PrimEx note #85

Photoproduction of pseudoscalar mesons on protons and light nuclei

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In this note we consider the updates done lately for π^0 production on Carbon and Silicon and discuss differences for $\pi, \eta, \eta\prime$ mesons photoproduction.

1 Introduction

The process of pseudoscalar meson $M = \pi, \eta, \eta\prime$ photoproduction on the nucleon

$$\gamma(k) + N(p_1) \to M(p) + N(p_2) \tag{1}$$

is determined by four amplitudes [1] depending on invariant variables $s = (k + p_1)^2$ and $t = (k - p)^2 = -\vec{q}^2 - \Delta^2 = -4|\vec{k}||\vec{p}|sin^2(\frac{\theta}{2}) - \frac{m_i^4}{4k^2}$; where m_i is the mass of the produced meson. The photoproduction on protons and light nuclei has advantage vs heavy nuclei: The Coulomb (Primakoff) and strong coherent peaks are well separated. In section 2 we cite expressions allowing to estimate the yield of pseudoscalar mesons in photoproduction off the proton Coulomb field. In section 3 we discuss the mesons production in the Coulomb field of any nuclei using the formula for electromagnetic form factor obtained without model approximations. We show how one can account for meson sizes and its impact on the value of lifetime. In Section 4 we consider any pseudoscalar meson photoproduction on a proton making use the result of JPAC collaboration [2, 3, 4] and estimated the relative contributions of different mesons. The correct strong form factor of nuclei (without optical models approximation) and shadowing of photon is considered in sections 5 and 6. Finally the cross section for pseudoscalar mesons incoherent photoproduction shortly discussed in section 7. In Appendix 1 we cite the expressions for Fourier-Bessel distribution. In Appendix 2 we point out changes needs to be applied to rewrite π^0 photoproduction formulas to η, η' photoproduction case.

2 Photoproduction of the proton electromagnetic field.

The amplitude for reaction (1) when meson production takes place in the Coulomb field of the proton or nuclei is:

$$T_C = Z\sqrt{8\alpha\Gamma_i}(\frac{\beta}{m_i})^{3/2}\frac{k^2 sin\theta}{q^2+\Delta_i^2}F_{em}(q,\Delta_i), \tag{2}$$

where $F_{em}(q,\Delta_i)$ is the electromagnetic form factor of proton or nuclei.
When the exchanged photon transverse momenta is equal to longitudinal one $q \equiv |\vec{q}| = \Delta = \frac{m_i^2}{2E}$ the electromagnetic cross section takes its maximum value (we adopt the normalization $\frac{d\sigma}{dt} = \frac{\pi}{k^2}\frac{d\sigma}{d\Omega} = \frac{\pi}{k^2}|T_C|^2$):

$$\frac{d\sigma}{dt}(q=\Delta) \sim \Gamma_i \frac{E^2}{m_i^7} \tag{3}$$

i.e. has a strong dependence on meson mass. In reality to estimate the yield of different mesons, it is more convenient to use the integrated cross section:

$$\sigma = \int \frac{d\sigma}{dt}dt \sim \Gamma_i \frac{log(\frac{E}{r_p m_i^2})}{m_i^3} \tag{4}$$

In deriving this expression, we took the matter distribution in a proton in Gaussian form:

$$\rho_p(r) = \frac{1}{\pi^{3/2}r_p^3}e^{-\frac{r^2}{r_0^2}}, \tag{5}$$

where the parameter $r_0^2 = \frac{2}{3} < r^2 >$. Such a parametrization allows to obtain a simple analytical expression for form factor:

$$F(q) = \int e^{i\vec{q}\vec{r}}\rho(r)d^3r = e^{-\frac{q^2<r^2>}{6}} \tag{6}$$

where $< r^2 >$ is the mean square radius of nucleon or nuclei.
Another frequently used formula for proton electromagnetic form factor is given by dipole formula: $F_{em}(q) = \frac{1}{(1+\frac{q^2}{\Lambda^2})^2}$ with $\Lambda^2 = 0.71GeV^2$.

