PRIMEX NOTE 37

Compton effect in kinematic conditions of PrimEx

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0.1 Born aproximations

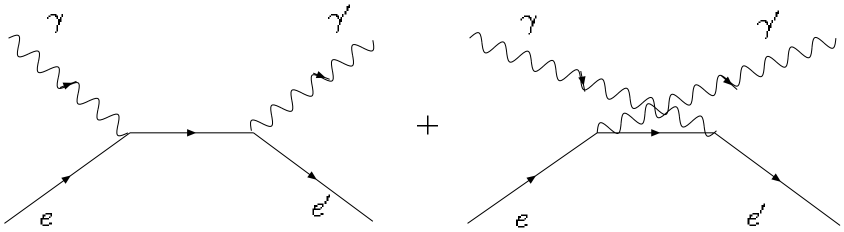


Figure 1: Compton scattering

Cross-section for Compton scattering is

$$\frac{d\sigma_0}{d\cos\theta_1} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega_1}{\omega_0}\right)^2 U_0 \tag{1}$$

$$U_0 = \left(\frac{\omega_1}{\omega_0} + \frac{\omega_0}{\omega_1} - \sin\theta_1{}^2\right) \tag{2}$$

where ω_0, ω_1 are energies of initial γ and scattered photons γ', θ_1 -angle between momentum of the scattered photon $\gamma\prime$ and initial photon γ,

$$\omega_1 = \frac{\omega_0}{1 + \frac{\omega_0}{m}(1 - \cos\theta_1)} \tag{3}$$

0.2 Virtual-photon radiative corrections

Fig.2 shows typical Feynman diagrams which by interference with the diagrams of Fig.1 give radiative corrections to first order in α. The complete set of diagrams also shown in

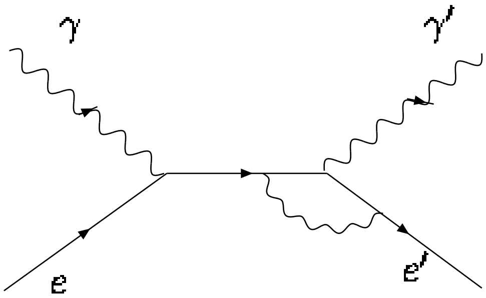


Figure 2: A typical diagram corresponding to the virtual radiative correction to single Compton scattering

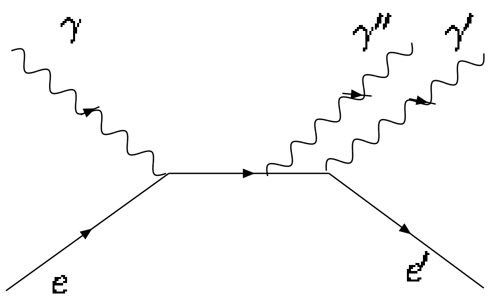


Figure 3: A typical Feynman diagram repesentating double Compton scattering

Fig.2(A)[2]. The corrections have been evaluated by Brawn end Feynman [1], accordindg to whom the cross section including the corrections(virtual corrections) is

$$d\sigma_0(1+\delta^c_{vir})$$

where

$$\delta^c_{vir} = -\frac{\alpha}{\pi U_0} Re U^c_{vir}(\chi1,\chi2) \tag{4}$$

$$U^c_{vir}(\chi1,\chi2) = P(\chi1,\chi2) + P(\chi2,\chi1)$$

$$P(\chi1,\chi2) = (1-2y\coth(2y))\ln\frac{\lambda}{m}U_0 - 2U_0\coth(2y)(2h(y)-h(2y))y-$$

$$4\left(\frac{1}{2}-\frac{1}{\chi1}\right)\tanh(y)y - \frac{4\left(-\frac{3\chi2^2}{4\chi1}-\frac{7\chi1}{4}+\frac{2}{\chi1}\right)y^2}{\chi1+\chi2}+$$

$$4\left(\frac{1}{\chi2}+\frac{1}{\chi1}\right)^2 + \frac{\frac{\chi1}{\chi2}+\frac{1}{2}}{\chi1-1} + \frac{2\left(\frac{\chi1^2}{\chi2}+\frac{\chi1}{\chi2}+\chi1+\frac{\chi2}{2}-\frac{3}{\chi2}-1+\frac{2}{\chi1}+\frac{\chi2}{\chi1^2}\right)(F(\chi1-1)-F(-1))}{\chi1}+$$

$$\ln(|\chi1|)\left(\frac{3\chi2}{2\chi1}+\frac{3\chi2}{2\chi1^2}+4y\coth(2y)\left(\frac{4\cosh^2(y)}{\chi1\chi2}-\frac{\chi2}{2\chi1}+\frac{(\chi1-6)\mathrm{sech}(2y)}{2\chi2}\right.\right.$$

$$\left.-\frac{1}{\chi1}-\frac{\chi1}{\chi2}+\frac{4}{\chi1^2}-1\right)+\frac{8}{\chi1}-\frac{8}{\chi1^2}+1-\frac{2\chi1^2+\chi2}{2(\chi1-1)^2\chi2}+\frac{-\chi2\chi1^2+2\chi1-\chi2^2}{2(\chi1-1)\chi1^2\chi2}-$$

$$\left.\frac{7}{\chi1\chi2}+\frac{3}{\chi2}\right)+h(y)\left(2y\coth(y)-\frac{4y(2-\cosh(2y))\sinh(2y)}{\chi1\chi2}\right)-\frac{12}{\chi1}-\frac{3\chi1}{2\chi2}-\frac{2\chi1}{\chi2^2}, \tag{5}$$

where

$$F(x) = -Li_2(-x),$$

and (in the lab system)

$$\chi1 = -\frac{2\omega_0}{m}, \chi2 = \frac{2\omega_1}{m}.$$

0.3 Soft-photon radiative corrections

In order to take in to accounnt all radiative corrections to first order it is necessary to consider the radiation of the two photon.Fig.3 shows typical Feynman diagrams for double Compton scattering.

