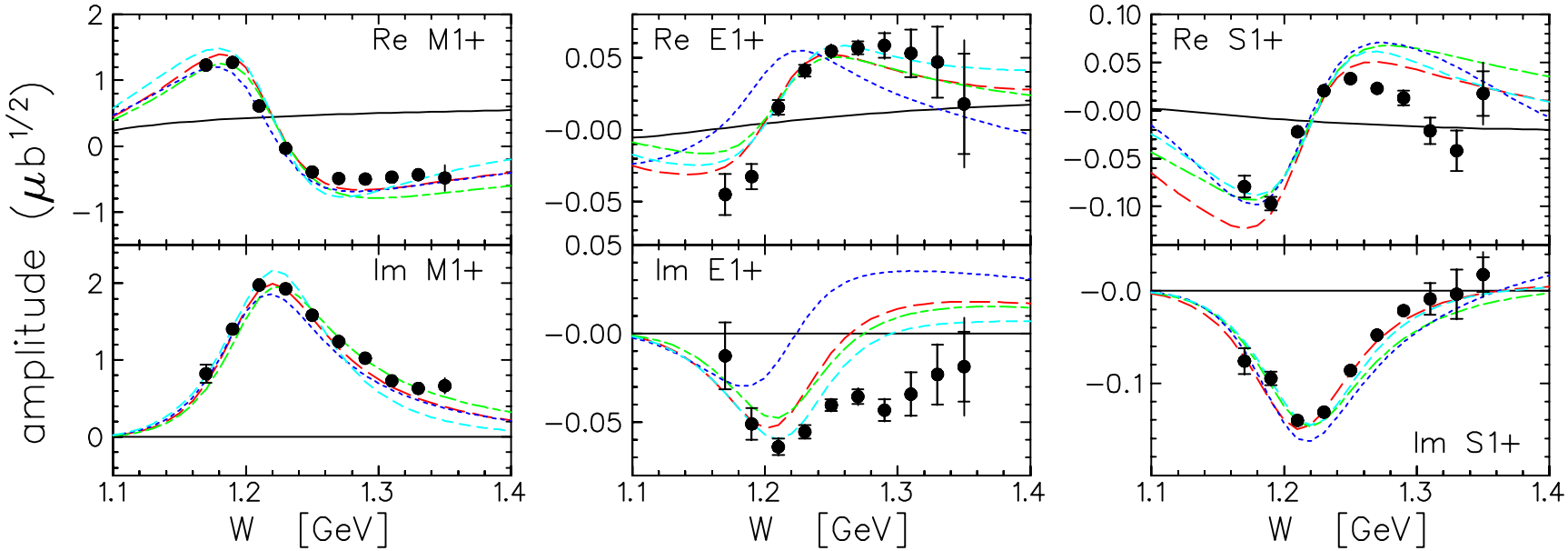
Truncation dependence: $\ell_\pi = 2$



Optimal model space

- baseline model = pseudovector Born + ρ, ω
  - minimizes bias
  - qualitative distinction between resonances and FSI modifications of background
  - results based upon MAID, DMT, or SL practically indistinguishable
  - some $\ell_\pi \geq 2$ amplitudes too large in SAID WI03
- vary: real and imaginary $\ell_\pi \leq 1$ multipoles, except $\mathrm{Im}M_{1-}$, plus real $2-$ multipoles $\Longrightarrow$ 16 parameters
  - correlation between $\mathrm{Im}M_{1-}, \mathrm{Im}S_{1-}$ not resolved by present data
  - model multipoles for $\ell_\pi \geq 2$ practically real, no experimental evidence for significant $\mathrm{Im}2-$
  - variation of $2+$ multipoles increases uncertainties without improving fits

$1+$ multipole amplitudes



- Systematic uncertainties small.
- Good agreement with models except for $\mathrm{Im}E_{1+}$
- Insensitive to choice of baseline (except SAID)
- Only $\mathrm{Im}E_{1+}$ sensitive to correlations with higher multipoles