3 Photoproduction in the Coulomb field of nuclei.

Using Glauber theory of multiple scattering [6] the electromagnetic form factor for light nuclei in (2) can be written as [7]:

$$\begin{aligned} F_{em}(q) &= 2\pi\frac{q^2+\Delta^2}{q}\int J_1(qb)\frac{b^2dbdz}{(b^2+z^2)^{3/2}}e^{i\Delta z}(1-G(b,z))^{A-1}\int_0^{\sqrt{b^2+z^2}} x^2\rho_{ch}(x)dx \\ G(b,z) &= \frac{f_s(0)}{ika_s}\int_z e^{-\frac{(\vec{b}-\vec{s}')^2}{2a_s}}\rho(s',z')d^2s'dz' = \frac{\sigma'}{2a_s}\int_z e^{-\frac{b^2+s'^2}{2a_s}}I_0(\frac{bs'}{a_s})\rho(s',z')s'ds'dz' \end{aligned} \tag{7}$$

Here $J_1(x), I_0(x)$ are Bessel functions of real and imaginary arguments; $\rho(r)$ the nuclear density, $\sigma' = \sigma\cdot\left(1-i\frac{Ref_s(0)}{Imf_s(0)}\right) = \frac{4\pi}{ik}f_s(0)$, $\sigma\equiv\sigma(MN)$ is the total meson nucleon cross section. In deriving this expression, the commonly used parametrization for elastic $MN\to MN$ amplitude is adopted: $f_s(q) = f_s(0)\cdot exp(-a_s q^2/2)$. Here we want to make two comments:

1) In some sense the size of produced meson is taken into account as the expression (7) is beyond the optical model approximations. The size of meson has an impact through the value of $\sigma(MN)$ and the slope of elastic amplitude a_s. The expression (7) has been modified multiplying the electromagnetic form factor (7) by the monopole pion form factor

$$F_\pi = (1-a_\pi\frac{t}{m_\pi^2})^{-1}$$

with the slope value [8] $a_\pi = 31.5\times10^{-3}$ The calculated impact of pion form factor on pion decay width has the order of $\sim 10^{-4}$.

2) Strictly speaking the two nuclear distributions in (7) should be different. The charge density distribution $\rho_{ch}(r)$ can be obtained from nucleons distribution in nucleus $\rho(r)$ responsible for mesons absorption by its convolution with proton charge distribution:

$$\rho_{ch}(r) = \int d^3r\prime\rho(r\prime)\rho_p(|\vec{r}-\vec{r}\prime|) \tag{8}$$

For the light nuclei, like carbon, the nuclear density distribution $\rho(r)$ corresponding to the harmonic oscillator potential well is widely used in the literature [6]:

$$\rho(r) = \frac{4}{\pi^{3/2}a_0^3A}(1+\frac{A-4}{6a_0^2}r^2)e^{-\frac{r^2}{a_0^2}} \tag{9}$$

Substituting expressions (5) and (9) in (8) for nuclear charge distribution we obtain:

$$\rho_{ch}(r) = \int d^3r\prime\rho(r\prime)\rho_p(|\vec{r}-\vec{r}\prime|)$$
$$= \frac{4}{\pi^{3/2}A(a_0^2+r_p^2)^{3/2}}\left(1+\frac{(A-4)}{6}[\frac{3r_p^2}{2(a_0^2+r_p^2)}+\frac{a_0^2r^2}{(a_0^2+r_p^2)^2}]\right)e^{-\frac{r^2}{r_p^2+a_0^2}} \quad (10)$$

Thus in case of the Gaussian parametrization, the effective radius of nuclear density (a_0 in expression (9)) requires a larger radius for the charge density ($\sqrt{a_0^2+r_p^2}$ in equation (10)).
For nuclear charge distribution, the Fourier-Bessel parametrization were used in PrimEx-II analysis. It has been extracted from the fit of the experimental data on electron scattering on nuclei (see appendix for details). For electromagnetic part we used the tabulated radius [9] as the global $dN/d\theta$ fit χ^2 sensitivity is an order of magnitude smaller for charged radius variation. The latest $dN/d\theta$ fit with the new slope value $a_p = 10GeV^{-2}$ requires the radius in strong part a bit smaller than tabulated (0.5%), which is in agreement with example above, where we show that the electromagnetic radius can be larger, then the strong one. From the other hand this difference is small (within the experimental error) and we can consider the same radii in strong and electromagnetic parts.