For the case in which the energy of one of the emitted photons is much smaller than the electron mass

$$\omega_2 \le \omega_{2max} << m$$

the cross section $d\sigma_{real,soft}$ in laboratory system was obtained by Brawn end Feynman and $d\sigma_{real,soft}$ is given by Eq.39[1].

$$d\sigma_{real,soft} = -\frac{\alpha}{\pi} d\sigma_0 \left[2(1 - 2y \tanh(2y)^{-1} \left(\ln \frac{2\omega_{2max}}{\lambda} - \frac{1}{2} \right) + 4y \tanh(2y)^{-1} [h(2y) - 1] \right], \tag{6}$$

where

$$h(y) = \frac{1}{y} \int_0^y u du \tanh(u)^{-1},$$

$$2 \sinh^2(y) = \frac{\omega_0 - \omega_1}{m}$$

0.4 Hard-photon double Compton effect

The differential cross section for the case of two hard photons emitted with four-momenta k_1 and k_2 has been calculated by Mandle and Skyrme [4].

$$d\sigma_{real} = \alpha r_0^2 \frac{d\Omega_1 d\Omega_2}{(4\pi)^2} \frac{\omega_1 \omega_2 d\omega_1}{m\omega_0} \frac{X}{T_1}, \tag{7}$$

with

$$T_1 = m + \omega_0(1 - \cos\theta_2) - \omega_1(1 - \cos\theta_{12})$$

The conservation laws $p_1 + k_0 = p_2 + k_1 + k_2$ can be used to express $d\sigma_{real}$ in terms of variables $\omega_1, \theta_1, \theta_2$ and ϕ(cf.Fig4).

In terms of this variables

$$\cos\theta_{12} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos\phi$$

and

$$\omega_2 = \frac{L}{T_1}, \tag{8}$$

$$L = m\omega_0 - \omega_1[m + \omega_0(1 - \cos\theta_1)].$$

If we combine the soft- and virtual-photon radiative corrections, the infrared divergent term associated with λ disappears, and we obtain the radiative corrections to Klein-Nishina formula.

0.5 Radiative tail

Consider Compton scattering in which we have recorded photons emitted into the given angle θ_1. The energy of the emitted photon in the single Compton effect $\omega_1 = \omega_s$ is given by

$$\omega_1 \to \omega_s = \frac{\omega_0}{1 + \frac{\omega_0}{m}(1 - \cos\theta_1)}.$$

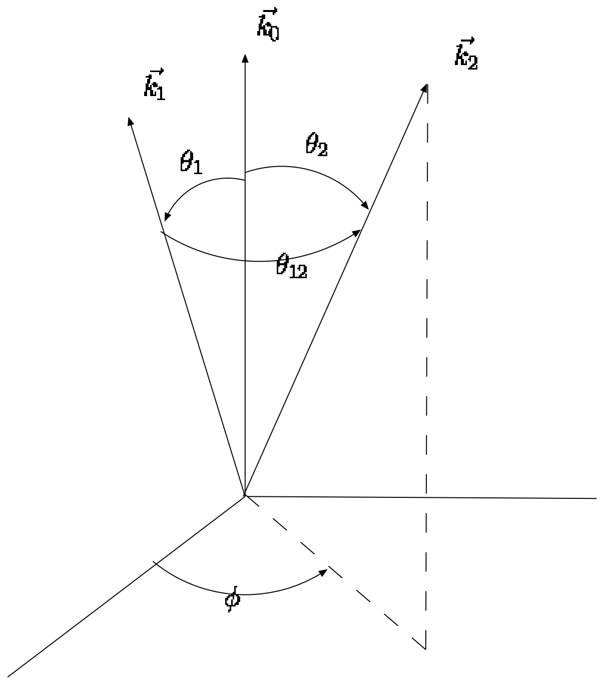


Figure 4: Relative directions in double Compton scattering

If the calorimeter has energy resolution ΔE, then not only single Compton photons are counted but also some from double Compton scattering

$$\omega_s - \Delta E < \omega_1 < \omega_s.$$

If $\Delta E << m$, then we need take in to account only soft-photon double Compton effect. Otherwise, we add the double Compton cross section in which ω_1 is integrated from $\omega_s - \Delta E$ to upper limit defined by $\omega_2 > \omega_{2max}$.

$$d\sigma_{real,hard} = \int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \int_{\omega_s-\Delta E}^{\omega_{1max}} d\omega_1 \frac{d\sigma_{real}}{d\Omega_2 d\omega_1}, \tag{9}$$

where the condition

$$\omega_2 = \frac{\omega_0(\omega_s - \omega_1)}{\omega_s[1 + \frac{\omega_0}{m}(1-\cos\theta_2) - \frac{\omega_1}{m}(1-\cos\theta_{12})]} \geq \omega_{2max}$$

defines

$$\omega_{1max} = \omega_s - \omega_{2max} f_{12}, \tag{10}$$

$$f_{12} = \frac{\left(\frac{\omega_0}{m}(1-\cos(\theta_1))+1\right)\left(\frac{\omega_0}{m}(1-\cos(\theta_2))+1\right) - \frac{\omega_0}{m}(1-\cos\theta_{12})}{\left(\frac{\omega_0}{m}(1-\cos(\theta_1))+1\right)\left(\frac{\omega_0}{m}(1-\cos(\theta_1)) - \frac{\omega_{2max}}{m}(1-\cos\theta_{12})+1\right)}.$$

