# BChPT x I/Nc: masses and currents 

## Jose L. Goity

Hampton University and Jefferson Lab

## OUTLINE

- General motivation
- The need for combining BChPT and I/Nc
- BChPT $\times \mathrm{I} / \mathrm{Nc}$ : brief basics
- Masses, sigma terms
- Vector currents in $\mathrm{SU}(3)$
- Axial currents in $\mathrm{SU}(3)$
- Summary, comments


## MOTIVATION

- QCD has small scales:
i) SChSB gives nearly massless GBs (small quark masses)
ii) emergent small scale at large Nc : $m_{\Delta}-m_{N}=\mathcal{O}\left(1 / N_{c}\right)$
- All the small scales MUST be treated consistently in any EFT: in QCD we need to combine ChPT with the I/Nc expansion!
- There is broad evidence that most aspects of the large Nc limit survive for Nc as small as 3: phenomenological observations and LQCD at $\mathrm{Nc}>3$.
- Enhanced symmetries in large $\mathrm{Nc}: \mathrm{SU}(3)$ to $\mathrm{U}(3)$ in mesons; spin-flavor $\mathrm{SU}(6)$ in baryons.
- meson-meson interactions suppressed; meson-baryon interactions can be enhanced by a factor $\sqrt{N_{c}}$
- Very interesting case BChPT and the I/Nc expansion: BChPT x I/Nc


## The need for combining BChPT and I/Nc

Ordinary BChPT (only $\mathrm{S}=1 / 2$ baryons) has poor convergence
$g_{\pi N}$ is large: need for large CTs
Inclusion of $\mathrm{S}=3 / 2$ baryons gives significant improvement in convergence: [Jenkins \& Manohar; many others]

QCD at large $N_{c}$ :

$$
\begin{aligned}
& F_{\pi}=\mathcal{O}\left(\sqrt{N_{c}}\right) \\
& m_{B}=\mathcal{O}\left(N_{c}\right) \\
& g_{A}=\mathcal{O}\left(N_{c}\right) \Rightarrow g_{\pi N}=\mathcal{O}\left(N_{c}{ }^{\frac{3}{2}}\right)
\end{aligned}
$$

well defined large $N_{c}$ limit imposes constraints!

Ordinary BChPT violates $1 / N_{c}$ power counting

- Emergent dynamical spin-flavor symmetry
[Gervais \& Sakita; Dashen \& Manohar] last millenium

$\sim \frac{k^{i} k^{\prime j}}{k_{0}} \frac{\stackrel{g}{g}_{A}^{2} N_{c}^{2}}{F_{\pi}^{2}}\left\langle B^{\prime}\right|\left[X^{i a}, X^{j b}\right]|B\rangle$
must be order $N_{c}^{0}$
$X^{i a}$ axial current
$\left[X^{i a}, X^{j b}\right]=\mathcal{O}\left(1 / N_{c}\right) \quad$ key requirement at large Nc
$\left\{T^{a}, S^{i}, X^{i a}\right\}$ generate contracted $S U\left(2 N_{f}\right)$ dynamical symmetry
classify baryons in multiplets of $S U\left(2 N_{f}\right)$ with generators $\left\{T^{a}, S^{i}, G^{i a}\right\}$

$$
G^{i a}=N_{c} X^{i a}
$$


ground state baryons: tower with $S=\frac{1}{2} \cdots \frac{N_{c}}{2}$
$\frac{1}{N_{c}}$ expansion as spin-flavor operator product expansion
$\left\langle B^{\prime}\right| \hat{O}_{Q C D}|B\rangle=\sum_{n} C_{n} \frac{1}{N_{c}^{\nu-1}}\left\langle B^{\prime}\right| \hat{O}_{n}|B\rangle$
$O_{n}$ : tensor operator product of spin-flavor generators and momenta $\nu_{n}$ : spin-flavor n-bodyness of $O_{n}$