4 Mesons photoproduction on nucleons

After seminal work [1] a numerous manuscripts were published devoted to the considered processes. We shortly discuss the recent works of JPAC collaboration on π meson [2], η meson [3] and η, η' mesons [4] photoproduction off protons in Regge model. Our choice is due to the fact that these are most recent works on this subject and what is more important are clear enough and can be easily applied to the estimation of yield of pseudoscalar mesons. The cross section of reaction (1) is determined by four invariant amplitudes A_i, which can be parameterized as Regge exchanges [3, 4]. Later on we will consider only two amplitudes A_1, A_4 which are determined by ρ, ω, ϕ exchanges with natural parity $P(-1)^J = 1$ in t-channel as the contribution of unnatural parity exchanges $P(-1)^J = -1$ (b, h, h' mesons) is small [4].

$$\begin{aligned}
\frac{d\sigma}{dt} &= \frac{1}{32\pi}\left(|A_1|^2 - t|A_4|^2\right) \\
A_{1,4}(s,t) &= \beta_{1,4}(t)\frac{1-e^{-i\pi\alpha(t)}}{sin(\pi\alpha(t))}(s/s_0)^{\alpha(t)-1} \\
\beta_1(t) &= g_{VM\gamma}g_{1V}te^{bt}; \quad \beta_4(t) = g_{VM\gamma}g_{4V}e^{bt} \\
s &= 2m_Nk + m_N^2; s_0 = 1GeV^2 \qquad (11)
\end{aligned}$$

The Regge trajectories for ρ, ω, ϕ exchanges are taken as [4]: $\alpha_\omega(t) = \alpha_\rho(t) = 0.5 + 0.9\,t; \quad \alpha_\phi = 0.9\,t$. The coupling constants of natural parity exchanges with nucleons [4]:

$$\begin{aligned}
g_{1\omega} &= 0; \quad g_{1\rho} = 13.49GeV^{-3}; \quad g_{1\phi} = 0 \\
g_{4\omega} &= 7.28GeV^{-2}; \quad g_{4\rho} = 2.3GeV^{-2}; \quad g_{4\phi} = 9.38GeV^{-2}, \qquad (12)
\end{aligned}$$

where g_{1V} corresponds to spin-flip at nucleon vertex, while g_{4V} is relevant to spin non-flip at nucleon vertex.

Couplings of vector exchanges at top vertex can be determined from vector mesons radiative decays [4]:

$$\begin{aligned}
g_{\rho\pi\gamma} &= 0.252GeV^{-1}; \quad g_{\omega\pi\gamma} = 0.696GeV^{-1}; \quad g_{\phi\pi\gamma} = 0.04GeV^{-1} \\
g_{\rho\eta\gamma} &= 0.48GeV^{-1}; \quad g_{\omega\eta\gamma} = 0.135GeV^{-1}; \quad g_{\phi\eta\gamma} = 0.21GeV^{-1} \\
g_{\rho\eta'\gamma} &= 0.398GeV^{-1}; \quad g_{\omega\eta'\gamma} = 0.127GeV^{-1}; \quad g_{\phi\eta'\gamma} = 0.216GeV^{-1} \qquad (13)
\end{aligned}$$

With these values we can calculate the differential cross section for any pseudoscalar meson photoproduction off nucleon by using equations (11).
It is well known, that the amplitude of meson photoproduction due to isovector exchange with I=1 (in our case ρ exchange) has the opposite signs on proton and neutron. This leads to the absence of ρ exchange contribution in the coherent amplitude of mesons photoproduction on the nuclei with the equal number of protons and neutrons like ^{12}C and ^{28}Si. The relative contributions from ρ and ω exchanges in A_4 amplitude are dominating at small t (11) and π^0, η, η' photoproduction on nucleon can be estimated with the following terms:

$$\begin{aligned}
g_{\rho\pi\gamma}g_{4\rho} &= 0.58GeV^{-3}; \quad g_{\omega\pi\gamma}g_{4\omega} = 5.07GeV^{-3} \\
g_{\rho\eta\gamma}g_{4\rho} &= 1.1GeV^{-3}; \quad g_{\rho\pi\gamma}g_{4\omega} = 0.983GeV^{-3} \\
g_{\rho\eta'\gamma}g_{4\rho} &= 0.92GeV^{-3}; \quad g_{\rho\pi\gamma}g_{4\omega} = 0.925GeV^{-3} \qquad (14)
\end{aligned}$$