The differential cross section $\frac{d\sigma_{real}}{d\Omega_2 d\omega_1}$ in the Eq.(9) consist of the two parts-one of them has logarithmic singularity,when $\omega_1 \to \omega_s$.

$$\frac{d\sigma_{real}}{d\Omega_2 d\omega_1} = f_1(\omega_1, \theta_1, \theta_2, \varphi) + \frac{f_2(\omega_1, \theta_1, \theta_2, \varphi)}{\omega_s - \omega_1}. \tag{11}$$

Rewrite Eq. (11) in the form

$$\frac{d\sigma_{real}}{d\Omega_2 d\omega_1} = f_1(\omega_1, \theta_1, \theta_2, \varphi) + \frac{f_2(\omega_1, \theta_1, \theta_2, \varphi) - f_2(\omega_s, \theta_1, \theta_2, \varphi)}{\omega_s - \omega_1} + \frac{f_2(\omega_s, \theta_1, \theta_2, \varphi)}{\omega_s - \omega_1}. \tag{12}$$

Terms in the end of the Eq.(12) may be integrated for ω_1

$$\int_{\omega_s - \Delta E}^{\omega_{1max}} \frac{f_2(\omega_s, \theta_1, \theta_2, \varphi)}{\omega_s - \omega_1} = f_2(\omega_s, \theta_1, \theta_2, \varphi) \ln \frac{\omega_{2max} f_{12}}{\Delta E} \tag{13}$$

After analitical integration the infrared part with $\ln \omega_{2max}$ for variables $\theta_1, \theta_2, \varphi$, this part canceled with terms in the soft-photon double Compton cross section Eq.(6). And then

$$d\sigma^r_{real,hard} =$$

$$\int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \int_{\omega_s - \Delta E}^{\omega_s} d\omega_1 (f_1(\omega_1, \theta_1, \theta_2, \varphi) + \frac{f_2(\omega_1, \theta_1, \theta_2, \varphi) - f_2(\omega_s, \theta_1, \theta_2, \varphi)}{\omega_s - \omega_1}) +,$$

$$\int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\varphi (f_2(\omega_s, \theta_1, \theta_2, \varphi) \ln \frac{f_{12}}{\Delta E}) \tag{14}$$

This cross section Eq.(14)for double Compton scattering does not contain the singularities and now may be integrated numerically.

If in the experiments the measuring apparatus registers the photons with energy greater than δE, then we need to modify limits of the inegration in Eq.(14). Then,for

$$0 \le \theta_1 \le \theta_{1max1}$$

$$\int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \int_{\omega_s - \Delta E}^{\omega_s} d\omega_1,$$

and for

$$\theta_{1max1} \le \theta_1 \le \theta_{1max2}$$

$$\int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\varphi \int_{\delta E}^{\omega_s} d\omega_1.$$

Where

$$\theta_{1max1} = \arccos \frac{(\Delta E + \delta E)(\omega_0 + m) - \omega_0 m}{(\Delta E + \delta E)\omega_0},$$

$$\theta_{1max2} = \arccos(1 - \frac{m}{\delta E} + \frac{m}{\omega_0}).$$

The accuracy of numerically integration is improved by use of variables t_1, t_2 instead $\cos\theta_1, \cos\theta_2$

$$\cos\theta_1 = \frac{1}{\beta} \tanh(\beta t_1)$$

$$\cos\theta_2 = \frac{1}{\beta} \tanh(\beta t_2),$$

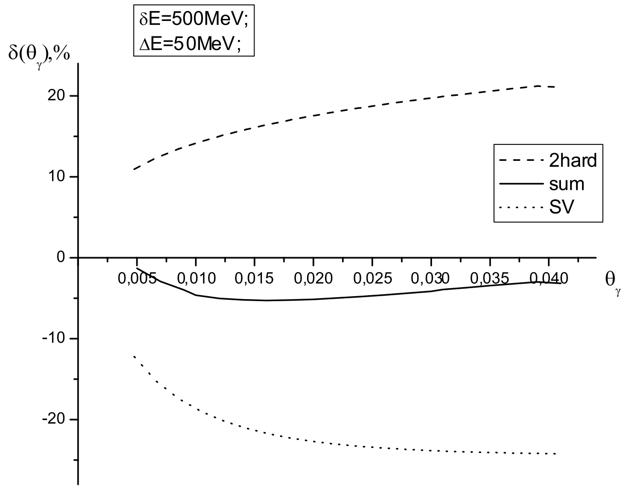


Figure 5: Dependence of the value $\delta^{SV} = \frac{d\sigma^{SV}/d\theta_1}{d\sigma_0/d\theta_1}$, $\delta^{2hard} = \frac{d\sigma^{2hard}/d\theta_1}{d\sigma_0/d\theta_1}$, $\delta^{sum} = \frac{d\sigma^{sum}/d\theta_1}{d\sigma_0/d\theta_1}$ on the $\theta_1(rad)(\omega_0 = 5GeV)$

with

$$\beta = \sqrt{1 + \frac{2m}{\omega_0}}.$$

The results of the numeric integration are shown in Fig.5,6

If we have incertainty in the measurement of the angle of the scattering photons $\Delta\theta_1$, is connected with incertainty of the energy photons

$$\Delta E = \frac{d\omega_1}{d\theta_1}\Delta\theta_1 = \frac{\omega_s^2 \sin\theta_1}{m}\Delta\theta_1. \tag{15}$$

Results of numeric calculation in this case are shown in Fig.7

If we have both ΔE and $\Delta\theta_1$,then

$$\Delta E \to \Delta E + \frac{\omega_s^2 \sin\theta_1}{m}\Delta\theta_1. \tag{16}$$

Results of numeric calculation in this case are shown in Fig.8

If we registered both photons and electrons in the Compton scattering, tnen energy of the two photons in the double Compton scattering must be restricted then

$$\omega_2 \leq 2\Delta E. \tag{17}$$

Results of numeric calculation in this case are showh in Figs.9-11.

