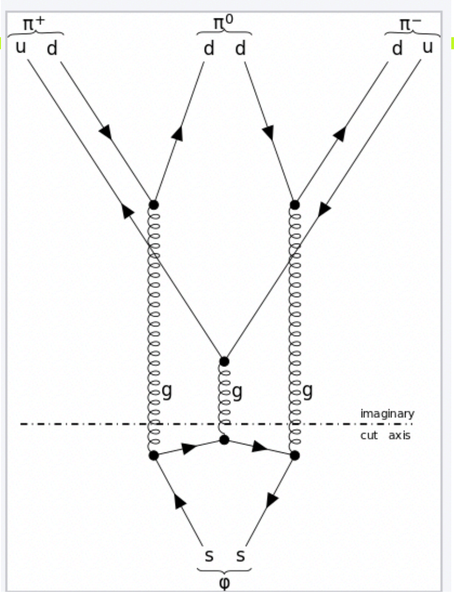
Excited J- - resonances from meson-meson scattering at the SU(3) flavor point in lattice QCD.

CHRIS JOHNSON, FEB 15 2021

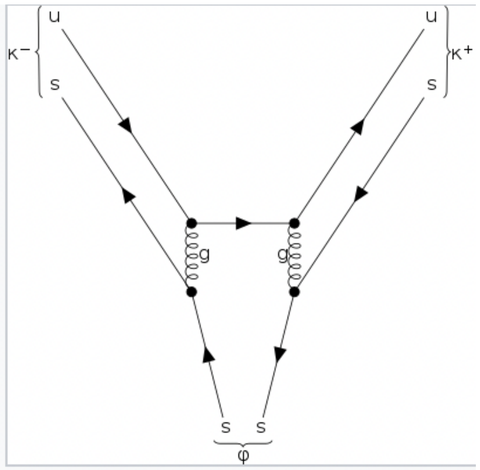
Experimental Status



The lightest vector ($J^{PC} = 1^{--}$) mesons are the ρ(770), ω(782), ϕ(1020)

States are well understood in e^+e^- annihilation due to their narrow widths and little background into decay into simple states like $\pi\pi$, $\pi\pi\pi$, $K\bar{K}$.

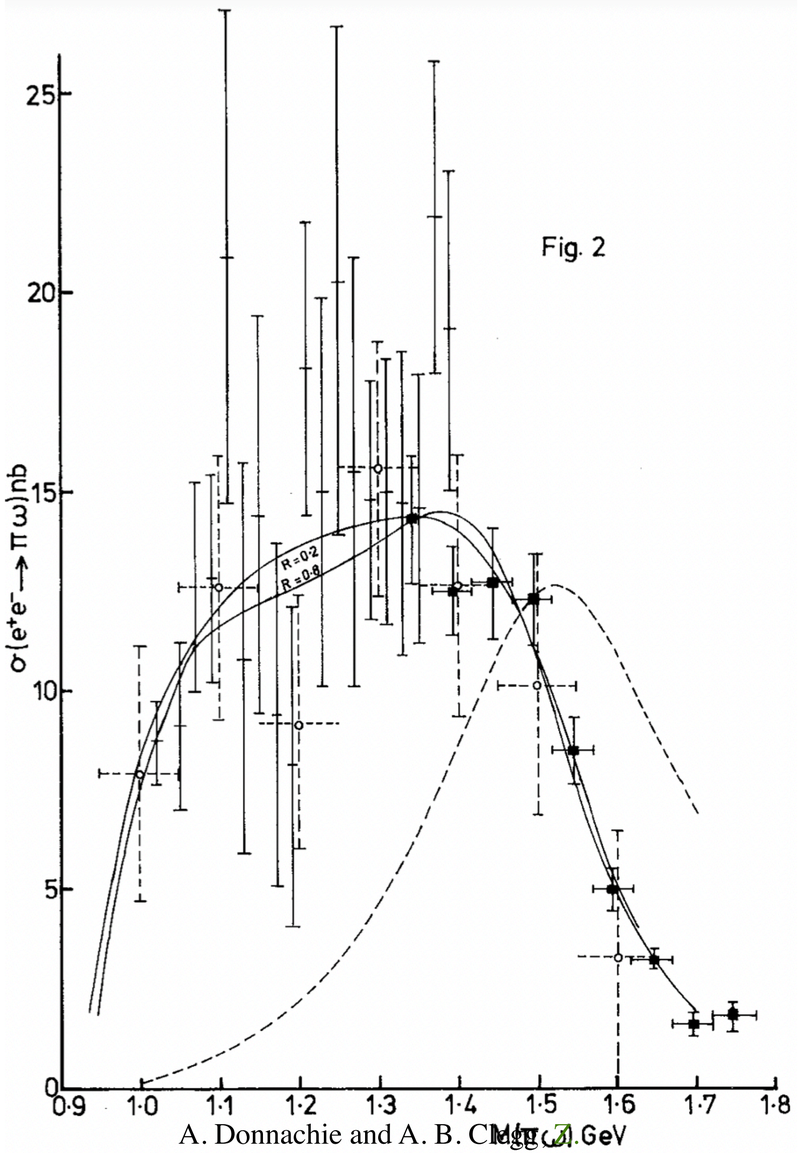
ω and ϕ states separated via decay channels $\pi\pi\pi$ vs $K\bar{K}$ (OZI)



Excited vector states picture:

I=1 : There appears to be two states $\rho(1450)$, $\rho(1700)$

I=0: Three states $\omega(1420)$, $\omega(1650)$, $\phi(1680)$



A. Donnachie and A. B. Clegg, Z. Phys. C 51, 689 (1991)

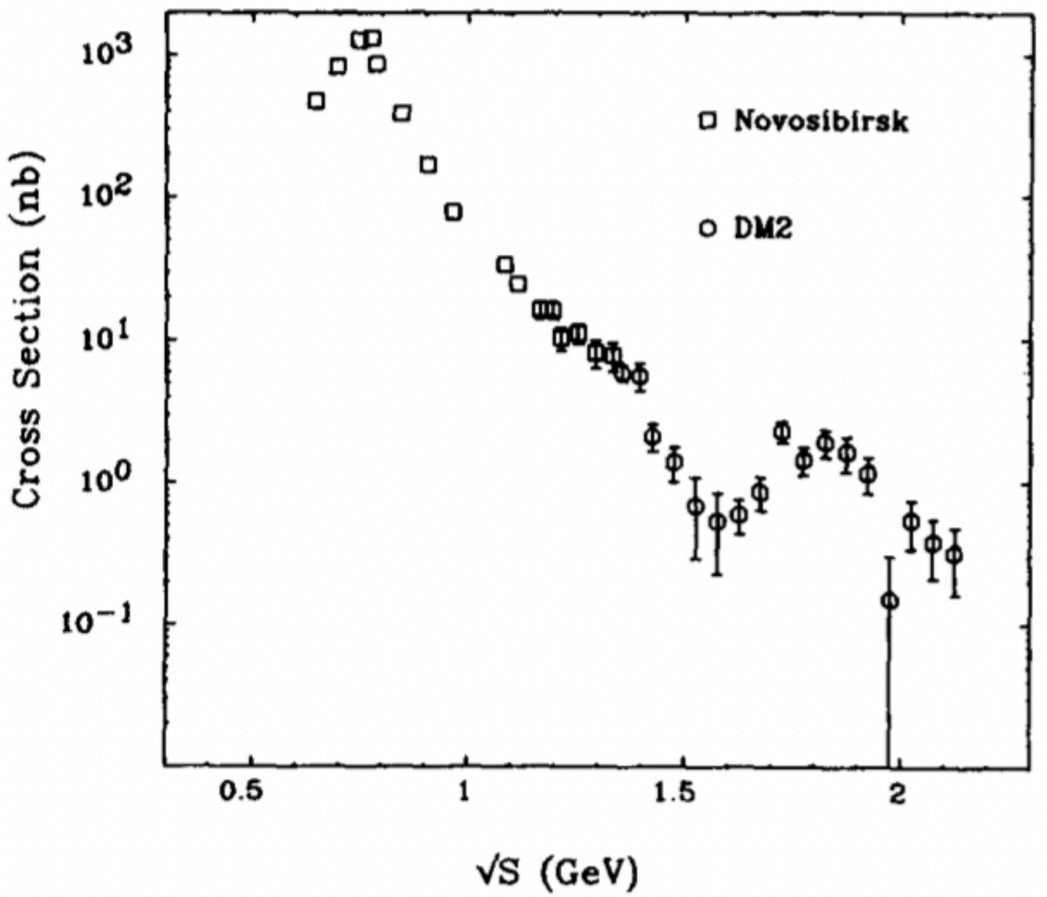
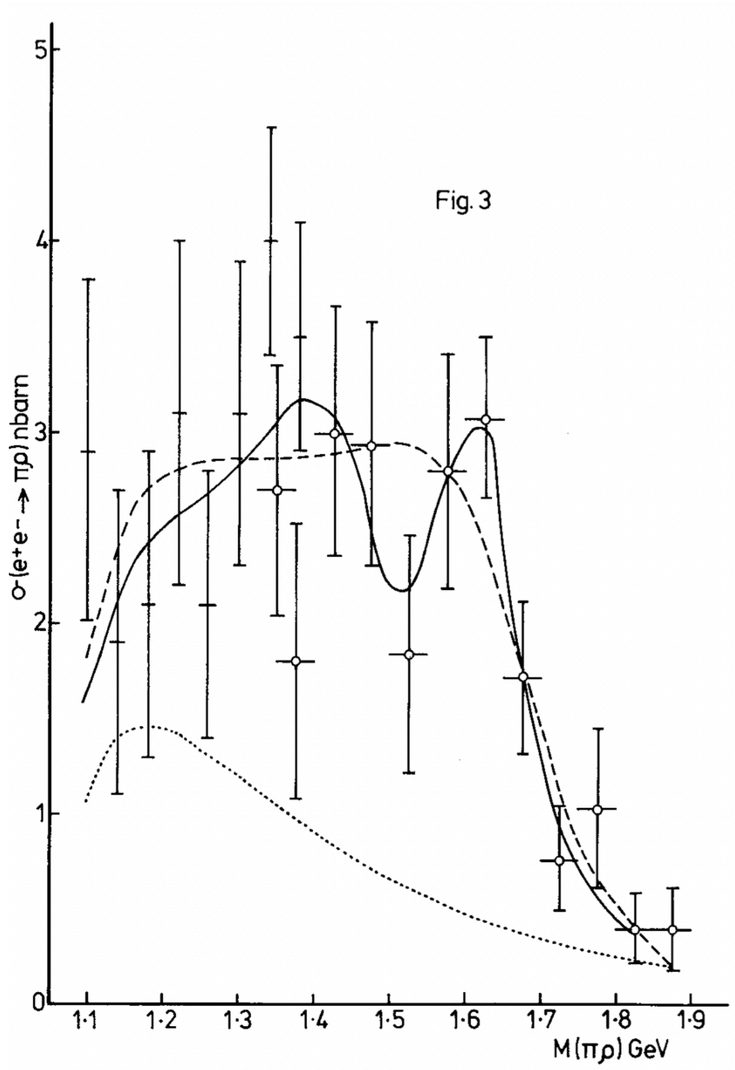


Fig. 5. $e^+e^- \to \pi^+\pi^-$ cross section versus $\sqrt{s}$. The Novosibirsk points are from ref. [2]. D. Bisello et al. (DM2), Phys. Lett. B 220, 321 (1989)



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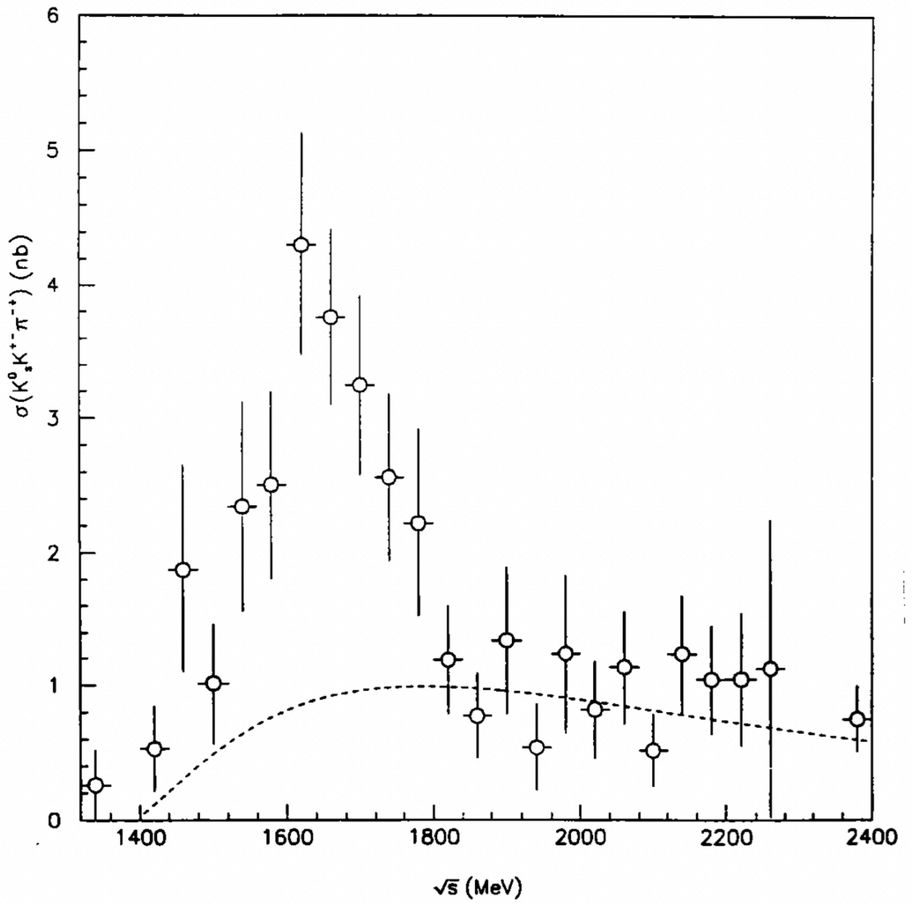


Fig. 2 $K_s^0 K^{\pm}\pi^{\mp}$ cross section. The dashed line shows the ρ, ω, ϕ tail contribution.

D. Bisello et al., Z. Phys. C 52, 227 (1991)

Experimental Status

Presence of two states in 1^{--} from quark model it is natural to interpret these states as a radial excitation in S-wave $[2^3S_1]$, and an orbital excitation in D-wave $[^3D_1]$ (or some linear combination of the two).

One would then expect three nearly degenerate D-wave states $[^3D_{1,2,3}]$.

	J^P
$\ell = 0$	1^-
$\ell = 1$	$(0,1,2)^+$
$\ell = 2$	$(1,2,3)^-$
...	...

What has been claimed in the PDG:

Isovector : $\rho(1400), \rho(1700), \rho_3(1690)$

Isoscalar: $\omega(1420), \omega(1650), \omega_3(1670)$ / $\phi(1680), \phi_3(1850)$.

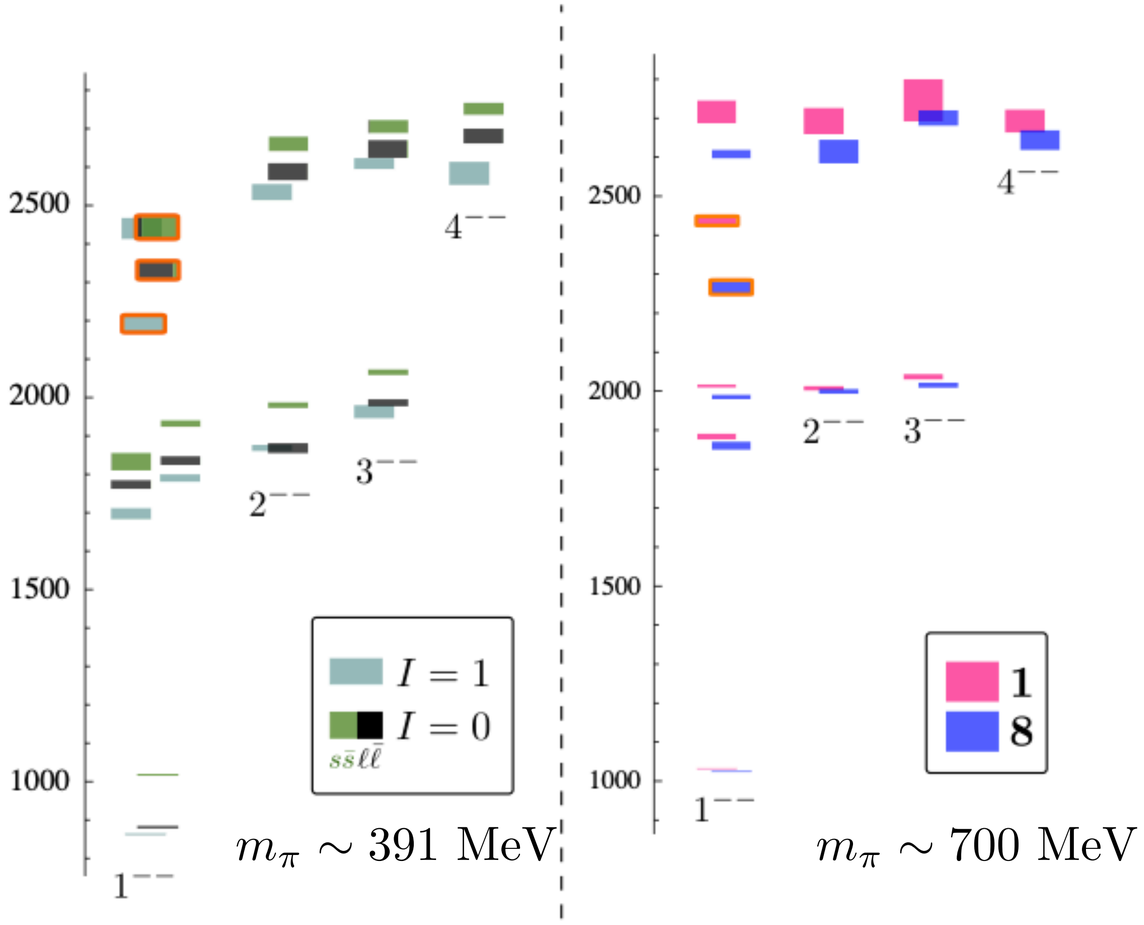
A Place to start

Energy spectrum calculation by *hadpsec* of states with basis of single meson operators

Great picture of states we expect

These states actually feature as resonances.

We can do better.



J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas (Hadron Spectrum), Phys. Rev. D88, 094505 (2013), arXiv:1309.2608 [hep-lat].

Outline

These J^{--} states are resonances which appear as poles in the scattering amplitude

$\Rightarrow$ need a reliable description of the scattering amplitude.

$\Rightarrow$ we make use of a variety of parameterizations that satisfy basic properties (i.e. unitarity, analyticity) and fit to the finite volume spectrum via Luscher's Quantization Condition.

$\Rightarrow$ calculate large basis of correlation functions in LQCD to produce finite volume spectrum

Finite-volume spectrum $\leftrightarrow$ scattering amplitude.

