New event generators for the PRad experiment

Alexander Gramolin

Budker Institute of Nuclear Physics, Novosibirsk, Russia

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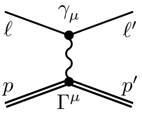
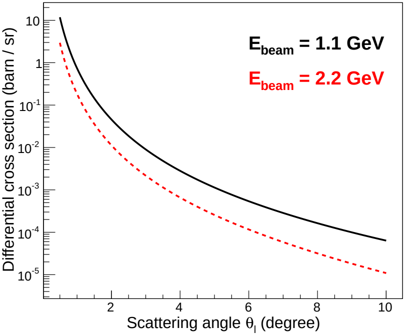
Elastic $e^- p$ and $e^- e^-$ scattering cross sections (lab frame)

$e^- p$ scattering (Rosenbluth, 1950):

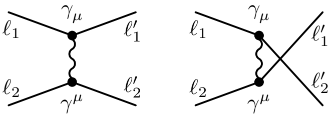


$$\frac{d\sigma}{d\Omega_\ell} = \frac{\alpha^2 E'_\ell}{4E^3_{\text{beam}}} \frac{\cos^2(\theta_\ell/2)}{\sin^4(\theta_\ell/2)} \frac{\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2)}{\varepsilon(1+\tau)}$$

$$\tau = Q^2/4M^2, \; \varepsilon = \left[1 + 2(1+\tau)\tan^2(\theta_\ell/2)\right]^{-1}$$

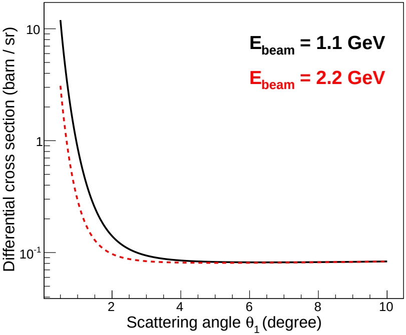


$e^- e^-$ scattering (Møller, 1932):

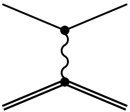


$$\frac{d\sigma}{d\Omega_1} = \frac{2\alpha^2 \cos\theta_1}{(E_{\text{beam}} \sin^2\theta_1 + 2m)^2} \left[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} + \frac{2s^2}{tu}\right]$$

$$s = (\ell_1 + \ell_2)^2, \; t = (\ell'_1 - \ell_1)^2, \; u = (\ell'_1 - \ell_2)^2$$

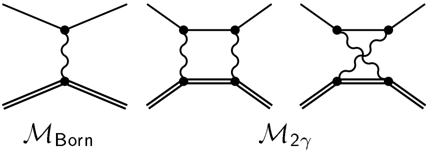


$e^- p$ scattering in the next-to-leading order

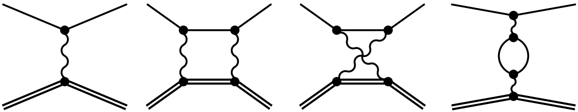


$\mathcal{M}_{\text{Born}}$

$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2$$



$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^\dagger_{\text{Born}} \mathcal{M}_{2\gamma}\right)$$



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$

$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^\dagger_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+\, 2\,\text{Re}\left(\mathcal{M}^\dagger_{\text{Born}}\mathcal{M}_{\text{vac}}\right)$$

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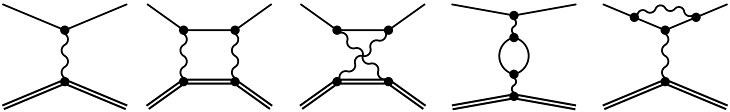

