JLab Cake seminars

Phenomenological analysis of 3D nucleon structure

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1 May 2019

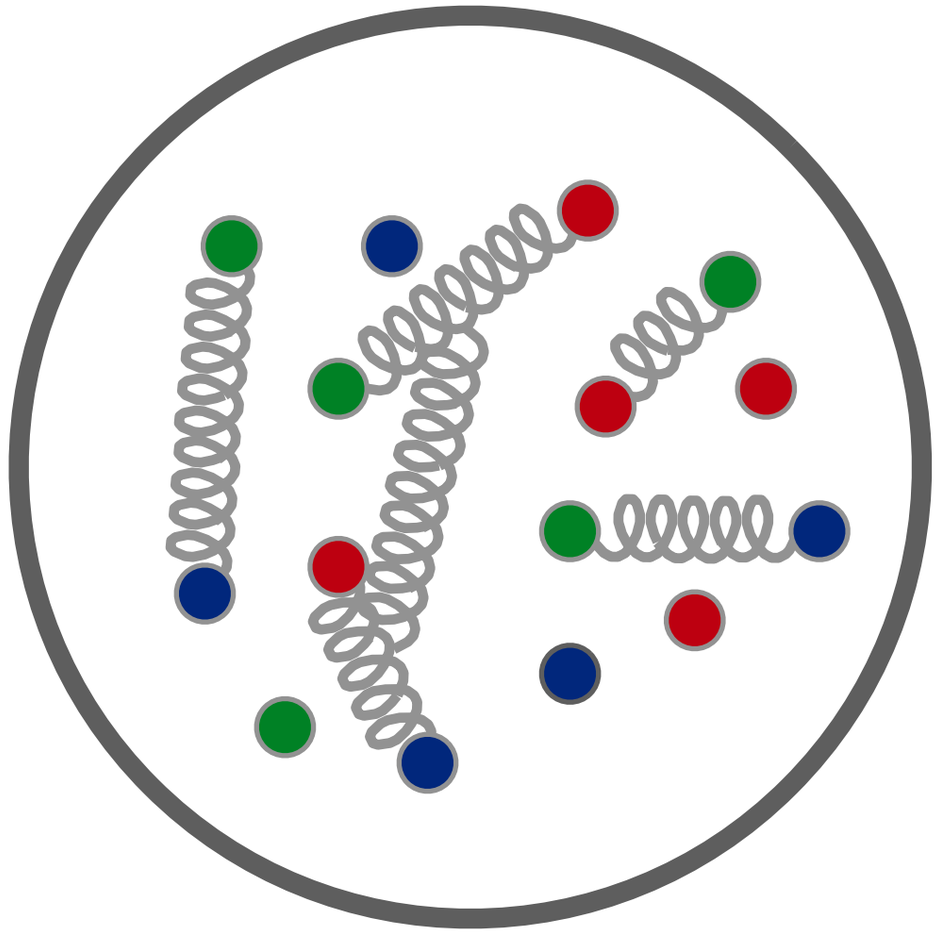
Outline

- Introduction to phenomenology of TMDs
- Extraction of partonic unpolarized TMDs: global fit
  - Relation between experimental observables and TMDs
  - Our choices for parametrization
  - Overview of experiments and data considered
  - Results and comparisons
- Extractions of Sivers function
  - Relation between Sivers distribution and unpolarized TMDs
  - Data considered
  - Results
- Outlook

Investigating nucleon internal structure

Test what we know about QCD

→ perturbative and lattice



Measure what we don't know about QCD

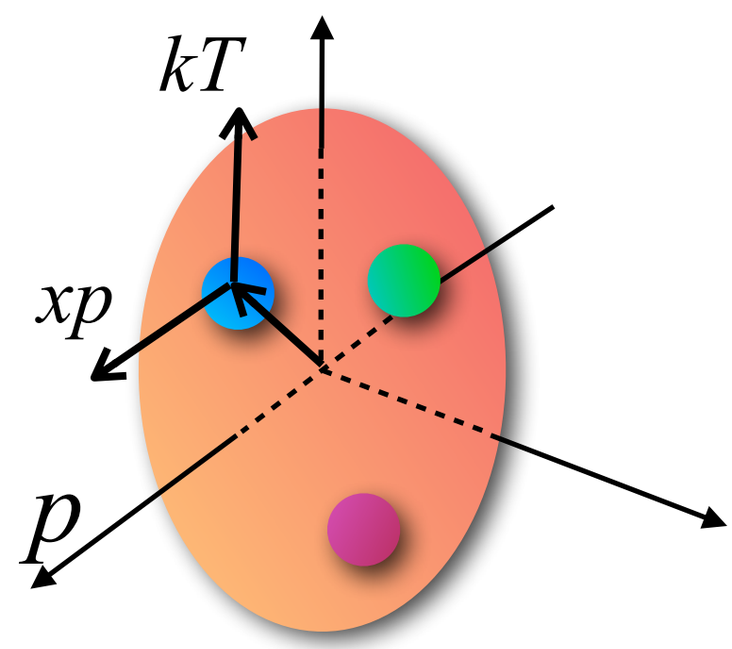
→ extraction from data

Multidimensional imaging of nucleon

Momentum and Position: how partons move inside the nucleon and distribution dependence on x

Flavor: how different flavors affect partonic distributions.

Spin: correlation between parton movement (OAM) and overall nucleon properties (missing spin budget).



Information summarized as
Parton Distribution
Function

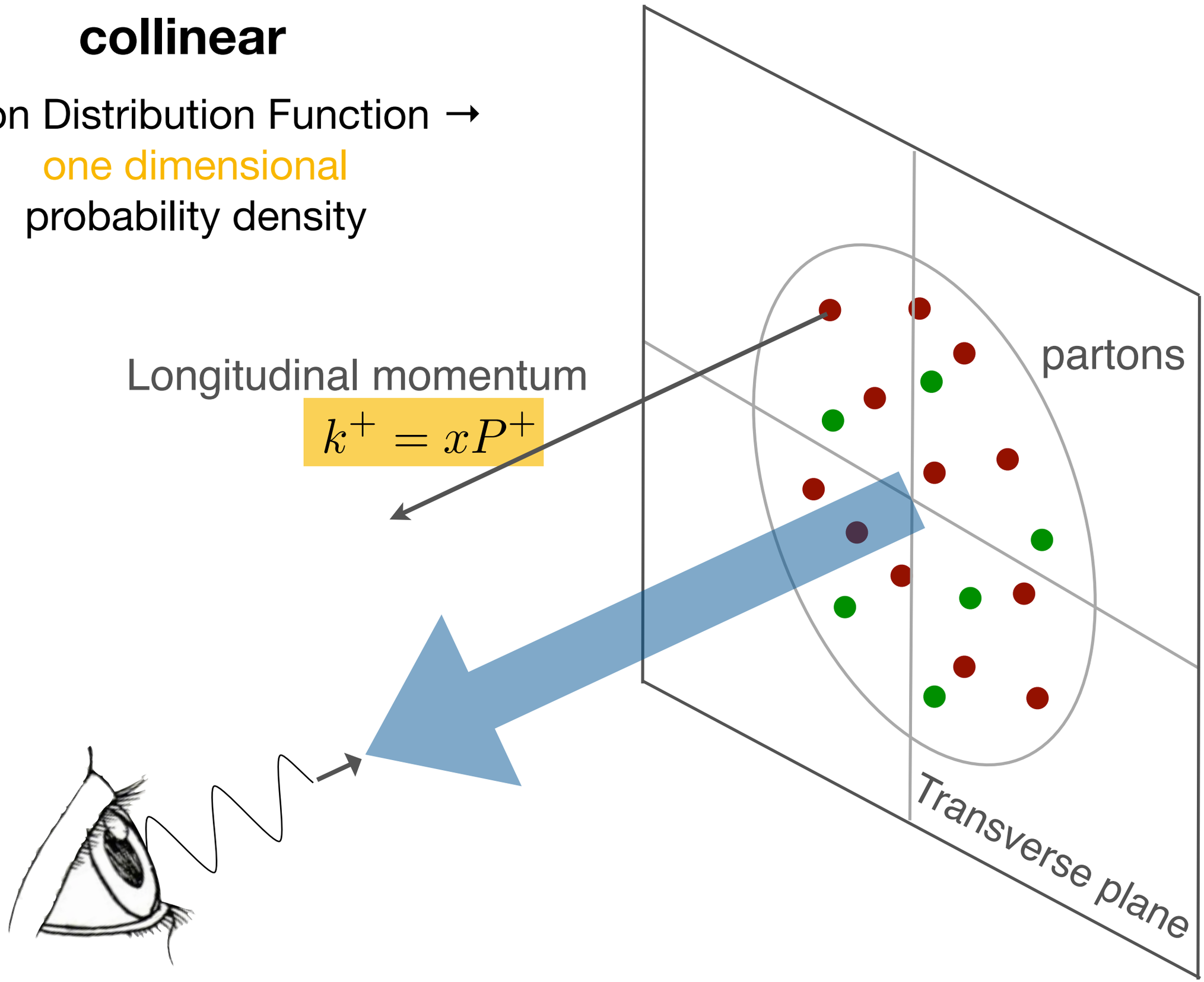
1D picture of the nucleon: PDF

collinear

Parton Distribution Function →
one dimensional
probability density

Longitudinal momentum

$$k^+ = xP^+$$



3Dimensional structure

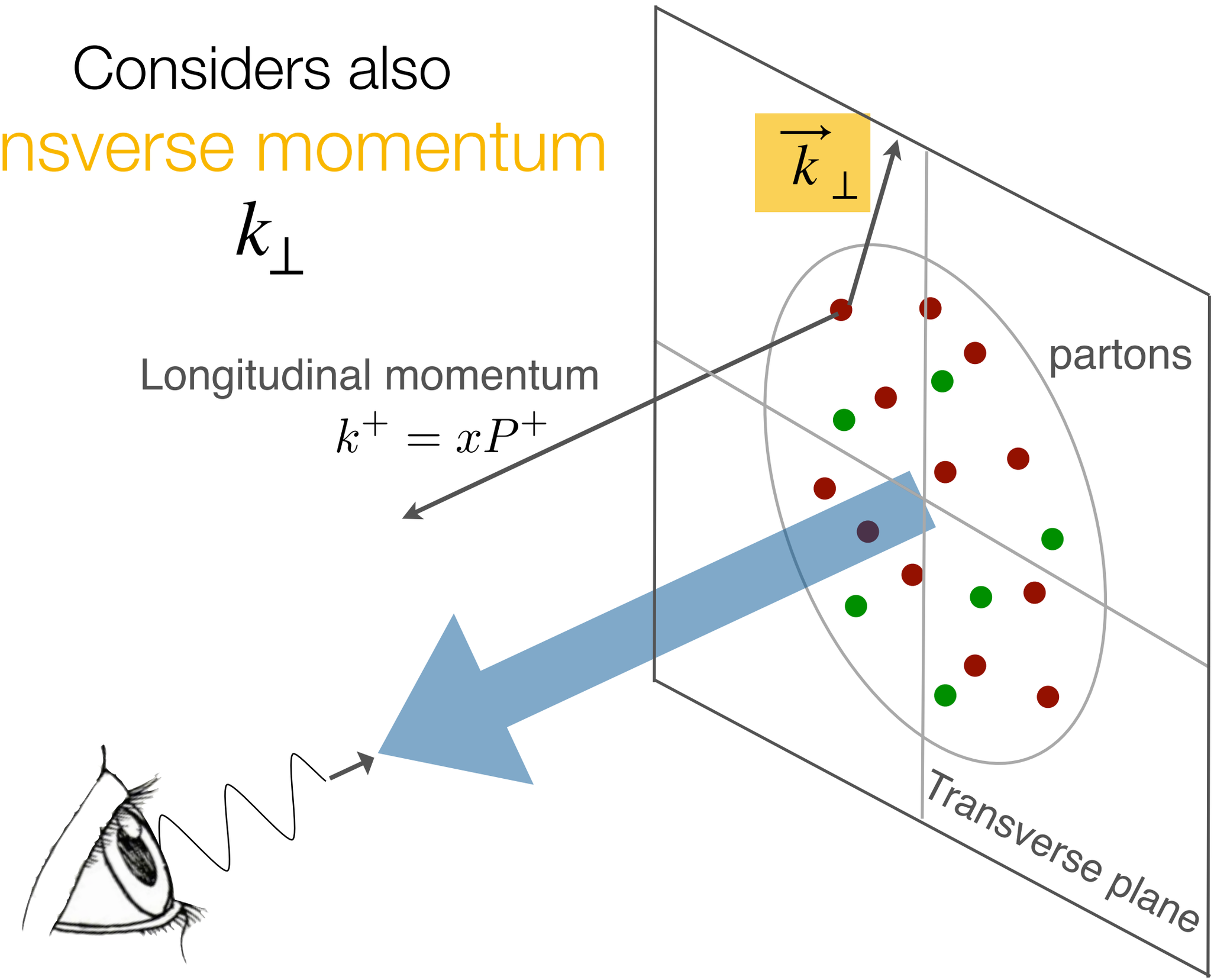
Considers also

transverse momentum

$k_\perp$

Longitudinal momentum

$k^+ = xP^+$



Transverse Momentum Distributions

quark polarization

nucleon polarization

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

Transverse Momentum Distributions

quark polarization

nucleon polarization

	U	L	T
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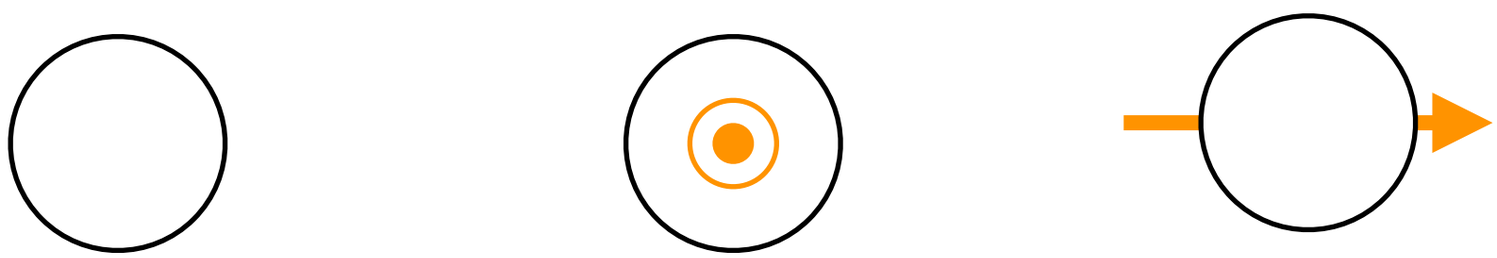
Unpolarized **Longitudinal** **Transverse**

Momentum direction
perp. screen



Transverse Momentum Distributions

Unpolarized

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
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Sivers function

TMD Parton Distribution Functions

Unpolarized

quark pol.

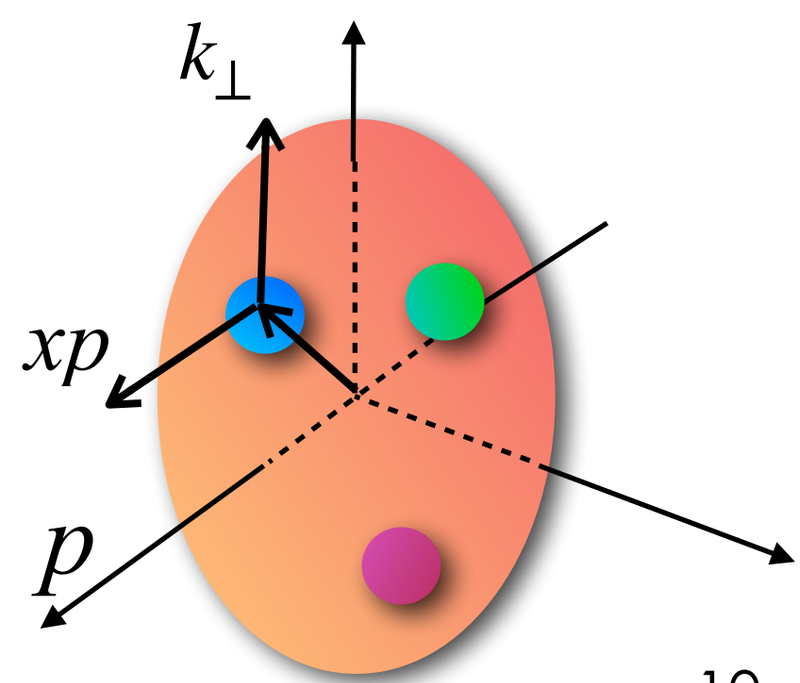
nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

dependence on:

longitudinal momentum fraction x

transverse momentum $k_\perp$

energy scale



TMD Parton Distribution Functions

Unpolarized

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U	f_1		$h_1^\perp$
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T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Why study unpolarized TMDs?

Nucleon tomography
High-energy phenomenology
Necessary to describe polarized processes

Why studying unpolarized TMDs?

Nucleon tomography

High-energy phenomenology

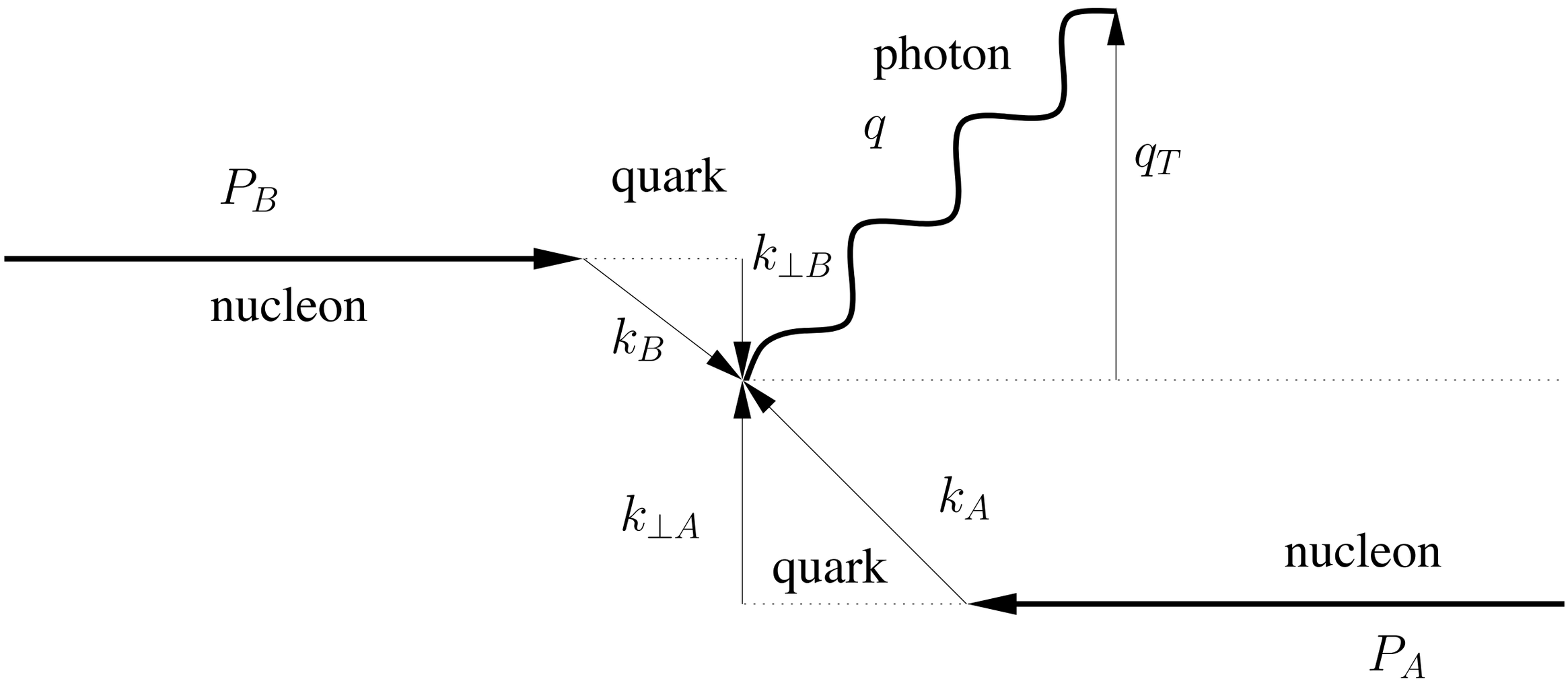
Necessary to describe also polarized processes

Open questions :

1. What is the functional form of TMDs at low transverse momentum ? And their kinematic and flavor dependence?
2. How can we separate the descriptions at low and high transverse momenta ?
3. How can we match TMD and collinear factorization ?
4. Can we test the generalized universality of TMDs ?
5. Can we perform a global fit of TMDs ?

Extraction from SIDIS & Drell-Yan

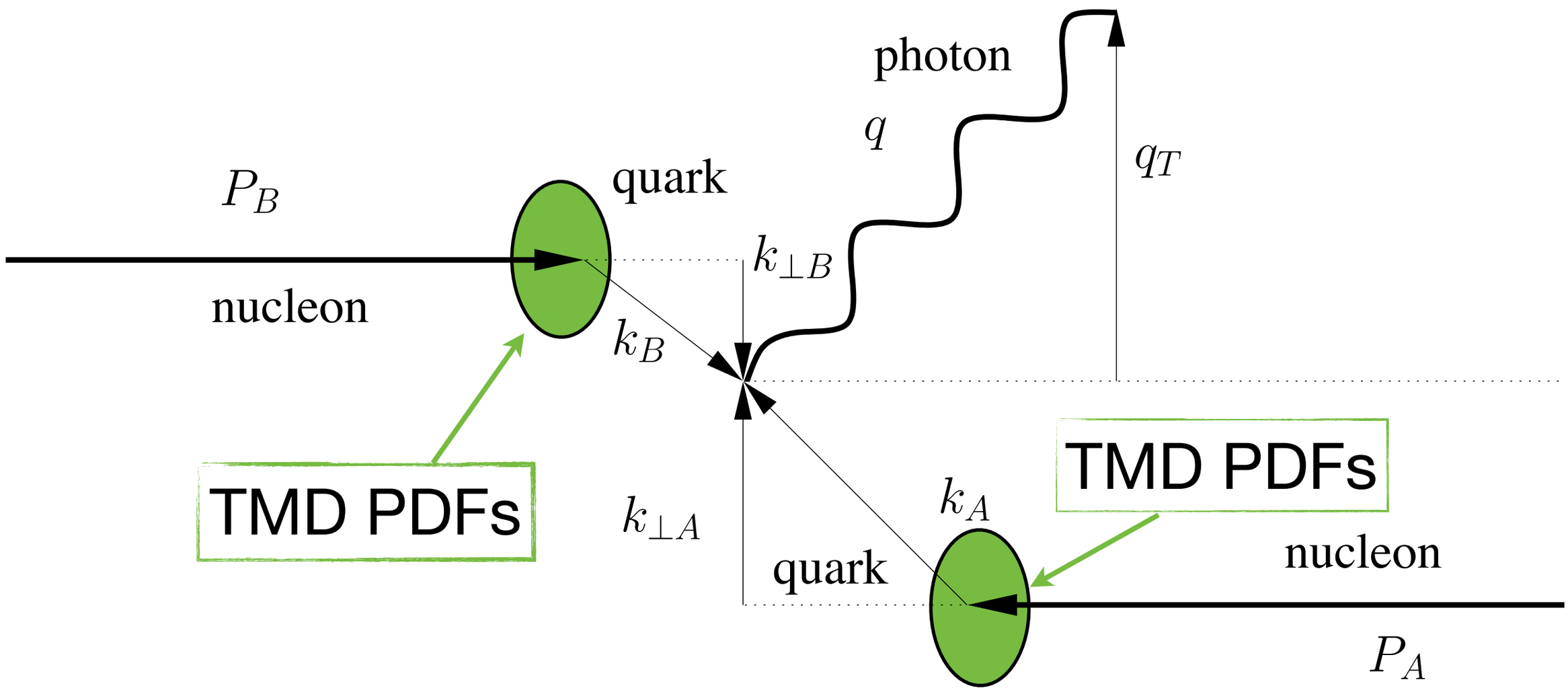
Drell-Yan / Z production



$$A + B \to \gamma^*/Z \to l^+ l^-$$

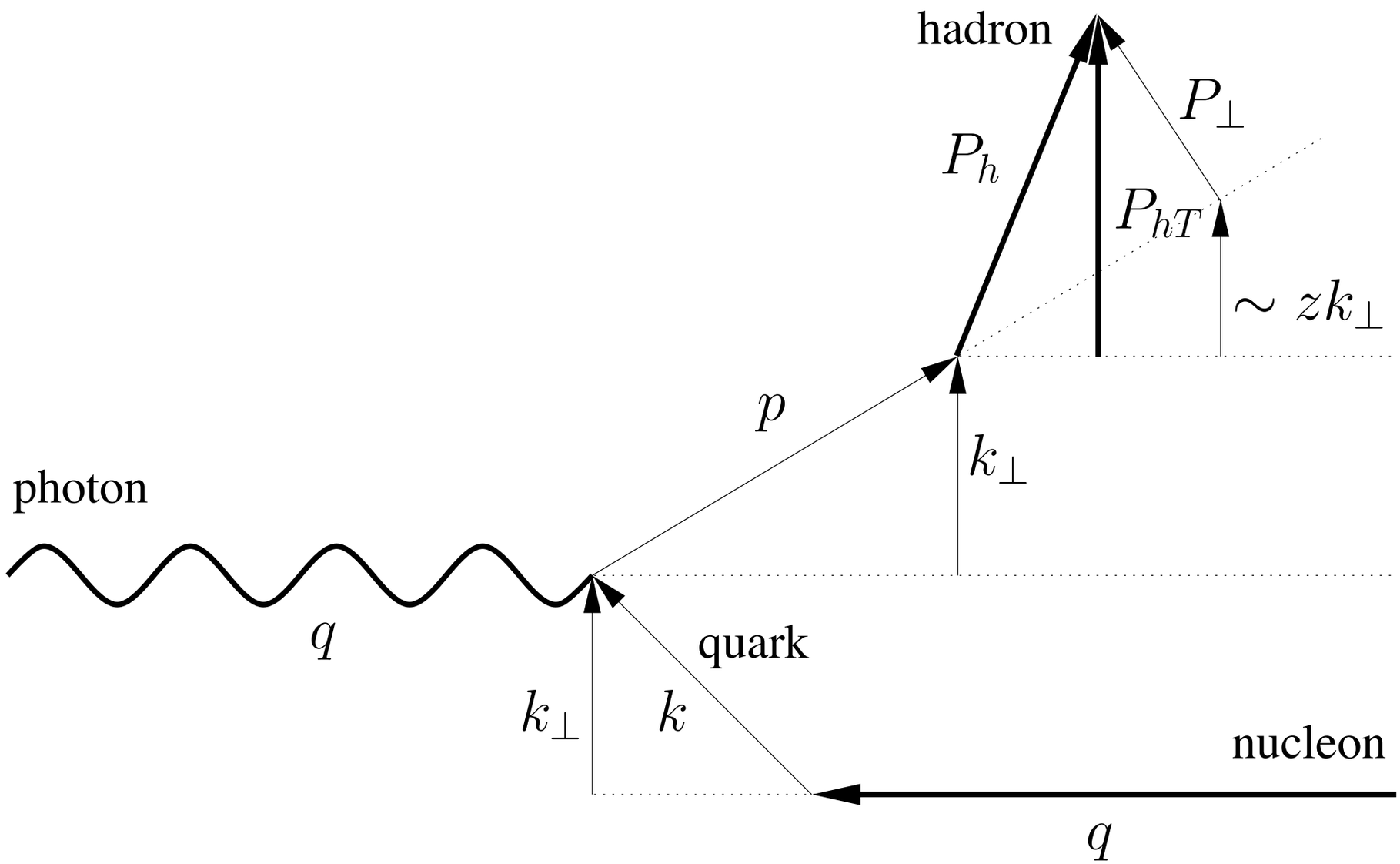
Extraction from SIDIS & Drell-Yan

Drell-Yan / Z production



$$A + B \to \gamma^*/Z \to l^+ l^-$$

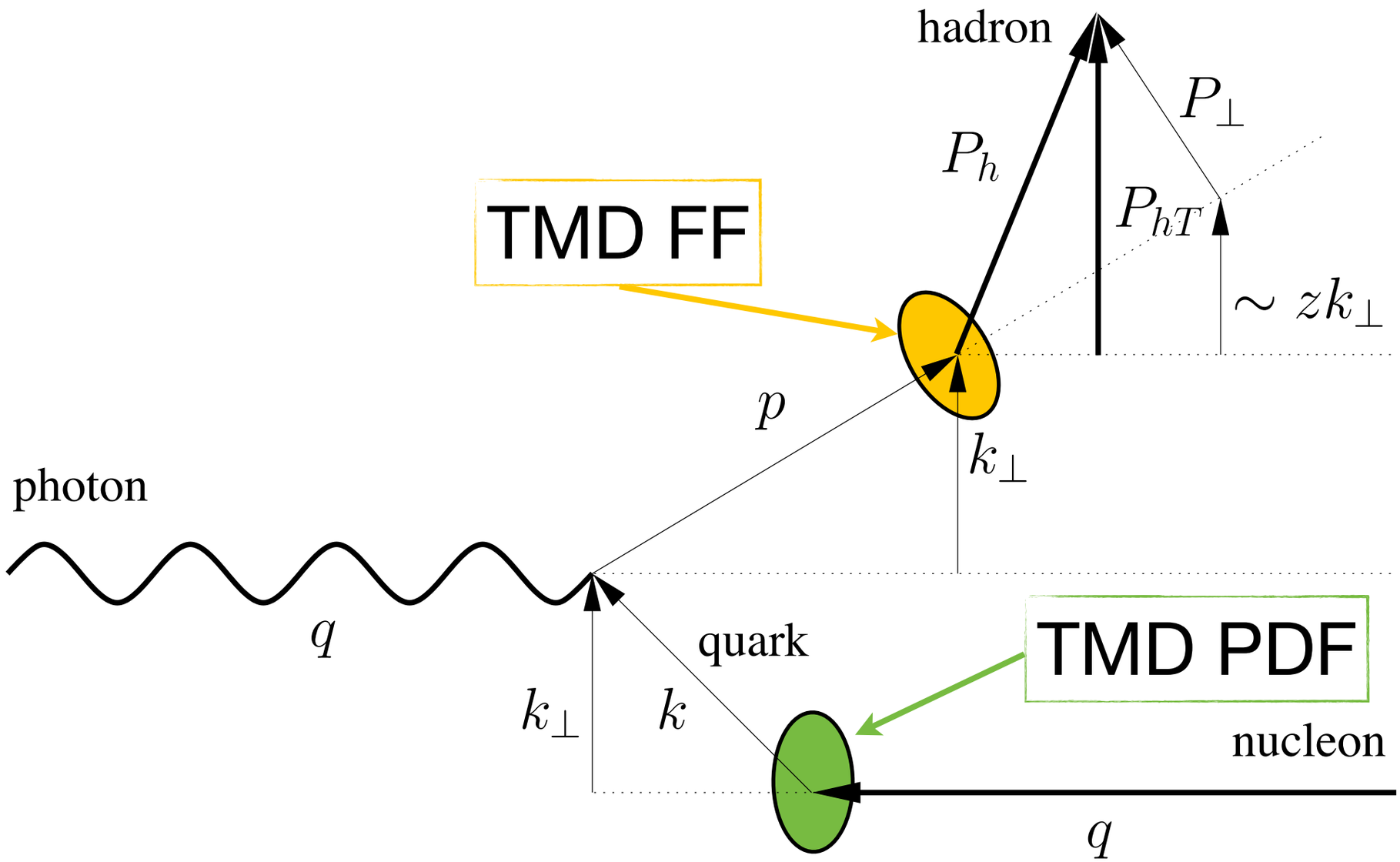
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

