



### **BChPT x I/Nc:** masses and currents

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#### OUTLINE

- General motivation
- The need for combining BChPT and I/Nc
- BChPT x I/Nc: brief basics
- Masses, sigma terms
- Vector currents in SU(3)
- Axial currents in SU(3)
- Summary, comments

#### MOTIVATION

- QCD has small scales:
  - i) SChSB gives nearly massless GBs (small quark masses)
  - ii) emergent small scale at large Nc:  $m_{\Delta}-m_{N}=\mathcal{O}(1/N_{c})$
- All the small scales MUST be treated consistently in any EFT: in QCD we need to combine ChPT with the I/Nc expansion!
- There is broad evidence that most aspects of the large Nc limit survive for Nc as small as 3: phenomenological observations and LQCD at Nc>3.
- Enhanced symmetries in large Nc: SU(3) to U(3) in mesons; spin-flavor SU(6) in baryons.
- $\bullet$  meson-meson interactions suppressed; meson-baryon interactions can be enhanced by a factor  $\sqrt{N_c}$
- Very interesting case BChPT and the I/Nc expansion: BChPT x I/Nc

# The need for combining BChPT and I/Nc

Ordinary BChPT (only S=1/2 baryons) has poor convergence

 $g_{\pi N}$  is large: need for large CTs

Inclusion of S=3/2 baryons gives significant improvement in convergence: [Jenkins & Manohar; many others]

QCD at large  $N_c$ :

$$F_{\pi} = \mathcal{O}(\sqrt{N_c})$$

$$m_B = \mathcal{O}(N_c)$$

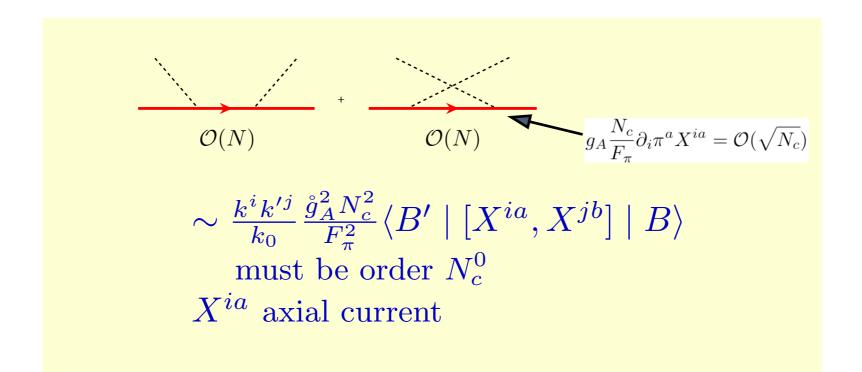
$$g_A = \mathcal{O}(N_c) \Rightarrow g_{\pi N} = \mathcal{O}(N_c^{\frac{3}{2}})$$

well defined large  $N_c$  limit imposes constraints!

Ordinary BChPT violates  $1/N_c$  power counting

# • Emergent dynamical spin-flavor symmetry

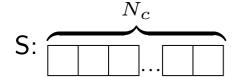
[Gervais & Sakita; Dashen & Manohar] last millenium



$$[X^{ia},X^{jb}]=\mathcal{O}(1/N_c)$$
 key requirement at large Nc

 $\{T^a, S^i, X^{ia}\}$  generate contracted  $SU(2N_f)$  dynamical symmetry

classify baryons in multiplets of  $SU(2N_f)$  with generators  $\{T^a, S^i, G^{ia}\}$  $G^{ia} = N_c X^{ia}$ 



ground state baryons: tower with  $S = \frac{1}{2} \cdots \frac{N_c}{2}$ 

 $\frac{1}{N_c}$  expansion as spin-flavor operator product expansion

$$\langle B' \mid \hat{O}_{QCD} \mid B \rangle = \sum_{n} C_n \frac{1}{N_c^{\nu_n - 1}} \langle B' \mid \hat{O}_n \mid B \rangle$$

 $O_n$ : tensor operator product of spin-flavor generators and momenta

 $\nu_n$ : spin-flavor n-bodyness of  $O_n$ 

Example: mass operator in chiral limit:

$$H_{QCD} \Rightarrow N_c m_0 + C_{HF} \frac{1}{N_c} \hat{S}^2 + \mathcal{O}(\frac{1}{N_c^3}) \hat{S}^4 + \cdots$$
  
expansion is in  $1/N_c^2$ ,  $m_{\Delta} - m_N = \mathcal{O}(\frac{1}{N_c})$ 

A test: 
$$g_A s$$

$$\frac{g_A^{N\Delta}}{g_A^N} = 1 + \mathcal{O}(\frac{1}{N_c^2}) \text{ [Dashen \& Manohar]}$$

$$g_A^N = -1.2724 \pm 0.0023$$
  $g_A^{N\Delta} = -1.235 \pm 0.011$ 

## BChPT x I/Nc: brief basics

- $m_B = \mathcal{O}(N_c) \Rightarrow \text{HB expansion is a } 1/N_c \text{ expansion}$
- Lagrangians built with chiral and spin-flavor tensor operators:

$$\mathbf{B}^{\dagger} T_{\chi} \otimes T_{SF} \mathbf{B}$$

$$\mathbf{B} = \begin{pmatrix} B_{S=1/2} \\ B_{S=3/2} \\ \vdots \\ B_{S=N_c/2} \end{pmatrix}$$
 GS tower of baryon fields

 $T_{\chi}$  chiral tensor  $T_{SF}$  spin-flavor tensor product of SU(6) generators chiral and  $1/N_c$  power counting determined by operators LECs: chosen to be  $\mathcal{O}(N_c^0)$ , have a  $1/N_c$  expansion themselves each Lagrangian term has a well defined leading chiral and  $1/N_c$  power need to link chiral and  $1/N_c$  expansions: small mass scale  $\Delta_{HF} = m_{3/2} - m_{1/2}$   $\xi$  expansion:  $\xi = \mathcal{O}(1/N_c) = \mathcal{O}(p)$ 