Lattice QCD

Introduces three fundamental changes:

Lattice spacing → does not likely play a big role

Lattice volume → tool we need for scattering

Quark mass → feature we make use of increasing pion mass

Compute correlation functions $C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$ to extract the finite volume spectrum

$$\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$$

Lattice QCD

Optimized operator constructed from applying the eigenvectors extracted from applying the variational method $h^\dagger = \sum_i v_i O_i$

Finite volume spectrum $\Rightarrow C_{ij}(t) = \sum_\alpha \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$

Momentum is quantized $\vec{p} = \frac{2\pi}{L}\vec{n}$

Meson-meson operators: $\sum_{\vec{p}_1 + \vec{p}_2 = \vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

No interactions

$$E = \sqrt{m_1^2 + \left(\frac{2\pi \vec{n}_1}{L}\right)^2} + \sqrt{m_2^2 + \left(\frac{2\pi \vec{n}_2}{L}\right)^2}$$

Scattering in a finite volume

$2 \to 2$ scattering amplitudes are related to the finite volume spectrum via Luscher's quantization condition: $\det\left[1 + i\boldsymbol{\rho} \cdot \mathbf{t} \cdot \left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$

$\rho_i(E) = \dfrac{2k_i}{E}$ is diagonal matrix of the phase space

$t_{ij}(E)$ is the symmetric scattering matrix satisfying unitarity $\mathrm{Im}(t_{ij}^{-1}) = -i\delta_{ij}\rho_i$

$M_{ij}(E, L)$ contains the finite volume pieces

Calculations have been done for elastic, coupled channel, and coupled channels with spinning particles.

Elastic scattering in a finite volume

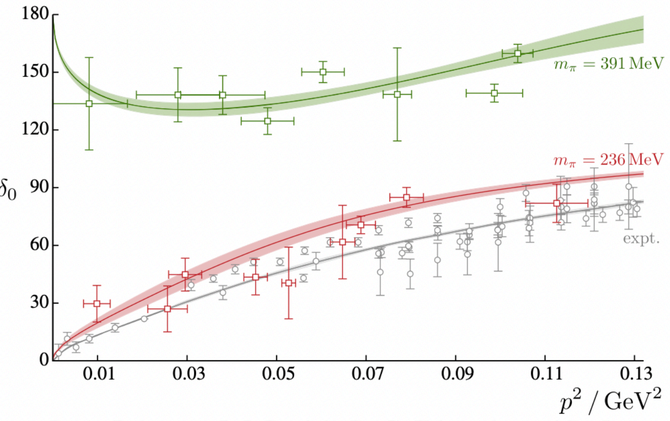
$$\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$$

Can describe the amplitude through a phase shift.

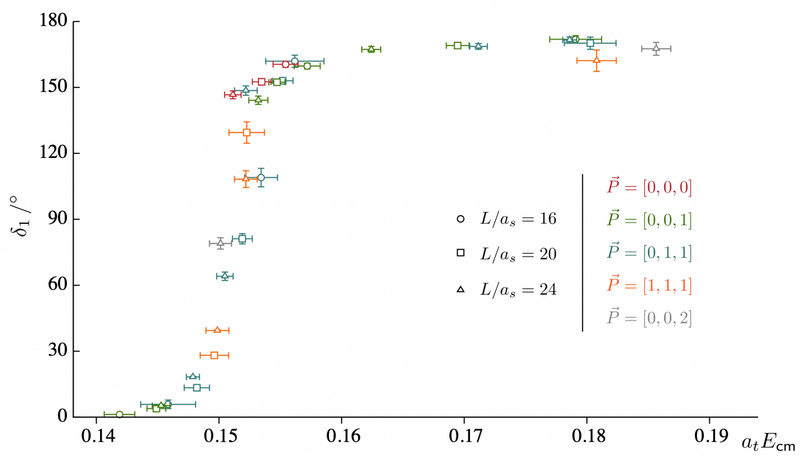
For a single elastic channel (one partial wave) $\Rightarrow t(E) = \dfrac{1}{\rho(\cot\delta(E) - i)}$

Reduces to a single equation $cot\delta(E) = M(E, L)$

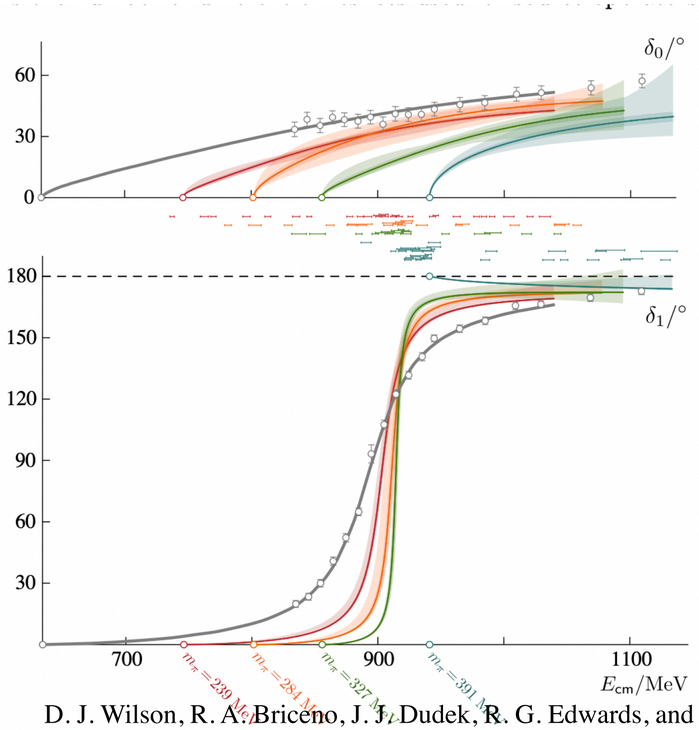
Has been done in numerous examples



R. A. Briceño, J. J. Dudek, R. G. Edwards, and D. J. Wilson, Phys. Rev. Lett. 118, 022002 (2017), arXiv:1607.05900 [hep-ph]



J. J. Dudek, R. G. Edwards, and C. E. Thomas, Phys. Rev. D86, 034031 (2012), arXiv:1203.6041 [hep-ph]



D. J. Wilson, R. A. Briceno, J. J. Dudek, R. G. Edwards, and C. E. Thomas, Phys. Rev. Lett. 123, 042002 (2019), arXiv:1904.03188 [hep-lat].

Coupled-channel

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

$\mathrm{Im}\boldsymbol{I} = -\boldsymbol{\rho}$

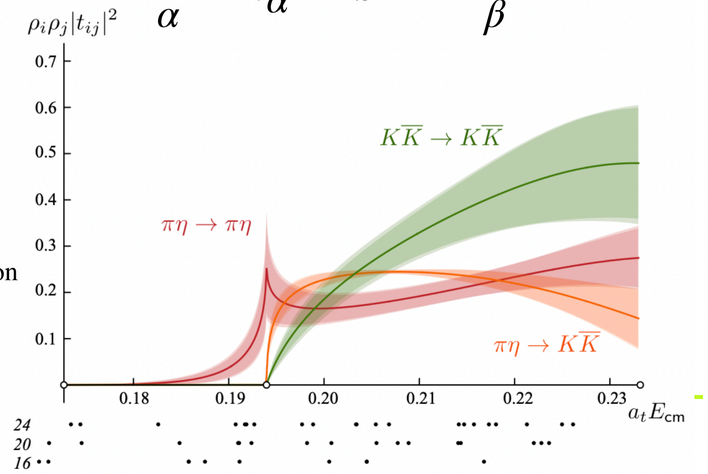
$$\det\left[1 + i\boldsymbol{\rho} \cdot \mathbf{t} \cdot \left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$$

More challenging, can no longer just write in terms of a single phase shift.

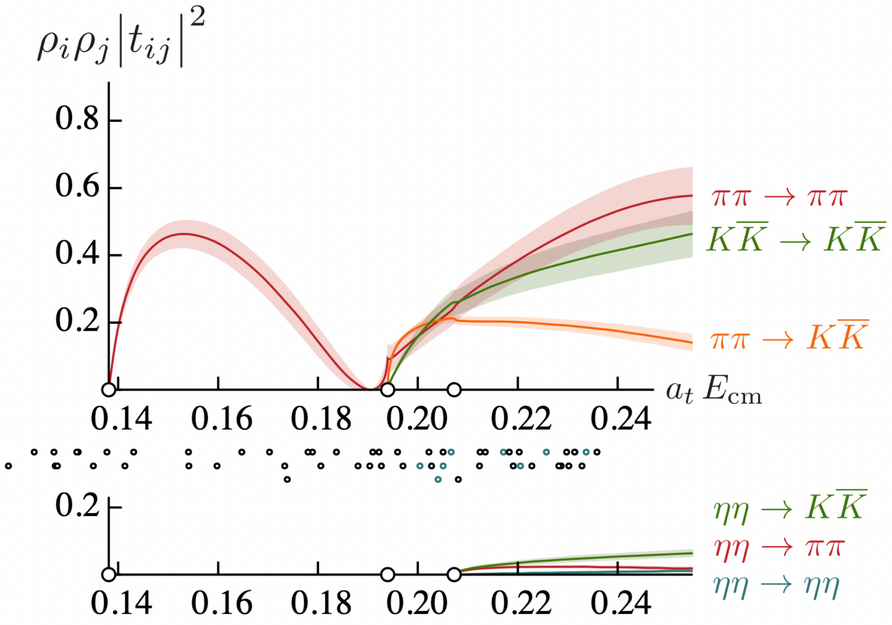
Solutions follow from K-matrix parameterizations of the amplitude : $\mathbf{t}^{-1} = \mathbf{K}^{-1} + \mathbf{I}$

K-matrix real and symmetric $K_{ij}(s) = \sum_{\alpha} \frac{g_i^{(\alpha)} g_j^{(\alpha)}}{m_\alpha^2 - s} + \sum_{\beta} s^{\beta} \gamma_{ij}$

⇒guarantees unitarity



J. J. Dudek, R. G. Edwards, and D. J. Wilson (Hadron Spectrum), Phys. Rev. D93, 094506 (2016), arXiv:1602.05122 [hep-ph]



R. A. Briceno, J. J. Dudek, R. G. Edwards, and D. J. Wilson, Phys. Rev. D97, 054513 (2018), arXiv:1708.06667 [hep-lat]

Coupled channel with nonzero spin

Orbital and angular momentum couple $\boldsymbol{\ell} \otimes \boldsymbol{S} \to \boldsymbol{J}$

Can use K-matrix to handle this (ex. 0^{-+},1^{--} scattering in $J^P = 1^+$)

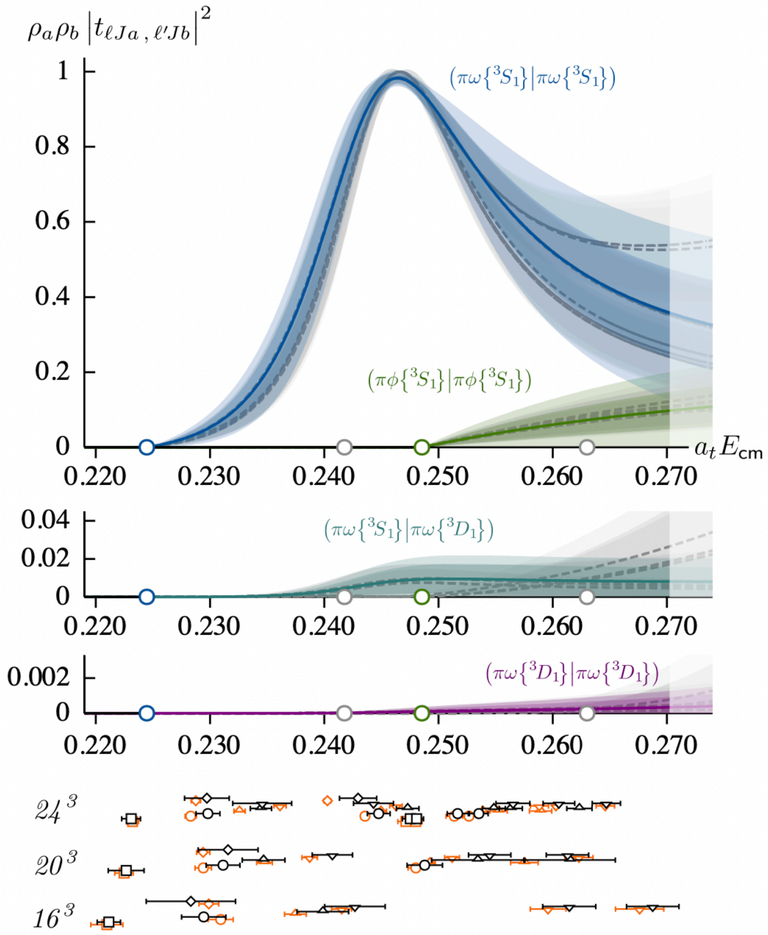
$$K_{1^+} = \begin{pmatrix} \{^3S_1|^3S_1\} & \{^3S_1|^3D_1\} \\ \{^3S_1|^3D_1\} & \{^3D_1|^3D_1\} \end{pmatrix}$$

ℓ	J^P
0	1^+
1	$(0,1,2)^-$
2	$(1,2,3)^+$
3	$(2,3,4)^-$
...	

Done in both non-resonant and resonant systems:

"Dynamically-coupled partial-waves in $\rho\pi$ isospin-2 scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek

"The b_1 resonance in coupled $\pi\omega, \pi\phi$ scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek



SU(3) Flavor Ensembles

This project studies the isoscalar J^{--}excited mesons at the SU(3) flavor point

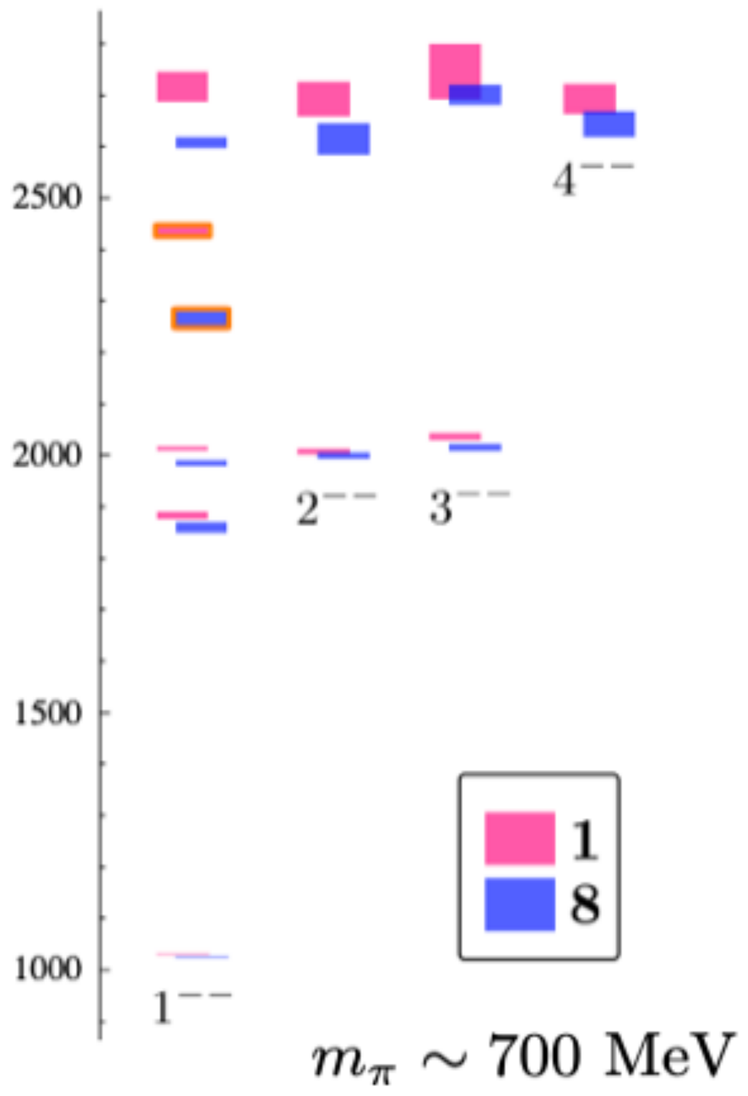
Advantages:

⇒Heavier light quark masses allow us to probe higher energy regions:

first three-particle threshold gets moved higher up

resonant states at lighter quark masses feature as stable particles

⇒Fewer channels (ex. $\pi, K, \bar{K}, \eta$ are all just η^8)



This work (excited J^{--} resonances...)

Old:

- Elastic scattering ✓
- Coupled channel ✓
- Spinning hadrons ✓

New:

- Multiple resonances in the same partial waves and irreps ⇒ a proper test of the finite volume formalism

SU(3) Flavor

Two neutral members basis states $I = I_z = Y = 0$

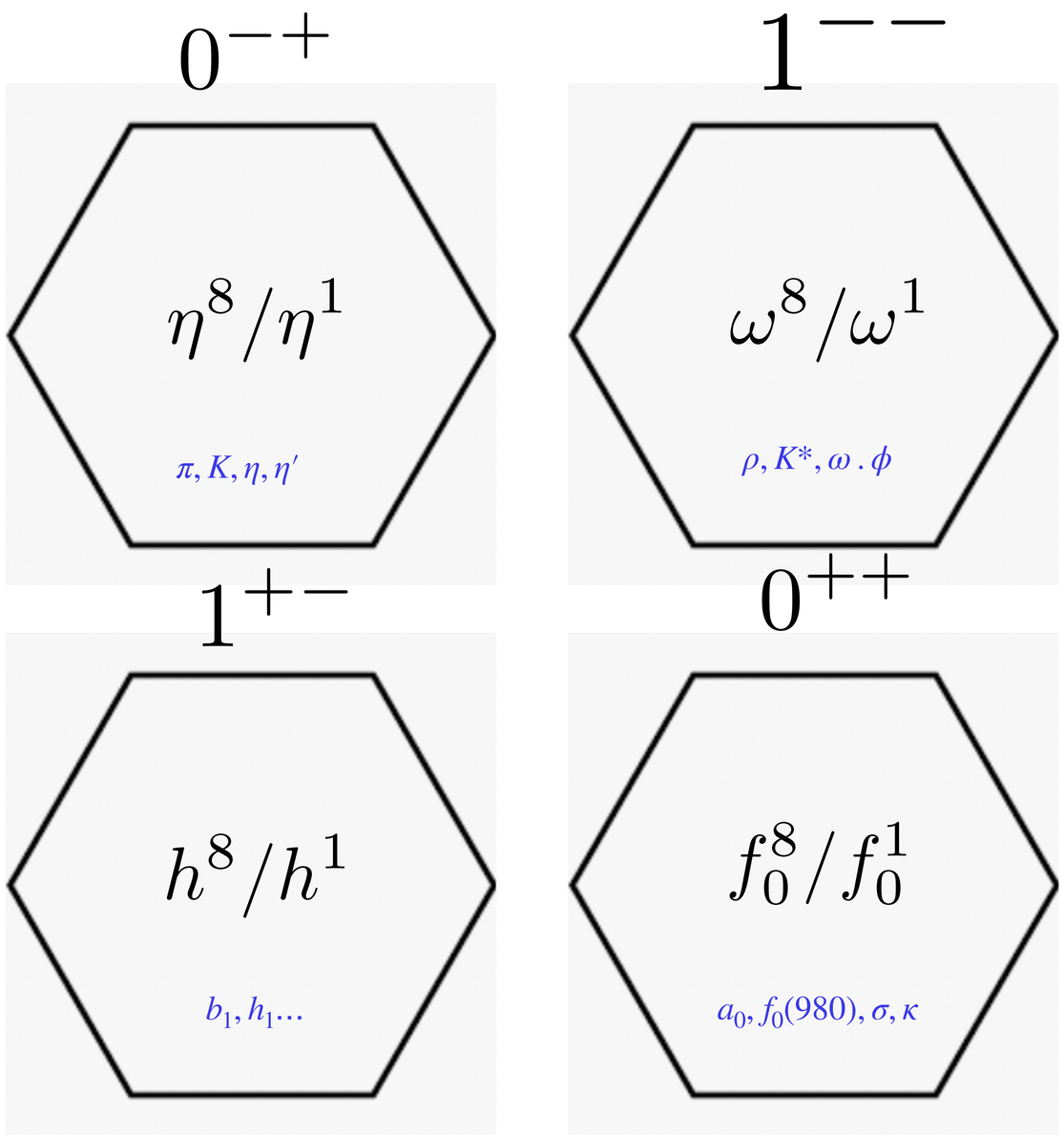
$$|\mathbf{1}\rangle = \frac{1}{\sqrt{3}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle\right)$$
$$|\mathbf{8}\rangle = \frac{1}{\sqrt{6}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle\right)$$