TMDs: Fragmentation Function

quark polarization

Unpolarized

U	L	T
D_1		$H_1^\perp$

TMD Fragmentation Functions
(TMD FFs)

dependence on:

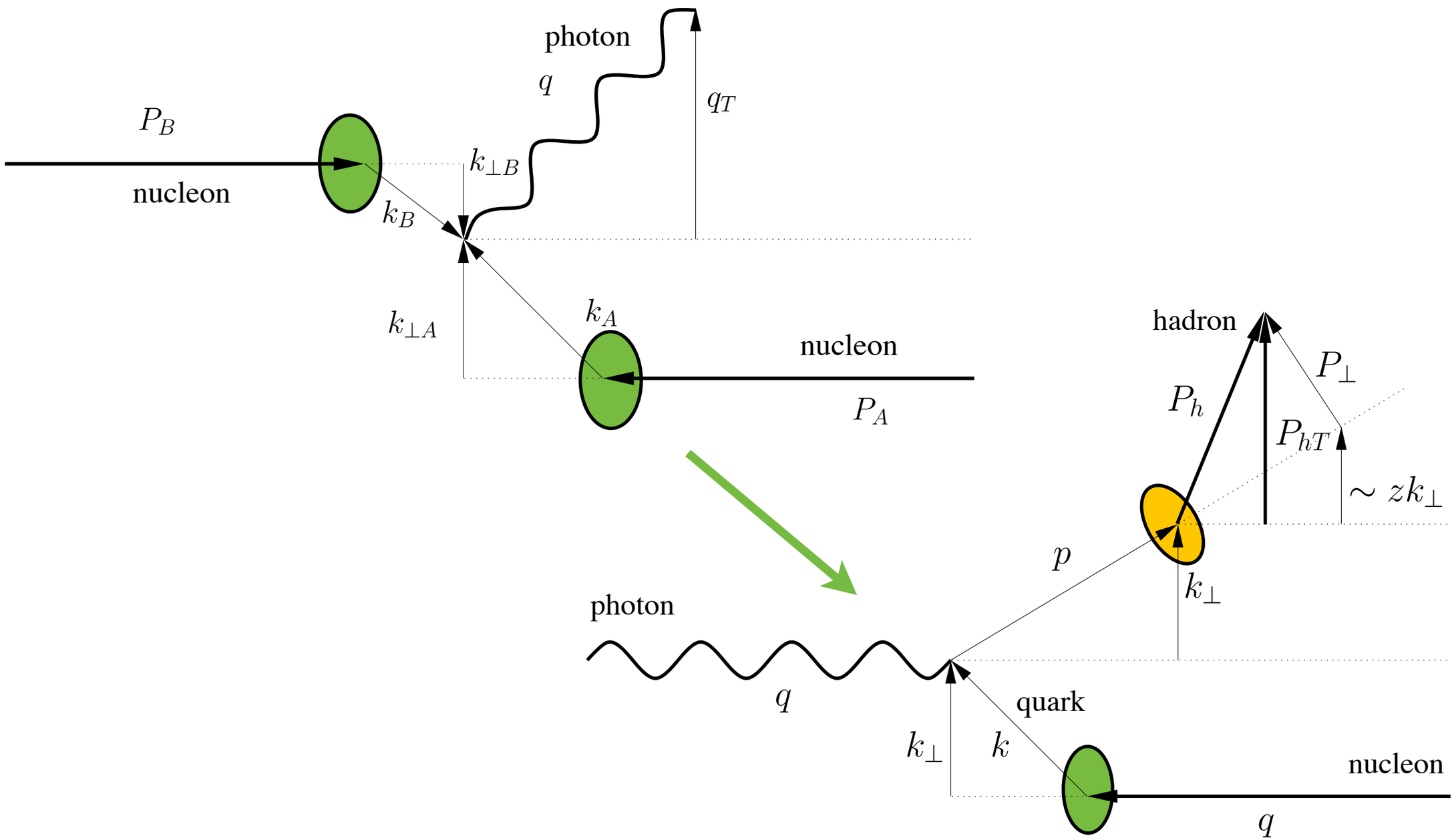
longitudinal momentum fraction **z**

transverse momentum $\boldsymbol{P}_\perp$

energy scale

Extraction from SIDIS & Drell-Yan

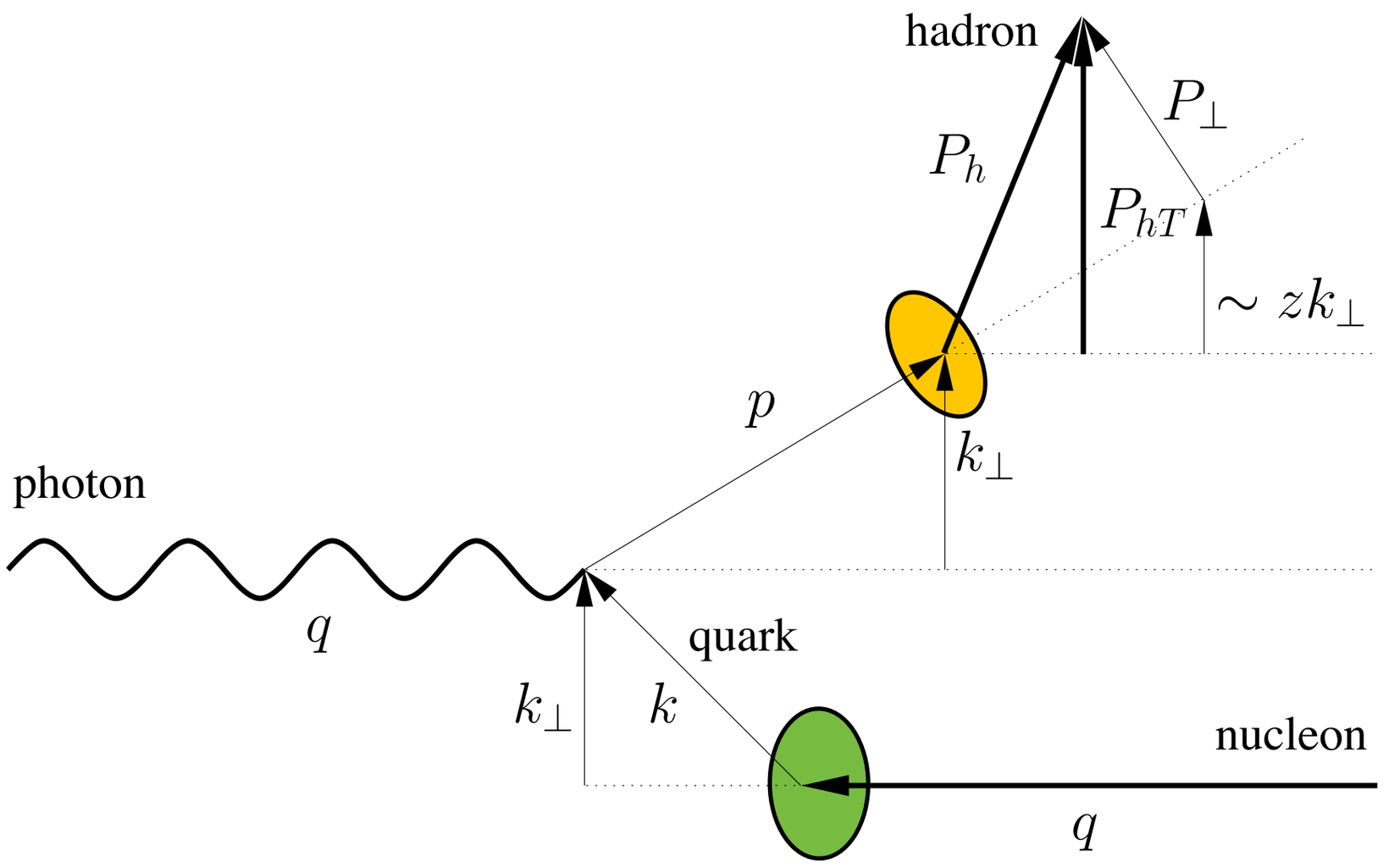
universality



Structure functions and TMDs: SIDIS

multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dxdzd\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dxdQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



Structure functions and TMDs

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TMD factorization

$$F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\sum_a\mathscr{H}^a\left(Q^2\right)x\int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp f_1^a(x,\boldsymbol{k}_\perp^2;Q^2)D_1^{a\to h}(z,\boldsymbol{P}_\perp^2;Q^2)$$
$$\times\,\delta^2(z\boldsymbol{k}_\perp-\boldsymbol{P}_{hT}+\boldsymbol{P}_\perp)+Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2)+\mathcal{O}(M^2/Q^2)$$

Structure functions and TMDs

At our accuracy level:

Leading Order (expansion in α_S)

Next-to Leading Log (corrections in $\alpha_S \log\left(z^2 Q^2 / P_{hT}^2\right)$)

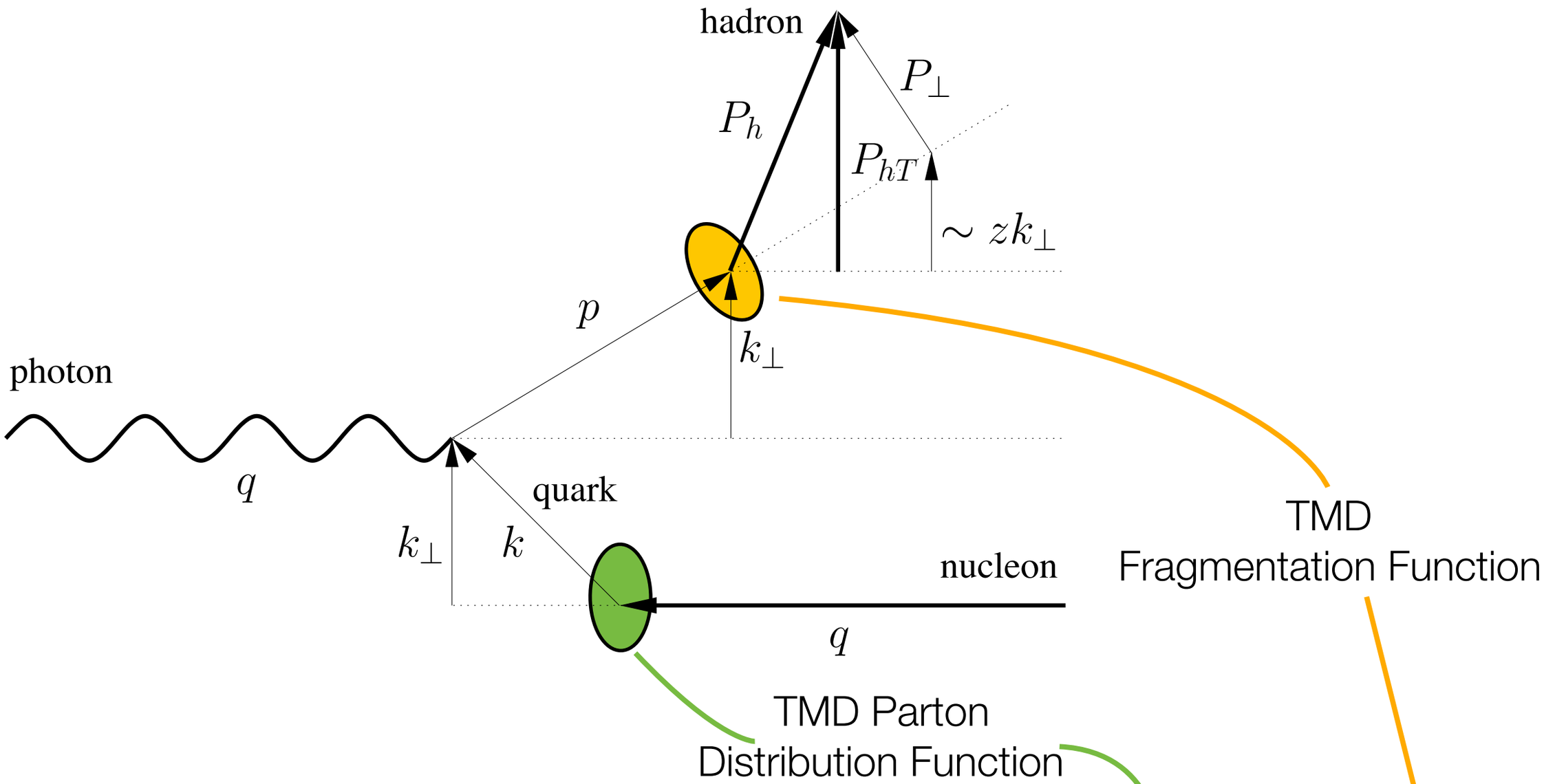
$$\simeq \mathcal{O}\left(\alpha_s^0\right)$$

$$F_{UU,T}\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right) = \sum_a \mathscr{H}^a\left(Q^2\right) x \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp f_1^a(x, \boldsymbol{k}_\perp^2; Q^2) D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)$$

$$\times\, \delta^2(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2, \boldsymbol{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

$$\simeq 0$$

Structure functions and TMDs

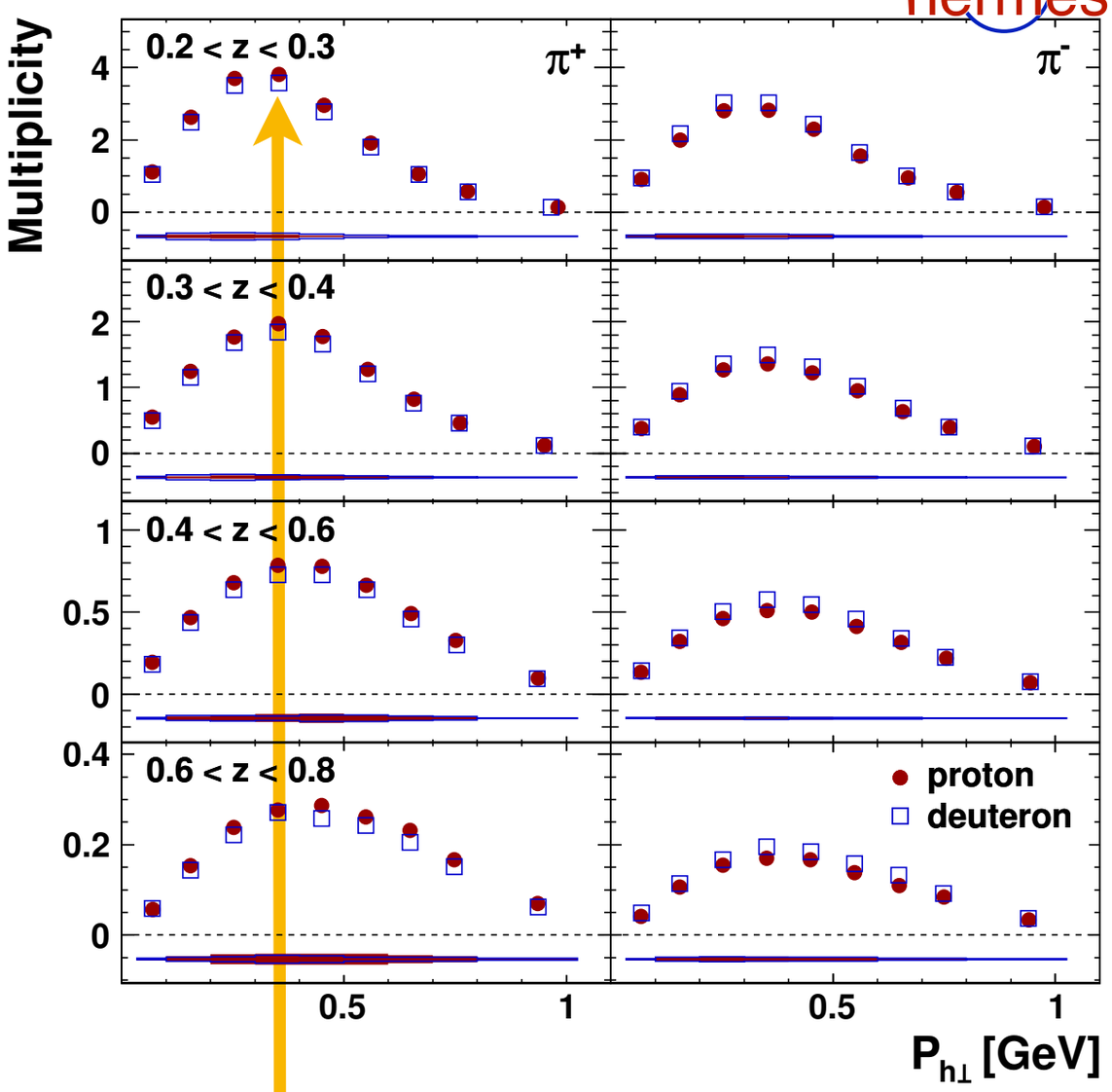


$$F_{UU,T}\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right) \simeq \sum_a x \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp f_1^a(x, \boldsymbol{k}_\perp^2; Q^2) D_1^{a\to h}(z, \boldsymbol{P}_\perp^2; Q^2)$$

$$\times\, \delta^2(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp)$$

TMD Evolution

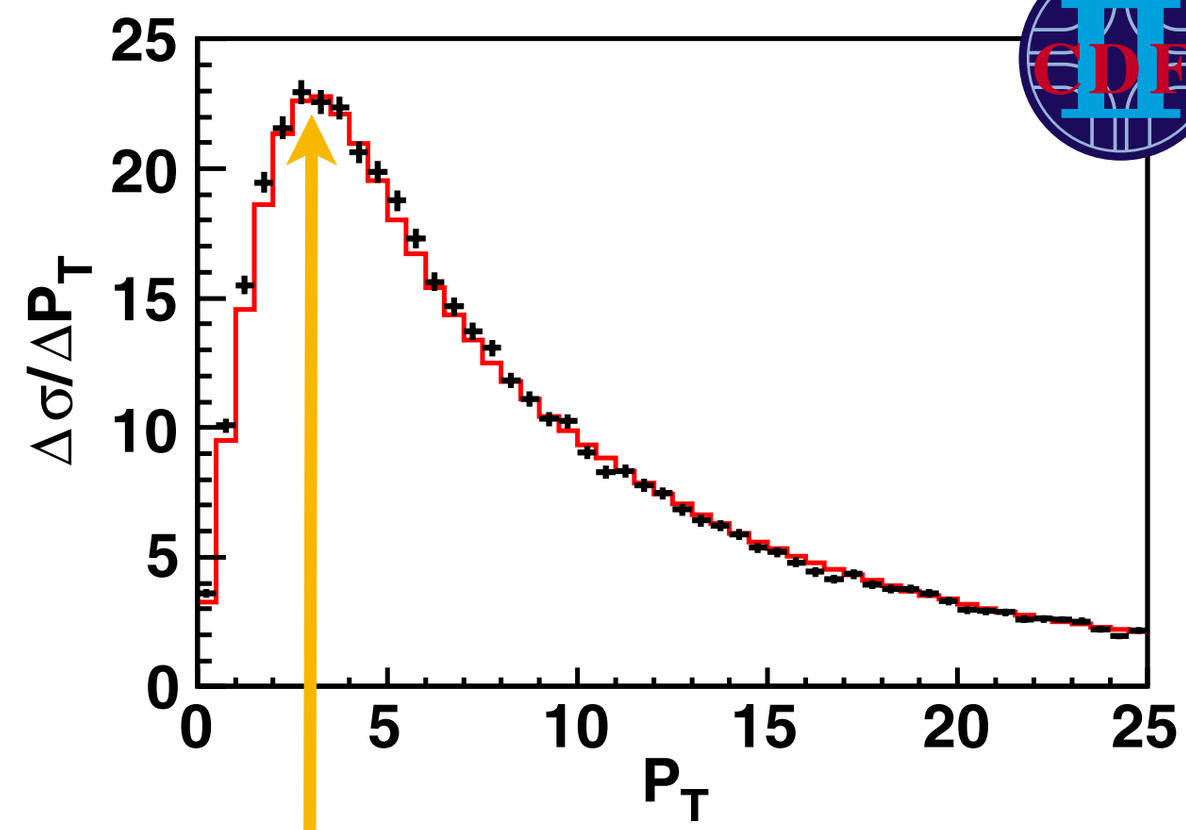
HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

reproduce shift of
TMD peak with energy scale

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude → EVOLUTION

Evolved TMDs

Fourier transform: ξ_T space

$$\tilde{f}_1^a\left(x,\xi_T;\mu^2\right) =$$

$$= \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)\left(x,\bar{\xi}_*;\mu_b\right) e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)} e^{g_K(\xi_T)\ln(\mu/\mu_0)} \hat{f}_{NP}^a\left(x,\xi_T\right)$$

nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

nonperturbative part of TMD

CSS formalism

Evolved TMDs

Fourier transform: ξ_T space

$$\tilde{f}_1^a\left(x,\xi_T;\mu^2\right) =$$

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nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

nonperturbative part of TMD

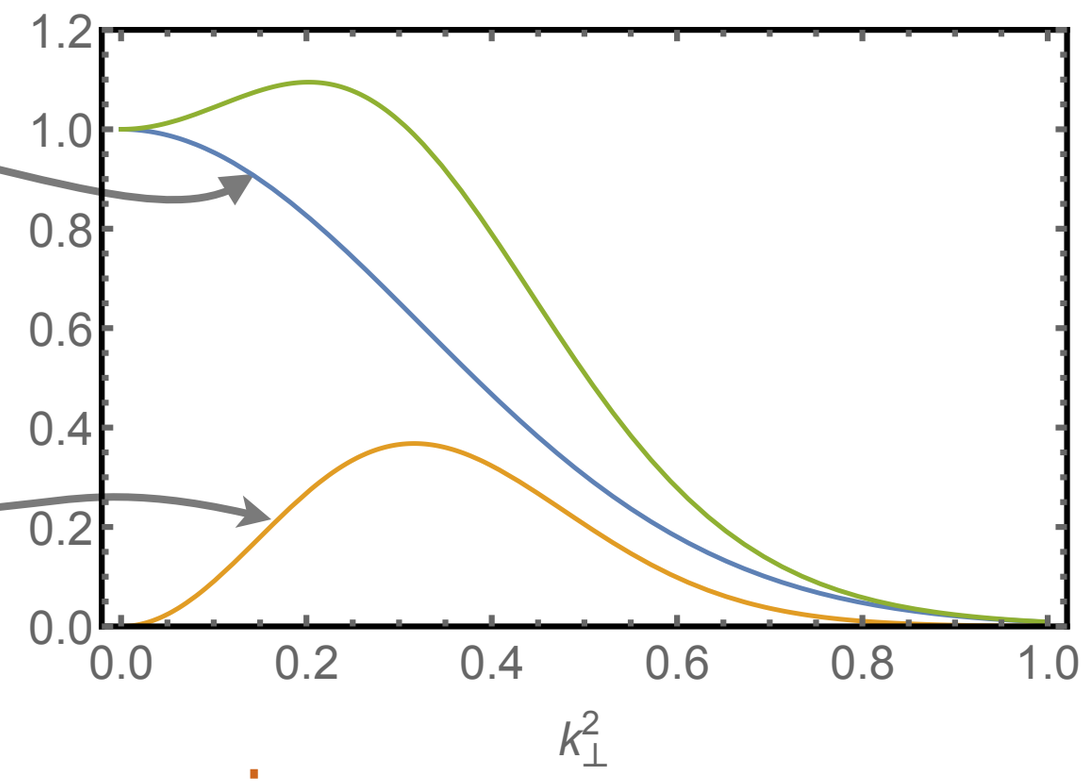
Non-perturbative contributions have to be extracted from experimental data, after parametrization