> Example: mass operator in chiral limit: $H_{Q C D} \Rightarrow N_{c} m_{0}+C_{H F} \frac{1}{N_{c}} \hat{S}^{2}+\mathcal{O}\left(\frac{1}{N_{c}^{3}}\right) \hat{S}^{4}+\cdots$ expansion is in $1 / N_{c}^{2}, m_{\Delta}-m_{N}=\mathcal{O}\left(\frac{1}{N_{c}}\right)$

$$
\begin{aligned}
& \text { A test: } g_{A} s \\
& \frac{g_{A}^{N \Delta}}{g_{A}^{N}}=1+\mathcal{O}\left(\frac{1}{N_{c}^{2}}\right) \text { [Dashen \& Manohar] } \\
& g_{A}^{N}=-1.2724 \pm 0.0023 \quad g_{A}^{N \Delta}=-1.235 \pm 0.011
\end{aligned}
$$

## - BChPT x I/Nc: brief basics

- $m_{B}=\mathcal{O}\left(N_{c}\right) \Rightarrow \mathrm{HB}$ expansion is a $1 / N_{c}$ expansion
- Lagrangians built with chiral and spin-flavor tensor operators:

$$
\begin{aligned}
& \mathbf{B}^{\dagger} T_{\chi} \otimes T_{S F} \quad \mathbf{B} \\
& \mathbf{B}=\left(\begin{array}{l}
B_{S=1 / 2} \\
B_{S=3 / 2} \\
\vdots \\
B_{S=N_{c} / 2}
\end{array}\right) \quad \text { GS tower of baryon fields }
\end{aligned}
$$

$T_{\chi}$ chiral tensor $\quad T_{S F}$ spin-flavor tensor product of $\mathrm{SU}(6)$ generators
chiral and $1 / N_{c}$ power counting determined by operators
LECs: chosen to be $\mathcal{O}\left(N_{c}^{0}\right)$, have a $1 / N_{c}$ expansion themselves each Lagrangian term has a well defined leading chiral and $1 / N_{c}$ power need to link chiral and $1 / N_{c}$ expansions: small mass scale $\Delta_{H F}=m_{3 / 2}-m_{1 / 2}$
$\xi$ expansion: $\xi=\mathcal{O}\left(1 / N_{c}\right)=\mathcal{O}(p)$