$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$

$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right)$$

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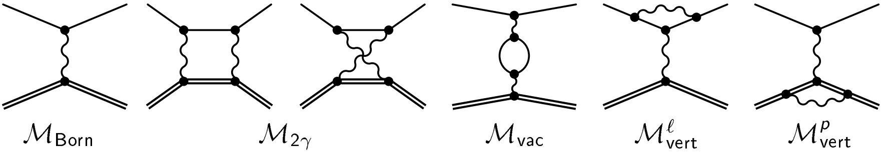
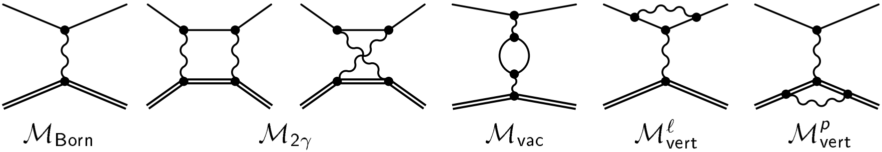


$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right)$$

Elastic scattering ($e^- p \to e^- p$):



$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$
$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right)$$

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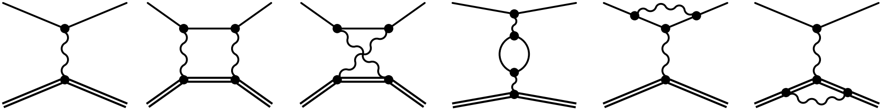
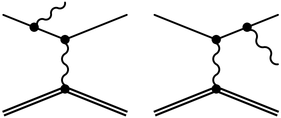


$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

First-order bremsstrahlung ($e^- p \to e^- p\, \gamma$):



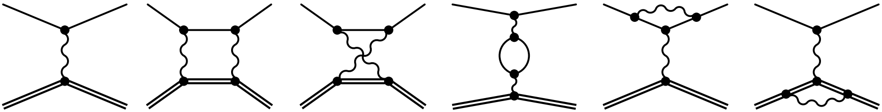
$\mathcal{M}^{\ell}_{\text{brems}}$

$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

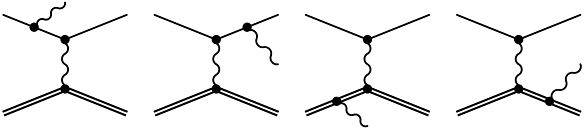
$$+ |\mathcal{M}^{\ell}_{\text{brems}}|^2$$

Elastic scattering ($e^- p \to e^- p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

First-order bremsstrahlung ($e^- p \to e^- p\, \gamma$):



$\mathcal{M}^{\ell}_{\text{brems}}$ $\mathcal{M}^{p}_{\text{brems}}$

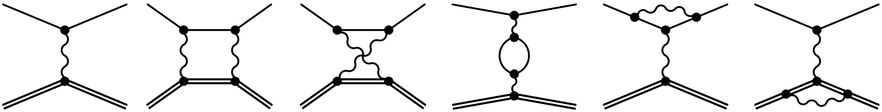
$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) + $$

$$+ 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+ |\mathcal{M}^{\ell}_{\text{brems}}|^2 + |\mathcal{M}^{p}_{\text{brems}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brems}}\mathcal{M}^{p}_{\text{brems}}\right)$$

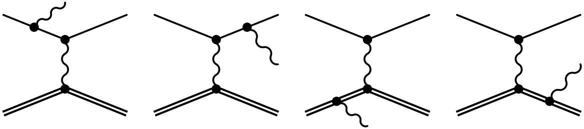
$e^- p$ scattering in the next-to-leading order

Elastic scattering ($e^- p \to e^- p$):



$\mathcal{M}_{\text{Born}}$ $\mathcal{M}_{2\gamma}$ $\mathcal{M}_{\text{vac}}$ $\mathcal{M}^{\ell}_{\text{vert}}$ $\mathcal{M}^{p}_{\text{vert}}$

First-order bremsstrahlung ($e^- p \to e^- p\,\gamma$):



$\mathcal{M}^{\ell}_{\text{brems}}$ $\mathcal{M}^{p}_{\text{brems}}$

$$\sigma(e^- p) = |\mathcal{M}_{\text{Born}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{2\gamma}\right) +$$