Model: non perturbative elements

input TMD PDF @ Q^2=1GeV^2

$$\tilde{f}^a_{NP} = \mathcal{F}.\mathcal{T}. \text{ of}$$

$$\left(\exp\left(\frac{-k_\perp^2}{g_1}\right) + \lambda k_\perp \exp\left(\frac{-k_\perp^2}{g_1}\right)\right)$$



sum of two different gaussians

with kinematic dependence on transverse momenta

$$g_1(x) = N_1 \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$

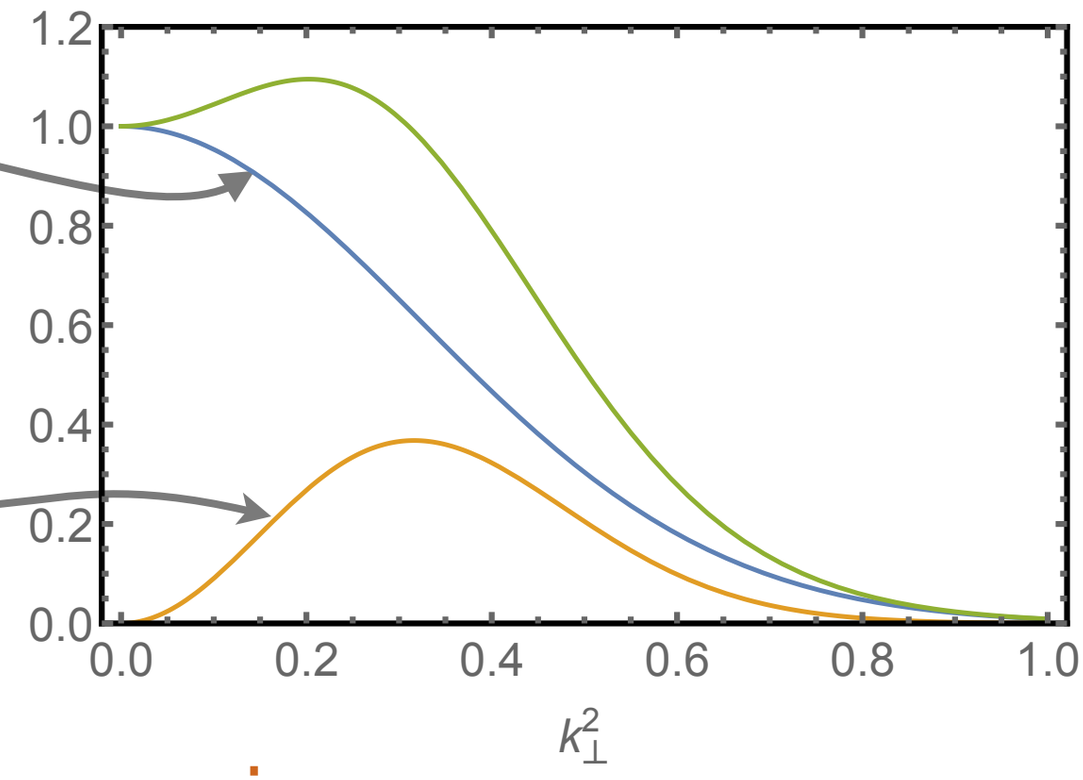
$$\hat{x} = 0.1$$

Model: non perturbative elements

input TMD PDF @ Q^2=1GeV^2

$$\tilde{f}^a_{NP} = \mathcal{F}.\mathcal{T}. \text{ of}$$

$$\left(\exp\left(\frac{-k_\perp^2}{g_1}\right) + \lambda k_\perp \exp\left(\frac{-k_\perp^2}{g_1}\right)\right)$$



sum of two different gaussians

with kinematic dependence on transverse momenta

For the FF we use two different variances: $g_3(z), g_4(z)$

Model: non perturbative elements

Free parameters

$$N_1,\ \alpha,\ \sigma,\ \lambda$$

4 for TMD PDF

$$N_3, N_4, \beta, \delta, \gamma, \lambda_F$$

6 for TMD FF

$$g_K = -\, g_2 \frac{\xi_T^2}{2}$$

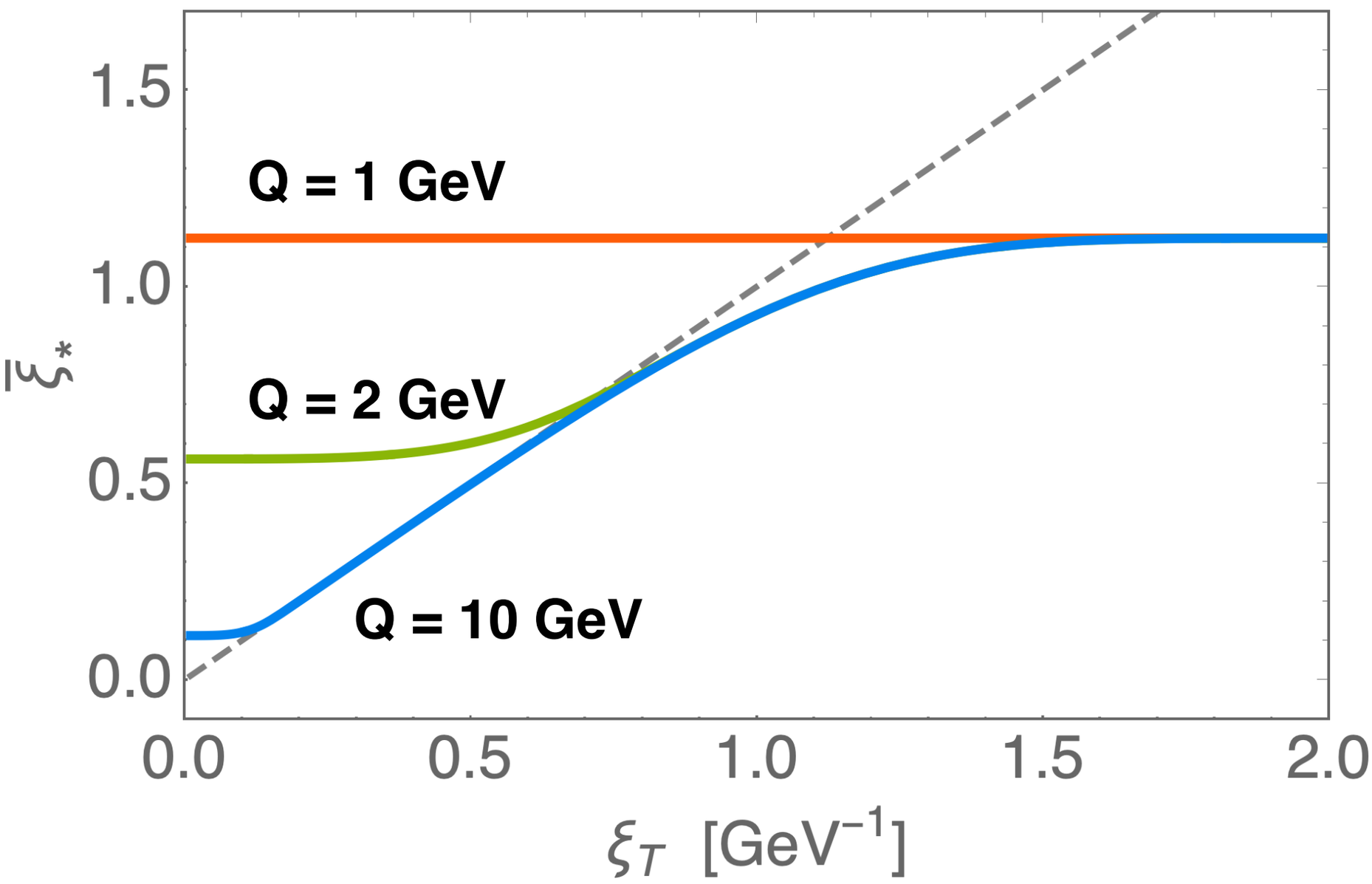
1 for NP contribution to TMD evolution

Total: 11 parameters

Evolution and ξ_T regions

$$\mu_b = 2e^{-\gamma_E}/\bar{\xi}_*$$

$$\bar{\xi}_*\left(\xi_T, \xi_{min}, \xi_{max}\right) = \xi_{max}\left[\frac{1-\exp(\xi_T^4/\xi_{max}^4)}{1-\exp(\xi_T^4/\xi_{min}^4)}\right]^{1/4}$$



$$\xi_{max} = 2e^{-\gamma_E}$$

$$\xi_{min} = 2e^{-\gamma_E}/Q$$

alternative notation:
b_T

Experimental data

SIDIS μN

6252
data points

SIDIS eN

1514
data points

Total: **8059** data

E288
E605

Drell-Yan

203
data points

Z Production

90
data points

Data selection and analysis

$Q2 > 1.4\ GeV^2$

$0.2 < z < 0.7$

$P_{hT}\ ,\ q_T < Min[0.2Q\ ,\ 0.7Qz] + 0.5\ GeV$

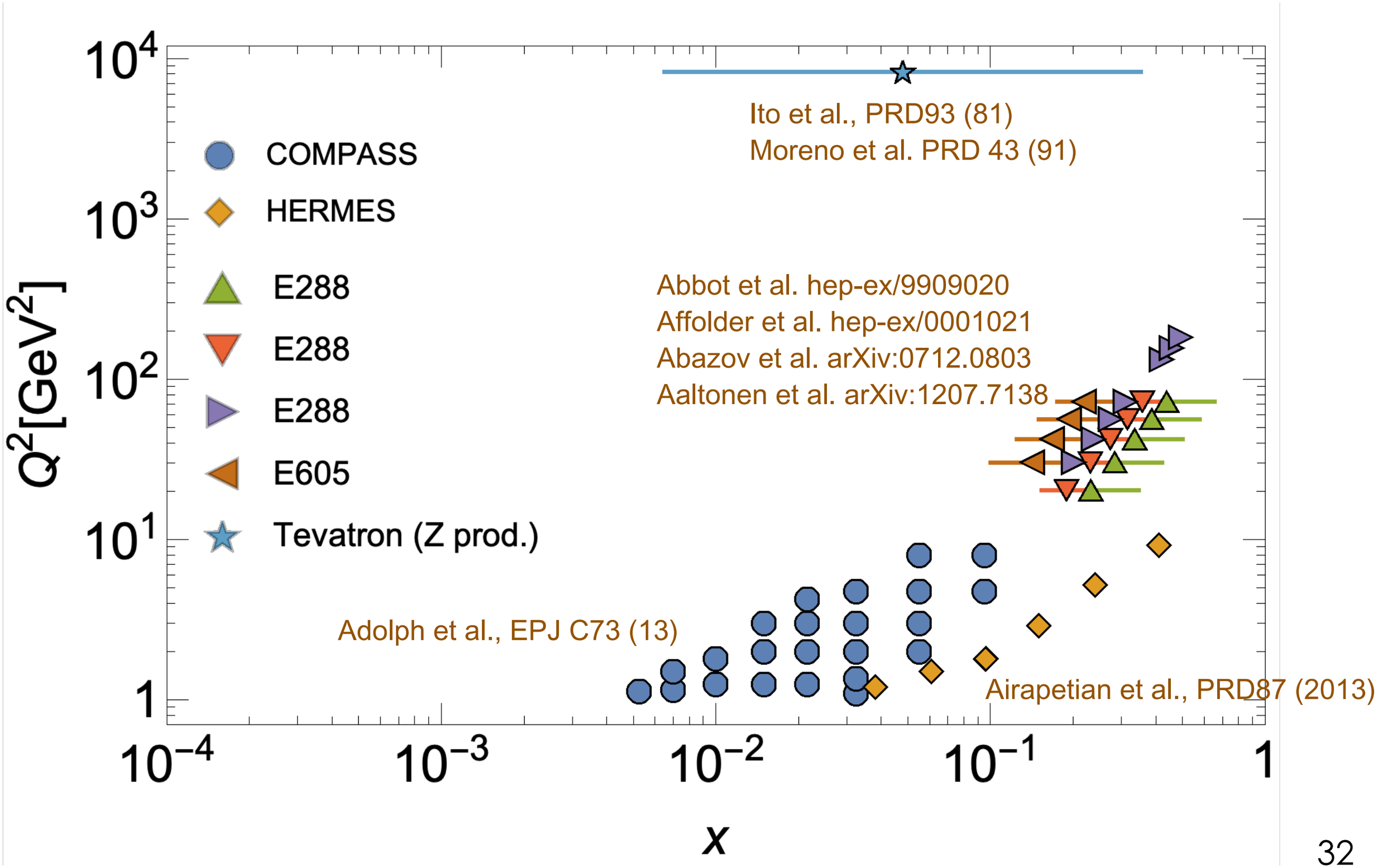
Motivations behind kinematical cuts

TMD factorization ($Ph_T/z << Q^2$)

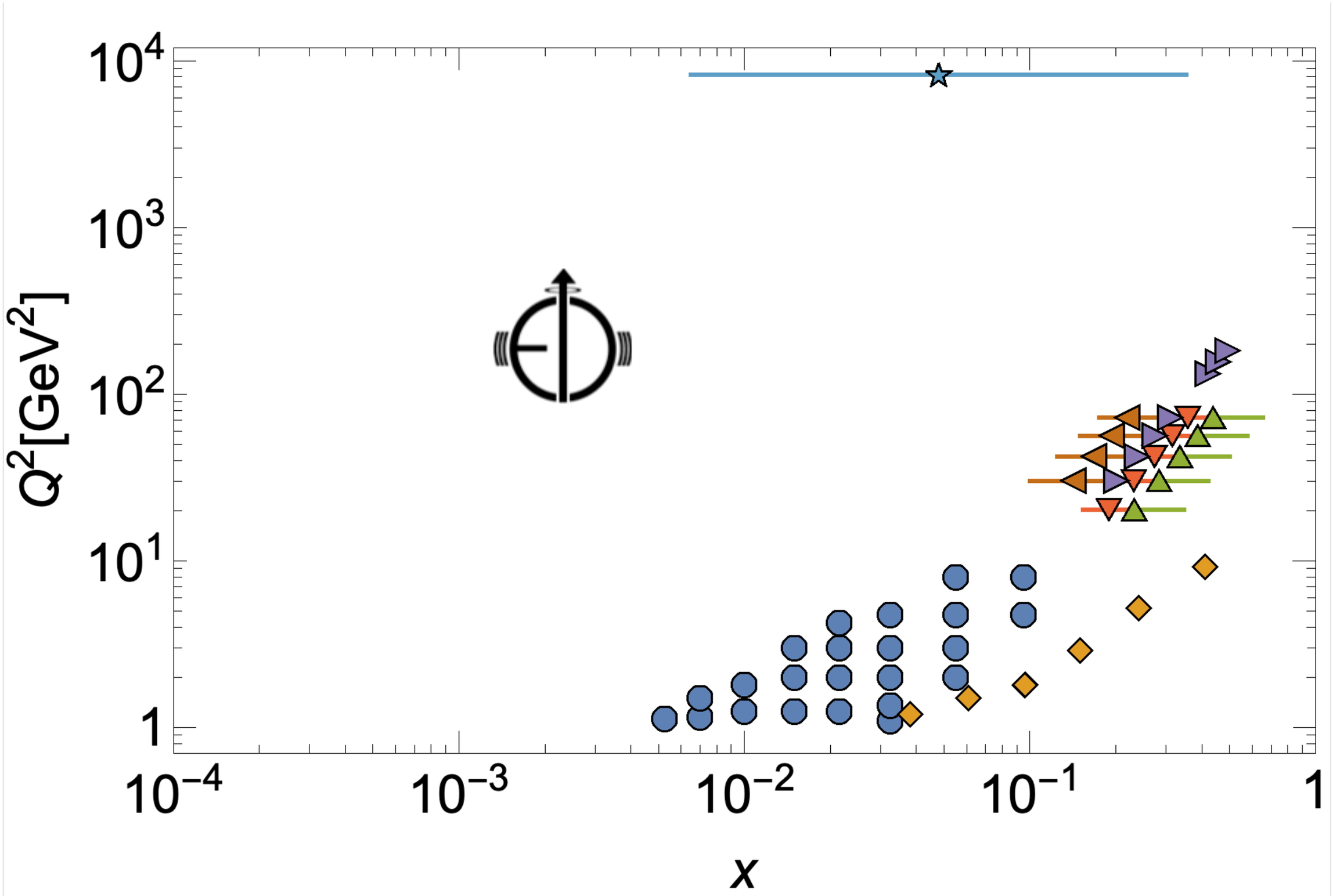
Avoid target fragmentation (low z)

and exclusive contributions (high z)

Data regions



Data regions



An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

published in

[JHEP06(2017)081]

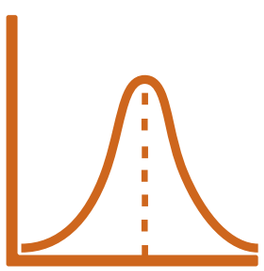
Summary of results

Total number of data points: 8059

Total number of free parameters: 11

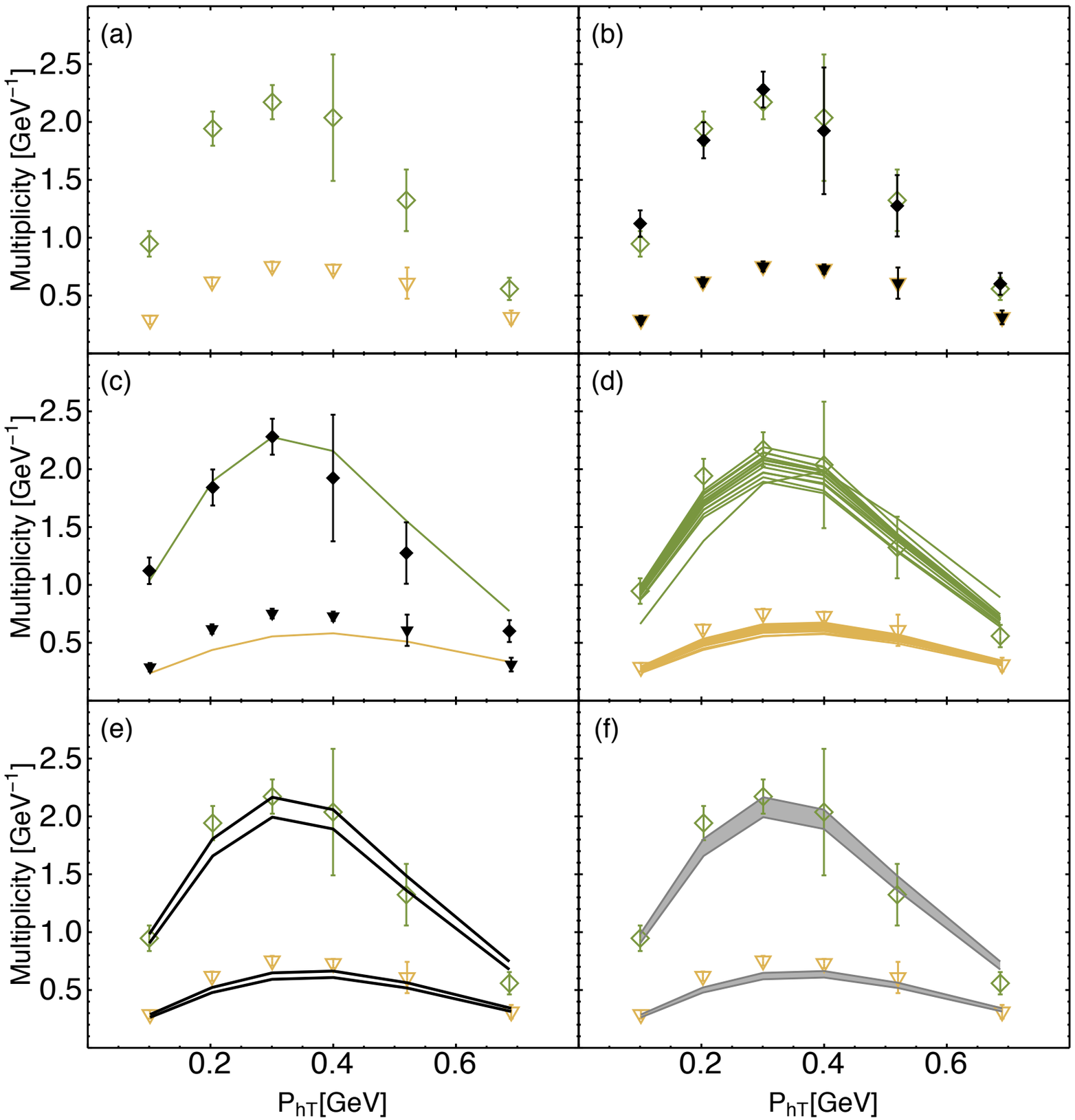
➛ 4 for TMD PDFs ➛ 6 for TMD FFs

➛ 1 for TMD evolution



$$\chi^2/d.of. = 1.55 \pm 0.05$$

Replica Methodology



a) Example of original data (two bins)

b) Data are replicated with Gaussian noise

c) The fit is performed on the replicated data

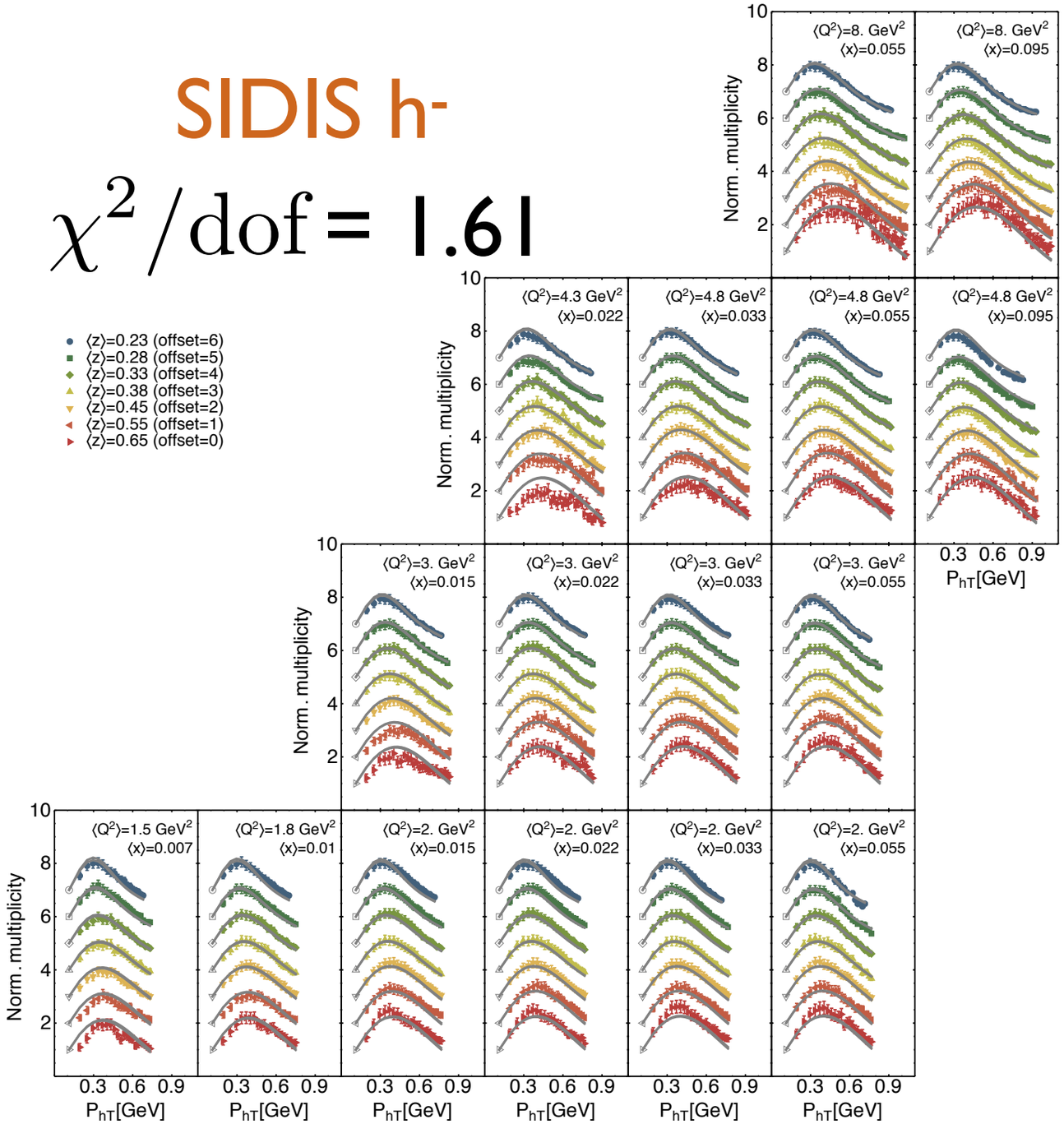
d) The procedure is repeated 200 times

e) For each point a 68% confidence level is identified

f) These point connects to create a 68% C.L. band

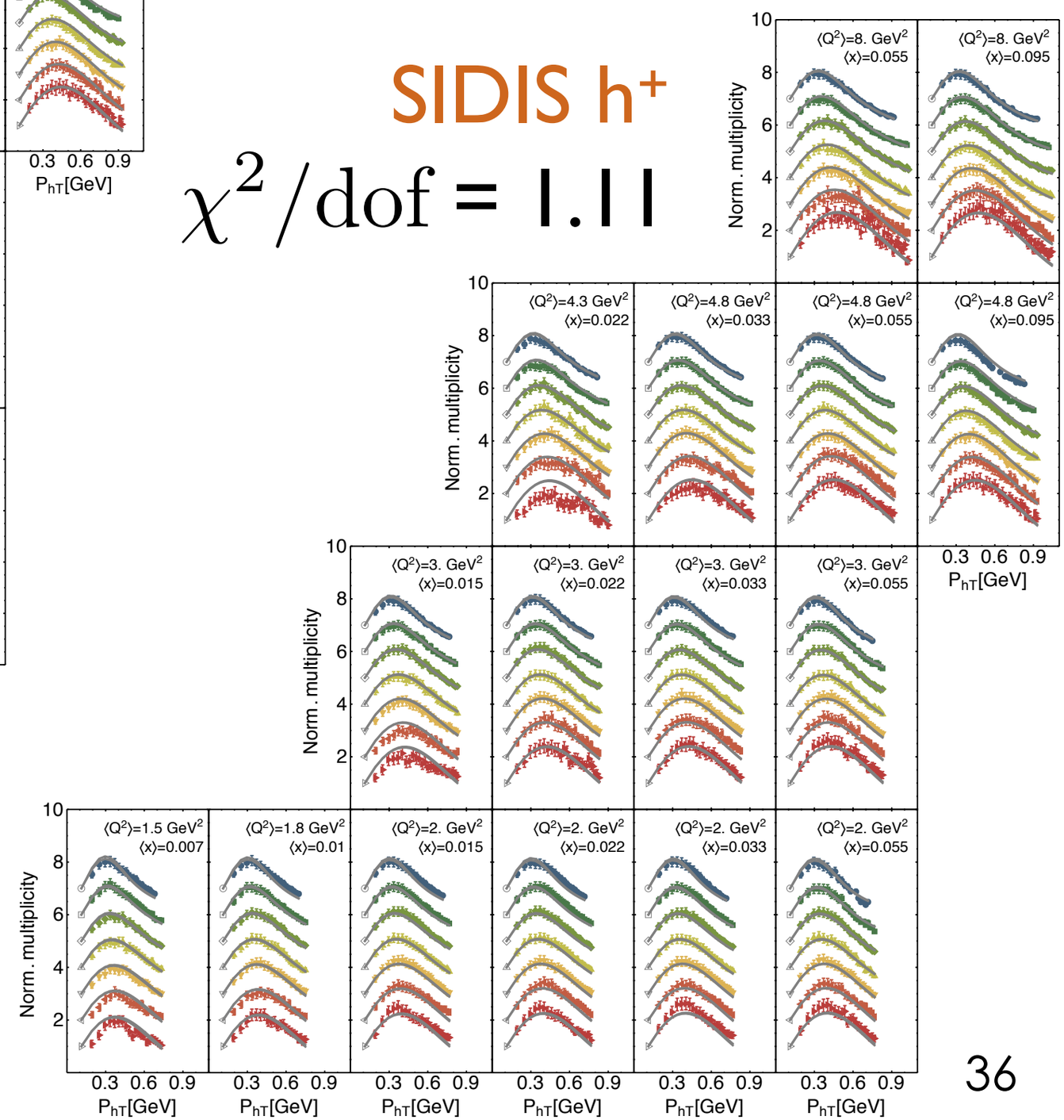
SIDIS h-

$$\chi^2/\mathrm{dof} = 1.61$$

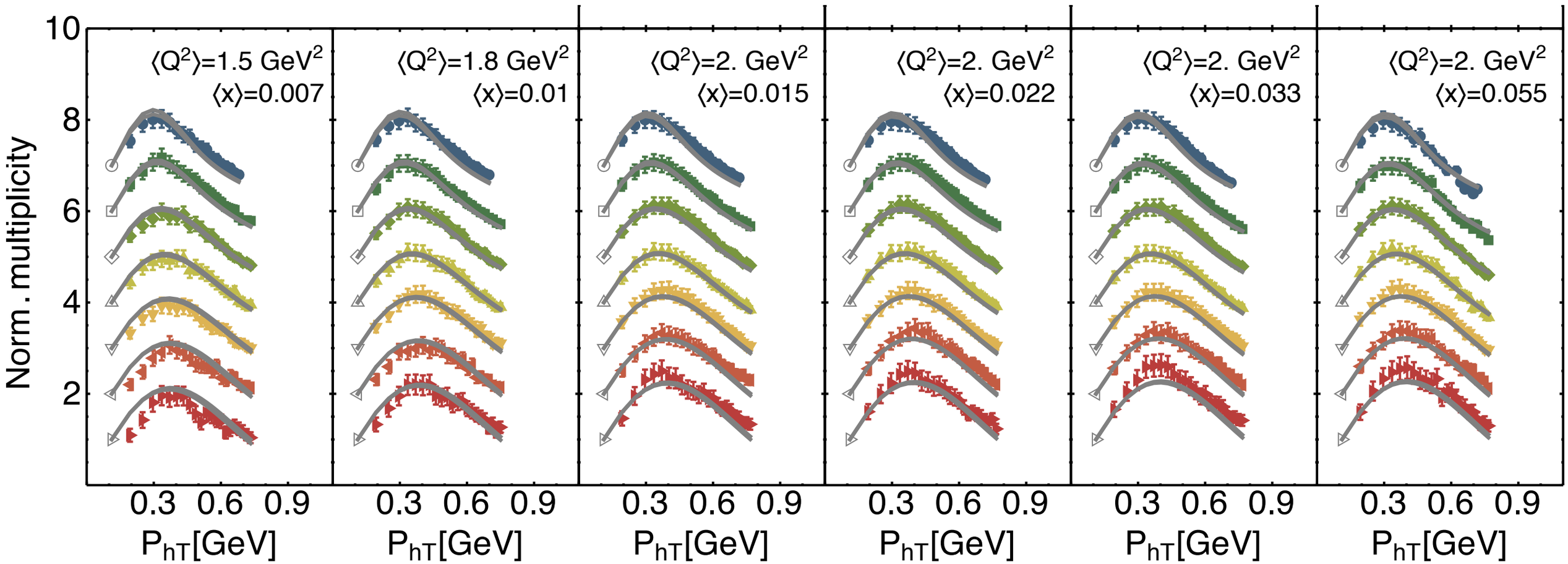


SIDIS h+

$$\chi^2/\mathrm{dof} = 1.11$$



COMPASS data
SIDIS h^+



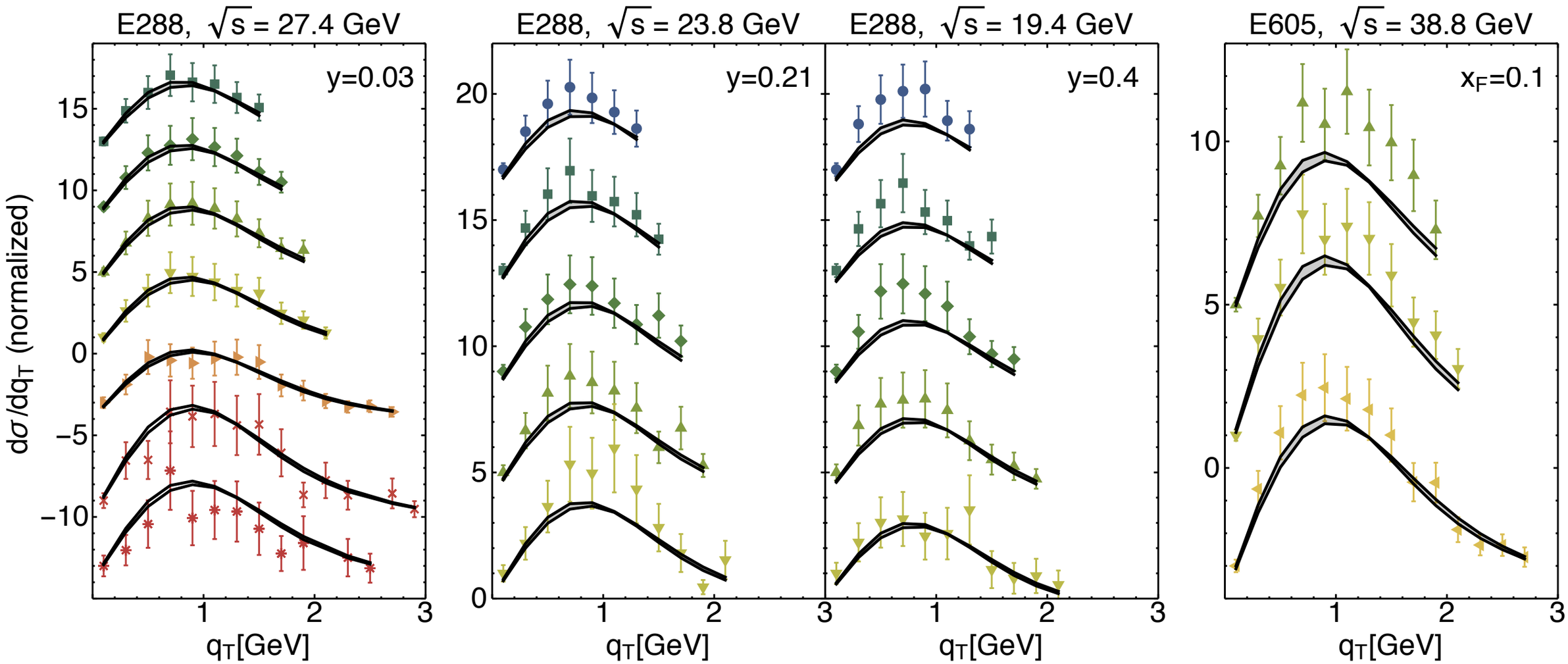
to avoid known problems
with Compass data normalization:

Observable

$$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

Drell-Yan data

χ^2/dof 0.32 0.84 0.99 1.12

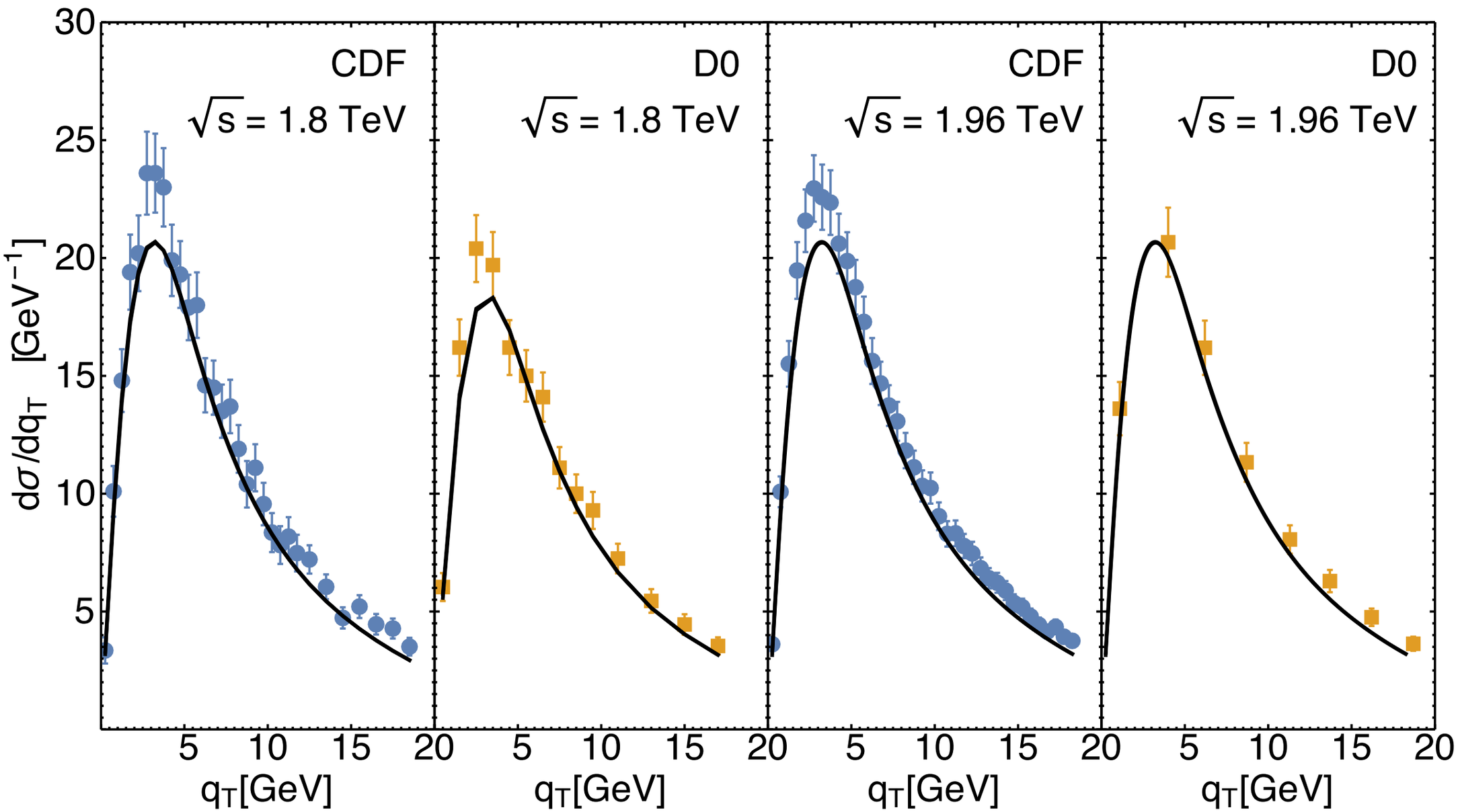


Fermilab

Q^2 Evolution: The peak is now at about 1 GeV, it was at 0.4 GeV for SIDIS

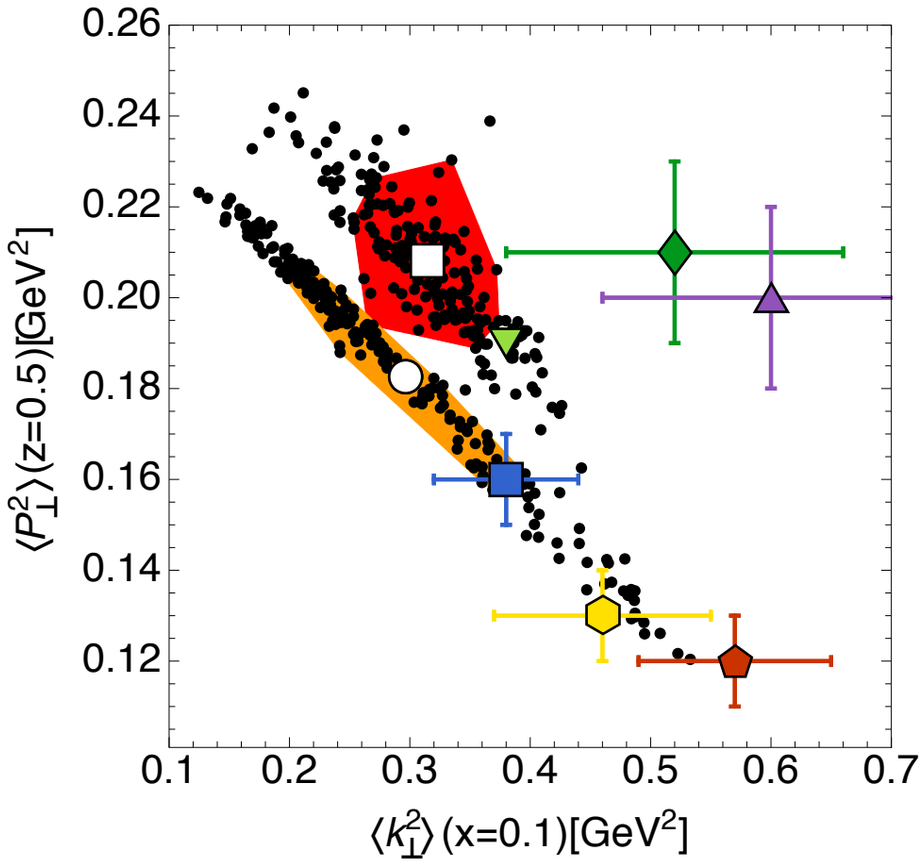
Z-boson production data

χ^2/dof 1.36 1.11 2.00 1.73



Q^2 Evolution: The peak is now at about 4 GeV

Best fit value: transverse momenta



Bacchetta, Delcarro, Pisano, Radici, Signori JHEP06(2017)081

Signori, Bacchetta, Radici, Schnell arXiv:1309.3507

Schweitzer, Teckentrup, Metz, arXiv:1003.2190

Anselmino et al. arXiv:1312.6261 [HERMES]

Anselmino et al. arXiv:1312.6261 [HERMES, high z]

Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]

Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]

Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 (Q = 1.5 GeV)

Red/orange regions: **68% CL** from replica method

Inclusion of DY/Z diminishes the correlation

Inclusion of Compass increases the $\langle P_\perp^2 \rangle$ and reduces its spread

e+e- would further reduce the correlation

Stability of our results

Test of our default choices

How does the χ^2 of a single replica change if we modify them?

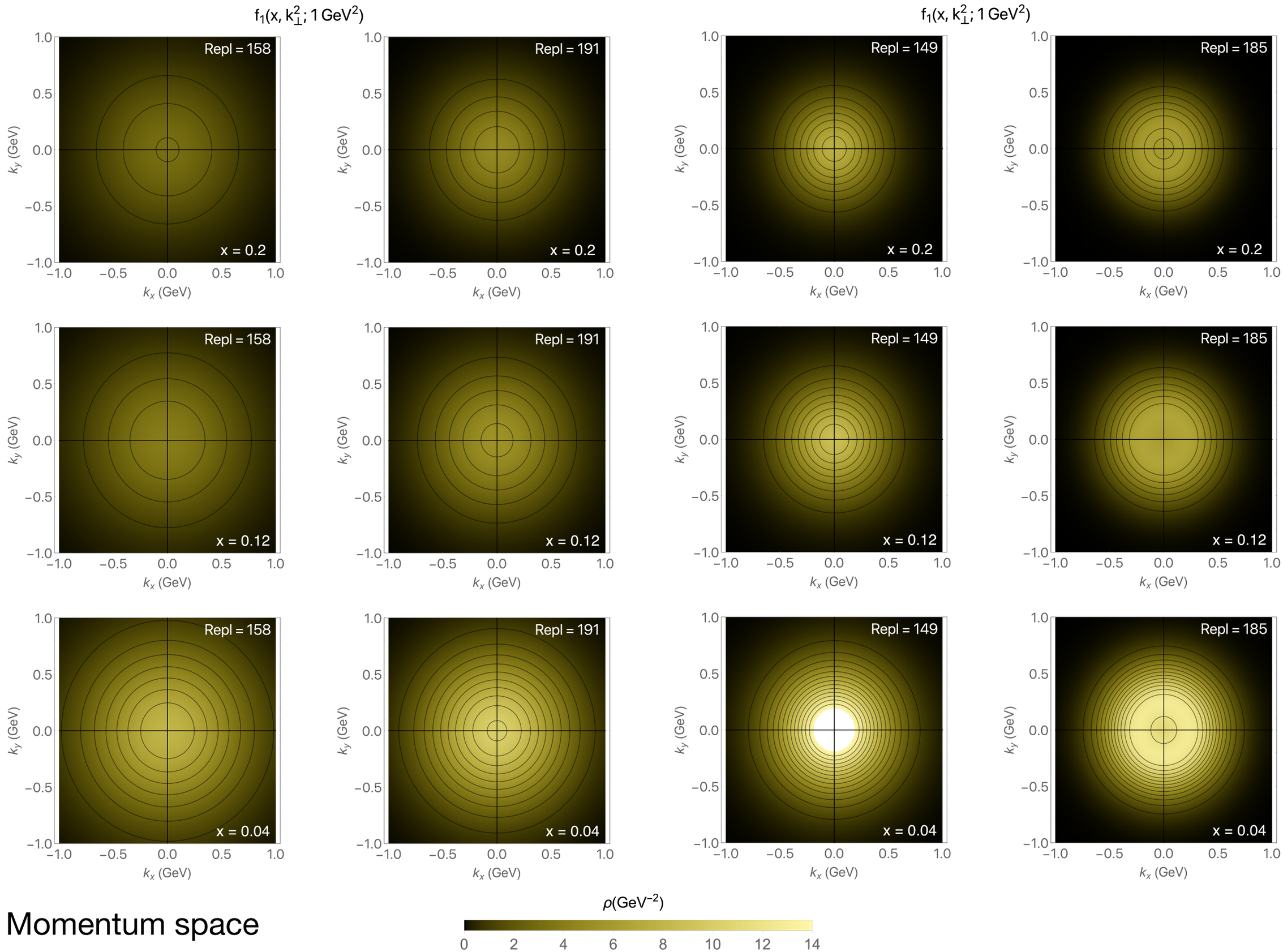
Original χ^2/dof = **1.51**

Normalization of HERMES data as done for COMPASS:
χ^2/dof = 1.27

Parametrizations for collinear PDFs
(NLO GJR 2008 default choice):
NLO MSTW 2008 (1.84), NLO CJ12 (1.85)

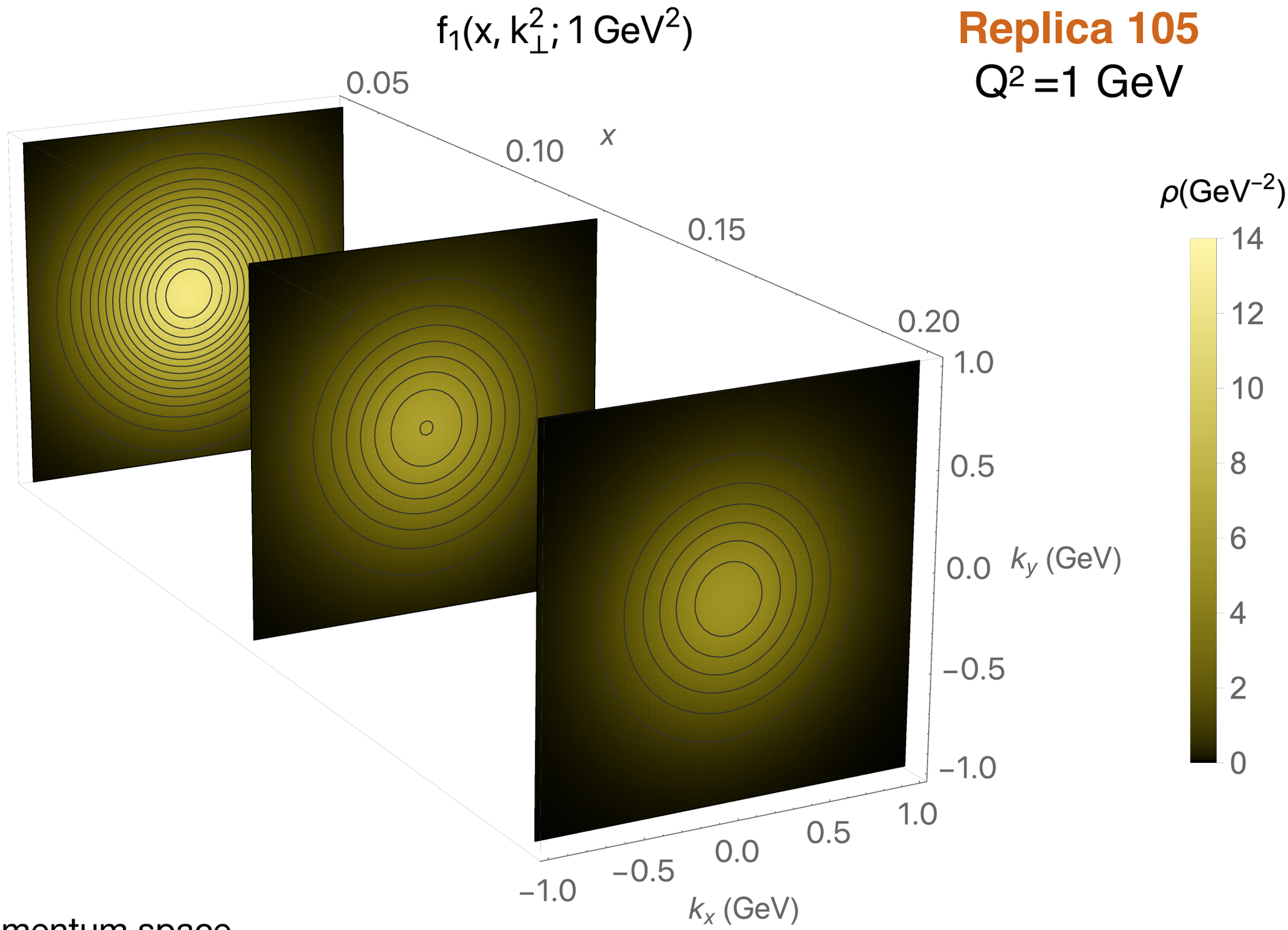
More stringent cuts
(TMD factorization better under control) χ^2/dof $\rightarrow$ 1
Ex: Q2 > 1.5 GeV^2; 0.25 < z < 0.6; PhT < 0.2Qz $\Rightarrow$ χ^2/dof = 1.02 (477 bins)

Visualization of TMDs: PDF 3D structure



Momentum space

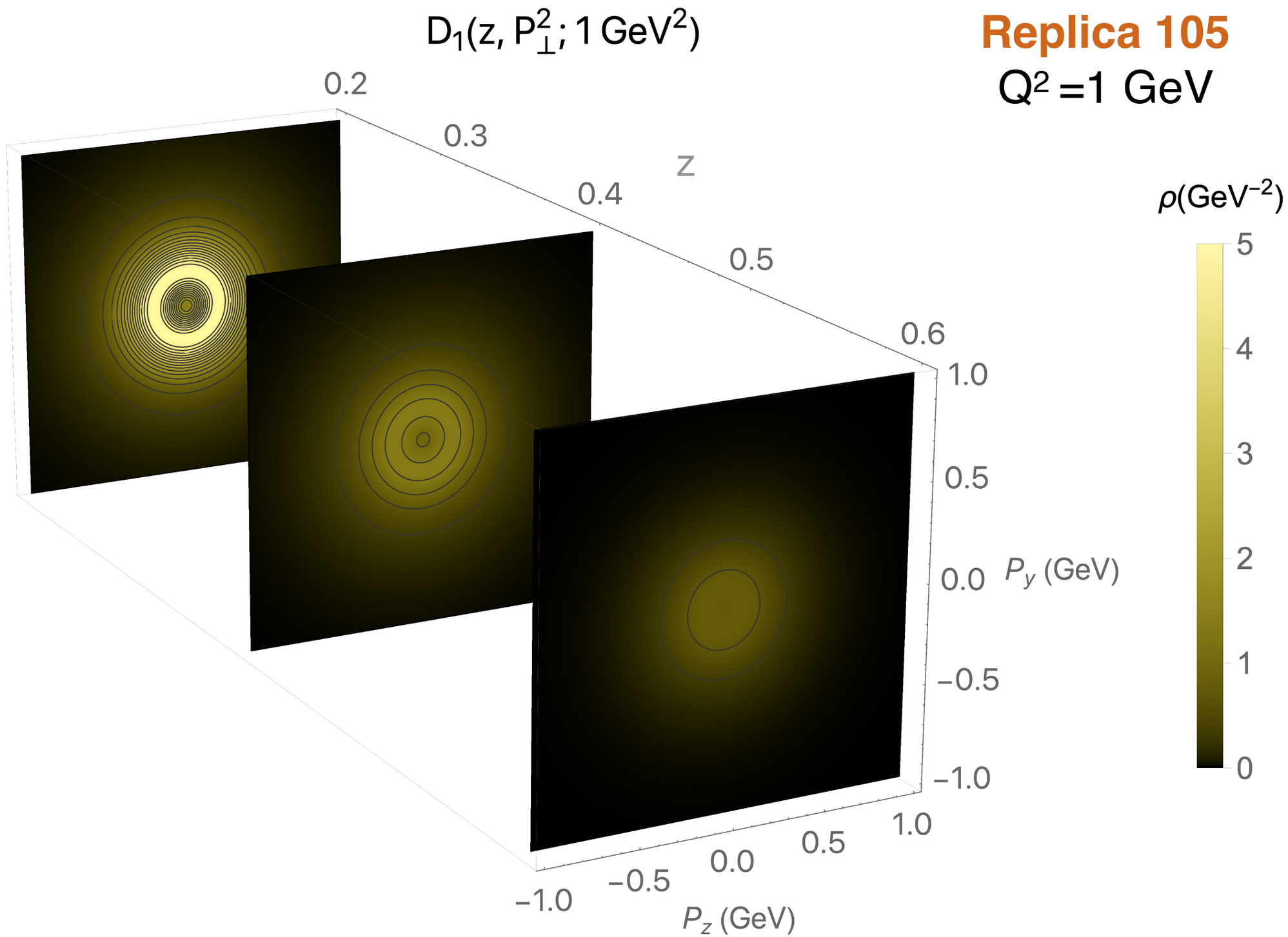
Visualization of TMDs: PDF 3D structure



Momentum space

Visualization of TMDs: FF 3D structure

TMD Fragmentation Function



Momentum space

Transverse Momentum Distributions

Unpolarized ✓

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Sivers function

TMDs: Sivers function distribution

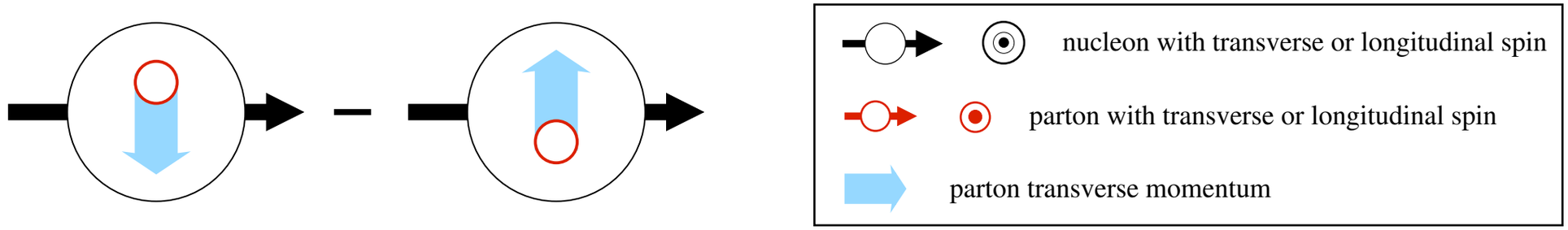
quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Sivers function

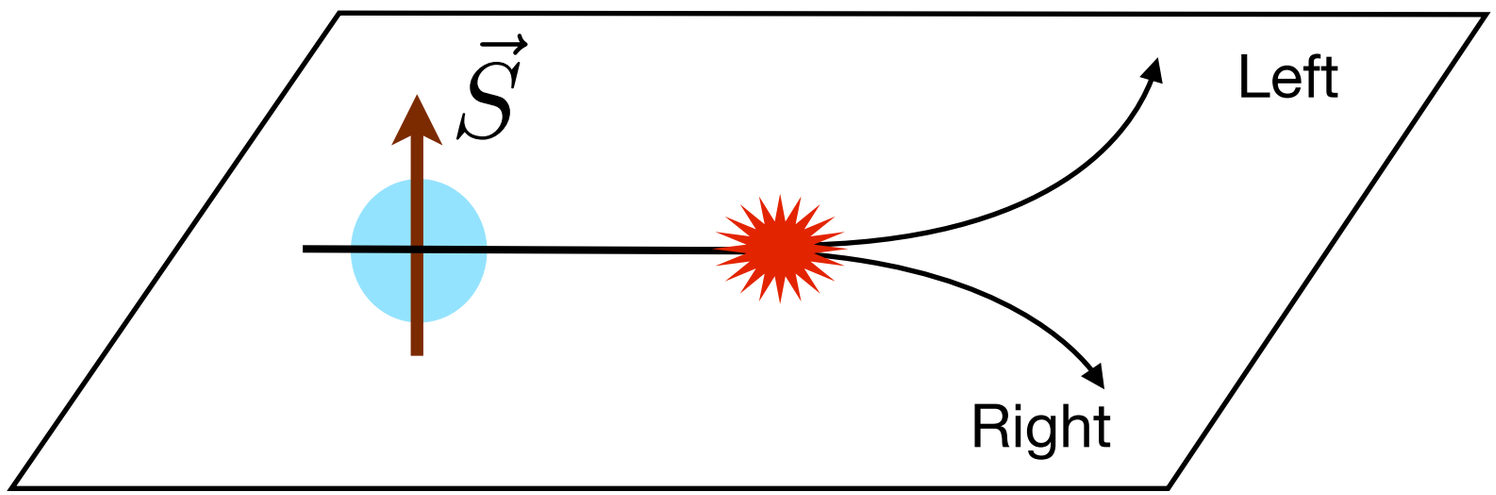
→number density of unpolarized partons inside a transversely polarized nucleon

$f_{1T}^\perp =$



Spin and quark motion correlation: SSA

scattering of transversely polarized proton off an unpolarized proton or electron



The asymmetry is defined as

$$A_N\left(x_F, k_\perp\right) \equiv \frac{L-R}{L+R} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

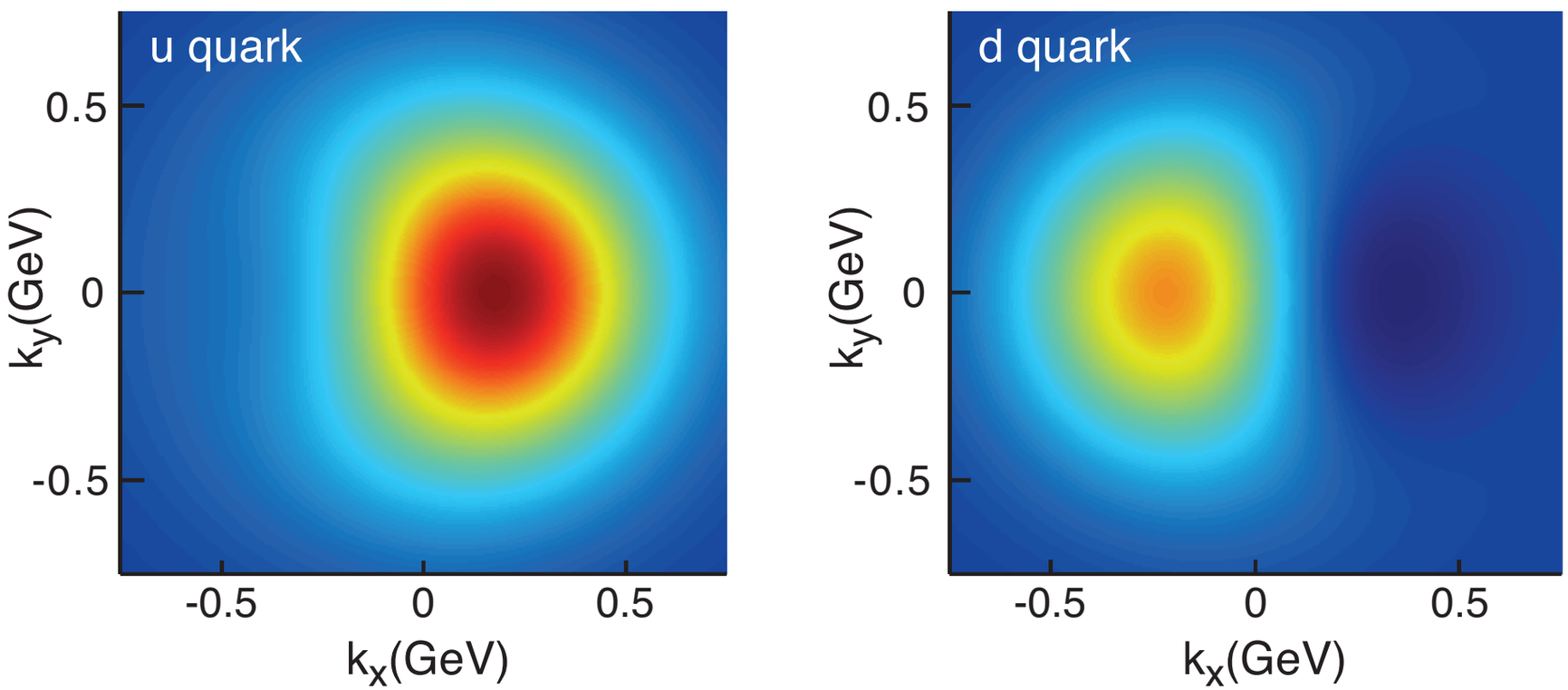
positive A_N means that for upward polarization, the pions tend to go to the left.

