Theoretical foundation of lifetime extraction from Primex data

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Precise experimental data require "state of art" theory.

$$\frac{d\sigma}{d\Omega} = \frac{k^2}{\pi}\frac{d\sigma}{dt} = \mid F_C + e^{i\varphi} F_S \mid^2 + \frac{d\sigma_{inc}}{d\Omega} \tag{1}$$

$\frac{d\sigma_{inc}}{d\Omega}$ - incoherent cross section.
φ - relative phase between Coulomb F_S and strong F_S amplitudes.

1 Coulomb amplitude F_C

1.G. Morpurgo (1964) Coulomb amplitude F_C with absorption of pion in the strong nucleus field was obtained in the framework of distorted wave approximation.

2 G. Faldt (1972) The general expression for particles production in Coulomb field using Glauber multiple scat-

tering theory.
He cite final expression for Coulomb amplitude only for the case of equal absorption of primarily and final particles.

3. S. Gevorkyan (2003) The expression for Coulomb form factor has been obtained for pseudoscalar mesons photoproduction:

$$\frac{d\sigma}{dt} = 8\pi\alpha Z^2\Gamma(\frac{\beta}{\mu})^3\frac{k^2 sin^2\theta}{(q^2+\Delta^2)^2} \mid F_{em}(q) \mid^2 \qquad (2)$$

$$F_{em}(q) = 2\pi\frac{q^2+\Delta^2}{q}\int J_1(qb)\frac{b^2dbdz}{(b^2+z^2)^{3/2}}e^{i\Delta z}$$
$$\times \exp(-\frac{\sigma\prime A}{2}\int_z^\infty \rho(b,z\prime)dz\prime)\int_0^{\sqrt{b^2+z^2}} x^2\rho(x)dx \qquad (3)$$

$$t = (k-p)^2 = -q^2-\Delta^2 = -4kpsin^2(\frac{\theta}{2}) - (\frac{\mu^2}{2E})^2$$

$$\sigma' = \sigma(1 - i\frac{Ref(0)}{Imf(0)}) = \frac{4\pi}{ik}f_s(0)$$

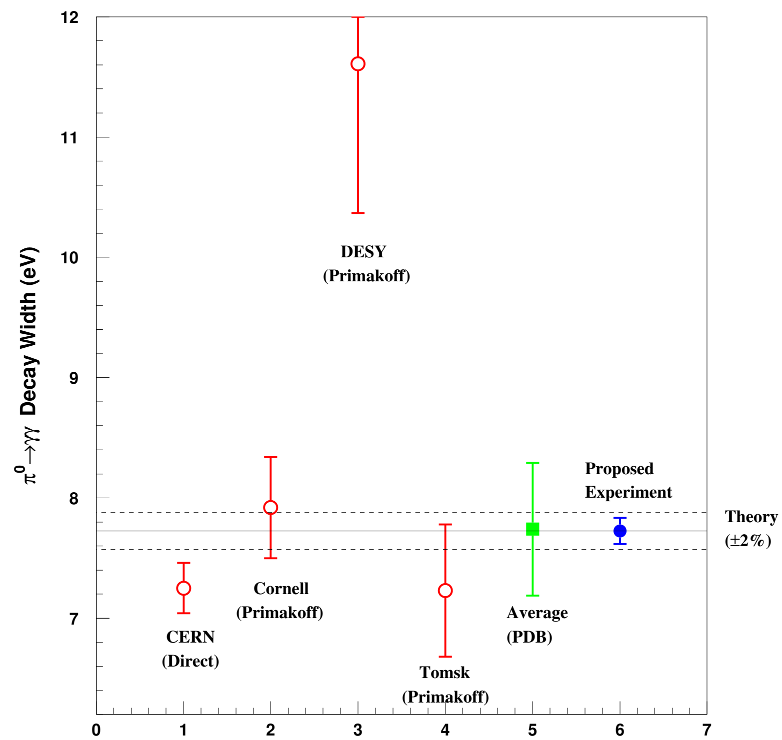

Figure 1: $\pi^0 \rightarrow \gamma\gamma$ decay width in eV. The horizontal line is the prediction of the axial anomaly(Eq. 2)[1, 2] with an estimated 2% error[7]. The experimental results with errors are for : 1) the direct method[9]; 2,3,4) the Primakoff method [11, 12, 13]; 5) Particle Data Book Average [3]; 6) the expected error for our future experiment, arbitrarily plotted to agree with the predicted value.