Pseudoscalar have small mixing angle from SU(3) states $\sim -10^\circ$

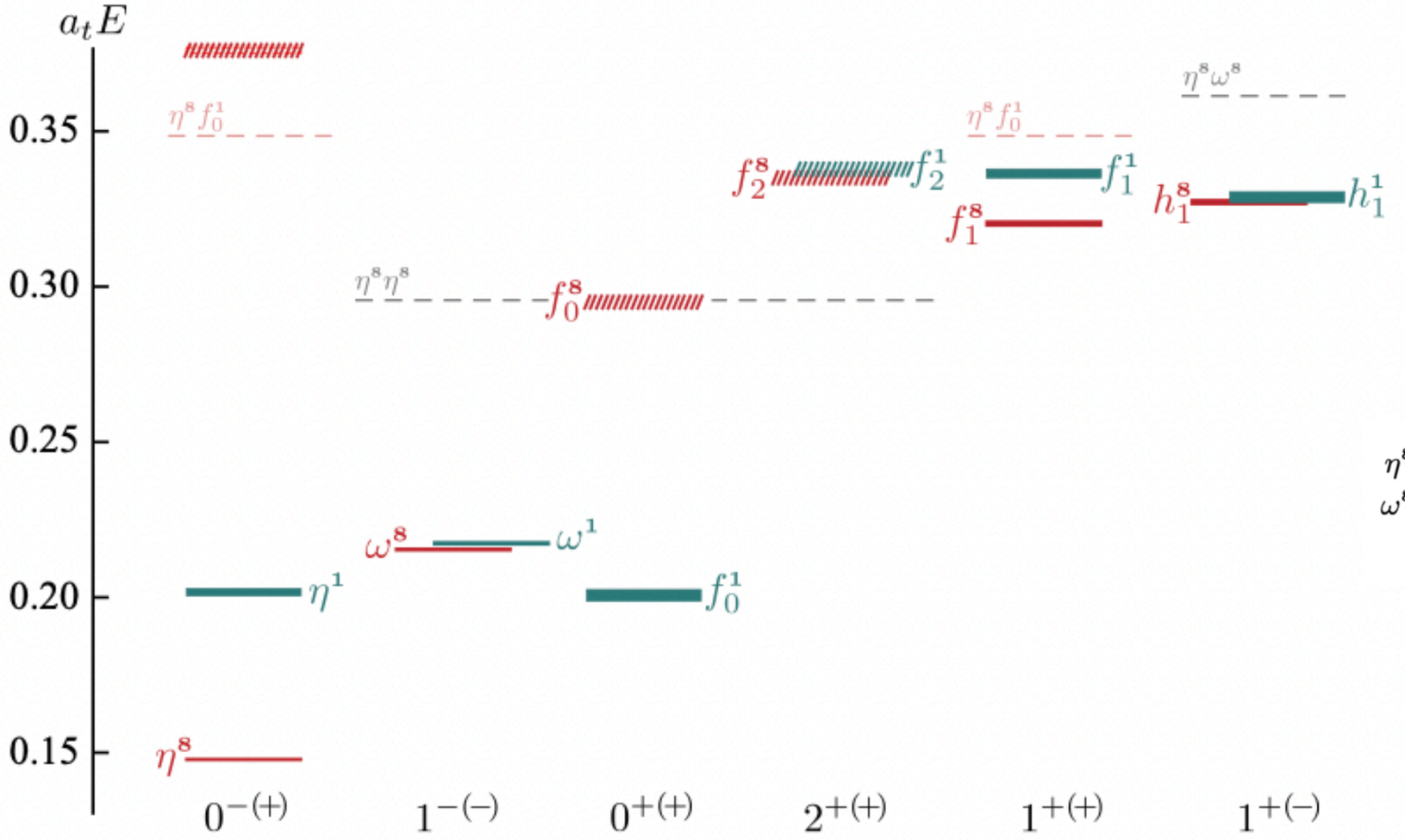
$$|\eta\rangle \sim |\eta^8\rangle \qquad |\eta'\rangle \sim |\eta^1\rangle$$

Mixing splits into light and strange quarks (OZI)

$$|\omega\rangle \sim \tfrac{1}{\sqrt{2}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle\right) \qquad |\phi\rangle \sim |\bar{s}s\rangle$$



SU(3) Flavor



η^8	0.1478(1)	η^1	0.2017(11)
ω^8	0.2154(2)	ω^1	0.2174(3)
		f_0^1	0.2007(18)

Channels in SU(3) Flavor

Conventional $\bar{q}q$ mesons live in either a singlet ($\bar{3} \otimes 3 \to 1$) or octet ($\bar{3} \otimes 3 \to 8$) representations.

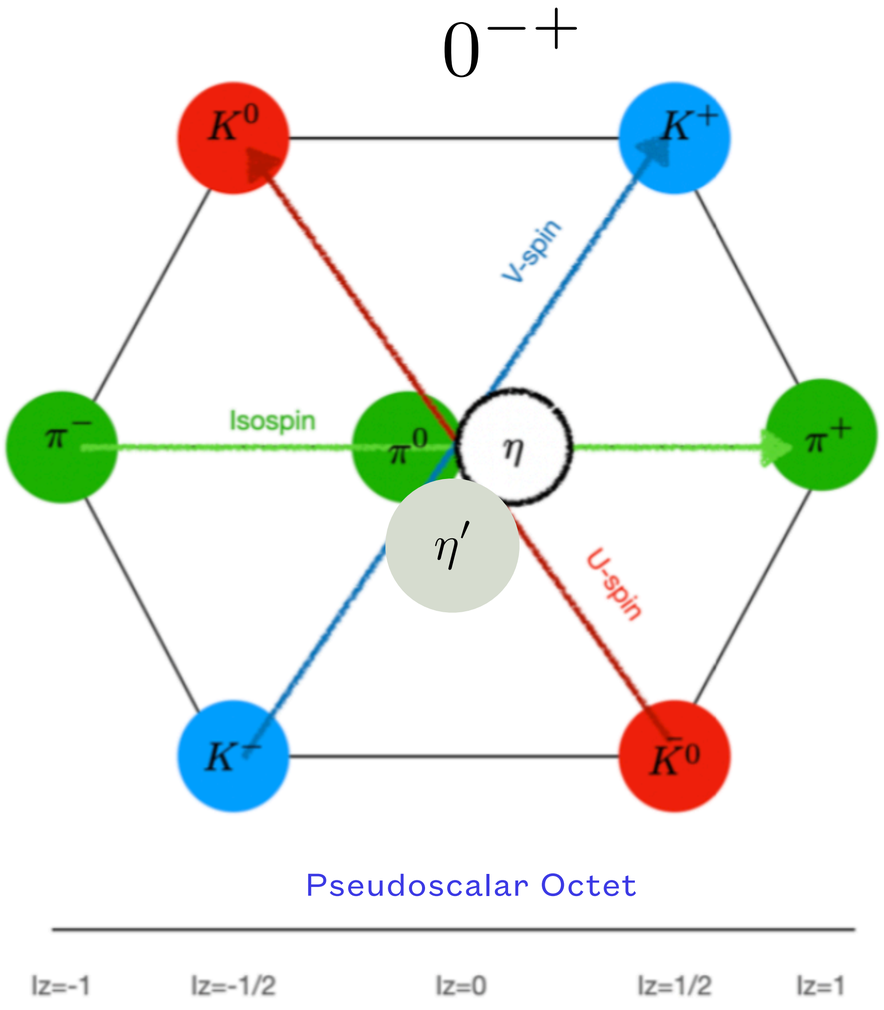
Two ways to project to flavor singlet $8 \otimes 8 \to 1$, and trivially $1 \otimes 1 \to 1$.

Charge conjugation in neutral member of the octet $|I = I_z = Y = 0\rangle$ for $8 \otimes 8 \to 1$:

$$\hat{C}(|8_1, C_1\rangle \otimes |8_2, C_2\rangle) \to C_1 C_2 (|8_1, C_1\rangle \otimes |8_2, C_2\rangle)$$

$\Rightarrow$ channels with C=- :
$\eta^8(0^{-+})\omega^8(1^{--}), f_0^1(0^{++})\omega^1(1^{--}), \eta^1(0^{-+})\omega^1(1^{--})$

$\Rightarrow$ can't have identical particles with C=-



Channels

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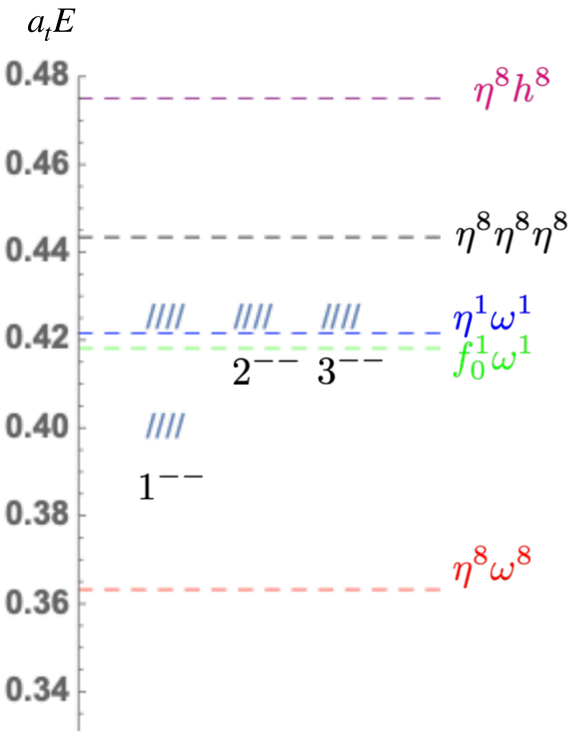
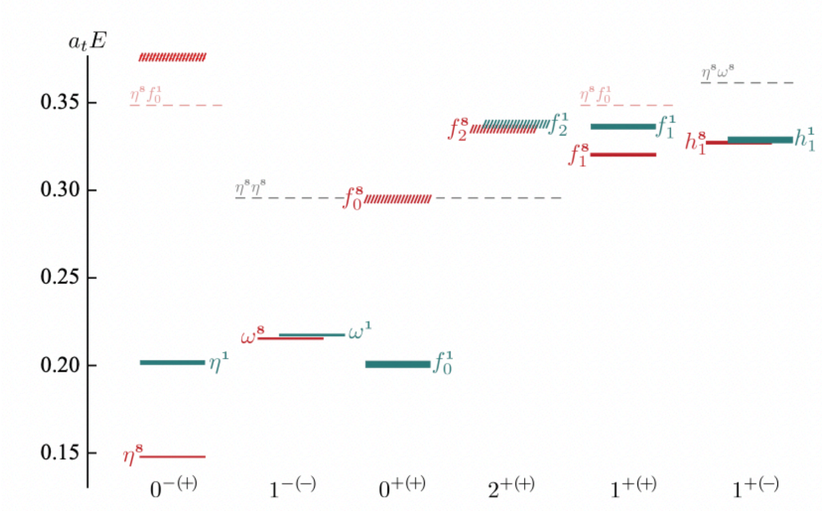


J=1: $\eta^8\omega^8\{^3P_1\}, f_0^1\omega^1\{^3S_1, ^3D_1\}, \eta^1\omega^1\{^3P_1\}$

J=2: $\eta^8\omega^8\{^3P_2, ^3F_2\}, f_0^1\omega^1\{^3D_2\}, \eta^1\omega^1\{^3P_2, ^3F_2\}$

J=3: $\eta^8\omega^8\{^3F_3\}, f_0^1\omega^1\{^3D_3, ^3G_3\}, \eta^1\omega^1\{^3F_3\}$

Threshold behavior $t_\ell(s) \sim k^{2\ell}$ will suppress some channels in this region



$\Rightarrow$ J=1: $\eta^8\omega^8\{^3P_1\}, f_0^1\omega^1\{^3S_1\}, \eta^1\omega^1\{^3P_1\}$

$\Rightarrow$ J=2: $\eta^8\omega^8\{^3P_2, ^3F_2\}, \eta^1\omega^1\{^3P_2\}$

$\Rightarrow$ J=3: $\eta^8\omega^8\{^3F_3\}$

Lattice QCD

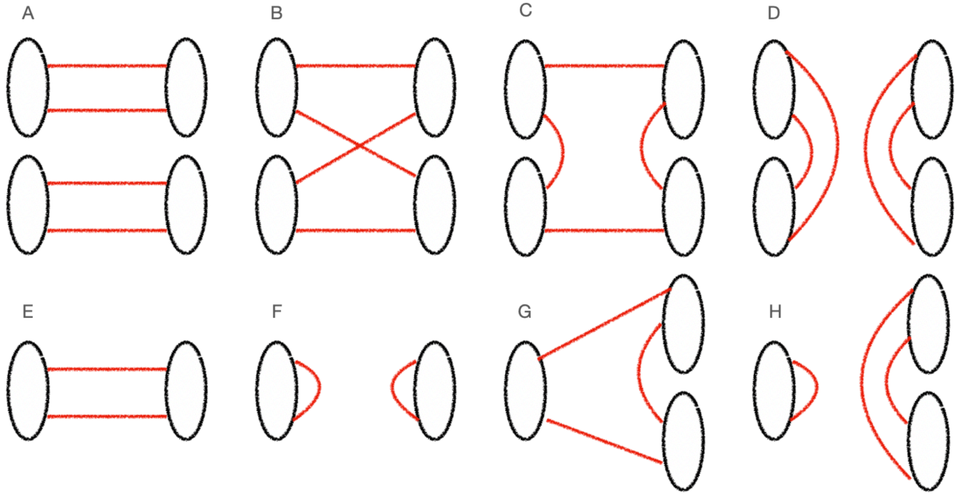
Finite volume spectrum $\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$

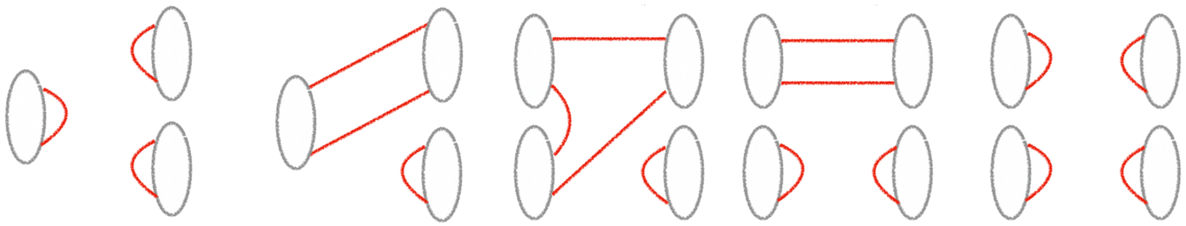
Meson-meson operators: $\sum_{\vec{p}_1 + \vec{p}_2 = \vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

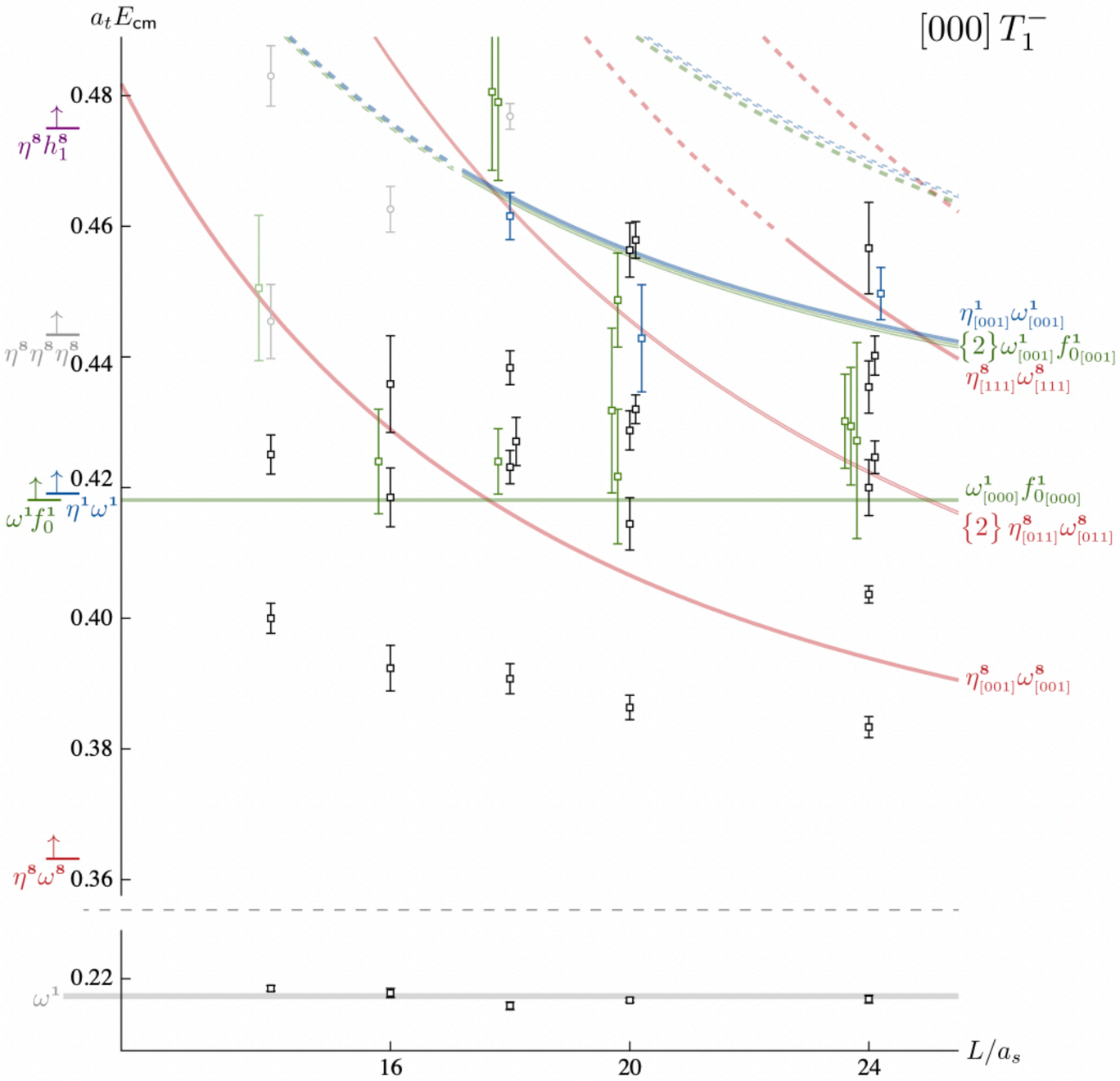
Will include $\eta^8(\vec{p_1})\omega^8(\vec{p_2}), \eta^1(\vec{p_1})\omega^1(\vec{p_2}), f_0^1(\vec{p_1})\omega^8(\vec{p_2})$

