# Measurement of Compton scattering cross section 

 at a few GeV electron energy
## By

Li Ye

A Dissertation<br>Submitted to the Faculty of Mississippi State University in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Nuclear Physics in the Department of Physics and Astronomy

Mississippi State, Mississippi

August 2018

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2018

# Measurement of Compton scattering cross section 

at a few GeV electron energy

## By

Li Ye
Approved:

Dipangkar Dutta
(Major Professor)

| James A. Dunne <br> (Committee Member) |
| :---: |
| Jeff A. Winger <br> (Committee Member) |
| Lamiaa El Fassi |
| (Committee Member) |
| (Committee Member) |

Hendrik F. Arnoldus
(Graduate Coordinator)

Jason M. Keith
Dean
Bagley College of Engineering

Name: Li Ye

Date of Degree: August 10, 2018
Institution: Mississippi State University

Major Field: Nuclear Physics
Major Professor: Dr. Dipangkar Dutta
Title of Study: Measurement of Compton scattering cross section at a few GeV electron energy

Pages of Study: 182

Candidate for Degree of Doctor of Philosophy

Electron Compton scattering is the best known fundamental QED process, however, a precision measurement of its cross section for a beam energy above 1 GeV has been lacking up to now. An updated high precision measurement of the neutral pion lifetime via the Primakoff effect (PrimEx-II) experiment was performed in Hall B of Jefferson Lab in 2010. The experiment used small angle coherent photoproduction of $\pi^{0}$,s in the Coulomb field of a nucleus, i.e. the Primakoff effect, to determine the lifetime with a precision of less than $1.5 \%$. It therefore requires thorough understanding of the underlying systematic uncertainties. To facilitate that data for well known electromagnetic processes were taken concurrently with the photoproduction data. This analysis pertains to measuring the Compton scattering cross section, which occurs with similar kinematics to the primary process. The combination of the well established theory for this process with large collected statistics allowed to extract this cross section with high precision in an energy region of $4-5 \mathrm{GeV}$ for ${ }^{12} \mathrm{C}$ and ${ }^{28} \mathrm{Si}$ targets. The results of this analysis will be presented.

## DEDICATION

This thesis is dedicated to my wife, Lei Xing who has always been supporting and encouraging me in the past six years, without her bibi, I would never go this far. To my parents, Fei-jun Ye and Bao-hua Wang, who give me unconditional love. Also to my little girl, Elaine Ye, who brings me so much happiness and energy to the tough Ph.D life.

## ACKNOWLEDGEMENTS

Firstly, I would like to express my sincere gratitude to my advisor Prof.Dr. Dipangkar Dutta for his continuous support of my Ph.D study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in the whole research and writing of this thesis. I am so lucky to have him as my Ph.D advisor and mentor.

My sincere thanks also goes to Prof. Ashot Gasparian and Prof. Liping Gan, who provided me a great opportunity to join in their team, to learn from and work with them. Without their support, it would not be possible to conduct this research.

I also want to thank Dr. Ilia Larin, Dr. Pawel Ambrozewicz, Dr. Yang Zhang, Dr. Maxime Levillain, who helped me get started with data analysis and always be willing to answer my questions. I had a great time to work with them.

Last but not the least, I would like to thank my parents and my wife for making me a better man.

This work is supported in part by the U.S. Department of Energy under Contacts No. DE-FG02-03ER41528, NSF MRI PHY-0079840, Thomas Jefferson National Laboratory and PrimEx collaboration

I thank my committee for their comments on this dissertation, and I thank Dr. Dipangkar Dutta for directing this research.

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## CHAPTER I

## INTRODUCTION

### 1.1 Physics Motivation

Compton scattering, discovered by Arthur Holly Compton, is the scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton effect. Part of the energy of the photon is transferred to the recoiling electron [49]. Electron Compton scattering is one of the most fundamental and the best understood reactions in QED. In the lab frame, we can write the four-momenta of the initial and final states of the photon [16]:

$$
\begin{gather*}
\boldsymbol{K}=(k, 0,0,-k)  \tag{1.1}\\
\overrightarrow{k^{\prime}}=\left(k^{\prime}, k^{\prime} \sin \theta_{\gamma}, 0, k^{\prime} \cos \theta_{\gamma}\right) \tag{1.2}
\end{gather*}
$$

where, $k$ is the incident photon momentum (energy), $k^{\prime}$ is the scattered photon momentum (energy), and $\theta_{\gamma}$ is the scattered photon angle. Similarly, we can write the four-momenta of the initial and final states of the electron:

$$
\begin{gather*}
\boldsymbol{P}=(E, 0,0, p)  \tag{1.3}\\
\overrightarrow{p^{\prime}}=\left(E^{\prime}, p^{\prime} \sin \theta_{e}, 0, p^{\prime} \cos \theta_{e}\right) \tag{1.4}
\end{gather*}
$$

where, $E$ is the incident electron energy, $p$ is the incident electron momentum, $E^{\prime}$ is the scattered electron energy, $p^{\prime}$ is the scattered electron momentum, and $\theta_{e}$ is the scattered electron angle.

Using conservation of momentum and energy, the relationship for scattered photon energy in terms of the incident electron and photon energy can be written as:

$$
\begin{equation*}
k^{\prime}=k \frac{E+p}{E+k+(k-p) \cos \theta_{\gamma}} \tag{1.5}
\end{equation*}
$$

If we define a kinematic parameter $\alpha$ as:

$$
\begin{equation*}
\alpha=\left(1+\frac{4 k E}{m^{2}}\right)^{-1} \tag{1.6}
\end{equation*}
$$

and use the ultra relativistic approximation: $p \simeq E\left(1-\frac{1}{2 \gamma^{2}}\right)$, where $\gamma$ is the Lorentz factor $\gamma=1 / \sqrt{1-v^{2} / c^{2}}$, we can further simplify the Eq. (1.5) to [7]:

$$
\begin{equation*}
\frac{k^{\prime}}{k}=\frac{4 \alpha \gamma^{2}}{1+\alpha \theta_{\gamma}^{2} \gamma^{2}} \tag{1.7}
\end{equation*}
$$

One can see from Eq. (1.7), that the momentum of the scattered photon is directly related to the scattering angle, and the maximum momentum transfer between electron and photon will happen for a completely backscattered photon $\left(\theta_{\gamma}=0\right)$. This kinematic limit is called the Compton edge, and we can write the maximum scattered photon energy:

$$
\begin{equation*}
k_{\max }^{\prime}=4 \alpha k \frac{E^{2}}{m^{2}} \tag{1.8}
\end{equation*}
$$

Then the maximum scattered photon energy $k_{\max }^{\prime}$ corresponds to the minimum scattered electron energy:

$$
\begin{equation*}
E_{\min } \simeq E-4 \alpha k \frac{E^{2}}{m^{2}} \tag{1.9}
\end{equation*}
$$

We can also calculate the Compton scattering cross section in QED. The leading order Feynman diagrams (Figure 1.1) were first calculated by Klein and Nishina in 1929 [30].


Figure 1.1
The lowest-order Feynman diagrams for single Compton scattering.

The Klein-Nishina formula for the differential cross section in the laboratory frame is given by:

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{r_{e}^{2}}{2} \frac{1}{\left[1+\gamma\left(1-\cos \theta_{\gamma}\right)\right]^{2}}\left[1+\cos ^{2} \theta_{\gamma}+\frac{\gamma^{2}\left(1-\cos \theta_{\gamma}\right)^{2}}{1+\gamma\left(1-\cos \theta_{\gamma}\right)}\right] \tag{1.10}
\end{equation*}
$$

Later higher order corrections including radiative corrections and double Compton scattering were calculated by Mandl and Skyrme [40]. Figure 1.2 shows Feynman diagrams of these two processes. The interference between the lowest-order single Compton scattering amplitude and the higher order corrections have been studied in the literature [3][14][40][43][45]. These studies suggested that the higher order corrections to the lowest-order Klein-Nishina formula should be about $5-10 \%$ when the beam energy is larger than 1 GeV [24]. Figure 1.3 shows the magnitude of these corrections as a function
of electron energy. More detailed calculations and formulas are described in Chapter 3 (Theoretical calculations and simulations).

(a)


Figure 1.2
Typical (a) radiative correction and (b) double Compton scattering contributions to single Compton scattering.

Most experiments have been performed in the energy region below 0.1 GeV and only a few experiments are in the energy range $0.1-1.0 \mathrm{GeV}$ with an accuracy of 10 to $15 \%$ [4][23][24][34]. Only one experiment [24] measured the Compton scattering total cross section up to 5.0 GeV using a bubble chamber detection method but with 20 $70 \%$ uncertainty in the above 1 GeV region. Therefore, the higher order corrections to the Klein-Nishina formula have never been tested experimentally. The PrimEx-II experimental measured the Compton scattering cross section in the $4.0-5.7 \mathrm{GeV}$ region in the forward angles with a projected accuracy of $\sim 2 \%$ which will fill this important gap in experimental knowledge.


Figure 1.3
Radiative corrections(dashed curve), double-scattering contributions(dotted curve), and the total percentage correction (solid curve) to the Klein-Nishina formula (horizontal solid curve) [24].

In addition, the PrimEx-II experiment aimed to perform a $1.5 \%$ level measurement of the absolute cross section for the photo-production of neutral pions in the Coulomb field of a nucleus as a test of Chiral Perturbation Theory. Such a high precision relies on good control of systematic uncertainties. The Compton experiment has the same experimental set up as the PrimEx-II experiment, hence by measuring the Compton scattering cross section, we can validate the systematic error for PrimEx-II at the few percentage level.

### 1.2 Measurement of the $\pi^{0}$ lifetime

### 1.2.1 Introduction

The measurement of the two photon decay width of the neutral pion $\left(\Gamma_{\pi^{0} \rightarrow \gamma \gamma}\right)$ is a stringent test of the predictions of the $\mathrm{U}(1)$ axial anomaly in quantum chromodynamics. In QCD, there are several observable phenomena that originate from anomalies. One is connected with the couplings of the quarks to the gluons. This is the so called axial anomaly by which the conservation of the axial $\mathrm{U}(1)$ symmetry of the classical Lagrangian of QCD is broken even in the limit where two or more quarks are massless, and the so called anomalous divergence of the corresponding axial-vector current becomes proportional to the product $\overrightarrow{E^{a}} \cdot \overrightarrow{B^{a}}$ of the chromo-electric and chromo-magnetic fields. The axial anomaly of interest to us involves the corresponding coupling of the quarks to photons [1][10]. In the limit of exact isospin symmetry, the $\pi^{0}$ couples only to the isotriplet axial-vector current $\bar{q} I_{3} \gamma_{\mu} \gamma_{5} q$, where $q=(u, d), \gamma_{\mu}$ and $\gamma_{5}$ are the Dirac matrices, and $I_{3}$ is the third isospin generator. If we limit ourselves to two quark flavors, the electromagnetic current is given by $\bar{q}\left(1 / 6+I_{3} / 2\right) \gamma_{\mu} q$. When coupling to the photon, the isosinglet and isotriplet components of the electromagnetic current lead to an anomaly that explicitly breaks the symmetry associated with the axial-vector current $\bar{q} I_{3} \gamma_{\mu} \gamma_{5} q$, and this in turn directly affects the coupling of the $\pi^{0}$ to two photons.

In the limit of vanishing quark masses, the anomaly leads to the $\pi^{0} \rightarrow \gamma \gamma$ decay amplitude [10]:

$$
\begin{equation*}
A^{\prime}\left(\pi^{0} \rightarrow \gamma \gamma\right)=\frac{\alpha_{e m}}{4 \pi F_{\pi}} \epsilon_{\mu v \rho \sigma} k^{\mu} k^{\prime v} \epsilon^{* \rho} \epsilon^{* \sigma} \tag{1.11}
\end{equation*}
$$

or the reduced amplitude:

$$
\begin{equation*}
A_{\gamma \gamma}=\frac{\alpha_{e m}}{\pi F_{\pi}}=(2.513 \pm 0.007) \times 10^{-2} G e V^{-1} \tag{1.12}
\end{equation*}
$$

where, $F_{\pi}=(92.42 \pm 0.25) \mathrm{MeV}$ [8] is the pion decay constant, $\alpha_{e m}$ is the fine structure constant, $\epsilon_{\mu v \rho \sigma}$ is the four-dimensional Levi-Cevita symbol and $k$ and $\epsilon$ are respectively photon momenta and polarization vectors.

The width of the $\pi^{0} \rightarrow \gamma \gamma$ decay predicted by this amplitude is [8]:

$$
\begin{equation*}
\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=M_{\pi^{0}}^{3} \frac{\left|A_{\gamma \gamma}\right|^{2}}{64 \pi}=7.725 \pm 0.044 \mathrm{eV} \tag{1.13}
\end{equation*}
$$

with a $0.6 \%$ uncertainty due to the experimental error in $F_{\pi}$. The crucial aspect of this expression is that it has no free parameters that need to be determined phenomenologically. In addition, since the mass of the $\pi^{0}$ is the smallest in the hadron spectrum, higher order corrections to this prediction are small and can be calculated with a sub-percent accuracy.

For an unpolarized photon beam, the $\pi^{0}$ cross section via Primakoff effect is given by [11]:

$$
\begin{equation*}
\frac{d \sigma_{p}}{d \Omega}=\Gamma_{\pi^{0} \rightarrow \gamma \gamma} \frac{8 \alpha_{e m} Z^{2}}{M_{\pi^{0}}^{3}} \frac{\beta^{3} E^{4}}{Q^{4}}\left|F_{e . m .}(Q)\right|^{2} \sin ^{2} \theta_{\pi} \tag{1.14}
\end{equation*}
$$

where $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}$ is $\pi^{0}$ decay width, $Z$ is the atomic number of the target nucleus, $M_{\pi^{0}}, \beta$ and $\theta_{\pi}$ are the mass, the velocity and the angle of the $\pi^{0}$, respectively, $E$ is the incident photon energy, $Q$ is the momentum transfer, and $F_{\text {e.m. }}$ is the electromagnetic form factor of the nucleus.

The $\pi^{0}$ photon-production in the few GeV energy region not only comes from the Primakoff effect, but also comes from the nuclear coherent effect, incoherent contribution and interference contribution. These background contributions must be taken into account to properly identify and subtract the Primakoff peak. The full cross section for $\pi^{0}$ photonproduction in the forward direction(up to $\sim 3-4^{\circ}$ ) is given by [5]:

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{d \sigma_{P}}{d \Omega}+\frac{d \sigma_{C}}{d \Omega}+\frac{d \sigma_{I}}{d \Omega}+2 \sqrt{\frac{d \sigma_{P}}{d \Omega} \frac{d \sigma_{C}}{d \Omega}} \cos \left(\phi_{1}+\phi_{2}\right) \tag{1.15}
\end{equation*}
$$

Here the Primakoff cross section, $\frac{d \sigma_{P}}{d \Omega}$, is given by Eq. (1.2). The nuclear coherent cross section $\frac{d \sigma_{C}}{d \Omega}$ is given by [5] [20][22]:

$$
\begin{equation*}
\frac{d \sigma_{C}}{d \Omega}=C A^{2}\left|F_{N}(Q)\right|^{2} \sin ^{2} \theta_{\pi^{0}} \tag{1.16}
\end{equation*}
$$

and $\frac{d \sigma_{I}}{d \Omega}$ is the incoherent cross section [21][25]:

$$
\begin{equation*}
\frac{d \sigma_{I}}{d \Omega}=\xi A(1-G(Q)) \frac{d \sigma_{H}}{d \Omega} \tag{1.17}
\end{equation*}
$$

where A is the nucleon number, $F_{N}(Q)$ is the form factor for the distribution of nuclear matter(corrected for pion final state interactions), the factor $C \sin ^{2} \theta_{\pi^{0}}$ in Eq. (1.16) is the square of the spin and isospin independent part of the $\pi^{0}$ photon-production amplitude on a single nucleon, $\xi$ is the absorption factor for incoherently produced pions, $(1-G(Q))$ is a suppression factor which reduces the cross section at small momentum transfer due to the Pauli exclusion principle, and $\frac{d \sigma_{H}}{d \Omega}$ is the $\pi^{0}$ photo-production cross section on a single nucleon. The relative phase between the Primakoff and nuclear coherent amplitudes without final state interactions is given by $\phi_{1}$, and the phase shift of the outgoing pion due to final state interactions is given by $\phi_{2}$.


Figure 1.4
Angular behavior of the electromagnetic and nuclear $\pi^{0}$ photo-production cross sections for ${ }^{12} \mathrm{C}$ in the forward direction.

One can separate the Primakoff effect from other photo-pion production mechanisms via kinematical considerations. The Primakoff cross section is zero in the forward incident photon direction, it has a sharp maximum at an angle $\theta_{\pi} \sim m_{\pi^{0}}^{2} / 2 E_{\pi}^{2}$, and falls rapidly to zero at larger angles. It is proportional to $Z^{2}$, and its peak value is roughly proportional to $E^{4}$ [5]. The nuclear coherent cross section for spin zero nuclei is also zero in the forward direction, but has a broad maximum at larger angular region than the Primakoff effect, and falls at larger angles as shown in Figure 1.4, where the amplitudes are normalized to the previous data [13], and distortion effects are included. The angular dependence of the Primakoff signal is different from the background processes, allowing $\Gamma\left(\pi^{0} \rightarrow \gamma \gamma\right)$ to be extracted from a fit to the angular distribution of photo-produced $\pi^{0}$. Measurements of the nuclear effects at larger angles are necessary to determine the unknown parameters in the production mechanism and thus make an empirical determination of the nuclear contribution in the Primakoff peak region. Consequently, any experimental measurement of the $\pi^{0}$ lifetime requires a $\pi^{0}$ detector with good angular resolution to eliminate nuclear coherent production, and good energy resolution in the decay photon detection to enable an invariant mass cut to suppress multi-photon backgrounds [5].

### 1.2.2 Previous Experiments

In the past, three experimental methods have been used to measure the $\pi^{0}$ decay width with varying degrees of success: the Direct Method, $\gamma * \gamma *$-collisions and the Primakoff method.
(1) Direct Method: Measuring the distance between the $\pi^{0}$ production and its decay points. This method was used at the CERN PS in 1963 and reached a precision of $17 \%$ [18]. In 1985, an improved version of this technique was used at the CERN SPS with a beam of 450 GeV photons incident on a tungsten foil to generate relativistic pions and a second tungsten foil was used to convert the photons from $\pi^{0} \rightarrow \gamma \gamma$ decay into electronposition pairs. They reported the neutral pion decay width result of $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=(7.34 \pm$ $0.18 \pm 0.11) \mathrm{eV}[6]$.
(2) $\gamma * \gamma *$ collisions: The neutral pions were generated in electron-positron collisions, i.e., $e^{+} e^{-} \rightarrow e^{+} e^{-} \gamma^{*} \gamma^{*} \rightarrow e^{+} e^{-} \pi^{0} \rightarrow e^{+} e^{-} \gamma \gamma$. A result has been published in 1988 by a group from DESY. The Crystal Ball collaboration reported a neutral pion decay width of $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=7.7 \pm 0.5 \pm 0.5 \mathrm{eV}[50]$.
(3) The Primakoff method: Measuring the cross section for the Primakoff process to obtain the pion decay width. Numbers of experiments were made using this method. In 1974 Browman measured the cross-section for the Primakoff process on several nuclei, with a bremsstrahlung photon beam of energies 4.4 and 6.6 GeV at Cornell, obtaining a pion decay width of $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=(8.02 \pm 0.42) \mathrm{eV}$ [13]. Groups from DESY [11] and Tomsk
[33] used $1.1 \mathrm{GeV}, 1.5 \mathrm{GeV}$ and 2.0 GeV bremsstrahlung photon beams obtaining pion decay widths of $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=11.7 \pm 1.2 \mathrm{eV}$ and $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=(7.32 \pm 0.5) \mathrm{eV}$, respectively.

In conclusion, the current average experimental value is $7.84 \pm 0.56 \mathrm{eV}$ and is in good agreement with the predicted value with the chiral limit amplitude. The error of $7 \%$ quoted by the Particle Data Book (PDG) [8] is most likely too low since each of the quoted experiments appears to have understated their errors and also, from the much larger dispersion between the different measurements. Even at the $7 \%$ level, the accuracy is not sufficient for a test of such a fundamental quantity, and in particular for the new calculations which take the finite quark masses into account.

### 1.2.3 The PrimEx Experiments

There are two experiments at JLab which have been performed using the Primakoff effect to measure the $\pi^{0}$ decay width, PrimEx-I and PrimEx-II. The first experimental data set (PrimEx-I) was collected in 2004 at JLab Hall B for incident photon energies of 4.9-5.5 GeV , the experiment measured $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}=7.82 \pm 0.14$ (stat.) $\pm 0.17$ (syst.) eV [36], with $2.8 \%$ total uncertainty. The result was a factor of 2.5 more precise than the PDG average of this fundamental quantity and it was consistent with theoretical predictions.

Compared with the previous experiments (introduced in previous sub-section) using the same method (measure the $\Gamma_{\pi^{0} \rightarrow \gamma \gamma}$ with Primakoff effect), the experimental equipment and technology of PrimEx-I experiment have been greatly improved, thanks to the rapid development of accelerator and detector technologies in recent years. As we know, the
tagging technique could enable a significantly more accurate knowledge of the photon flux, for the PrimEx-I experiment the photon flux uncertainty was about $1 \%$. Furthermore, the photon flux uncertainty was the main contribution of the total uncertainties for the previous experiment that used the same method, therefore this $1 \%$ level of accuracy for photon flux was the high light in experimental progress. In addition, PrimEx-I collaboration designed and built a high-energy and high-resolution electromagnetic calorimeter (HyCal) to detect the two decay photons from neutral pion decay and get the cross section for this process. The PrimEx-I experiment used $5 \%$ radiation length ${ }^{12} \mathrm{C}$ and ${ }^{208} \mathrm{~Pb}$ targets to measure the decay width of $\pi^{0}$.