From this table one can see that the ratio of ρ to ω exchanges in the amplitude of π^0 photoproduction on nucleon is $\sim 10\%$, and in the case of η and η'

photoproduction is ~ 1. Because of the opposite signs of the photoproduction amplitude on proton and neutron for isovector exchange, in the coherent photoproduction case for all nuclei with the same number of protons and neutrons, the contribution from ρ exchange is cancelled. So the nucleon amplitude containing ρ and ω contribution, which are about the same for η, η' will loose ρ part (about one half) for such nuclei. For the π^0 case the ρ part is small for nucleon and the reduction for nuclei (due to cancellation of this part) will not be significant. For the Coulomb photoproduction this is not the case. Therefore the relative ratio of strong and electromagnetic parts of the cross section on nuclei in the case of η, η' photoproduction should be much smaller, than for the π^0 production case (approximately four times) and consequently the relative contribution of the strong part in photoproduction of η, η' is much smaller, than for the case of π^0 photoproduction on nuclei.

4.1 Parameterization of meson photoproduction amplitude on nucleon. Slopes and forward amplitudes.

The amplitude of mesons coherent photoproduction on nucleus reads:

$$T_S(q, q_L) = A(\vec{h} \cdot \vec{q})\phi(0)F_S(q, q_L) \tag{15}$$

The nuclear form factor $F_S(q, q_L)$ has a strong dependence on the slope a_p of the production amplitude on the nucleon (see eq. (25) later on). We parameterized the elementary amplitude of meson production on nucleon $f_p(t)$ and scattering $f_s(t)$ in the usual way:

$$\begin{aligned} f_p &= f_p(0)\ (\vec{h} \cdot \vec{q})e^{-\frac{a_p q^2}{2}} \\ f_s &= f_s(0)\ e^{-\frac{a_s q^2}{2}} \end{aligned} \tag{16}$$

Here $f_p(0), f_s(0)$ and a_p, a_s are the forward elementary amplitudes and their slopes, $\vec{h} = [\vec{k} \times \vec{\epsilon}]$. We adopt the following normalization of differential cross sections

$$\frac{d\sigma}{d\Omega} = \frac{k^2}{\pi}\frac{d\sigma}{dt} = |f_{p,s}(t)|^2; \quad |\vec{h} \cdot \vec{q}|^2 = q^2 = |t| \tag{17}$$

For the scattering slope of elastic process $M + N \to M + N$ we put the well known value (see for example [5] table XII on page 317, the slope parameter B value for 4 GeV for $\rho^0, \omega + p \to \rho^0, \omega + p$) $a_s = 7GeV^{-2} = 0.28fm^2$, whereas for the slope in photoproduction $\gamma + N \to M + N$ we use the value

$a_p = 10GeV^{-2} = 0.4fm^2$. This value has been obtained by few independent ways:
1) From (16) and (17) the photoproduction on nucleon reads:

$$\frac{d\sigma}{d\Omega} = q^2|f_p(0)|^2 e^{-a_p q^2} \tag{18}$$

From the fit with the formula (with two free parameters a_p and $f_p(0)$) to describe the experimental data on π^0 photoproduction on nucleon measured at DESY [10] with the result:

$$a_p = 10GeV^{-2} = 0.4fm^2; f_p(0) = 15.8\sqrt{\mu b}/GeV$$

On the fig.1 we show the differential cross section calculated using this parameters by the formula (18) (solid line) and calculations (dashed line) by Regge theory accounting for the poles and Regge cuts [11]. The similarity is impressing.
2) The same slope and forward amplitude can be extracted from the expressions (11). Noting that in the coherent amplitude only A_4 is contributing (A_1 is the amplitude with spin flip of nucleon) and rewriting the energy dependence as $(s/s_0)^{\alpha(t)-1} = e^{(\alpha(0)-1+0.9t)log(s/s_0)}$, the slope in photoproduction is:

$$b + 0.9log(s/s_0) = 3 + 0.9 \times 2.3 \approx 5GeV^{-2}$$

The slope we use is doubled (see (18)) compared to this expression, thus our slope $a_p = 10GeV^{-2} = 0.4fm^2$.
3) The slope b in (11) is a result of finite sizes of nucleon and meson. To see this, we adopt the gaussian parametrization for nucleon and meson form factors (see [13] eq. 37) $F(q) = e^{-\frac{<r^2>q^2}{6}}$, where $< r^2 >$ is a mean square radius of meson or nucleon. Choosing $< r_p^2 >= 0.64fm^2; < r_M^2 >= 0.36fm^2$ one has $b = \frac{<r_p^2>+<r_M^2>}{6} = \frac{1}{6}fm^2 \approx 4GeV^{-2}$. This value is not far from $b = 3GeV^{-2}$ in (11) due to the fit.
The optical model approximations ($a_p = a_s = 0$ in (16)) leads to b=0, i.e. meson and nucleon are considered as point like particles. The term $0.9, log(s/s_0)$ considered above is relevant to the interaction radius expansion with energy and has nothing with proton and nucleon dimensions.
It is not a big deal to estimate the forward amplitude $f_p(0)$ for any meson production using (11) and values of relevant vertex. As an example let's

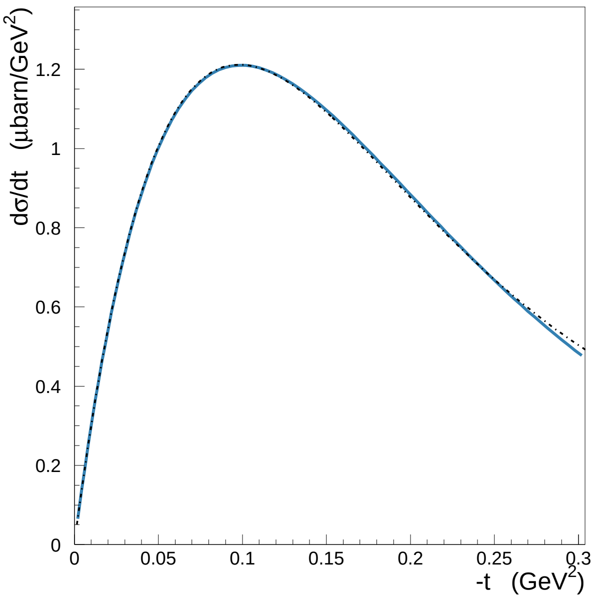

Figure 1: Differential cross section for π^0 photoproduction on nucleon calculated by formula (18) (dashed curve) and by Regge model with cuts [11] (solid curve)

estimate it for π^0 photoproduction at k=5.2 GeV:

$$\begin{aligned} f_p(0) &= \frac{k}{4\sqrt{2}\pi} A_4(s,0) \\ &= \frac{k}{4\sqrt{2}\pi} \left(g_{\rho\pi\gamma} g_{4\rho} + g_{\omega\pi\gamma} g_{4\omega}\right) \frac{1-e^{-i\pi\alpha(0)}}{sin(\pi\alpha(0))} (s/s_0)^{\alpha(0)-1} \\ &= \frac{k}{4\sqrt{2}\pi} 5.65(1+i)(s/s_0)^{-0.5} \approx 0.51(1+i) GeV^{-2} \qquad (19) \end{aligned}$$

Thus Regge theory predicts the same real and imaginary parts of the production amplitudes, i.e. the phase $\varphi = 0.785$. Considering that $GeV^{-2} =$

$0.04 fm^2 = 400\mu b$ one gets $|f_p(0)|^2 \approx 208 \mu b GeV^{-2}$ not far from the result shown above (right after (18)).