$\eta^8\omega^8$



$\eta^1\omega^1 / f_0^1\omega^1$





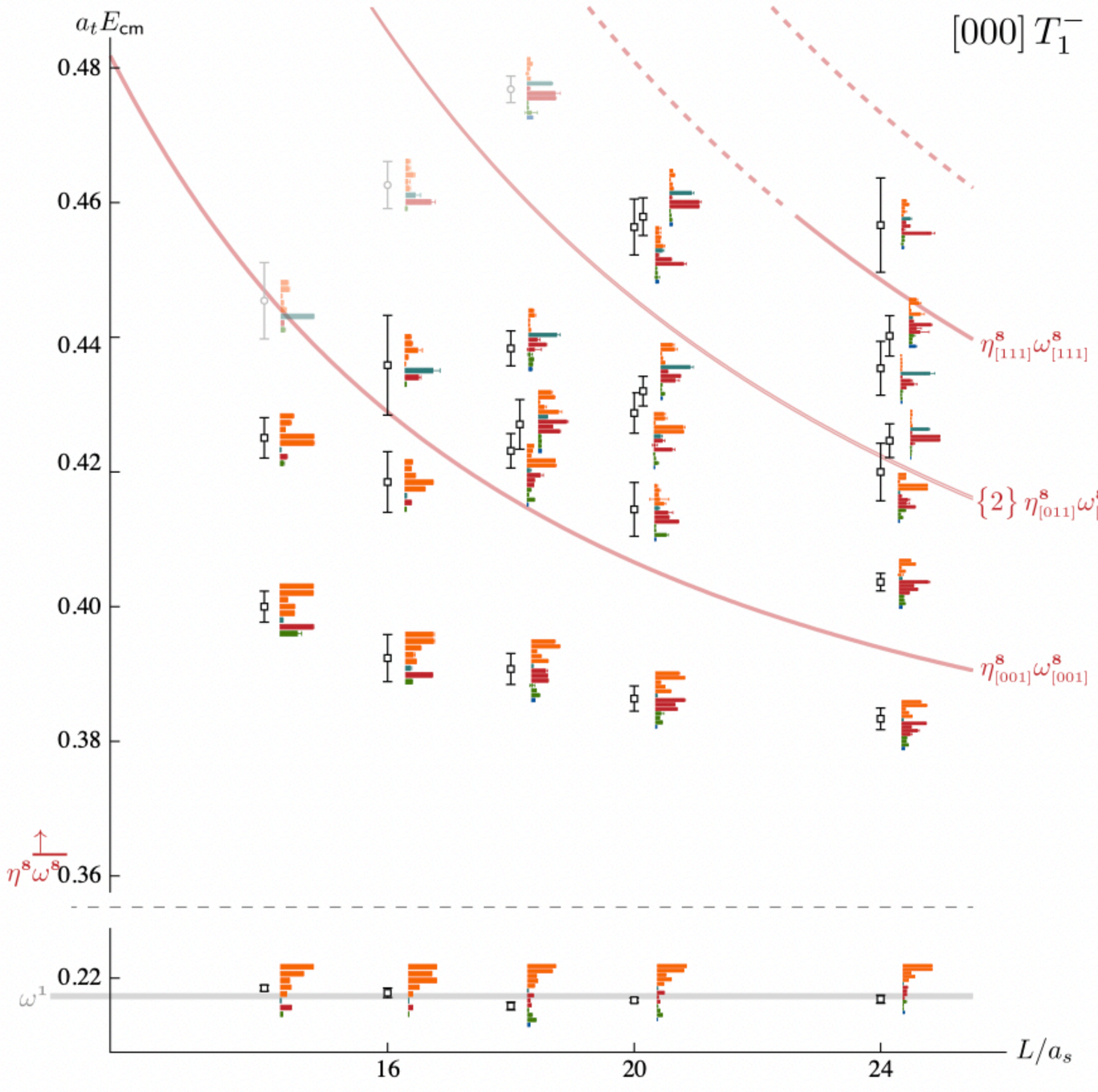
$J^P = (1,3,\ldots)^-$

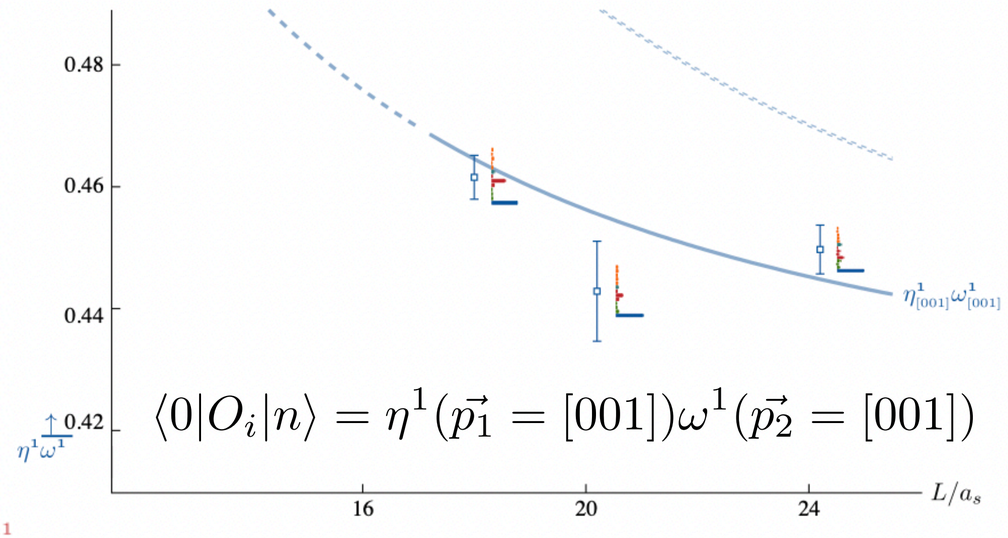
Three resonances in a single irrep.

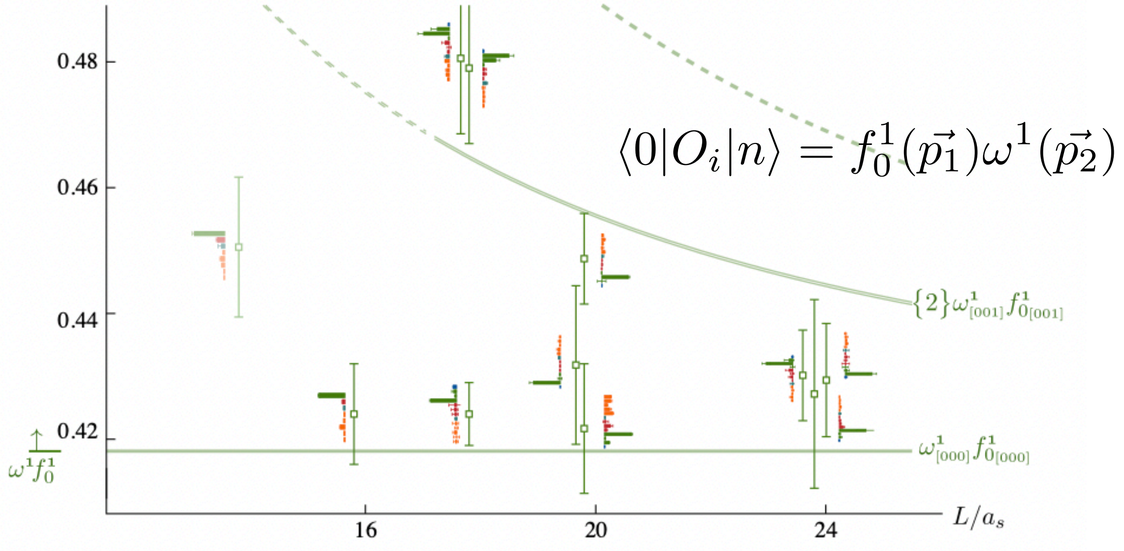
$$\Rightarrow \rho\{^3 2S_1\}, \rho\{^3 D_1\}, \rho\{^3 D_3\}$$

Much messier than what we are typically used to working with.

Appears to be a decoupling within the heavier channels $f_0^1\eta^1, \eta^1\omega^1$


$a_t E_{\rm cm}$
[000] T_1^-
0.48
0.46
0.44
0.42
0.40
0.38
0.36
$\eta^8\omega^8$
$\eta^8_{[111]}\omega^8_{[111]}$
$\{2\}\ \eta^8_{[011]}\omega^8_{[011]}$
$\eta^8_{[001]}\omega^8_{[001]}$
0.22
ω^1
16
20
24
L/a_s


0.48
0.46
0.44
0.42
$\eta^1\omega^1$
$\eta^1_{[001]}\omega^1_{[001]}$
$\langle 0|O_i|n\rangle = \eta^1(\vec{p_1} = [001])\omega^1(\vec{p_2} = [001])$
16
20
24
L/a_s


0.48
0.46
0.44
0.42
$\omega^1 f_0^1$
$\langle 0|O_i|n\rangle = f_0^1(\vec{p_1})\omega^1(\vec{p_2})$
$\{2\}\omega^1_{[001]}f^1_{0[001]}$
$\omega^1_{[000]}f^1_{0[000]}$
16
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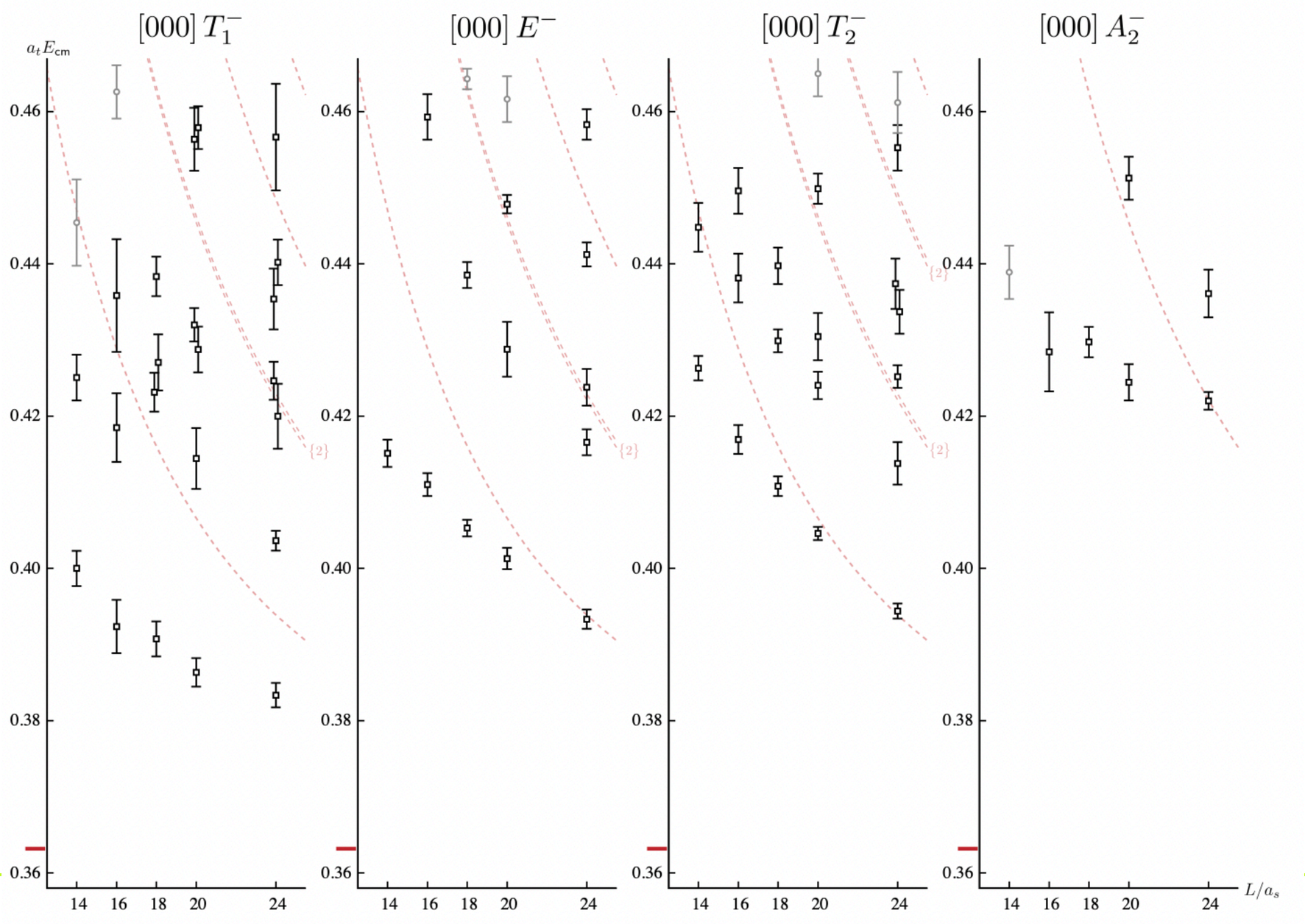
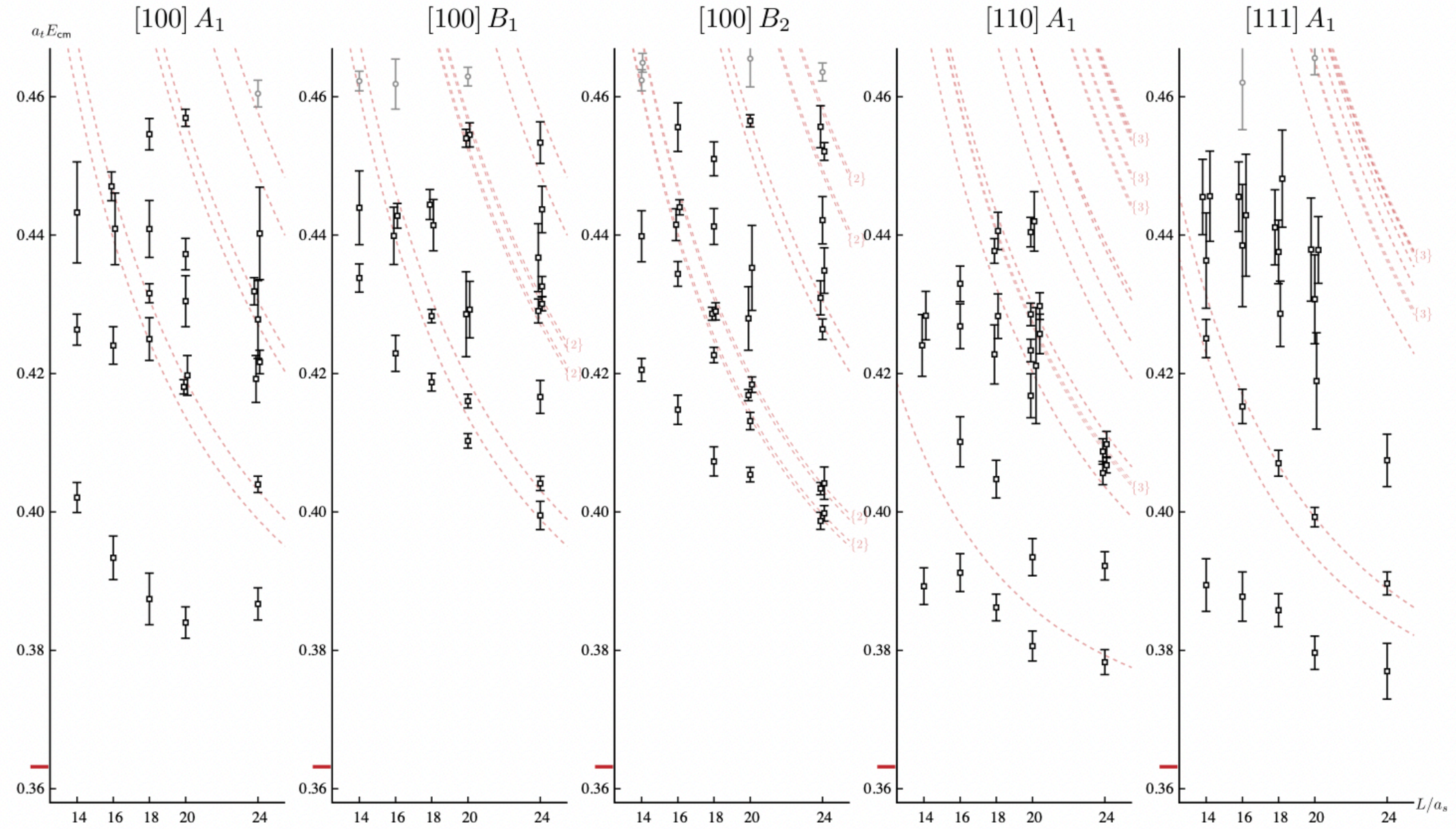
L/a_s



$J^P = (1, 3, ...)^-$ $J^P = (2, ...)^-$ $J^P = (2, 3, ...)^-$ $J^P = (3, ...)^-$



$J^P = 0^+, 1^-, 2^+, 3^-, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^+, 3^\pm, \ldots$

Plan of attack

Carry forward with elastic scattering in $\eta^8\omega^8$

$\Rightarrow$ fit to amplitudes of J=2,3 simultaneously ($T_2^-[000], E^-[000], A_2^-[000], B_1[001], B_2[001]$)

$\Rightarrow$ fix J=3 amplitude and fit for the J=1 amplitude ($T_1^-[000], A_1[001], A_1[111]$)

Perform analysis of $\eta^1\omega^1, f_0^1\omega^1$ as if channels were non-resonant and totally decoupled

Later relax the elastic assumption and allow $\eta^1\omega^1$ to couple

$\Rightarrow$ we find the change is rather insignificant

Parameterizations, J=2,3

J=2 dynamically coupled in P- and F-waves

$$K_{ij} \to (2k_i)^\ell K_{ij}^{\ell\ell'} (2k_j)^{\ell'}$$

	J^P
$\ell = 0$	1^+
$\ell = 1$	$(0, 1, 2)^-$
$\ell = 2$	$(1, 2, 3)^+$
$\ell = 3$	$(2, 3, 4)^-$
...	...