# Lagrangians in $\xi$ expansion

$$\mathcal{L}_{B}^{(1)} = \mathbf{B}^{\dagger} (iD_{0} - \frac{C_{HF}}{N_{c}} \hat{S}^{2} - \mathring{g}_{A} u^{ia} G^{ia} + \frac{c_{1}}{2\Lambda} \hat{\chi}_{+}) \mathbf{B}$$

$$\mathcal{L}_{B}^{(2)} = \mathbf{B}^{\dagger} \left\{ \left( -\frac{1}{2N_{c}m_{0}} + \frac{w_{1}}{\Lambda} \right) \vec{D}^{2} + \left( \frac{1}{2N_{c}m_{0}} - \frac{w_{2}}{\Lambda} \right) \tilde{D}_{0}^{2} + \frac{c_{2}}{\Lambda} \chi_{+}^{0} \right. \\
+ \frac{C_{1}^{A}}{N_{c}} u^{ia} S^{i} T^{a} + \frac{C_{2}^{A}}{N_{c}} \epsilon^{ijk} u^{ia} \left\{ S^{j}, G^{ka} \right\} \\
+ \kappa \epsilon^{ijk} F_{+ij}^{a} G^{ka} + \rho_{0} F_{-0i}^{0} S^{i} + \rho_{1} F_{-0i}^{a} G^{ia} \\
+ \frac{\tau_{1}}{N_{c}} u_{0}^{a} G^{ia} D_{i} + \frac{\tau_{2}}{N_{c}^{2}} u_{0}^{a} S^{i} T^{a} D_{i} + \frac{\tau_{3}}{N_{c}} \nabla_{i} u_{0}^{a} S^{i} T^{a} + \tau_{4} \nabla_{i} u_{0}^{a} G^{ia} + \cdots \right\} \mathbf{B}$$

$$\mathcal{L}_{B}^{(3)} = \mathbf{B}^{\dagger} \left\{ \frac{c_{3}}{N_{c}\Lambda^{3}} \hat{\chi}_{+}^{2} + \frac{h_{1}\Lambda}{N_{c}^{3}} \hat{S}^{4} + \frac{h_{2}}{N_{c}^{2}\Lambda} \hat{\chi}_{+} \hat{S}^{2} + \frac{h_{3}}{N_{c}\Lambda} \chi_{+}^{0} \hat{S}^{2} + \frac{h_{4}}{N_{c}\Lambda} \chi_{+}^{a} \{S^{i}, G^{ia}\} \right. \\
+ \frac{C_{3}^{A}}{N^{2}} u^{ia} \{\hat{S}^{2}, G^{ia}\} + \frac{C_{4}^{A}}{N_{c}^{2}} u^{ia} S^{i} S^{j} G^{ja} \\
+ \frac{D_{1}^{A}}{\Lambda^{2}} \chi_{+}^{0} u^{ia} G^{ia} + \frac{D_{2}^{A}}{\Lambda^{2}} \chi_{+}^{a} u^{ia} S^{i} + \frac{D_{3}^{A}(d)}{\Lambda^{2}} d^{abc} \chi_{+}^{a} u^{ib} G^{ic} + \frac{D_{3}^{A}(f)}{\Lambda^{2}} f^{abc} \chi_{+}^{a} u^{ib} G^{ic} \\
+ g_{1}^{E} \left[ D_{i}, E_{+ia} T^{a} \right] + \frac{1}{N_{c}} g_{2}^{E} \left[ D_{i}, E_{+ia} S^{j} G^{ja} \right] + \kappa_{1} \frac{1}{N_{c}} B_{+}^{ia} S^{i} T^{a} + \cdots \right\} \mathbf{B}$$

$$\mathcal{L}_{B}^{(4)} = \mathbf{B}^{\dagger} \frac{1}{2\Lambda} \left\{ \frac{1}{\Lambda^{2}} (\kappa_{2} \chi_{+}^{0} B_{+}^{ia} G^{ia} + \kappa_{3} d^{abc} \chi_{+}^{a} B_{+}^{ib} G^{ic} + \kappa_{4} \chi_{+}^{a} B_{+}^{ia} S^{i}) \right.$$

$$\left. + \kappa_{5} \frac{1}{N_{c}^{2}} B_{+}^{ia} \{ \hat{S}^{2}, G^{ia} \} + \kappa_{6} \frac{1}{N_{c}} B_{+}^{ia} S^{i} T^{a} + \kappa_{7} \frac{1}{N_{c}^{2}} B_{+}^{ia} S^{i} S^{j} G^{ja} + \cdots \right\} \mathbf{B}$$

# Loops and the non-commutativity of the expansions

$$= \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2 - M_\pi^2} \frac{i}{p^0 + k^0 - (m_{B'} - m_B)} \times \text{vertex factors}$$

$$\mathcal{O}(1/N_c)$$

contains non-analytic terms:

$$(M_{\pi}^2 - (m_{\Delta} - m_N)^2)^{\frac{3}{2}}, \ tanh^{-1} \left( \frac{(m_{\Delta} - m_N)}{\sqrt{1/(-M_{\pi}^2 + (m_{\Delta} - m_N)^2}} \right)$$

link  $1/N_c$  and chiral expansions:

$$\xi$$
 expansion:  $\xi = \mathcal{O}(1/N_c) = \mathcal{O}(p)$ 

equivalent to not expanding non-analytic terms

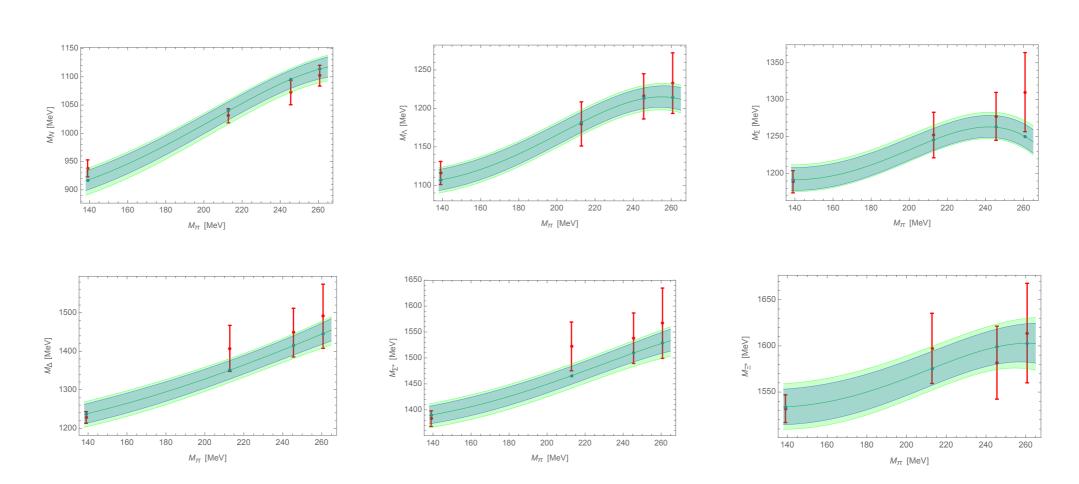
$$\nu_{\xi} = 1 + 3L + \frac{n_{\pi}}{2} + \sum_{i} n_{i} \left( \nu_{O_{i}} + \nu_{p_{i}} - 1 \right)$$