## Lagrangians in $\xi$ expansion

$$
\begin{aligned}
\mathcal{L}_{B}^{(1)}= & \mathbf{B}^{\dagger}\left(i D_{0}-\frac{C_{H F}}{N_{c}} \hat{S}^{2}-\stackrel{\circ}{g}_{A} u^{i a} G^{i a}+\frac{c_{1}}{2 \Lambda} \hat{\chi}_{+}\right) \mathbf{B} \\
\mathcal{L}_{B}^{(2)} & =\mathbf{B}^{\dagger}\left\{\left(-\frac{1}{2 N_{c} m_{0}}+\frac{w_{1}}{\Lambda}\right) \vec{D}^{2}+\left(\frac{1}{2 N_{c} m_{0}}-\frac{w_{2}}{\Lambda}\right) \tilde{D}_{0}^{2}+\frac{c_{2}}{\Lambda} \chi_{+}^{0}\right. \\
& +\frac{C_{1}^{A}}{N_{c}} u^{i a} S^{i} T^{a}+\frac{C_{2}^{A}}{N_{c}} \epsilon^{i j k} u^{i a}\left\{S^{j}, G^{k a}\right\} \\
& +\kappa \epsilon^{i j k} F_{+i j}^{a} G^{k a}+\rho_{0} F_{-0 i}^{0} S^{i}+\rho_{1} F_{-0 i}^{a} G^{i a} \\
& \left.+\frac{\tau_{1}}{N_{c}} u_{0}^{a} G^{i a} D_{i}+\frac{\tau_{2}}{N_{c}^{2}} u_{0}^{a} S^{i} T^{a} D_{i}+\frac{\tau_{3}}{N_{c}} \nabla_{i} u_{0}^{a} S^{i} T^{a}+\tau_{4} \nabla_{i} u_{0}^{a} G^{i a}+\cdots\right\} \mathbf{B} \\
\mathcal{L}_{B}^{(3)} & =\mathbf{B}^{\dagger}\left\{\frac{c_{3}}{N_{c} \Lambda^{3}} \hat{\chi}_{+}^{2}+\frac{h_{1} \Lambda}{N_{c}^{3}} \hat{S}^{4}+\frac{h_{2}}{N_{c}^{2} \Lambda} \hat{\chi} \hat{S}^{2}+\frac{h_{3}}{N_{c} \Lambda} \chi_{+}^{0} \hat{S}^{2}+\frac{h_{4}}{N_{c} \Lambda} \chi_{+}^{a}\left\{S^{i}, G^{i a}\right\}\right. \\
& +\frac{C_{3}^{A}}{N_{2}^{2}} u^{i a}\left\{\hat{S}^{2}, G^{i a}\right\}+\frac{C_{4}^{A}}{N_{c}^{2}} u^{i a} S^{i} S^{j} G^{j a} \\
& +\frac{D_{1}^{A}}{\Lambda^{2}} \chi_{+}^{0} u^{i a} G^{i a}+\frac{D_{2}^{A}}{\Lambda^{2}} \chi_{+}^{a} u^{i a} S^{i}+\frac{D_{3}^{A}(d)}{\Lambda^{2}} d^{a b c} \chi_{+}^{a} u^{i b} G^{i c}+\frac{D_{3}^{A}(f)}{\Lambda^{2}} f^{a b c} \chi_{+}^{a} u^{i b} G^{i c} \\
& \left.+g_{1}^{E}\left[D_{i}, E_{+i a} T^{a}\right]+\frac{1}{N_{c}} g_{2}^{E}\left[D_{i}, E_{+i a} S^{j} G^{j a}\right]+\kappa_{1} \frac{1}{N_{c}} B_{+}^{i a} S^{i} T^{a}+\cdots\right\} \mathbf{B} \\
\mathcal{L}_{B}^{(4)} & =\mathbf{B}^{\dagger} \frac{1}{2 \Lambda}\left\{\frac{1}{\Lambda^{2}}\left(\kappa_{2} \chi_{+}^{0} B_{+}^{i a} G^{i a}+\kappa_{3} d^{a b c} \chi_{+}^{a} B_{+}^{i b} G^{i c}+\kappa_{4} \chi_{+}^{a} B_{+}^{i a} S^{i}\right)\right. \\
& \left.+\kappa_{5} \frac{1}{N_{c}^{2}} B_{+}^{i a}\left\{\hat{S}^{2}, G^{i a}\right\}+\kappa_{6} \frac{1}{N_{c}} B_{+}^{i a} S^{i} T^{a}+\kappa_{7} \frac{1}{N_{c}^{2}} B_{+}^{i a} S^{i} S^{j} G^{j a}+\cdots\right\} \mathbf{B}
\end{aligned}
$$

## Loops and the non-commutativity of the expansions


contains non-analytic terms:

$$
\left(M_{\pi}^{2}-\left(m_{\Delta}-m_{N}\right)^{2}\right)^{\frac{3}{2}}, \tanh ^{-1}\left(\frac{\left(m_{\Delta}-m_{N}\right)}{\sqrt{1 /\left(-M_{\pi}^{2}+\left(m_{\Delta}-m_{N}\right)^{2}\right.}}\right)
$$

link $1 / N_{c}$ and chiral expansions:

$$
\xi \text { expansion: } \xi=\mathcal{O}\left(1 / N_{c}\right)=\mathcal{O}(p)
$$

equivalent to not expanding non-analytic terms

$$
\nu_{\xi}=1+3 L+\frac{n_{\pi}}{2}+\sum_{i} n_{i}\left(\nu_{O_{i}}+\nu_{p_{i}}-1\right)
$$

## - Masses, sigma terms: $\mathrm{SU}(3)$

WF renormalization factor is $\mathcal{O}\left(N_{c}\right)$ ! plays key role in $N_{c}$ power counting consistency in loops

- mass corrections are $\mathcal{O}\left(N_{c}\right)$ (terms proportional to $M_{G B}^{3}$ )
- SU(3) mass splitting of course $\mathcal{O}\left(N_{c}^{0}\right)$
$M_{\pi}$ dependency from LQCD $\left(M_{K} \sim 500 \mathrm{MeV}\right)$ :
poor convergence above $M_{\pi} \sim 250 \mathrm{MeV}$