Phenomenology of Sivers function

⇒ presence of a non-zero Sivers function

will induce a dipole deformation of f_1

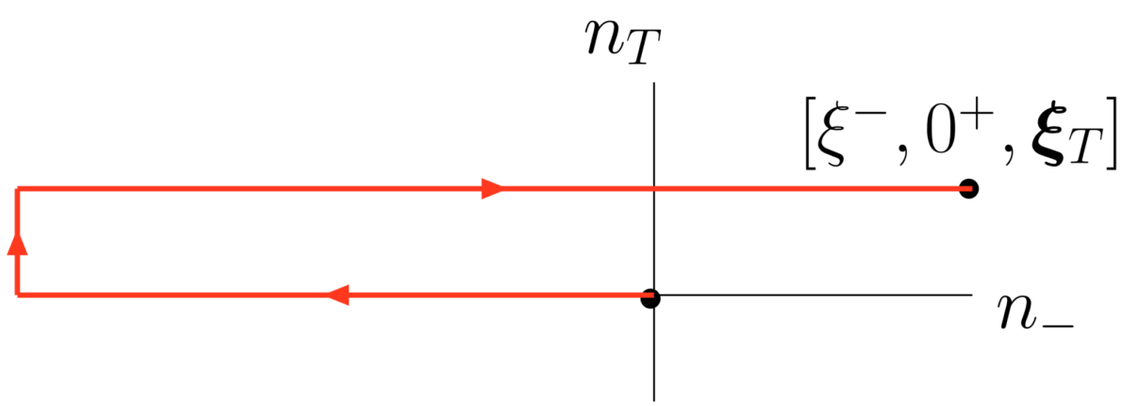
$x\, f_1(x, k_T, S_T)$



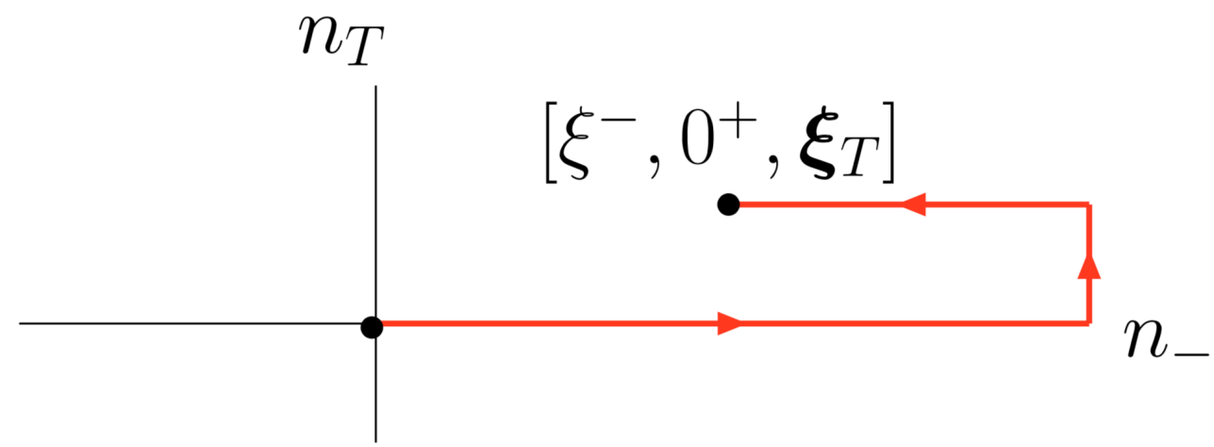
[EIC White Paper]

Sivers function sign change

vanishing Sivers function? → Final state interactions and Wilson lines to consider



DY
past-pointing



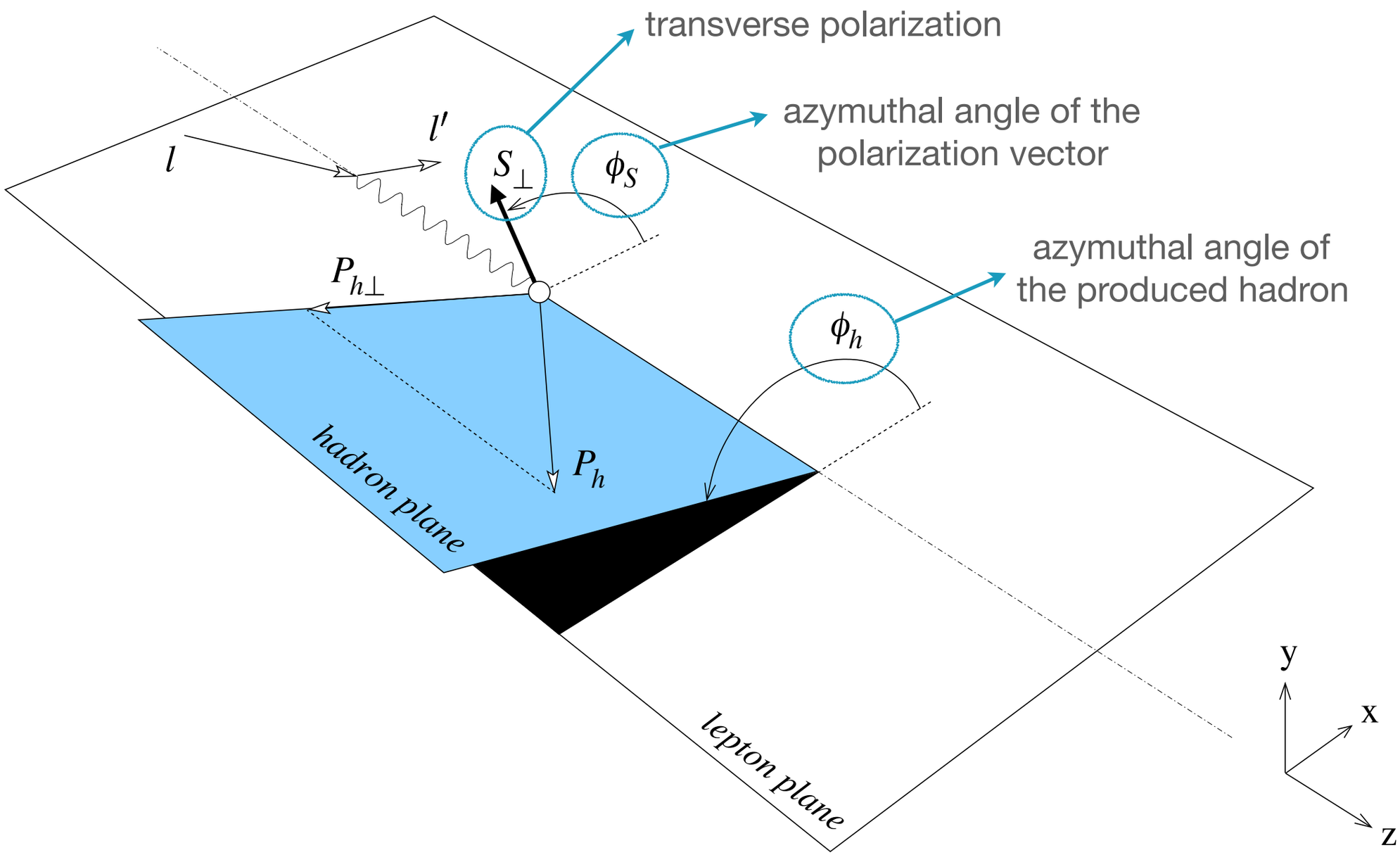
SIDIS
future-pointing

Sign change in Sivers function

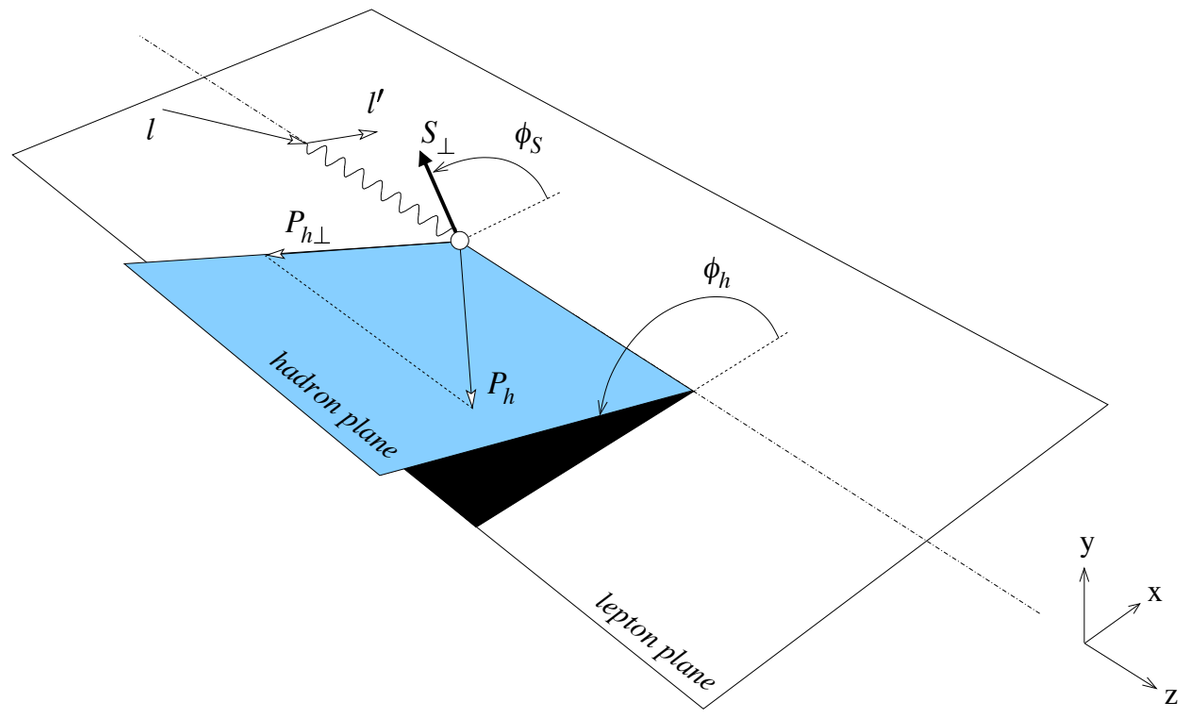
$$f_{1T,DIS}^{\perp} = -f_{1T,DY}^{\perp}$$

Extraction of Sivers Function

The Sivers function can be determined through its contributions to the cross section of the polarized SIDIS process.



Extraction of Sivers Function



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h d\boldsymbol{P}_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}$$

$$+\sin(\phi_h-\phi_S)\,|\boldsymbol{S}_T|\left[F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right]+\cdots\Big\}$$

contributions from other spin structure functions

the spin structure function $F_{UT}^{\sin(\phi_h-\phi_S)}$ is a convolution of the Sivers function $f_{1T}^{\perp}$ with the unpolarized fragmentation function $D_{h/q}$

Extraction of Sivers Function

Isolating the terms relevant to the $\sin(\phi_h - \phi_S)$ modulation

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\ \sin(\phi_h - \phi_S)}{\int d\phi_S\, d\phi_h\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

in terms of structure functions

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}}{F_{UU,T} + \varepsilon F_{UU,L}}$$

we will consider only the terms at order α_S^0

LO - NLL

written in terms of convolutions of TMDs

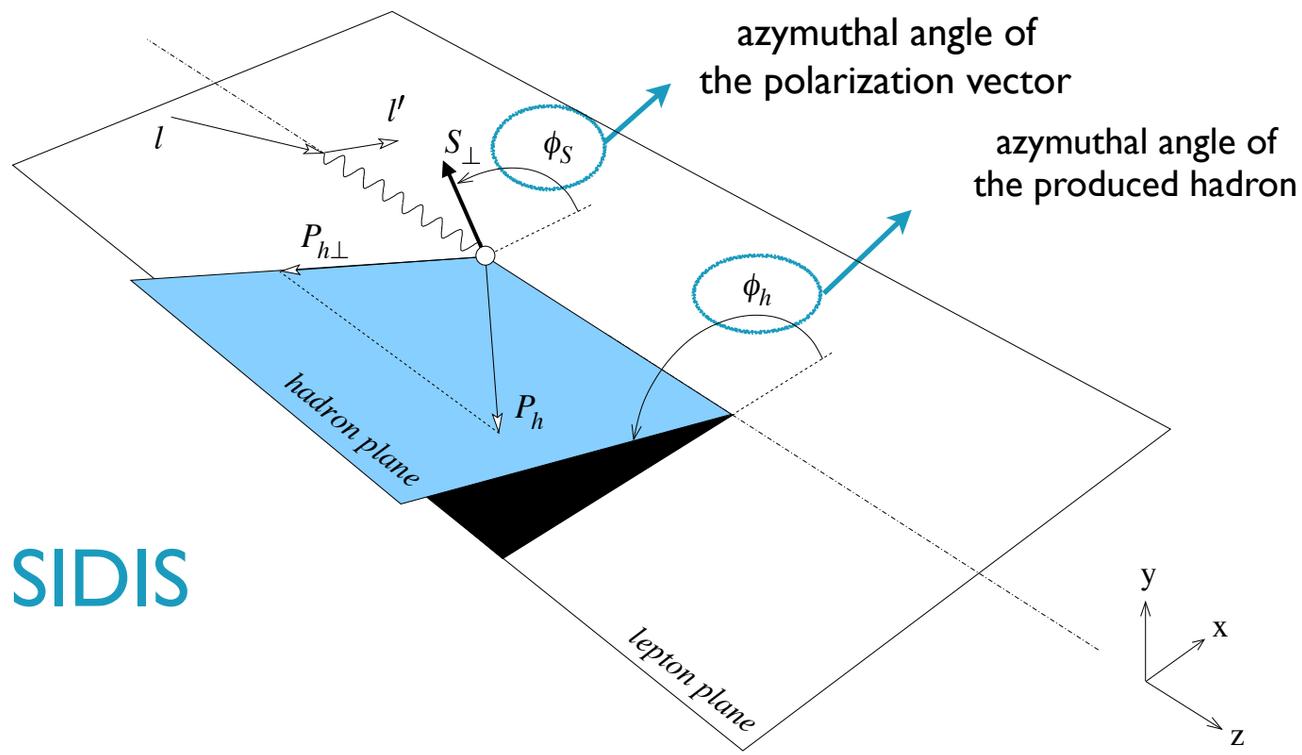
$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathscr{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M} f_{1T}^{\perp} D_1\right]$$

$$F_{UU,T} = \mathscr{C}\left[f_1 D_1\right]$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0$$

$$F_{UU,L} = \mathscr{O}\left(M^2/Q^2, P_{hT}^2/Q^2\right) = 0$$

Extraction of Sivers Function



SIDIS

$$A_{UT}^{\sin(\phi_h-\phi_S)} \sim \frac{\mathscr{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_\perp}{M} f_{1T}^{\perp} D_1\right]}{\mathscr{C}\left[f_1 D_1\right]}$$

universality

first Sivers extraction with unpolarized TMDs extracted from data
and inclusion of TMD evolution

Parametrization of Sivers function

Sivers function can be parametrized in terms of its first moment

$$f_{1T}^{\perp}(x, k_\perp^2) = f_{1T}^{\perp(1)}(x) f_{1TNP}^{\perp}(x, k_\perp^2)$$

Its nonperturbative part is arbitrary, but constrained by the positivity bound.

$$f_{1TNP}^{\perp}(x, k_\perp^2) = \frac{1}{\pi K_f} \frac{(1 + \lambda_S k_\perp^2)}{(M_1^2 + \lambda_S M_1^4)} e^{-k_\perp^2/M_1^2} f_{1NP}(x, k_\perp^2)$$

following the definition of the nonperturbative part of the unpolarized TMD distribution

$$f_{1NP}(x, k_\perp^2) = \frac{1}{\pi} \frac{(1 + \lambda k_\perp^2)}{(g_1 + \lambda g_1^2)} e^{-k_\perp^2/g_1}$$

Free parameters λ_S, M_1

Parametrization of Sivers function

normalization
(abs.value <1)

$T_n(x)$ Chebyshev polynomials

$$f_{1T,a}^{\perp(1)}(x) = \frac{N_{Siv}^a}{G_{max}^a} x^{\alpha_a}(1-x)^{\beta_a}\left(1 + A_a T_1(x) + B_a T_2(x)\right) f_1^a(x, Q^2)$$

maximum value
of the function

Free parameters $N_{Siv}^a, \alpha_a, \beta_a, A_a, B_a$

Flavor dependent: different for up, down, sea

Evolution of Sivers

We simply assume that $f_{1T}^{\perp(1)}$ evolves in the same way as unpolarized f_1

Difference in the Wilson coefficients: $C^i \to C^{Siv}$

At our accuracy level (LO): $C^{Siv(0)} = \delta(1-x)\delta^{ai}$

The evolved Sivers function first moment becomes

$$\tilde{f}_{1T}^{\perp(1)a}(x, \xi_T^2; Q^2) = f_1^a(x; \mu_b^2)\ e^{S(\mu_b^2, Q^2)}\ e^{g_K(\xi_T)\ln(Q^2/Q_0^2)}\ \tilde{f}_{1T\mathrm{NP}}^{\perp(1)a}(x, \xi_T^2)$$

same choices used for evolved unpolarized TMDs

Experimental data

hermes

proton [H]

95
data points

Jefferson Lab

neutron [^{3}He]

6
data points

COMPASS 2009

deuteron [^{6}LiD]

88
data points

COMPASS 2017

Proton [NH_3]

111
data points

Same kinematic cuts applied to unpolarized

x, z, P_{hT} data projections

Experimental data

hermes

proton [H]

~~95~~
data points
30

Jefferson Lab

neutron [^{3}He]

6
data points

COMPASS 2009

deuteron [^{6}LiD]

~~88~~
data points
32

COMPASS 2017

Proton [NH_3]