# Masses, sigma terms: SU(3)

WF renormalization factor is  $\mathcal{O}(N_c)$  ! plays key role in  $N_c$  power counting consistency in loops

- ullet mass corrections are  $\mathcal{O}(N_c)$  (terms proportional to  $M_{GB}^3$ )
- ullet SU(3) mass splitting of course  $\mathcal{O}(N_c^0)$

 $M_{\pi}$  dependency from LQCD ( $M_{K}\sim500$  MeV): poor convergence above  $M_{\pi}\sim250$  MeV



[Alexandrou et al (2014), ETMC LQCD Coll. octet and decuplet baryon masses

### Mass relations

#### **GMO**

$$\begin{split} \Delta_{GMO} &= \text{ Th: } \left(\frac{g_A^N(LO)}{g_A^N}\right)^2 44 \pm 5 \text{ MeV vs Exp: } 25.6 \pm 1.5 \text{ MeV} \\ \Delta_{GMO} &= - \left(\frac{\mathring{g}_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3} (M_K^3 - \frac{1}{4} M_\pi^3 - \frac{2}{\sqrt{3}} (M_K^2 - \frac{1}{4} M_\pi^2)^{\frac{3}{2}}) \right. \\ &+ \frac{2C_{HF}}{N_c} \left(-M_K^2 \log M_K^2 + \frac{1}{4} M_\pi^2 \log M_\pi^2 + (M_K^2 - \frac{1}{4} M_\pi^2) \log (\frac{4}{3} M_K^2 - \frac{1}{3} M_\pi^2)\right) \right) + \mathcal{O}(1/N_c^3) \\ &= 37 \text{ MeV} + \mathcal{O}(1/N_c^3) \\ &= 10 \text{ in large } N_C, \ \Delta_{GMO} \text{ is } \mathcal{O}(1/N_c) \end{split}$$

ES

$$\Delta_{ES} = m_{\Xi^*} - 2m_{\Sigma^*} + m_{\Delta} =$$
Th: 
$$-\left(\frac{g_A^N(LO)}{g_A^N}\right)^2 6.5 \text{MeV vs Exp: } -4 \pm 7 \text{MeV} = \mathcal{O}(1/N_c)$$

GR

$$\Delta_{GR} = m_{\Xi^*} - m_{\Sigma^*} - (m_{\Xi} - m_{\Sigma}) = 0, \quad \text{Exp: } 21 \pm 7 \text{ MeV},$$

$$\Delta_{GR} = \frac{h_2}{\Lambda} \frac{12}{N_c} (M_K^2 - M_\pi^2) + \underbrace{O(1/N_c) \text{ UV finite no-analytic terms}}_{\sim 68 \text{ MeV} \times \left(\frac{g_A^N(LO)}{g_A^N}\right)^2}$$

#### $\pi N$ $\sigma$ -term

Feynman-Hellmann Theorem

$$\sigma_q(B) = \frac{\partial}{\partial m_q} M_B = m_q \langle B \mid \bar{q}q \mid B \rangle$$

$$\hat{\sigma}(B) = m_q \langle B \mid \bar{u}u + \bar{d}d - 2\bar{s}s \mid B \rangle \qquad \qquad \sigma_{\pi N} = \hat{\sigma} + \frac{2\hat{m}}{m_s} \sigma_{s}$$

 $\sigma_{\pi N} \sim 60 \text{ MeV from } \pi - N \text{ analysis}$ 

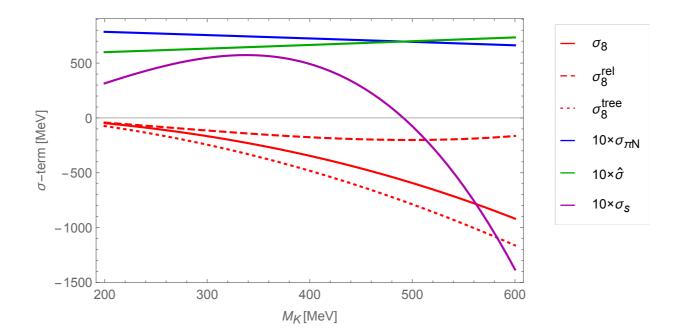
$$\hat{\sigma} = \underbrace{\frac{\hat{m}}{m_{s} - \hat{m}} (\frac{N_{c} + 3}{6} m_{\Xi} + \frac{2N_{c} - 3}{3} m_{\Sigma} - \frac{5N_{c} - 3}{6} m_{N})}_{\text{Q}(N_{c})} + \Delta \hat{\sigma} \leftarrow \mathcal{O}(N_{c})$$

$$\frac{\hat{\sigma}}{N_{c} - \hat{m}} (\frac{N_{c} + 3}{6} m_{\Xi} + \frac{2N_{c} - 3}{3} m_{\Sigma} - \frac{5N_{c} - 3}{6} m_{N}) + \Delta \hat{\sigma} \leftarrow \mathcal{O}(N_{c})$$

$$2.3 \times 10^{5} \text{MeV}^{3} \times \frac{g_{A}^{2}}{F_{\pi}^{2}} \sim 40 \text{ MeV}$$