Can handle this with the K-matrix $\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$

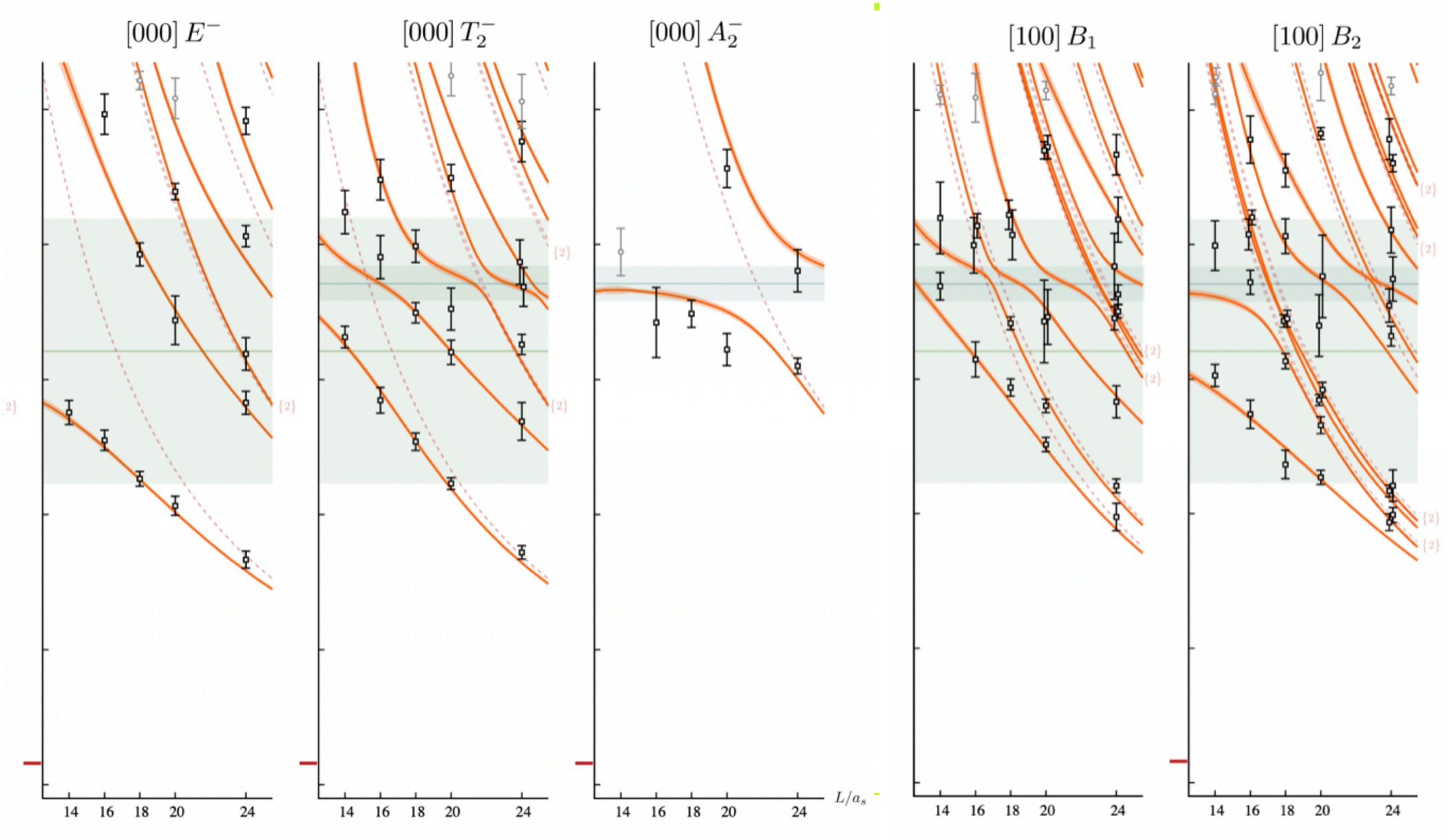
$$K_{J=2} = \begin{bmatrix} ({}^3P_2|{}^3P_2) & ({}^3P_2|{}^3F_2) \\ ({}^3P_2|{}^3F_2) & ({}^3F_2|{}^3F_2) \end{bmatrix}$$

J=3 Breit-Wigner parameterization

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

Im$\boldsymbol{I} = -\boldsymbol{\rho}$


[000] E^-
[000] T_2^-
[000] A_2^-
[100] B_1
[100] B_2
14
16
18
20
24
L/a_s
{2}

$2^{--}, 3^{--}$

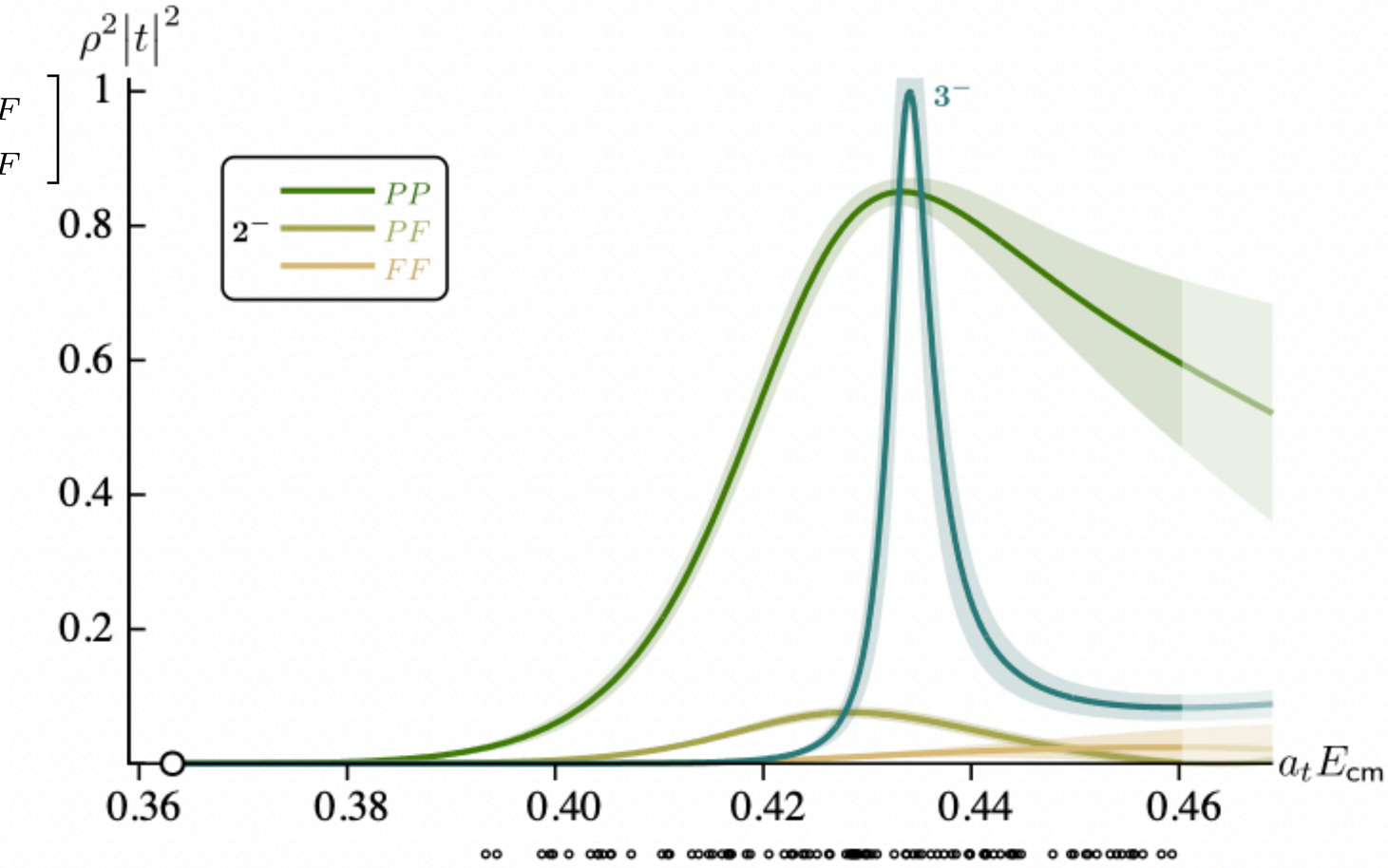
$$K_{J=2} = \frac{1}{m_R^2 - s} \begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & \gamma_{FF} \end{bmatrix}$$

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$

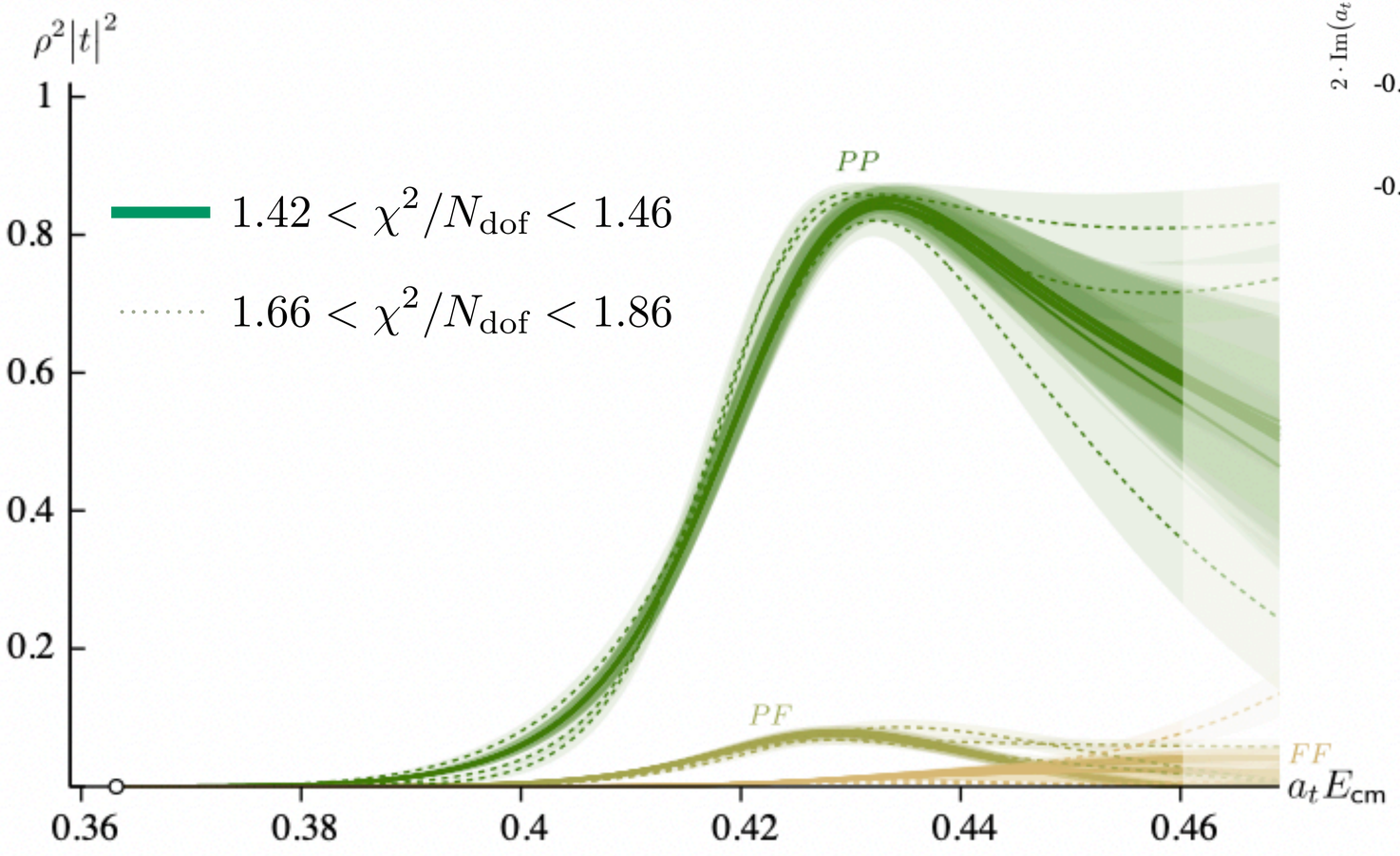
$$\begin{array}{l} J=2 \left\{ \begin{array}{l} m = 0.4322(15) \cdot a_t^{-1} \\ g_P = 0.753(37) \\ g_F = -4.13(29) \cdot a_t^2 \\ \gamma_{PP} = 0.1(33) \cdot a_t^2 \\ \gamma_{PF} = -110(17) \cdot a_t^4 \\ \gamma_{FF} = 143(322) \cdot a_t^6 \end{array} \right. \\ J=3 \left\{ \begin{array}{l} m = 0.4341(9) \cdot a_t^{-1} \\ g = 4.85(28) \cdot a_t^2 \end{array} \right. \end{array}$$

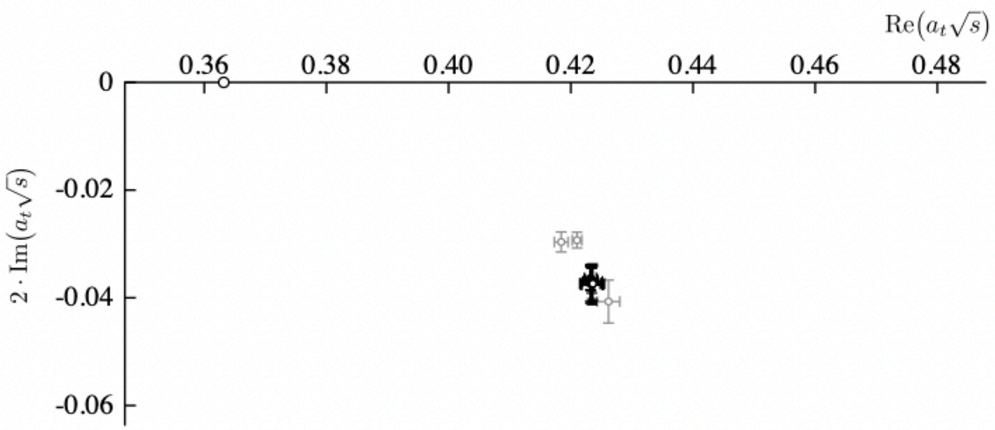
$$\begin{bmatrix} 1 & 0.31 & 0.29 & 0.13 & -0.37 & 0.31 & 0.19 & 0.07 \\ & 1 & -0.08 & -0.70 & 0.04 & 0.48 & 0.07 & -0.23 \\ & & 1 & 0.21 & -0.15 & -0.18 & -0.01 & -0.12 \\ & & & 1 & -0.34 & -0.34 & -0.16 & 0.23 \\ & & & & 1 & -0.23 & -0.03 & -0.05 \\ & & & & & 1 & 0.02 & 0.05 \\ & & & & & & 1 & -0.04 \\ & & & & & & & 1 \end{bmatrix}$$

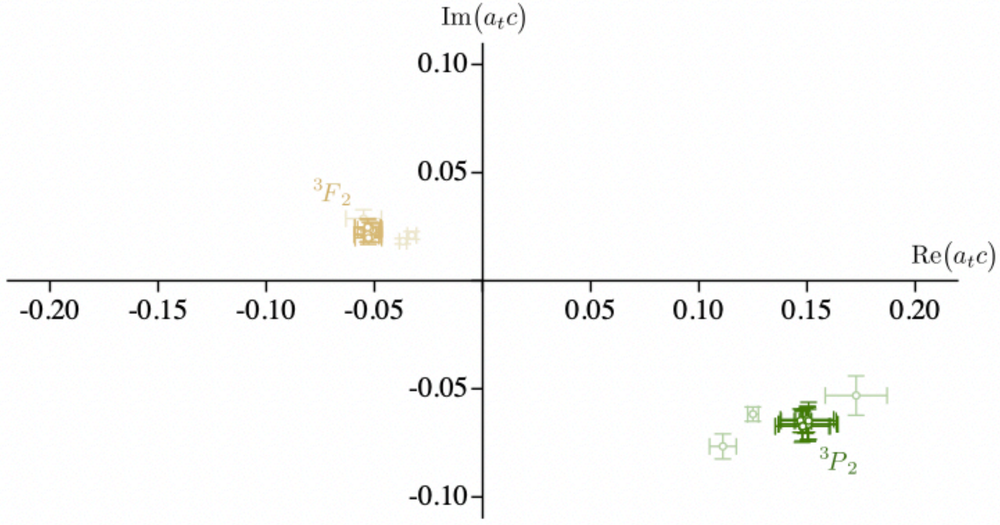
$$\chi^2/N_{\text{dof}} = \frac{120.3}{91-8} = 1.45$$



2^{--}


$\rho^2|t|^2$
$1.42 < \chi^2/N_{\rm dof} < 1.46$
$1.66 < \chi^2/N_{\rm dof} < 1.86$
PP
PF
FF
$a_t E_{\rm cm}$
1
0.8
0.6
0.4
0.2
0.36
0.38
0.4
0.42
0.44
0.46


$\mathrm{Re}(a_t\sqrt{s})$
$2\cdot\mathrm{Im}(a_t\sqrt{s})$
0.36
0.38
0.40
0.42
0.44
0.46
0.48
0
-0.02
-0.04
-0.06


$\mathrm{Im}(a_t c)$
$\mathrm{Re}(a_t c)$
$^3F_2$
$^3P_2$
0.10
0.05
-0.05
-0.10
-0.20
-0.15
-0.10
-0.05
0.05
0.10
0.15
0.20

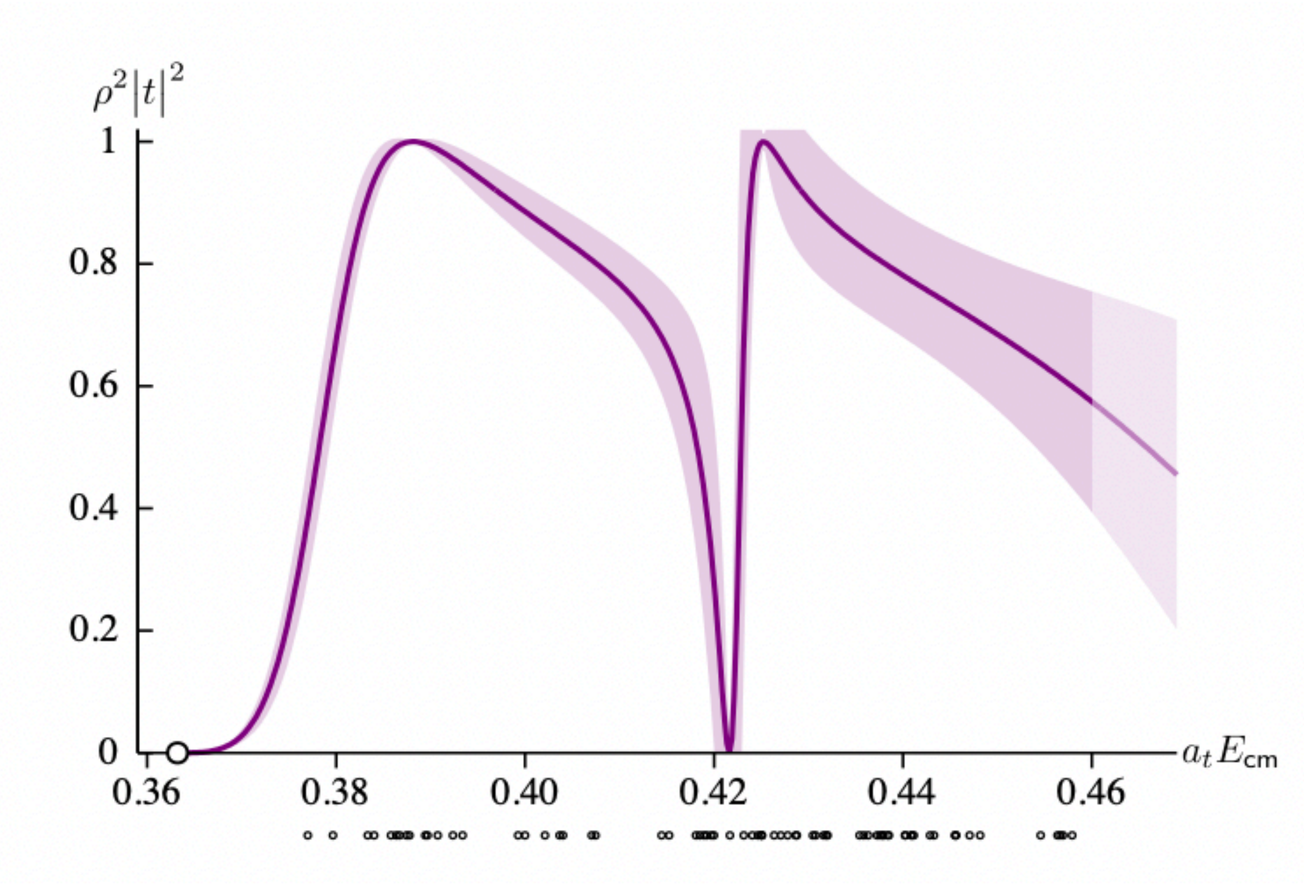
1^{--}

$$K_{J=1} = \frac{g_{\mathsf{a}}^2}{m_{\mathsf{a}}^2 - s} + \frac{g_{\mathsf{b}}^2}{m_{\mathsf{b}}^2 - s} + \gamma$$