~~111~~
data points
52

Same kinematic cuts applied to unpolarized

x, z, P_{hT} data projections

Results

LO - NLL

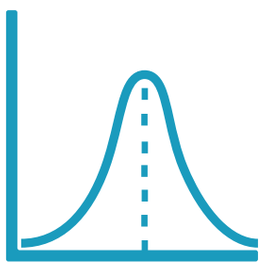
Replica method

Summary of results

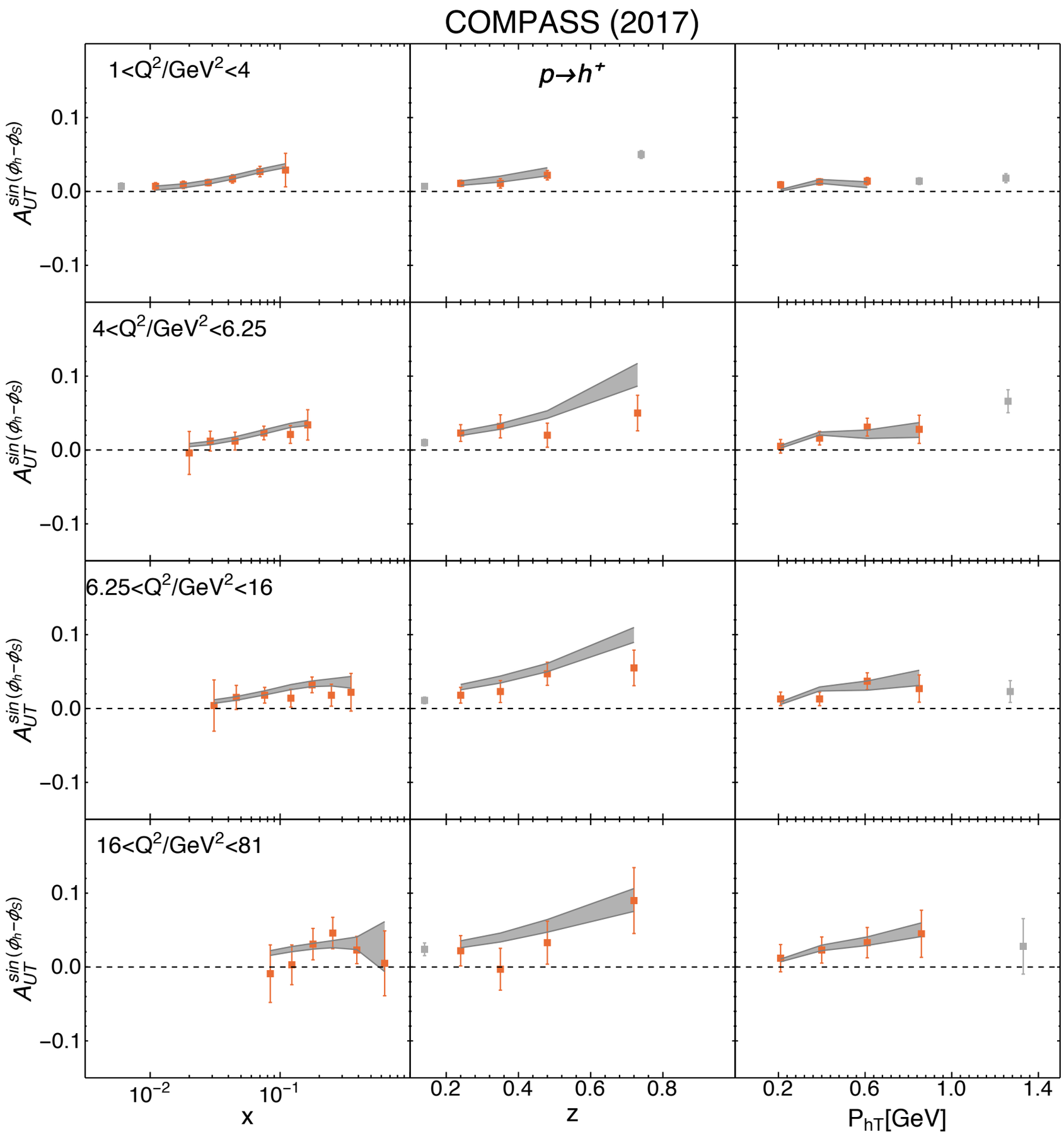
Total number of data points: 118

Total number of free parameters: 14

➛ for 3 different flavors



$$\chi^2/d.o.f = 1.06 \pm 0.12$$

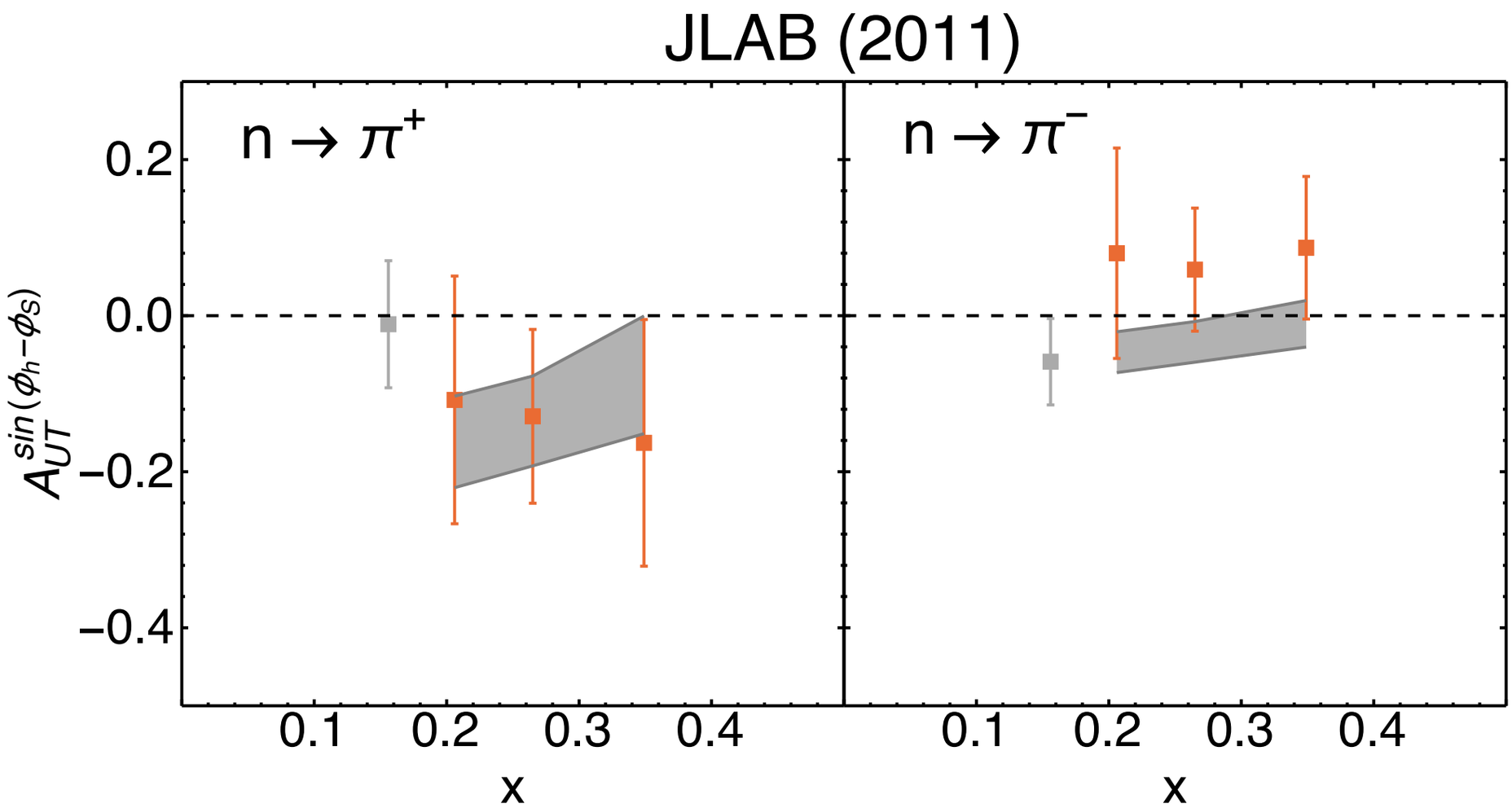


proton

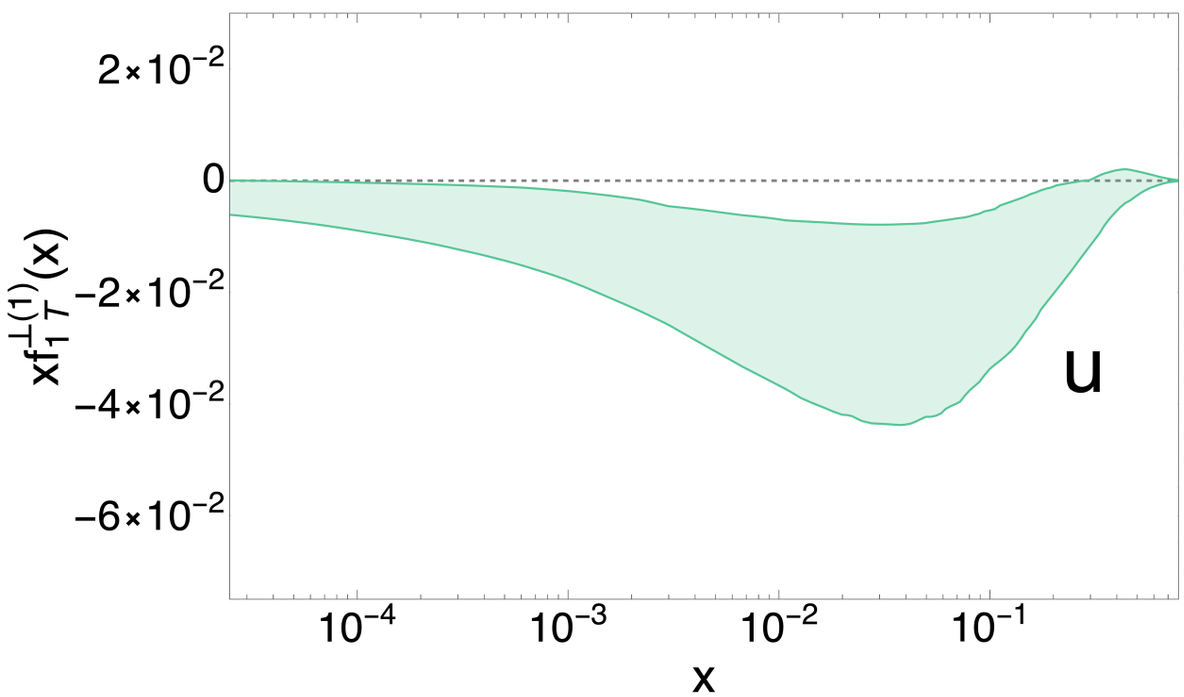
positive hadron

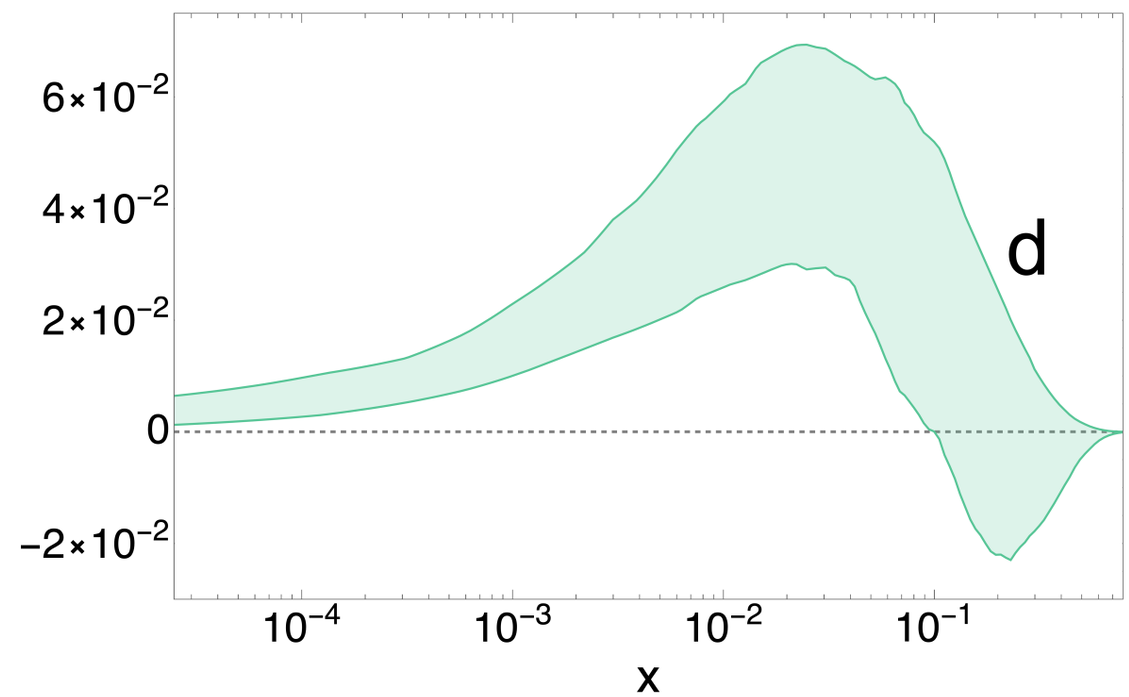
Jefferson Lab

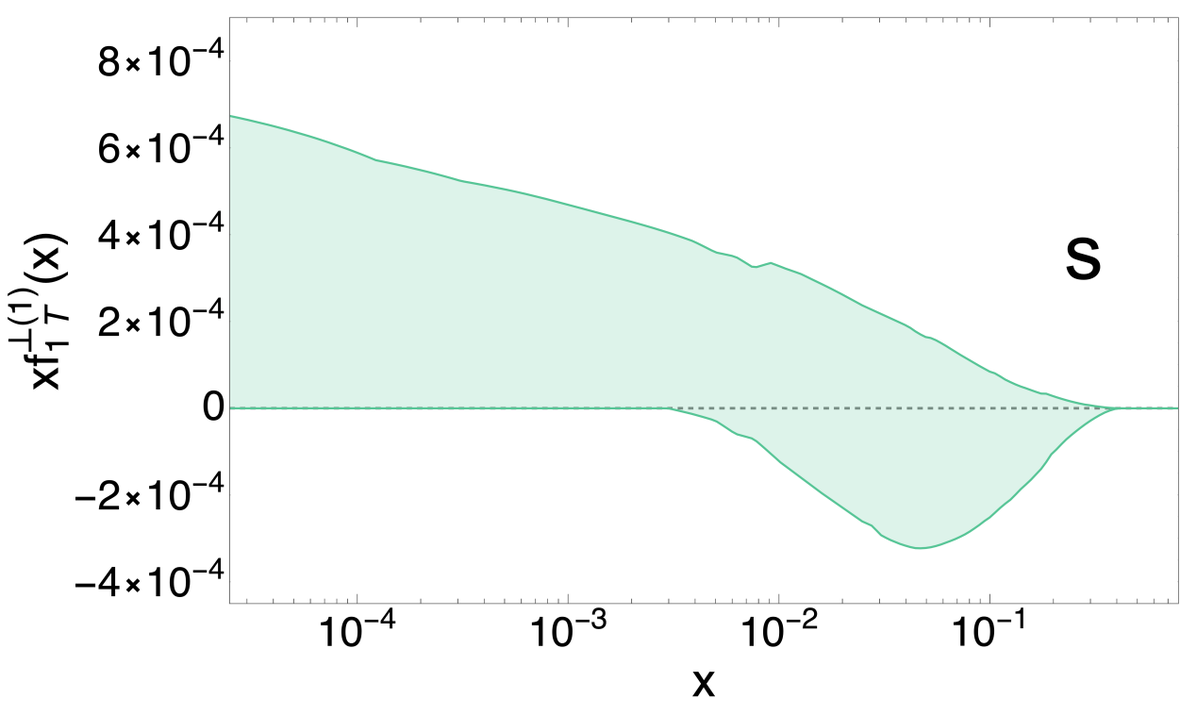
neutron



Sivers function first moment

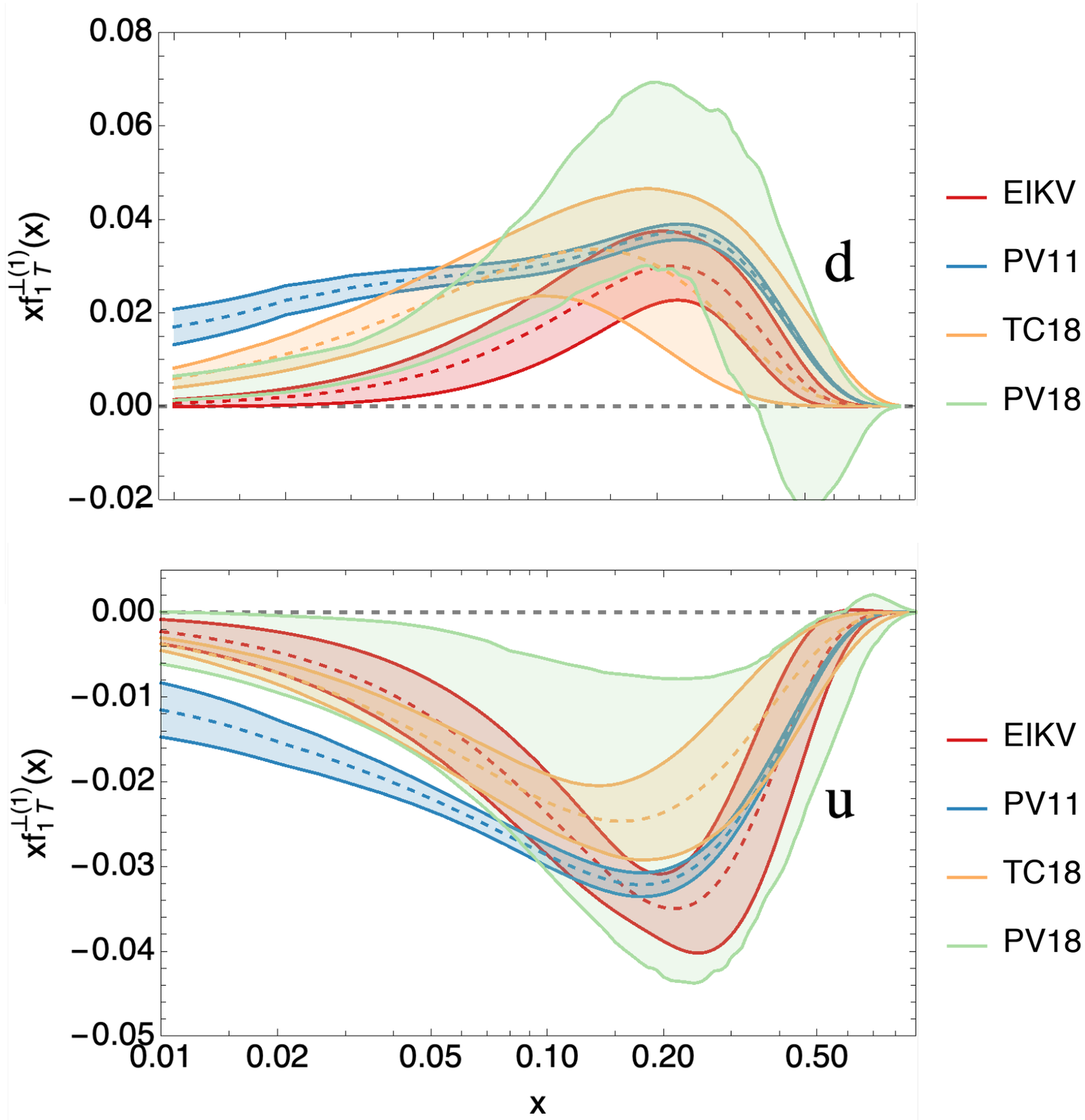




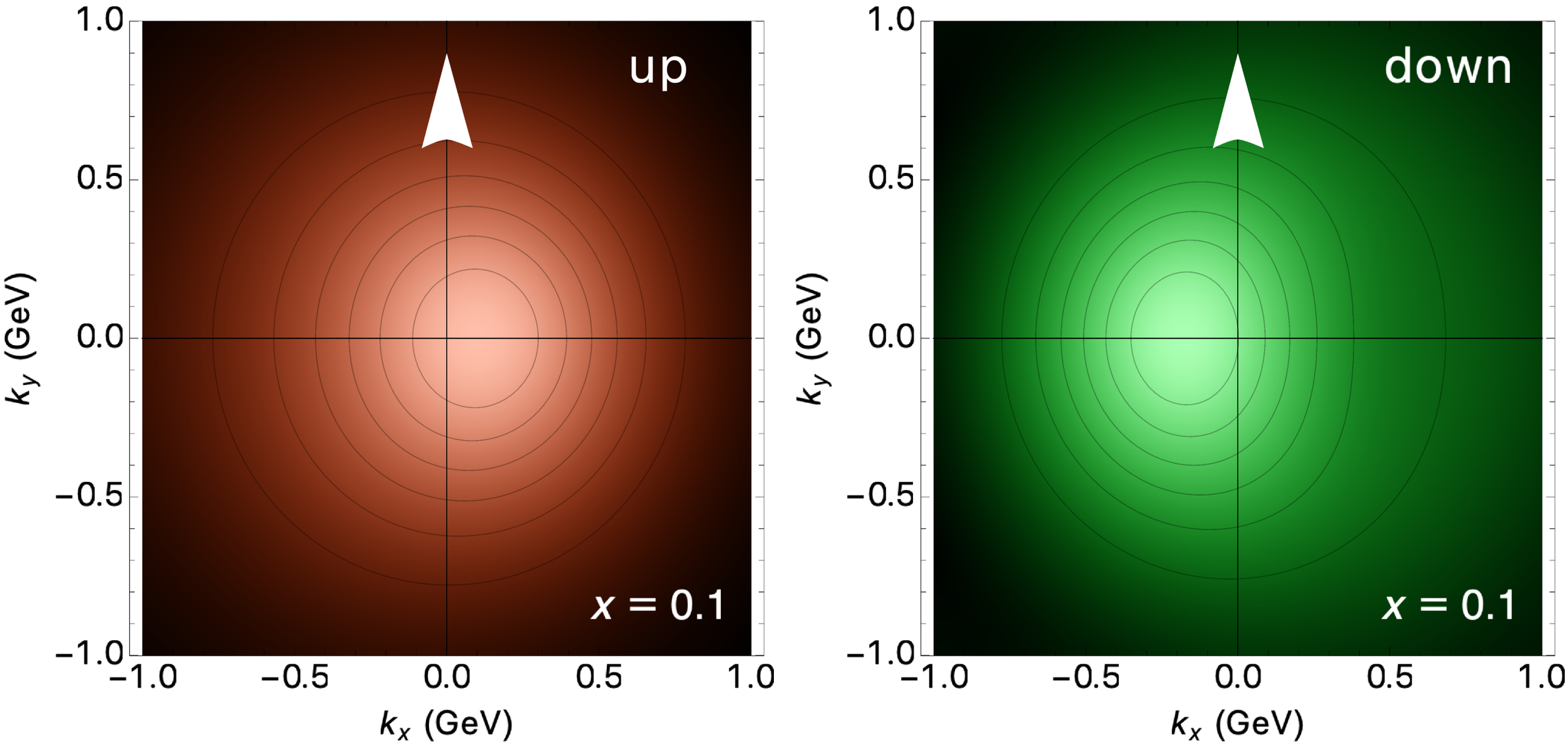


logarithmic plots of 68% C.L bands for first moment of Sivers function for down, up and s quarks

Results comparison



Internal structure deformation



$$xf_1(x, k_\perp^2; Q^2) - xf_{1T}^\perp(x, k_\perp^2; Q^2)$$

Conclusions

First global extraction of evolved partonic unpolarized TMDs from SIDIS, DY and Z boson

Determination of Sivers function featuring evolution and extracted unpolarized TMDS

Test of the universality and evolution formalism of partonic TMDs

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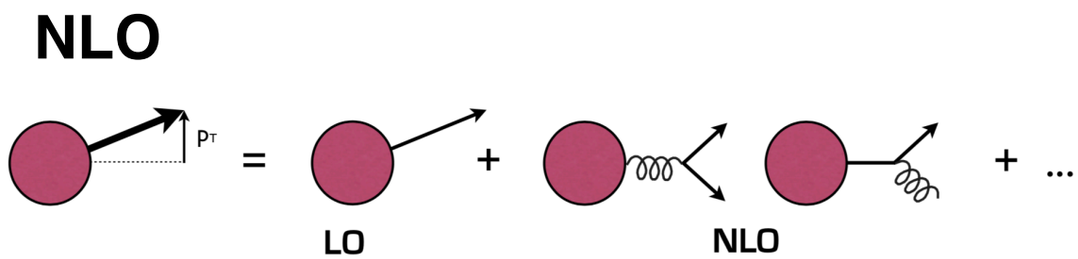
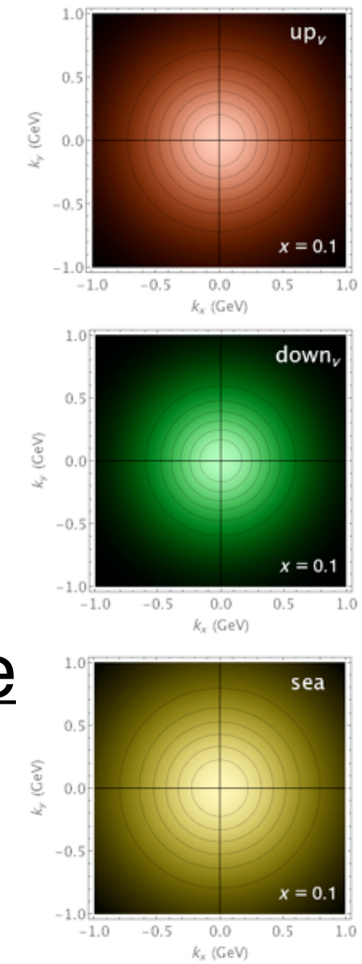
Thanks and Buon Primo Maggio!

Future outlooks: unpolarised





Higher accuracy

Flavor dependence

ANALYSIS IMPROVEMENTS

Additional data

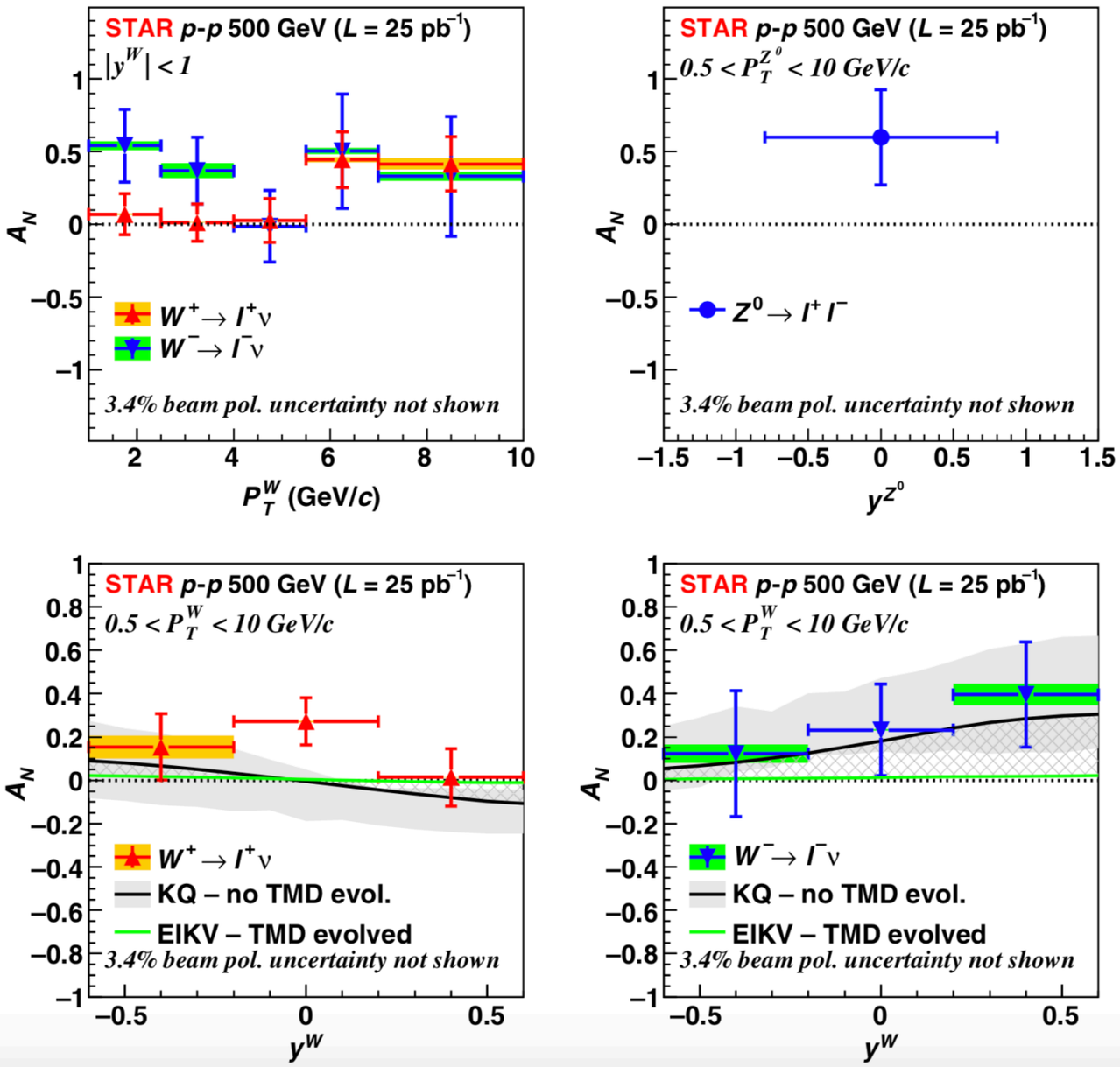
COMPASS 2017

CMS

LHCb

ATLAS EXPERIMENT

Future outlooks: Sivers



Predictions of
A_N asymmetries
for W/Z production

Anomalous magnetic moment
(testing Pavia2011 hypothesis)

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx x[H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)].$$

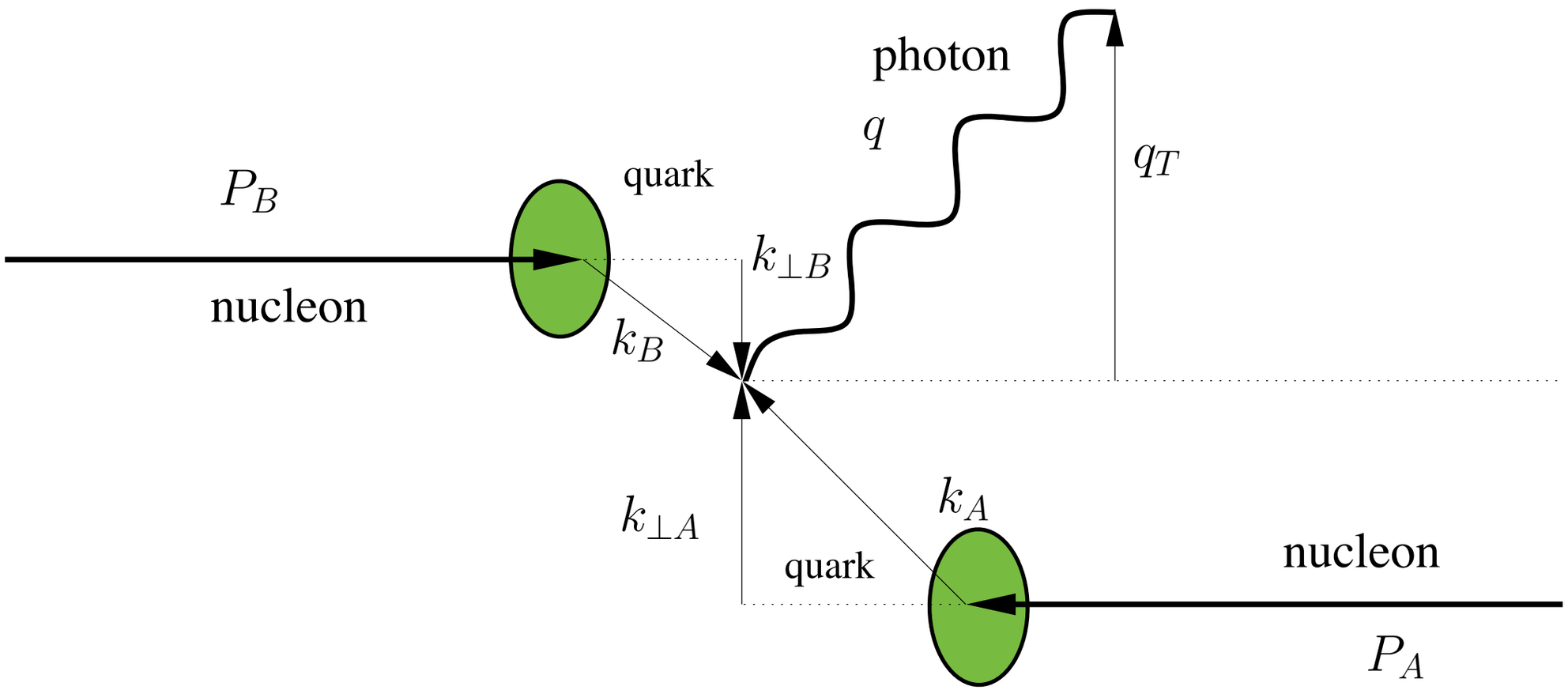
Higher accuracy
(after unpol. TMD improved fit)

BACKUP

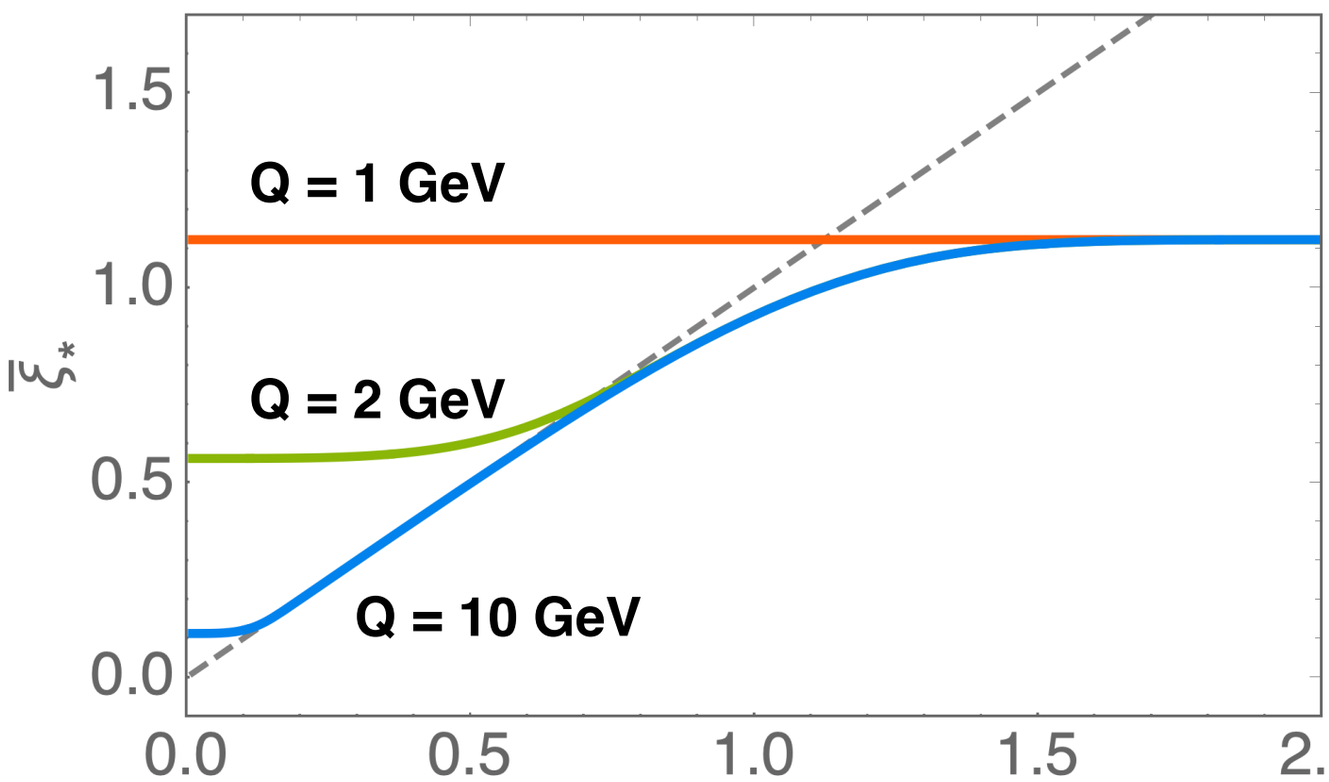
Structure functions and TMDs: DY

Differential cross section

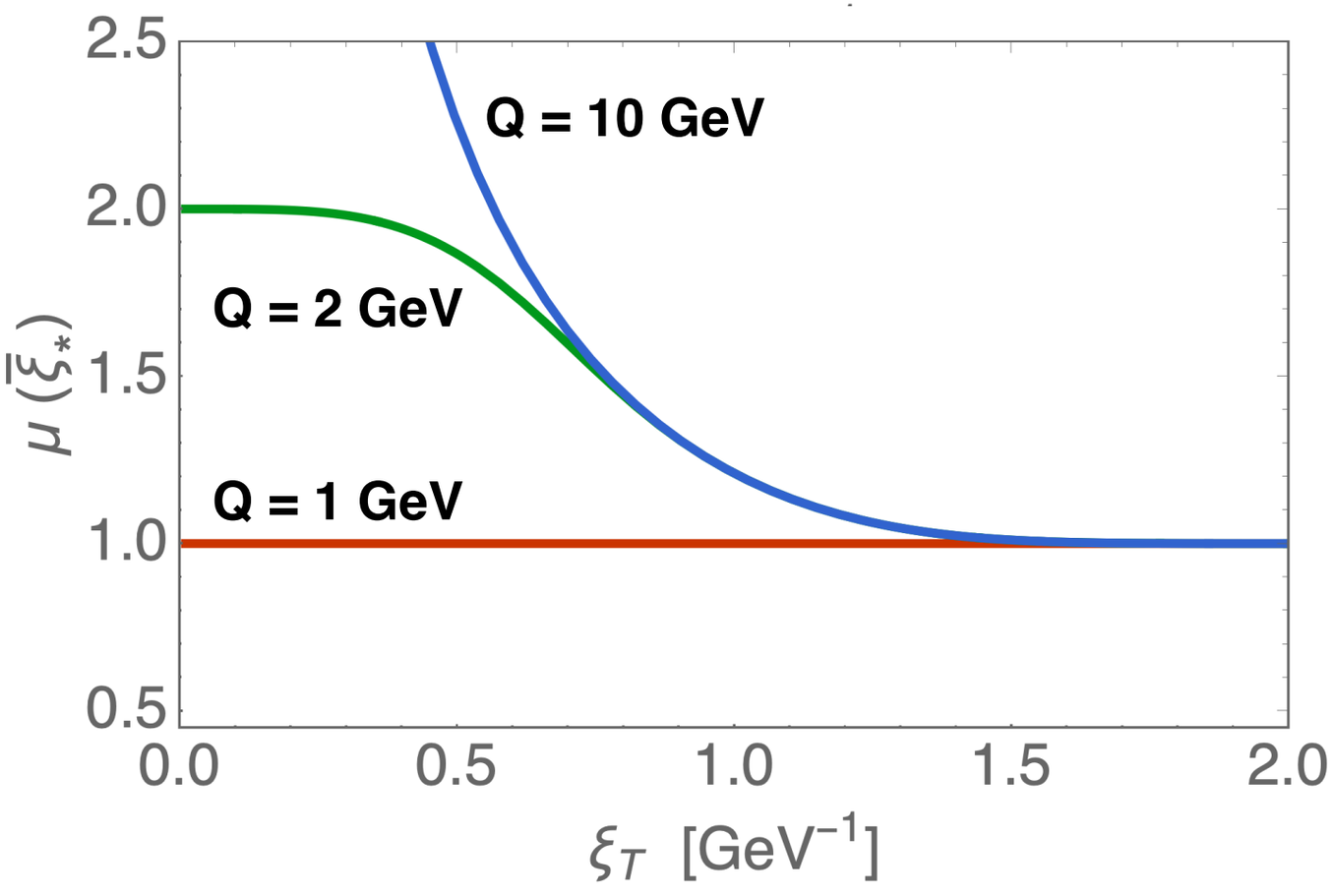
$$\frac{d\sigma}{dQ^2 dq_T^2 d\eta} = \sigma_0^{\gamma,Z}\left(F_{UU}^1 + \frac{1}{2}F_{UU}^2\right)$$