5 Coherent photoproduction in the strong nuclear field

The main impact on the lifetime extraction from the measured differential cross sections comes from our knowledge of the strong amplitude T_S in the coherent process of the reaction:

$$\gamma + A \to M + A. \tag{20}$$

In the Glauber theory of multiple scattering [6] this coherent photoproduction amplitude is given by:

$$\begin{aligned} T_S(q,\Delta) &= A\frac{ik}{2\pi}\int e^{i\vec{q}\cdot\vec{b}+i\Delta z}\Gamma_p(\vec{b}-\vec{s})\rho(\vec{s},z) \\ &\times [1-\int \Gamma_s(\vec{b}-\vec{s}\prime)\rho(\vec{s}\prime,z\prime)d^2s\prime dz\prime]^{A-1}d^2bd^2sdz \end{aligned} \tag{21}$$

Two dimensional vectors $\vec{b}$ and $\vec{s}$ are impact parameter and nucleon coordinate in the plane transverse to the incident photon momentum; z – longitudinal nucleon coordinate inside nucleus. The profile functions $\Gamma_{p,s}(\vec{b}-\vec{s})$ are the two dimensional Fourier transformation of elementary amplitudes for the η photoproduction off the nucleon $f_p = f(\gamma + N \to \eta + N)$ and elastic pion-nucleon scattering $f_s = f(\eta + N \to \eta + N)$:

$$\Gamma_{p,s}(\vec{b}-\vec{s}) = \frac{1}{2\pi ik}\int e^{i\vec{q}\cdot(\vec{b}-\vec{s})}f_{p,s}(q)d^2q. \tag{22}$$

We parameterized the elementary production amplitudes in the classical way:

$$\begin{aligned} f_p &= \phi(0)\ (\vec{h}\cdot\vec{q})e^{-\frac{a_pq^2}{2}} \\ f_s &= f_s(0)\ e^{-\frac{a_sq^2}{2}} \end{aligned} \tag{23}$$

Here $\phi(0), f_s(0)$ and a_p, a_s are the forward elementary amplitudes and their slopes, $\vec{h} = [\vec{k}\times\vec{\epsilon}]$, $\vec{k}$ and $\vec{\epsilon}$ are beam photon momentum and polzarization. The relevant profile functions are:

$$\begin{aligned} \Gamma_p(\vec{b}-\vec{s}) &= \frac{\vec{h}\cdot(\vec{b}-\vec{s})}{ka_p^2}\phi(0)e^{-\frac{(\vec{b}-\vec{s})^2}{2a_p}} \\ \Gamma_s(\vec{b}-\vec{s}) &= \frac{f_s(0)}{ika_s}e^{-\frac{(\vec{b}-\vec{s})^2}{2a_s}} \end{aligned} \tag{24}$$

Substituting expressions in (21) we obtain [12]:

$$
\begin{aligned}
T_S(q,q_L) &= A(\vec{h}\cdot\vec{q})\phi(0)F_S(q,q_L) \\
F_S(q,q_L) &= \frac{2\pi}{qa_p^2}\int J_1(qb)(bI_0(\frac{bs}{a_p}) \\
&- sI_1(\frac{bs}{a_p}))e^{i\Delta z}e^{-\frac{b^2+s^2}{2a_p}}\rho(s,z)(1-G(b,z))^{A-1}bdbsdsdz \\
G(b,z) &= \frac{f_s(0)}{ika_s}\int_z e^{-\frac{(\vec{b}-\vec{s}')^2}{2a_s}}\rho(s',z')d^2s'dz' \\
&= \frac{\sigma'}{2a_s}\int_z e^{-\frac{b^2+s'^2}{2a_s}}I_0(\frac{bs'}{a_s})\rho(s',z')s'ds'dz' \qquad (25)
\end{aligned}
$$

Here I_0, I_1 are the zero and first order Bessel functions of imaginary argument. In these equations the finite size of meson is taken into account through the slope in photoproduction off nucleon a_p and elastic scattering a_s. This causing strong form factor accounting for slope in the elementary amplitudes fall off steeper than in the case of optical model approximation.