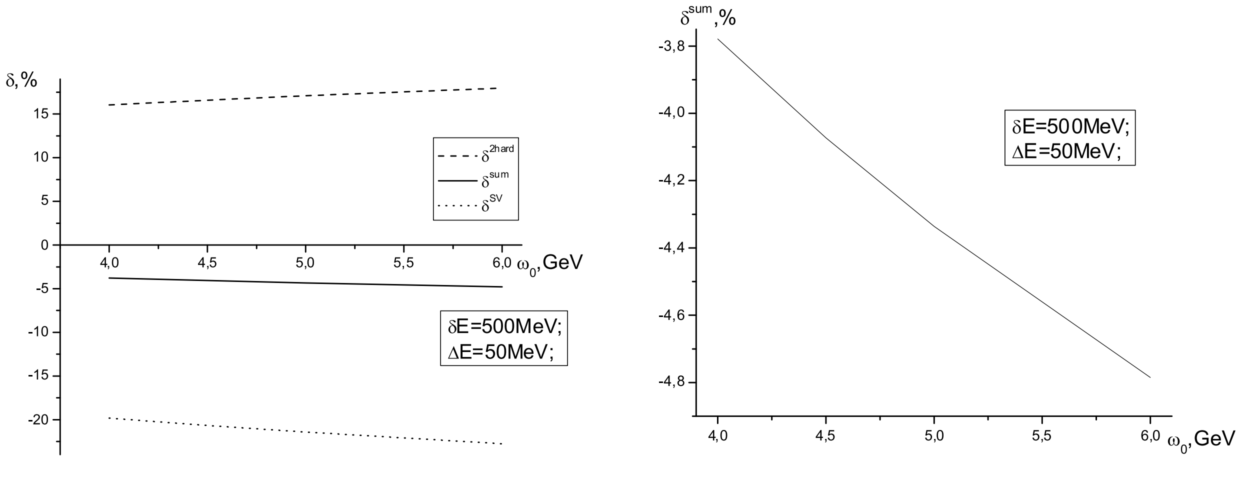


Figure 6: Dependence the integrated over θ_1 value δ^{SV}, δ^{2hard}, δ^{sum} from ω_0.