2 Coherent π^0 production amplitude F_S

$$\gamma A \to \pi^0 A \tag{4}$$

The basic of our calculations is the Glauber multiple scattering theory
Two well grounded assumption:
1. The overall phase-shift function $\lambda(\vec{b}, \vec{s}_1, \vec{s}_2, ...\vec{s}_A)$ is the sum of the phase-shift functions λ_j for collisions with individual nucleons.
2. The ground state of the nucleus can be described by means of the independent particle model, i.e.

$$\mid \Psi(\vec{r}_1, \vec{r}_2, ...\vec{r}_A) \mid^2 = \prod_{j=1}^{A} \rho_j(r_j) \tag{5}$$

$$\begin{aligned} F_S(q, q_L) &= A\frac{ik}{2\pi}\int e^{i\vec{q}\vec{b}+iq_L z}\Gamma_p(\vec{b}-\vec{s})\rho(\vec{s}, z) \\ &\times [1 - \int \Gamma_s(\vec{b}-\vec{s}\prime)\rho(\vec{s}\prime, z\prime)d^2s\prime dz\prime]^{A-1}d^2bd^2sdz \end{aligned} \tag{6}$$

$$\begin{aligned} \Gamma_p(\vec{b}-\vec{s}) &= \frac{1}{2\pi ik}\int e^{i\vec{q}(\vec{b}-\vec{s})}f_p(q)d^2q \\ \Gamma_s(\vec{b}-\vec{s}) &= \frac{1}{2\pi ik}\int e^{i\vec{q}(\vec{b}-\vec{s})}f_s(q)d^2q \end{aligned} \tag{7}$$

1.Nuclear densities are slowly changing functions in respect to profile functions.

$$\rho(r) = \rho(0)e^{-\frac{r^2}{R^2}}$$

For $f(q) = f(0)e^{-aq^2}$ profile function

$$\Gamma(\vec{b}-\vec{s}) \approx e^{\frac{(\vec{b}-\vec{s})^2}{2a}}$$

$$R^2 >> 2a \approx 0.5f^2$$

$$\int \Gamma_s(\vec{b}-\vec{s})\rho(s,z)d^2sdz = \frac{\sigma\prime}{2}\int \rho(\vec{b},z)dz \qquad (8)$$

$\sigma\prime = \sigma(1 - i\frac{Ref(0)}{Imf(0)}) = \frac{4\pi}{ik}f_s(0)$.

$$\int \Gamma_p(\vec{b}-\vec{s})\rho(s,z)d^2sdz = \frac{2\pi}{k}f(0)\vec{h}\int \frac{\partial\rho(\vec{b},z)}{\partial\vec{b}}dz \quad (9)$$

2. Replace the binomial to exponent using the relation $(1-\frac{x}{A})^A \approx exp(-x)$ valid for A¿10

Basic expression can be cast in the factorized form:

$$\begin{aligned} F_S(q) &= A(\vec{h}\vec{q})f(0)F_A \\ F_A &= -\frac{2\pi}{q}\int J_1(qb)\frac{\partial\rho(b,z)}{\partial b}bdbdze^{i\Delta z}\exp(-\frac{\sigma\prime A}{2}\int_z^\infty \rho(b,z\prime)dz\prime) \end{aligned} \qquad (10)$$

$$\begin{aligned} F_A &= \int e^{i\vec{q}\vec{b}}\rho(b,z)bdbdze^{i\Delta z}\exp(-\frac{\sigma\prime A}{2}\int_z^\infty \rho(b,z\prime)dz\prime) \\ &- \frac{A\pi\sigma}{q}\int J_1(qb)\rho(b,z_1)\frac{\partial\rho(b,z_2)}{\partial b}bdbdz_1dz_2e^{i\Delta z_1} \\ &\times \exp(-\frac{\sigma\prime A}{2}\int_{z_1}^\infty \rho(b,z')dz') \end{aligned} \qquad (11)$$

The first term is the usual nuclear form factor and the second one is so called Faldt correction.It is positive ($\frac{\partial\rho(b,z)}{\partial b} < 0$) and can be interpreted as the result of final state interaction of pions produced at nonzero angles coming to forward direction after multiple scatterings.