## Mass relations

## GMO

$$
\begin{aligned}
& \Delta_{G M O}=\mathrm{Th}:\left(\frac{g_{A}^{N}(L O)}{g_{A}^{N}}\right)^{2} 44 \pm 5 \mathrm{MeV} \text { vs Exp: } 25.6 \pm 1.5 \mathrm{MeV} \\
& \Delta_{G M O}=-\left(\frac{\mathfrak{g}_{A}}{4 \pi F_{\pi}}\right)^{2}\left(\frac{2 \pi}{3}\left(M_{K}^{3}-\frac{1}{4} M_{\pi}^{3}-\frac{2}{\sqrt{3}}\left(M_{K}^{2}-\frac{1}{4} M_{\pi}^{2}\right)^{\frac{3}{2}}\right)\right. \\
& \left.+\frac{2 C_{H F}}{N_{c}}\left(-M_{K}^{2} \log M_{K}^{2}+\frac{1}{4} M_{\pi}^{2} \log M_{\pi}^{2}+\left(M_{K}^{2}-\frac{1}{4} M_{\pi}^{2}\right) \log \left(\frac{4}{3} M_{K}^{2}-\frac{1}{3} M_{\pi}^{2}\right)\right)\right)+\mathcal{O}\left(1 / N_{c}^{3}\right) \\
& =37 \mathrm{MeV}+\mathcal{O}\left(1 / N_{c}^{3}\right) \\
& \quad \text { in large } N_{c}, \Delta_{G M O} \text { is } \mathcal{O}\left(1 / N_{c}\right)
\end{aligned}
$$

ES

$$
\begin{aligned}
& \Delta_{E S}=m_{\Xi^{*}}-2 m_{\Sigma^{*}}+m_{\Delta}= \\
& \text { Th: }-\left(\frac{g_{A}^{N}(L O)}{g_{A}^{N}}\right)^{2} 6.5 \mathrm{MeV} \text { vs Exp: }-4 \pm 7 \mathrm{MeV}=\mathcal{O}\left(1 / N_{c}\right)
\end{aligned}
$$

## GR

$\Delta_{G R}=m_{\Xi^{*}}-m_{\Sigma^{*}}-\left(m_{\Xi}-m_{\Sigma}\right)=0, \quad$ Exp: $21 \pm 7 \mathrm{MeV}$,
$\Delta_{G R}=\frac{h_{2}}{\Lambda} \frac{12}{N_{c}}\left(M_{K}^{2}-M_{\pi}^{2}\right)+\underbrace{\mathrm{O}\left(1 / N_{c}\right) \text { UV finite no-analytic terms }}_{\sim 68 \mathrm{MeV} \times\left(\frac{g_{\Lambda}^{N}(L O)}{g_{\Lambda}^{A}}\right)^{2}})$
$\pi N \quad \sigma$-term

$$
\begin{gathered}
\sigma_{q}(B)=\frac{\partial}{\partial m_{q}} M_{B}=m_{q}\langle B| \bar{q} q|B\rangle \\
\hat{\sigma}(B)=m_{q}\langle B| \bar{u} u+\bar{d} d-2 \bar{s} s|B\rangle \quad \sigma_{\pi N}=\hat{\sigma}+\frac{2 \hat{m}}{m_{s}} \sigma_{s} . \\
\sigma_{\pi N} \sim 60 \mathrm{MeV} \text { from } \pi-N \text { analysis } \\
\hat{\sigma}=\underbrace{\searrow}_{\substack{\uparrow \\
\underbrace{\frac{\hat{m}-\hat{m}}{m_{s}}\left(\frac{N_{c}+3}{6} m_{\Xi}+\frac{2 N_{c}-3}{3} m_{\Sigma}-\frac{5 N_{c}-3}{6} m_{N}\right)}_{\mathcal{O}\left(N_{c}\right)}+\Delta \hat{\sigma} \\
@ N_{c}=3: \sim 23 \mathrm{MeV}} \mathcal{O}\left(N_{c}\right)} \quad 2.3 \times 10^{5} \mathrm{MeV}^{3} \times \frac{g_{A}^{2}}{F_{\tilde{\pi}}} \sim 40 \mathrm{MeV} \\
40 \% \text { from } 8 \text { in loop and } 60 \% \text { from } 10
\end{gathered}
$$