Sensitivity to specific amplitudes

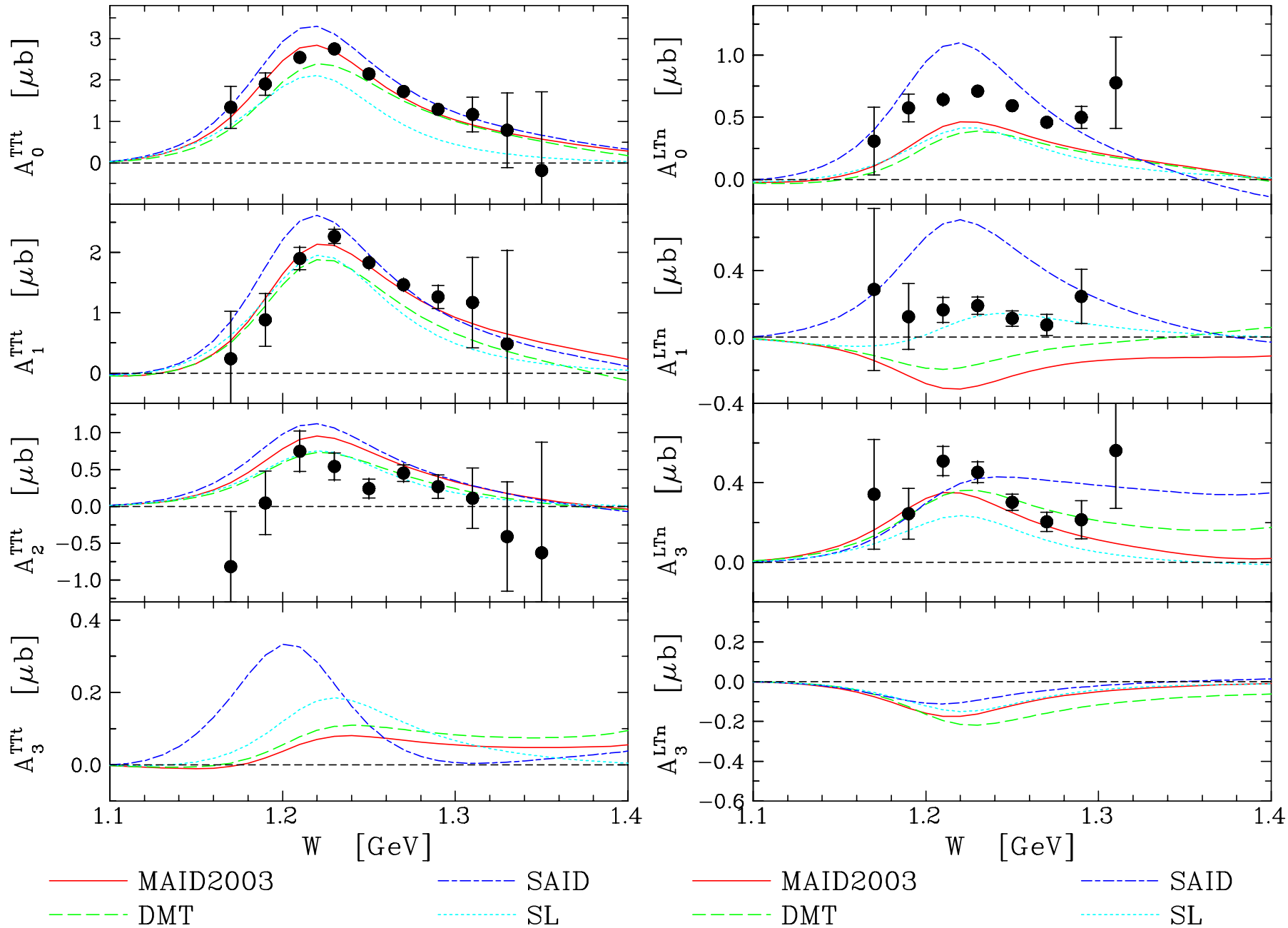
$$A = b + ir \Longrightarrow A_1^* A_2 = \underbrace{(b_1 b_2 + r_1 r_2)}_{\text{R-type}} + i \underbrace{(b_1 r_2 - b_2 r_2)}_{\text{I-type}}$$

Sensitivity to nonresonant or nondominant multipoles found in specific Legendre coefficients. Using sp truncation and M_{1+} dominance:

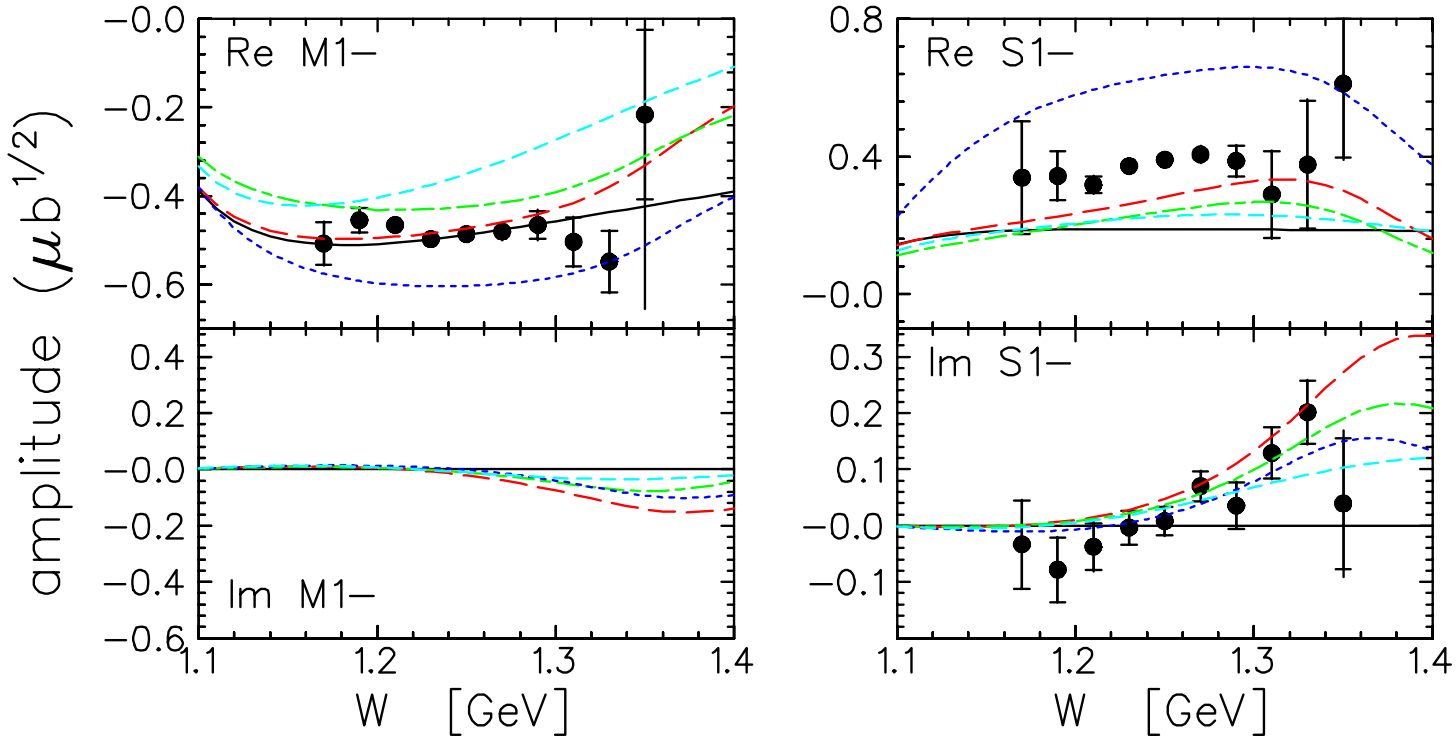
- $\mathrm{Im} M_{1-} \approx 0$, $A_0^{TTt} \approx 3 \mathrm{Im} M_{1+}^* M_{1-}$
  $\Longrightarrow A_0^{TTt} \sim -3(\mathrm{Re} M_{1-})(\mathrm{Im} M_{1+}^*)$
- $\mathrm{Re} S_{1-} > \mathrm{Im} S_{1-}$, $A_0^{LTn} \approx -\mathrm{Im} M_{1+}^* S_{1-}$
  $\Longrightarrow A_0^{LTn} \sim (\mathrm{Re} S_{1-})(\mathrm{Im} M_{1+}^*)$
- S_{0+} isolated in 5 R-type, 5 I-type Legendre coefficients

Note: qualitative guidance only. Multipole analysis does not rely on these relationships.