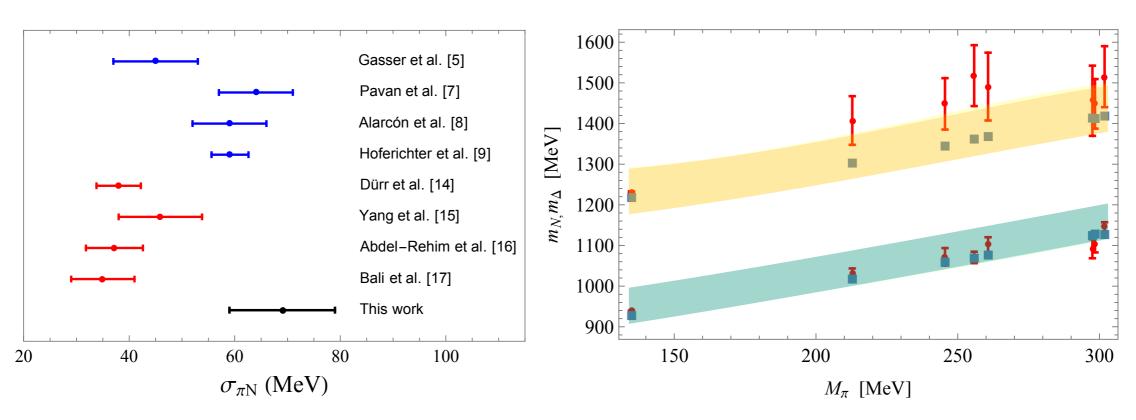
40 % from 8 in loop and 60 % from 10

- $\frac{\Delta \sigma_8}{\Delta_{GMO}} \sim -13$ : independent of  $g_A/F_\pi$ , virtually independent of  $C_{HF}$ , mild dependence on  $M_K, M_\pi$ !
- $\frac{\Delta \sigma_8}{\Delta_{GMO}}$  changes little if one turns off decuplet! but  $g_A$  from  $\Delta_{GMO}$  too large, clashes with axial couplings



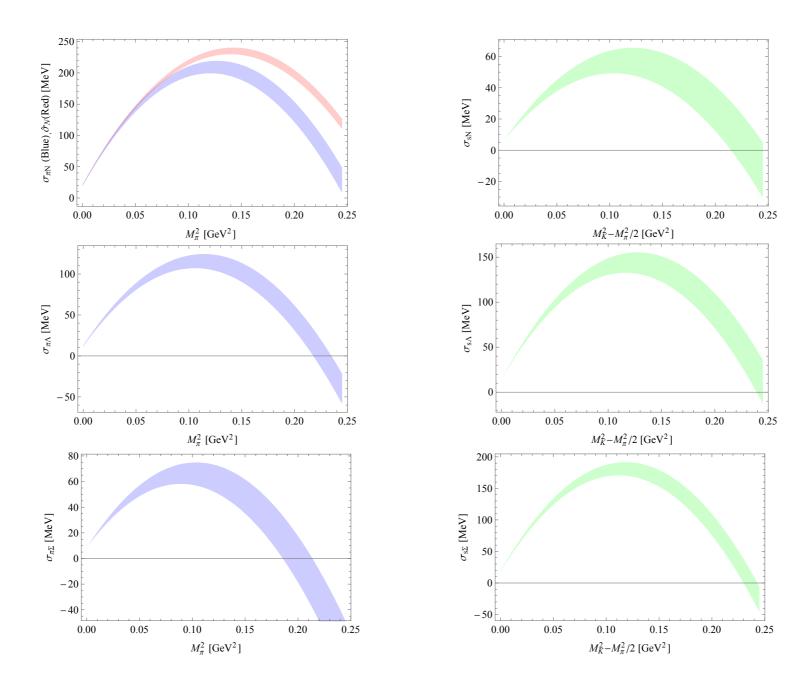
 $\hat{\sigma} = 70 \pm 9 \text{ MeV} \oplus LQCD \quad \sigma_{\pi N} = 69 \pm 10 \text{ MeV}$ 

[LQCD: Alexandrou et al (2016)]



[J.M.Alarcon, I.Fernando & JLG (2018)]

#### Quark mass dependencies of $\sigma$ terms



Historically misleading statement:

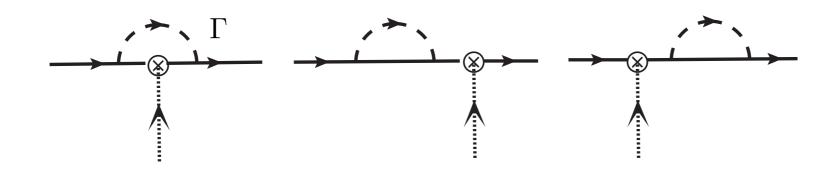
" $\sigma$  terms gives the quark mass contribution to the baryon mass" only true in the linear regime  $\sigma \propto m_q$ 

	$\frac{\mathring{g}_A}{F_\pi}$	$\frac{M_0}{N_c}$	$C_{HF}$	$c_1$	$c_2$	$h_2$	$h_3$	$h_4$	α	β
Fit	$MeV^{-1}$	MeV	MeV						MeV	MeV
1	0.0126(2)	364(1)	166(23)	-1.48(4)	0	0	0.67(9)	0.56(2)	-1.63(24)	2.16(22)
2	0.0126(3)	213(1)	179(20)	-1.49(4)	-1.02(5)	-0.018(20)	0.69(7)	0.56(2)	-1.62(24)	2.14(22)
3	0.0126*	262(30)	147(52)	-1.55(3)	-0.67(8)	0	0.64(3)	0.63(3)	-1.63*	2.14*
	$\Delta_{GMO}^{ ext{phys}}$	$\sigma_8$	$\Delta\sigma_8$	ô	$\sigma_{\pi N}$	$\sigma_s$	$\sigma_3$	$\sigma_{u+d}(p-n)$	)	
	MeV	MeV	MeV	MeV	MeV	MeV	MeV	MeV		
1	25.6(1.1)	-583(24)	-382(13)	70(3)(6)	-	_	-1.0(3)	-1.6(6)		
2	25.5(1.5)	-582(55)	-381(20)	70(7)(6)	69(8)(6)	-3(32)	-1.0(4)	-1.6(8)		
3	25.8*	-615(80)	-384(2)	74(1)(6)	65(15)(6)	-121(15)	_	_		

NNLO tree level relation between  $\sigma$  terms GMO, ES and Gursey-Radicati should be very good additional ones not suppressed in  $1/N_c$ : need test-- LQCD some day...

