PrimEx-II $\Gamma(\pi^0 \to \gamma\gamma)$ width: two analyses result and combined PrimEx-I and PrimEx-II result

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Abstract

This writeup describes details of two PrimEx-II analyses uncertainties and the way to combine two targets and two analyses results into the joint PrimEx-II result. Some systematic inputs, which were previosly conservativly estimated, have been revisited. In the second part of this note we discuss PrimEx-I and PrimEx-II systematics correlation, the way to combine two PrimEx run results and the final $\Gamma(\pi^0 \to \gamma\gamma)$ value with uncertainties.

1 Two PrimEx-II $\Gamma(\pi^0 \to \gamma\gamma)$ result values and errors

The table below represents two analyses and two targets results reported at June-2018 PrimEx-II Collaboration meeting for $\Gamma(\pi^0 \to \gamma\gamma)$ values. The result of analysis (C) has been updated (shown in bold font in the table) with the recent modification of the strong amplitude described in [1] and its syst. uncertainty has been updated as discussed in the section 3.

Table 1: $\Gamma(\pi^0 \to \gamma\gamma)$ result table

Target	Value	Stat. error	Syst. error
	[eV]		
(H)ybrid mass analyis [2]			
^{28}Si	7.831	0.060	0.124
^{12}C	7.783	0.120	0.137
(C)onstraint mass analyis			
^{28}Si	7.766	0.064	0.132
^{28}Si	7.781	0.064	0.120
^{12}C	7.753	0.134	0.141
^{12}C	7.742	0.134	0.130

2 Correlation between components of PrimEx-II systematic uncertainty

The following table gives the list of the items contributing to the systematic uncertainty and whether they are the different or the same for different targets or analyses. In case if the item is mostly correlated for both analyses, we consider it entirely correlated to simplify assessment.

Table 2: $\Gamma(\pi^0 \to \gamma\gamma)$ systematic uncertainty components

Item	Whether it is correlated for both	
	targets	analyses
Beam parameters	yes	yes
Photon beam flux	yes	yes
Target parameters	No	yes
DAQ efficiency	yes	yes
Event selection	yes	yes
Monte-Carlo Simulation	yes	yes
Yield Extraction	yes	No
Applied theory parameters	No	yes

3 Itemized components of PrimEx-II systematic uncertainty

In this section we are going through the all main items of the systematic uncertainty. Items which were previously conservatively estimated, have been revisited as was suggested on June 2018 PrimEx-II Collaboration meeting, attention have been paid to give more close estimation to not overinflate their contribution. We give values for each item which were used in both (H)ybrid mass and (C)onstrained mass analyses, revised value (if it was updated) with the short description. The values are given in % unless otherwise specified. Few missing items in one analysis have been filled from another one.

3.1 Beam parameters

Beam parameters uncertainty effect on $\Gamma(\pi^0 \to \gamma\gamma)$ systematics has been estimated by parameter variation by certain value.

For beam energy it was taken 0.13% systematic uncertainty from the Tagger calibration and 0.05% from unknown energy distribution within the single 0.1% wide E-channel. As it was mentioned on the June 2018 Collaboration meeting, calibration uncertainty should be much less since special attention has been paid to the energy calibration before PrimEx-II run. We used for revised value 0.05% energy uncertainty from energy distribution within E-channel and the same value for the miscalibration, in total it gives twice smaller input value than it has been used conservatively before.

Beam position uncertainty value of $2mm$ has been taken in analysis (H). This item has not been attributed in this analysis to the beam parameters but to the HyCal geometry. We put it in both analyses to the beam parameter table here. For analysis (C) $1mm$ beam deviation from the database position has been used. Beam position has been accurately monitored during the run by EPICS data [3] and single arm compton events. We don't expect deviation between the database and the actual value more than $0.5mm$ for the selected events (even this value is still conservative and most likely is too high). Events with bad beam conditions were rejected. We used $0.5mm$ deviation value here (even though RMS value should be used instead of the maximum possible deviation). Since effect on $\Gamma(\pi^0 \to \gamma\gamma)$ uncertainty is quadratic for this item, the revised value should be at least 4 times less, than it was used before. Since we have two independent contributions from x and y direction, this item was scaled by the factor of $\sqrt{2}$.