Sensitivity to $1-$ amplitudes

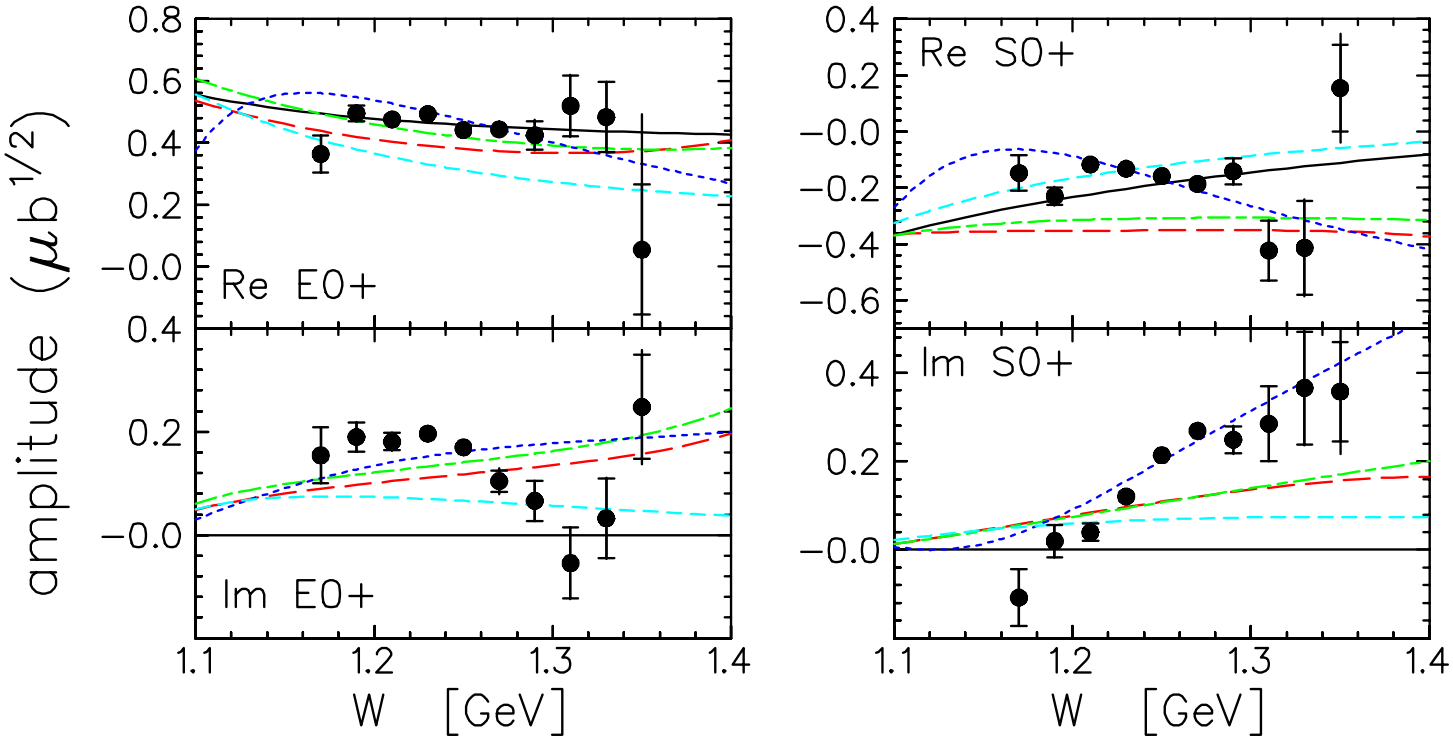


$1-$ multipole amplitudes



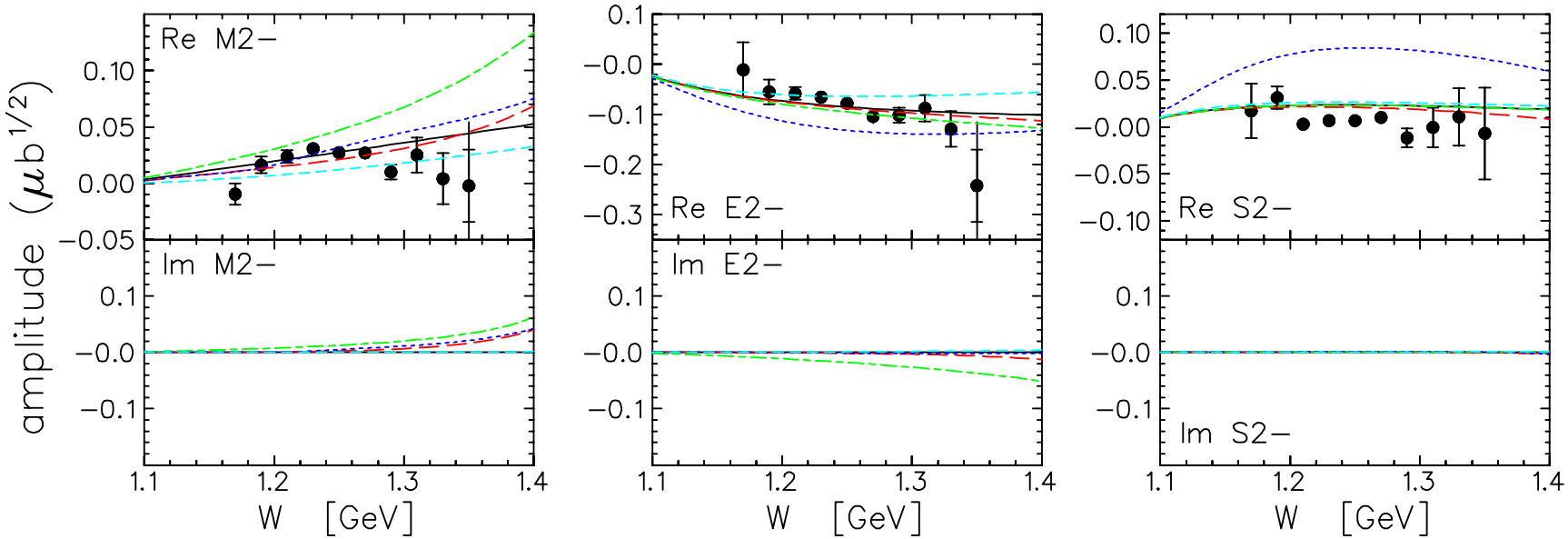
- Qualitatively consistent with Legendre analysis.
- Model variations especially large for $\mathrm{Re}M_{1-}$. Data near PV Born.
- $\mathrm{Re}S_{1-}$ data strong on low-W side of Roper suggests radial excitation.