To further improve the experimental precision, the PrimEx collaboration performed the second experiment (PrimEx-II) in 2010 at JLab Hall B. There were several improvements made for PrimEx-II based on PrimEx-I experience, which include:
(1) The photon beam energy range expanded to $4.4-5.3 \mathrm{GeV}$;
(2) The cross section of the reaction is proportional to the square of the atomic number of the target nucleus, and the cross section for $\pi^{0}$ production from other processes (the physical background) decreases with increasing the atomic number, the PrimEx-II experiment used a ${ }^{28} \mathrm{Si}$ target instead of a ${ }^{208} \mathrm{~Pb}$ target to reduce the systematic uncertainty;
(3) The thickness of the target increased from $5 \%$ to $10 \%$ radiation length to reduce the statistical uncertainty.

The challenging goal of the PrimEx collaboration, to measure the neutral pion lifetime with a precision of $\sim 1.4 \%$, imposed an absolute normalization of the cross-section with an unprecedented precision of $1 \%$ or better. Table 1.1 lists the major contributions to
the projected error for the PrimEx-II experiment. As one can see, the total uncertainty is dominated by the $1 \%$ photon flux uncertainty. Therefore, it is very important to verify this systematic uncertainty by using another approach, which is the measurement of the Compton scattering cross section with the same detector settings.

Table 1.1
Summary of major contributions to the projected experimental error for PrimEx-II.

| Statistical | $0.40 \%$ |
| :---: | :---: |
| Target thickness | $0.70 \%$ |
| Photon flux | $1.00 \%$ |
| $\pi^{0}$ detector acceptance | $0.40 \%$ |
| Background subtraction | $0.20 \%$ |
| Beam energy | $0.10 \%$ |
| Distorted form factor calibration errors | $0.40 \%$ |
| TOTAL ERROR | $1.40 \%$ |

The work presented in this dissertation describes the systematic uncertainty verification procedure for the PrimEx-II experiment, which is accomplished by measuring the absolute cross section for a well known QED process, Compton scattering, thus achieving the $1 \%$ level precision required.

### 1.3 The Compton cross section measurement in PrimEx-II Experiment

During the PrimEx-II experiment, a few Compton runs were also performed. For the Compton runs, all the experimental setup was kept the same as the PrimEx-II experiment except for the dipole magnet being turned off. The PrimEx-II experiment detected two photons from pion decay and used the dipole magnet to bend the background charged particles out of the HyCal acceptance. The dipole magnet was turned off in the Compton measurement to detect both scattered electrons and photons. Both measurements used the same incident photon beam energy range from 4.4 GeV to 5.3 GeV , the new ${ }^{12} \mathrm{C}$-II and ${ }^{28}$ Si targets, and the ${ }^{12} \mathrm{C}$-I target that was used in the PrimEx-I experiment.

Figure 1.5 shows an example of a single Compton event that was detected in the Hybrid Calorimeter (HyCal). One incident photon (with initial energy $E_{0}$ and momentum $\vec{k}_{0}$ ) interacts with an electron in the target (with initial energy $m_{e}$ and initially at rest), then the scattered photon (with final energy $E_{\gamma}^{\prime}$ and momentum $\vec{k}^{\prime}$ ) and electron (with final energy $E_{e}^{\prime}$ and momentum $\vec{p}$ ) hit the calorimeter ( HyCal ) modules. Certain modules of HyCal around each hit point will be fired, and we have a so called "cluster"(defined in Section (2.6)) for each hit point, then each fired module will generate the output signal to further reconstruct the energy and position of the detected cluster using "Island" algorithm described in Section (2.6).


Figure 1.5
Detection of a single Compton event in HyCal.


Figure 1.6
Kinematics of Compton scattering process.

Kinematics for the processes of interest is shown in Figure 1.6, where $m_{e}$ is the electron mass and

- $k=\left(E_{0}, \overrightarrow{k_{0}}\right)$ is the 4-momentum vector of the incident photon,
- $k^{\prime}=\left(E_{\gamma}^{\prime}, \overrightarrow{k^{\prime}}\right)$ is the 4-momentum vector of the emergent photon,
- $p_{0}=\left(m_{e}, \overrightarrow{0}\right)$ is the 4-momentum vector of the atomic electron,
- $p=\left(E_{e}^{\prime}, \vec{p}\right)$ is the 4-momentum vector of the outgoing electron.

The incident photon energy $E_{0}$, the scattered photon angle with respect to the incident photon direction, $\theta_{\gamma}$, determines uniquely the kinematics of a Compton event, where the scattered photon energy is given by:

$$
\begin{equation*}
E_{\gamma}^{c a l}=\frac{E_{0}}{1+\frac{E_{0}}{m_{e}}\left(1-\cos \left(\theta_{\gamma}\right)\right)} \tag{1.18}
\end{equation*}
$$

and the scattered electron energy is defined as:

$$
\begin{equation*}
E_{e}^{c a l}=\frac{E_{0}+m_{e}}{1+\frac{E_{0}}{m_{e}}\left(1-\cos \left(\theta_{e}\right)\right)} \tag{1.19}
\end{equation*}
$$

where $E_{0}$ is the incident photon energy as measured by the Hall-B photon tagger(TAG)(see section (2.3)) system, and $\theta_{i}(i=\gamma, e)$ are calculated as:

$$
\begin{equation*}
\theta_{i}=\arctan \left(\frac{\sqrt{x_{i}^{2}+y_{i}^{2}}}{z}\right) \tag{1.20}
\end{equation*}
$$

where $x_{i}, y_{i}$ are the measured cluster position, $z$ is the distance between HyCal and the target. In this experiment the $z$ position was 706 cm and $E_{0}$ varied from 4.4 to 5.4 GeV while the small electron mass ( $\mathrm{me} \sim 0.511 \times 10^{-3} \mathrm{GeV}$ ) was ignored. So we can calculate the scattered electron and photon energy using Eq. (1.18) for both case. Since the HyCal can not identify an electron from a photon, and we used the same equation to calculate the
energy, we just labeled the calculated energy as $E_{1}^{\text {cal }}$ and $E_{2}^{\text {cal }}$ for cluster-1 and cluster-2, respectively. This calculation error was found to be less than $0.02 \%$. The calculated cluster energy $E_{1}^{c a l}$ and $E_{2}^{c a l}$ were then used to select Compton scattering events.

There are 6 variables that we can use to select Compton scattering events,

1. Cluster's positions: $x_{i}, y_{i},(\mathrm{i}=1,2)$,
2. $\Delta T$ : time difference between the HyCal trigger and the Master Or (MOR) trigger (see Section (3.1)),

$$
\begin{equation*}
\Delta T=T_{M O R}-T_{H y C a l} \tag{1.21}
\end{equation*}
$$

3. $R_{\text {min }}$ : cluster separation, the distance between two clusters on HyCal ,

$$
\begin{equation*}
R_{\text {min }}=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}} \tag{1.22}
\end{equation*}
$$

4. $\Delta \phi$ : azimuthal angle difference of two clusters on HyCal ,

$$
\begin{equation*}
\Delta \phi=\left|\phi_{1}-\phi_{2}\right| \tag{1.23}
\end{equation*}
$$

where $\phi_{1}, \phi_{2}$ are the azimuthal angles for the cluster- 1 and the cluster- 2 , respectively,
5. $\Delta E$ : elasticity, the difference between photon beam energy and the sum of the cluster's energy on HyCal,

$$
\begin{equation*}
\Delta E=E_{0}-\left(E_{1}+E_{2}\right) \tag{1.24}
\end{equation*}
$$

where $E_{0}$ is the photon beam energy, $E_{1}, E_{2}$ are the reconstructed cluster energies for the cluster- 1 and the cluster- 2 , respectively,
6. $\Delta K$ : kinematic energy difference, the difference between the sum of the calculated cluster energy and photon beam energy.

$$
\begin{equation*}
\Delta K=\left(E_{1}^{c a l}+E_{2}^{c a l}\right)-E_{0} \tag{1.25}
\end{equation*}
$$

where $E_{1}^{c a l}, E_{2}^{c a l}$ are the calculated cluster energies for the cluster-1 and the cluster-2, respectively, using Eq. (1.18).

After applying appropriate cuts on these six variables, the $\Delta K$ distribution was used to extract the Compton events. The main contributions of the background were identified as accidental events and pair production events. Even though most of the pair production background pass through the central hole of the HyCal , still some of the pair production
events would pass the cuts and get into our region of interest due to multiple scattering. Hence, to extract the measured Compton yield the data were fitted with a weighted sum of the simulated Compton events, simulated pair production events and the accidental events. The total Compton scattering cross section over all energies can then be extracted from the measured yield using:

$$
\begin{equation*}
\sigma=\frac{1}{n_{e} \Gamma_{\gamma}} \frac{N_{\text {Compton }}}{A} \tag{1.26}
\end{equation*}
$$

where,

- $n_{e} \Gamma_{\gamma}$ is the integrated experimental luminosity, where $n_{e}$ is the number of electrons per $\mathrm{cm}^{2}$ and $\Gamma_{\gamma}$ is the experimental photon flux (there was a blind number added into flux which was only un-blinded after the whole analysis process being done).
- $N_{\text {Compton }}$ is the extracted Compton yield.
- $A$ is the simulated acceptance factor.

The percentage of accidental coincidence events, $C_{\text {accidental }}$, in the data sample was estimated using the events in the tails of the time difference $\Delta T$ distribution as

$$
\begin{equation*}
C_{a c c i d e n t a l}=\frac{p^{f i t} \times N^{\text {bins }}}{M^{\text {data }}} \tag{1.27}
\end{equation*}
$$

where $p^{f i t}$ is the the average number of accidental coincidences in each bin, which is obtained by fitting the tails of the $\Delta T$ distribution integrated over the entire $\Delta T$ range, shown by the red line in Figure 1.7, $N^{\text {bins }}$ is the number of bins in the $\Delta T$ cut range $( \pm 6.5 n s)$, and $M^{\text {data }}$ is the total number of events in the $\Delta T$ cut range.

The simulated Compton events were used to calculate the acceptance factor as:

$$
\begin{equation*}
A=\frac{N_{\text {simulation }}}{N_{0}^{\text {generated }}} \tag{1.28}
\end{equation*}
$$

where $N_{\text {simulation }}$ is the number of events reconstructed or accepted in HyCal, while $N_{0}^{\text {generated }}$ is the number of generated events.


Figure 1.7
Time difference distribution for Energy bin 1 for ${ }^{12} \mathrm{C}$-I target.

In summary, this dissertation describes the extraction of the Compton scattering cross sections for carbon and silicon targets. It is the first measurement of the Compton scattering cross section for few GeV photons with percent level precision. The higher order QED corrections to the Klein-Nishina formula is also about a few percent, however previous experimental results at this energy range had too large uncertainties to constrain this theoretical prediction. The high precision results of this experiment will help to verify the high order QED corrections prediction. Moreover, the agreement of the extracted cross sections with the theoretical predictions would refine the systematic uncertainties of the $\pi^{0}$ lifetime measurement experiment (PrimEx-II) to be less than $2 \%$.

## CHAPTER II

## EXPERIMENTAL SETUP

### 2.1 Thomas Jefferson Lab National Accelerator Facility

The Thomas Jefferson National Accelerator Facility (TJNAF), commonly known as Jefferson Lab or JLab, is located in Newport News, Virginia and it is one of 17 national laboratories funded by the U.S. Department of Energy. The laboratory's main research facility is the Continuous Electron Beam Accelerator Facility (CEBAF) accelerator, which consists of a polarized electron source, an injector, and a pair of superconducting Radio Frequency (SRF) linear accelerator (linacs) that are 7/8-mile (1400 m) long connected to each other by two arc sections that contain steering magnets. As the electron beam makes up to five successive passes through the linacs, its energy is increased up to 6 GeV (after the recent upgrade, the beam energy can reach a maximum of 12 GeV ).

The design of CEBAF allows the electron beam to be continuous rather than the pulsed beam that is typical of ring shaped accelerators. (There is some beam structure, but the pulses are very much shorter and closer together) The electron beam can be directed onto three potential targets (see Figure 2.1), located in four experimental halls. As part of the 12 GeV upgrade, the fourth experimental hall was added. One of the distinguishing features of Jefferson Lab is the continuous nature of the electron beam, with a bunch length of less
than 1 picosecond. Another is the Jefferson Lab's use of SRF technology, which uses liquid helium to cool niobium to approximately $4 \mathrm{~K}\left(452.5^{\circ} \mathrm{F}\right)$, removing electrical resistance and allowing the most efficient transfer of energy to electrons. To achieve this, Jefferson Lab houses the world's largest liquid helium refrigerator [28], that is considered as one of the first large-scale implementations of SRF technology. The accelerator is buried 8 meters approximately 25 feet underground, and its tunnels' walls are 2 feet thick.

The beam end's stations are four experimental halls, labeled Hall-A, Hall-B, Hall-C, and Hall-D. Each hall contains specialized spectrometers to record the products of collisions between the parent electron beam or the child tagged bremsstrahlung photons and a stationary target. This allows physicists to study the structure of the atomic nucleus, specifically the interaction of the quarks and gluons that make up protons and neutrons in the nucleus.

In each circulation around the accelerator, the electron beam make up to five passes through the two linacs, and get bent using a different set of bending magnets in semicircular arcs at the ends of the linacs. Then, it gets split and sent to each experimental hall.

When the target' nucleus is hit by an electron beam, an "interaction", or "event", occurs, and the products particles scatter into the hall detection spectrometer. Each hall contains an array of particle detectors that track the physical properties of the particles produced in each event. The detectors generate electrical pulses that are converted into digital values by analog to digital converters (ADCs), time to digital converters (TDCs) and pulse counters (scalers).

The digital data are recorded on-line using the data acquisition system and stored so that the physicist can analyze it off-line to extract the physics observable of their interest.


Figure 2.1
JLab CEBAF accelerator.

### 2.2 The PrimEx-II Experimental Setup

The PrimEx-II experiment was conducted at Jefferson Lab in the experimental Hall-B using the Continuous Wave (CW) electron beam from the CEBAF accelerator. A schematic of the PrimEx-II experimental setup is shown in Figure 2.2. The main elements of the PrimEx-II experiment beam-line are [5]:

1. the Hall-B photon tagger,
2. a collimator,
3. targets,
4. a pair production luminosity monitor,
5. a Helium bag,
6. a hybrid electromagnetic calorimeter containing a high resolution insertion in the central region near the beam,
7. a total absorption lead-glass counter.

The following sections describe in detail the Hall-B photon tagger, the pair production luminosity monitor, the novel electromagnetic hybrid calorimeter and the total absorption counter.

### 2.3 The Hall-B Photon Tagger

The main components of the Hall-B photon tagger were a thin ( $\sim 10^{-4}$ rad. length) bremsstrahlung converter foil (the "radiator"), a dipole magnet capable of producing a full field strength of 1.75 T , and two rows of plastic scintillator hodoscopes acting as "Ecounters" and "T-counters" (energy and timing counters) [46].


Figure 2.2
Schematic of the PrimEx experimental beam-line setup.


Figure 2.3

The overall schematic of the Hall B tagging system. The electron trajectories (red-dashed) are labeled according to the fraction of the incident energy that was transferred to the photon (blue-dashed).

The radiator located on the electron beam-line and is made of a high atomic number ( Z ) material such as Au of $10^{-4}$ radiation length that was used for PrimEx-II experiment. After the electron beam with an initial energy $E_{0}$ (in PrimEx-II $E_{0}=5.76 \mathrm{GeV}$ ) hit the radiator, the electron looses energy in the electromagnetic field of a nucleus and emits an energetic photon. The number of photons with energies from $E_{\gamma}$ to $E_{\gamma}+d E_{\gamma}$ is proportional to the $Z^{2}$ of the radiator but inversely proportional to $E_{\gamma}$ [12]. Neglecting the energy transfer from the electron to the nucleus, due to the relatively small electron mass, and using energy conservation one gets :

$$
\begin{equation*}
E_{\gamma}=E_{0}-E_{e} \tag{2.1}
\end{equation*}
$$

where $E_{\gamma}$ is the energy of the bremsstrahlung photon, and $E_{e}$ is the energy of the scattered electron after radiating. $E_{0}$ is the incident electron energy which is defined by the accelerator. Therefore, one can calculate the photon energy by measuring the scattered electron energy. The photon produced by the radiator travels straight through the tagger toward the target, to serve as a photon beam in the experiment. A tagger dipole magnet is located downstream of the radiator in order to bend the electrons away from the beam-line, its field is set to allow full energy electrons which did not radiate to follow a certain path (red-dashed line in Figure 2.4) toward the shielded beam dump [46]. The electrons which loose energy in the bremsstrahlung process, will get detected by the hodoscope counters (blue-dashed line in Figure 2.4).

There are 384 overlapping 4 mm thick scintillators (called E-counters) that lie along a flat focal plane downstream from the straight edge of the tagger dipole magnet to measure the bent electron energies (see Figure 2.3). The E-counters have a varying width (from

6 mm to 18 mm ) to cover a constant energy interval of about $3 \times 10^{-3} E_{0}$. By using the overlapping E-counters, 767 fine E-channels of width $10^{-3} E_{0}$ are defined via coincidences between adjacent E-counters [46]. There are 61 T -counters with designed timing resolution of 300 ps (the best resolution achieved is 110 ps [46]) located directly under the E-counters' plane to provide the beam bucket the timing information. Each T-counter is 2 cm thick plastic scintillator read out with double sided photomultipliers (PMTs). In order to improve the detection efficiency and to ensure that there are no gaps, all T-counters and E-counters have a geometrical overlapping region. Using these T-counters, 121 non-overlapping Tchannels are defined using an intelligent algorithm identifying coincidence between two adjacent counters. The scheme for T-counters and T-channels is presented in Figure 2.4. In the PrimEx-II experiment, only 93 E-counters (id:1-93) and 19 T-counters (id:1-19) were used because these detectors covered most of the high energy photon beam.


Figure 2.4
Top: Scheme of the T-counter layout, Bottom: Scheme of the coincidence T-channels.

### 2.4 The Targets

There were three different targets used in the PrimEx-II experiment including two carbon, ${ }^{12} \mathrm{C}$-I and ${ }^{12} \mathrm{C}$-II, and one ${ }^{28} \mathrm{Si}$ targets. The ${ }^{12} \mathrm{C}$-II and ${ }^{28} \mathrm{Si}$ targets were used in the $\pi^{0}$ decay width measurement, and all three targets were used for the Compton cross section measurement.
${ }^{12} \mathrm{C}$-I target is a highly ordered/oriented pyrolytic graphite (HOPG) target, which was produced using high temperature chemical vapor deposition (CVD) furnace technology. This process creates atomic layers of carbon oriented to each other in a crystalline form [41]. It has a $0.9662 \pm 0.0001 \mathrm{~cm}$ central thickness and $2.1979 \pm 0.0003 \mathrm{~g} / \mathrm{cm}^{3}$ density, which corresponds to about $5 \%$ radiation length.
${ }^{12} \mathrm{C}$-II target is a combination of "Block\#1" and "Block \#2". The Block\#1 is identical with the ${ }^{12} \mathrm{C}$-I target and Block \#2 is "normal" graphite, with a $0.9417 \pm 0.0001 \mathrm{~cm}$ central thickness and $1.4938 \pm 0.0006 \mathrm{~g} / \mathrm{cm}^{3}$ density. Block\#1 was attached to Block\#2 with mylar tape, and has in total about $8 \%$ radiation length [26].

The ${ }^{28}$ Si target consists of 10 disks, each disk is a mono-crystal silicon semiconductor wafer with 1 mm thickness. It has $1.0015 \pm 0.0003 \mathrm{~cm}$ central thickness and $2.316 \pm$ $0.008 \mathrm{~g} / \mathrm{cm}^{3}$ density, which corresponds to about $10 \%$ radiation length [27].

Table 2.1 lists the detailed information on the three targets used.
Table 2.1

|  | Target ${ }^{12} \mathbf{C}$-I | Target ${ }^{12} \mathbf{C}$-II <br> Block \#1 | Target ${ }^{12} \mathbf{C}$-II <br> Block \#2 | ${\text { Target }{ }^{28} \mathbf{S i}}^{\mid \text {Density, } \rho,\left(\mathrm{g} / \mathrm{cm}^{3}\right)}$ Central Thickness, T, $(\mathrm{cm})$ |
| :--- | :---: | :---: | :---: | :---: |
| 2.1979 | $0.9662 \pm 0.0003$ | $1.4938 \pm 0.0006$ | $2.1979 \pm 0.0003$ | $2.3160 \pm 0.0080$ |
| $\rho \mathbf{T}\left(\mathrm{~g} / \mathrm{cm}^{2}\right)$ | $2.1236 \pm 0.0004$ | $1.4068 \pm 0.0001$ | $0.9662 \pm 0.0001$ | $1.0015 \pm 0.0003$ |
| Fraction Uncertainty in $\rho \mathbf{T}$ | $0.02 \%$ | $0.04 \%$ | $2.1236 \pm 0.0004$ | $2.3195 \pm 0.0080$ |

### 2.5 The Pair Spectrometer

During the PrimEx-II experiment, when the high energy photon beam interacts with the target, not only $\pi^{0}$ s but also $e^{+} e^{-}$pairs were generated. Therefore, a pair spectrometer (PS) was constructed for the purpose of monitoring these $e^{+} e^{-}$pairs and thereby indirectly monitor the relative tagged photon flux. It includes a $1.98 \mathrm{~T} \cdot \mathrm{~m}$ dipole magnet and two symmetric arms on each side of the beam line (left and right). Each arm of the pair spectrometer has two rows of scintillator hodoscopes, and each row has 8 plastic scintillator hodoscopes, for 32 detectors in total. Schematic views of the pair spectrometer are shown in Figure 2.5 and 2.6. The scintillators of the front row are $2.4 \times 7.5 \times 0.5 \mathrm{~cm}^{3}$ in size, and made of BC420 plastic. The back row detectors are $3.1 \times 9.3 \times 2.0 \mathrm{~cm}^{3}$ in size and are also made of BC420 plastic [47]. The thickness of the front row detectors is only onefourth of the back row detectors' to minimize the change in the trajectory of electrons and positrons due to multiple scattering. We used 4 -fold timing coincidence between the two arms of the pair spectrometer (left-front, left-back, right-front, right-back) to define a pair spectrometer event which greatly reduces the rate of accidental coincidences.