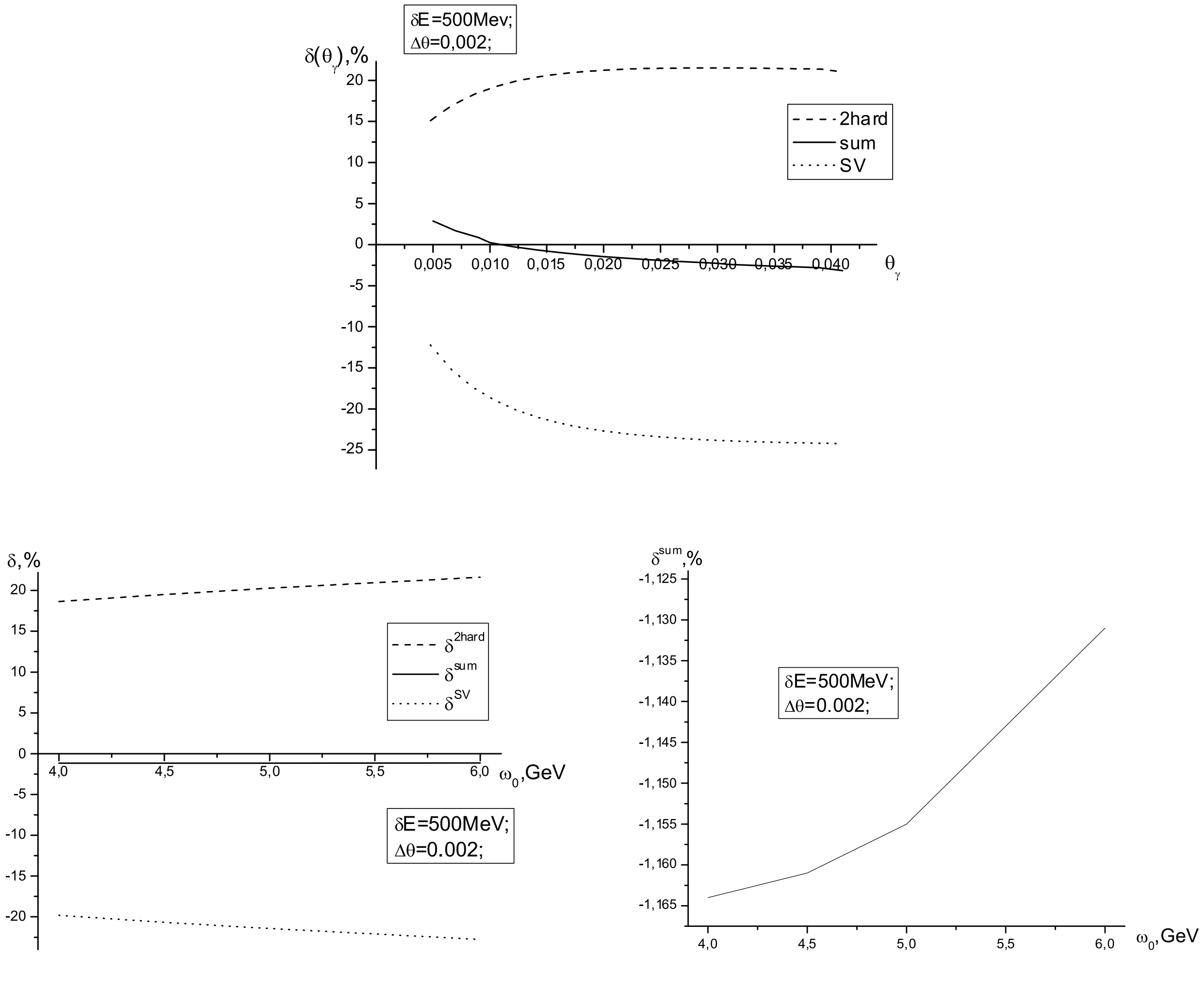


Figure 7: The functions δ for the case (15).