For the beam width (σ) variation we conservatively used $\frac{1}{8}$ of the nominal value. It was shown, that overall effect of such a variation is equivalent to about 0.7 of the effect of 10% π^0 angular resolution variation, that giving us revised value for analysis (H) 0.05% contribution and for analysis (C) 0.14%.

For the beam slope uncertainty we took the value of $0.1mrad$ which is from one hand is a good conservative estimation from the ratio of position uncertainty to the distance between Tagger radiator and HyCal, and from another hand is the reasonable estimation from EPICS BPM analysis [3, 4].

Table 3: Beam parameters itemized uncertainty

Item	analysis (H)	analysis (C)	revised (H)	revised (C)
Beam Energy	0.273	0.3	0.14	0.15
Beam Position	0.3	0.2	0.03	0.07
Beam Width	0.2		0.05	0.18
Beam Slope	0.09	0.15	0.09	0.08
Total	0.46	0.44	0.18	0.26

3.2 Photon beam flux

Photon beam flux is the same for both analyses. We are not revising this item here, but giving a short description of subitems.

Tagging ratio has been studied in detail for runs after TAC module replacement. PrimEx note [5] gives a detailed description of this item error budget breakdown.

Electron counting subitem here is directly associated with DAQ dead time estimation. Dead time (in the units of %) has been estimated by few different ways: assuming fixed electronics dead time of $13\mu sec$ [6], from the number of clock trigger events recorded, from the ratio of gated and ungated scaler counts in the intervals between scaler events. We used level of the agreement (RMS) between these methods as an uncertanty for this subitem.

Beam position effect on collimation (collimator scraping in case of beam missteering) has been adopted from the thesis [7] (page 92). Obtained in [7] value 0.45% is related to 1.3mm beam shift. As it has already been mentioned, the observed beam position stability is within 0.5mm. We conservatively extrapolated this value to 0.13%.

Relative tagging ratio subitem was taken from the large statistics beam intensity study (results published in [8]). Elastic π^0 yield to photon beam flux ratio has been studied independtly and was found stable within the statistical error [9].

Other subitems like T-counter's TDC time unit value or generator freqency precision assumed negligible (less than 0.1%) based on discussions with hardware experts and relative scaler rates observed in the recorded events.

Table 4: Photon beam flux itemized uncertainty

Item	Value
Tagging Ratio	0.37
Electron Counting	0.55
Beam collimation	0.18
Relative tagging ratio	0.4
Total	0.80

3.3 Target

Target parameters have been precisely measured. Methods and results of these measurements have been documented in PrimEx notes [10, 11, 12] in details and published in [13]. Effect of admixture of other chemical elements in graphite target has been previously estimated in PrimEx-I analysis as 0.1% and mostly contributed from oxygen admixture. Silicon target has very high chemical purity and this sytematic contribution can be omitted.

Table 5: Target itemized uncertainty

Item	Value (^{28}Si)	Value (^{12}C)
Target density	0.35	0.020
Target thickness	0.03	0.007
Chemical purity	¡0.01	0.10
Total	0.35	0.10

3.4 DAQ

Effect of DAQ and hardware on the PrimEx-II measurement systmatics can be caused by: 1) inefficiency of HyCal trigger; 2) accuracy of estimation of the fraction of rejected events with ADC error bit set; and from 3) discrepancy between the status of HyCal channels during the runtime and used in the simulation values from the databse. Trigger efficiency has been studied by the similar with PrimEx-I method during the snake scan and inefficiency has been estimated within 0.1% level [14]. Events with ADC errors and HyCal modules status [15] have been monitored and measured for every run. We estimate contrubution from the deviation of these two parameters from the average run value during the run as less than 0.1%.