The dipole magnet is used as a sweeping magnet, during the $\pi^{0}$ lifetime production data to insure that we do not get charged particle background in the calorimeter. However during the Compton production data taking the dipole magnet was turned off in order to detect the scattered electrons from the Compton scattering process in the calorimeter.


Figure 2.5
Layout of the pair spectrometer.


Figure 2.6
A picture of the pair spectrometer hodoscopes mounted on aluminum frame.

### 2.6 The Hybrid Calorimeter(HyCal)

The energies and positions of the photons and electrons from the Compton scattering process are measured by the HyCal. It has an inner part and a outer part.

The inner part is a $34 \times 34$ array of 1152 lead-tungstate $\left(\mathrm{PbWO}_{4}\right)$ crystals with a $4.15 \times$ $4.15 \mathrm{~cm}^{2}$ central hole left open to let the incident photon beam pass through. Each $\mathrm{PbWO}_{4}$ crystal has a radiation length of 0.89 cm and a Moliere radius of 2.20 cm with dimensions of $2.075 \times 2.075 \times 21.2 \mathrm{~cm}^{3}$, which is wrapped in $63 \mu \mathrm{~m}$ thick light reflective foil (Type VM-2000) to improve the light collection efficiency. One end of each crystal is connected to a Hamamatsu R4125A PMT to collect the light and then transform it into electrical signals.

The outer part is six layers of 576 lead-glass (TF1) modules surrounding the inner crystals, each module has a radiation length of 2.74 cm and a Moliere radius of 4.70 cm with dimensions of $3.815 \times 3.815 \times 34.0 \mathrm{~cm}^{3}$. A photograph of the lead-tungstate and the lead-glass is shown in Figure 2.7.

The HyCal is $119.0 \times 119.0 \mathrm{~cm}^{2}$ in the direction transverse to the beam and was located about 7.06 meters downstream of the target. A photograph of the HyCal is shown in Figure 2.8.

The size of the cross section of each module is governed by its moliere radius, and designed to reconstruct the position of the incident particle based on the lateral spreading of the shower between the modules caused by the incident particle. The central area of the HyCal constructed from $\mathrm{PbWO}_{4}$ crystals has very good energy and position resolutions.


Figure 2.7
$\mathrm{PbWO}_{4}$ crystal (small one) and Lead-Glass module used in HyCal .


Figure 2.8
The $\mathrm{PbWO}_{4}$ inset and the Lead-Glass of HyCal in the frame enclosure.


Figure 2.9
Example of a real event, hot channel and cosmic ray in the HyCal.

When a high energy particle (mostly photons and electrons in this experiment) hits the modules of the HyCal, electromagnetic interactions (pair production) generate a positron and an electron, which due to the Bremsstrahlung effect, generate radiative photons while traveling through the crystals. These photons then pair produce to generate more positrons and electrons, creating a shower of charged particles and photons. This positron-electron and photons production cycle will continue until the particles do not have enough energy to produce each other, then the number of generated photons will be proportional to the incident particle energy. For almost all the case, the number of generated photons has maximum at "central module", which is hit by the incident particle, while decrease gradually at the surrounding modules. By counting the number of generated photons via PMTs installed at the end of module, we can calculate the energy deposition for each fired module. All the fired modules caused by this incident particle consist a "cluster", the cluster energy and the cluster position are the information that we want to get as close to the true values as possible. Ideally, the sum of energy deposition of the fired modules would be the cluster energy, but in reality there are noise signals from all the modules without any generated photon, and we have to set a cutoff value for the ADCs, so the fired modules (ADC value passes the cutoff value) are not necessarily caused by incident particles but by the fluctuations of the noise, the accidental events (cosmic rays for example) or the "hot" channels (always fired due to the incorrect ADC cut off). Figure 2.9 shows the example of these cases and what they look like in the HyCal. The calibration process did not require a very high accuracy of the cluster energy, so we used sum of the energy deposition of the fired modules in a 6 by 6 modules area as the cluster energy during the calibration. Figure 2.10
shows the energy resolution for different combinations of modules for 4.3 GeV incident electrons.

The gains of individual counters were determined by sending a low intensity tagged photon beam directly into the detector as part of a calibration procedure. During the calibration, the calorimeter was installed on a transporter in order to expose every module of the detector, one by one, to the beam (see Figure 2.11). The energy dependent resolution can be described by [9] :

$$
\begin{equation*}
\sigma_{E} / E=p 0+p 1 / \sqrt{E}+p 2 / E \tag{2.2}
\end{equation*}
$$

where $E$ is the energy of the incident photon in GeV . The constant $p 0$ accounts for calibration errors, shower leakage and non-uniformity in light collection efficiency along the length of the crystals. The parameter $p 1$ arises from statistical fluctuations in the electromagnetic shower and photon statistics in the PMT, and the term with the constant $p 2$ is due to the electronics detection noise. The data from the so-called "snake" calibration runs were used to obtain a resolution function for the crystal part of the calorimeter (see Figure 2.12).

The yield of light, produced by scintillation, within the crystal is highly dependent on temperature ( $\sim 2 \% /{ }^{\circ} \mathrm{C}$ ). Therefore, the calorimeter is thermally isolated and surrounded on all four sides by water cooled copper plates in order to stabilize the temperature with a precision of $\pm 0.1^{\circ} \mathrm{C}$.

Energy Resolution


Figure 2.10
$\mathrm{PbWO}_{4}$ crystal module energy resolution for 4.3 GeV incident electrons: single module (left blue), $3 \times 3$ modules (mid green), $6 \times 6$ modules (right red).


Figure 2.11
Schematic view of HyCal on the transporter. The shaded purple region depicts the lead tungstate modules, and the light blue region depicts the lead glass modules.


Figure 2.12
HyCal energy resolution function obtained from "snake" calibration data.

After the calibration, the accuracy of the cluster energy became critical for the data analysis purpose, therefore we used so called "island" method to define a cluster, followed by the following steps
(1) Split the HyCal map into different sectors because the HyCal consisted by two types of modules ( $\mathrm{PbWO}_{4}$ crystal and Lead-glass).
(2) Search for a maxima in the energy deposition in the modules of each sector and form a cluster with all the possible fired modules around maxima for this sector.
(3) Test if a single hit can be split into two close by hits by increasing the ADC cutoff values.
(4) Test if the fired modules from different sectors can be considered as one cluster.
(5) Merge the hits together for different sectors if it satisfy the test in step 4.

A typical example of a cluster on the HyCal defined by "island" method is shown in Figure 2.13. We then used the sum of the module energy deposition of the cluster as the cluster energy and the center of gravity of the cluster as the cluster position.


Figure 2.13
Example of a cluster on the HyCal defined by island method.

### 2.7 Total Absorption Counter

At the beginning of the experiment, the total absorption counter (TAC) consisted of a single $20 \times 20 \times 40 \mathrm{~cm}^{3}$ lead glass block (SF5, $L=17 X_{0}$ ) attached with a single $5 "$ Hamamatsu PMT (R1250, with a rise time of $\sim 2.5 \mathrm{~ns}$ ), and was instrumented with both an ADC and TDC. Later on, because of severe damage to the detector by radiation, we replaced this detector with a single $15 \times 15 \times 35 \mathrm{~cm}^{3}$ lead glass block (TF1). The TAC was mounted on a vertical linear stage behind the HyCal. The vertical stage enabled the placement of the TAC out of the path of the primary beam during high intensity runs.

During the PrimEx-II calibration period (TAC Run), TAC was used to provide the "absolute tagging ratio", $R_{\text {absolute }}$, of each tagged bremsstrahlung photon for each T-counter. $R_{\text {absolute }}$ is the ratio between the number of TAC events that were coincident with a Tcounter $\left(N_{T A C}^{\text {taged }}\right)$ and the number of total T-counter detected events $\left(N_{e}\right)$ :

$$
\begin{equation*}
R_{\text {absolute }}=\frac{N_{T A C}^{\text {tagged }}}{N_{e}} \tag{2.3}
\end{equation*}
$$

This ratio was used to calculate the photon flux incident on the experimental targets. TAC measurements were only performed during the low intensity runs ( $\sim 100 p A$ ), because at low intensity condition, the tagger usually has only one triggered signal and the TAC also detects only one photon event. This situation greatly reduced accidental coincident events between tagger and TAC, so that we can assume the TAC has $100 \%$ detection efficiency. Figure 2.14 shows the absolute tagging ratio of each T-counter for different beam intensi-
ties during TAC the runs. One can see that the ratio $R_{\text {absolute }}<1$. There are three reasons for this ratio being less than one:
(1) part of the Bremsstrahlung photons generated by Tagger were absorbed before reaching the TAC;
(2) at the radiator area of the tagger, some of the electrons due to the Moller scattering were detected by the T-counter but these electrons will not generate any photons;
(3) the background events in Hall B, for example some cosmic events or the scattered electrons from upstream beam-line, could also trigger a T-counter.

During the PrimEx-II experiment production runs, the beam intensity was high ( $\sim$ $100 n A$ ). By setting the $N_{e}$ counts from each T-counter we can calculate the absolute normalization of the photon flux as:

$$
\begin{equation*}
N_{T A C}^{\text {tagged }}=N_{e} \times R_{\text {absolute }} \tag{2.4}
\end{equation*}
$$

where $R_{\text {absolute }}$ is measured during the TAC calibration runs [35].


Figure 2.14
The $R_{\text {absolute }}$ of each T-counter for different beam intensities of TAC runs [35].

## CHAPTER III

## DETECTOR CALIBRATION

The Hall-B tagger and the HyCal are two very important detectors for the PrimEx-II experiment. The tagger provided the energy and the timing information of the incident photon beam, while the HyCal provided the energy and the position information on the photons from $\pi^{0}$ decay. This chapter describes the calibration procedure for the HyCal ADCs, the timing information obtained from the tagger detector, and the position information obtained from the HyCal .

### 3.1 Tagger Timing and Energy Calibration

The tagger construction is shown in Chapter 2, Figure 2.3 on page 27. It consists of 61 "T-counter" timing detectors and 384 "E-counter" energy detectors. The Master Or (MOR) trigger, which is the logical "OR" of the T-counters [15], coincident with the HyCal trigger was used for the event selection during the data analysis. To precisely reconstruct the timing and energy information of incident photon, these detectors need to be well calibrated. All the T-counters and E-counters time was against a signal received from the RF drive of the accelerator [15].

### 3.1.1 Calibration of T-counter Double-sided PMT Signals

As depicted in Figure 3.1, the tagger T-counters are plastic scintillators with a PMT installed at each end, such that each PMT generates a signal when an incident electron passes through the plastic scintillator hodoscope. We identify one event by the coincidence between the signals from the two PMTs. The Tagger timing calibration procedure involves shifting the time difference between the two PMTs such that it is a distribution centered around zero.


Figure 3.1
T-counter with PMTs at each end.

The calibration process in the software is done as follows. First, we get the time difference distribution from the two PMT signals, which is typically a Gaussian distribution, and we find its mean value $X_{L R}$ by fitting the distribution with a Gaussian function. Second, we fix the result for one signal and shift the other signal value by $X_{L R}$, to make the mean value of the difference coincides with zero. Figure 3.2 shows the calibrated T-counters co-
incidence time between its left and right PMTs. The TDC offsets for all T-counters already set in the calibration database of PrimEx-II before the TDC alignment work was started. This TDC alignment procedure aimed to update those TDC offsets [38].

### 3.1.2 Calibration of Coincident Signals from Two Neighboring T-counters

There is $\sim 10 \%$ geometrical overlap between neighboring T-counters as shown in Figure 3.3. After the calibration for each T-counter, the two signals from the two ends of a T-counter are aligned, but the signals between different T-counters may still be misaligned. Therefore, we needed to follow the same procedure to calibrate the timing information for all T-counters. Let's take T-1 and T-2 as an example, first we get the time difference distributions for T-1 and T-2, then we fix the mean value for T-counter which has the smaller id number, then we add the same correction value $X_{\text {adjacent }}$ to both ends of the T-2 counter to make the average value of the difference between T-1 and T-2 coincides with zero. Then, we do the same for all the remaining T-counters and we have all T-counters calibrated (see Figure 3.4).

### 3.1.3 Calibration for Coincident Signals of T-counters and E-counters

The same calibration procedure as described in the previous subsection was used to align the timing difference between the T-counters and the E-counters. The time difference for a calibrated set of E-counters is shown in Figure 3.5


Figure 3.2
Calibrated time difference distribution for T-counters (ps.the ID $=16$ T-counter had issues and was excluded from the data) [38].


Figure 3.3
Electron passing through the overlapping area of adjacent T-counters.


Figure 3.4
Calibrated time difference distributions of all T-counters [38].


Figure 3.5
Calibrated time difference between T-counters and E-counters [38].

### 3.1.4 Energy Calibration for E-counters

The old TAC detector was used for the tagger energy calibration during the TAC runs (see Section (2.7)) and it was pre-calibrated. From previous section (2.3) we know that the radiated electrons followed different path and reached different E-counters, while associate photons carried different energies. By measuring the photon energy $\left(E_{\gamma}\right)$ with the TAC, we can calculate its associated radiated electron energy $\left(E_{e}\right)$ using the equation $E_{e}=E_{0}-E_{\gamma}$, where $E_{0}$ is the incident electron energy defined by the accelerator. Then we known that the E-counter hit by this radiated electron with energy $E_{e}$ corresponds to a photon with energy $E_{\gamma}$. With large statistics, we were able to calibrate all the E-counters hence we known what was the photon energy when a E-counter was hit by its associate radiated electron. The technical details of the calibration procedure can be found in Ref. [15].

### 3.2 HyCal Trigger Timing Alignment

We used total sum of the module's dynode signal as the HyCal trigger signal, the cutoff ADC value was set to be equivalent to 2.5 GeV (HyCal already calibrated), which means the HyCal would be triggered whenever the total sum of the energy deposition exceeded 2.5 GeV. During the calibration runs, the HyCal trigger time aligned with MOR trigger and the observed time difference between the signals of the HyCal trigger and the MOR trigger $\left(T_{\text {HyCal }}-T_{M O R}\right)$ during the calibration runs is shown in Figure 3.6. This time difference also called " $\Delta T$ ", which has already defined in Section (1.3).


Figure 3.6
Time difference between the HyCal trigger and the MOR trigger. The fit to the distribution is performed with a function consisting of a double gaussian and a linear background. The timing resolution is calculated to be 1.2 ns from the fitting parameters.

### 3.3 HyCal ADC Performance Validation

HyCal contains 576 lead-glass modules and 1152 lead-tungsten modules. The end of each module is attached to one PMT, and the signal from the PMT is split into two using a custom made divider circuit. One signal (anode) is read out with an ADC and provides the energy information, while the other (dynode) is read out with a TDC to provide the timing information used to form the trigger. In order to check the performance of each signal during the experiment, we defined two test functions F1 and F2. The test functions were evaluated for each HyCal module and their run-to-run variation was monitored as a measure of detector stability. The test function F1 was used to monitor the anode signal or ADC signal of each HyCal module, and it was defined as:

$$
\begin{equation*}
F 1(i)=\frac{N 1(i)}{\sum_{i=1}^{2156} N 1(i)} \tag{3.1}
\end{equation*}
$$

where, $N 1(i)$ is the number of events where the i-th module collected the largest amount of shower energy compared to the other modules that formed the cluster for that event. For all functional modules, $F 1(i)$ should stay constant with time within statistical variation.

The second test function, F2, was used to monitor the dynode signal or TDC signal of each HyCal module. It was defined as:

$$
\begin{equation*}
F 2(i)=\frac{N 2(i)}{N 1(i)} \tag{3.2}
\end{equation*}
$$

where, $N 2(i)$ has the same definition as $N 1(i)$ but using the dynode signal instead of the anode signal used to evaluate $N 1(i)$. We used PrimEx-II $\pi^{0}$ production runs to test the F1 and F2 stabilities, and we choose the events that have cluster energy larger than 1.5 GeV .

Typically, both test functions for neighboring modules were very similar to each other, and they did not vary from run-to-run, these modules were considered as normal channels. Figure 3.7 shows a example of F1 for module (874) and Figure 3.9 shows a example of F2 for module (1702). Some modules had much lower F1, F2 values than the other modules for all the runs or certain runs as shown in Figure 3.8 and Figure 3.10. These modules were considered as abnormal channels.


Figure 3.7
A example of F1 stability of a normal module (874) [37].


Figure 3.8
A example of F1 stability of a abnormal module (877) [37].


Figure 3.9
A example of F2 stability of a normal module (1702) [37].

## RunNumber_F2[1703]



Figure 3.10
A example of F2 stability of a abnormal module (1703) [37].

### 3.4 Calibration for HyCal Position Reconstruction

The purpose of this calibration is to align the center of $\mathrm{HyCal}(0,0)$ to the beam target interaction point, and make the longitudinal cross section of HyCal perpendicular to the beam direction. Before the experiment started, the JLab survey group aligned and surveyed the HyCal relative to the beam line. Still we need to verify those results by analysing the data from the experiment, and furthermore monitor the stability of the HyCal coordinate reconstruction during the whole experiment. We used two methods. The first method used the projected yield which involves the distribution of $\pi^{0}$ production in the $\theta_{x}$ and $\theta_{y}$ axes. The second method used the "single arm" Compton events (only scattered photon from Compton scattering is detected by HyCal ).

### 3.4.1 HyCal Coordinate Calibration by Using $\pi^{0}$ Production

The $\pi^{0} \mathrm{~s}$ produced from electrons scattering on the nuclear target decay into two photons. The angle of the $\pi^{0}$ projected along the X -direction $\left(\theta_{x}\right)$ and Y -direction $\left(\theta_{y}\right)$ can be reconstructed from the position and the energy of the two decaying photons and are given by :

$$
\begin{align*}
& \sin \theta_{x}=\frac{P_{x}}{|\vec{p}|}=\frac{\frac{E_{1} x_{1}}{r_{1}}+\frac{E_{2} x_{2}}{r_{2}}}{\sqrt{E_{1}^{2}+E_{2}^{2}+2 E_{1} E_{2} \cos \theta_{12}}}  \tag{3.3}\\
& \sin \theta_{y}=\frac{P_{y}}{|\vec{p}|}=\frac{\frac{E_{1} y_{1}}{r_{1}}+\frac{E_{2} y_{2}}{r_{2}}}{\sqrt{E_{1}^{2}+E_{2}^{2}+2 E_{1} E_{2} \cos \theta_{12}}} \tag{3.4}
\end{align*}
$$

where,

$$
\begin{equation*}
r_{i}=\sqrt{x_{i}^{2}+y_{i}^{2}+z^{2}},(i=1,2) \tag{3.5}
\end{equation*}
$$

and

$$
\begin{align*}
\cos \theta_{12} & =\frac{r_{1}^{2}+r_{2}^{2}-r_{12}^{2}}{2 r_{1} r_{2}} \\
& =\frac{x_{1} x_{2}+y_{1} y_{2}+z^{2}}{\sqrt{x_{1}^{2}+y_{1}^{2}+z_{1}^{2}} \sqrt{x_{2}^{2}+y_{2}^{2}+z_{2}^{2}}} \tag{3.6}
\end{align*}
$$

where, $\vec{p}$ is the momentum of $\pi^{0}, P_{x}, P_{y}$ are the projected momenta along the X and Y axes, $x_{1}, y_{1}, x_{2}, y_{2}$ are the reconstructed positions of the two photons from $\pi^{0}$ decay, $E_{1}, E_{2}$ are the energies of the two photons, $z$ is the distance between the target and the HyCal surface, $r_{1}, r_{2}$ are the distance between detected photon position and target center, respectively, and $\theta_{12}$ is the angle between the two photons.

The distribution of $\pi^{0}$ yield versus $\theta_{x}, \theta_{y}$ are Gaussian distributions, if the HyCal center $(0,0)$ is located on the beam line and the longitudinal cross section of the HyCal is perpendicular to the beam direction, the mean value of these distributions should be at zero. Figure 3.11 shows these distributions after the calibration.

Because the limited statistics, we combined a few runs to do the calibration. Based on the photon beam stability, we divided the $\pi^{0}$ production data (Run 64716-65112) into 5 groups (see Table 3.1 [39].)

### 3.4.2 HyCal Reconstruction Fine Tuning

To perform high precision measurement of energy and direction of neutral pions in the PrimEx experiment, extensive study of resolution function of the HyCal calorimeter


Figure 3.11
Distributions of $\pi^{0}$ production yield versus $\theta_{x}, \theta_{y}$ after the calibration [39].