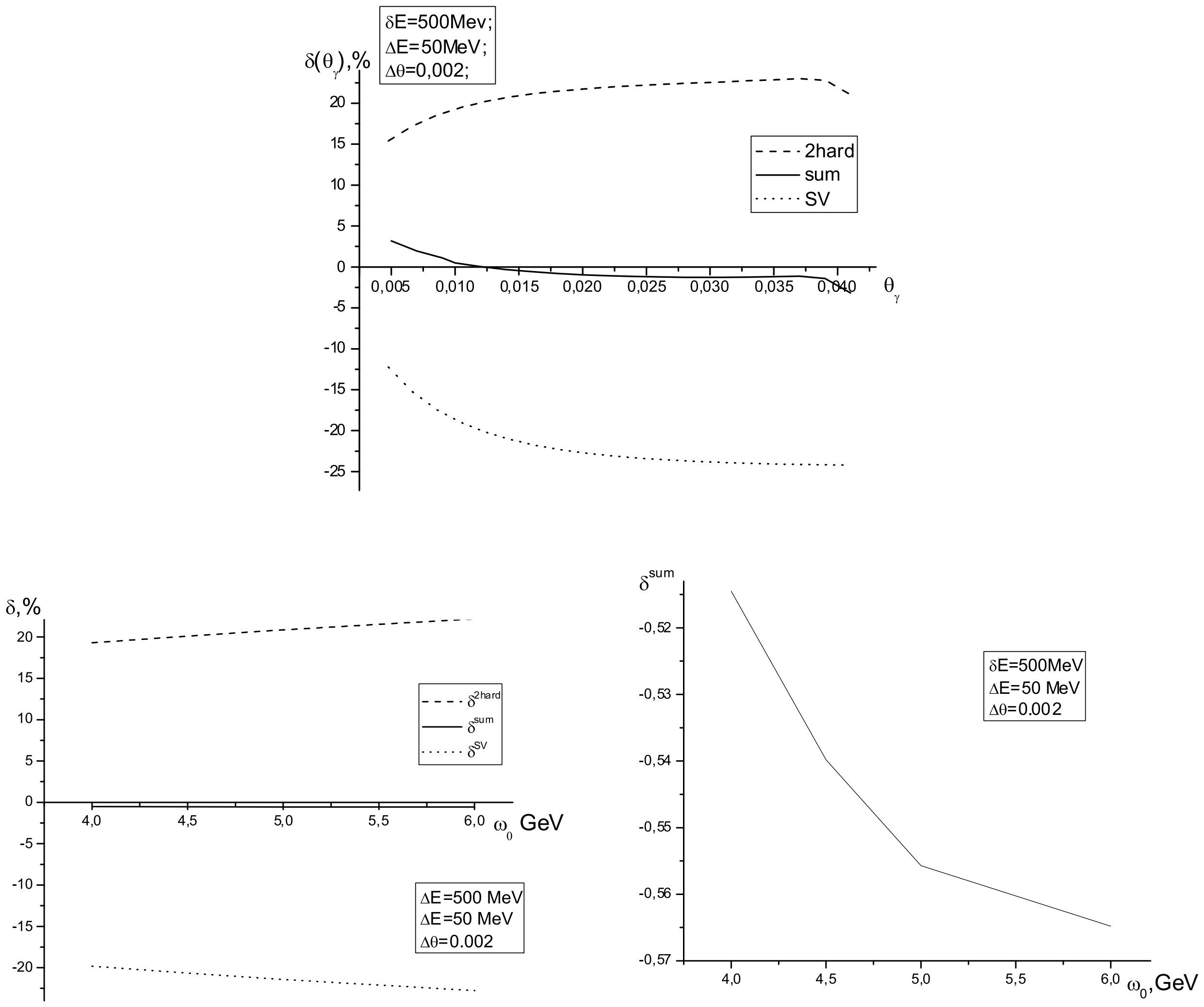
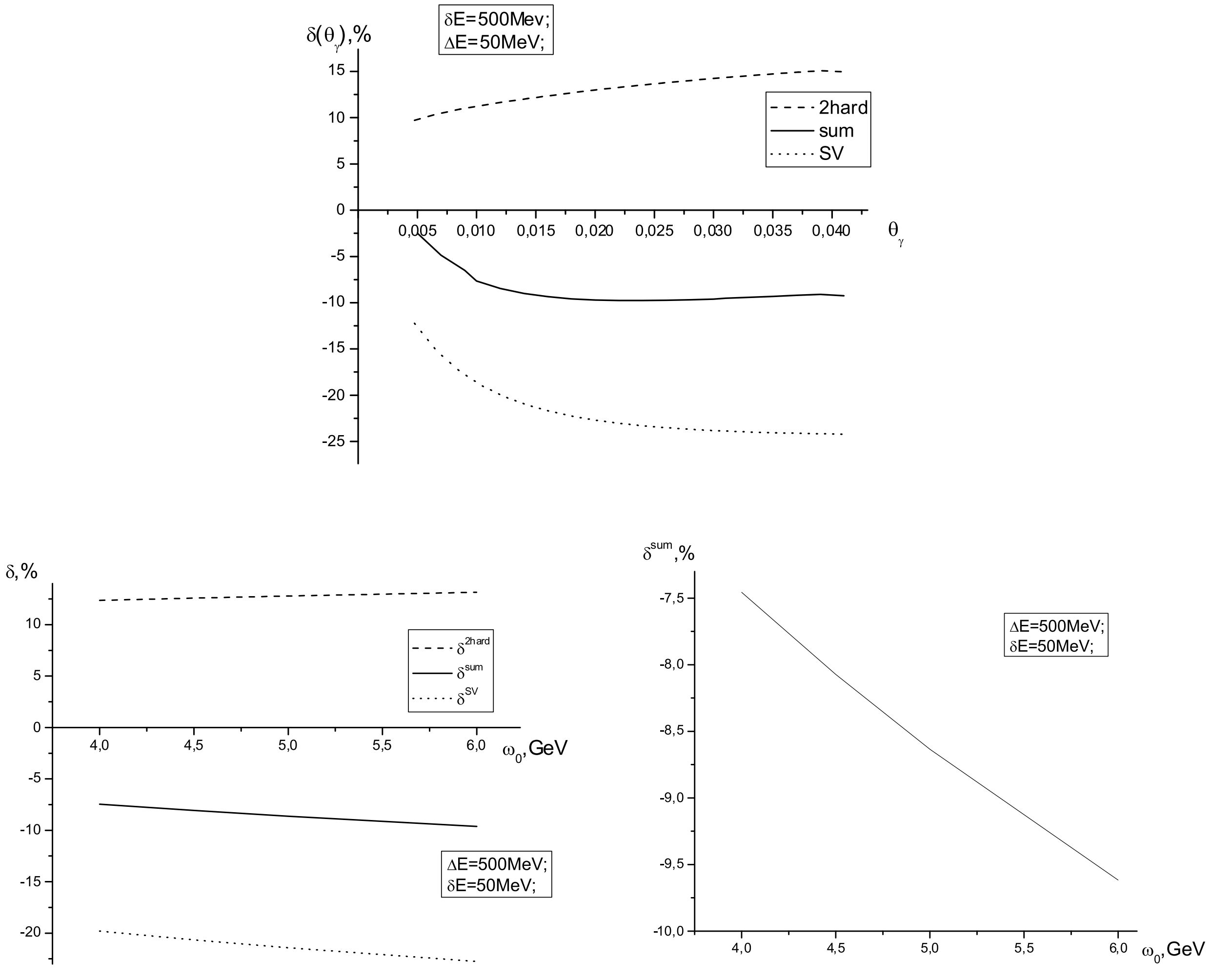