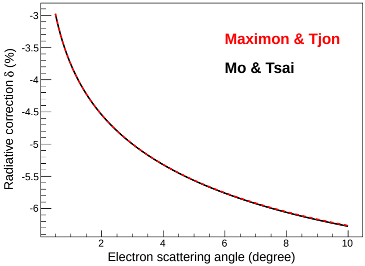
$$+\,2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}_{\text{vac}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{\ell}_{\text{vert}}\right) + 2\,\text{Re}\left(\mathcal{M}^{\dagger}_{\text{Born}}\mathcal{M}^{p}_{\text{vert}}\right) +$$

$$+\,|\mathcal{M}^{\ell}_{\text{brems}}|^2 + |\mathcal{M}^{p}_{\text{brems}}|^2 + 2\,\text{Re}\left(\mathcal{M}^{\ell\dagger}_{\text{brems}}\mathcal{M}^{p}_{\text{brems}}\right)$$

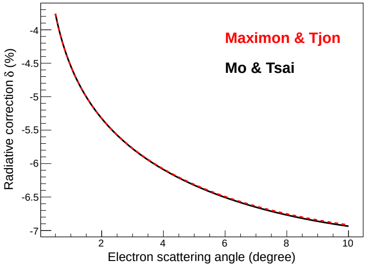
✓ Cancellation of infrared divergences (corresponding terms are marked in color)

Radiative corrections to $e^- p$ scattering

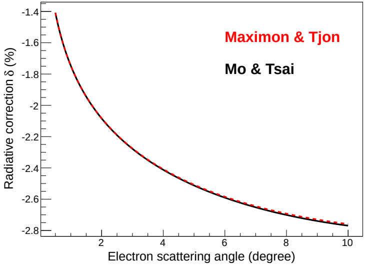
$E_{\text{beam}} = 1.1$ GeV, $\Delta E = 0.1 E_{\text{beam}}$:



$E_{\text{beam}} = 2.2$ GeV, $\Delta E = 0.1 E_{\text{beam}}$:



$E_{\text{beam}} = 1.1$ GeV, $\Delta E = 0.2 E_{\text{beam}}$:



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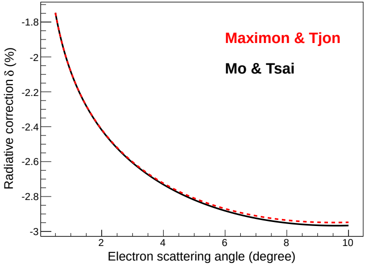


A new event generator for $e^- p$ scattering

A new multipurpose event generator ($\ell^\pm p \to \ell^\pm p$ and $\ell^\pm p \to \ell^\pm p \gamma$), called `ESEPP` (Elastic Scattering of Electrons and Positrons by Protons), has been developed for the Monte Carlo simulation of unpolarized elastic scattering of electrons and positrons (as well as muons and antimuons) on a hydrogen target.

The source code of the generator is freely available (under the GNU GPL license) on the page `http://www.inp.nsk.su/~gramolin/esepp/` (see arXiv:1401.2959 for a detailed description).

The main features of `ESEPP`:

✓ Four types of incident particles are possible: e^-, e^+, μ^-, μ^+;

✓ All the kinematic parameters of the final particles are known ⇒ `ESEPP` is a multipurpose generator;

✓ An accurate calculation for first-order bremsstrahlung is used instead of the common soft-photon/peaking approximations;

✓ Not only the lepton bremsstrahlung is considered, but also the proton bremsstrahlung and the interference term (important for the TPE experiments);

✓ The proton form factors are taken into account (several different models);

✓ The ultrarelativistic approximation $m^2 \ll Q^2$ is not used (except for the lepton vertex correction).

$\ell^\pm p \to \ell^\pm p\,\gamma$: the soft-photon approximation

- The differential cross section ($z=-1$ for e^-/μ^- and $z=+1$ for e^+/μ^+):

$$\frac{d\sigma_{\rm brems}}{dE_\gamma\, d\Omega_\gamma\, d\Omega_\ell} = -\frac{\alpha E_\gamma}{4\pi^2}\left[z\,\frac{\ell}{k\cdot\ell} - z\,\frac{\ell'}{k\cdot\ell'} + \frac{p}{k\cdot p} - \frac{p'}{k\cdot p'}\right]^2 \frac{d\sigma_{\rm Born}}{d\Omega_\ell}.$$