$$\sigma_{Nm_s} = \frac{m_s}{8\hat{m}} \left( -4(N_c - 1)\sigma_{N\hat{m}} + (N_c + 3)\sigma_{\Lambda\hat{m}} + 3(N_c - 1)\sigma_{\Sigma\hat{m}} \right) 
\sigma_{\Lambda m_s} = \frac{m_s}{8\hat{m}} \left( -4(N_c - 3)\sigma_{N\hat{m}} + (N_c - 5)\sigma_{\Lambda\hat{m}} + 3(N_c - 1)\sigma_{\Sigma\hat{m}} \right) 
\sigma_{\Sigma m_s} = \frac{m_s}{8\hat{m}} \left( -4(N_c - 3)\sigma_{N\hat{m}} + (N_c + 3)\sigma_{\Lambda\hat{m}} + (3N_c - 11)\sigma_{\Sigma\hat{m}} \right) 
\sigma_{\Delta m_s} = \frac{m_s}{8\hat{m}} \left( -4(N_c - 1)\sigma_{\Delta\hat{m}} - 5(N_c - 3)(\sigma_{\Lambda\hat{m}} - \sigma_{\Sigma\hat{m}}) + 4N_c\sigma_{\Sigma^*\hat{m}} \right) 
\sigma_{\Sigma^* m_s} = \frac{m_s}{8\hat{m}} \left( -(N_c - 3)(4\sigma_{\Delta\hat{m}} + 5\sigma_{\Lambda\hat{m}} - 5\sigma_{\Sigma\hat{m}}) + 4(N_c - 2)\sigma_{\Sigma^*\hat{m}} \right).$$

# $1/N_c$ power counting for currents at one-loop



$$\begin{array}{lll} \text{UV div} & = & \frac{1}{(4\pi)^2} (\frac{\mathring{g}_A}{F_\pi})^2 \Big\{ \frac{1}{2} (\lambda_\epsilon + 1) M_{ab}^2 [G^{ia}, [G^{ib}, \Gamma]] \\ & + & \frac{1}{3} (\lambda_\epsilon + 2) (2 [[G^{ia}, \Gamma], [\delta \hat{m}, [\delta \hat{m}, G^{ia}]]] + [[\Gamma, [\delta \hat{m}, G^{ia}]], [\delta \hat{m}, G^{ia}]]) \Big\} \end{array}$$

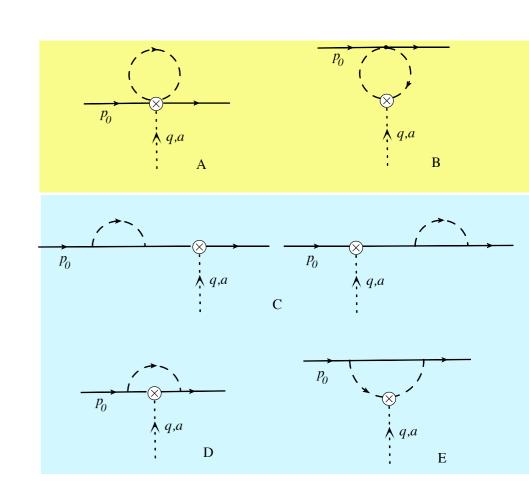
- ullet Individual diagrams violate  $N_c$  power counting
- WF renormalization key for consistency
- ullet Diagrams where current couples to GBs or vertices baryon-GB do not violate  $N_c$  power counting

## **Vector Currents**

SU(3) breaking corrections to the vector currents:

Ademollo-Gatto theorem at  $\mathcal{O}(\xi^2)$  non-analytic calculable corrections to AGTh  $\mathcal{O}(N_c^0)$  ,

different spin baryons in loop give  $\mathcal{O}(N_c)$  terms! key cancellations give  $N_c$  consistency



# SU(3) breaking to vector charges

Charge	$\frac{f_1}{f_1^{SU(3)}}$	$\frac{f_1}{f_1^{SU(3)}} - 1$			
		[Flores-Mendieta & JLG:2014]	[Villadoro:2006]	[Lacour et al:2007]	[Geng et al:2009]
		${\tt HBChPT}{\times}1/N_{\it C}$	HBChPT with 8 and 10	HBChPT only 8	RBChPT with 8 and 10
$\overline{\Lambda p}$	0.952	-0.048	-0.080	-0.097	-0.031
$\Sigma^- n$	0.966	-0.034	-0.024	0.008	-0.022
$\Xi^-\Lambda$	0.953	-0.047	-0.063	-0.063	-0.029
$\Xi^-\Sigma^0$	0.962	-0.038	-0.076	-0.094	-0.030

 $Q^2 = \Delta m_B^2$ 

FIG. 5. Comparison of percentage SU(3)-breaking in  $f_1$  determined in this work, highlighted by the shaded bands, with that of other calculations. The error bands for our results are those given in Table II combined in quadrature. Blue squares, green circles and orange triangles denote results of quark model [10, 11],  $1/N_c$  expansion [12] and chiral perturbation theory [14, 16, 40] approaches respectively, while the pink diamonds show results from lattice QCD [18, 20]. The red stars show the results of this work at  $Q^2 = 0$  (solid line), where we have corrected from  $\vec{q} = 0$  to  $Q^2 = 0$  using the dipole form given in Eq. (29), and at  $Q^2 = -(M_B - M_{B'})^2$  (dotted line).

FIG. 5. Grapprison of person type SU(3) glust kindeter on the SU(3)-breaking effects, we reglect these differences. Explicitly, we find the differof other calculations calculations of other calculations calculations are of the calculations of the calcu

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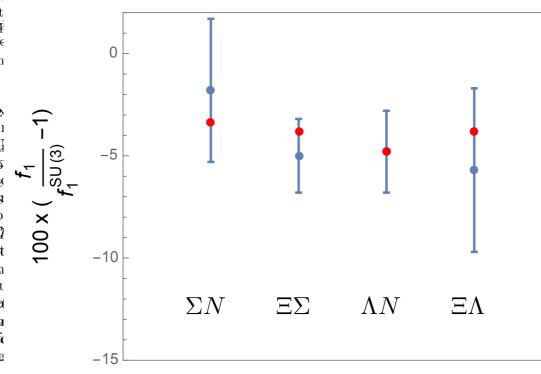


Figure 4 Fi all results alrest the rest length of the second se baryon effects on Wife attribution the configuration of the value of t our use of the FRA scheme.