3 Intermediate channels

High energy photons are shadowed in nuclei.For our case it lead to two step π^0 production through the cell

$$\gamma + N \to \rho + N \to \pi^0 + N$$

$$F_I(q) = -\frac{\pi\sigma A}{q}\int J_1(qb)\rho(b,z_1)\frac{\partial\rho(b,z_2)}{\partial b}\theta(z_2 - z_1)bdbdz_1dz_2$$
$$\times e^{i\Delta_\rho(z_1 - z_2) + i\Delta z_2}\exp(-\frac{\sigma\prime A}{2}\int_{z_1}^{\infty}\rho(b,z\prime)dz) \quad (12)$$

$\Delta_\rho = \frac{m^2\rho}{2E}$ -longitudinal momentum transfer in the ρ photoproduction. This expression and Fäldt correction are very similar and only difference is in their dependence on longitudinal momenta transfer.

In the limit of infinite energies:

$$F_A = \frac{2}{\sigma\prime A}\int d^2be^{i\vec{q}\vec{b}}(1 - \exp(-\frac{\sigma\prime A}{2}\int\rho(b,z\prime)dz)) \quad (13)$$

This expression looks as the coherent production of π^0 by primarily ρ the fact well known for diffractive processes.

4 Nuclear density parameterization

For charge and nuclear densities we use the Fermi parameterization:

$$\rho(r) = \rho_0 \frac{1 + w\frac{r^2}{R^2}}{1 + \exp(\frac{r-R}{c})}; r^2 = b^2 + z^2$$

$$\frac{\partial\rho(b,z)}{\partial b} = \frac{b\rho_0}{crR^2(1+\exp(\frac{r-R}{c}))} \times [2cwr + (2cwr - R^2 - wr^2)\exp(\frac{r-R}{c})] \quad (14)$$

with the following parameters:

Carbon(A=12):w=-0.149;R=2.355f;c=0.5224f;$\rho_0 = 0.01519 f^{-3}$

Lead (A=208):w=0;R=6.624f;c=0.549f;$\rho_0 = 0.0007696 f^{-3}$

5 Strong form factor for light nuclei

1.Born approximation ($\sigma_{tot}(\pi N) = 0$

2.Gauss parameterizations for nucleon density distribution $\rho(r) = \rho(0)e^{-\frac{r^2}{R^2}}$ and for elementary amplitudes $f(q) \approx e^{-aq^2}$