- The cross section integrated over all photon directions and energies $E_\gamma < E_\gamma^{\rm cut}$:

$$\left.\frac{d\sigma_{\rm brems}}{d\Omega_\ell}\right|_{E_\gamma<E_\gamma^{\rm cut}} = \frac{-\alpha}{4\pi^2}\,\frac{d\sigma_{\rm Born}}{d\Omega_\ell}\int\limits_{E_\gamma<E_\gamma^{\rm cut}} \frac{d^3k}{E_\gamma}\left[z\,\frac{\ell}{k\cdot\ell} - z\,\frac{\ell'}{k\cdot\ell'} + \frac{p}{k\cdot p} - \frac{p'}{k\cdot p'}\right]^2$$

$$= -2\alpha\,\frac{d\sigma_{\rm Born}}{d\Omega_\ell}\sum_{i,j}\Theta(p_i)\Theta(p_j)B\left(p_i, p_j, E_\gamma^{\rm cut}\right),$$

where

$$B\left(p_i, p_j, E_\gamma^{\rm cut}\right) = \frac{1}{8\pi^2}\int\limits_{E_\gamma<E_\gamma^{\rm cut}} \frac{d^3k}{\sqrt{|\mathbf{k}|^2+\lambda^2}}\,\frac{p_i\cdot p_j}{(k\cdot p_i)(k\cdot p_j)}$$

$$= \frac{p_i\cdot p_j}{4\pi}\int_0^1 \frac{dx}{p_x^2}\left(\ln\frac{4\left(E_\gamma^{\rm cut}\right)^2}{p_x^2} + \frac{p_x^0}{|\mathbf{p}_x|}\ln\frac{p_x^0-|\mathbf{p}_x|}{p_x^0+|\mathbf{p}_x|} + \ln\frac{p_x^2}{\lambda^2}\right),$$

$$p_x = \left(p_x^0,\, \mathbf{p}_x\right) = x p_i + (1-x) p_j,$$

$\Theta(\ell)=z$, $\Theta(\ell')=-z$, $\Theta(p)=1$, $\Theta(p')=-1$, and λ is a fictitious mass of the photon.

$$\left.\frac{d\sigma_{\text{elast}}}{d\Omega_\ell} + \frac{d\sigma_{\text{brems}}}{d\Omega_\ell}\right|_{E_\gamma < E_\gamma^{\text{cut}}} = \left(1 + \delta_{\text{virt}} + \delta_{\text{brems}}\right) \frac{d\sigma_{\text{Born}}}{d\Omega_\ell},$$

where

$$\delta_{\text{virt}} = \delta_{\text{vac}}^{e} + \delta_{\text{vert}},$$

$$\delta_{\text{brems}} = \delta_{\text{brems}}^{\ell\ell} + \delta_{\text{brems}}^{pp} - z\,\delta_{\text{brems}}^{\ell p},$$

$$\delta_{\text{vac}}^{e} = \frac{2\alpha}{\pi}\left(-\frac{5}{9} + \frac{1}{3}\ln\frac{-q^2}{m_e^2}\right),$$

$$\delta_{\text{vert}} = \frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{-q^2}{m^2} - 2\right),$$

$$\delta_{\text{brems}}^{\ell\ell} = -2\alpha\left[\widetilde{B}\left(\ell,\ell,E_\gamma^{\text{cut}}\right) - 2\widetilde{B}\left(\ell,\ell',E_\gamma^{\text{cut}}\right) + \widetilde{B}\left(\ell',\ell',E_\gamma^{\text{cut}}\right)\right],$$

$$\delta_{\text{brems}}^{pp} = -2\alpha\left[\widetilde{B}\left(p,p,E_\gamma^{\text{cut}}\right) - 2\widetilde{B}\left(p,p',E_\gamma^{\text{cut}}\right) + \widetilde{B}\left(p',p',E_\gamma^{\text{cut}}\right)\right],$$