at non-ateriorizations values; of he Q2 result results naccentrerely Following Following klin Rocks. i 12 Refs., [25], 4 by 42 e who are alsown sho facility after little and soparist on with rother work and are able to usbletto discathex third lax to a polarical isor malitane to detect a modular including Dain (2D) with two attempts to the chiral lax to a polarical form of the chiral lax to a polarical lax to a mine the refrect be affect reported and the restriction of the refrective of the restriction of the res

 $m_u \neq m_u$  of our results the weight characteristic from the property of the

e in the quantity  $(f_1/f_1^{\mathrm{SU}(3)}-1)\times 100$  for  $\Sigma^-\to n$   $\mid \Sigma^0\to p$  and also  $\Xi^0\to \Sigma^+$  and  $\Xi^-\to \Sigma^0$  to be in the ge 0.03-0.04, which is an order of magnitude smaller n the statistical uncertainties of our calculation. Finally, to estimate the magnitude of the effect caused the non-zero values of  $Q^2$  used in our analysis, we re corrected from  $Q^2 = -(M_{B_1} - M_{B_2})^2$  to  $Q^2 = 0$ ng the standard dipole parameterisation which is used fit experimental results [43]:

$$f_1(Q^2) = \frac{f_1(0)}{(1 + Q^2/M_V^2)^2},$$
 (29)

ere  $M_V = 0.97$  GeV is chosen, generally universally oss the baryon octet, for strangeness-changing (and 4 GeV for strangeness-conserving) decays [44]. These nbers may be more directly compared with the results previous analyses as shown in Fig. 5. It is clear that the ve extrapolation in  $Q^2$  by Eq. (29) causes a significant cancement of the SU(3)-breaking in our results, particrly for the  $\Sigma \to N$  transition where in our calculation value of  $Q^2$  is the largest. We emphasize that our Daryons attended in Table II and obtained

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[S. Sasaki, (2017)]

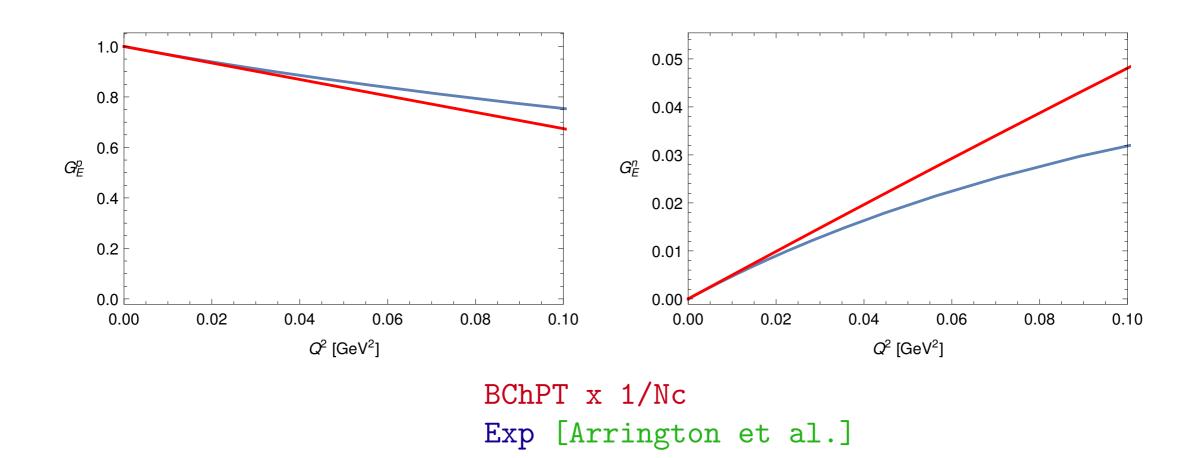
## Charge radii

Two possible counter terms  $\propto T^a \ {\rm needed} \ {\rm to} \ {\rm subtract} \ {\rm UV} \ {\rm div} \\ \propto S^i G^{ia} \ {\rm finite} \ {\rm renormalization}$ 

fix LECs with p and n charge radii

	$g_1^E = 1.48$	$g_2^E = 0.74$		Baryon	$\langle r^2 \rangle_{\mathrm{Th}} [fm^2]$	$\langle r^2 \rangle_{\rm CT}(\mu = m_\rho)[fm^2]$
Baryon p n Σ	$\langle r^2 \rangle_{\rm Th} [fm^2] \ 0.7658 \ -0.1161 \ 0.74$	$\begin{aligned} & \text{Exp} \\ & 0.7658 \pm 0.01068 \\ & -0.1161 \pm 0.0022 \\ & 0.61 \pm 0.16 \end{aligned}$	CT 0.66 -0.049 0.61	$p$ $n$ $\Sigma^+$ $\Sigma^0$ $\Sigma^ \Lambda$ $\Xi^0$ $\Xi^-$	0.7658 -0.1161 0.801 0.029 -0.741 -0.029 -0.016 -0.692	0.655 $-0.049$ $0.655$ $0.024$ $-0.606$ $-0.024$ $-0.049$ $-0.606$

# Charge form factors at low $Q^2$



Interesting lesson:

Curvature of FFs ( $\frac{d^2}{dQ^{2^2}}G^E(Q^2)$ ) from loop non-analytic terms Correct signs, but too small!; cancellation between diags B and E

Detailed long distance (peripheral) charge distribution consistent with  $N_c$  power counting [Alarcon, Granados, Weiss]

# Magnetic moments

LO: only one operator :  $\kappa \, \mu_N G^{ia}$ 

Ratio	Exp	LO
$\overline{p/n}$	-1.46	-1.5
$\Sigma^+/\Sigma^-$	-2.12	-3.
$\Lambda/\Sigma^+$	-0.25	-1/3
$p/\Sigma^+$	1.14	1.
$\Xi^0/\Xi^-$	1.92	2.
$p/\Xi^0$	-2.23	-1.5
$\Delta^{++}/\Delta^{+}$	1.37	2.
$\Omega^{-}/\Delta^{+}$	-0.75	-1.
$p/\Delta^+$	1.03	1.