- $\frac{\Delta \sigma_{8}}{\Delta_{G M O}} \sim-13$ : independent of $g_{A} / F_{\pi}$, virtually independent of $C_{H F}$, mild dependence on $M_{K}, M_{\pi}$ !
- $\frac{\Delta \sigma_{8}}{\Delta_{G M O}}$ changes little if one turns off decuplet!
but $g_{A}$ from $\Delta_{G M O}$ too large, clashes with axial couplings


$$
\hat{\sigma}=70 \pm 9 \mathrm{MeV} \quad \oplus \quad L Q C D \quad \sigma_{\pi N}=69 \pm 10 \mathrm{MeV}
$$

[LQCD: Alexandrou et al (2016)]


## Quark mass dependencies of $\sigma$ terms



Historically misleading statement:
" $\sigma$ terms gives the quark mass contribution to the baryon mass" only true in the linear regime $\sigma \propto m_{q}$

|  | $\begin{aligned} & \hline \frac{\dot{g}_{A}}{F_{\pi}} \end{aligned}$ | $\frac{M_{0}}{N_{c}}$ | $C_{H F}$ | $c_{1}$ | $c_{2}$ | $h_{2}$ | $h_{3}$ | $h_{4}$ | $\alpha$ | $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Fit | $\mathrm{MeV}^{-1}$ | MeV | MeV |  |  |  |  |  | MeV | MeV |
| 1 | 0.0126(2) | 364(1) | 166(23) | -1.48(4) | 0 | 0 | 0.67(9) | 0.56(2) | -1.63(24) | 2.16(22) |
| 2 | 0.0126(3) | 213(1) | 179(20) | -1.49 (4) | $-1.02(5)$ | -0.018(20) | 0.69(7) | 0.56(2) | -1.62(24) | 2.14(22) |
| 3 | 0.0126* | 262(30) | 147(52) | $-1.55(3)$ | -0.67 (8) | 0 | 0.64(3) | 0.63(3) | -1.63* | 2.14* |
|  | $\Delta_{G M O}^{\text {phys }}$ | $\sigma_{8}$ | $\Delta \sigma_{8}$ | $\hat{\sigma}$ | $\sigma_{\pi N}$ | $\sigma_{s}$ | $\sigma_{3}$ | $\sigma_{u+d}(p-n)$ |  |  |
|  | MeV | MeV | MeV | MeV | MeV | MeV | MeV | MeV |  |  |
| 1 | 25.6(1.1) | -583(24) | -382(13) | $70(3)(6)$ | - | - | -1.0(3) | -1.6(6) |  |  |
| 2 | 25.5(1.5) | -582(55) | -381 (20) | 70(7)(6) | 69(8)(6) | -3(32) | -1.0(4) | -1.6(8) |  |  |
| 3 | 25.8* | -615(80) | -384(2) | 74(1)(6) | 65(15)(6) | $-121(15)$ | - | - |  |  |

## NNLO tree level relation between $\sigma$ terms

GMO, ES and Gursey-Radicati should be very good additional ones not suppressed in $1 / N_{c}$ : need test-- LQCD some day...

$$
\begin{aligned}
& \sigma_{N m_{s}}=\frac{m_{s}}{8 \hat{m}}\left(-4\left(N_{c}-1\right) \sigma_{N \hat{m}}+\left(N_{c}+3\right) \sigma_{\Lambda \hat{m}}+3\left(N_{c}-1\right) \sigma_{\sum \hat{m}}\right) \\
& \sigma_{\Lambda m_{s}}=\frac{m_{s}}{8 \hat{m}}\left(-4\left(N_{c}-3\right) \sigma_{N \hat{m}}+\left(N_{c}-5\right) \sigma_{\Lambda \hat{m}}+3\left(N_{c}-1\right) \sigma_{\Sigma \hat{m}}\right) \\
& \sigma_{\Sigma m_{s}}=\frac{m_{s}}{8 \hat{m}}\left(-4\left(N_{c}-3\right) \sigma_{N \hat{m}}+\left(N_{c}+3\right) \sigma_{\Lambda \hat{m}}+\left(3 N_{c}-11\right) \sigma_{\Sigma \hat{m}}\right) \\
& \sigma_{\Delta m_{s}}=\frac{m_{s}}{8 \hat{m}}\left(-4\left(N_{c}-1\right) \sigma_{\Delta \hat{m}}-5\left(N_{c}-3\right)\left(\sigma_{\Lambda \hat{m}}-\sigma_{\Sigma \hat{m}}\right)+4 N_{c} \sigma_{\Sigma^{*} \hat{m}}\right) \\
& \sigma_{\Sigma^{*} m_{s}}=\frac{m_{s}}{8 \hat{m}}\left(-\left(N_{c}-3\right)\left(4 \sigma_{\Delta \hat{m}}+5 \sigma_{\Lambda \hat{m}}-5 \sigma_{\Sigma \hat{m}}\right)+4\left(N_{c}-2\right) \sigma_{\Sigma^{*} \hat{m}}\right) .
\end{aligned}
$$