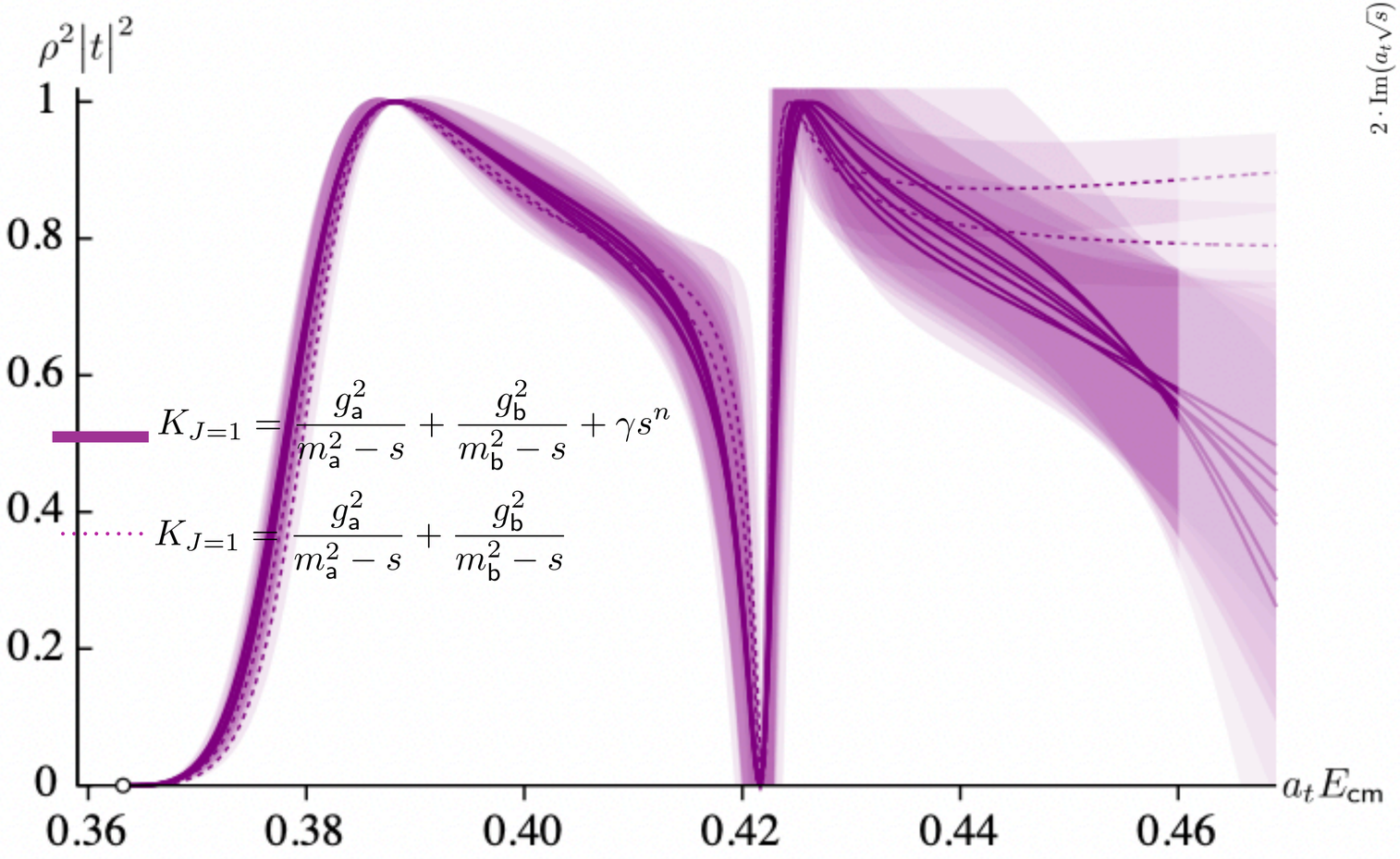
$$\begin{aligned} m_{\mathsf{a}} &= 0.3881(14) \cdot a_t^{-1} \\ g_{\mathsf{a}} &= 1.46(10) \\ m_{\mathsf{b}} &= 0.4242(17) \cdot a_t^{-1} \\ g_{\mathsf{b}} &= -0.36(13) \\ \gamma &= 20.9(86) \cdot a_t^2 \end{aligned}$$

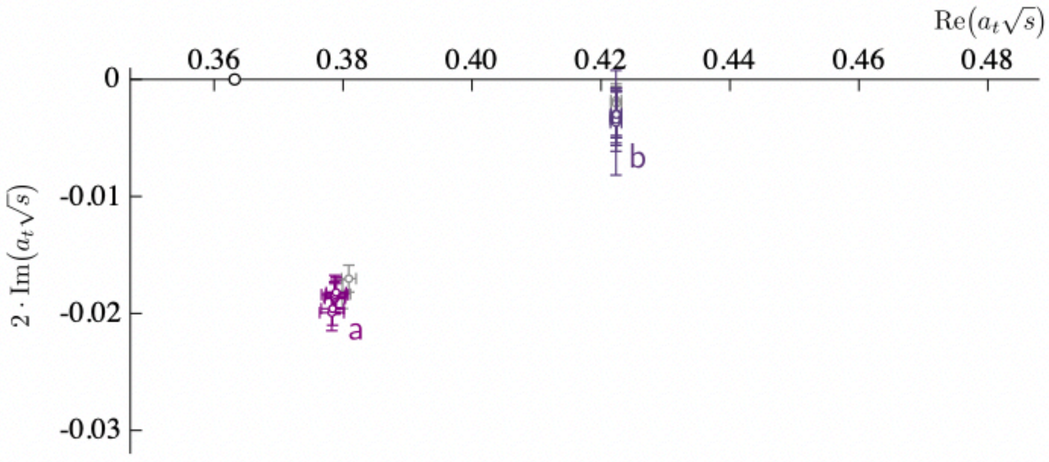
$$\begin{bmatrix} 1 & 0.08 & 0.43 & -0.33 & 0.19 \\ & 1 & 0.37 & -0.46 & 0.81 \\ & & 1 & -0.86 & 0.49 \\ & & & 1 & -0.57 \\ & & & & 1 \end{bmatrix}$$

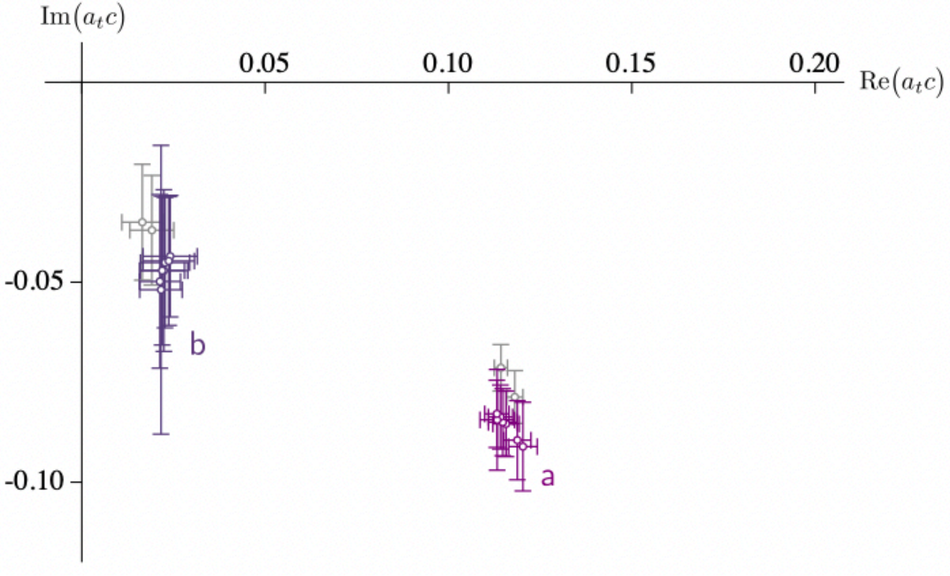
$$\chi^2/N_{\text{dof}} = \frac{91.3}{72-5} = 1.36$$



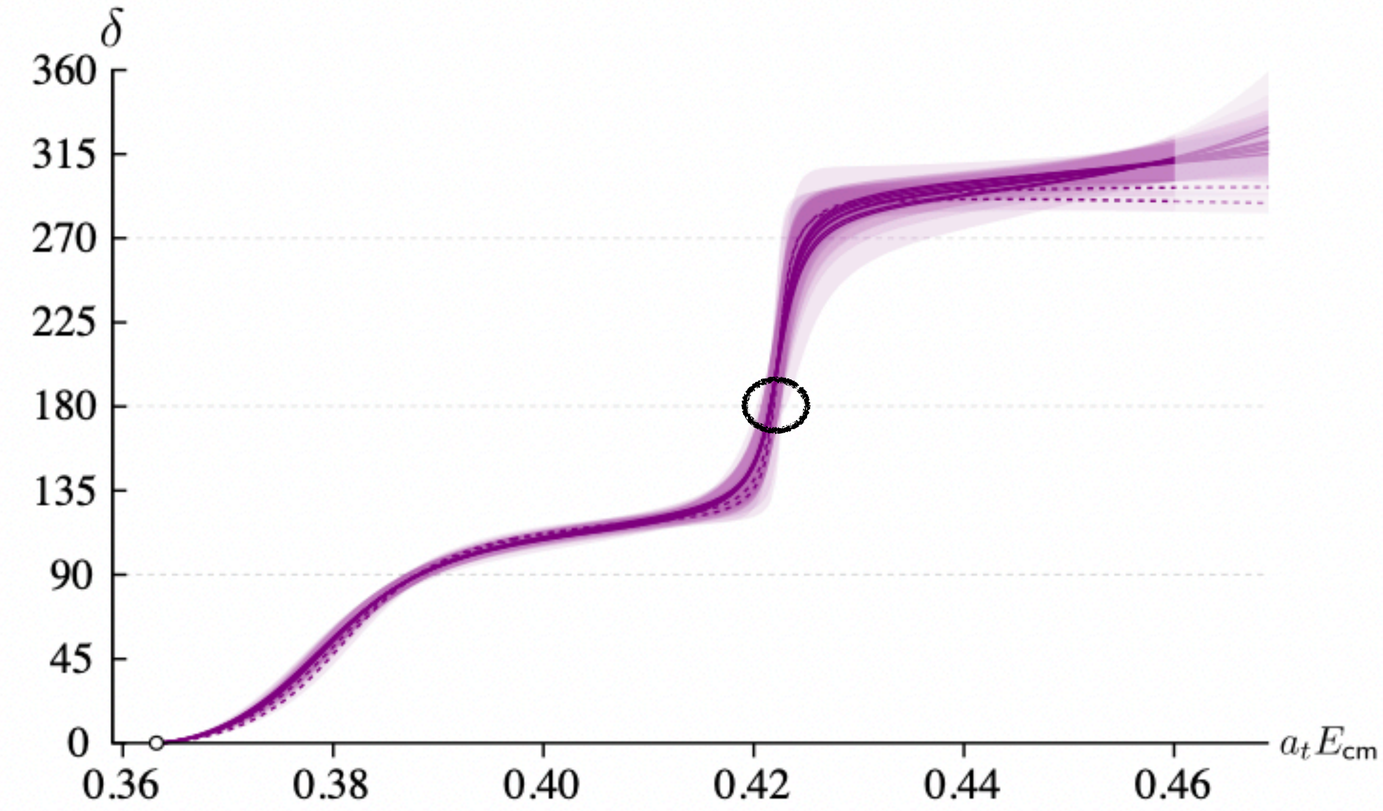
1^{--}

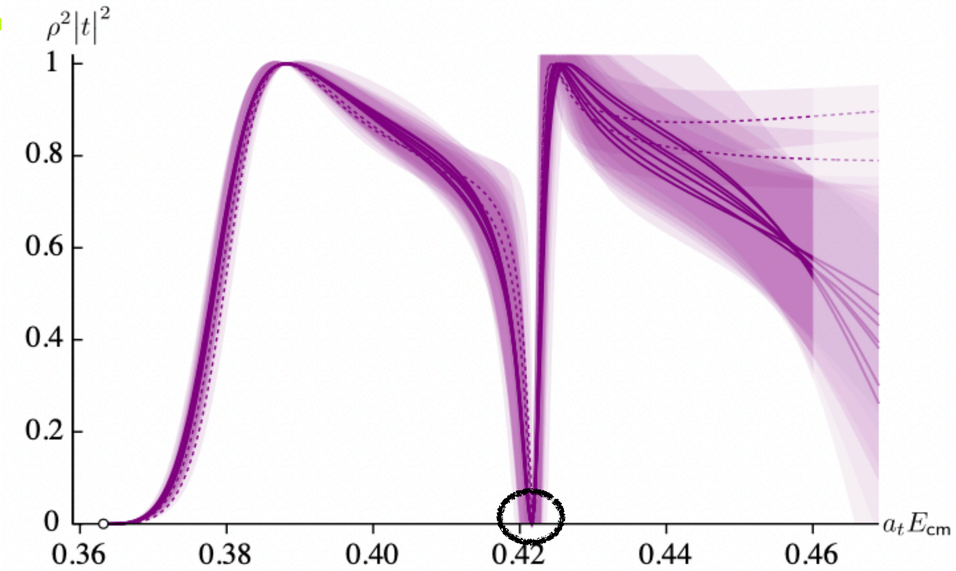

$\rho^2|t|^2$
1
0.8
0.6
0.4
0.2
0
0.36
0.38
0.40
0.42
0.44
0.46
$a_t E_{\mathrm{cm}}$
$K_{J=1} = \frac{g_a^2}{m_a^2 - s} + \frac{g_b^2}{m_b^2 - s} + \gamma s^n$
$K_{J=1} = \frac{g_a^2}{m_a^2 - s} + \frac{g_b^2}{m_b^2 - s}$


$\mathrm{Re}(a_t\sqrt{s})$
0.36
0.38
0.40
0.42
0.44
0.46
0.48
0
-0.01
-0.02
-0.03
$2 \cdot \mathrm{Im}(a_t\sqrt{s})$
a
b


$\mathrm{Im}(a_t c)$
0.05
0.10
0.15
0.20
$\mathrm{Re}(a_t c)$
-0.05
-0.10
a
b

Elasticity





Zero is a feature of elastic unitarity

$$t = \frac{1}{\rho(\cot\delta - i)}$$

Cannot generate with an effective range

$$k^3 \cot\delta = \frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

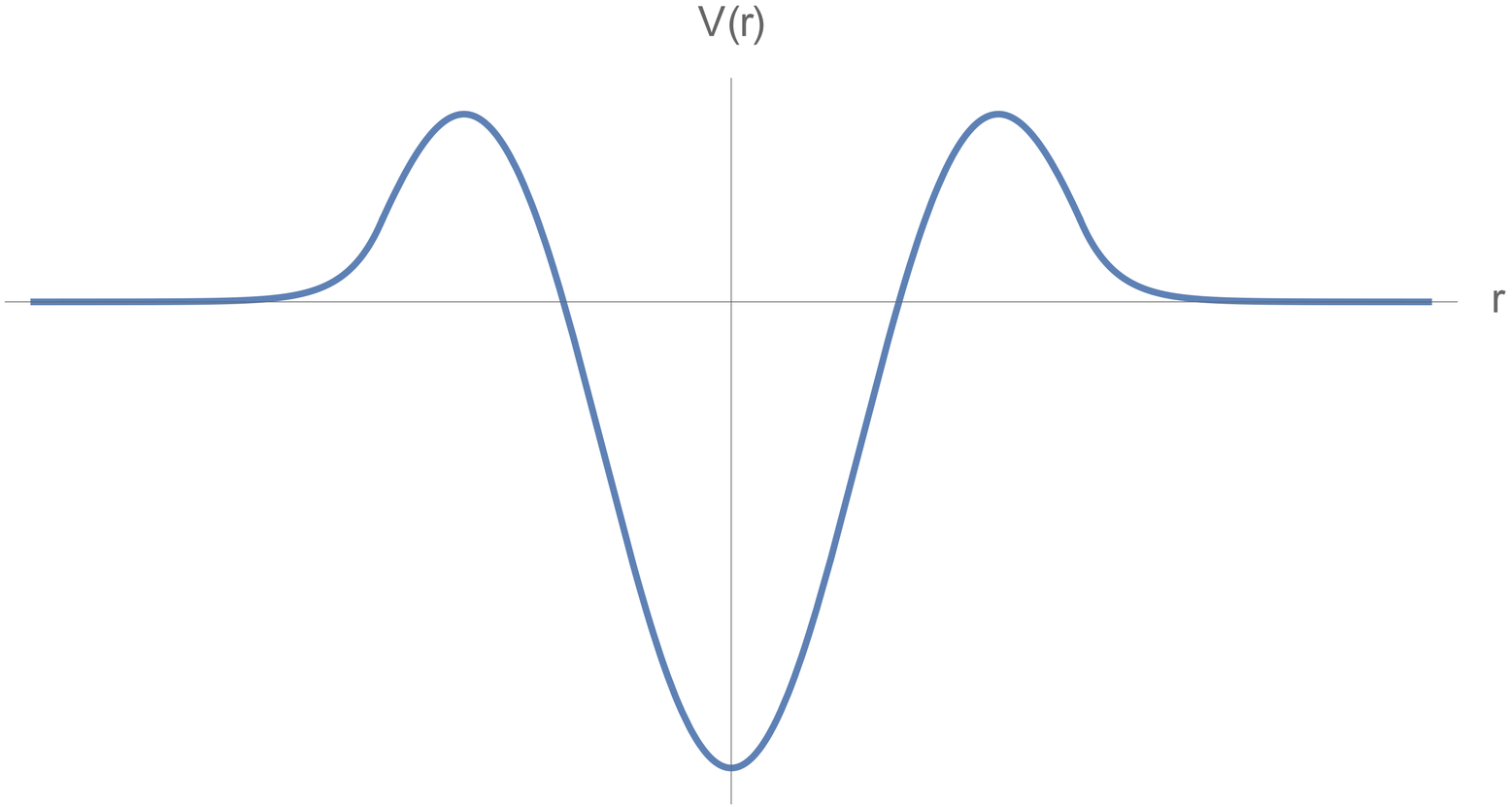
Resonance interpretation

In N.R. scattering, the interactions are generated by a potential.

Lighter resonance has further to tunnel through the barrier to decay.