Table 6: DAQ itemized uncertainty

Item	Value
Trigger inefficiency (due to hardware)	¡0.1
Event with ADC errors accounting	¡0.1
HyCal channels status	¡0.1
Total	0.1

3.5 Event selection

Single γ energy cut subitem has been obtained by the cut variation in analysis (H). In analysis (C) this contribution has been estimated by two ways. First, cut value as well as Monte-Carlo efficiency have been modified many times within 0.2GeV and cross section values have been stored in the histogram for each modified cut value. Then RMS of the obtained distribution has been evaluated. Second, maximum deviation between measured photon energy in the calorimeter and actual value due to nonlineratity has been conservatively estimeted as $0.05GeV$ for $0.5GeV$ energy value. This deviation has been multiplied by single γ energy (from π^0 decay) probability distribution density near $0.5GeV$ value and by acceptance. This product gives acceptance variation due to energy cut shift by $\pm 0.05GeV$ near $0.5GeV$ value of 0.2%. γ pair energy cut subitem has been obtained again by the cut variation in analysis (H). In analysis (C) this contribution has been found negligible (this cut can be easily dropped with no effect on analisys). Time difference cut subitem in analysis (H) has been estimated by the same cut variation technique. For analysis (C) this subitem as well as wrong beam candidate selection subitem for both analyses have been taken as a statistical precision of the observed fraction of rejected by the cut events.

Table 7: Event selection itemized uncertainty

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
Single γ energy cut	0.05		0.2	
γ pair energy cut	0.05		¡0.1	
Hycal trigger and Tagger beam candidate time difference cut	0.025		0.06	0.34
Wrong beam candidate selection	0.1	0.2	0.09	0.34
Total	0.13	0.22	0.23	0.52

3.6 Monte Carlo acceptance and resolution simulations

Target absorption uncertainty has been conservatively estimated based on 2% conversion cross section uncertainty for all three γs traveling through the target. We set the same values for both analyses here.

π^0 angular resolution subitem has been attributed to Monte Carlo since the only thing affecting total uncertainty here is the discrepancy between real and simulated values. The contribution itself has been obtained by variation of the coordinate and energy resolutions in Monte-Carlo by 10% (which is less than observed in PrimEx-I discrepancy for compton on thin 0.5% r.l. beryllium foil angular resolution of 3%). This trick gave us the values shown in the table.

π^0 branching ratio subitem is taken from PDG but small and could be safely removed from the table.

HyCal distance to target has been conservatively estimated in analysis (C) by the solid angle change by $1cm$ distance (uncertainty value). In analysis (H) this subitem has been analysed in more detail way: the slope of acceptance with distance shift has been obtained, then the uncertainty value of $1cm$ has been multiplied by the slope. This way gives more precise and smaller value (0.05 vs 0.2), which we will use for both analyses.

HyCal energy response is the main subitem in this table and connetced with the fraction of events in the energy tail (with energy deposited in HyCal modules less than 80% of the incident γ energy). The discrpenacy between Monte-Carlo and data will contribute to this subitem. The value has been estimated from the comparison of the data and low intensity HyCal snake scan [16].

Next subitem is simply Monte-Carlo statistical accuracy: 50 million events have been generated in large MC sample for each small (0.001°) π^0 production angular bin and all 180 E-channels together. Having in mind that only half (odd) E-channels are mostly presented in statistics we get the shown value.

Table 8: Monte Carlo itemized uncertainty

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
Target absorption	0.24	0.20	0.24	0.20
π^0 angular resolution	0.07		0.26	0.12
π^0 branching ratio	0.034			
HyCal distance to target	0.05			
HyCal energy response	0.45			
Limited MC statistics	0.02			
Total	0.52	0.50	0.58	0.51

3.7 Yield Extraction

Multiple tests have been performed to check yield extraction stability vs histogramming, fitting and background subtraction procedure parameters: function describing background in mass spectrum has been varied between 1st, 2nd and 3rd order polynomial; mass fitting range has been changed from 20MeV to 40MeV (analysis C) and between 0.07 and 0.15 (analysis H); two ways to extract number of events in signal peak have been tried: from the fitting function parameter and from the data histogram with fitted background subtracted; fit parameters have been changed by their errors and fixed, then yield parameter value variation have been checked; empty target and ω,ρ background amplitudes in the fit function have been changed by $\pm 20\%$; bin migration near the zero production angle value has been varied by its stat. error (analysis C). In addition yiled was extracted for three different production angle binning: 0.015°, 0.02°, 0.03° (analysis H) and 0.01°, 0.02°, 0.03° (analysis C). Eventually 500 MC samples have been generated with pi0 production angle distribution corresponding to certain known parameter values: Γ, ϕ, C_S, C_I. These parameters then were extracted the same way as for the data and compared with the generated values. The deviation of the fitted mean value obtained from 500 Γ results has been added as the yield systematics item (the RMS of obtained distribution was in agreement with the statistical error).