## $1 / N_{c}$ power counting for currents at one-loop



$$
\begin{aligned}
\mathrm{UV} \operatorname{div} & =\frac{1}{(4 \pi)^{2}}\left(\frac{\hat{g}_{A}}{F_{\pi}}\right)^{2}\left\{\frac{1}{2}\left(\lambda_{\epsilon}+1\right) M_{a b}^{2}\left[G^{i a},\left[G^{i b}, \Gamma\right]\right]\right. \\
& \left.+\frac{1}{3}\left(\lambda_{\epsilon}+2\right)\left(2\left[\left[G^{i a}, \Gamma\right],\left[\delta \hat{m},\left[\delta \hat{m}, G^{i a}\right]\right]\right]+\left[\left[\Gamma,\left[\delta \hat{m}, G^{i a}\right]\right],\left[\delta \hat{m}, G^{i a}\right]\right]\right)\right\}
\end{aligned}
$$

- Individual diagrams violate $N_{c}$ power counting
- WF renormalization key for consistency
- Diagrams where current couples to GBs or vertices baryon-GB do not violate $N_{c}$ power counting


## Vector Currents

SU(3) breaking corrections to the vector currents:


## SU(3) breaking to vector charges

| Charge | $\frac{f_{1}}{f_{1}^{S U(3)}}$ | $\begin{gathered} \frac{f_{1}}{f_{1}^{S U(3)}}-1 \\ \substack{\text { [Flores-Mendieta \& JLG: 2014] } \\ \text { HBChPT } \times 1 / N_{c}} \end{gathered}$ | [Villadoro:2006] HBChPT with 8 and 10 | [Lacour et al: 2007] | [Geng et al:2009] RBChPT with 8 and 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Lambda p$ | 0.952 | -0.048 | -0.080 | -0.097 | -0.031 |
| $\Sigma^{-} n$ | 0.966 | -0.034 | -0.024 | 0.008 | -0.022 |
| $\Xi^{-} \Lambda$ | 0.953 | -0.047 | -0.063 | -0.063 | -0.029 |
| $\Xi^{-} \Sigma^{0}$ | 0.962 | -0.038 | -0.076 | -0.094 | -0.030 |

## LQCD


[Shanahan et al, (2015)]

$$
f_{1}^{\Sigma \rightarrow N}(0)=-0.9662(43), \quad f_{1}^{\Xi \rightarrow \Sigma}(0)=+0.9742(28)
$$

[S. Sasaki, (2017)]

## Charge radii

Two possible counter terms $\propto T^{a}$ needed to subtract UV div $\propto S^{i} G^{i a}$ finite renormalization

$$
\text { fix LECs with } p \text { and } n \text { charge radii }
$$

| $g_{1}^{E}=1.48$ |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $g_{2}^{E}=0.74$ |  |  |
| Baryon | $\left\langle r^{2}\right\rangle_{\mathrm{Th}}\left[\mathrm{fm}^{2}\right]$ | $\operatorname{Exp}$ | CT |
| p | 0.7658 | $0.7658 \pm 0.01068$ | 0.66 |
| n | -0.1161 | $-0.1161 \pm 0.0022$ | -0.049 |
| $\Sigma^{-}$ | 0.74 | $0.61 \pm 0.16$ | 0.61 |