⇒ should be narrow compared to heavier resonance

⇒ must explain through relativistic treatment



Resonance interpretation

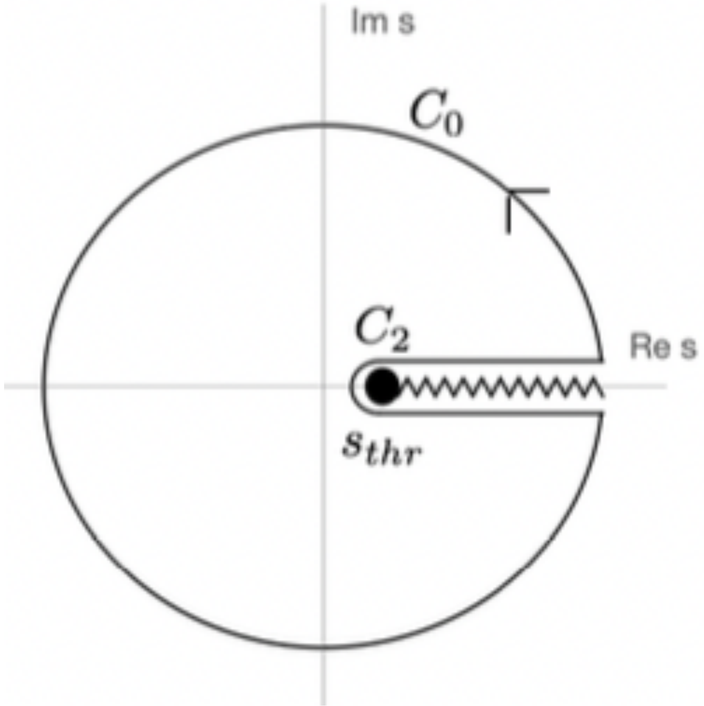
$$t(s) = \frac{N(s)}{D(s)}$$

Write dispersively $\frac{1}{2\pi i}\oint \frac{D(s')}{s'-s} = D(s_0) - \frac{s-s_0}{\pi}\int_{s_{thr}}^{\infty} \frac{N(s')\rho(s')}{(s'-s)(s'-s_0)} ds'$

⇒ can add poles to D(s) that feature as zeros in t(s)

⇒ create nearby poles in t(s)

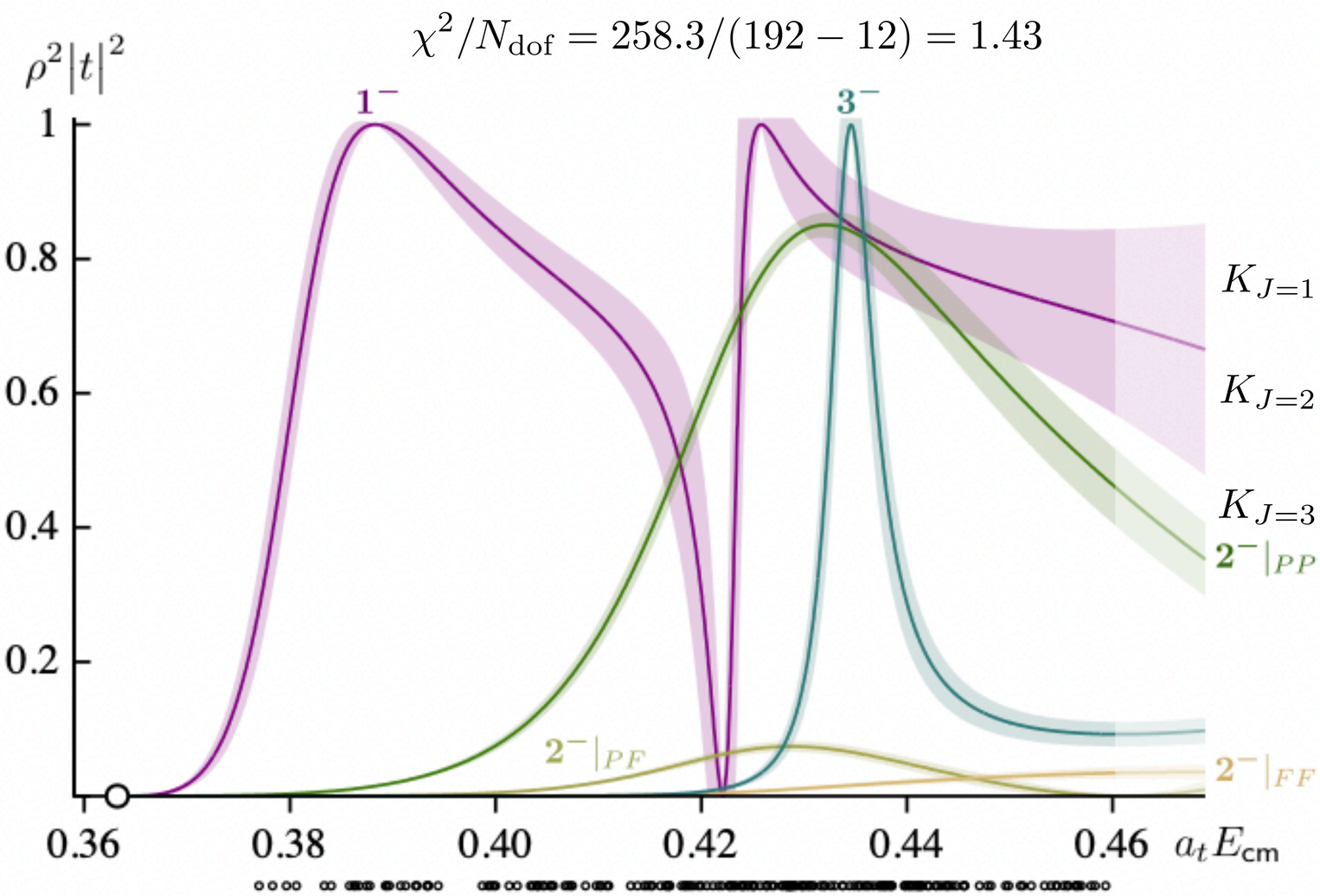
⇒ these "CDD" poles have an interpretation that they would be stable particles if there were not lighter mesons for which it to decay



$1^{--}, 2^{--}, 3^{--}$

Add the $[011]A_1$ irreps and fit all simultaneously

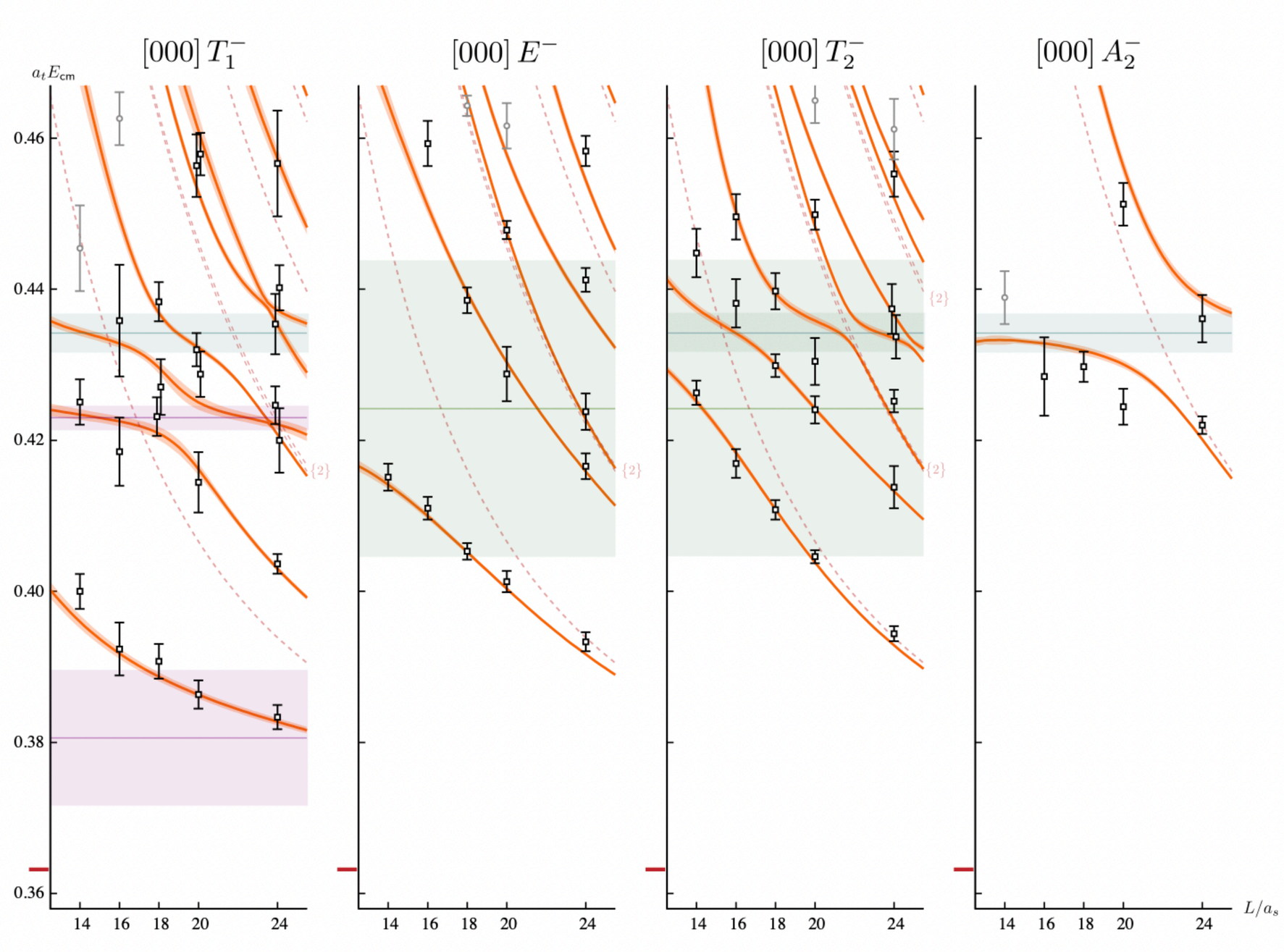
Very good constraint $N_{\text{dof}} = 180$

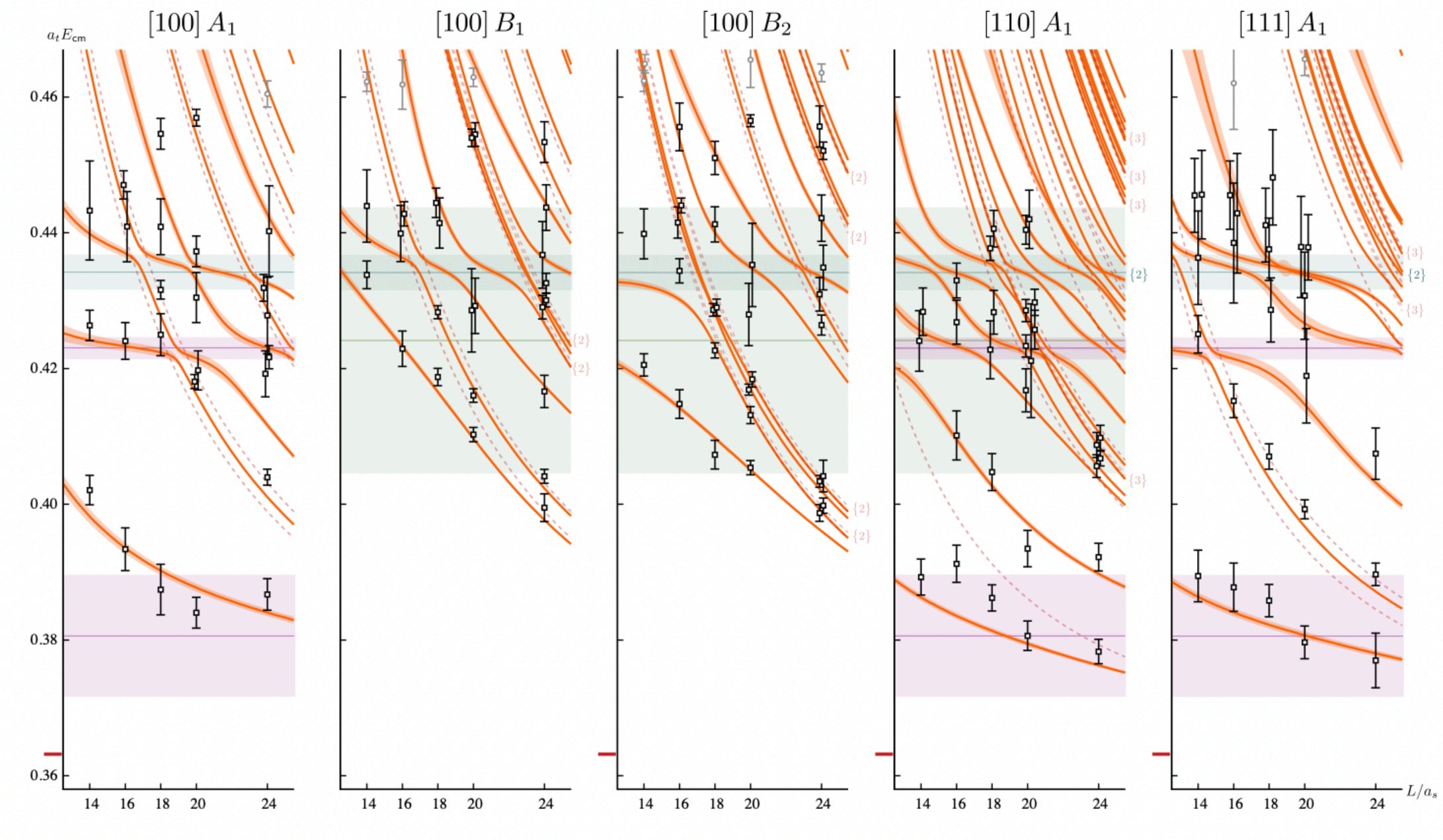


$$K_{J=1} = \frac{g_{\mathsf{a}}^2}{m_{\mathsf{a}}^2 - s} + \frac{g_{\mathsf{b}}^2}{m_{\mathsf{b}}^2 - s} + \gamma$$

$$K_{J=2} = \frac{1}{m_R^2 - s}\begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & 0 \end{bmatrix}$$

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$


[000] T_1^-
[000] E^-
[000] T_2^-
[000] A_2^-
$a_t E_{\rm cm}$
0.46
0.44
0.42
0.40
0.38
0.36
{2}
14
16
18
20
24
L/a_s


[100] A_1
[100] B_1
[100] B_2
[110] A_1
[111] A_1
$a_t E_{\rm cm}$
0.46
0.44
0.42
0.40
0.38
0.36
14
16
18
20
24
L/a_s
{2}
{3}

Comparing to the ω_J^*, ϕ_J^*

Relationship to familiar meson states:

$$\mathbf{1} = \frac{1}{2\sqrt{2}}\left(K^+\bar{K}^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0} + \pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right)$$

Pseudoscalar states have little mixing from SU(3) eigenstates $\eta \sim \eta_8, \eta' \sim \eta_1$

Vector states are nearly ideally flavor mixed:

$$\omega = \sqrt{\frac{2}{3}}\omega_1 + \sqrt{\frac{1}{3}}\omega_8\,;\,\phi = \sqrt{\frac{1}{3}}\omega_1 - \sqrt{\frac{2}{3}}\omega_8$$

$$\Gamma = g^2\frac{\rho}{M}$$

Isoscalar states are admixtures of octet and singlet, but we haven't computed the octet

$$|I = I_z = Y = 0, \mathbf{8}\rangle = \sqrt{\frac{1}{20}}\left(K^+\bar{K}^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0}\right) - \sqrt{\frac{1}{5}}\left(\pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right)$$

$$\mathbf{8} = \eta_8\omega_1$$

Comparing to the ω_J^*, ϕ_J^*

Can enforce OZI to find octet contributions $g^1 = c^{\omega_1}_{\eta^8\omega^8}$; $g^8 = c^{\omega_8}_{\eta^8\omega^8}$; $h^8 = c^{\omega^8}_{\eta^8\omega^1}$

$\phi^* \to \pi\rho, \eta\omega$ give the constraints $g^8 = -\dfrac{\sqrt{5}}{4} g^1$; $h^8 = -\dfrac{1}{2\sqrt{2}} g^1$

$$\Gamma = g^2 \frac{\rho}{M}$$

$$\Gamma(\omega^\star \to \pi\rho) = 3\,\tfrac{\rho}{M}\,\tfrac{3}{16}\left(g^1\right)^2$$
$$\Gamma(\omega^\star \to K\bar{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{64}\left(g^1\right)^2$$
$$\Gamma(\omega^\star \to \eta\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{16}\left(g^1\right)^2$$

$$\Gamma(\phi^\star \to K\bar{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{32}\left(g^1\right)^2$$
$$\Gamma(\phi^\star \to \eta\phi) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{4}\left(g^1\right)^2$$

$$\Gamma(\rho^\star \to \pi\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{3}{16}\left(g^1\right)^2$$
$$\Gamma(\rho^\star \to K\bar{K}^*) = 2\,\tfrac{\rho}{M}\,\tfrac{3}{32}\left(g^1\right)^2,$$

Can predict ρ_J^* decays from OZI

$$|I = I_z = 1, \mathbf{8}\rangle = -\sqrt{\frac{3}{10}}\left(K^+\bar{K}^{*0} + \bar{K}^0 K^{*+}\right) + \frac{1}{\sqrt{5}}\pi^+\omega_8 + \frac{1}{\sqrt{5}}\eta_8\rho^+$$

Comparing to the ω_J^*, ϕ_J^*

J=3:

$$\Gamma(\rho_3 \to \pi\omega, K\overline{K}^*) = 22,2\,\text{MeV}$$
$$\Gamma(\omega_3 \to \pi\rho, K\overline{K}^*, \eta\omega) = 62,2,1\,\text{MeV}$$
$$\Gamma(\phi_3 \to K\overline{K}^*, \eta\phi) = 20,3\,\text{MeV}\,,$$

$$\Gamma^{tot}_{\omega_3} \sim 168(10)\ \text{MeV}$$
$$\Gamma^{tot}_{\phi_3} \sim 87(25)\ \text{MeV}$$
$$\Gamma^{\pi\omega}_{\rho_3} \sim 30(10)\ \text{MeV}$$
$$\Gamma^{K\bar{K}\pi}_{\rho_3} \sim 7\ \text{MeV}$$

J=2:

$$\Gamma(\rho_2 \to \pi\omega, K\overline{K}^*) = 125,36\,\text{MeV}$$
$$\Gamma(\omega_2 \to \pi\rho, K\overline{K}^*, \eta\omega) = 365,36,17\,\text{MeV}$$
$$\Gamma(\phi_2 \to K\overline{K}^*, \eta\phi) = 148,44\,\text{MeV}\,,$$

Comparing to the ω_J^*, ϕ_J^*

J=1:

$$\Gamma(\rho_b \to \pi\omega, K\bar{K}^*) = 9,3\,\text{MeV}$$
$$\Gamma(\omega_b \to \pi\rho, K\bar{K}^*, \eta\omega) = 25,3,1\,\text{MeV}$$
$$\Gamma(\phi_b \to K\bar{K}^*, \eta\phi) = 13,5\,\text{MeV}\,,$$

$\longleftrightarrow$

$\omega^\star(1606)$ $\quad \Gamma_{\pi\rho} \sim 84\,\text{MeV}$

$\rho^\star(1730)$ $\quad \Gamma_{\pi\omega} \sim 0\,,$

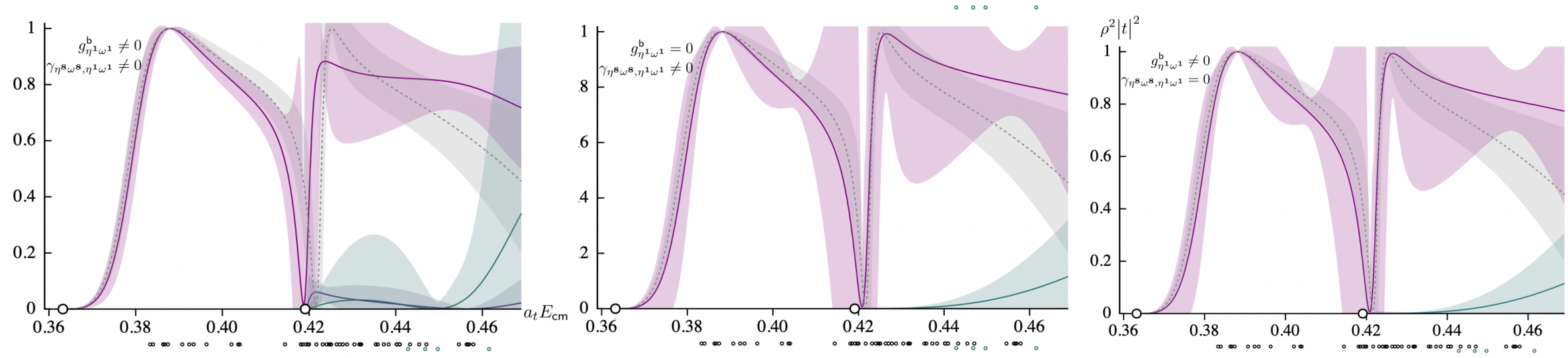
$$\Gamma(\rho_a \to \pi\omega, K\bar{K}^*) = 133,9\,\text{MeV}$$
$$\Gamma(\omega_a \to \pi\rho, K\bar{K}^*, \eta\omega) = 384,4,5\,\text{MeV}$$
$$\Gamma(\phi_a \to K\bar{K}^*, \eta\phi) = 154,25\,\text{MeV}\,,$$