$$\delta_{\text{brems}}^{\ell p} = 4\alpha\left[\widetilde{B}\left(\ell,p,E_\gamma^{\text{cut}}\right) - \widetilde{B}\left(\ell,p',E_\gamma^{\text{cut}}\right) - \widetilde{B}\left(\ell',p,E_\gamma^{\text{cut}}\right) + \widetilde{B}\left(\ell',p',E_\gamma^{\text{cut}}\right)\right],$$

$$\widetilde{B}\left(p_i,p_j,E_\gamma^{\text{cut}}\right) = \frac{p_i\cdot p_j}{4\pi}\int_0^1 \frac{dx}{p_x^2}\left(\ln\frac{4\left(E_\gamma^{\text{cut}}\right)^2}{p_x^2} + \frac{p_x^0}{|\mathbf{p}_x|}\ln\frac{p_x^0 - |\mathbf{p}_x|}{p_x^0 + |\mathbf{p}_x|}\right).$$

A new event generator for fixed-target Møller scattering

Radiative corrections to Møller scattering are similar to those in $e^- p$ scattering, but:

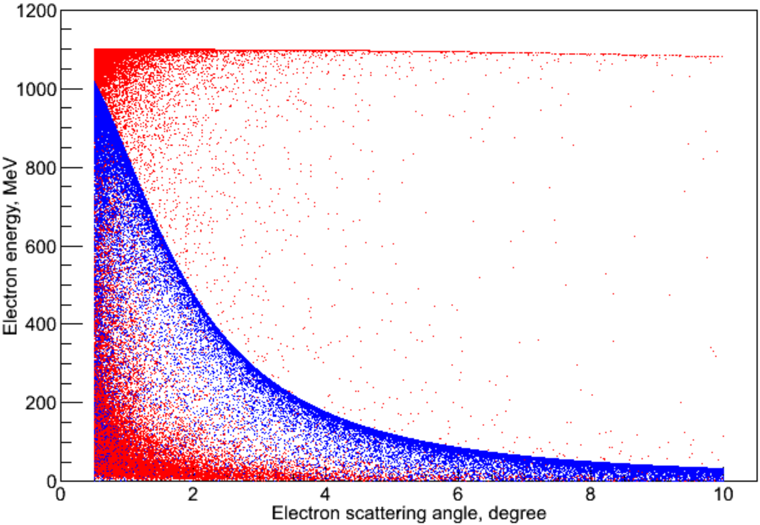
- all particles are structureless;
- there are more Feynman diagrams involved (due to the fact that we have two identical particles in the final state).

For example, first-order soft-photon bremsstrahlung is described as

$$\frac{d\sigma_{e^-e^-\to e^-e^-\gamma}}{dE_\gamma\, d\Omega_\gamma\, d\Omega_1} = -\frac{\alpha E_\gamma}{4\pi^2}\left[\frac{\ell_1}{k\cdot\ell_1} + \frac{\ell_2}{k\cdot\ell_2} - \frac{\ell_1'}{k\cdot\ell_1'} - \frac{\ell_2'}{k\cdot\ell_2'}\right]^2 \frac{d\sigma_{\text{Møller}}}{d\Omega_1}.$$

A new event generator for the fixed-target unpolarized Møller/Bhabha scattering is developing by the author. A preliminary version (based on the soft-photon approximation for first-order bremsstrahlung) of the Møller generator is ready and available at `http://www.inp.nsk.su/~gramolin/mbs/`. Implementation of more accurate QED formulas for first-order bremsstrahlung is in progress.

Example: E vs θ for $e^- p$ and $e^- e^-$ scattering



$E_{\text{beam}} = 1.1 GeV$, $e^- p$ scattering, $e^- e^-$ scattering

Conclusions:

- Radiative corrections to $e^- p$ and $e^- e^-$ scattering are important for the PRad experiment.
- The best way to take into account these corrections is to perform a realistic Monte Carlo simulation of the detector used in the experiment.
- We will have two independent sets of event generators in the experiment. It is useful to be more confident.
- The `ESEPP` generator for $e^- p$ scattering is ready and its source code is publicly available at http://www.inp.nsk.su/~gramolin/esepp/ (see arXiv:1401.2959 for a detailed description).
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Thank you for your attention!

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