Table 9: Yield itemized uncertainty

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
Inv. mass background	*	*	0.4	
Inv. mass range			0.4	
Signal accounting	0.56	0.80	0.55	
Inv. mass fit parameters			0.5	
Empty target subtraction			0.2	
ω/ρ background	0.14	0.16	0.04	0.06
Bin migration near zero angle			0.2	
Production angle binning	0.05	0.1	0.12	0.49
Realistic MC test	0.65		0.2	
Total	0.87	1.05	1.00	1.11

3.8 π^0 Photoproduction Theory

The main contribution from theoretical photoproduction descrption to analysis result uncertainty is due to unknown strong nucleus radius, which was found by increasing it against the electromagnetic radius and minimizing χ^2 of the global fit. Other values which have been checked in analysis (C): sensitivity to pion-nucleon interaction cross section and amplitude phase; shadowng effect amplitude (by variation of nominal value $\xi = 0.25 \pm 0.25$); strong coherent amplitude on nucleon parameterization (Cornell [17, 18], Laget [19], Sibirtsev [20]); incoherent model (Glauber and MCMC model [21] for carbon; original Glabuer model [22] and modified with the exponential parameter slope fitted to describe higher angle data obtained with the Lead Glass part included). These items have been added here to analysis (H) systematics budget (marked by = sign in the table below). Spin effects from non zero spin admixture of ^{29}Si in the silicon target assumed to be negligible [23].

Table 10: Applied theory itemized uncertainty

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
π-N interaction	¡0.1			
Shadowing effect amplitude (ξ)	=		0.31	0.18
Stong coherent amplitude	=		0.08	0.05
Incoherent model variation	=		0.08	0.12
Nonzero isotope admixture	negligible			
Strong nucleus radius increasing	0.25	0.53	0.24	0.52
Total	0.43	0.58	0.42	0.57

3.9 Total PrimEx-II systematic uncertainty

This table gives all mentioned above systematic items together and their quadratic sum. For the purpose of comparison with PrimEx-I uncertainty we will exclude here ”strong nucleus radius” subitem from the theory uncertainty part and give it as an separate entry.

Table 11: Total systematic uncertainty

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
Beam parameters	0.18	0.26	0.18	0.26
Photon beam flux	0.80			
Target parameters	0.35	0.10	0.35	0.10
DAQ efficiency	0.10			
Event selection	0.13	0.22	0.23	0.52
Monte-Carlo Simulation	0.52	0.50	0.58	0.51
Yield Extraction	0.87	1.05	1.00	1.11
Applied theory parameters (w/o ”radius”)	0.34	0.24	0.34	0.24
Strong nucleus radius increasing	0.25	0.53	0.24	0.52
Total systematic	1.43	1.57	1.54	1.68

4 Combining PrimEx-II analyses result

For two different analyses and the same target stat. errors are correlated, and syst. errors could be split into correlated and uncorrelated parts. Systematic error of combined result can be calculated according to the following formula:

$$\sigma_{syst}(12) = \sqrt{(w_1\sigma_{syst\ corr}(1) + w_2\sigma_{syst\ corr}(2))^2 + w_1^2\sigma^2_{syst\ UNcorr}(1) + w_2^2\sigma^2_{syst\ UNcorr}(2)} \tag{1}$$

where symbols 1, 2, 12 designate 1st, 2nd and combined analyses; $w_{1,2}$ - analysis weights to be used for averaging: $\Gamma_{12} = w_1\Gamma_1 + w_2\Gamma_2$; $corr$ and $UNcorr$ stand for correlated and uncorrelated parts of syst. error. Equal weights ($w_{1,2} = \frac{1}{2}$) for both analyses can be used in this case (see for example [24]).