Evolution and ξ_T regions



$$\bar{\xi}_* \left(\xi_T, \xi_{\min}, \xi_{max}\right) = \xi_{\max} \left[\frac{1 - \exp\left(\xi_T^4/\xi_{\max}^4\right)}{1 - \exp\left(\xi_T^4/\xi_{\min}^4\right)}\right]^{1/4}$$



$$\mu_b = 2e^{-\gamma_E}/\bar{\xi}_*$$

μ and b_* prescriptions

Choice Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv \dfrac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$ *Collins, Soper, Sterman, NPB250 (85)*

$\mu_b = 2e^{-\gamma_E}/b_*$ $\quad b_* \equiv b_{\rm max}\left(1 - e^{-\frac{b_T^4}{b_{\rm max}^4}}\right)^{1/4}$ *Bacchetta, Echevarria, Mulders, Radici, Signori arXiv:1508.00402*

$\mu_b = Q_0 + q_T$ $\quad b_* = b_T$ *DEMS 2014*

Complex-b prescription *Laenen, Sterman, Vogelsang, PRL 84 (00)*

Pavia 2017 perturbative ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1\left(\mathcal{O}(\alpha_S^1)\right)$ ✔	$A_2\left(\mathcal{O}(\alpha_S^2)\right)$ ✔	$A_3\left(\mathcal{O}(\alpha_S^3)\right)$	$\dots$
	$B_1\left(\mathcal{O}(\alpha_S^1)\right)$ ✔	$B_2\left(\mathcal{O}(\alpha_S^2)\right)$	$\dots$
$C_0\left(\mathcal{O}(\alpha_S^0)\right)$ ✔	$C_1\left(\mathcal{O}(\alpha_S^1)\right)$	$C_2\left(\mathcal{O}(\alpha_S^2)\right)$	$\dots$
$H_0\left(\mathcal{O}(\alpha_S^0)\right)$ ✔	$H_1\left(\mathcal{O}(\alpha_S^1)\right)$	$H_2\left(\mathcal{O}(\alpha_S^2)\right)$	$\dots$
	$Y_1\left(\mathcal{O}(\alpha_S^1)\right)$	$Y_2\left(\mathcal{O}(\alpha_S^2)\right)$	$\dots$

Model: non perturbative elements

input TMD FF (Q^2=1 GeV^2)

$$\hat{D}_{1NP}^{a\to h} = \text{ F.T. of } \frac{1}{g_{3a\to h} + (\lambda_F/z^2) g_{4a\to h}^2} \left(e^{-\frac{\boldsymbol{P}_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{\boldsymbol{P}_\perp^2}{z^2} \, e^{-\frac{\boldsymbol{P}_\perp^2}{g_{4a\to h}}} \right)$$

sum of two different gaussians
with different variance
with kinematic dependence on transverse momenta

width z-dependence

$$g_{3,4}(z) = N_{3,4} \, \frac{(z^\beta + \delta) \, (1-z)^\gamma}{(\hat{z}^\beta + \delta) \, (1-\hat{z})^\gamma}$$

where

$$N_{3,4} \equiv g_{3,4}(\hat{z})$$

$$\hat{z} = 0.5$$

Average transverse momenta

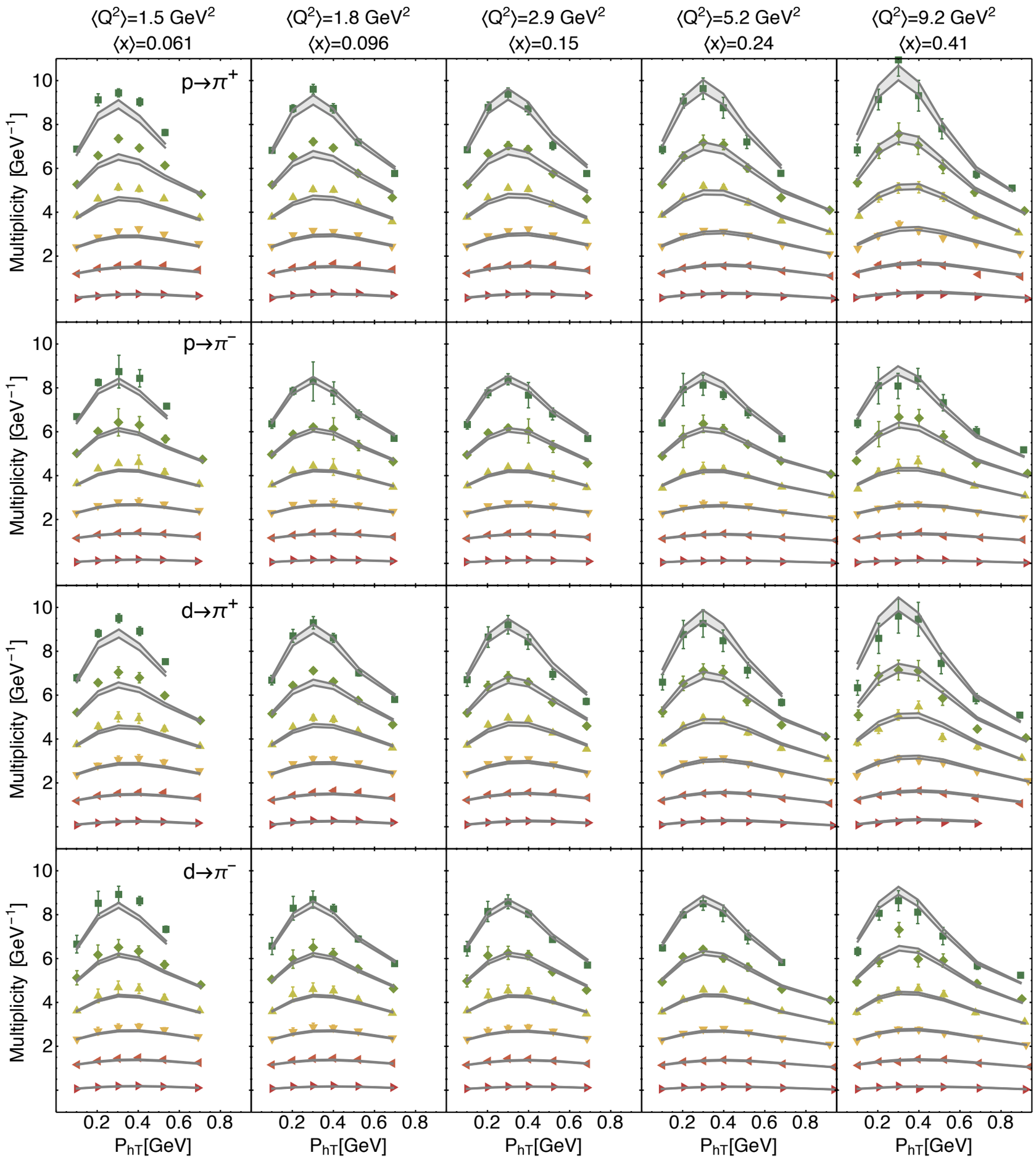
$$\langle \boldsymbol{k}_\perp^2 \rangle (x) = \frac{g_1(x) + 2\lambda g_1^2(x)}{1 + \lambda g_1(x)}$$

$$\langle \boldsymbol{P}_\perp^2 \rangle (z) = \frac{g_3^2(z) + 2\lambda_F g_4^3(z)}{g_3(z) + \lambda_F g_4^2(z)}$$

Quark unpol. TMD: extractions

		Framework	HERMES	COMPASS	DY	Z boson production	N° of points
PAST	KN 2006 hep-ph/0506225	NLL/NLO	✘	✘	✔	✔	98
PAST	Pavia 2013 arXiv:1309.3507	No evo	✔	✘	✘	✘	1538
PAST	Torino 2014 arXiv:1312.6261	No evo	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
PAST	DEMS 2014 arXiv:1407.3311	NNLL/NLO	✘	✘	✔	✔	223
PAST	EIKV 2014 arXiv:1401.5078	NLL/LO	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
PRESENT	Pavia 2016 arXiv:1703.10157	NLL/LO	✔	✔	✔	✔	8059
PRESENT	SV 2017 arXiv:1706.01473	NNLL/ NNLO	✘	✘	✔	✔	309

Hermes data pion production



χ^2/dof

4.83

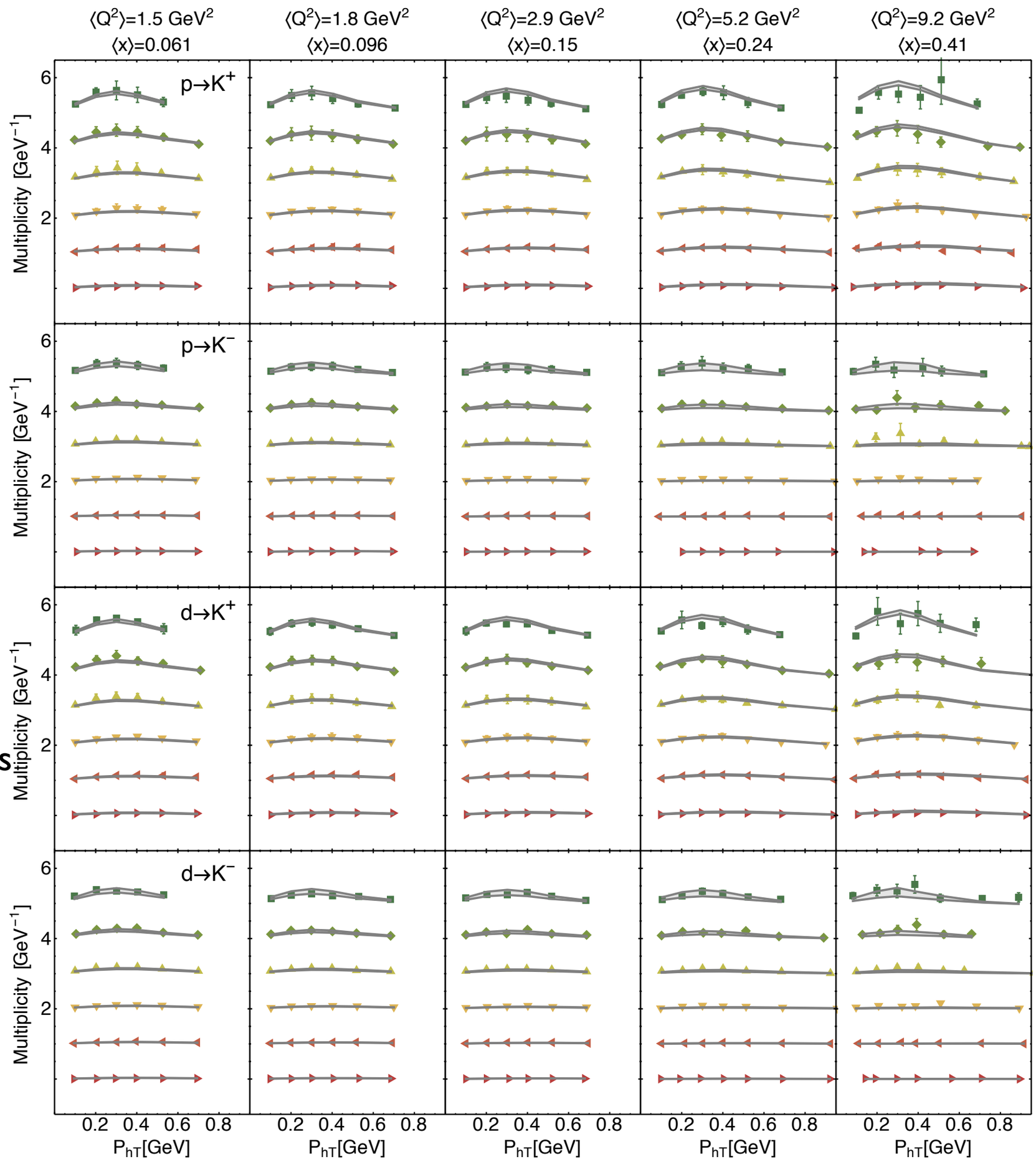
2.47

3.46

2.00

π

Hermes data pion production



χ^2/dof

0.91

0.82

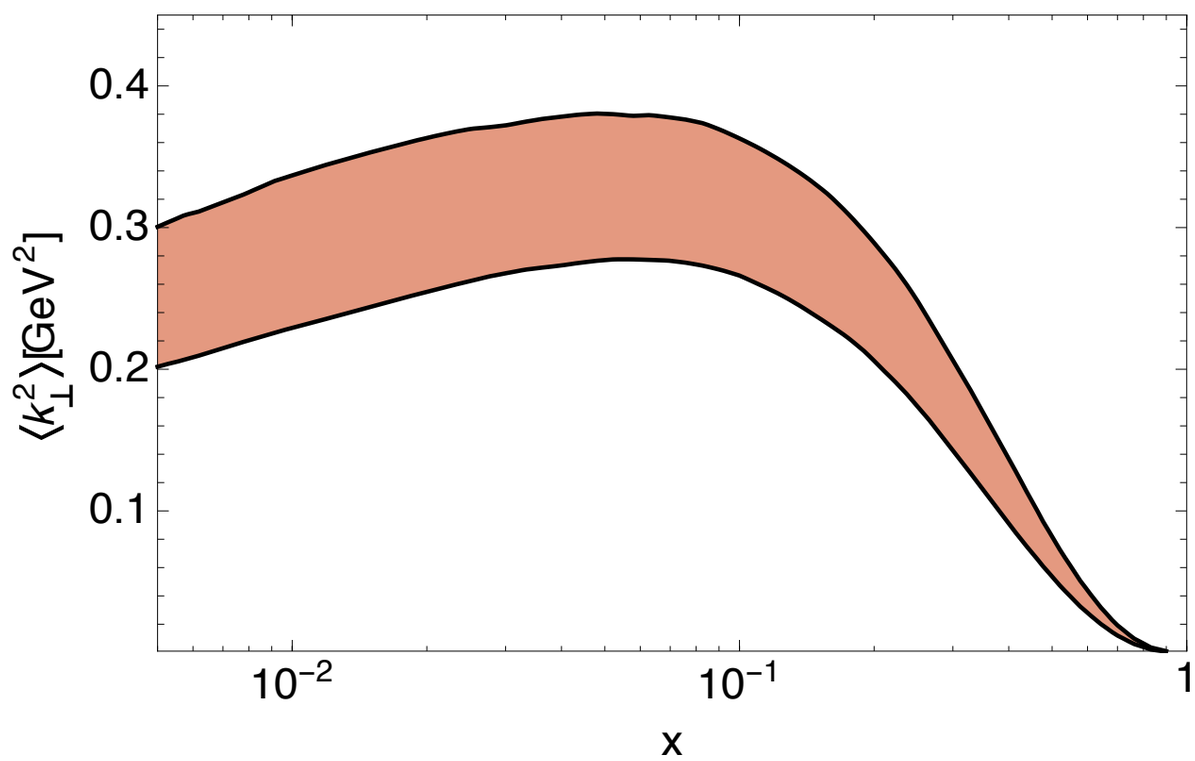
1.31

2.54

K

better agreement than pions: larger uncertainties form FFs

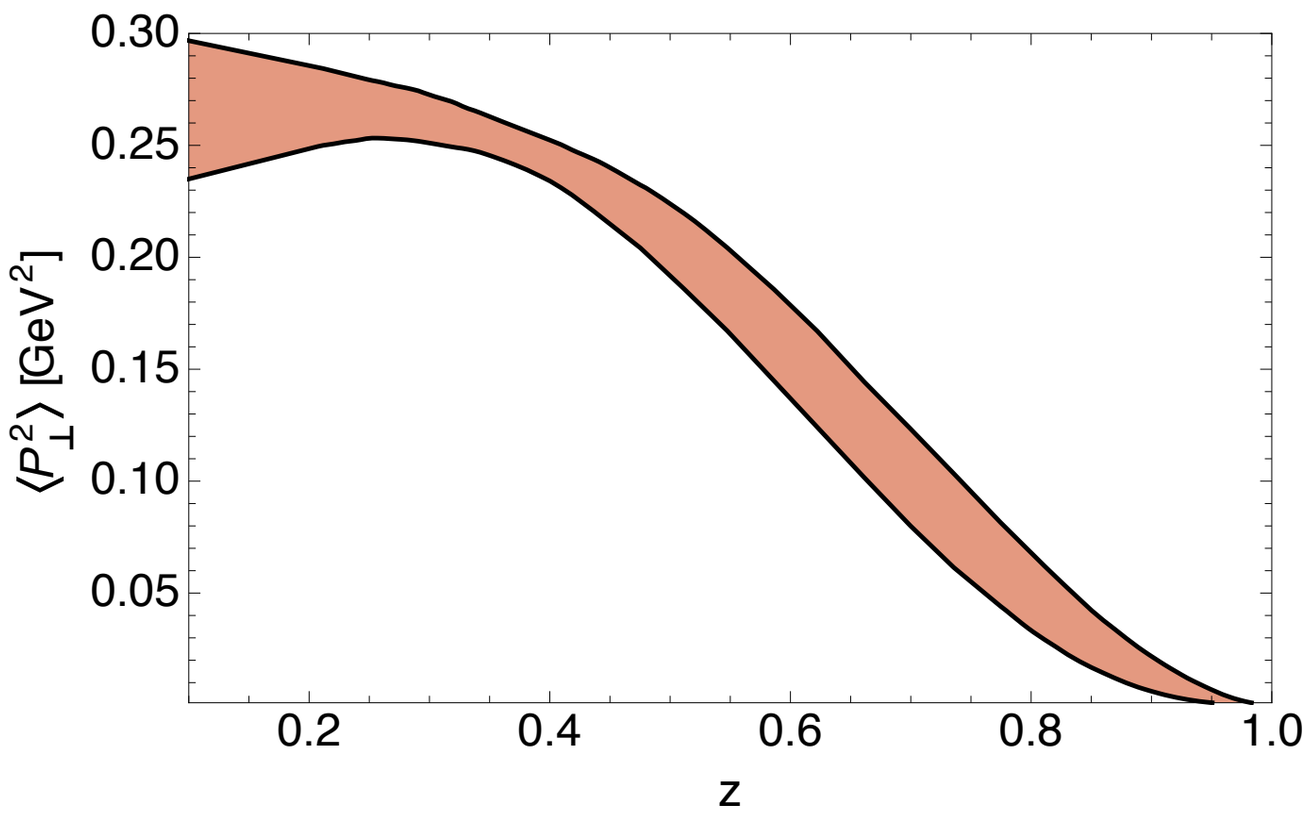
Mean transverse momentum



Change in TMD width
x-dependence

in TMD PDF

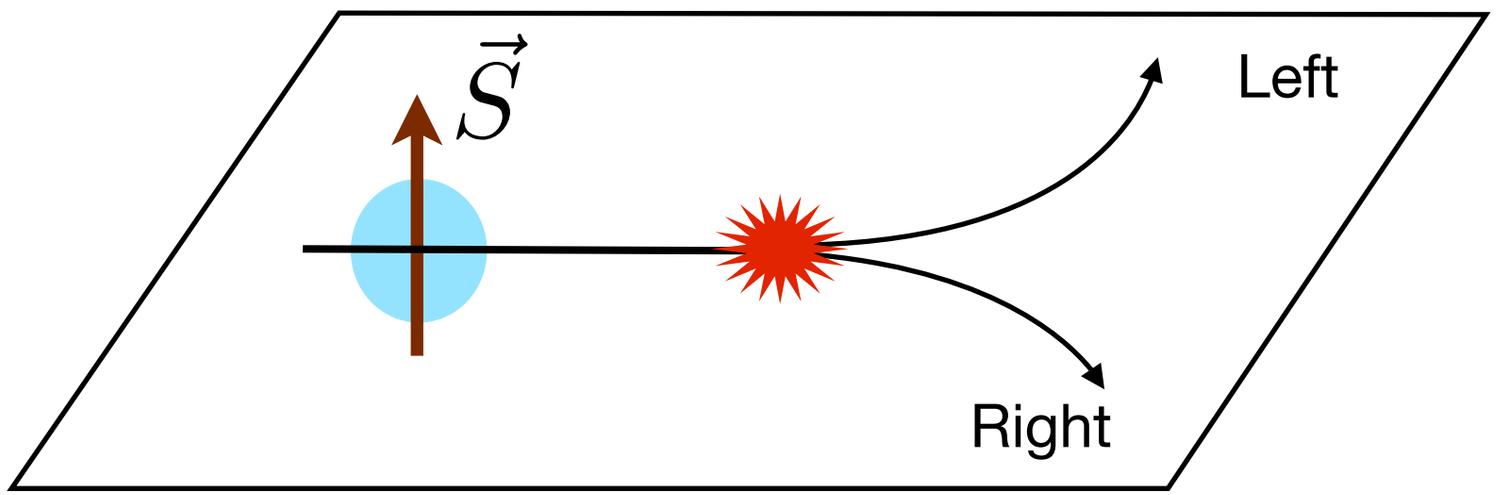
Q^2=1GeV^2



in TMD FF

Spin and quark motion correlation: SSA

scattering of transversely polarized proton off an unpolarized proton or electron



The asymmetry is defined as

$$A_N\left(x_F, k_\perp\right) \equiv \frac{L-R}{L+R} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

positive A_N means that for upward polarization, the pions tend to go to the left.

Evolution of Sivers

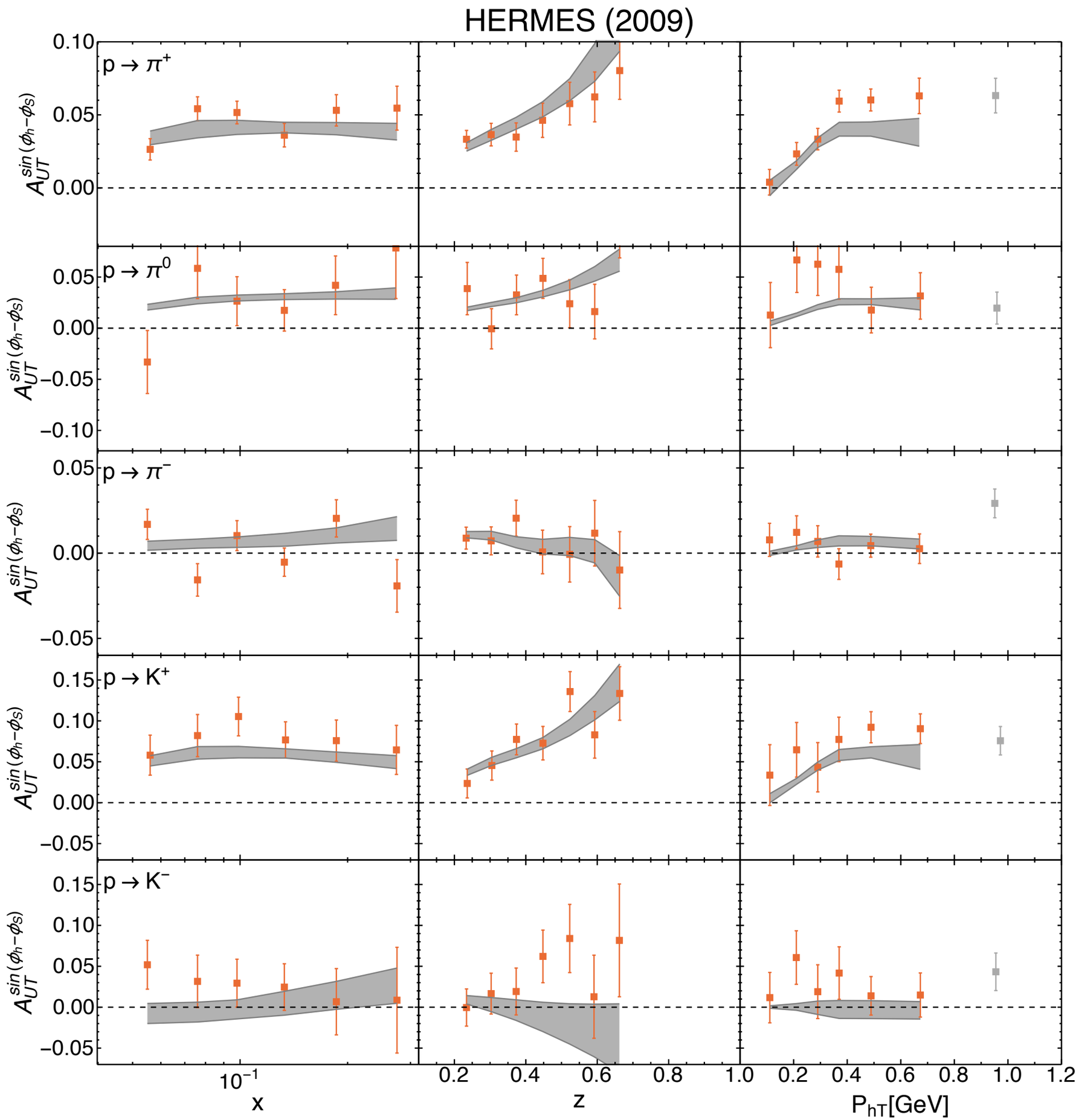
The evolved Sivers function first moment becomes

$$\tilde{f}_{1T}^{\perp(1)a}(x, \xi_T^2; Q^2) = f_1^a(x; \mu_b^2)\ e^{S(\mu_b^2, Q^2)}\ e^{g_K(\xi_T)\ln(Q^2/Q_0^2)}\ \tilde{f}_{1T\mathrm{NP}}^{\perp(1)a}(x, \xi_T^2)$$