SU(3) breaking is important

#### NNLO Counterterms

$$\frac{1}{\Lambda}\kappa_{1}\frac{1}{N_{c}}B_{+}^{ia}S^{i}T^{a} + \frac{1}{2\Lambda}\left\{\frac{1}{\Lambda^{2}}(\kappa_{2}\chi_{+}^{0}B_{+}^{ia}G^{ia} + \kappa_{3}d^{abc}\chi_{+}^{a}B_{+}^{ib}G^{ic} + \kappa_{4}\chi_{+}^{a}B_{+}^{ia}S^{i}) + \kappa_{5}\frac{1}{N_{c}^{2}}B_{+}^{ia}\{\hat{S}^{2},G^{ia}\} + \kappa_{6}\frac{1}{N_{c}^{2}}B_{+}^{ia}S^{i}S^{j}G^{ja}\right\}$$

## NNLO Magnetic moments: 1-loop

LECs	
12	2.00
$\kappa \ \kappa_1$	$\frac{2.00}{3.36}$
$\kappa_1 \ \kappa_2$	0
$\kappa_3$	1.69
$\kappa_4$	0.61
$\kappa_5$	-5.67
$\kappa_6$	0

Octet	$\mu_{\mathrm{Th}}[\mu_N]$	$\mu_{\mathrm{Exp}}[\mu_N]$
p	2.724	2.79285
$\mathbf{n}$	-1.92	-1.91304
$\Sigma+$	2.457	2.458
$\Sigma 0$	0.717	_
$\sum$ -	-1.02	-1.16
$\Lambda$	-0.60	-0.61
$\Xi 0$	-1.29	-1.25
Ξ-	-0.65	-0.65

Decuplet	$\mu_{\mathrm{Th}}[\mu_N]$	$\mu_{\mathrm{Exp}}[\mu_N]$
$\Delta^{++}$	5.1	3.7
$\Delta^+$	$\frac{3.1}{2.5}$	3.7 2.7
$\Delta^0$	-0.13	_
$\Delta^-$	-2.8	_
$\Sigma^{*+}$ $\Sigma^{*0}$	$\frac{2.7}{0.1}$	_
$\sum_{*}$	-2.5	_
$\Xi^{*0}$	0.3	_
$\Xi^{*-}$	-2.2	_
Ω	-2.0	-2.0

## Magnetic radii

$$\kappa_0 = 10.5$$

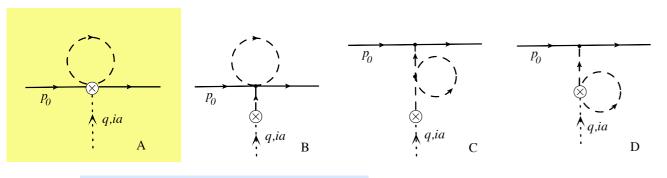
Baryon	$< r^2 >_{\text{Exp}} [\text{fm}^2]$	$< r^2 >_{\mathrm{Th}} [\mathrm{fm}^2]$	$< r^2 >_{\text{Loop}} (\mu = m_{\rho}) [\text{fm}^2]$
p n	$0.78 \\ 0.87$	$0.86 \\ 0.86$	$0.28 \\ 0.32$

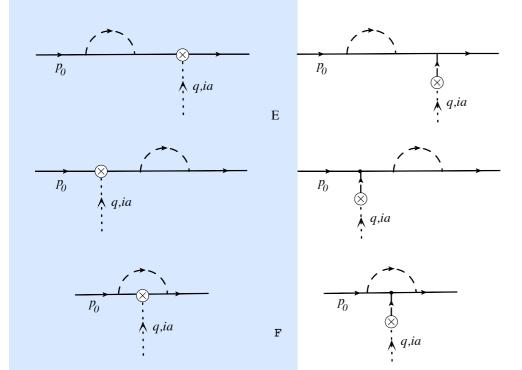
# Axial-vector currents

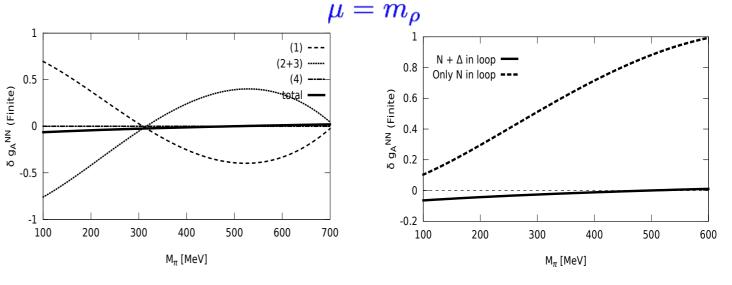
[Flores-Mendieta, Hernandez & Hofmann; Fernando & JLG] [SU(2): A. Calle-Cordon & JLG]

#### Definition of axial couplings

$$\langle B' \mid A^{ia} \mid B \rangle = \frac{6}{5} g_A^{aBB'} \langle B' \mid G^{ia} \mid B \rangle$$







cancellations to accuracy  $1/N_c^2$  in large  $N_c$  persist at  $N_c=3$ 

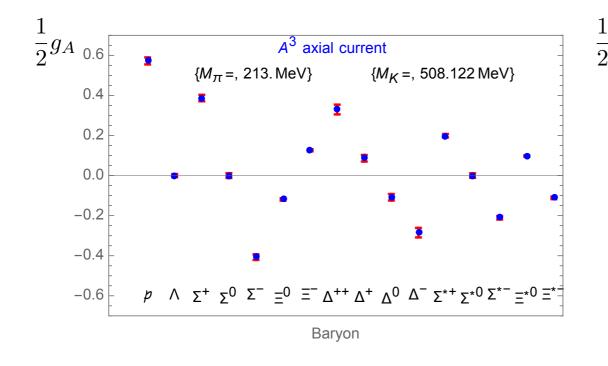
# Fit to SU(3) LQCD $g_A$ 's

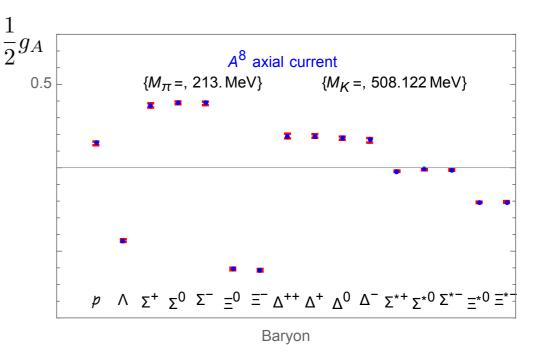
Key observed feature:0 fixed  $M_K$ ,  $g_A$ 's have little dependence on  $M_\pi$ 