Figure 8: The functions δ for the case (16).



δE=500Mev;
ΔE=50MeV;
δ(θ_γ),%
15
10
5
0
-5
-10
-15
-20
-25
0,005
0,010
0,015
0,020
0,025
0,030
0,035
0,040
θ_γ
2hard
sum
SV
δ,%
10
5
0
-5
-10
-15
-20
4,0
4,5
5,0
5,5
6,0
ω_0, GeV
δ^2hard
δ^sum
δ^SV
ΔE=500MeV;
δE=50MeV;
δ^sum,%
-7,5
-8,0
-8,5
-9,0
-9,5
-10,0
4,0
4,5
5,0
5,5
6,0
ω_0, GeV
ΔE=500MeV;
δE=50MeV;

Figure 9:

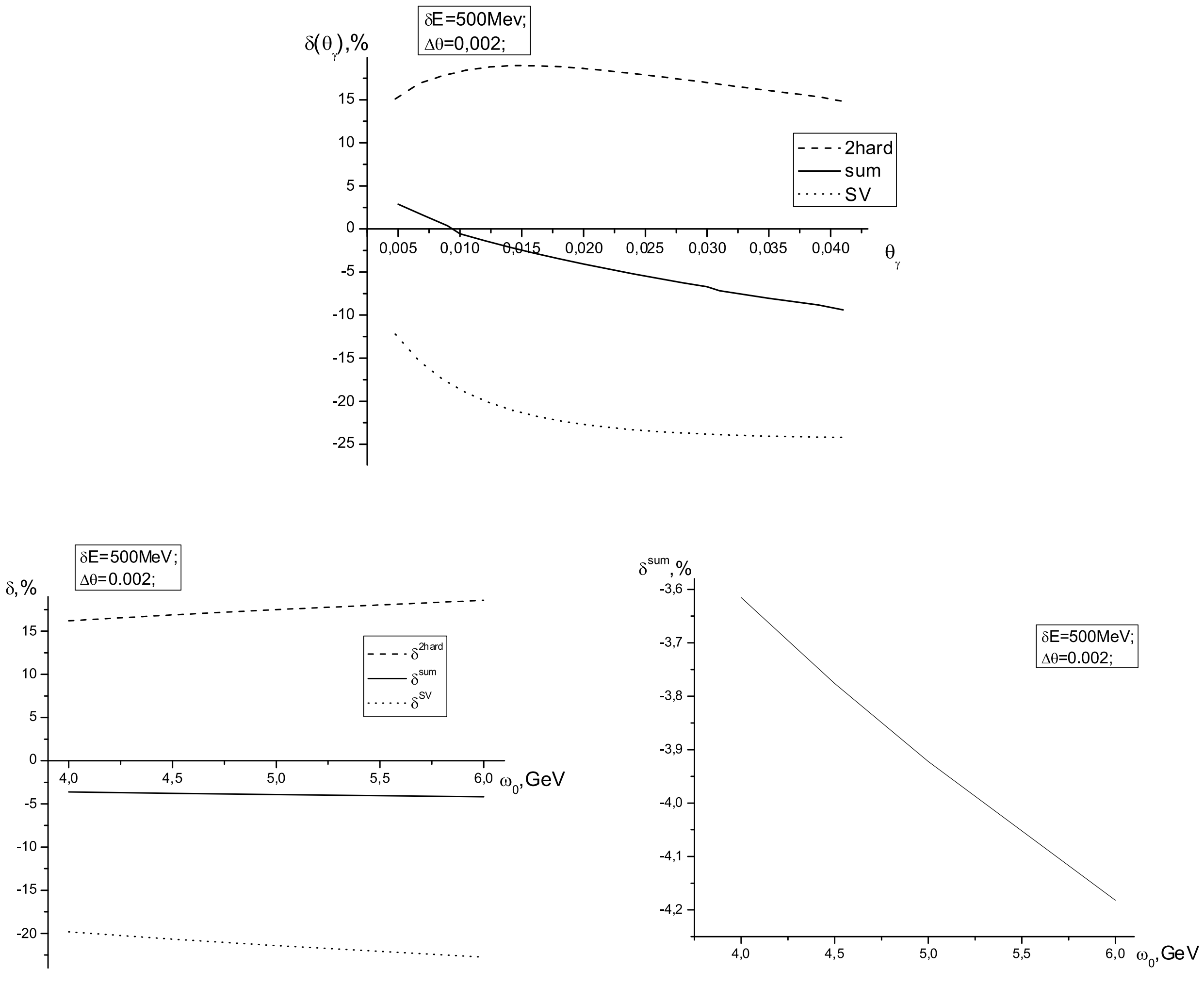
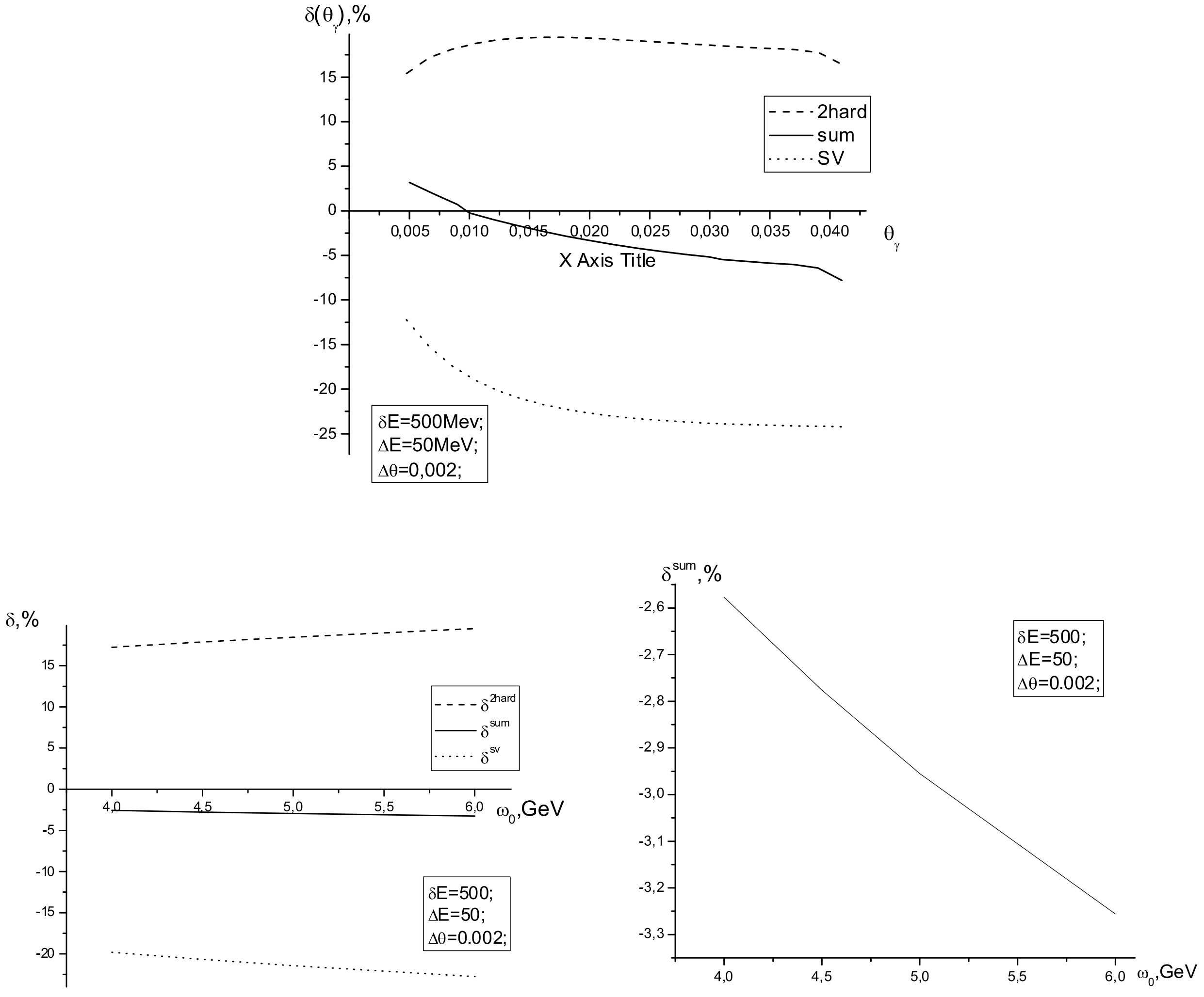