$0+$ multipole amplitudes



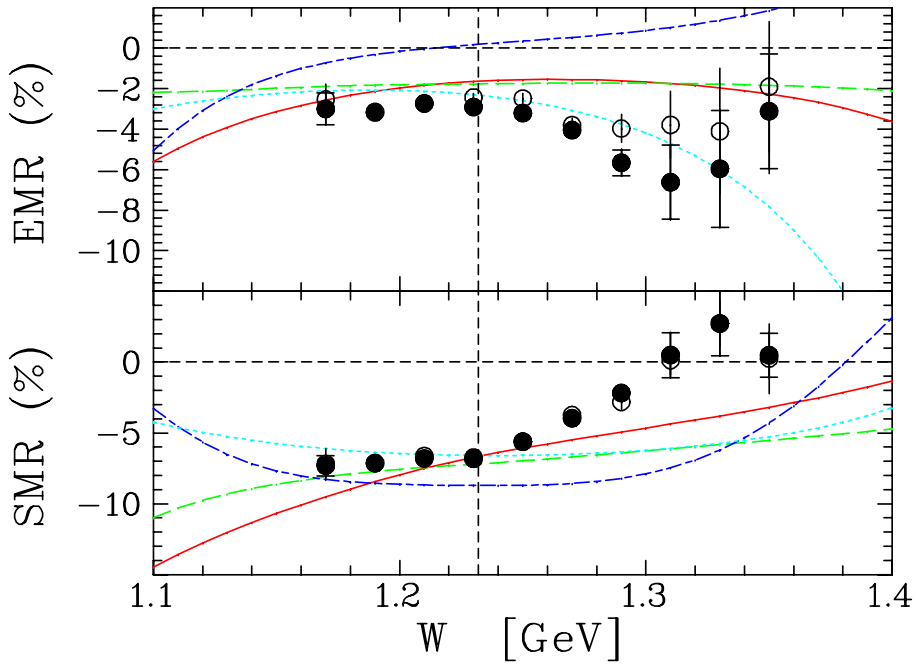
- Successful fits requires significant modification of S_{0+}
  - Appreciable slope in $\mathrm{Im}S_{0+}$ only present in SAID.
  - Negative slope in $\mathrm{Re}S_{0+}$ only present in SAID.
  - Nonresonant πNN effect not described well by current dynamical models (DMT or SL).

$2-$ multipole amplitudes



- No evidence for appreciable imag $2-$ here.
- Relatively strong $\mathrm{Re}E_{2-}$ consistent with PV Born.

EMR,SMR: multipole method



$$R_{EM} = \frac{\mathrm{Re}E_{1+}\mathrm{Re}M_{1+} + \mathrm{Im}E_{1+}\mathrm{Im}M_{1+}}{\mathrm{Re}M_{1+}\mathrm{Re}M_{1+} + \mathrm{Im}M_{1+}\mathrm{Im}M_{1+}}$$

$$R_{SM} = \frac{\mathrm{Re}S_{1+}\mathrm{Re}M_{1+} + \mathrm{Im}S_{1+}\mathrm{Im}M_{1+}}{\mathrm{Re}M_{1+}\mathrm{Re}M_{1+} + \mathrm{Im}M_{1+}\mathrm{Im}M_{1+}}$$

Does not assume sp truncation or M_{1+} dominance.

EMR,SMR: truncation dependence

variables	SMR, %	EMR, %	χ^2_ν
$0+, 1+, 1-$	-6.73 ± 0.24	-2.43 ± 0.19	1.69
$0+, 1+, 1-, 2-, 2+$	-6.95 ± 0.49	-3.19 ± 0.79	1.64
$0+, 1+, 1-, \mathrm{Re}2-$	-6.85 ± 0.27	-2.73 ± 0.20	1.65
above except $\mathrm{Im}M_{1-}$	-6.84 ± 0.15	-2.91 ± 0.19	1.65

- use pseudovector Born baseline
- uncertainties increase with number of fitting parameters
- results almost independent of truncation scheme (within uncertainties)
- minimum uncertainty upon elimination of $\mathrm{Im}M_{1-}$, but affects EMR

EMR,SMR: baseline variation

baseline	SMR, %	EMR, %	χ^2_ν
Born	-6.84 ± 0.15	-2.91 ± 0.19	1.65
MAID2003	-6.90 ± 0.15	-2.79 ± 0.19	1.67
DMT	-6.82 ± 0.15	-2.70 ± 0.19	1.67
SL	-6.79 ± 0.15	-2.81 ± 0.19	1.64
SAID	-7.38 ± 0.15	-2.53 ± 0.20	1.85

- optimum parameter set
- SAID WI03 unsuitable as baseline because oscillations due to $\ell_\pi \geq 2$ too strong
- EMR, SMR insensitive to baseline (except SAID)

$I = 1/2$ contamination

- Need both $n\pi^+$ and $p\pi^0$ for isospin analysis, but models predict negligible contamination. For example,

		$p\pi^0$	$I = \frac{3}{2}$
MAID2003	EMR	-1.65%	-1.62%
	SMR	-6.73%	-6.71%

- Or, assume Born baseline describes background, such that

$$R_{SM}^{(3/2)} \approx \mathrm{Re}\frac{\Delta S_{1+}}{\Delta M_{1+}} = -6.81\%$$

$$R_{EM}^{(3/2)} \approx \mathrm{Re}\frac{\Delta E_{1+}}{\Delta M_{1+}} = -3.12\%$$

Contamination negligible for SMR, within uncertainty for EMR.