SU(3) calculation by Cyprus Group [Alexandrou et al, (2016)]  $g_A^{3BB}$  and  $g_A^{8BB}$ 

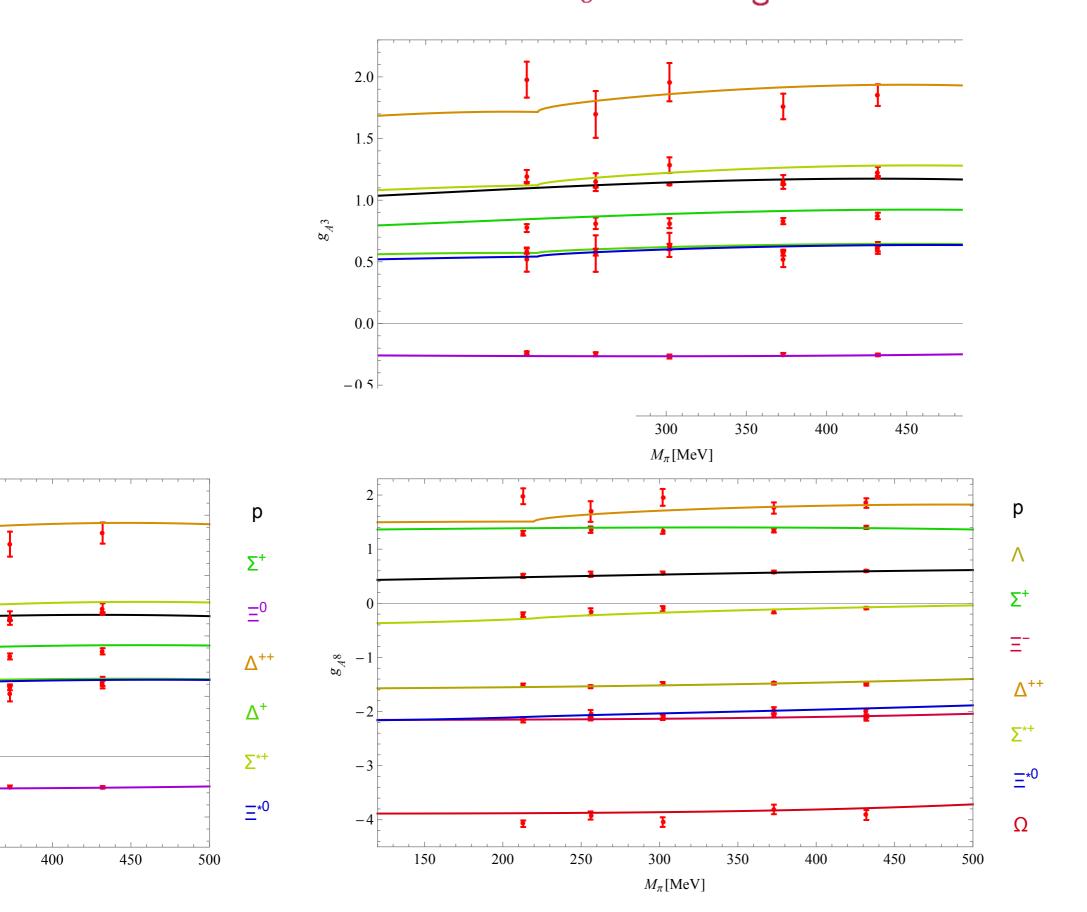
		<u>'</u>									
Fit	$\chi^2_{ m dof}$	$\overset{\circ}{g}_{\!\scriptscriptstyle A}$	$\delta \overset{\circ}{g}_{A}$	$C_1^A$	$C_2^A$	$C_3^A$	$C_4^A$	$D_1^A$	$D_2^A$	$D_3^A$	$D_4^A$
LO	3.9	1.35	•••	•••	•••	•••	•••	•••	•••	•••	•••
NLO Tree	0.91	1.42	•••	-0.18	•••	•••	•••	•••	0.009	•••	•••
NLO Full	1.08	1.02	0.15	-1.11	0.	1.08	0.	-0.56	-0.02	-0.08	0.
	1.13	1.04	0.08	-1.17	0.	1.15	0.	-0.59	-0.02	-0.09	0.
	1.19	1.06	0.	-1.23	0.	1.21	0.	-0.62	-0.03	-0.09	0.

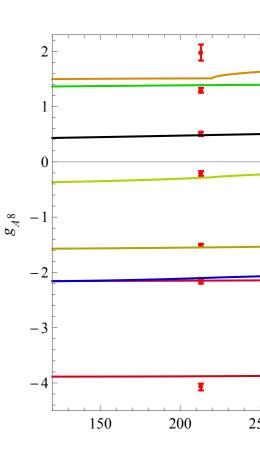
#### [I. Fernando & JLG (2018)]





Mild  $M_\pi$  dependence of axial couplings cannot be described without the cancellations of  $N_c$  violating terms





#### Observations on axial couplings

- show most prominently the need for theory consistent with  $1/N_c$  expansion
- natural fit at one-loop of the axial couplings from SU(3) LQCD
- impossible to fit  $g_A$ s of octet when turning off decuplet
- in SU(3):  $g_A^N(LO)/g_A^N \sim 0.8$  from fit to axial couplings and from  $\Delta_{GMO}$
- numerous relations among axial couplings with calculable corrections

# Summary and comments

- BChPT x 1/Nc improves convergence by eliminating large Nc power violating terms from loop corrections.
- In baryons it requires implementing a dynamical spin-flavor symmetry, broken at sub-leading orders in 1/Nc: use to implement BChPT x 1/Nc
- It affects every observable
- Convergence improvement is especially important in SU(3).
- ullet New insights on  $\sigma$  terms.
- Axial couplings are particularly important tests of the approach.
- New results for the vector currents.
- ullet Need for more LQCD results at different values of  $m_{u,d,s}$ .
- Works in progress:
  - i) Compton scattering [with Ishara Fernando and Cintia Willemyns].
  - ii)  $\pi N$  scattering [with Dulitha Jayakodige].