Table 3.1

HyCal coordinate calibration constant

| Run Number | Target | $\mathbf{X}(\mathrm{cm})$ | $\mathbf{Y}(\mathrm{cm})$ |
| :--- | :---: | :---: | :---: |
| $64716-64830$ | ${ }^{28} S i$ | $-0.41( \pm 0.02)$ | $-0.11( \pm 0.02)$ |
| $64831-64900$ | ${ }^{28} S i$ | $-0.35( \pm 0.02)$ | $-0.17( \pm 0.02)$ |
| $64901-64988$ | ${ }^{28} S i$ | $-0.39( \pm 0.01)$ | $-0.19( \pm 0.02)$ |
| $65006-65080$ | ${ }^{12} C-I I$ | $-0.40( \pm 0.02)$ | $-0.19( \pm 0.02)$ |
| $65081-65112$ | ${ }^{12} C-I I$ | $-0.43( \pm 0.04)$ | $-0.15( \pm 0.04)$ |

was performed using data obtained during dedicated runs with the tagged photon beam [17]. The analysis was based on PrimEx-I data, which were taken in 2004 and the island reconstruction algorithm was added to the analysis for the PrimEx-II runs.

### 3.4.2.1 Position Reconstruction

The simplest estimation for the cluster coordinate is to take the center of gravity of an electromagnetic shower, which is:

$$
\begin{equation*}
x=\sum x_{i} w_{i} \tag{3.7}
\end{equation*}
$$

where $x_{i}$ is the coordinate of the center of i-th module, the weight $w_{i}$ is the fraction of the shower energy deposited in the i-th module $\left(E_{i}\right)$ to the total energy: $w_{i}=E_{i} / \sum E_{i}$.

This method gives unbiased estimation only in the center and at the edge of the module (see Figure 3.12), because it assumes only one module absorbs most of the energy, and it dominates the position calculation in cases when the deposited energy is varies linearly. But the radial energy falloff of the showers is approximately exponential, therefore, we used logarithmic weights instead of linear weights, defined as [17]:

$$
\begin{equation*}
w_{i}=\max \left\{0, w_{0}+\ln \left(E_{i} / \sum E_{i}\right)\right\} \tag{3.8}
\end{equation*}
$$

where $E_{i}$ is the energy deposited in i-th module and $w_{0}$ is a free parameter to be found. This method had much smaller deviation in the reconstructed coordinate from the actual value, compared with linear center of gravity method (see Figure 3.13). To further eliminate the
majority of the remaining bias and improve the reconstruction algorithm, we applied a correction function $g$ to the logarithmic method [17]:

$$
\begin{equation*}
x_{\text {corrected }}=x_{\text {rec }}+g\left(x_{\text {rec }}\right) \tag{3.9}
\end{equation*}
$$

where the correction function $g$ was chosen as an odd order polynomial. The details for this correction can be found in Ref. [17]. After this correction, the reconstructed hit position is much more uniformly distributed, as one can see from Figure 3.14 (left) and the reconstructed position as a function of actual position is shown in Figure 3.14 (right).


Figure 3.12
Deviation of reconstructed coordinate occupancy (left) and mean value of the deviation (right) from an actual value as a function of position inside a module for the center of gravity method [17].


Figure 3.13

Deviation of reconstructed coordinate occupancy (left) and mean value of the deviation (right) from an actual value as a function of position inside a module for the logarithmic method [17].


Figure 3.14
(left): Deviation of reconstructed coordinate occupancy, (right blue solid): mean value of the deviation from the true value as a function of position inside a module for the corrected logarithmic method, (right brown open): same quantity before the correction.
[17].

### 3.4.2 2 Position Resolution

The position resolution was extracted directly during the snake runs. Since the incident coordinate represents position of the photon on the HyCal face, the distribution of the difference between the reconstructed coordinate and calculated beam center position is a result of the convolution of the HyCal response and the beam spot profile [17]. The beam spot profile was generated by GEANT and verified with the data from super-harp scan [2]. We observed $5-10 \%$ variation of the width of the beam spot and the effect of such variation on the obtained (unfolded) resolution was taken as our systematic uncertainty. The position resolution as a function of energy fitted with $\alpha \sqrt{E}$ function is shown in Figure 3.15. For the PrimEx-II energy range $(4-5 \mathrm{GeV})$, the position resolution for $\mathrm{PbWO}_{4}$ module is about 0.15 cm .


Figure 3.15
HyCal position resolution as a function of energy [17].

## CHAPTER IV

## MONTE CARLO SIMULATIONS

There are two Monte Carlo simulations used in the extraction of the experimental yield.
(1) A Compton generator along with a Geant3 based simulation of the experimental setup to simulate the Compton events.
(2) Simulation of the pair production events using the default Geant3 generator along with the simulation of the experimental setup.

The simulated events were then reconstructed using the same algorithm as the experimental data.

### 4.1 The Compton Scattering Simulation

### 4.1.1 Event generator

The event generator uses a cross section model with corrections to first order in $\alpha$ [14], that was adapted for numerical simulation by Tkabladze, Konchatnyi and Prok [48]. A brief description of the various components of the model is reproduced from Ref. [48], as described below.

### 4.1.1.1 Born approximations



Figure 4.1
The lowest-order Feynman diagrams for single Compton scattering [42].

The Compton scattering cross section in the lowest order Born approximation (shown by the Feynman diagram in Figure 4.1), is described by the Klein-Nishina formalism [14], which is also given in Section (1.1) :

$$
\begin{equation*}
\frac{d \sigma}{d \Omega_{\gamma}}=\frac{r_{e}^{2}}{2} \frac{1}{\left[1+\gamma\left(1-\cos \theta_{\gamma}\right)\right]^{2}}\left[1+\cos ^{2} \theta_{\gamma}+\frac{\gamma^{2}\left(1-\cos \theta_{\gamma}\right)^{2}}{1+\gamma\left(1-\cos \theta_{\gamma}\right)}\right] \tag{4.1}
\end{equation*}
$$

where $\gamma=\frac{E_{\gamma}}{m c^{2}}, \theta_{\gamma}$ is the photon scattering angle, $E_{\gamma}$ is the incident photon energy, $m$ is the electron mass, $r_{e}$ is the classical electron radius and $d \Omega=2 \pi \sin \theta d \theta$.

### 4.1.1.2 Higher order corrections



Figure 4.2
Typical (a) radiative correction and (b) double Compton scattering contributions to single Compton scattering [42].

There are two types of higher order corrections (in the first order of $\alpha$ ) that were applied to the Born cross section. The first type of correction is the Virtual-photon radiative correction due to the possibility of emission and reabsorption of a virtual photon by an electron during the scattering process. The second type of correction is the so called double Compton scattering where a secondary photon is emitted. The interference of this process with Born term also gives rise of corrections of order $\alpha$.

The virtual corrections have been evaluated by Brown and Feynman [14], but the virtual corrections alone do not have a physical meaning because of the infrared divergence they contain. In fact a part of the second type of correction (double Compton scattering) where a very soft secondary photon is emitted must be considered simultaneously with the virtual correction. When corrections due to virtual and real soft photon emission, with energy
much less than the electron mass, are combined, the divergences cancel out and we have a physically meaningful finite cross section as [31]:

$$
\begin{equation*}
d \sigma_{0}\left(1+\delta_{S V}\right) \tag{4.2}
\end{equation*}
$$

where $\delta_{S V}$ is the combined virtual and soft corrections as calculated in Ref. [31]. For this calculation the soft photon energy $k<k_{\max }$, where $k_{\max }$ is the energy of photons that were inaccessible due to the resolution of HyCal .

The remaining correction is due to double Compton scattering with a secondary photon of energy greater than $k_{\text {max }}$. This correction was calculated in Ref. [43] and [40]. An expression for the differential cross section $\sigma\left(k ; k_{1}, k_{2}\right) d k_{1} d k_{2}$ for an energy in $\left(k_{1}, k_{1}+\right.$ $d k_{1}$ ), emitted into an element of solid angle $d \Omega_{1}$ in the direction $\theta_{1}$, and a second photon with energy in $\left(k_{2}, k_{2}+d k_{2}\right)$ emitted into an element of solid angle $d \Omega_{2}$ in the direction $\theta_{2}$ can be express as a function of $k_{1}, \theta_{1}, \theta_{2}, \phi$, by using conservation of energy and momentum as [40]:

$$
\begin{equation*}
d \sigma\left(k ; k_{1}, \theta_{1}, \theta_{2}, \phi\right)=\alpha r_{0}^{2} \frac{d \Omega_{1} d \Omega_{2}}{(4 \pi)^{2}} \frac{k_{1} k_{2} d k_{1}}{m_{2} k} \frac{X}{T_{1}} \tag{4.3}
\end{equation*}
$$

where $\phi$ is the azimuthal angle between the two planes $\left(\vec{k}, \overrightarrow{k_{1}}\right)$ and $\left(\vec{k}, \overrightarrow{k_{2}}\right), d \Omega_{1}, d \Omega_{2}$ are the solid angle of two hard photons and

$$
\begin{equation*}
T_{1}=m_{e}+k\left(1-\cos \theta_{2}\right)-k_{1}\left(1-\cos \theta_{12}\right) \tag{4.4}
\end{equation*}
$$

The conservation laws $p_{1}+k_{0}=p_{2}+k_{1}+k_{2}$ was used to express $d \sigma$ in terms of variables $k_{1}, \theta_{1}, \theta_{2}$ and $\phi$ as shown in Figure 4.3. Finally,

$$
\begin{equation*}
\cos \theta_{12}=\cos \theta_{1} \cos \theta_{2}+\sin \theta_{1} \sin \theta_{2} \cos \phi \tag{4.5}
\end{equation*}
$$

and

$$
\begin{equation*}
k_{2}=\frac{L}{T_{1}} \tag{4.6}
\end{equation*}
$$

where

$$
\begin{equation*}
L=m_{e} k-k_{1}\left[m_{e}+k\left(1-\cos \theta_{1}\right)\right] \tag{4.7}
\end{equation*}
$$

Combining the virtual and soft corrections with the hard double Compton corrections, the total correction to the Born term takes the form of : $d \sigma_{t o t}=d \sigma_{0}\left[1+\delta_{S V}+\delta_{d h}\right]$ where, $\delta_{d h}$ is the correction to hard double Compton scattering as calculated in Ref. [31].


Figure 4.3
Relative directions in double Compton scattering.

### 4.1.1.3 Total radiative cross section for Compton scattering

In the Compton simulation, we applied two correction terms to the Born level cross section $d \sigma_{0}$, with the total radiative cross section given by:

$$
\begin{equation*}
d \sigma_{t o t}=d \sigma_{0}\left(1+\delta_{S V}+\delta_{d h}\right) \tag{4.8}
\end{equation*}
$$

The virtual and soft corrections, $\delta_{S V}$, involves a numerical integration over a 1-dimensional differential cross section while the hard double Compton correction, $\delta_{d h}$, involves a numerical integration over a 4-dimensional differential cross section. The corrections are separated into these two types based on whether the energy of the secondary emitted photon is less than or greater than a parameter, $\omega_{2 \max }$. The parameter $\omega_{2 \max }$ is arbitrary but must be less than the electron mass. The integrations were carried out using Monte Carlo methods and it was verified that the final result of the integration is independent of the value of $\omega_{2 \max }$ [48].

The calculated total radiative cross sections for different energies are presented in Table 4.1 and Figure 4.4. These cross sections were found to be consistent with an alternative calculation of M. Konchatnyi and a fraction of a percent less than the values obtained from the XCOMP database of the National Institute of Standards and Technology (NIST) [48]. The cross sections were then fitted by a third degree polynomial, which is:

$$
\begin{equation*}
d \sigma_{f i t}=1.060-0.3255 E+0.0456 E^{2}-0.0024 E^{3} \tag{4.9}
\end{equation*}
$$

and this function was used to calculate the corresponding cross sections which have the same energies as the data points. The so called "Theory" in the later on chapter is referring to the Function (4.14).

Table 4.1
Total radiative cross section for Compton scattering on carbon at various photon energies [48].

| Photon energy $(\mathrm{GeV})$ | $\sigma_{\text {tot }}(\mathrm{mb})$ |
| :---: | :---: |
| 3.5 | 0.3770 |
| 4.0 | 0.3346 |
| 4.5 | 0.3012 |
| 5.0 | 0.2741 |
| 5.5 | 0.2517 |
| 6.0 | 0.2329 |



Figure 4.4
Total radiative cross section of Compton scattering on carbon at various photon energies.

### 4.1.2 Compton Scattering Event Generation and Validation

Using the cross section model described in the previous section, an event generator was built for Compton scattering with the full radiative cross section. The event generator was created using the BASES/SPRING simulation package [29]. The BASES/SPRING simulation package is used to obtain total cross sections and to generate events for elementary processes in high energy physics. It is able to integrate singular functions and to generate events with 50 independent variables [48].

This package was used to create two generators : (1) to generate events according to radiative corrected Klein-Nishina formula, and (2) to generate events according to the double Compton scattering cross section formula. The BASES/SPING package uses the probability information supplied by BASES to generate events with four-momentum vectors of
final state particles. In our case, we have 2-particle final states for the soft-virtual generator, and 3-particle final states for the double Compton generator. Since we know the relative contribution of each process to the total cross section, we generate events according to that composition [48].

We generated one million events for each energy bin, associated with the "E-counters". Since we have a total of 180 E-counters in the experiment, a total of 180 million events were generated by the simulation. Then we used these generated events as the initial incident events incident on the target and the detectors package in the GEANT-3 framework. The position and energy information of the scattered particles in HyCal were recorded in the simulation and these events were then reconstructed using the same algorithm and decoder as used to reconstruct the experimental data.

Figure 4.5 shows the reconstructed beam position distributions projected on the X and Y axes using the simulated data. We can see that the mean of the Gaussian fit to the X and Y positions are 0.010 cm and 0.007 cm , respectively, which is within the position resolution $(\sim 0.15 \mathrm{~cm})$ of HyCal relative to the expected value $(0,0)$. The $\sigma$ in the plot is larger $(\sim 0.36 \mathrm{~cm})$ because it is the convoluted resolution with the beam profile. This gives us the confidence that HyCal coordinate system was calibrated well and the reconstructing algorithm was working properly.

There were 6 cuts applied (for definition see Sub-section (1.3)) to select Compton events:

1. HyCal fiducial cut :

- $-3.90 \mathrm{~cm}<x_{i}<4.55 \mathrm{~cm}$, and $-4.00 \mathrm{~cm}<y_{i}<4.30 \mathrm{~cm}$ is the area being removed due to the central hole of HyCal ,


Figure 4.5
Reconstructed beam X (left) and Y (right) positions from simulation data.

- $-15.0 \mathrm{~cm}<x_{i}<-8.0 \mathrm{~cm}$, and $-11.5 \mathrm{~cm}<y_{i}<-5.0 \mathrm{~cm}$ is the area being removed due to dead modules,
- $33.232 \mathrm{~cm}<\left|x_{i}\right|$, and $33.210 \mathrm{~cm}<\left|y_{i}\right|$ is cut used to exclude the lead-glass region,

2. $\Delta T$ time difference (ns): $|\Delta T|<6.5$,
3. $\Delta \phi$ azimuthal angle difference (Degree): $|\Delta \phi-180.0|<5 \sigma_{\phi}^{i}$,
with the width of the distribution of the azimuthal angle, $\sigma_{\phi}^{i}$ for each target is given by: $\sigma_{\phi}^{C-I}=3.99, \sigma_{\phi}^{C-I I}=4.37$, and $\sigma_{\phi}^{S i}=4.70$,
4. $R_{\text {min }}$ cluster separation (cm): $R(E)<R_{\text {min }}$,
where $R(E)$ is a function of beam energy defined as:
$R(E)=19.00+1.95 \times\left(4.85-E_{0}\right)$,
5. $\Delta E$ elasticity $(\mathrm{GeV}):|\Delta E|<5 \sigma_{E}^{i}$,
with the width of the distribution of the elasticity, $\sigma_{E}^{i}$ for each target is given by: $\sigma_{E}^{C-I}=0.078, \sigma_{E}^{C-I I}=0.078$, and $\sigma_{E}^{S i}=0.080$,
6. $\Delta K$ kinematic energy difference $(\mathrm{GeV}):|\Delta K|<4.0 \sigma_{K}^{i}$,
with the width of the distribution of the kinematic energy distributions, $\sigma_{K}^{i}$ for each target is given by: $\sigma_{K}^{C-I}=0.127, \sigma_{K}^{C-I I}=0.136$, and $\sigma_{K}^{S i}=0.172$.

The positions of the reconstructed clusters on HyCal for the simulated data on Compton scattering from the ${ }^{12} \mathrm{C}$-I target were also verified and the results are shown in Figure 4.6 and Figure 4.7. The two square blank blocks are due to the same HyCal fiducial cut applied to the experimental data. The central block cut is to remove the inner layer modules because of the larger uncertainties associated with these blocks, the other cut is because of dead modules. Figure 4.6 is for the clusters with higher energies while Figure 4.7 is for the clusters with lower energies. One can see that for both clusters there are edges, that is because the higher energy clusters can not exist beyond a certain angle due to the Compton scattering kinematics.

Next, we examine the simulated Compton events by reconstructing the six variables that are used to select the Compton events for the experimental data. Figure 4.8 shows the $R_{\text {min }}, \Delta \phi, \Delta E$ and $\Delta K$ distributions in the simulated Compton data. Figure 4.9 and Figure 4.10 shows the two-dimensional distributions of these parameters before and after cuts applied to the simulated Compton data. Thus, the simulated Compton data was validated and made ready to use in the extraction of the Compton cross section.

Reconstructed Cluster-1 position on HyCal


Figure 4.6
Reconstructed hit position on HyCal of simulated Compton scattering for cluster-1, and the ${ }^{12} \mathrm{C}$-I target (cuts applied).


Figure 4.7
Reconstructed hit position on HyCal of simulated Compton scattering for cluster-2, and the ${ }^{12} \mathrm{C}$-I target (cuts applied).

$\Delta \mathrm{E}$ Distribution


$\Delta \mathrm{K}$ Distribution


Figure 4.8
Distribution of the reconstructed variables for the simulated Compton data. The variables are used to identify Compton events in the data. The different colors show the distributions after cuts are applied on the four variables. The cuts used were $R_{\min }(\mathrm{pink})$, $\Delta \phi$ (green), $\Delta K$ (blue), and $\Delta E$ (orange).


Figure 4.9

The two dimensional plots of the simulated Compton data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.


Figure 4.10

The two dimensional plots of the simulated Compton data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.

### 4.1.3 Cross section Model for the Simulation of Electron-positron pair production

The pair-production cross section model was built by Alexandr Korchin from the Kharkov Institute of Physics and Technology [32]. The calculated total cross section for electronpositron production on ${ }^{12} \mathrm{C}$ target at 5 GeV region was validated with data from NIST [44]. The following brief description of the various components of the model is reproduced from Ref. [32].

We consider photo-production of $e^{+} e^{-}$pairs on a nucleus with atomic weight $A$ and atomic number $Z$ :

$$
\begin{equation*}
\gamma+A \rightarrow e^{+}+e^{-}+\gamma^{\prime}+A \tag{4.10}
\end{equation*}
$$

At a few GeV photon energies and very small momentum transfer relevant for the PrimExII experiment, the cross section for this process consists of the following contributions in order of importance :

1. Bethe-Heitler mechanism of pair production on the nucleus (coherent process) with screening effects due to atomic electrons and Coulomb distortion.
2. Pair production on atomic electrons with excitation of all atomic states. It contains correlation effects due to the presence of other electrons and nuclei.
3. Quantum Electro-Dynamical (QED) radiative corrections (of order $\alpha / \pi$ with respect to dominant contributions): (i) virtual-photon loops and (ii) real photon process $\gamma+$ $A \rightarrow e^{+}+e^{-}+A+\gamma^{\prime}$, where the final photon has the energy $\omega^{\prime} \leq \delta \omega$ (energy resolution in experiment).
4. Nuclear incoherent contribution quasi-elastic, or quasi-free process on the proton $\gamma+p \rightarrow e^{+}+e^{-}+p$.
5. Nuclear coherent contribution, or virtual Compton Scattering two-step mechanism $\gamma+A \rightarrow \gamma^{*}+A \rightarrow e^{+}+e^{-}+A$.