To combine two results with a correlated part of error one can use formula (1) and weights minimizing total uncertainty $\sigma_{total}(12) = \sigma_{total}(w_1\Gamma(1)+w_2\Gamma(2))$. This can be done by solving the simple differential equation: $\frac{\partial\sigma^2_{total}(12)}{\partial w_1} = 0$, where $\sigma^2_{total}(12) = (w_1\sigma_{corr}(1)+ w_2\sigma_{corr}(2))^2 + w_1^2\sigma^2_{UNcorr}(1) + w_2^2\sigma^2_{UNcorr}(2)$ and w_1+w_2=1. Statistical error will be part of uncorrelated error in case of averaging different targets and a part of the correletaed error in case of averaging two analyses using the same statistics. The result for $w_{1,2}$ can be written as:

$$w_1 \sim \frac{\sigma^2_{2,total} - \sigma_{1,corr}\sigma_{2,corr}}{\sigma^2_{1,total} + \sigma^2_{2,total} - 2\sigma_{1,corr}\sigma_{2,corr}}; \quad w_2 = 1 - w_1. \tag{2}$$

The same result for weights can be obtained from [25] formula (6) and [26] formula (27) with error matrix: $\begin{bmatrix} \sigma^2_{1,total} & \sigma_{1,corr}\sigma_{2,corr} \\ \sigma_{2,corr}\sigma_{1,corr} & \sigma^2_{2,total} \end{bmatrix}$. Weights out of (0;1) range would mean, that one of results has too large error, which is highly correlated this another result with much smaller error. A result with such a high correlated error should not be used for combining.

"Yield extraction" subitems (Table 9, page 8) are partly correlated for both analyses, and "Theory" subitems (Table 10, page 9) are partly correlated for both targets. For "Yield extraction" the following subitems can be considered uncorrelated: "Inv. mass range variation" (different shape of background), "Inv. mass fit parameters uncertainty" and "Production angle binning" (independent invariant mass fit procedures and some angular bin sizes), "Realistic MC test" (deviation of the fit result from the pregenerated value). For the analysys (H) items "Inv. mass range variation" and "Inv. mass fit parameters uncertainty" are merged with others, we estimate their input proportionally to the observed in the analysis (C). For the "Photoproduction Theory" subitems, we consider "$\pi - N$" interaction cross section and the "Shadowing effect amplitude (ξ)" items to be correleated between two targets, since they are related to the nucleon interaction. The other items have been assumed to be uncorrelated for both targets since they are nucleus specific. The correlated and uncorreletated parts of these two items are presented in the following table:

Table 12: Correlated and uncorrelated portions of "Yield extraction" and "Photoproduction theory" items

Item	analysis (H)		analysis (C)	
	Value (^{28}Si)	Value (^{12}C)	Value (^{28}Si)	Value (^{12}C)
"Yield extraction" correletated	0.44	0.62	0.74	0.74
"Yield extraction" uncorreletated	0.75	0.85	0.68	0.83
"Photoproduction theory" correletated	0.33	0.20	0.33	0.20
"Photoproduction theory" uncorreletated	0.28	0.55	0.27	0.54

Using correlated part of the syst. uncertainy as defined in tables 2 and 12, Γ values from table 1 and syst. error breakdown from table 11 we obtained the following results and errors for the procedure of combining targets and analyses (we are keeping extra digits here, which could be rounded later):

1) Combining two targets within the same analysis (considering part of "photoproduction" item and statistics uncorrelated) we obtained the following values:

$w_C(analysis\ H) = 0.1898$; $w_{Si}(analysis\ H) = 0.8102$;
$w_C(analysis\ C) = 0.1850$; $w_{Si}(analysis\ C) = 0.8150$;
$\Gamma(C + Si\ analysis\ H) = 7.8219eV \pm 0.0537eV(\text{stat.}) \pm 0.1118eV(\text{syst.})$;
$\Gamma(C + Si\ analysis\ C) = 7.7738eV \pm 0.0578eV(\text{stat.}) \pm 0.1195eV(\text{syst.})$;

2) Combining two analyses for the same target (considering part of "Yield" item uncorrelated and statistics correlated) we obtained the following values:

$w_{analysis\ H} = 0.5$; $w_{analysis\ C} = 0.5$;
$\Gamma(Si\ combined\ HC) = 7.8060eV \pm 0.0620eV(\text{stat.}) \pm 0.1085eV(\text{syst.})$;
$\Gamma(C\ combined\ HC) = 7.7625eV \pm 0.1270eV(\text{stat.}) \pm 0.1172eV(\text{syst.})$;

3) For the purpose of the 2nd step combination of results, we split systematic uncertainty of the combined in the 1st step results into correlated and uncorrelated parts again. For combining two analyses, with target results already combined at the 1st step, we took as uncorrelated the portion of the "Yield extraction" entry from the table 12, since it is the only uncorrelated item between two analyses. To combine two target results, which were already combined between two analyses at the 1st step, we took "Target" item and part of "Photoproduction theory" entry from the table 12 as uncorrelated. Using these assumptions with the formulas (1,2) we obtained the 2nd step combination results:

a) Combining $\Gamma(C + Si\ analysis\ H)$ and $\Gamma(C + Si\ analysis\ C)$ with the equal weights:

$$\Gamma(C + Si\ H + C + Si\ C) = 7.7978eV \pm 0.0557eV(\text{stat.}) \pm 0.1087eV(\text{syst.}); \pm 1.5667\%(\text{total});$$

b) Combining $\Gamma(Si\ combined\ HC)$ and $\Gamma(C\ combined\ HC)$ with weights $w_C = 0.1788$, w_{Si} =0.8212:

$$\Gamma(Si\ comb.\ HC + C\ comb.\ HC) = 7.7982eV \pm 0.0557eV(\text{stat.}) \pm 0.1087eV(\text{syst.}); \pm 1.5712\%(\text{total});$$

4) Obtained values for the combined result in sections 3.a and 3.b are very close. Rounding them to three and two decimal digits we got the final PrimEx-II result:

$$\Gamma(PrimEx\text{-}II) = 7.798eV \pm 0.715\%(\text{stat.}) \pm 1.392\%(\text{syst.}); \pm 1.569\%(\text{total});$$

or

$$\Gamma(PrimEx\text{-}II) = 7.798eV \pm 0.056eV(\text{stat.}) \pm 0.109\text{eV}(\text{syst.}); \pm 0.122\text{eV}(\text{total});$$

and

$$\Gamma(PrimEx\text{-}II) = 7.80eV \pm 0.72\%(\text{stat.}) \pm 1.39\%(\text{syst.}); \pm 1.57\%(\text{total});$$

or

$$\Gamma(PrimEx\text{-}II) = 7.80eV \pm 0.06eV(\text{stat.}) \pm 0.11\text{eV}(\text{syst.}); \pm 0.12\text{eV}(\text{total}).$$

5 The Final PrimEx result: combining PrimEx-I and PrimEx-II

To combine PrimEx-I and PrimEx-II results we will go through two experiments systematic uncertainty budgets and split them into correlated and uncorrelated parts, then average two final results into one. For PrimEx-I final result we will use:

$\Gamma(PrimEx\text{-}I) = 7.82eV \pm 0.14eV(\text{stat.}) \pm 0.17\text{eV}(\text{syst.})$ value reported in [27].

5.1 Correlation between PrimEx-I and PrimEx-II systematic uncertainty

For PrimEx-I systematics we will use table 5.4 from [28] page 217, and for PrimEx-II systematics tables presented previously in this note averaging two analyses values. The following systematics table is organized in the way it is presented in [28], and shows the correlation between two PrimEx runs systematics:

Table 13: Correlation between two PrimEx runs systematics

Item	PrimEx-I value	PrimEx-II value	Correlation
Target parameters	0.3	0.28	No
Photon beam flux	0.97	0.80	No
Target absorption	0.1	0.23	No
DAQ efficiency	0.1	0.1	No
Event selection	0.45	0.22	yes
Br($\pi^0 \to \gamma\gamma$)	0.03	0.03	yes
HyCal distance to target	0.3	0.05	No
HyCal response function	0.45	0.45	yes
Monte-Carlo stat. precision	0.23	0.02	No
Yield extractoion, correlated	1.12	0.61	yes
Yield extractoion, uncorrelated	1.10	0.52	No
Yield extractoion, total	1.6	0.80	partly
Accidentals	0.15		yes
ω and ρ background	0.3		yes
π^0 angular resolution	0.2	0.15	yes
Beam parameters	0.43	0.32	No
π^0 photoproduction theory, corr.	0.52	0.31	yes
π^0 photoproduction theory, unc.	0.27	0.25	No
π^0 photoproduction theory, total	0.59	0.39	partly
Total correlated	1.44	0.81	yes
Total uncorrelated	1.635	1.144	No

PrimEx-I and PrimEx-II use mostly the different targets and target parameters and absorption have been considered uncorrelated. Photon beam flux has been estimated independent way for two PrimEx runs using the different DAQ and Tagger electronics, this item is also considered uncorrelated as well as DAQ efficiency. Event selection is mostly based on HyCal energy linearity effect on the actual single gamma energy cut value. This entry as well as HyCal energy response function and Branching ratio were marked as correlated. HyCal distance to target has been changed between two runs and has an independent uncertainty. Monte-Carlo samples have been simulated with the different geometry and have independent statistics. Yield extraction entry is the largest one and have been split into two parts the same way it was done for averaging two PrimEx-II analyses (table 12). Accidentals and ω/ρ background items have been conservatively considered correlated; first one is a part of the event selection subitem for PrimEx-II analysis and the second one is yield extraction subitem (were already accounted for). π^0 angular resolution defined by the possible difference between simulations and the data and could be correlated. Beam parameters were different for two run conditions (PrimEx-II has more stable beam and a better energy calibration for the Tagger), we accounted them here as uncorrelated. For the photoproduction theory we split it again into correletaed and uncorrelated parts according to the table 12.

5.2 Averaging PrimEx-I and PrimEx-II result

Using formula (2) (page 10) and errors from the table 13 we have obtained the weights to combine PrimEx-I and PrimEx-II results: $w_I = 0.1607$, $w_{II} = 0.8393$, which give us combined PrimEx-I and PrimEx-II result:

$$\boxed{\Gamma(PrimExI{+}II){=}7.802eV{\pm}0.052eV\text{(stat.)}{\pm}0.105\text{eV(syst.)}; \pm0.117\text{eV}(1.50\%)\text{(total)},}$$

or rounding to two digits:

$$\boxed{\Gamma(PrimExI{+}II){=}7.80eV{\pm}0.05eV\text{(stat.)}{\pm}0.11\text{eV(syst.)}; \pm0.12\text{eV}(1.5\%)\text{(total).}}$$

The plot below is showing the total error of the combined result as weight using to average results (reproduced by the simple simulation made for the error values of PrimEx-I and PrimEx-II):

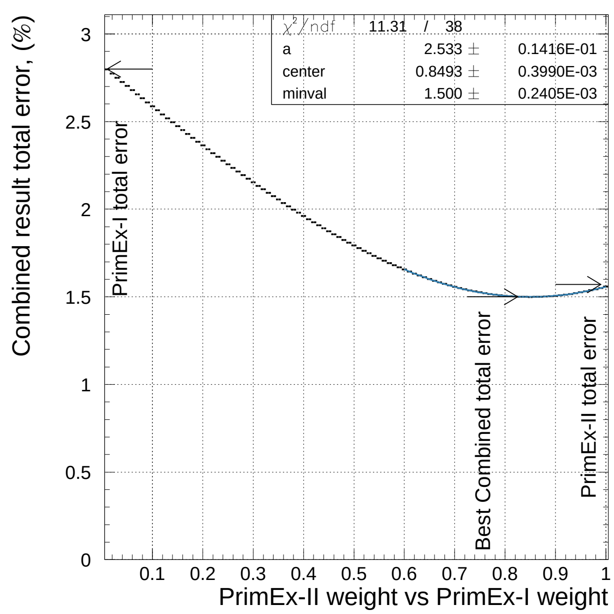

Figure 1: Simulated PrimEx-I and PrimEx-II averaged result total error as a function of the weights used to combine two results.

References

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