$$F_A(q) = e^{-\frac{(R^2+2a)q^2}{4}} \quad (15)$$

$$a = 6GEV^{-2} = 0.24 f^2; \quad R_c \approx 2f$$

$$\exp(-x) \to (1 - \frac{x}{A})^A$$

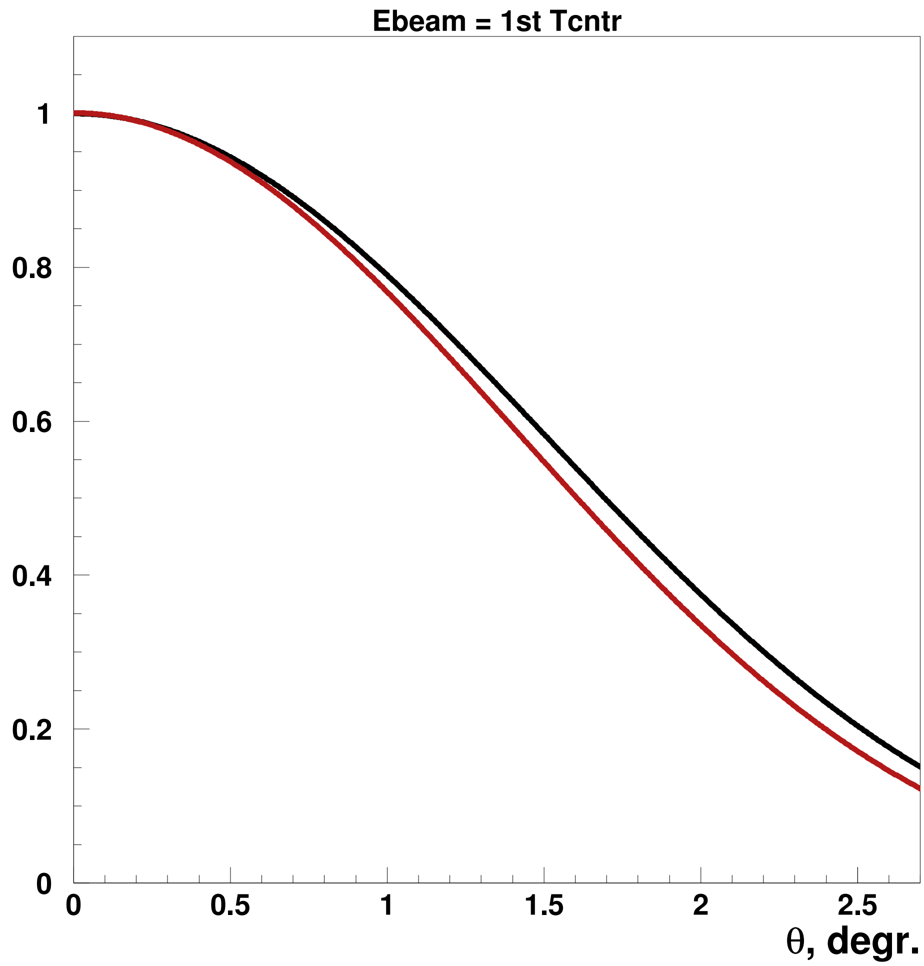

Figure 1: The strong form factor F_A angle dependence with approximations 1,2 (top curve) and without it (bottom one)

6 Photoproduction of π^0 on nucleon

Discuss shortly the process of π^0 photoproduction off the nucleon i.e.

$$\gamma + N \to \pi^0 + N \tag{16}$$

a) The energy dependence.
The amplitude of this process try to zero at small angles and is proportional to the photon energy $f = Nksin\theta$. Such energy dependence is the straight result of approximate energy independence of the invariant product $s^2\frac{d\sigma}{dt}$ $s = (k + P_N)^2 = M_N^2 + 2M_N k$ (lab.system).

$$k^2\frac{d\sigma}{dt} = \pi\frac{d\sigma}{d\Omega} = \pi N^2 k^2 sin^2\theta \approx (t_{min} - t) \tag{17}$$

On the other hand the experimental data at small angles has a tendency to have a stronger than linear dependence of amplitude in accordance with Regge theory.
b) Phase.

According to Regge theory the real and imaginary parts of amplitude are determined by signature factor, which in our case is determined by Regge trajectory α_ω

$$\frac{1 - e^{-i\pi\alpha_\omega}}{2\sin(\pi\alpha_\omega)} = \frac{e^{\frac{i\pi\alpha_\omega}{2}} - e^{\frac{-i\pi\alpha_\omega}{2}}}{2\sin(\pi\alpha_\omega)}e^{\frac{-i\pi\alpha_\omega}{2}} = \frac{\sin(\frac{\pi\alpha_\omega}{2})}{\sin(\pi\alpha_\omega)}e^{\frac{i\pi(1-\alpha_\omega)}{2}} \tag{18}$$

Thus the relative phase factor between Coulomb and strong amplitudes becomes $e^{\frac{i\pi(1-\alpha_\omega)}{2}}$. For $\alpha_\omega(0)$=0.44 $\varphi_\omega =$

$\frac{\pi(1-\alpha_\omega)}{2}$=0.88 in agreement with the value obtained from the fit of experimental data of Cornell experiment.

7 What to do

1) To finish the Coulomb and Strong Coherent parts: A) Revised with another nuclear density.
1. Density which corresponds to the harmonic oscillator potential.
2. Fourier-Bessel parametrization of the nucleons density.
B) Investigate the primary process $\gamma + N \to \pi^0 + N$

It allows to obtain the right dependence of this amplitude on photon energy which can have impact (small?) on lifetime extraction.
Besides its knowledge is crucial for incoherent part of nuclear cross section.
2) Incoherent photoproduction of π^0.