### 4.1.3.1 Pair production on a nucleus

The exclusive cross section on the nucleus is given by :

$$
\begin{equation*}
\frac{d^{4} \sigma_{A}}{d \epsilon_{+} d \theta_{-} d \theta_{+} d \phi}=Z^{2} \frac{\alpha^{3}}{2 \pi \omega^{3} \vec{Q}^{4}}\left|F_{A}\left(\vec{Q}^{2}\right)-f_{a t}\left(\vec{Q}^{2}\right)\right|^{2}|T|^{2} \tag{4.11}
\end{equation*}
$$

where, $\alpha=1 / 137.036$ is the fine-structure constant, $\theta_{+}, \theta_{-}$are the lepton polar angles, $\phi$ is the azimuthal angle between the plane spanned by the momenta $\vec{k}, \overrightarrow{p_{+}}$and the plane spanned by $\vec{k}, \vec{p}_{-}, k=(\omega, \vec{k})$ is the photon 4-momentum, $p_{+}=\left(\epsilon_{+}, \vec{p}_{+}\right)$and $p_{-}=\left(\epsilon_{-}, \vec{p}_{-}\right)$ is the positron and electron 4-momentum, respectively, and $m_{e}$ is the electron mass. $\vec{Q}=$ $\vec{k}-\vec{p}_{+}-\vec{p}_{-}$is the 3-momentum transferred to the nucleus, $Z$ is the atomic number, $\left|T^{2}\right|$ is a kinematic factor, $f_{a t}\left(\vec{Q}^{2}\right)$ is the atomic form factor describing charge distribution of electrons $\rho_{a t}(r)$, and $F_{A}\left(\vec{Q}^{2}\right)$ is the nuclear charge form factor (Fourier transform of $\rho_{A}(r)$ ) which behaves like

$$
\begin{equation*}
F_{A}\left(\vec{Q}^{2}\right) \approx 1-\frac{1}{6} \vec{Q}^{2}\left\langle r^{2}\right\rangle_{A} \tag{4.12}
\end{equation*}
$$

where $\left\langle r^{2}\right\rangle_{A}$ is the mean squared radius of the nucleus.
After integration, the cross section has the form [Bethe-Heitle] [12] :

$$
\begin{align*}
\frac{d \sigma_{A}}{d \epsilon_{+}}= & \int \frac{d^{4} \sigma_{A}}{d \epsilon_{+} d \theta_{-} d \theta_{+} d \phi} d \theta_{+} d \theta_{-} d \phi \\
= & Z^{2} \frac{\alpha^{3}}{m_{e}^{2} \omega^{3}}\left[\left(\epsilon_{+}^{2}+\epsilon_{-}^{2}\right)\left(\phi_{1}-\frac{4}{3} \log Z-4 f\right)\right.  \tag{4.13}\\
& \left.+\frac{2}{3} \epsilon_{+} \epsilon_{-}\left(\phi_{2}-\frac{4}{3} \log Z-4 f\right)\right]
\end{align*}
$$

where, $f=f\left((\alpha Z)^{2}\right)$ is the Coulomb distribution function [Bethe and Maximon] [19],

$$
\begin{equation*}
f\left((\alpha Z)^{2}\right)=(\alpha Z)^{2} \sum_{n=1}^{\infty} \frac{1}{n\left[n^{2}+(\alpha Z)^{2}\right]} \tag{4.14}
\end{equation*}
$$

For the ${ }^{12} \mathrm{C}$ nucleus $f \approx 2.3 \times 10^{-3}$.

### 4.1.3.2 Pair production on atomic electrons

The corresponding cross section has the form:

$$
\begin{equation*}
\frac{d^{4} \sigma_{e}}{d \epsilon_{+} d \theta_{-} d \theta_{+} d \phi}=Z \frac{\alpha^{3}}{2 \pi \omega^{3} \vec{Q}^{4}} H\left(\vec{Q}^{2}\right)|T|^{2} \tag{4.15}
\end{equation*}
$$

After the calculation, we have the energy distribution of positrons has the form:

$$
\begin{equation*}
\frac{d \sigma_{e}}{d \epsilon_{+}}=Z \frac{\alpha^{3}}{m_{e}^{2} \omega^{3}}\left[\left(\epsilon_{+}^{2}+\epsilon_{-}^{2}\right)\left(\psi_{1}-\frac{8}{3} \log Z\right)+\frac{2}{3} \epsilon_{+} \epsilon_{-}\left(\psi_{2}-\frac{8}{3} \log Z\right)\right] \tag{4.16}
\end{equation*}
$$

### 4.1.3.3 Total cross section for pair production

Other than the contributions mentioned in previous two sub-sections, we also have radiative corrections, nuclear incoherent contribution and nuclear coherent contribution. But these contributions are relatively small, Table 4.2 is an example of the results of a calculation at a photon energy of $4.91 \mathrm{GeV}, x_{+}=0.4$, and $x_{-}=0.6$.

Table 4.2
Various contributions to cross section at photon energy 4.91 GeV [32].

| Mechanism | Contribution (\%) |
| :--- | :---: |
| Nuclear Bethe-Heitler | 82.789 |
| Atomic electrons | 17.185 |
| Nuclear incoherent (quasielastic) | 0.026 |
| Nuclear coherent (virtual CS) | $\sim 10^{-5}$ |
| Total | 100.000 |

Finally, we can write the total cross section :

$$
\begin{equation*}
\sigma_{t o t}=d \sigma_{A}+d \sigma_{e} \tag{4.17}
\end{equation*}
$$

and the calculated cross section for different energies is presented in Table 4.3. These values are $\sim 2 \%$ different from the NIST value ( 353.6 mb ) [44].

## Table 4.3

Total cross section of pair production on carbon at various photon energies [32].

| Photon energy $(\mathrm{GeV})$ | $\sigma_{\text {tot }}(\mathrm{mb})$ |
| :---: | :---: |
| 4.91 | 348.8 |
| 4.97 | 348.9 |
| 5.03 | 349.1 |
| 5.08 | 349.0 |
| 5.13 | 349.1 |
| 5.18 | 349.3 |
| 5.23 | 349.2 |
| 5.28 | 349.5 |
| 5.34 | 349.3 |
| 5.41 | 349.5 |
| 5.46 | 349.5 |

### 4.1.4 Pair production generation and validation

The default event generator of the GEANT-3 and GEANT-4 framework were used in this simulation. The difference between the two generators is less than $0.1 \%$ for all the distributions that were used in the analysis. The total cross section for pair production in the energy range of the experiment ( $4.4 \mathrm{GeV}-5.3 \mathrm{GeV}$ ) was also validated by comparing
to the NIST data. For the ${ }^{12} \mathrm{C}$ target, the total cross section extracted from simulation is $\sim 351 \mathrm{mb}$ (NIST value is 353.6 mb ), and for the ${ }^{28} \mathrm{Si}$ target, the extracted value is $\sim 1340 \mathrm{mb}$ (NIST value is 1384 mb [44]), as shown in Figure 4.11 and Figure 4.12. The difference between the simulation values and the accepted NIST values was considered as a systematic effect and is included in the systematic uncertainty analysis in Section (5.5).


Figure 4.11
Comparison of $e^{+} e^{-}$cross section between the extracted value from simulation and the NIST value on ${ }^{12} \mathrm{C}$-I target.

The reconstructed cluster positions on HyCal for the simulated pair production on the ${ }^{12} \mathrm{C}$-I target was also validated and the results are shown in Figure 4.13 and Figure 4.14. The central hole and the left bottom blank are due to the HyCal fiducial cut and we can see the distributions are more uniformly distributed comparing with the Compton simulation. This occurs because these pair production background events get into the HyCal


Figure 4.12
Comparison of $e^{+} e^{-}$cross section between the extracted value from simulation and the NIST value on ${ }^{28}$ Si target.
acceptance due to multiple scattering, so they do not have special structures like Compton events.

Next, we examine the simulated pair production events by reconstructing the six variables that are used to select the Compton events for the experimental data. Figure 4.15 shows the $R_{\text {min }}, \Delta \phi, \Delta E$ and $\Delta K$ distributions in the simulated pair production data. Figure 4.16 and Figure 4.17 shows the two-dimensional distributions of these parameters before and after cuts applied to the simulated pair production data. Thus, the simulated pair production data was validated and made ready to use in the extraction of the Compton cross section.


Figure 4.13
Reconstructed hit position on HyCal of simulated $e^{+} e^{-}$background for Cluster-1, and the ${ }^{12} \mathrm{C}$-I target (cuts applied).


Figure 4.14
Reconstructed hit position on HyCal of simulated $e^{+} e^{-}$background for Cluster-2, and the ${ }^{12} \mathrm{C}$-I target (cuts applied).


Figure 4.15
Distribution of the reconstructed variables for the simulated pair production data. The variables are used to identify Compton events in the data. The different colors show the distributions after cuts are applied on the four variables. The cuts used were $R_{\min }$ (pink), $\Delta \phi$ (green), $\Delta K$ (blue), and $\Delta E$ (orange).


Figure 4.16
The two dimensional plots of the simulated pair production data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.


Figure 4.17
The two dimensional plots of the simulated pair production data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.

## CHAPTER V

## MEASUREMENT OF THE INTEGRATED COMPTON CROSS SECTION

Extraction of the cross section for Compton scattering requires knowledge about the experimental luminosity, the acceptance of the the detector system and the experimental yield. In this chapter we will discuss each of these elements in detail and finally present the extracted cross section. The experimental luminosity depends on the photon flux and the target thickness. The target thickness was already discussed in Sec. 2.4, here we begin with a discussion of the photon flux.

The energy bins were determined by combining 10 E-counters. As described in Section (3.1.3). The PrimEx -II experiment used 180 E-counters during the Compton scattering data collection, and hence there are a total of 18 energy bins.

### 5.1 Photon Flux for Each Energy Bin

After the tagger ADC and TDC alignment and calibration, the tagged photon flux on the target can be obtained using Eq. (2.4). The absolute tagging ratio is already discussed
in Section (2.7) and in this section we will introduce the number of tagged electrons on E-counters or T-counters, $N_{e}$. The formulas to calculate $N_{e}$ are given by:

$$
\begin{gather*}
N_{e}=\frac{T \times l t \times N_{i}}{N_{5} \times t_{O O T}}  \tag{5.1}\\
T=\frac{N_{\text {ungated }}}{\nu_{\text {gen }}}  \tag{5.2}\\
l t=\frac{N_{\text {gated }}}{N_{\text {ungated }}} \tag{5.3}
\end{gather*}
$$

where, $T$ is the time if interval between two scaler events ( 10 sec ), $l t$ is the DAQ livetime (dimensionless), $N_{i}$ is the number of hits seen in a selected time window for the i-th Tcounter, $N_{5}$ is the number of clock triggers recorder in that same interval, $t_{O O T}$ is the size of the time window ( $2 \mu s e c$ ), $N_{(u n) g a t e d}$ is the (un)gated scaler counts during the interval and $\nu_{g e n}$ is the generator frequency used for the scaler.

The flux results and its uncertainties for each energy bin are listed in Table 5.1, and this information was used in the cross section calculation (note that there was a blind number added to the flux which was only un-blind after the analysis was done, the results in this table are the true flux values).

Table 5.1
Photon flux and associated uncertainty for each energy bin.

| Energy Bin | Energy <br> $(\mathrm{GeV})$ | ${ }^{12}$ C-I Target <br> Flux $\left(\times 10^{9}\right)$ | ${ }^{12}$ C-II Target <br> Flux $\left(\times 10^{9}\right)$ | ${ }^{28}$ Si Target <br> Flux $\left(\times 10^{9}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| E01 | 5.281 | $1.3485(134)$ | $0.2438(24)$ | $0.1187(12)$ |
| E02 | 5.241 | $1.5863(140)$ | $0.2830(25)$ | $0.1374(12)$ |
| E03 | 5.195 | $1.6705(138)$ | $0.2984(25)$ | $0.1456(12)$ |
| E04 | 5.145 | $1.5935(136)$ | $0.2829(24)$ | $0.1370(12)$ |
| E05 | 5.093 | $1.6871(139)$ | $0.3035(25)$ | $0.1475(12)$ |
| E06 | 5.041 | $1.8276(148)$ | $0.3280(27)$ | $0.1593(13)$ |
| E07 | 4.988 | $1.9705(160)$ | $0.3533(29)$ | $0.1719(14)$ |
| E08 | 4.937 | $1.8133(154)$ | $0.3261(28)$ | $0.1586(13)$ |
| E09 | 4.883 | $1.8274(148)$ | $0.3258(27)$ | $0.1587(13)$ |
| E10 | 4.827 | $1.7065(136)$ | $0.3062(24)$ | $0.1490(12)$ |
| E11 | 4.774 | $1.8903(152)$ | $0.3355(27)$ | $0.1626(13)$ |
| E12 | 4.726 | $1.5150(123)$ | $0.2720(22)$ | $0.1320(11)$ |
| E13 | 4.673 | $0.7827(061)$ | $0.1392(11)$ | $0.0675(05)$ |
| E14 | 4.609 | $1.8114(143)$ | $0.3237(26)$ | $0.1573(12)$ |
| E15 | 4.554 | $1.9283(151)$ | $0.3448(27)$ | $0.1677(13)$ |
| E16 | 4.503 | $1.7338(139)$ | $0.3100(25)$ | $0.1499(12)$ |
| E17 | 4.459 | $1.9689(152)$ | $0.3541(27)$ | $0.1723(13)$ |
| E18 | 4.400 | $1.4775(117)$ | $0.2617(21)$ | $0.1274(10)$ |
| ALL | 4.840 | $30.1391(2471)$ | $5.3918(442)$ | $2.6201(215)$ |

### 5.2 Event Selection



Figure 5.1
Detection of a single Compton event in HyCal

This analysis used special Compton scattering data collected during the PrimEx-II experiment, on three targets, including a 5\% radiation length ${ }^{12} \mathrm{C}$-I target (runs 64876 64883), a $8 \%$ radiation length ${ }^{12} \mathrm{C}$-II target (runs $65080-65081$ ) and a $10 \%$ radiation length ${ }^{28}$ Si target (runs 65078-65079). Data were collected with an incident photon energy ranged from 4.4 to 5.3 GeV , and different targets. A schematic of Compton events is shown in Figure 5.1. For each event, the photon beam energy $E_{\gamma}$, clusters energies $E_{i}$, clusters positions $x_{i}$ and $y_{i}$, and clusters time on $\operatorname{HyCal} T_{i}(i=1,2,3)$ were measured.

To validate the data quality, first we reconstructed the beam position to check for offsets in the HyCal coordinate system (see Figure 5.2). The mean values of a Gaussian fit for the X and Y positions are 0.067 cm and 0.004 cm , respectively. Relative to the expected values $(0,0)$, this is within the position resolution of the HyCal. This gives us the confidence that HyCal coordinate system was calibrated well and the reconstructing algorithm was working properly.


Figure 5.2
Reconstructed beam X (left) and Y (right) positions from experimental data.

The positions of the reconstructed clusters on HyCal for the experiment data from the ${ }^{12} \mathrm{C}$-I target were also verified and the results are shown in Figure 5.3 (higher energy) and Figure 5.4 (lower energy). The two square blank blocks are due to the same HyCal fiducial cut applied on the experimental data. The central block cut is to remove the inner layer modules because of the larger uncertainties associated with these blocks, the other cut is because of dead modules.

Reconstructed Cluster-1 position on HyCal


Figure 5.3
Reconstructed hit position on HyCal for cluster-1, and the ${ }^{12} \mathrm{C}$-I target data (cuts applied).


Figure 5.4
Reconstructed hit position on HyCal for cluster-2, and the ${ }^{12} \mathrm{C}$-I target data (cuts applied).

The following cuts are then applied (for definition see sub-section (1.3)) to select
Compton events :

1. HyCal fiducial cut :

- $-3.90 \mathrm{~cm}<x_{i}<4.55 \mathrm{~cm}$, and $-4.00 \mathrm{~cm}<y_{i}<4.30 \mathrm{~cm}$ is the area being removed due to the central hole of HyCal ,
- $-15.0 \mathrm{~cm}<x_{i}<-8.0 \mathrm{~cm}$, and $-11.5 \mathrm{~cm}<y_{i}<-5.0 \mathrm{~cm}$ is the area being removed due to dead modules,
- $33.232 \mathrm{~cm}<\left|x_{i}\right|$, and $33.210 \mathrm{~cm}<\left|y_{i}\right|$ is cut used to exclude the lead-glass region,

2. $\Delta T$ time difference (ns): $|\Delta T|<6.5$,
3. $\Delta \phi$ azimuthal angle difference (Degree): $|\Delta \phi-180.0|<5 \sigma_{\phi}^{i}$,
with the width of the distribution of the azimuthal angle, $\sigma_{\phi}^{i}$ for each target is given by: $\sigma_{\phi}^{C-I}=3.99, \sigma_{\phi}^{C-I I}=4.37$, and $\sigma_{\phi}^{S i}=4.70$,
4. $R_{\text {min }}$ cluster separation (cm): $R(E)<R_{\min }$,
where $R(E)$ is a function of beam energy defined as:
$R(E)=19.00+1.95 \times\left(4.85-E_{0}\right)$,
5. $\Delta E$ elasticity (GeV): $|\Delta E|<5 \sigma_{E}^{i}$,
with the width of the distribution of the elasticity, $\sigma_{E}^{i}$ for each target is given by: $\sigma_{E}^{C-I}=0.078, \sigma_{E}^{C-I I}=0.078$, and $\sigma_{E}^{S i}=0.080$,
6. $\Delta K$ kinematic energy difference $(\mathrm{GeV}):|\Delta K|<4.0 \sigma_{K}^{i}$, with the width of the distribution of the kinematic energy distributions, $\sigma_{K}^{i}$ for each target is given by: $\sigma_{K}^{C-I}=0.127, \sigma_{K}^{C-I I}=0.136$, and $\sigma_{K}^{S i}=0.172$.

Figure 5.5 shows all the distributions that were used in this analysis with the red arrows showing the range used to select the Compton events. Figure 5.6 shows the $R_{m i n}, \Delta \phi, \Delta E$ and $\Delta K$ distributions of the experiment data after applying these cuts. One can see that some of the background events being excluded from the data. Figure 5.7 and Figure 5.8 shows the two-dimensional distributions of these parameters before and after the cuts are


Figure 5.5
(1) XY position, (2) $\Delta T$, (3) $\Delta \phi$, (4) $R_{\min }$, (5) $\Delta E$ and (6) $\Delta K$ distributions with cutting range (red arrows).

$\Delta \mathrm{E}$ Distribution

$\Delta \phi$ Distribution

$\Delta \mathrm{K}$ Distribution


Figure 5.6
Distribution of the reconstructed variables for the data. The variables are used to identify Compton events in the data. The different colors show the distributions after cuts are applied on the four variables. The cuts used were $R_{\min }$ (pink), $\Delta \phi$ (green), $\Delta K$ (blue), and $\Delta E$ (orange).


Figure 5.7
The two dimensional plots of the experimental data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.


Figure 5.8
The two dimensional plots of the experimental data for variables used to select Compton events. The left plots are for all simulated events while the right plots show the same distributions after the cuts.
applied to the data. The HyCal fiducial cut removes the 10 central modules around the central hole of the HyCal and the dead module area. These modules were discarded because they have worse energy and position resolution compared with the other modules. The timing cut is 6.5 ns (same for all three targets) because the width $\sigma$ of the time difference distribution is about 1.2 ns , which gives $5.5 \sigma$ of about 6.5 ns . The timing cut was set to be slightly larger than $5 \sigma$ to include additional background contribution to help study and better understand the fitting procedure used to extract the Compton cross section. The $\Delta \phi$ and $\Delta E$ cuts were set to about $\pm 5 \sigma$ of the distribution (different for the three targets), the $R_{\min }$ cut function was set to be parallel to the fit line of the highest density distribution of the $R_{\min }$ versus energy bins 2-D distribution, the normalization constant for the cut function was set to about 19 cm which includes the tail of the distribution and has the best signal/background ratio, see Figure 5.9. The $\Delta K$ cut was set to about $\pm 4 \sigma$ (different for the three targets) and is designed to limit the pair production background, this is the most sensitive distribution that can separate the background from the Compton events.


Figure 5.9
$R_{\min }$ versus $E_{\gamma}$ distribution, and the red line is the cut applied to select the Compton events.

The cut efficiencies were studied by varying the range of the cuts. For example, the cut efficiency of the $\Delta \phi$ angle is defined as: efficiency $=N_{\text {cut }} / N_{\text {nocut }}$, where $N_{\text {nocut }}$ is the number of events without the $\Delta \phi$ cut but all other cuts applied, and $N_{\text {cut }}$ is the number of events with the $\Delta \phi$ cut along with all other cuts applied. By changing the $\Delta \phi$ cut range from $\pm 10$ degree to $\pm 120$ degree, meanwhile keeping all the other cuts the same, we get the variation of the $\Delta \phi$ cut efficiency as shown in Figure 5.10. The red arrow shows the value of the cut that was used to get the final Compton yield. The same method was used on all other variables, with the cut efficiency results presented from Figure 5.11 to Figure 5.14. These studies gave us the hint where should we cut. The red points in these plots are the simulated Compton events which is our signal, while the blue points are the experiment data which contain some background events. We did not include the timing information in the simulation so the $\Delta T$ cut efficiencies for the simulation were constantly $100 \%$, and the smaller cut range for the data gave us lower cut efficiencies because it excluded some accidental events. Similarly for the other distributions, the efficiencies for the simulation are higher than the data's because simulation dose not have background. We wanted to keep as many Compton events as possible meanwhile excluding maximum number of background events. So we can not cut too tight otherwise we would lose too many good events (learned from red points) and we can not cut too loose otherwise we would have more background events (learned from blue points), that was how these cut efficiency studies drove us to the final cut range decision.
$\Delta \varphi$ Cut Efficiency


Figure 5.10
$\Delta \phi$ cut efficiency as a function of the range of the cut, and the red arrow indicates the value that was used to select the Compton events.


Figure 5.11
$\Delta T$ cut efficiency as a function of the range of the cut, and the red arrow indicates the value that was used to select the Compton events.


Figure 5.12
$R_{\text {min }}$ cut efficiency as a function of the range of the cut, and the red arrow indicates the value that was used to select the Compton events.
$\Delta \mathrm{E}$ Cut Efficiency


Figure 5.13
$\Delta E$ cut efficiency as a function of the range of the cut, and the red arrow indicates the value that was used to select the Compton events.
$\Delta \mathrm{K}$ Cut Efficiency


Figure 5.14
$\Delta K$ cut efficiency as a function of the range of the cut, and the red arrow indicates the value that was used to select the Compton events.