6 Photon shadowing in nuclei

The real photons at high energy are shadowed in nuclei [14]. The photon shadowing in mesons photoproduction is a result of the two step process [15]: initial photon produces a vector meson in the nucleus, which consequently produces the final meson after scattering. The main contribution to such a two step photoproduction comes from the ρ-mesons, as the cross section for ρ photoproduction on the nucleon is almost an order of magnitude higher than for the ω production. The amplitude relevant to the two step process $\gamma \to \rho \to M$ in multiple scattering theory is given by:

$$
\begin{aligned}
T_I(q) &= \frac{ik}{2\pi}A(A-1)\int e^{i\vec{q}\cdot\vec{b}}d^2b\Gamma_p(\vec{b}-\vec{s}_1)\Gamma_s(\vec{b}-\vec{s}_2)\rho(s_1,z_1)\rho(s_2,z_2) \\
&\times\ \theta(z_2-z_1)e^{i\Delta_\rho(z_1-z_2)+i\Delta z_2}(1-G(b,z_1))^{A-2}d^2s_1d^2s_2dz_1dz_2 \\
G_s(b,z_1) &= \int_{z_1}^{\infty}\Gamma_s(\vec{b}-\vec{s}')\rho(s',z')d^2s'dz, \qquad (26)
\end{aligned}
$$

where $\Delta_\rho = \frac{m_\rho^2}{2E}$ is the longitudinal momentum transfer in the elementary reaction $\rho + N \to M + N$. Using equations (16) this expression can be expressed in the form convenient for the numerical integration [12]

$$
\begin{aligned}
T_I(q) &= A(\vec{h} \cdot \vec{q}) f_p(0) F_{SI} \\
F_{SI} &= (A-1)\frac{\pi\sigma'}{q a_s a_p^2} \int J_1(qb) I_0(\frac{bs_2}{a_s})[bI_0(\frac{bs_1}{a_p}) - s_1 I_1(\frac{bs_1}{a_p})] \\
&\times\ \theta(z_2 - z_1)\rho(s_1, z_1)\rho(s_2, z_2) e^{-\frac{(a_p+a_s)b^2}{2a_p a_s}} e^{-\frac{s_1^2}{2a_p} - \frac{s_2^2}{2a_s}} e^{i\Delta_\rho(z_1-z_2)+i\Delta z_2} \\
&\times\ (1 - G(b, z_1))^{A-2} bdbs_1 ds_1 s_2 ds_2 dz_1 dz_2 \qquad (27)
\end{aligned}
$$

The complete strong amplitude accounting for the photon shadowing reads:

$$
T_S(q) = A(\vec{h} \cdot \vec{q})\phi(0)[F_S - wF_I]; \quad w = \frac{f(\gamma N \to \rho N) f(\rho N \to \pi N)}{f(\rho N \to \rho N) f(\gamma N \to \pi N)}, \qquad (28)
$$

where the range of the shadowing parameter w changes between zero (no shadowing) and one (Vector dominance model).

7 Incoherent photoproduction

Incoherent pion photoproduction is a production with the excitation or breakup of the target nucleus:

$$
\gamma + A \to M + A' \qquad (29)
$$

The general expression for the incoherent cross section established in the literature [18, 19] is given by:

$$
\frac{d\sigma_{inc}}{d\Omega} = \frac{d\sigma_0}{d\Omega}(q) N(0, \sigma)(1 - G(t)) \qquad (30)
$$

where $\frac{d\sigma_p}{d\Omega}$ is the elementary cross section on nucleon $\gamma + N \to M + N$, and

$$
N(0, \sigma) = \int \frac{1 - e^{-\sigma T(b)}}{\sigma} d^2 b. \qquad (31)
$$

Here $T(b) = A \int \rho(b, z) dz$. The factor $(1 - G(t))$ takes into account the suppression of pseudoscalar meson production at small angles due to the Pauli blocking principle [18]. For the light nuclei (like carbon) this factor can be expressed as

$$
G(t) = [1 + (\frac{q^2 R^2}{15})^2] e^{-\frac{2q^2 R^2}{15}}. \qquad (32)
$$

The factorization in (30) is valid only for the Born approximation (without meson absorption). As it was shown in the case of the proton elastic scattering on nuclei, the consideration of the absorption process in the final state changes substantially this expression. Assuming that the photoproduction cross section on the nucleon is completely determined by the single-flip process in the elementary amplitude:

$$\frac{d\sigma_p}{d\Omega} = c_p q^2 e^{-a_p q^2}, \tag{33}$$

the incoherent cross section can be expressed in the following form [20]:

$$\begin{aligned}
\frac{d\sigma_{inc}}{d\Omega} &= \frac{d\sigma_p}{d\Omega}(q)(N(0,\sigma) - \frac{|F_S(q)|^2}{A}) + c_p Q^2 \\
Q^2 &= \pi\sigma^2 \int \frac{\partial\rho(b,z_2)}{\partial b}\frac{\partial\rho(b,z_1)}{\partial b}\rho(b,z_3)\theta(z_2-z_1)\theta(z_3-z_2) \\
&\times \exp(-\frac{\sigma}{2}\int_{z_1}^{\infty}\rho(b,z\prime)dz\prime - \frac{\sigma}{2}\int_{z_2}^{\infty}\rho(b,z\prime)dz\prime)bdbdz_1dz_2dz_3
\end{aligned} \tag{34}$$

Here $F_S(q)$ is the nucleus strong form factor evaluated by the expression (25). Only in the case if absorption is absent ($\sigma = 0$), the factorization similar to the expression (30) takes place.

8 Summary

In this note we have shown that:
1) The process of meson photoproduction off light nuclei (C, Si) is well described by Fourier-Bessel charge and nuclear density distributions.
2) The strong form factor is sensitive to the slope a_p of elementary amplitude. The slope change from $0.4\,fm^2$ to $0.24\,fm^2$ leads to increase in the nuclear density distribution radius ar the level of percent.
The separate publication of strong part of our data can be motivated as follows: despite the bulk of data on vector mesons photoproduction off nuclei [14] our data are unique as it is the only experiment on coherent photoproduction of meson, which production amplitude on nucleon is zero at zero production angle. Such a behavior leads to the essential difference from diffraction processes. The diffraction (coherent production off nuclei) is determined by the nucleus form factors: $F(q) = \int e^{i\vec{q}\vec{r}}\rho(r)d^3r$, which is the Fourier transform of nuclear density $\rho(r)$. In our case the coherent amplitude is described by the specific form factor, which is a Fourier transform of nuclear density derivative $\frac{d\rho(r)}{dr}$. Such a peculiarity leads to additional an absorption (Faldt correction) and is more sensitive to the nuclear density parametrization, than the "usual" form factor.

9 Appendix 1

The nucleus charge density distribution can be expressed through the nucleus form factor $F(q)$:

$$\rho_{ch}(r) = \frac{1}{(2\pi)^3} \int F(q) e^{i\vec{q}\vec{r}} d^3q = \frac{1}{2\pi^2} \int F(q) \frac{sin(qr)}{qr} q^2 dq \tag{35}$$

It can be expanded as:

$$\rho(r) = \sum_{\nu} a_\nu j_0(\frac{\nu\pi r}{R}); \qquad r \leq R, \tag{36}$$

where $j_0(x) = \frac{sin(x)}{x}$ - zero order spherical Bessel function. Here there are two constrains:

$$Z = 4\pi \sum_{\nu} a_\nu (\frac{R}{\nu\pi})^3 \nu\pi(-1)^{\nu+1} \tag{37}$$

$$< r^2 >= \frac{4\pi}{Q} \sum_{\nu} a_\nu (\frac{R}{\nu\pi})^5 \nu\pi(-1)^{\nu} [6 - (\nu\pi)^2] \tag{38}$$

From [9] for silicon we have the mean square radius $\sqrt{< r^2 >} = 3.085(17) fm$.

10 Appendix 2

1) In the electromagnetic amplitude (2) one has to take a correct longitudinal transfered momentum $\Delta_i = \frac{m_i^2}{2E}$. The same has to be done in the electromagnetic form factor (7) with the obvious substitution: $\sigma(\pi N) \rightarrow \sigma(\eta N) = \sigma(\pi' N)$. The value of ηN total cross section can be calculated from relation given by quark model [21]

$$\sigma(\eta N) = \frac{1}{3}\left(\sigma(K^+p) + \sigma(K^-p) + \sigma(K^+n) + \sigma(K^-n)\right) - \frac{1}{6}\left(\sigma(\pi^+p) + \sigma(\pi^-p)\right)$$

This cross section has been measured many years ago [22] with a result of $\sigma(\eta N) \approx 20 mb$.
2) The same changes must be done in the strong form factor (25).

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