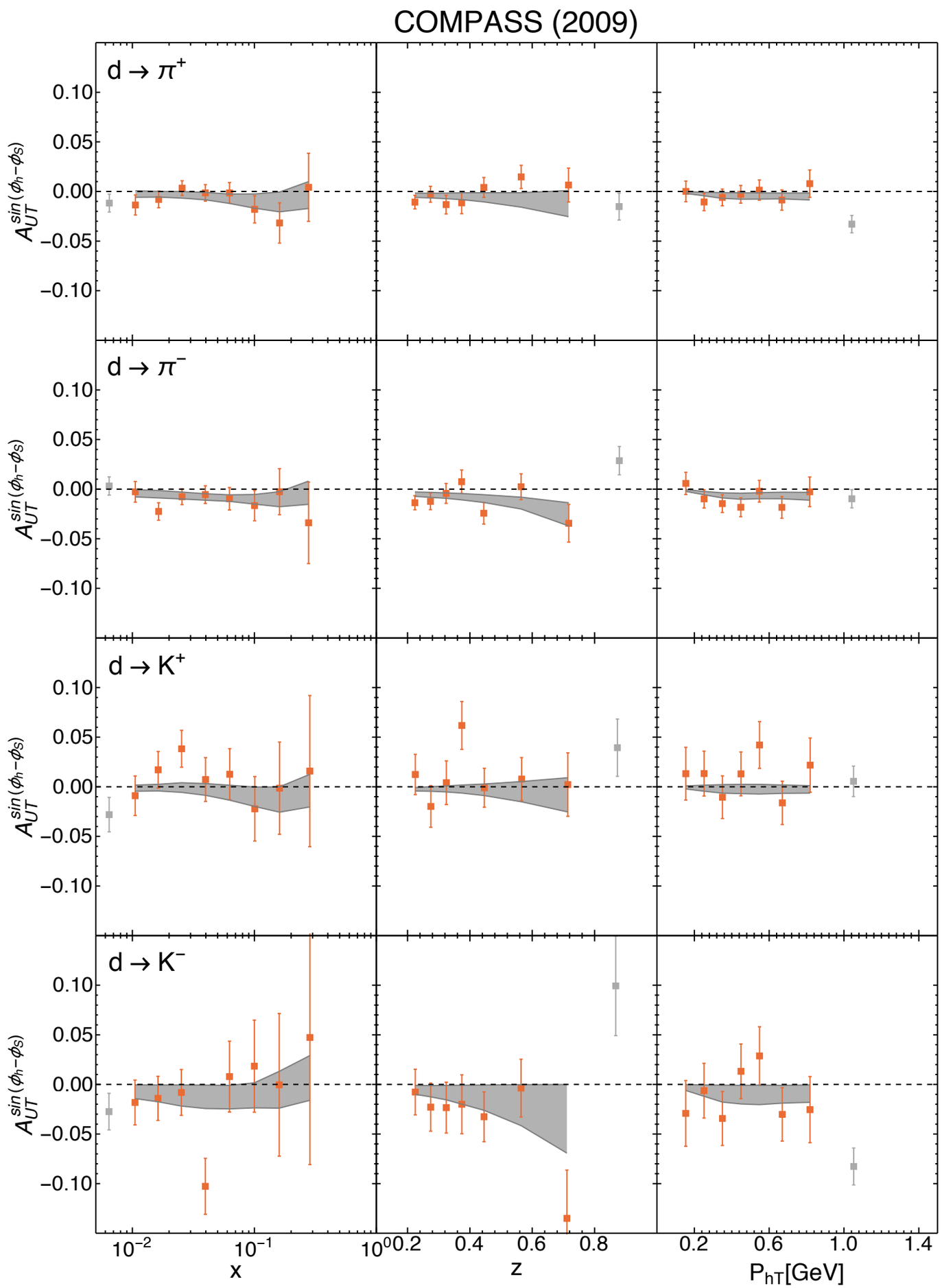
The first moment $f^{\perp(1)}(x)$ is related to the twist-3 Qiu-Sterman function by the following relation

$$f_{1T}^{\perp(1)}(x) = -\frac{1}{2M} T_F(x, x)$$

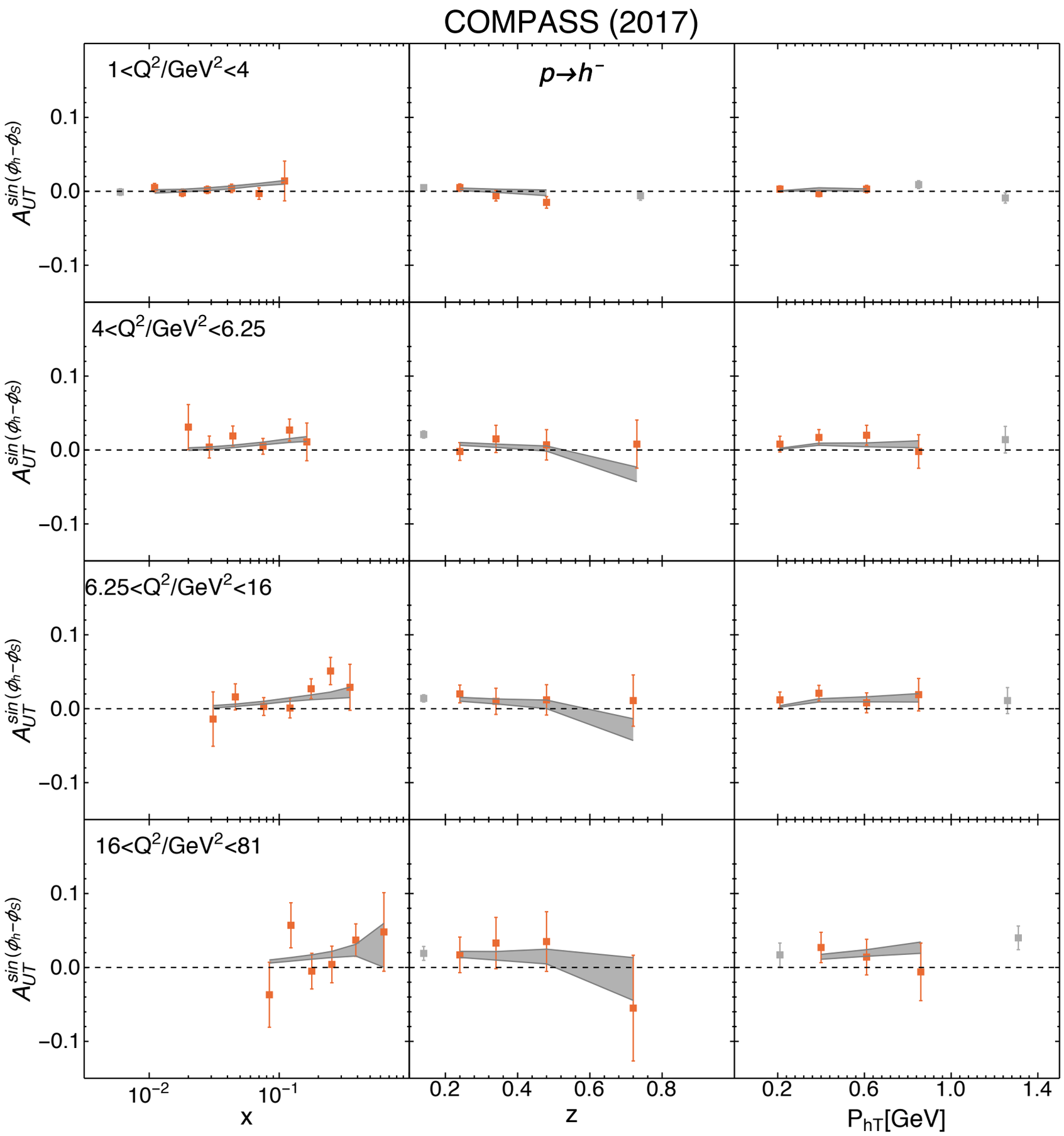
The μ_b evolution for this term follows the Efremov-Teryaev-Qiu-Sterman (ETQS) evolution equations



proton


COMPASS (2009)
d → π+
d → π−
d → K+
d → K−
$A_{UT}^{\sin(\phi_h-\phi_S)}$
0.10
0.05
0.00
−0.05
−0.10
10−2
10−1
100
x
0.2
0.4
0.6
0.8
z
0.2
0.6
1.0
1.4
PhT[GeV]

deuteron

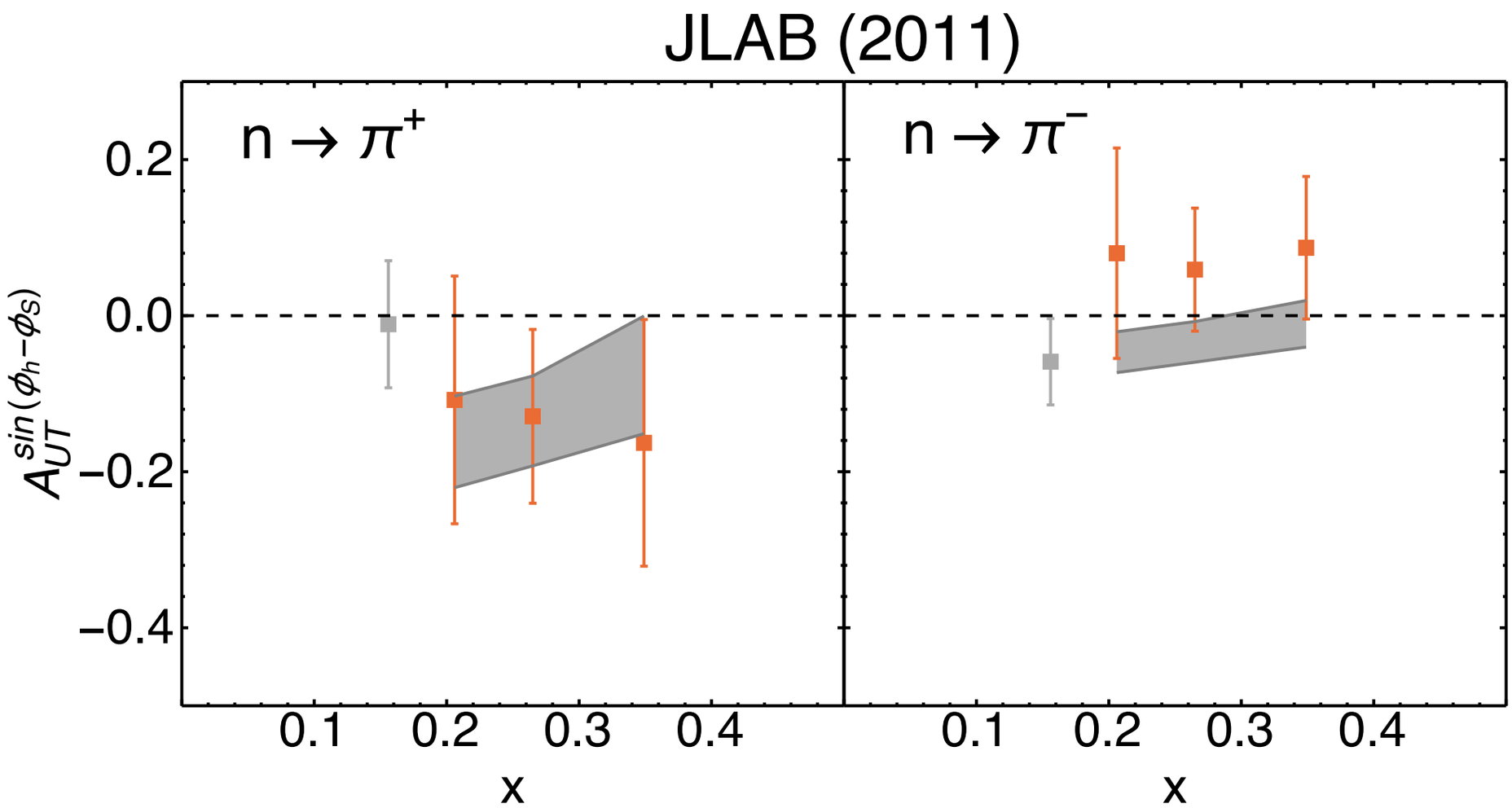


proton

negative hadron

Jefferson Lab

neutron



Sivers in coordinate space

ξ_T space

to apply
CSS formalism for evolution

Sivers distribution function

$$\tilde{f}_{1T}^{\perp(n)a}(x,\xi_T^2;Q^2) = n!\left(-\frac{-2}{M^2}\partial_{\xi_T^2}\right)^n \tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2) = \frac{n!}{(M^2)^n}\int_0^\infty d|\boldsymbol{k}_\perp||\boldsymbol{k}_\perp|\left(\frac{|\boldsymbol{k}_\perp|}{\xi_T}\right)^n J_n(\xi_T|\boldsymbol{k}_\perp|)\tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2)$$

first moment

$$\tilde{f}_{1T}^{\perp(1)a}(x,\xi_T^2;Q^2) = \frac{1}{M^2}\int_0^\infty d|\boldsymbol{k}_\perp||\boldsymbol{k}_\perp|\left(\frac{|\boldsymbol{k}_\perp|}{\xi_T}\right) J_1(\xi_T|\boldsymbol{k}_\perp|)\tilde{f}_{1T}^{\perp a}(x,\xi_T^2;Q^2)$$

Results comparison: Pavia 2011

Constraining Quark Angular Momentum through
Semi-Inclusive Measurements

Angular momentum

GPD

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx x[H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)].$$

$$f_1^a(x,Q^2)$$

no corresponding collinear pdf

$$\sum_q e_{q_v}\int_0^1 dx E^{q_v}(x,0,0) = \kappa,$$

[Bacchetta, Radici - PRL 107, 212001 (2011)

Results comparison: Pavia 2011

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no corresponding collinear pdf

$$\sum_q e_{q_v}\int_0^1 dx E^{q_v}(x,0,0) = \kappa,$$

..from theoretical consideration and spectator model results:

$$\rightarrow f_{1T}^{\perp(0)a}(x;Q_L^2) = -L(x)E^a(x,0,0;Q_L^2),$$

Lensing function

$$L(x) = \frac{K}{(1-x)^\eta}$$

[Bacchetta, Radici - PRL 107, 212001 (2011)

Results comparison: Pavia 2011

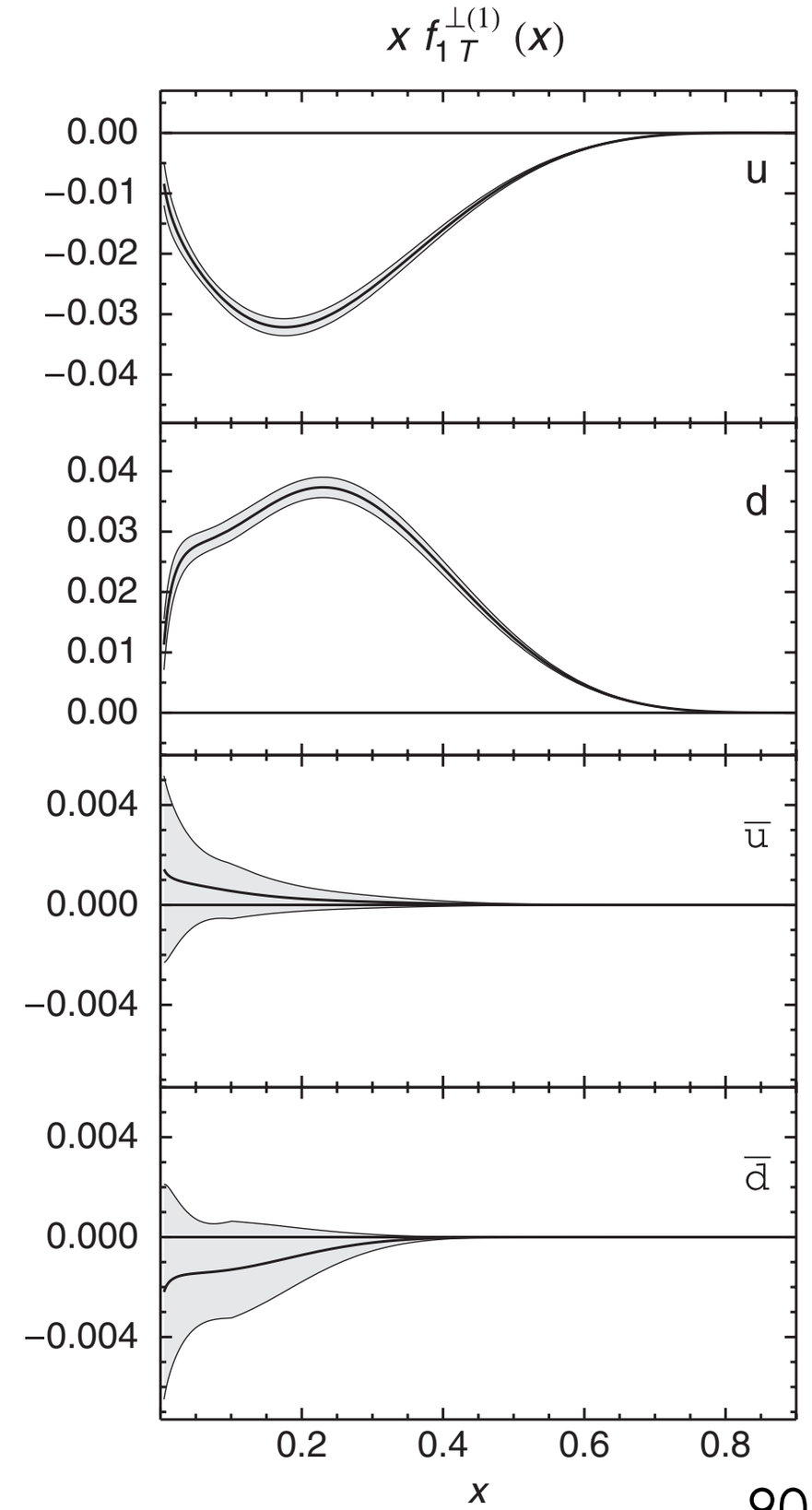
Azimuthal asymmetries

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x, z, P_T^2, Q^2)$$
$$= -\frac{M_1^2(M_1^2+\langle k_\perp^2\rangle)}{\langle P_{\rm Siv}^2\rangle^2}\,\frac{zP_T}{M}\left(z^2+\frac{\langle P_\perp^2\rangle}{\langle k_\perp^2\rangle}\right)^3 e^{-z^2P_T^2/\langle P_{\rm Siv}^2\rangle}$$
$$\times\frac{\sum_a e_a^2 f_{1T}^{\perp(0)a}(x;Q^2)D_1^a(z;Q^2)}{\sum_a e_a^2 f_1^a(x;Q^2)D_1^a(z;Q^2)},$$

Hermes, Compass, Jlab data

TABLE I. Best-fit values of the 8 free parameters for the case $C^{s_v} = C^{\bar{s}} = 0$. The final χ^2/d.o.f. is 1.323. The errors are statistical and correspond to $\Delta\chi^2 = 1$

C^{u_v}	C^{d_v}	$C^{\bar{u}}$	$C^{\bar{d}}$
-0.229 ± 0.002	1.591 ± 0.009	0.054 ± 0.107	-0.083 ± 0.122
M_1 (GeV)	K (GeV)	η	α^{u_v}
0.346 ± 0.015	1.888 ± 0.009	0.392 ± 0.040	0.783 ± 0.001

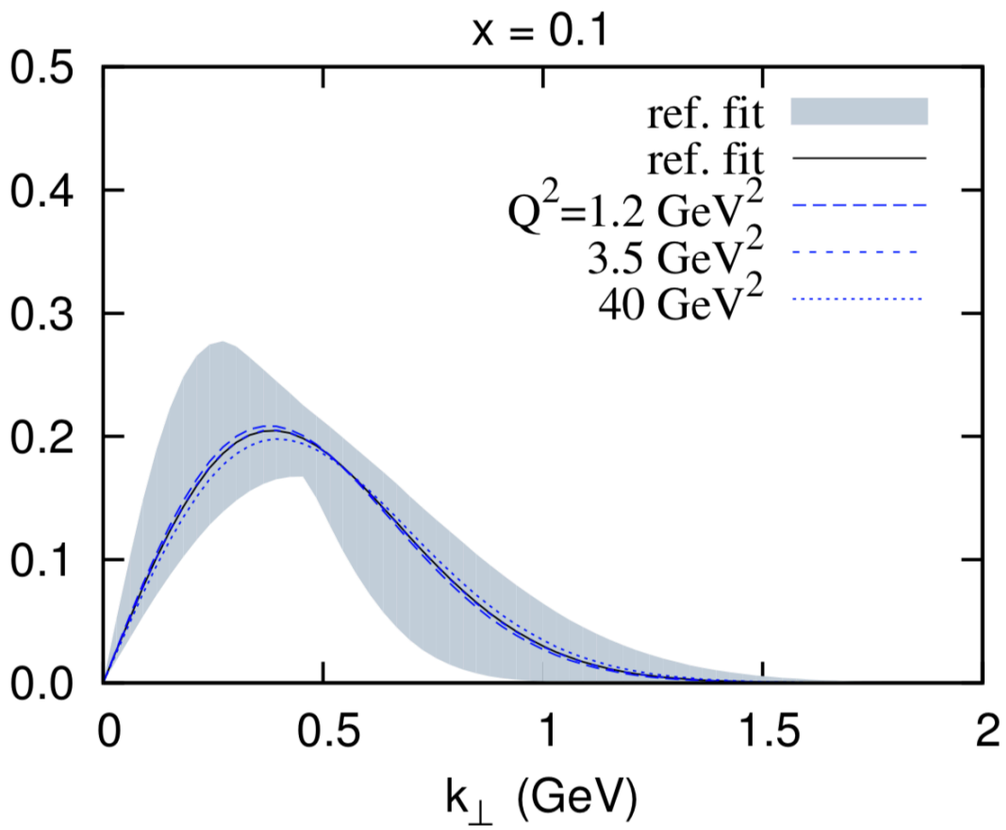


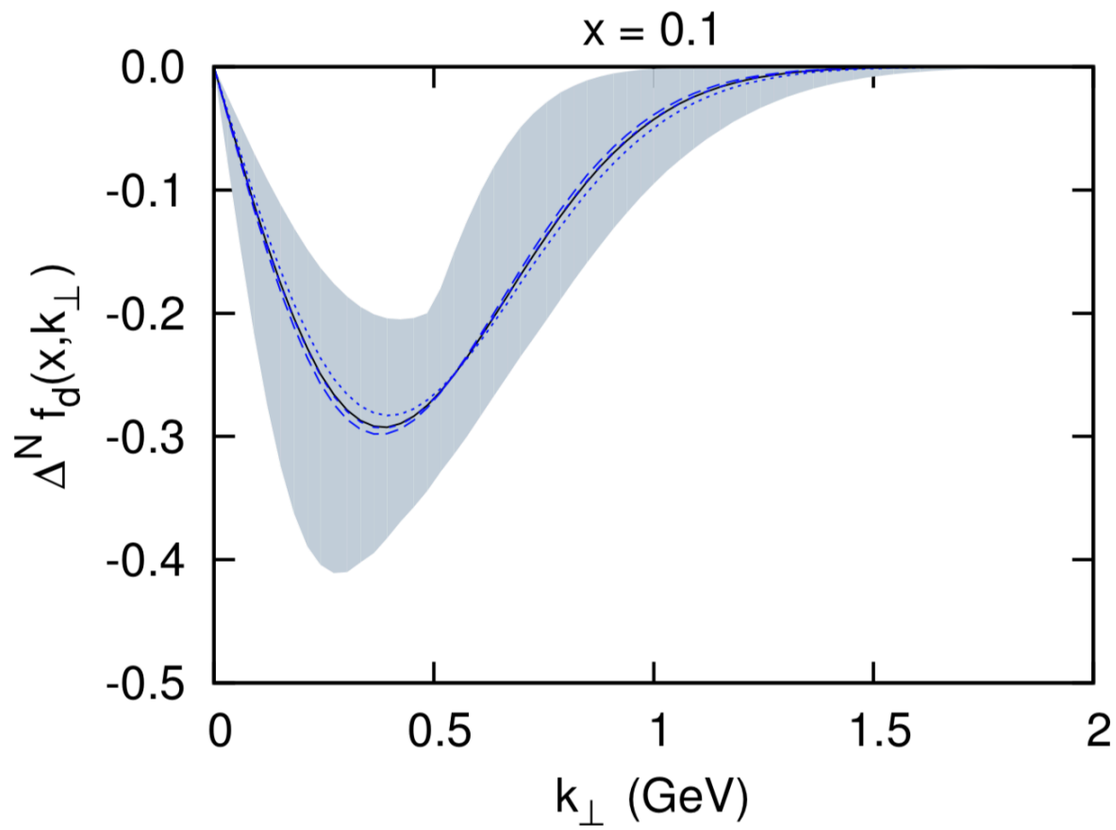
Results comparison: TO - CA group

Same selection of data, considering all projections

$A_{UT}^{sin(\phi_h - \phi_S)}$

3 cases for evolution: no evolution, collinear twist-3, TMD-like evolution





$\chi^2/dof \sim 0.94$

[Eur. Phys. J., A39:89– 100, 2009]

Results comparison: EIKV

Global fit of the HERMES, COMPASS and JLab experimental data on polarized reactions to extract the Sivers functions.

➔Hermes, Compass, Jlab data

➔using CSS evolution

➔relating the first moment of the Sivers function to the twist-three Qiu-Sterman quark-gluon correlation function

$$f_{1T,\mathrm{SIDIS}}^{\perp q(\alpha)}(x,b;Q) = \left(\frac{ib^{\alpha}}{2}\right) T_{q,F}(x,x,c/b_*) \exp\left\{-\int_{c/b_*}^{Q} \frac{d\mu}{\mu}\left(A \ln \frac{Q^2}{\mu^2} + B\right)\right\}$$

$$\times \exp\left\{-b^2\left(g_1^{\mathrm{sivers}} + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}$$

$$T_{q,F}(x,x,\mu) = N_q \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}} x^{\alpha_q}(1-x)^{\beta_q} f_{q/A}(x,\mu)$$

[Echevarria et al. - Phys. Rev. D.89.074013 (2014)]

Results comparison: EIKV

$T_{qF}(x, x, \mu)$ ➔ “collinear counterpart" of the Sivers function

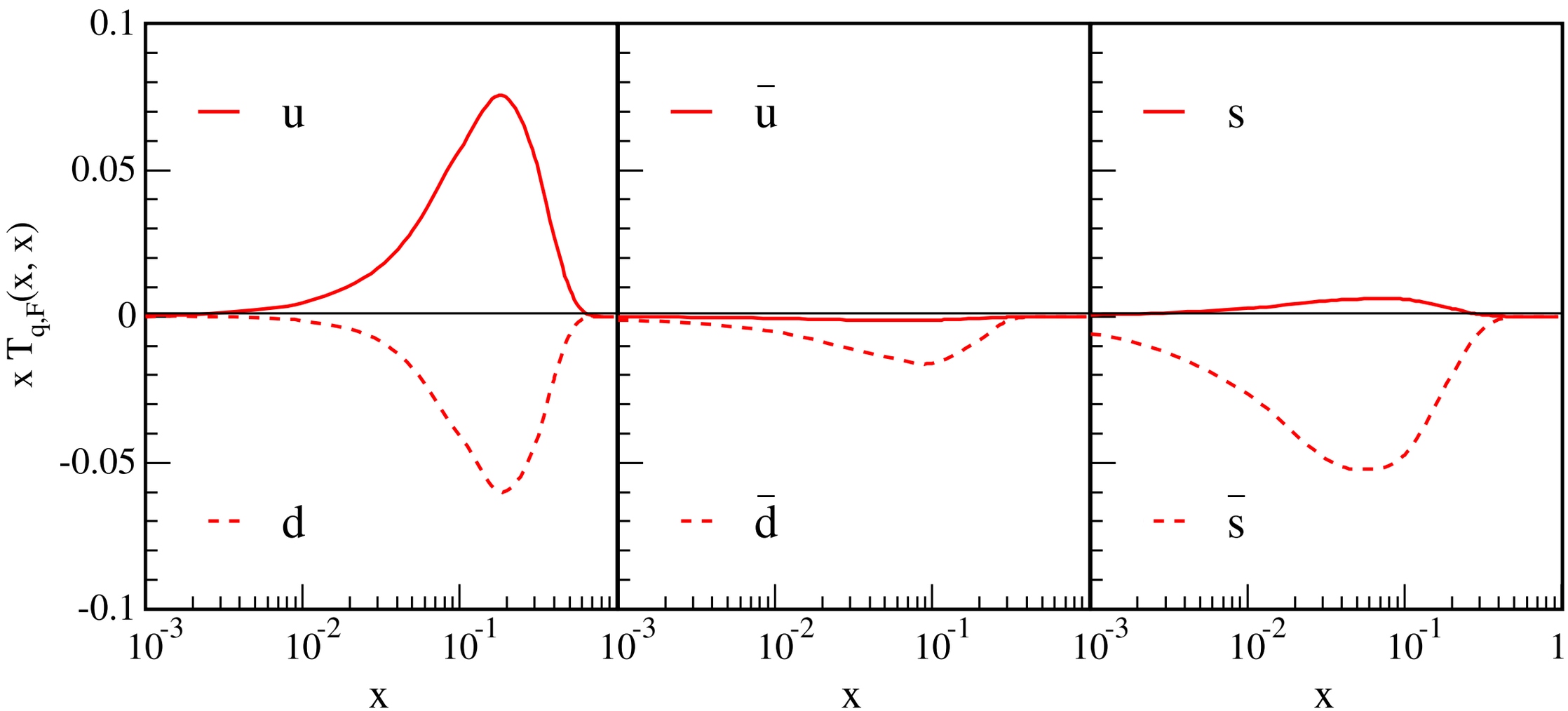


FIG. 11 (color online). Qiu-Sterman function $T_{q,F}(x, x, Q)$ for u, d and s flavors at a scale $Q^2 = 2.4$ GeV^2, as extracted by our simultaneous fit of JLab, HERMES and COMPASS data.

[Echevarria et al. - Phys. Rev. D.89.074013 (2014)]