### 5.3 Integrated Cross Section for All Energies

Events were selected using the cuts described above, and histogrammed as distributions of the kinematic energy difference $(\Delta K)$. We get two types of distributions from the data:

1. Raw data distribution - raw data after subtraction of empty target distribution.
2. Background distribution - accidental events. The accidental events mainly came from the pair production background after multiple scattering with the beamline elements, which usually have a delay when they reach the HyCal compared with the Compton events. So we can select these events by applying the timing cut ( $\pm 6.5 \mathrm{~ns}$ ) and we choose the events with $|\Delta T|>6.5 \mathrm{~ns}$ as the accidental events.

And from the Monte Carlo simulation described in Chapter 4 we get two distributions :

1. Signal distribution - simulated Compton events
2. Background distribution - simulated pair-production events

The data distributions were then fitted to the simulated distributions for each energy bin. Signal and background distributions were fit to data distributions using maximum likelihood fit (TFraction Fitter in Root). In order to have a better fit to the data, all signal and background distributions had to be shifted to better match the shape of the data distribution. The TFraction Fitter fits the data to the simulation bin by bin and finds the best solution. But all the distributions are approximately Gaussian in shape, making it hard to find a solution without constraining the fitting parameters. One such constraint was to normalize the simulated pair production distribution with the experimental flux and fix its strength parameter $\left(p_{2}(i)\right)$ as: $p_{2}(i)=P_{\text {NIST }} \pm 5 \%$, where $i$ is the energy-bin number and $P_{\text {NIST }}$ is the expected strength in $\%$ as determined from the NIST database for the pair production cross section [44] and the $\pm 5 \%$ accounts for the $5 \%$ total error quoted by the NIST database. The second constraint was on the strength parameter for the accidental background distribution, $p_{1}(i)$, which was fixed to be $p_{1}=C(i) \pm 0.01$, where $C(i)$ is the percentage of accidental events in the data for the i-th energy bin. Now we can extract the Compton yield, where "yield" means the total number of events.

The fitted distributions for all energy bins and the three targets are shown in Figure 5.15, Figure 5.16 and Figure 5.17.


Figure 5.15
Yield fit for ${ }^{12} \mathrm{C}$-I target.


Figure 5.16
Yield fit for ${ }^{12} \mathrm{C}$-II target.


Figure 5.17
Yield fit for ${ }^{28}$ Si target.

Fitting the simulated distributions to the data gives us 3 parameters ( $p_{0}, p_{1}$ and $p_{2}$ ) such that the yield from the data is given by :

$$
\begin{equation*}
Y_{f i t}=Y_{C}+Y_{a c c}+Y_{P}=p_{0} Y_{d a t a}+p_{1} Y_{d a t a}+p_{2} Y_{d a t a} \tag{5.4}
\end{equation*}
$$

where $Y_{C}$ is the simulated Compton yield, $Y_{a c c}$ is the yield from accidental events and $Y_{P}$ is the yield from the pair production simulation. Ideally, for a perfect fit, $Y_{\text {data }}=$ $Y_{\text {fit }}$, that gives us $p_{0} Y_{C}=Y_{\text {data }}\left(1-p_{1}-p_{2}\right)$. However, as seen in the plots shown in Figure 5.15 ,Figure 5.16 and Figure 5.17, the fits do not exactly match the data, especially at the shoulders of the distributions around $\pm 0.1$ to $\pm 0.3 \mathrm{GeV}$. Therefore, we use Eq.

$$
\begin{equation*}
Y_{\text {Compton }}=Y_{\text {data }}\left(1-p_{1}-p_{2}\right) \tag{5.5}
\end{equation*}
$$

to calculate the Compton yield $Y_{\text {Compton }}$. From the fit results for the three targets, one can see that the pair production background is about $3 \%$ for the ${ }^{12} C-I$ target, $4 \%$ for the ${ }^{12} \mathrm{C}-I I$ target and $12 \%$ for the ${ }^{28} \mathrm{Si}$ target. That is because the pair production cross section on silicon target is about three times higher than the one on the carbon target.

Using the Eq. (1.28) from Section (1.3), we can calculate the acceptance, which is equal to the number of events that were reconstructed on $\mathrm{HyCal}\left(N_{\text {simulation }}\right)$ divided by the number of events that were generated in the event generator ( $N_{0}^{\text {generator }}$ ), then using Eq. (1.26) we can get the integrated cross section for all energies and the three targets. The cross section results are listed in Table (5.2) with statistic uncertainties only.

Table 5.2
Integrated Cross Section for all targets

| Target | ${ }^{12} \mathbf{C}-\mathbf{I}$ <br> $(m b)$ | ${ }^{12} \mathbf{C}-\mathbf{I I}$ <br> $(m b)$ | ${ }^{28} \mathbf{S i}$ <br> $(m b)$ |
| :--- | :---: | :---: | :---: |
| Cross Section | $0.2806(3)$ | $0.2826(6)$ | $0.2806(11)$ |

The disagreement between the fitted $\Delta K$ distribution and the data indicates that either the pair production or the Compton simulation distribution may not represent the real shape. The pair production simulation was studied using Geant4 and Geant3 toolkits with their default pair production event generators and the results agreed with each other within $0.1 \%$ and were also consistent with the NIST database [44]. Even so, the tails of the pair production distributions that typically fall outside the HyCal acceptance can end up within the acceptance after multiple scattering. This would lead to incorrect reconstruct-
ed momentum and hence incorrect $\Delta K$ distributions for the pair production background. These contributions have not been studied well and introduce some uncertainty in our understanding of the pair production background. Besides, the Compton simulation may also have some uncertainty in the energy and position information. Therefore, the mean value and the width $(\sigma)$ of the simulated Compton distributions were adjusted in order to better fit the data. Figure 5.18, Figure 5.19 and Figure 5.20 show the original simulation and the adjusted simulation distributions for all three targets.


Figure 5.18
$\Delta K$ distributions of original simulation (blue) and adjusted simulation(yellow) for ${ }^{12} \mathrm{C}-\mathrm{I}$ target and the fit lines for both distributions using Gaussian distribution (red).


Figure 5.19
$\Delta K$ distributions of original simulation (blue) and adjusted simulation(yellow) for ${ }^{12} \mathrm{C}$-II target and the fit lines for both distributions using Gaussian distribution (red).


Figure 5.20
$\Delta K$ distributions of original simulation (blue) and adjusted simulation(yellow) for ${ }^{28} \mathrm{Si}$ target and the fit lines for both distributions using Gaussian distribution (red).

After adjusting the $\Delta K$ distributions for the simulated Compton events the fitting procedure was repeated and the new results are shown from Figure A. 1 to Figure 5.23. One can see the improvement of the reduced $\chi^{2}$ for these distributions. For example, for ${ }^{12} \mathrm{C}-\mathrm{I}$ target it improved from 29.7 to 4.4. The integrated cross section was recalculated from these new fits and the results (statistic uncertainties only) are listed in Table 5.3.


Figure 5.21
${ }^{12} \mathrm{C}$-I target $\Delta K$ distribution for all energy bins.
$\Delta \mathrm{K}$ Distribution For All Energy Bins


Figure 5.22
${ }^{12} \mathrm{C}$-II target $\Delta K$ distribution for all energy bins.


Figure 5.23
${ }^{28}$ Si target $\Delta K$ distribution for all energy bins.

Table 5.3

Integrated Cross Section for all targets

| Target | ${ }^{12} \mathbf{C}-\mathbf{I}$ <br> $(\mathrm{mb})$ | ${ }^{12} \mathbf{C}$-II <br> $(\mathrm{mb})$ | ${ }^{28} \mathbf{S i}$ <br> $(\mathrm{mb})$ |
| :--- | :---: | :---: | :---: |
| Cross Section | $0.2806(3)$ | $0.2824(6)$ | $0.2809(11)$ |

### 5.4 Integrated Cross Section for each energy bin

The consistency of the cross sections for the three different targets for all energies gave us the confidence about the event selection methods and the fitting procedures, the cross section value itself had no physical meaning at the point because the "blind number applied to the photon beam flux", so we can not judge. Then we can further divide the data into 18 energy bins, used the same event selection methods and fitting procedures which are described above to extract the Compton yield and calculate the cross sections for each energy bin. The following sub-sections describe the fit results of the $\Delta K$ distributions for the 18 energy bins.

### 5.4.1 Yield Extraction for ${ }^{12} \mathbf{C}$-I, ${ }^{12} \mathbf{C}$-II and ${ }^{28}$ Si Target

Examples of the fit histograms for the first energy bin and the three targets are shown in Figure 5.24, Figure 5.25 and Figure 5.26 as examples. The fit histograms for each of the remaining 17 energy bins and the three targets are shown in Appendix A. For the energy bin 01 of the Carbon-I target, the fit result gave us the Compton signal is $96.79 \%$, the pair
production background is $2.44 \%$ and the accidental background is $0.80 \%$. So the fit result is $96.79+2.44+0.80=100.03$, which means the difference between the fit yield and the data is about $0.03 \%$. And these fraction numbers vary for the different energy bins because the accidental background are different for each energy bin.


Figure 5.24
Yield fit for the ${ }^{12} \mathrm{C}$-I target and energy bin 1.


Figure 5.25
Yield fit for the ${ }^{12} \mathrm{C}$-II target and energy bin 1.


Figure 5.26
Yield fit for the ${ }^{28}$ Si target and energy bin 1.

### 5.4.2 Acceptance

Compton simulation data were used to calculate the acceptance factors as:

$$
\begin{equation*}
A(i)=\frac{N_{\text {simulation }}(i)}{N_{0}^{\text {generate }}(i)} \tag{5.6}
\end{equation*}
$$

where $N_{\text {simulation }}(i)$ is the number of events reconstructed and accepted on HyCal for a given energy bin $(i), N_{0}^{\text {generate }}(i)$ is the number of generated events for the given energy bin $(i)$.

The Figure 5.27 to Figure 5.29 show the acceptance value for different energy bins for all three targets.


Figure 5.27
Acceptance for different energy bins and the ${ }^{12} \mathrm{C}-\mathrm{I}$ target.


Figure 5.28
Acceptance for different energy bins and the ${ }^{12} \mathrm{C}$-II target.


Figure 5.29
Acceptance for different energy bins and the ${ }^{28}$ Si target.

### 5.4.3 Accidentals

By using events in the tails of the time difference distribution, one can estimate the percentage of accidental coincidence events in the data, as shown in Figure 5.30. The percentage of accidental coincidences is given by:

$$
\begin{equation*}
C_{i}^{\text {accidental }}=\frac{p_{i}^{f i t} \times N_{i}^{\text {bins }}}{M_{i}^{\text {data }}} \tag{5.7}
\end{equation*}
$$

where, $i$ is the energy bin number from 1 to $18, C_{i}^{\text {accidental }}$ is percentage of accidentals for a given energy bin $(i)$, and $p_{i}^{f i t}$ is the fitting parameter for a given $\Delta T$ cut, for example the range shown by the red arrows in Figure 5.30, which corresponds to the average number of events in each bin, $N_{i}^{\text {bins }}$ is the number of bins in the $\Delta T$ cut range ( $\pm 6.5 n s$ ), and $M_{i}^{\text {data }}$ is the total number of events in the $\Delta T$ cut range. Figure 5.31 to Figure 5.33 show the $C_{i}^{\text {accidental }}$ values versus energy for different energy bins and the three targets.


Figure 5.30
$\Delta T$ distribution for energy bin 1 and the ${ }^{12} \mathrm{C}$-I target.


Figure 5.31
$C_{i}^{\text {accidental }}$ for different energy bins and the ${ }^{12} \mathrm{C}$-I target.


Figure 5.32
$C_{i}^{\text {accidental }}$ for different energy bins and the ${ }^{12} \mathrm{C}$-II target.


Figure 5.33
$C_{i}^{\text {accidental }}$ for different energy bins and the ${ }^{28}$ Si target.

### 5.4.4 HyCal Response Function Correction

The response function of HyCal was studied during the calibration runs of the PrimExII experiment. In the calibration runs, we centered the beam on each module one by one to study the efficiency and the calibration constant for that module. Since we know the beam energy $E_{\text {Tagger }}$, and we have the reconstructed cluster energy from HyCal $E_{H y C a l}$, we can calculate the ratio $E_{H y C a l} / E_{\text {Tagger }}$, also known as the elasticity (see Figure 5.34). We call the elasticity distribution as the HyCal response function. We can also have the response function from the simulation data (see Figure 5.35). We split elasticity into four regions $(x=0-0.2, x=0.2-0.5, x=0.5-0.8, x=0.8-0.9)$ to study the difference between the calibration run data and the simulation. For a certain module, if the number of events in the calibration data was less than the number of events in the simulated data for the same elasticity region, it was considered a leakage of this module. The overall leakage of a HyCal module is about $0.45 \%$ with an estimate systematic uncertainty $0.50 \%$, therefore, in the cross section calculation we put a Hycal response function correction which is $C_{H R F}=1-0.0045=0.9955 \pm 0.0050$


Figure 5.34
HyCal response function from calibration data for module W2016.


Figure 5.35
HyCal response function from simulated data for module W2016.

### 5.4.5 Cross Sections

The integrated radiative cross section for each energy bin is calculated using:

$$
\begin{equation*}
\sigma_{i}=\frac{1}{n_{e} \Gamma_{\gamma}^{i}} \frac{Y_{i}^{\text {Compton }}}{A_{i} C_{H R F}} \tag{5.8}
\end{equation*}
$$

where,

- $i$ is the energy bin number from 1 to 18 ,
- $n_{e}$ is the number of electrons per $\mathrm{cm}^{2}$, and for each target is given by:
$n_{e}^{C-I}=2.1236(4) \times 6.0221(0) \times 10^{23} / 2=6.3943(13) \times 10^{23}$,
$n_{e}^{C-I I}=3.5304(7) \times 6.0221(0) \times 10^{23} / 2=1.0630(4) \times 10^{24}$,
$n_{e}^{S i}=2.3195(80) \times 6.0221(0) \times 10^{23} / 2=6.9842(244) \times 10^{23}$,
- $\Gamma_{\gamma}^{i}$ is the experimental photon flux as described in Section (5.1),
- $Y_{i}^{\text {Compton }}$ is the experimental yield extracted in previous Subsection as described in Section (5.4.1),
- $A_{i}$ is the simulated acceptance factors as described in Section (5.4.2),
- $C_{H R F}$ is the HyCal response function correction, which is $0.9955(50)$.

Using Eq. 5.7 and incorporating the corrections to the HyCal response function, we get the integrated radiative cross section for each energy bin and the three targets. The results for each energy bin and associate uncertainties (syst. and stat.) are listed in Table 5.4, Table 5.5 and Table 5.6, all the numbers that used to calculate the cross sections and associate uncertainties (syst. and stat.) are also listed in these tables. These cross sections are measured integrated radiative cross sections and we wanted to compare with the theoretical predictions which are the Klein-Nishina cross sections with higher order corrections. The calculations can be found in Section 4.1.1, we used the fitted third degree polynomial as
the "theory" (red line in Figure 5.36, Figure 5.39 and Figure 5.42) to compare with the measured cross sections (blue dots in Figure 5.36, Figure 5.39 and Figure 5.42). Then we can calculate the ratio of the experimental results to the theoretical predictions for the integrated cross section as a function of energies and the three targets by using:

$$
\begin{equation*}
\text { ratio }=\sigma_{\text {experiment }} / \sigma_{\text {theory }}-1 \tag{5.9}
\end{equation*}
$$

and the results are shown in Figure 5.37, Figure 5.40 and Figure 5.43. We can learn that the experimental results are agreed with the theoretical prediction within $2 \%$ for all energy bins. We also projected these ratio results on the Y-axis (see Figure 5.38, Figure 5.41 and Figure 5.44), we can get the RMS values : $0.70 \%, 1.06 \%$ and $1.15 \%$ for the ${ }^{12} \mathrm{C}-\mathrm{I}$, ${ }^{12} \mathrm{C}-\mathrm{II}$ and ${ }^{28} \mathrm{Si}$ targets, respectively, the fitted $\sigma$ values with Gaussian functions: $0.75 \%$, $0.76 \%$ and $0.84 \%$, respectively. These numbers gave us the information that how well the experimental results are agreed with the theory.


Figure 5.36

Comparison of the experimental results to the theoretical calculations for the ${ }^{12} \mathrm{C}$-I target.
Table 5.4
All the numbers used in the ${ }^{12} \mathrm{C}$-I target cross section calculation

| Energy Bin | Yield <br> $Y_{\text {Compton }}$ | Fit Par. <br> $\left(p_{0}\right)$ | Fit Par. <br> $\left(p_{1}\right)$ | Fit Par. <br> $\left(p_{2}\right)$ | Flux <br> $\left(\times 10^{9}\right)$ | Acceptance | Cross Section <br> $(\mathrm{mb})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | $32729(717)$ | $0.9679(1)$ | $0.0080(0)$ | $0.0244(0)$ | $1.3485(134)$ | $0.1493(4)$ | $0.2578(65)$ |
| E02 | $38651(376)$ | $0.9660(1)$ | $0.0086(0)$ | $0.0257(0)$ | $1.5863(141)$ | $0.1501(4)$ | $0.2574(39)$ |
| E03 | $41571(372)$ | $0.9666(1)$ | $0.0081(0)$ | $0.0255(0)$ | $1.6705(137)$ | $0.1509(4)$ | $0.2614(37)$ |
| E04 | $40333(593)$ | $0.9639(1)$ | $0.0081(0)$ | $0.0281(0)$ | $1.5935(135)$ | $0.1520(4)$ | $0.2641(49)$ |
| E05 | $43124(601)$ | $0.9638(1)$ | $0.0075(0)$ | $0.0286(0)$ | $1.6871(138)$ | $0.1529(4)$ | $0.2650(47)$ |
| E06 | $47761(351)$ | $0.9658(1)$ | $0.0071(0)$ | $0.0270(0)$ | $1.8276(148)$ | $0.1540(4)$ | $0.2691(35)$ |
| E07 | $52634(1026)$ | $0.9650(1)$ | $0.0068(0)$ | $0.0282(0)$ | $1.9705(160)$ | $0.1552(4)$ | $0.2729(61)$ |
| E08 | $49322(915)$ | $0.9657(1)$ | $0.0065(0)$ | $0.0279(0)$ | $1.8133(154)$ | $0.1563(4)$ | $0.2760(60)$ |
| E09 | $50319(1024)$ | $0.9659(1)$ | $0.0057(0)$ | $0.0283(0)$ | $1.8274(148)$ | $0.1575(4)$ | $0.2772(64)$ |
| E10 | $47903(562)$ | $0.9644(1)$ | $0.0067(0)$ | $0.0289(0)$ | $1.7065(137)$ | $0.1583(4)$ | $0.2810(45)$ |
| E11 | $54511(721)$ | $0.9632(1)$ | $0.0067(0)$ | $0.0300(0)$ | $1.8903(151)$ | $0.1594(4)$ | $0.2868(49)$ |
| E12 | $43703(553)$ | $0.9613(1)$ | $0.0075(0)$ | $0.0311(0)$ | $1.5150(123)$ | $0.1609(4)$ | $0.2842(48)$ |
| E13 | $23615(287)$ | $0.9587(1)$ | $0.0086(0)$ | $0.0326(0)$ | $0.7827(061)$ | $0.1622(4)$ | $0.2949(50)$ |
| E14 | $54757(397)$ | $0.9595(1)$ | $0.0100(0)$ | $0.0304(0)$ | $1.8114(143)$ | $0.1633(4)$ | $0.2934(38)$ |
| E15 | $59983(629)$ | $0.9534(1)$ | $0.0120(0)$ | $0.0341(0)$ | $1.9283(150)$ | $0.1644(4)$ | $0.2999(45)$ |
| E16 | $54536(405)$ | $0.9533(1)$ | $0.0130(0)$ | $0.0335(0)$ | $1.7338(139)$ | $0.1655(4)$ | $0.3013(39)$ |
| E17 | $63047(537)$ | $0.9523(1)$ | $0.0140(0)$ | $0.0338(0)$ | $1.9689(152)$ | $0.1666(4)$ | $0.3047(41)$ |
| E18 | $48170(424)$ | $0.9520(1)$ | $0.0150(0)$ | $0.0331(0)$ | $1.4775(117)$ | $0.1678(4)$ | $0.3080(43)$ |
| ALL | $843561(5941)$ | $0.9577(1)$ | $0.0092(0)$ | $0.0333(0)$ | $30.1390(2471)$ | $0.1581(4)$ | $0.2806(34)$ |