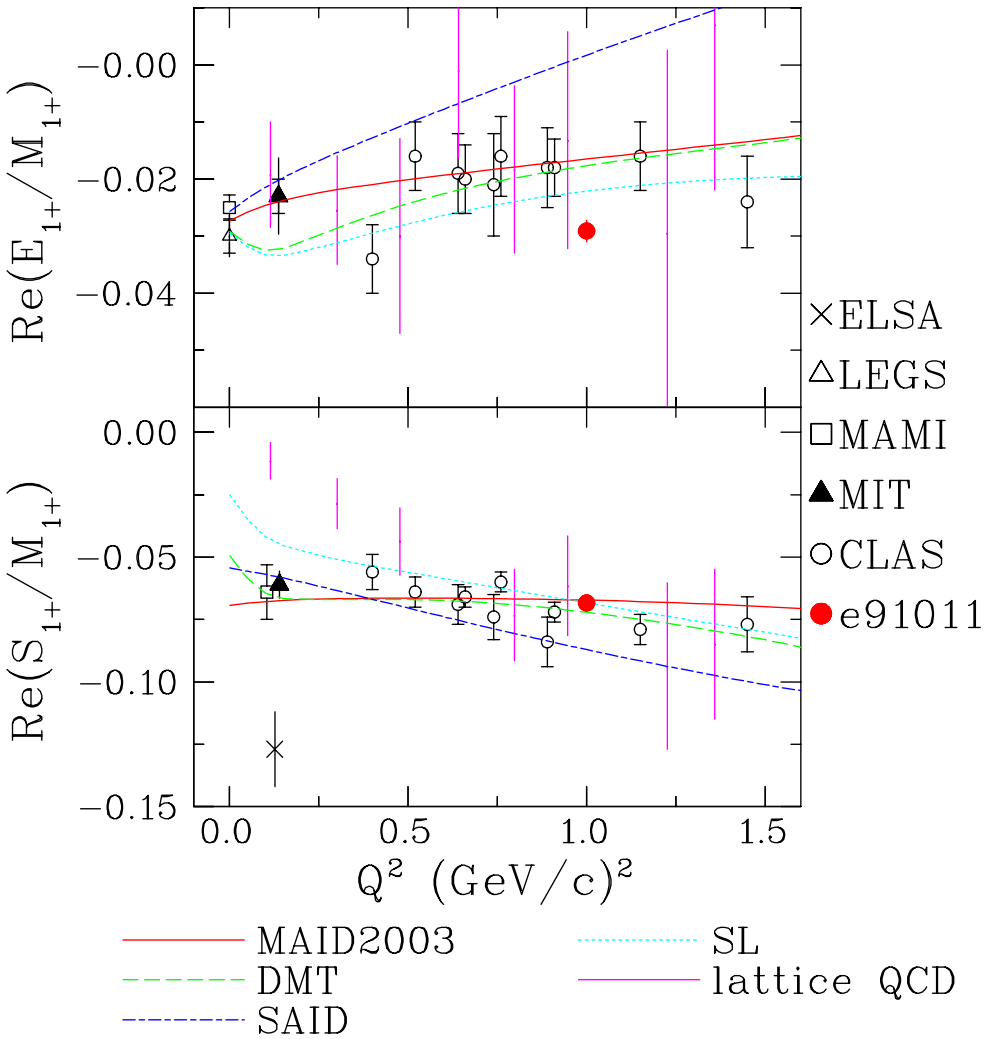
Legendre vs. multipole fit

method/baseline	SMR, %	EMR, %	χ^2_ν
Legendre sp	-6.07 ± 0.11	-2.04 ± 0.13	1.67
Legendre $sp+$	-6.11 ± 0.11	-1.92 ± 0.14	1.50
fit+Born	-6.84 ± 0.15	-2.91 ± 0.19	1.65
fit+MAID2003	-6.90 ± 0.15	-2.79 ± 0.19	1.67
fit+DMT	-6.82 ± 0.15	-2.70 ± 0.19	1.67
fit+SL	-6.79 ± 0.15	-2.81 ± 0.19	1.64
fit+SAID	-7.38 ± 0.15	-2.53 ± 0.20	1.85

- EMR more sensitive to extra terms.
- Both EMR,SMR larger for multipole analysis; relative error in EMR large.

Traditional Legendre analysis inadequate!