δE=500Mev;
Δθ=0,002;
δ(θγ),%
2hard
sum
SV
θγ
δE=500MeV;
Δθ=0.002;
δ,%
δ^2hard
δ^sum
δ^SV
ω0,GeV
δ^sum,%
δE=500MeV;
Δθ=0.002;
ω0,GeV

Figure 10:



δ(θγ),%
2hard
sum
SV
X Axis Title
θγ
δE=500Mev;
ΔE=50MeV;
Δθ=0,002;
δ,%
δ2hard
δsum
δsv
ω0,GeV
δE=500;
ΔE=50;
Δθ=0.002;
δsum,%
δE=500;
ΔE=50;
Δθ=0.002;
ω0,GeV

Figure 11:

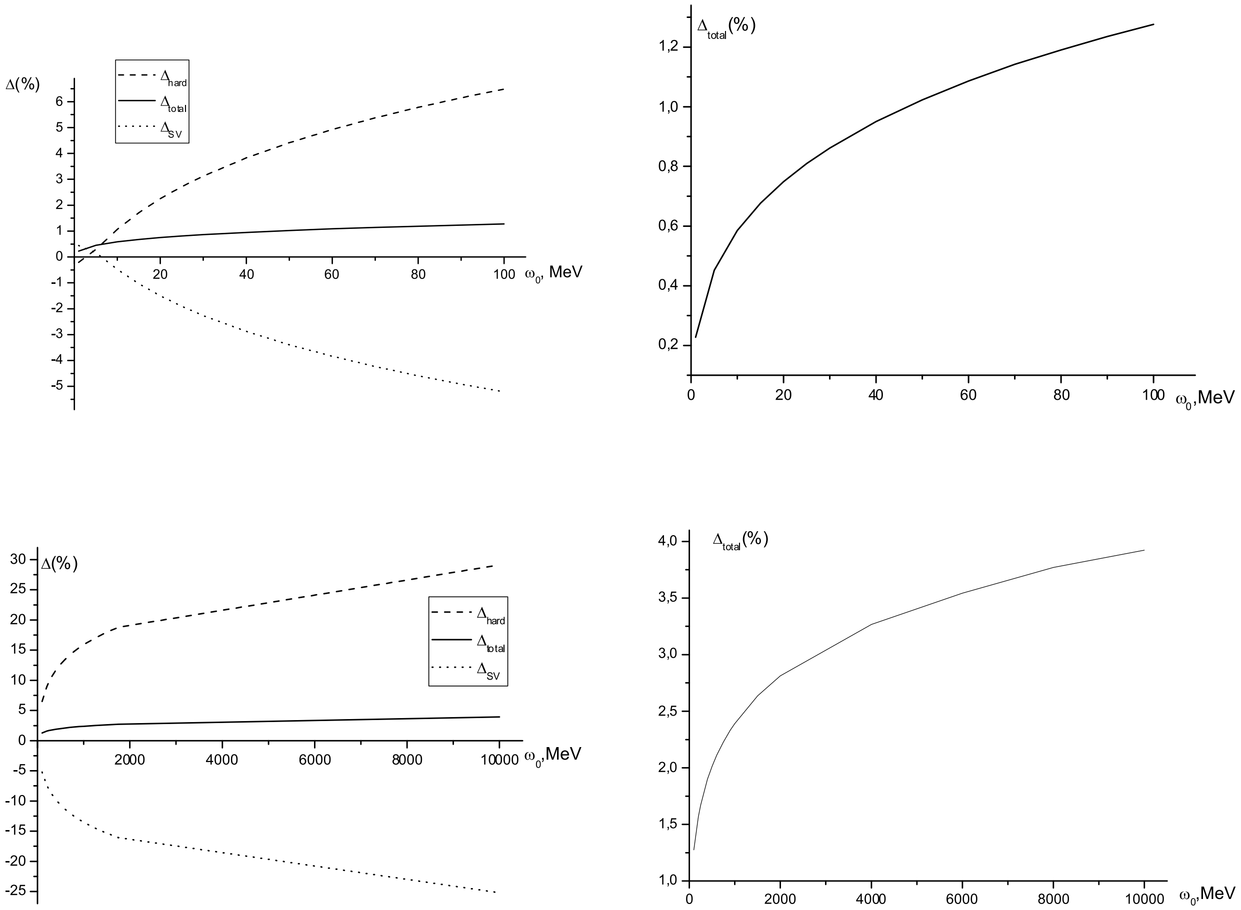


Figure 12:

0.6 Total cross section for double Compton effect.

In order to obtain the total cross for double Compton scattering in which both emitted photons are hard, we integrate the Eq.(7) over variables $\theta_1, \theta_2, \varphi, \omega_1$.

$$d\sigma^t_{real,hard} = \frac{1}{2}\alpha r_0^2 \frac{1}{8\pi}\int_{-1}^{1} d(\cos\theta_2)\int_0^{2\pi} d\varphi \int_{\omega_{2max}}^{\omega_{1max}} d\omega_1 \frac{\omega_1\omega_2}{m\omega_0}\frac{X}{T_1} \tag{18}$$

In Eq.(18)before to make the numerically integration needed to separate the infrared singularities and on the low and on the upper limits the integration over ω_1. Results of numerically calculation for total cross section doudle Compton scattering is shown in Fig.12.

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