Baryon $\left\langle r^{2}\right\rangle_{\mathrm{Th}}\left[f m^{2}\right] \quad\left\langle r^{2}\right\rangle_{\mathrm{CT}}\left(\mu=m_{\rho}\right)\left[f m^{2}\right]$

## Charge form factors at low $Q^{2}$



Interesting lesson:
Curvature of FFs ( $\frac{d^{2}}{d Q^{2}} G^{E}\left(Q^{2}\right)$ ) from loop non-analytic terms Correct signs, but too small!; cancellation between diags $B$ and $E$

Detailed long distance (peripheral) charge distribution consistent with $N_{c}$ power counting [Alarcon, Granados, Weiss]

## Magnetic moments

LO: only one operator : $\kappa \mu_{N} G^{i a}$

| Ratio | Exp | LO |
| :---: | :---: | :---: |
| $p / n$ | -1.46 | -1.5 |
| $\Sigma^{+} / \Sigma^{-}$ | -2.12 | -3. |
| $\Lambda / \Sigma^{+}$ | -0.25 | $-1 / 3$ |
| $p / \Sigma^{+}$ | 1.14 | 1. |
| $\Xi^{0} / \Xi^{-}$ | 1.92 | 2. |
| $p / \Xi^{0}$ | -2.23 | -1.5 |
| $\Delta^{++} / \Delta^{+}$ | 1.37 | 2. |
| $\Omega^{-} / \Delta^{+}$ | -0.75 | -1. |
| $p / \Delta^{+}$ | 1.03 | 1. |

SU(3) breaking is important

## NNLO Counterterms

$$
\begin{aligned}
& \frac{1}{\Lambda} \kappa_{1} \frac{1}{N_{c}} B_{+}^{i a} S^{i} T^{a}+\frac{1}{2 \Lambda}\left\{\frac{1}{\Lambda^{2}}\left(\kappa_{2} \chi_{+}^{0} B_{+}^{i a} G^{i a}+\kappa_{3} d^{a b c} \chi_{+}^{a} B_{+}^{i b} G^{i c}+\kappa_{4} \chi_{+}^{a} B_{+}^{i a} S^{i}\right)\right. \\
+\quad & \left.\kappa_{5} \frac{1}{N_{c}^{2}} B_{+}^{i a}\left\{\hat{S}^{2}, G^{i a}\right\}+\kappa_{6} \frac{1}{N_{c}^{2}} B_{+}^{i a} S^{i} S^{j} G^{j a}\right\}
\end{aligned}
$$

## NNLO Magnetic moments: 1-loop

| LECs |  |
| :---: | :---: |
|  |  |
| $\kappa$ | 2.00 |
| $\kappa_{1}$ | 3.36 |
| $\kappa_{2}$ | 0 |
| $\kappa_{3}$ | 1.69 |
| $\kappa_{4}$ | 0.61 |
| $\kappa_{5}$ | -5.67 |
| $\kappa_{6}$ | 0 |


| Octet | $\mu_{\mathrm{Th}}\left[\mu_{N}\right]$ | $\mu_{\operatorname{Exp}}\left[\mu_{N}\right]$ |
| :---: | :---: | :---: |
|  |  |  |
| p | 2.724 | 2.79285 |
| n | -1.92 | -1.91304 |
| $\Sigma+$ | 2.457 | 2.458 |
| $\Sigma 0$ | 0.717 | - |
| $\Sigma-$ | -1.02 | -1.16 |
| $\Lambda$ | -0.60 | -0.61 |
| $\Xi 0$ | -1.29 | -1.25 |
| $\Xi-$ | -0.65 | -0.65 |


| Decuplet | $\mu_{\mathrm{Th}}\left[\mu_{N}\right]$ | $\mu_{\operatorname{Exp}}\left[\mu_{N}\right]$ |
| :---: | :---: | :---: |
|  |  |  |
| $\Delta^{++}$ | 5.1 | 3.7 |
| $\Delta^{+}$ | 2.5 | 2.7 |
| $\Delta^{0}$ | -0.13 | - |
| $\Delta^{-}$ | -2.8 | - |
| $\Sigma^{*+}$ | 2.7 | - |
| $\Sigma^{* 0}$ | 0.1 | - |
| $\Sigma^{*-}$ | -2.5 | - |
| $\Xi^{* 0}$ | 0.3 | - |
| $\Xi^{*-}$ | -2.2 | - |
| $\Omega$ | -2.0 | -2.0 |