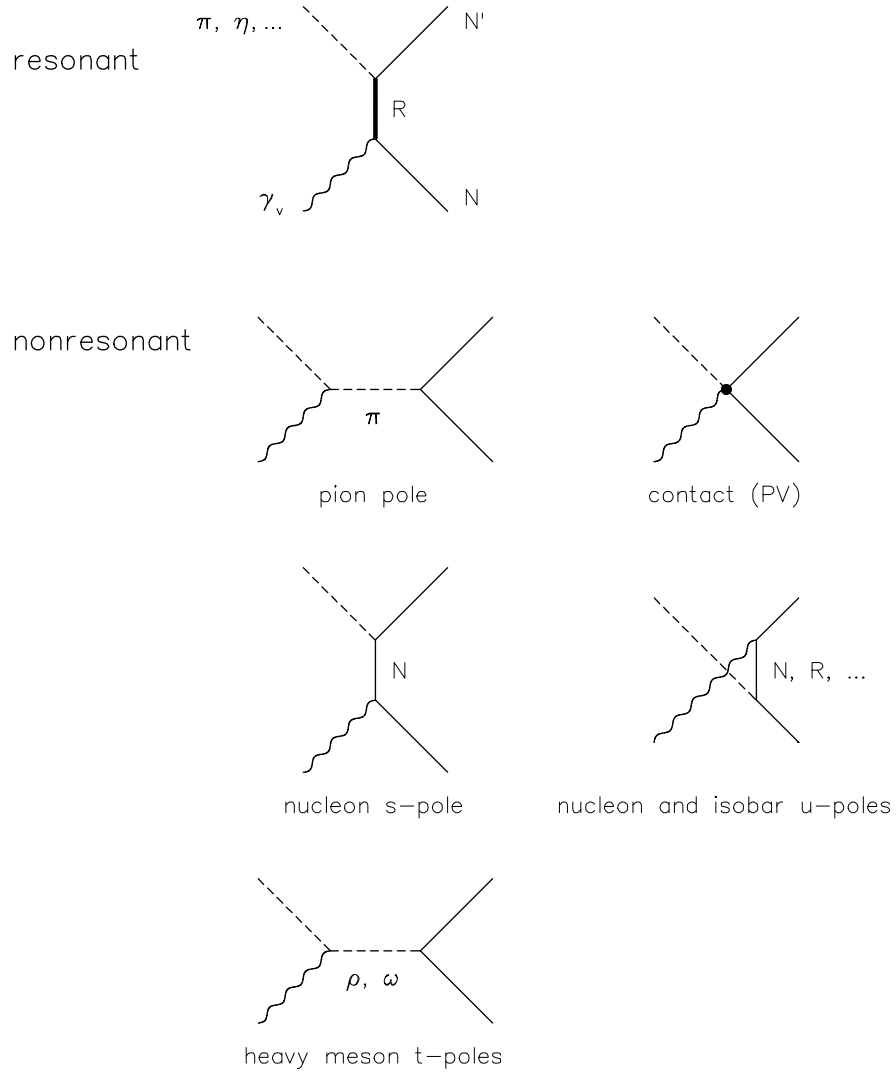
Quadrupole ratios, $Q^2 \lesssim 1.5$



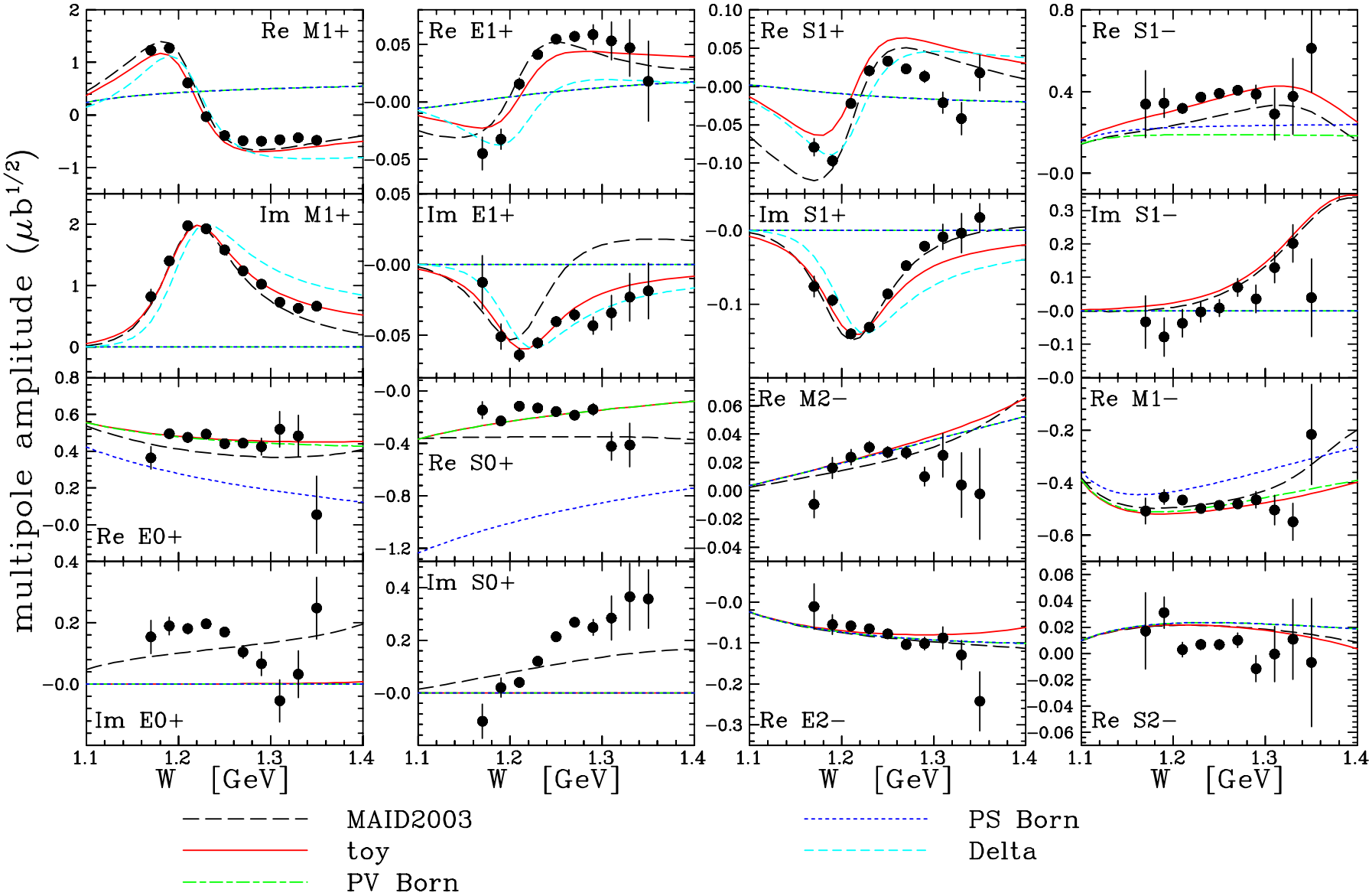
- MAID, DMT, SL fits included CLAS data.
- Disagree strongly with EMR from SAID-WI03.
- Both EMR and SMR appear flat over this range of Q^2.
- Quenching pionic contributions may produce slope in lattice QCD.

Toy model

- Based upon MAID
  - consider PV/PS mixing
  - include ρ, ω diagrams
  - include phases ϕ_M, ϕ_E, ϕ_S (here constant)
  - omit resonant u-pole
  - penetrabilities, widths, form factors, etc. similar to MAID98
- fit “by eye” to multipoles



Toy model: multipoles



Toy model: observations

- PV coupling preferred over PS; no obvious benefit from mixing
- $P_{33}(1232)$ phases
  - $\phi_M \approx \phi_E \approx \phi_S \approx 25^\circ$
  - ϕ_E depends on Q^2: at $Q^2 = 0$, MAID $\Longrightarrow \phi_E \approx 75^\circ$
- ${}_pA_{1/2} \approx 0$ at $Q^2 = 1$ (correlated with PV/PS mixture)
- strong ${}_pS_{1/2}$ shows radial Roper
  - consistent with CLAS $Q^2 = 0.4, 0.65$ (Aznauryan *et al.*)
  - additional phase small, $\sim 5^\circ$
- strong $\mathrm{Im}S_{0+}$ and $\mathrm{Im}E_{0+}$ require FSI

Conclusions

- First (nearly) complete set of polarization responses
  - OOP important for separation of responses
  - model variations large for I-type and smaller for R-type but increasing with $|W - m_\Delta|$
- Nearly model independent multipole analysis
  - good agreement for $1+$ multipoles except $\mathrm{Im}E_{1+}$; sensitive to unitarity phase
  - strong $\mathrm{Im}0+$ suggests FSI modification
  - strong S_{1-} — harbinger of radial Roper
  - PV coupling preferred
  - neither $\ell_\pi \leq 1$ nor M_{1+} dominance reliable for EMR,SMR
  - relative error in traditional EMR large

Obligatory conclusion: more research needed!

Outlook

- Experimental possibilities
  - repeat with smaller ϵ for Rosenbluth separation
  - higher Q^2 for Δ
  - higher W for Roper, etc.
  - η production near S_{11}
  - $p(\vec{e}, e'\vec{n})\pi^+$ near threshold for S_{0+}/E_{0+} and χPT
  - target polarization, both transverse & longitudinal
- Theoretical challenges
  - refine effective Lagrangian, dispersion theory, and/or coupled-channels (dynamical) models
  - connect with quark models
  - lattice QCD
    - improve statistics
    - eliminate quenching
    - extend Q^2