Table 5.5
All the numbers used in the ${ }^{12} \mathrm{C}$-II target cross section calculation

| Energy Bin | Yield <br> $Y_{\text {Compton }}$ | Fit Par. <br> $\left(p_{0}\right)$ | Fit Par. <br> $\left(p_{1}\right)$ | Fit Par. <br> $\left(p_{2}\right)$ | Flux <br> $\left(\times 10^{9}\right)$ | Acceptance | Cross Section <br> $(m b)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | $9468(98)$ | $0.9645(1)$ | $0.0058(0)$ | $0.0299(0)$ | $0.2438(24)$ | $0.1441(4)$ | $0.2571(48)$ |
| E02 | $11064(201)$ | $0.9660(1)$ | $0.0052(0)$ | $0.0294(0)$ | $0.2830(25)$ | $0.1448(4)$ | $0.2576(60)$ |
| E03 | $11942(221)$ | $0.9638(1)$ | $0.0052(0)$ | $0.0311(0)$ | $0.2984(24)$ | $0.1456(4)$ | $0.2621(60)$ |
| E04 | $11643(288)$ | $0.9642(1)$ | $0.0056(0)$ | $0.0297(0)$ | $0.2829(24)$ | $0.1466(4)$ | $0.2678(85)$ |
| E05 | $12583(119)$ | $0.9622(1)$ | $0.0046(0)$ | $0.0332(0)$ | $0.3035(25)$ | $0.1477(4)$ | $0.2678(44)$ |
| E06 | $13822(280)$ | $0.9622(1)$ | $0.0041(0)$ | $0.0336(0)$ | $0.3280(27)$ | $0.1485(4)$ | $0.2706(65)$ |
| E07 | $14805(201)$ | $0.9624(1)$ | $0.0047(0)$ | $0.0326(0)$ | $0.3533(29)$ | $0.1495(4)$ | $0.2673(50)$ |
| E08 | $14282(258)$ | $0.9621(1)$ | $0.0045(0)$ | $0.0333(0)$ | $0.3261(28)$ | $0.1507(4)$ | $0.2772(62)$ |
| E09 | $14450(196)$ | $0.9610(1)$ | $0.0046(0)$ | $0.0341(0)$ | $0.3258(26)$ | $0.1516(4)$ | $0.2790(52)$ |
| E10 | $13797(136)$ | $0.9606(1)$ | $0.0043(0)$ | $0.0346(0)$ | $0.3062(24)$ | $0.1527(4)$ | $0.2814(46)$ |
| E11 | $15501(390)$ | $0.9599(1)$ | $0.0044(0)$ | $0.0351(0)$ | $0.3355(27)$ | $0.1537(4)$ | $0.2867(81)$ |
| E12 | $12973(208)$ | $0.9584(1)$ | $0.0035(0)$ | $0.0376(0)$ | $0.2720(22)$ | $0.1550(4)$ | $0.2935(57)$ |
| E13 | $6692(105)$ | $0.9568(1)$ | $0.0042(0)$ | $0.0387(0)$ | $0.1392(11)$ | $0.1563(4)$ | $0.2933(69)$ |
| E14 | $16035(238)$ | $0.9537(1)$ | $0.0080(0)$ | $0.0380(0)$ | $0.3237(26)$ | $0.1574(4)$ | $0.3002(59)$ |
| E15 | $17337(348)$ | $0.9531(1)$ | $0.0082(0)$ | $0.0386(0)$ | $0.3448(27)$ | $0.1583(4)$ | $0.3029(71)$ |
| E16 | $15700(219)$ | $0.9532(1)$ | $0.0073(0)$ | $0.0393(0)$ | $0.3100(25)$ | $0.1595(4)$ | $0.3029(57)$ |
| E17 | $18212(346)$ | $0.9521(1)$ | $0.0088(0)$ | $0.0393(0)$ | $0.3541(27)$ | $0.1605(4)$ | $0.3057(69)$ |
| E18 | $13822(242)$ | $0.9480(1)$ | $0.0100(0)$ | $0.0412(0)$ | $0.2617(21)$ | $0.1616(4)$ | $0.3118(68)$ |
| ALL | $243402(2194)$ | $0.9558(1)$ | $0.0060(0)$ | $0.0381(0)$ | $5.3918(442)$ | $0.1525(4)$ | $0.2824(38)$ |

Table 5.6

| Energy Bin | Yield <br> $Y_{\text {Compton }}$ | Fit Par. <br> $\left(p_{0}\right)$ | Fit Par. <br> $\left(p_{1}\right)$ | Fit Par. <br> $\left(p_{2}\right)$ | Flux <br> $\left(\times 10^{9}\right)$ | Acceptance | Cross Section <br> $(\mathrm{mb})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | $3014(51)$ | $0.8918(1)$ | $0.0025(0)$ | $0.1051(0)$ | $0.1187(12)$ | $0.1423(4)$ | $0.2592(74)$ |
| E02 | $3446(64)$ | $0.8860(1)$ | $0.0035(0)$ | $0.1083(0)$ | $0.1374(12)$ | $0.1429(4)$ | $0.2548(72)$ |
| E03 | $3776(76)$ | $0.8890(1)$ | $0.0039(0)$ | $0.1051(0)$ | $0.1456(12)$ | $0.1437(4)$ | $0.2620(75)$ |
| E04 | $3545(61)$ | $0.8871(1)$ | $0.0024(0)$ | $0.1092(0)$ | $0.1370(12)$ | $0.1446(4)$ | $0.2598(70)$ |
| E05 | $4061(84)$ | $0.8866(1)$ | $0.0018(0)$ | $0.1095(0)$ | $0.1475(12)$ | $0.1454(4)$ | $0.2748(78)$ |
| E06 | $4367(81)$ | $0.8786(1)$ | $0.0021(0)$ | $0.1155(0)$ | $0.1593(13)$ | $0.1464(4)$ | $0.2718(72)$ |
| E07 | $4807(64)$ | $0.8830(1)$ | $0.0029(0)$ | $0.1122(0)$ | $0.1719(14)$ | $0.1473(4)$ | $0.2755(63)$ |
| E08 | $4439(61)$ | $0.8809(1)$ | $0.0029(0)$ | $0.1130(0)$ | $0.1586(13)$ | $0.1484(4)$ | $0.2738(65)$ |
| E09 | $4557(77)$ | $0.8855(1)$ | $0.0026(0)$ | $0.1107(0)$ | $0.1587(13)$ | $0.1495(4)$ | $0.2789(71)$ |
| E10 | $4413(77)$ | $0.8814(1)$ | $0.0019(0)$ | $0.1135(0)$ | $0.1490(12)$ | $0.1504(4)$ | $0.2859(74)$ |
| E11 | $4810(75)$ | $0.8722(1)$ | $0.0027(0)$ | $0.1201(0)$ | $0.1626(13)$ | $0.1513(4)$ | $0.2838(69)$ |
| E12 | $3960(70)$ | $0.8720(1)$ | $0.0039(0)$ | $0.1181(0)$ | $0.1320(11)$ | $0.1525(4)$ | $0.2854(76)$ |
| E13 | $2103(30)$ | $0.8751(1)$ | $0.0041(0)$ | $0.1192(0)$ | $0.0675(05)$ | $0.1537(4)$ | $0.2943(86)$ |
| E14 | $4927(78)$ | $0.8750(1)$ | $0.0024(0)$ | $0.1172(0)$ | $0.1573(12)$ | $0.1548(4)$ | $0.2938(71)$ |
| E15 | $5405(105)$ | $0.8756(1)$ | $0.0040(0)$ | $0.1173(0)$ | $0.1677(13)$ | $0.1555(4)$ | $0.3008(79)$ |
| E16 | $4871(84)$ | $0.8713(1)$ | $0.0055(0)$ | $0.1181(0)$ | $0.1499(12)$ | $0.1565(4)$ | $0.3014(76)$ |
| E17 | $5707(112)$ | $0.8729(1)$ | $0.0058(0)$ | $0.1186(0)$ | $0.1723(13)$ | $0.1575(4)$ | $0.3052(80)$ |
| E18 | $4235(85)$ | $0.8710(1)$ | $0.0061(0)$ | $0.1180(0)$ | $0.1274(10)$ | $0.1586(4)$ | $0.3043(85)$ |
| ALL | $76099(1100)$ | $0.8745(1)$ | $0.0034(0)$ | $0.1181(0)$ | $2.6202(215)$ | $0.1501(4)$ | $0.2809(51)$ |



Figure 5.37
Ratio of the experimental result to the theoretical prediction for the integrated cross section as a function of energy for the ${ }^{12} \mathrm{C}$-I target.


Figure 5.38
Percent deviation of experiment from theory for the ${ }^{12} \mathrm{C}$-I target.


Figure 5.39
Comparison of the experimental results to the theoretical calculation for the ${ }^{12} \mathrm{C}$-II target.


Figure 5.40
Ratio of the experimental result to the theoretical prediction for the integrated cross section as a function of energy for the ${ }^{12} \mathrm{C}$-II target.


Figure 5.41
Percent deviation of experiment from theory for the ${ }^{12} \mathrm{C}$-II target.


Figure 5.42
Comparison of the experimental results to the theoretical calculation for the ${ }^{28} \mathrm{Si}$ target.


Figure 5.43
Ratio of the experimental result to the theoretical prediction for the integrated cross section as a function of energy for the ${ }^{28} \mathrm{Si}$ target.


Figure 5.44
Percent deviation of experiment from theory for the ${ }^{28} \mathrm{Si}$ target.

### 5.5 Systematic Uncertainties

In order to determine the Compton cross section with highest possible precision, various systematic studies were performed. These studies include:

- photon beam flux,
- target density,
- acceptance,
- hyCal response function,
- cut Stability,
- signal/background separation.


### 5.5.1 Systematic Uncertainties in Photon Beam Flux

In order to get the photon flux uncertainty, the tagging ratio was measured during the "TAC (total absorption counter) run". The TAC run was a special run during the experiment to measure the photon flux by using hardware scalers counting the number of tagged photons incident on the physics target. The tagging ratio was calculated as a fraction of events within the selected set which have a TAC TDC signal close to tagger time (within $\pm 3 n s)$ [35]. As an alternative cut to select events we tried to require TAC energy deposition to be greater than $20 \%$ of the beam energy reported by tagger. We also got results without cutting out beam trips to check our sensitivity to them. There were 4 contributions studied for systematic uncertainty in the tagging ratio [35]:

1. the major contribution is from the long-term stability of the tagging ratio;
2. systematics associated with TAC geometry alignment, this was studied with data from run 64996 when the TDC was moved downstream by 5 cm ;
3. systematics from beam trips;
4. systematics from the ADC value studied by varying the cuts on the ADC.

The final results from the 4 sources listed above for each energy bin, are listed in the second column of the systematic uncertainties table (Table 5.8,Table 5.9 and Table 5.10).

### 5.5.2 Systematic Uncertainties in Target

The details on the target density measurement can be found in Section (2.4), where we had noted that the systematic uncertainty associated with the target density measurement is $0.02 \%, 0.02 \%$, and $0.35 \%$ for the three targets, respectively. Other than that, the systematic uncertainties associated with target thickness and chemical purity were also considered.

All the values are listed in Table 5.7 and they are all the same for different energy bins.

Table 5.7
Systematic uncertainties in target, all values are in \%.

|  | ${ }^{12} \mathbf{C}$-I Target | ${ }^{12} \mathbf{C}$-II Target | ${ }^{28}$ Si Target |
| :--- | :---: | :---: | :---: |
| Target density | 0.02 | 0.02 | 0.35 |
| Target thickness | 0.01 | 0.04 | 0.03 |
| Chemical purity | $<0.01$ | 0.10 | $<0.01$ |
| Total | 0.02 | 0.11 | 0.35 |

### 5.5.3 Systematic Uncertainties in Acceptance Factors

We calculated the acceptance factors by using the Compton simulation data, so there are some uncertainties coming from the geometry difference (for example the distance between target and the HyCal surface) between the simulation and the real experiment, and from the uncertainties of the Compton event generator. This is studied by changing the geometries in the simulation and estimating the error from the event generator. It was estimated to be $0.25 \%$ for all energy bins.

### 5.5.4 Systematic Uncertainties in HyCal Response Function

We estimated the systematic uncertainties in HyCal response function by comparing the data from the calibration runs to the simulated data for different region (see Section (5.4.4)). We had four regions in total for the elasticity distribution (see Fig. 5.34) and we compared the total number of events with the simulated distribution for each region. The largest difference between the calibration data and the simulated data is about $0.5 \%$. Therefore, we took $0.5 \%$ as the systematic uncertainties in HyCal response function.

### 5.5.5 Systematic Uncertainties in Event Selection

For the event selection systematic errors, there are 5 contributions were studied by varying the range of the event selection cuts.

The systematic uncertainty of the $\Delta T$ cut was studied by varying the $\Delta T$ cut by $\pm 1 \sigma$ while keeping all other cuts the same. The $\sigma$ for the $\Delta T$ is $\sim 1.2 n s$, and the nominal $\Delta T$ cut is $|\Delta T|<6.5 n s$, therefore the cut was varied between 5.3 ns and 7.7 ns . The final cross section was extracted for these 3 different values of the $\Delta T$ cut and largest difference in the cross section from the nominal values was taken as the systematic uncertainty ( $\xi(\Delta T)$ ) due to the $\Delta T$ cut.

Similarly the remaining 4 event selection cuts were varied one by one while keeping all the other cuts fixed. Each cut was varied by $\pm 1 \sigma$, their respective resolutions ( $\Delta \phi \sim$ $4.0 \mathrm{deg}, \Delta E \sim 0.08 \mathrm{GeV}, \Delta K \sim 0.15 \mathrm{GeV}$ and $\left.R_{\min } \sim 0.3 \mathrm{~cm}\right)$, and the cross section was recalculated. The largest difference in the cross section from the nominal value for each of these cuts was taken as the systematic uncertainty $\left(\xi\left(R_{\text {min }}\right), \xi(\Delta \phi), \xi(\Delta E), \xi(\Delta K)\right)$ due to that cut.

### 5.5.6 Systematic Uncertainties for Signal/Background Separation

For the signal/background separation study, there are two main contributions:
(1) The fitting error study: because we used an adjusted distribution to do the fitting, there are some uncertainties due to the difference in shape of the distribution, and these can be quantified in terms of the mean value $\mu$ and the standard deviation $\sigma$. For each energy bin $i$, the fit was redone after changing the final mean value $\mu_{\text {reshape }}$ by $\pm 1 \mu_{\text {error }}$ where $\mu_{\text {error }}$ is the fitting uncertainty obtained from the default TFraction Fitter in Root. The largest difference between the nominal cross section and the refitted cross section
was taken to be the systematic uncertainty due to the fitting error, $\xi(\mu)_{i}$. Using the same procedure we can have the error for standard deviation $\xi(\sigma)_{i}$. Then we have $\xi(\text { Fitting })_{i}=$ $\sqrt{\xi(\mu)_{i}^{2}+\xi(\sigma)_{i}^{2}}$
(2) The background histogram: by changing the total pair production yield $\pm 5 \%$ (NIST precision), we have the error $\xi\left(e^{+} e^{-} M C\right)_{i}$.

Finally, we can calculate the total systematic uncertainty by:
$\xi(\text { total })_{i}=\sqrt{\xi(F l u x)_{i}^{2}+\xi(T g t)_{i}^{2}+\xi(\text { Accp })_{i}^{2}+\xi(\text { Res.Fn. })_{i}^{2}+\xi(C u t s)_{i}^{2}+\xi(S g / B g)_{i}^{2}}$
where

$$
\begin{gather*}
\xi(\text { Cuts })_{i}=\sqrt{\xi(\Delta T)_{i}^{2}+\xi(\Delta \phi)_{i}^{2}+\xi(\Delta E)_{i}^{2}+\xi(\Delta K)_{i}^{2}+\xi\left(R_{\text {min }}\right)_{i}^{2}}  \tag{5.11}\\
\xi(S g / B g)_{i}=\sqrt{\xi(\text { Fitting })_{i}^{2}+\xi\left(e^{+} e^{-} M C\right)_{i}^{2}} \tag{5.12}
\end{gather*}
$$

The cut stability study was also extended to a larger variation in the cut range, the results are shown in Appendix B. Figure 5.45 is the $\Delta T$ cut stability as one example. These results were used to determine the appropriate range of the cuts. Systematic uncertainties for the different targets are listed in Table 5.8, Table 5.9 and Table 5.10.
Table 5.8
Systematic uncertainties for the ${ }^{12} \mathrm{C}$-I target. All values are in \%.

| Energy Bin | Flux | Tgt | Accp | HyCal | Event Selection Cuts |  |  |  | Signal/Bgd |  |  | Uncertainties |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Res.Fn. | $\Delta T$ | $\Delta \phi$ | $\Delta E$ | $\Delta K$ | $R_{\min }$ | Fitting | $e^{+} e^{-}$MC | Syst. | Stat. | Total |
| E01 | 0.99 | 0.02 | 0.25 | 0.50 | 0.99 | 0.97 | 0.99 | 0.97 | 0.97 | 0.01 | 0.12 | 2.46 | 0.56 | 2.53 |
| E02 | 0.89 | 0.02 | 0.25 | 0.50 | 0.31 | 0.31 | 0.61 | 0.52 | 0.31 | 0.01 | 0.13 | 1.43 | 0.52 | 1.52 |
| E03 | 0.82 | 0.02 | 0.25 | 0.50 | 0.22 | 0.28 | 0.59 | 0.50 | 0.24 | 0.01 | 0.13 | 1.34 | 0.50 | 1.43 |
| E04 | 0.85 | 0.02 | 0.25 | 0.50 | 0.59 | 0.62 | 0.73 | 0.75 | 0.56 | 0.01 | 0.14 | 1.79 | 0.51 | 1.86 |
| E05 | 0.82 | 0.02 | 0.25 | 0.50 | 0.57 | 0.59 | 0.68 | 0.69 | 0.56 | 0.01 | 0.14 | 1.71 | 0.49 | 1.78 |
| E06 | 0.81 | 0.02 | 0.25 | 0.50 | 0.11 | 0.18 | 0.46 | 0.51 | 0.08 | 0.01 | 0.13 | 1.23 | 0.47 | 1.31 |
| E07 | 0.81 | 0.02 | 0.25 | 0.50 | 0.81 | 0.86 | 0.91 | 0.95 | 0.81 | 0.01 | 0.14 | 2.18 | 0.44 | 2.23 |
| E08 | 0.85 | 0.02 | 0.25 | 0.50 | 0.79 | 0.81 | 0.86 | 0.88 | 0.79 | 0.01 | 0.14 | 2.12 | 0.46 | 2.17 |
| E09 | 0.81 | 0.02 | 0.25 | 0.50 | 0.85 | 0.87 | 0.99 | 0.96 | 0.86 | 0.01 | 0.14 | 2.26 | 0.45 | 2.30 |
| E10 | 0.80 | 0.02 | 0.25 | 0.50 | 0.42 | 0.44 | 0.61 | 0.67 | 0.41 | 0.01 | 0.14 | 1.53 | 0.47 | 1.60 |
| E11 | 0.80 | 0.02 | 0.25 | 0.50 | 0.47 | 0.55 | 0.71 | 0.69 | 0.47 | 0.01 | 0.15 | 1.64 | 0.44 | 1.70 |
| E12 | 0.81 | 0.02 | 0.25 | 0.50 | 0.46 | 0.49 | 0.69 | 0.66 | 0.46 | 0.01 | 0.16 | 1.60 | 0.49 | 1.68 |
| E13 | 0.78 | 0.02 | 0.25 | 0.50 | 0.43 | 0.48 | 0.66 | 0.63 | 0.45 | 0.01 | 0.16 | 1.55 | 0.67 | 1.69 |
| E14 | 0.79 | 0.02 | 0.25 | 0.50 | 0.18 | 0.20 | 0.44 | 0.47 | 0.13 | 0.01 | 0.15 | 1.21 | 0.44 | 1.28 |
| E15 | 0.78 | 0.02 | 0.25 | 0.50 | 0.42 | 0.40 | 0.53 | 0.55 | 0.39 | 0.01 | 0.17 | 1.43 | 0.42 | 1.49 |
| E16 | 0.80 | 0.02 | 0.25 | 0.50 | 0.18 | 0.20 | 0.43 | 0.50 | 0.12 | 0.01 | 0.17 | 1.22 | 0.44 | 1.30 |
| E17 | 0.77 | 0.02 | 0.25 | 0.50 | 0.27 | 0.27 | 0.53 | 0.46 | 0.24 | 0.01 | 0.17 | 1.27 | 0.41 | 1.34 |
| E18 | 0.79 | 0.02 | 0.25 | 0.50 | 0.28 | 0.27 | 0.49 | 0.55 | 0.23 | 0.01 | 0.17 | 1.31 | 0.47 | 1.39 |
| ALL | 0.82 | 0.02 | 0.25 | 0.50 | 0.10 | 0.17 | 0.46 | 0.46 | 0.07 | 0.01 | 0.17 | 1.22 | 0.11 | 1.22 |