Magnetic radii

| $\kappa_{0}=10.5$ |  |  |  |
| :---: | :---: | :---: | :---: |
| Baryon | $\left\langle r^{2}\right\rangle_{\text {Exp }}\left[\mathrm{fm}^{2}\right]$ | $\left.<r^{2}\right\rangle_{\mathrm{Th}}\left[\mathrm{fm}^{2}\right]$ | $<r^{2}>_{\text {Loop }}\left(\mu=m_{\rho}\right)\left[\mathrm{fm}^{2}\right]$ |
|  |  | 0.8 |  |
| p | 0.78 | 0.86 | 0.28 |
| n | 0.87 | 0.86 | 0.32 |

## Axial-vector currents

[Flores-Mendieta, Hernandez \& Hofmann; Fernando \& JLG] [SU(2): A. Calle-Cordon \& JLG]
Definition of axial couplings

$$
\left\langle B^{\prime}\right| A^{i a}|B\rangle=\frac{6}{5} g_{A}^{a B B^{\prime}}\left\langle B^{\prime}\right| G^{i a}|B\rangle
$$


cancellations to accuracy $1 / N_{c}^{2}$ in large $N_{c}$ persist at $N_{c}=3$

## Fit to $\operatorname{SU}(3) \mathrm{LQCD} g_{A}$ 's

Key observed feature:@ fixed $M_{K}, g_{A}$ 's have little dependence on $M_{\pi}$

```
SU(3) calculation by Cyprus Group [Alexandrou et al, (2016)]
g}\mp@subsup{A}{}{3BB}\mathrm{ and }\mp@subsup{g}{A}{8BB
```

| Fit | $\chi_{\text {dof }}^{2}$ | $\stackrel{\circ}{g}_{A}$ | $\delta g_{A}^{\circ}$ | $C_{1}^{A}$ | $C_{2}^{A}$ | $C_{3}^{A}$ | $C_{4}^{A}$ | $D_{1}^{A}$ | $D_{2}^{A}$ | $D_{3}^{A}$ | $D_{4}^{A}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LO | 3.9 | 1.35 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| NLO Tree | 0.91 | 1.42 | $\ldots$ | -0.18 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 0.009 | $\ldots$ | $\ldots$ |
| NLO Full | 1.08 | 1.02 | 0.15 | -1.11 | 0. | 1.08 | 0. | -0.56 | -0.02 | -0.08 | 0. |
|  | 1.13 | 1.04 | 0.08 | -1.17 | 0. | 1.15 | 0. | -0.59 | -0.02 | -0.09 | 0. |
|  | 1.19 | 1.06 | 0. | -1.23 | 0. | 1.21 | 0. | -0.62 | -0.03 | -0.09 | 0. |

[I. Fernando \& JLG (2018)]



Mild $M_{\pi}$ dependence of axial couplings cannot be described without the cancellations of $N_{c}$ violating terms



Observations on axial couplings

- show most prominently the need for theory consistent with $1 / N_{c}$ expansion
- natural fit at one-loop of the axial couplings from SU(3) LQCD
- impossible to fit $g_{A}$ s of octet when turning off decuplet
- in $\operatorname{SU}(3): g_{A}^{N}(L O) / g_{A}^{N} \sim 0.8$ from fit to axial couplings and from $\Delta_{G M O}$
- numerous relations among axial couplings with calculable corrections


## Summary and comments

- BChPT x 1/Nc improves convergence by eliminating large Nc power violating terms from loop corrections.
- In baryons it requires implementing a dynamical spin-flavor symmetry, broken at sub-leading orders in 1/Nc: use to implement BChPT x 1/Nc
- It affects every observable
- Convergence improvement is especially important in $\operatorname{SU}(3)$.
- New insights on $\sigma$ terms.
- Axial couplings are particularly important tests of the approach.
- New results for the vector currents.
- Need for more LQCD results at different values of $m_{u, d, s}$.
- Works in progress:
i) Compton scattering [with Ishara Fernando and Cintia Willemyns].
ii) $\pi-N$ scattering [with Dulitha Jayakodige].