Table 5.9

| Energy Bin | Flux | Tgt | Accp | HyCal | Event Selection Cuts |  |  |  | Signal/Bgd |  |  | Uncertainties |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Res.Fn. | $\Delta T$ | $\Delta \phi$ | $\Delta E$ | $\Delta K$ | $R_{\min }$ | Fitting | $e^{+} e^{-}$MC | Syst. | Stat. | Total |
| E01 | 0.99 | 0.11 | 0.25 | 0.50 | 0.13 | 0.29 | 0.83 | 0.47 | 0.22 | 0.01 | 0.15 | 1.54 | 1.050 | 1.87 |
| E02 | 0.89 | 0.11 | 0.25 | 0.50 | 0.86 | 0.70 | 0.96 | 0.77 | 0.74 | 0.01 | 0.15 | 2.10 | 0.968 | 2.32 |
| E03 | 0.82 | 0.11 | 0.25 | 0.50 | 0.92 | 0.76 | 0.91 | 0.78 | 0.74 | 0.01 | 0.16 | 2.10 | 0.932 | 2.30 |
| E04 | 0.85 | 0.11 | 0.25 | 0.50 | 1.00 | 1.11 | 1.20 | 1.12 | 1.09 | 0.01 | 0.15 | 2.68 | 0.944 | 2.84 |
| E05 | 0.82 | 0.11 | 0.25 | 0.50 | 0.25 | 0.12 | 0.76 | 0.42 | 0.18 | 0.01 | 0.17 | 1.37 | 0.909 | 1.65 |
| E06 | 0.81 | 0.11 | 0.25 | 0.50 | 0.74 | 0.91 | 0.92 | 0.96 | 0.96 | 0.01 | 0.17 | 2.25 | 0.867 | 2.41 |
| E07 | 0.81 | 0.11 | 0.25 | 0.50 | 0.31 | 0.54 | 0.93 | 0.60 | 0.45 | 0.01 | 0.16 | 1.68 | 0.838 | 1.88 |
| E08 | 0.85 | 0.11 | 0.25 | 0.50 | 0.90 | 0.78 | 0.72 | 0.84 | 0.77 | 0.01 | 0.17 | 2.08 | 0.853 | 2.24 |
| E09 | 0.81 | 0.11 | 0.25 | 0.50 | 0.61 | 0.42 | 0.86 | 0.58 | 0.44 | 0.01 | 0.17 | 1.68 | 0.848 | 1.88 |
| E10 | 0.80 | 0.11 | 0.25 | 0.50 | 0.35 | 0.28 | 0.70 | 0.45 | 0.22 | 0.01 | 0.17 | 1.39 | 0.868 | 1.64 |
| E11 | 0.80 | 0.11 | 0.25 | 0.50 | 1.20 | 1.12 | 1.06 | 1.11 | 1.11 | 0.01 | 0.18 | 2.70 | 0.820 | 2.82 |
| E12 | 0.81 | 0.11 | 0.25 | 0.50 | 0.64 | 0.56 | 0.84 | 0.59 | 0.44 | 0.01 | 0.19 | 1.73 | 0.897 | 1.95 |
| E13 | 0.78 | 0.11 | 0.25 | 0.50 | 0.82 | 0.70 | 0.84 | 0.81 | 0.67 | 0.01 | 0.19 | 1.99 | 1.250 | 2.35 |
| E14 | 0.79 | 0.11 | 0.25 | 0.50 | 0.43 | 0.64 | 0.85 | 0.78 | 0.50 | 0.01 | 0.19 | 1.78 | 0.808 | 1.95 |
| E15 | 0.78 | 0.11 | 0.25 | 0.50 | 0.96 | 0.80 | 0.93 | 0.91 | 0.86 | 0.01 | 0.19 | 2.23 | 0.778 | 2.36 |
| E16 | 0.80 | 0.11 | 0.25 | 0.50 | 0.38 | 0.51 | 0.85 | 0.75 | 0.46 | 0.01 | 0.20 | 1.70 | 0.817 | 1.89 |
| E17 | 0.77 | 0.11 | 0.25 | 0.50 | 0.72 | 0.93 | 0.93 | 0.73 | 0.89 | 0.01 | 0.20 | 2.13 | 0.759 | 2.26 |
| E18 | 0.79 | 0.11 | 0.25 | 0.50 | 0.79 | 0.64 | 0.95 | 0.79 | 0.68 | 0.01 | 0.21 | 2.00 | 0.873 | 2.19 |
| ALL | 0.82 | 0.11 | 0.25 | 0.50 | 0.22 | 0.19 | 0.70 | 0.44 | 0.09 | 0.01 | 0.19 | 1.35 | 0.207 | 1.36 |

Table 5.10

| Energy Bin | Flux | Tgt | Accp | HyCal | Event Selection Cuts |  |  |  | Signal/Bgd |  |  | Uncertainties |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Res.Fn. | $\Delta T$ | $\Delta \phi$ | $\Delta E$ | $\Delta K$ | $R_{\text {min }}$ | Fitting | $e^{+} e^{-}$MC | Syst. | Stat. | Total |
| E01 | 0.99 | 0.35 | 0.25 | 0.50 | 0.54 | 0.62 | 0.79 | 0.94 | 0.67 | 0.01 | 0.53 | 2.08 | 1.93 | 2.84 |
| E02 | 0.89 | 0.35 | 0.25 | 0.50 | 0.95 | 0.74 | 0.73 | 0.83 | 0.73 | 0.01 | 0.54 | 2.18 | 1.81 | 2.83 |
| E03 | 0.82 | 0.35 | 0.25 | 0.50 | 0.94 | 0.96 | 0.73 | 0.81 | 0.89 | 0.01 | 0.53 | 2.28 | 1.72 | 2.85 |
| E04 | 0.85 | 0.35 | 0.25 | 0.50 | 0.56 | 0.68 | 0.93 | 0.82 | 0.60 | 0.01 | 0.55 | 2.03 | 1.78 | 2.70 |
| E05 | 0.82 | 0.35 | 0.25 | 0.50 | 0.97 | 0.76 | 0.92 | 0.85 | 0.92 | 0.01 | 0.55 | 2.31 | 1.66 | 2.85 |
| E06 | 0.81 | 0.35 | 0.25 | 0.50 | 0.64 | 0.82 | 0.72 | 0.95 | 0.76 | 0.01 | 0.58 | 2.12 | 1.61 | 2.66 |
| E07 | 0.81 | 0.35 | 0.25 | 0.50 | 0.04 | 0.50 | 0.70 | 0.82 | 0.22 | 0.01 | 0.56 | 1.69 | 1.53 | 2.28 |
| E08 | 0.85 | 0.35 | 0.25 | 0.50 | 0.22 | 0.49 | 0.78 | 0.78 | 0.21 | 0.01 | 0.57 | 1.74 | 1.60 | 2.36 |
| E09 | 0.81 | 0.35 | 0.25 | 0.50 | 0.38 | 0.49 | 0.96 | 0.97 | 0.52 | 0.01 | 0.55 | 1.98 | 1.57 | 2.53 |
| E10 | 0.80 | 0.35 | 0.25 | 0.50 | 0.71 | 0.69 | 0.78 | 0.76 | 0.77 | 0.01 | 0.57 | 2.04 | 1.60 | 2.59 |
| E11 | 0.80 | 0.35 | 0.25 | 0.50 | 0.42 | 0.65 | 0.80 | 0.78 | 0.47 | 0.01 | 0.60 | 1.87 | 1.54 | 2.42 |
| E12 | 0.81 | 0.35 | 0.25 | 0.50 | 0.51 | 0.76 | 0.92 | 0.89 | 0.57 | 0.01 | 0.59 | 2.06 | 1.70 | 2.67 |
| E13 | 0.78 | 0.35 | 0.25 | 0.50 | 0.16 | 0.57 | 0.46 | 1.00 | 0.28 | 0.01 | 0.60 | 1.74 | 2.33 | 2.91 |
| E14 | 0.79 | 0.35 | 0.25 | 0.50 | 0.40 | 0.74 | 0.77 | 0.76 | 0.50 | 0.01 | 0.59 | 1.88 | 1.52 | 2.41 |
| E15 | 0.78 | 0.35 | 0.25 | 0.50 | 0.92 | 0.77 | 0.75 | 0.75 | 0.94 | 0.01 | 0.59 | 2.20 | 1.45 | 2.64 |
| E16 | 0.80 | 0.35 | 0.25 | 0.50 | 0.58 | 0.81 | 0.90 | 0.80 | 0.45 | 0.01 | 0.59 | 2.02 | 1.53 | 2.53 |
| E17 | 0.77 | 0.35 | 0.25 | 0.50 | 0.96 | 0.79 | 0.92 | 0.79 | 0.72 | 0.01 | 0.59 | 2.22 | 1.41 | 2.63 |
| E18 | 0.79 | 0.35 | 0.25 | 0.50 | 0.86 | 0.95 | 0.88 | 0.79 | 0.79 | 0.01 | 0.59 | 2.25 | 1.64 | 2.79 |
| ALL | 0.82 | 0.35 | 0.25 | 0.50 | 0.07 | 0.51 | 0.69 | 0.99 | 0.14 | 0.01 | 0.59 | 1.79 | 0.39 | 1.83 |



Figure 5.45
Differences in yield relative to the final result for $\Delta T$ cut and the ${ }^{12} \mathrm{C}$-I target

### 5.6 Extracted Cross Sections

The extracted Compton scattering radiative cross sections for ${ }^{12} \mathrm{C}$ and ${ }^{28} \mathrm{Si}$ targets integrated over all energy bins are listed in Table 5.11, and the cross sections for each energy bin are listed in Table 5.12, Table 5.13 and Table 5.14. The tables also show the theoretical predictions and the deviation of the measured cross sections form the predictions. The statistical uncertainty, the systematic uncertainty and the total uncertainty (quadrature sum of the two) are also shown.

Table 5.11

Integrated cross sections

| Target | Energy <br> $(\mathrm{GeV})$ | Cross Section <br> $(\mathrm{mb})$ | Theory <br> $(\mathrm{mb})$ | Deviation <br> $(\%)$ | Syst. error <br> $(\%)$ | Stat. error <br> $(\%)$ | Total Error <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{12} \mathbf{C}-\mathbf{I}$ | 4.84 | 0.2806 | 0.2822 | -0.57 | $\pm 1.22$ | $\pm 0.11$ | $\pm 1.22$ |
| ${ }^{12} \mathbf{C}-\mathbf{I I}$ | 4.84 | 0.2824 | 0.2822 | 0.19 | $\pm 1.34$ | $\pm 0.21$ | $\pm 1.36$ |
| ${ }^{28} \mathbf{S i}$ | 4.84 | 0.2809 | 0.2822 | -0.46 | $\pm 1.79$ | $\pm 0.39$ | $\pm 1.83$ |

### 5.7 Summary and Conclusions

The Compton scattering cross section measurement in the PrimEx-II experiment successfully extracted the total Compton scattering cross section for $4.4-5.3 \mathrm{GeV}$ photons with less than $2 \%$ uncertainty, which is the first measurement at this energy range to date. The extracted cross section, integrated over all energies, is in agreement with the theoretical prediction at the level of the uncertainty $(1.3 \%-1.8 \%)$ for all three targets.

In this dissertation, the details of the experiment, the data analysis process and the results were presented. These results imply that the higher order corrections to the KleinNishina cross section are in good agreement with the experimental results (within $2 \%$ ) for few GeV photons. It further establishes the systematic uncertainty for the measurement of the $\pi^{0}$ decay width by the PrimEx-II experiment to be less than $2 \%$.
Table 5.12
Cross sections for the ${ }^{12} \mathrm{C}$-I target.

| Energy Bin | Energy <br> $(G e V)$ | Cross Section <br> $(m b)$ | Theory <br> $(m b)$ | Deviation(\%) <br> $(\%)$ | Syst. Error <br> $(\%)$ | Stat. Error <br> $(\%)$ | Total Error <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | 5.281 | 0.2578 | 0.2603 | -0.96 | 2.46 | 0.562 | 2.53 |
| E02 | 5.241 | 0.2574 | 0.2621 | -1.79 | 1.43 | 0.518 | 1.52 |
| E03 | 5.195 | 0.2614 | 0.2642 | -1.05 | 1.34 | 0.499 | 1.43 |
| E04 | 5.145 | 0.2641 | 0.2665 | -0.93 | 1.79 | 0.507 | 1.86 |
| E05 | 5.093 | 0.2650 | 0.2690 | -1.52 | 1.71 | 0.490 | 1.78 |
| E06 | 5.041 | 0.2691 | 0.2716 | -0.95 | 1.23 | 0.466 | 1.31 |
| E07 | 4.988 | 0.2729 | 0.2743 | -0.53 | 2.18 | 0.444 | 2.23 |
| E08 | 4.937 | 0.2760 | 0.2770 | -0.36 | 2.12 | 0.458 | 2.17 |
| E09 | 4.883 | 0.2772 | 0.2798 | -0.93 | 2.26 | 0.454 | 2.30 |
| E10 | 4.827 | 0.2810 | 0.2829 | -0.65 | 1.53 | 0.465 | 1.60 |
| E11 | 4.774 | 0.2868 | 0.2858 | 0.32 | 1.64 | 0.436 | 1.70 |
| E12 | 4.726 | 0.2842 | 0.2886 | -1.52 | 1.60 | 0.488 | 1.68 |
| E13 | 4.673 | 0.2949 | 0.2916 | 1.11 | 1.55 | 0.665 | 1.69 |
| E14 | 4.609 | 0.2934 | 0.2954 | -0.70 | 1.21 | 0.436 | 1.28 |
| E15 | 4.554 | 0.2999 | 0.2988 | 0.38 | 1.43 | 0.418 | 1.49 |
| E16 | 4.503 | 0.3013 | 0.3019 | -0.21 | 1.22 | 0.439 | 1.30 |
| E17 | 4.459 | 0.3047 | 0.3047 | -0.02 | 1.27 | 0.408 | 1.34 |
| E18 | 4.400 | 0.3080 | 0.3085 | -0.18 | 1.31 | 0.467 | 1.39 |
| ALL | 4.840 | 0.2806 | 0.2822 | -0.57 | 1.22 | 0.111 | 1.22 |

Table 5.13

| Energy Bin | Energy <br> $(\mathrm{GeV})$ | Cross Section <br> $(\mathrm{mb})$ | Theory <br> $(\mathrm{mb})$ | Deviation(\%) <br> $(\%)$ | Syst. Error <br> $(\%)$ | Stat. Error <br> $(\%)$ | Total Error <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | 5.281 | 0.2571 | 0.2603 | -1.20 | 1.54 | 1.050 | 1.87 |
| E02 | 5.241 | 0.2576 | 0.2621 | -1.70 | 2.10 | 0.968 | 2.31 |
| E03 | 5.195 | 0.2621 | 0.2642 | -0.78 | 2.10 | 0.932 | 2.30 |
| E04 | 5.145 | 0.2677 | 0.2665 | 0.46 | 2.69 | 0.944 | 2.85 |
| E05 | 5.093 | 0.2678 | 0.2690 | -0.46 | 1.37 | 0.909 | 1.65 |
| E06 | 5.041 | 0.2706 | 0.2716 | -0.38 | 2.25 | 0.867 | 2.41 |
| E07 | 4.988 | 0.2673 | 0.2743 | -2.56 | 1.68 | 0.838 | 1.88 |
| E08 | 4.937 | 0.2772 | 0.2770 | 0.098 | 2.08 | 0.853 | 2.25 |
| E09 | 4.883 | 0.2790 | 0.2798 | -0.31 | 1.67 | 0.848 | 1.88 |
| E10 | 4.827 | 0.2814 | 0.2829 | -0.52 | 1.39 | 0.868 | 1.64 |
| E11 | 4.774 | 0.2867 | 0.2858 | 0.30 | 2.69 | 0.820 | 2.81 |
| E12 | 4.726 | 0.2935 | 0.2886 | 1.70 | 1.73 | 0.897 | 1.95 |
| E13 | 4.673 | 0.2933 | 0.2916 | 0.57 | 1.98 | 1.250 | 2.34 |
| E14 | 4.609 | 0.3002 | 0.2954 | 1.61 | 1.78 | 0.808 | 1.95 |
| E15 | 4.554 | 0.3029 | 0.2988 | 1.37 | 2.23 | 0.778 | 2.36 |
| E16 | 4.503 | 0.3029 | 0.3019 | 0.32 | 1.70 | 0.817 | 1.89 |
| E17 | 4.459 | 0.3057 | 0.3047 | 0.31 | 2.13 | 0.759 | 2.26 |
| E18 | 4.400 | 0.3118 | 0.3085 | 1.05 | 2.00 | 0.873 | 2.18 |
| ALL | 4.840 | 0.2824 | 0.2822 | 0.09 | 1.34 | 0.207 | 1.36 |

Table 5.14

| Energy Bin | Energy <br> $(\mathrm{GeV})$ | Cross Section <br> $(\mathrm{mb})$ | Theory <br> $(\mathrm{mb})$ | Deviation(\%) <br> $(\%)$ | Syst. Error <br> $(\%)$ | Stat. Error <br> $(\%)$ | Total Error <br> $(\%)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E01 | 5.281 | 0.2592 | 0.2603 | -0.42 | 2.08 | 1.93 | 2.84 |
| E02 | 5.241 | 0.2548 | 0.2621 | -2.76 | 2.18 | 1.81 | 2.83 |
| E03 | 5.195 | 0.2620 | 0.2642 | -0.81 | 2.28 | 1.72 | 2.85 |
| E04 | 5.145 | 0.2598 | 0.2665 | -2.54 | 2.03 | 1.78 | 2.70 |
| E05 | 5.093 | 0.2748 | 0.2690 | 2.12 | 2.31 | 1.66 | 2.85 |
| E06 | 5.041 | 0.2718 | 0.2716 | 0.08 | 2.12 | 1.61 | 2.66 |
| E07 | 4.988 | 0.2755 | 0.2743 | 0.45 | 1.69 | 1.53 | 2.28 |
| E08 | 4.937 | 0.2738 | 0.2770 | -1.14 | 1.74 | 1.60 | 2.36 |
| E09 | 4.883 | 0.2789 | 0.2798 | -0.34 | 1.98 | 1.57 | 2.53 |
| E10 | 4.827 | 0.2859 | 0.2829 | 1.05 | 2.04 | 1.60 | 2.59 |
| E11 | 4.774 | 0.2838 | 0.2858 | -0.70 | 1.87 | 1.54 | 2.42 |
| E12 | 4.726 | 0.2854 | 0.2886 | -1.09 | 2.06 | 1.70 | 2.67 |
| E13 | 4.673 | 0.2943 | 0.2916 | 0.90 | 1.74 | 2.33 | 2.91 |
| E14 | 4.609 | 0.2938 | 0.2954 | -0.56 | 1.88 | 1.52 | 2.41 |
| E15 | 4.554 | 0.3008 | 0.2988 | 0.66 | 2.20 | 1.45 | 2.64 |
| E16 | 4.503 | 0.3014 | 0.3019 | -0.17 | 2.02 | 1.53 | 2.53 |
| E17 | 4.459 | 0.3052 | 0.3047 | 0.15 | 2.22 | 1.41 | 2.63 |
| E18 | 4.400 | 0.3043 | 0.3085 | -1.36 | 2.25 | 1.64 | 2.79 |
| ALL | 4.840 | 0.2809 | 0.2822 | -0.46 | 1.79 | 0.387 | 1.83 |

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## APPENDIX A

YIELD FITTING HISTOGRAMS OF EACH ENERGY BIN FOR THE THREE

TARGETS

## A. 1 Yield fit histograms of each energy bin and the ${ }^{12} \mathbf{C}$-I target



Figure A. 1


Figure A. 2
Yield fit of ${ }^{12} \mathrm{C}$-I target and energy bin 1-6


Figure A. 3
Yield fit of ${ }^{12} \mathrm{C}$-I target and energy bin 7-12


Figure A. 4
Yield fit of ${ }^{12} \mathrm{C}$-I target and energy bin 13-18
A. 2 Yield fit histograms of each energy bin and the ${ }^{12} \mathbf{C}$-II target


Figure A. 5
Yield fit of ${ }^{12} \mathrm{C}$-II target and energy bin 1-6


Figure A. 6
Yield fit of ${ }^{12} \mathrm{C}$-II target and energy bin 7-12


Figure A. 7
Yield fit of ${ }^{12} \mathrm{C}$-II target and energy bin 13-18
A. 3 Yield fit histograms of each energy bin and the ${ }^{28}$ Si target


Figure A. 8
Yield fit of ${ }^{28}$ Si target and energy bin 1-6


Figure A. 9
Yield fit of ${ }^{28}$ Si target and energy bin 7-12


Figure A. 10
Yield fit of ${ }^{28}$ Si target and energy bin 13-18

## APPENDIX B

DIFFERENCES IN YIELD RELATIVE TO THE FINAL RESULT FOR THE THREE TARGETS

## B. 1 Differences in yield relative to the final result for ${ }^{12} \mathbf{C}$-I target



Figure B. 1
Differences in yield relative to the final result for $\Delta T$ cut and ${ }^{12} \mathrm{C}$-I target


Figure B. 2
Differences in yield relative to the final result for $\Delta \phi$ cut and ${ }^{12} \mathrm{C}$-I target


Figure B. 3
Differences in yield relative to the final result for $\Delta E$ cut and ${ }^{12} \mathrm{C}-\mathrm{I}$ target


Figure B. 4
Differences in yield relative to the final result for $R_{\text {min }}$ cut and ${ }^{12} \mathrm{C}$-I target


Figure B. 5
Differences in yield relative to the final result for $\Delta K$ cut and ${ }^{12} \mathrm{C}-\mathrm{I}$ target

## B. 2 Differences in yield relative to the final result for ${ }^{12} \mathbf{C}$-II target



Figure B. 6
Differences in yield relative to the final result for $\Delta T$ cut and ${ }^{12} \mathrm{C}$-II target


Figure B. 7
Differences in yield relative to the final result for $\Delta \phi$ cut and ${ }^{12} \mathrm{C}$-II target


Figure B. 8
Differences in yield relative to the final result for $\Delta E$ cut and ${ }^{12} \mathrm{C}$-II target


Figure B. 9
Differences in yield relative to the final result for $R_{\text {min }}$ cut and ${ }^{12} \mathrm{C}$-II target


Figure B. 10
Differences in yield relative to the final result for $\Delta K$ cut and ${ }^{12} \mathrm{C}$-II target

## B. 3 Differences in yield relative to the final result for ${ }^{28} \mathbf{S i}$ target



Figure B. 11
Differences in yield relative to the final result for $\Delta T$ cut and ${ }^{28}$ Si target


Figure B. 12
Differences in yield relative to the final result for $\Delta \phi$ cut and ${ }^{28}$ Si target


Figure B. 13
Differences in yield relative to the final result for $\Delta E$ cut and ${ }^{28}$ Si target


Figure B. 14
Differences in yield relative to the final result for $R_{\text {min }}$ cut and ${ }^{28}$ Si target


Figure B. 15
Differences in yield relative to the final result for $\Delta K$ cut and ${ }^{28}$ Si target