$\longleftrightarrow$

$\omega^\star(1440)$ $\quad \Gamma_{\pi\rho} \sim 240\,\text{MeV}$

$\rho^\star(1463)$ $\quad \Gamma_{\pi\omega} \sim 52 - 78\,\text{MeV}$

Coupled-Channel $\eta^8\omega^8 - \eta^1\omega^1$



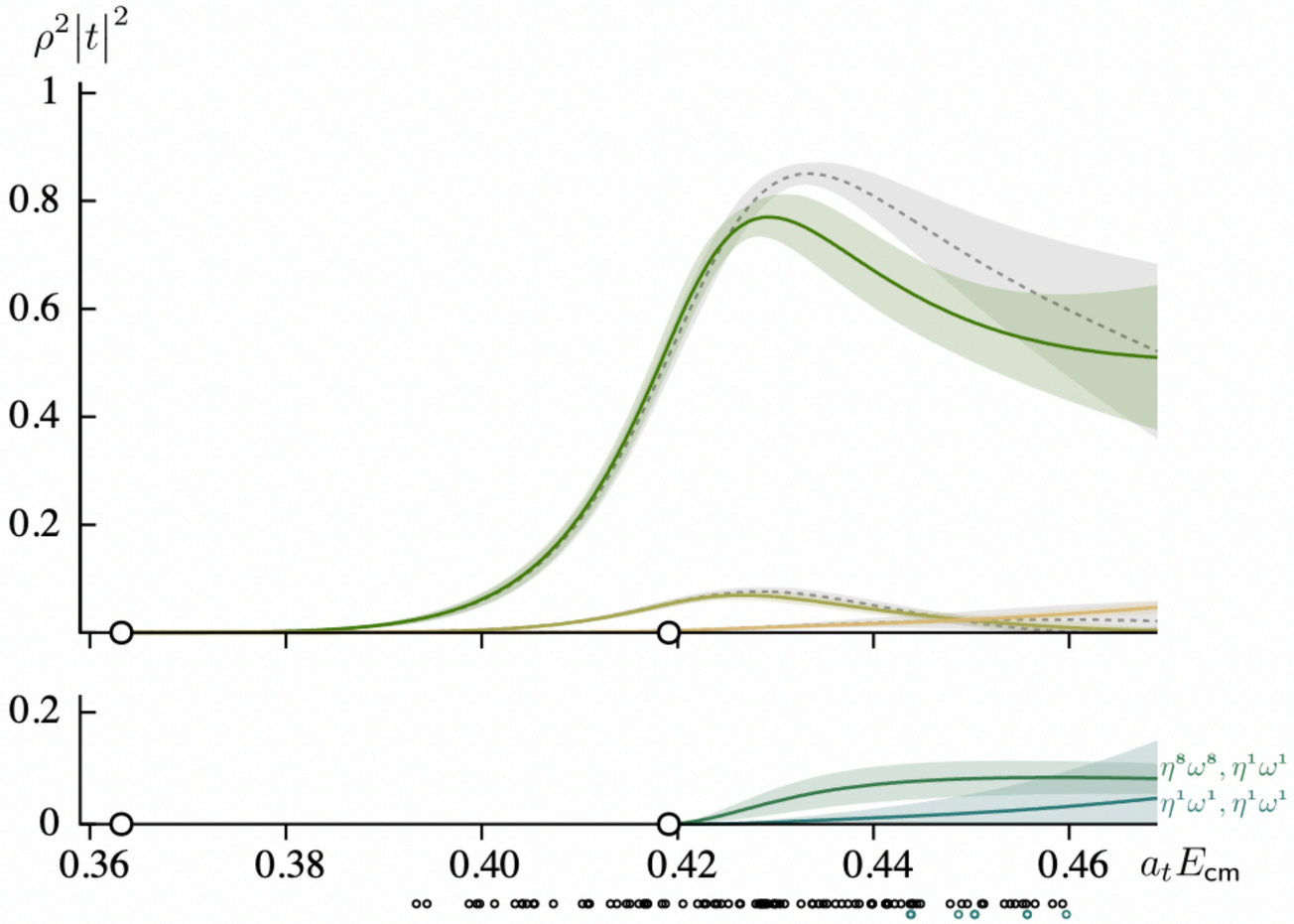
Only 4 levels with large $\eta^1\omega^1$ overlap.

Only real difference in fit-1 which features two $\eta^1\omega^1$ parameters.

Potentially a small coupling $c_{\eta^1\omega^1} \lesssim 0.04$ does not change overall width.

Statistical uncertainties on $f_0^1\omega^1$ energy levels prevent a proper C.C. analysis with this channel.

c.c. $2^{--}, \eta^8\omega^8 - \eta^1\omega^1$



Mild changes in the amplitude.

$a_t|c_{\eta^1\omega^1}| \sim 0.07(2)$ is small and comparable to F-wave coupling.

Additional singularities

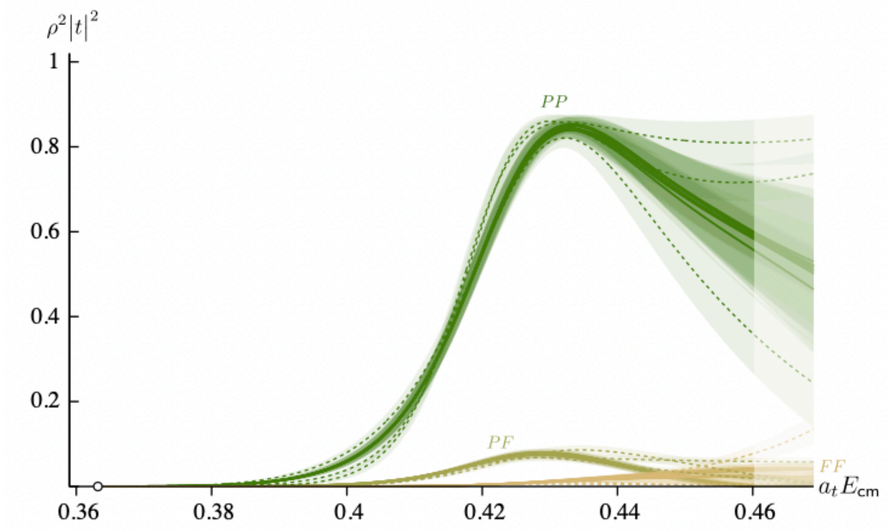
Unphysical sheet real axis pole $a_t\sqrt{s} \sim 0.23$ on many parameterizations

⇒ wanders a bit and remains far from physical scattering

Additional real axis pole $a_t\sqrt{s} \sim 0.24$ for simple phase space parameterization

⇒ not surprising this parameterization has poorer analytic properties

⇒ residue is real, a true p-wave bound state has imaginary coupling



Amplitude analytic structure

The full scattering amplitude T(s,t) relates all scattering channels s,t,u- through an analytic continuation.

s-channel unitarity constrains the “right hand cut” to form $2^{N_{chan}}$ Riemann sheets

⇒ built into our parameterizations

Analyticity requires poles off axis real valued poles be on unphysical sheets.

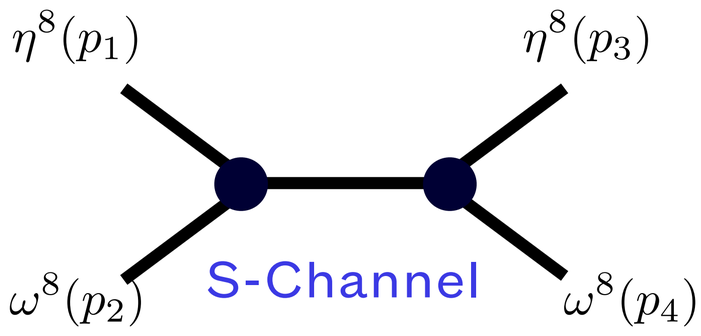
⇒ reject parameterizations that have these

t,u-channel unitarity manifests themselves in the form of a “left hand cut”

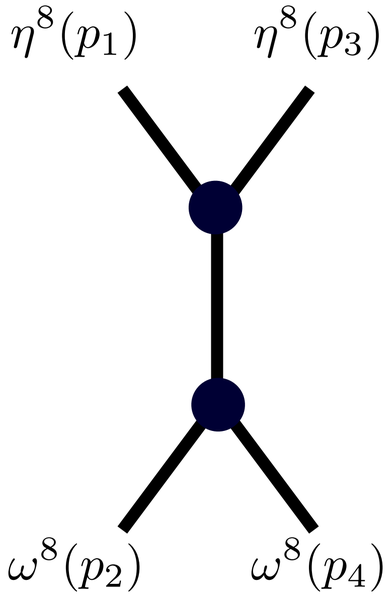
⇒ not described but we know where they are

⇒ hope is we remain far enough away

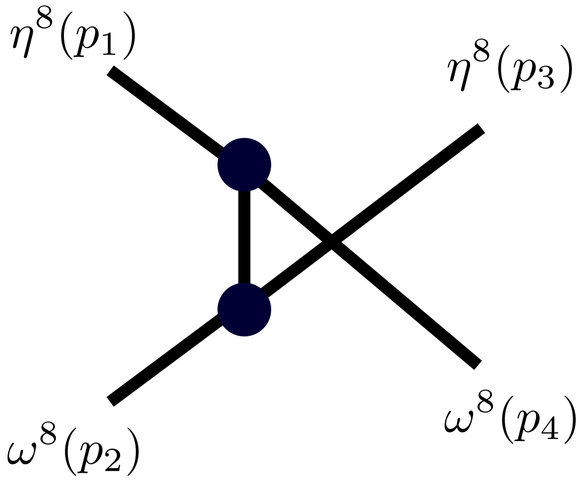
Cross Channels



S-Channel

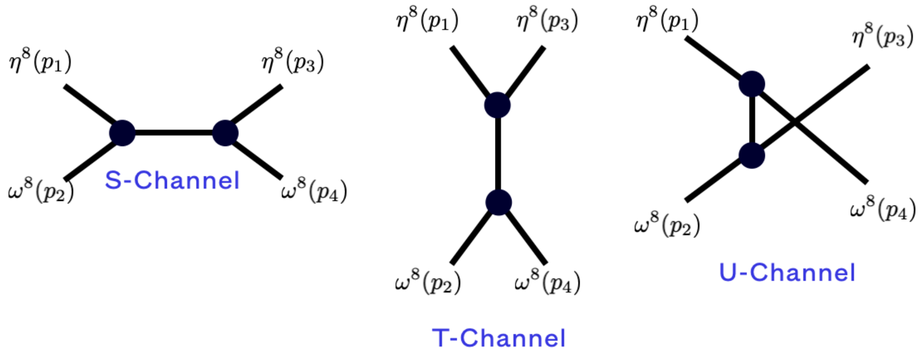


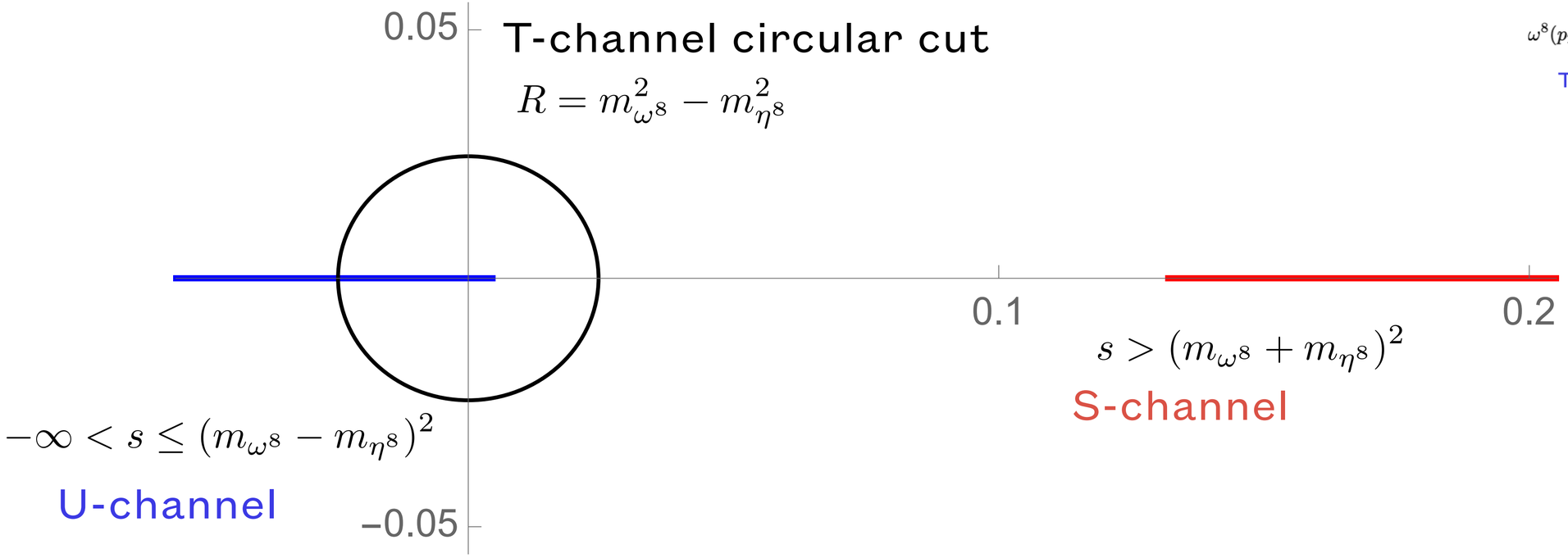
T-Channel



U-Channel

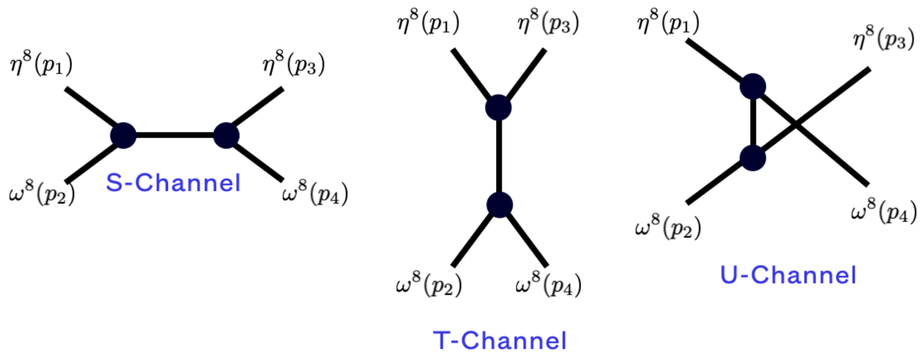
Cuts

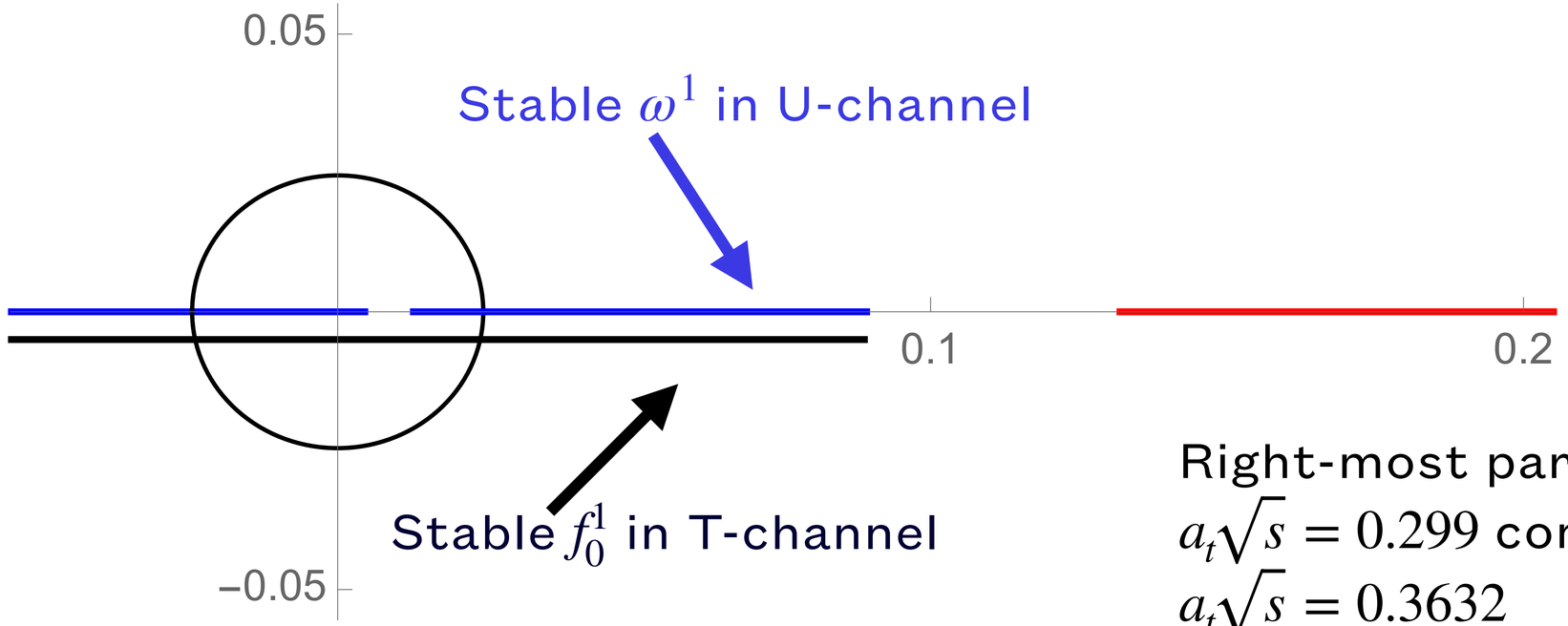

$\eta^8(p_1)$
$\eta^8(p_3)$
$\omega^8(p_2)$
S-Channel
$\omega^8(p_4)$
$\eta^8(p_1)$
$\eta^8(p_3)$
$\omega^8(p_2)$
$\omega^8(p_4)$
T-Channel
$\eta^8(p_1)$
$\eta^8(p_3)$
$\omega^8(p_2)$
$\omega^8(p_4)$
U-Channel


0.05
T-channel circular cut
$R = m_{\omega^8}^2 - m_{\eta^8}^2$
0.1
0.2
$s > (m_{\omega^8} + m_{\eta^8})^2$
S-channel
$-\infty < s \leq (m_{\omega^8} - m_{\eta^8})^2$
U-channel
−0.05

Cuts

Stable particles in cross-channels add additional singularities



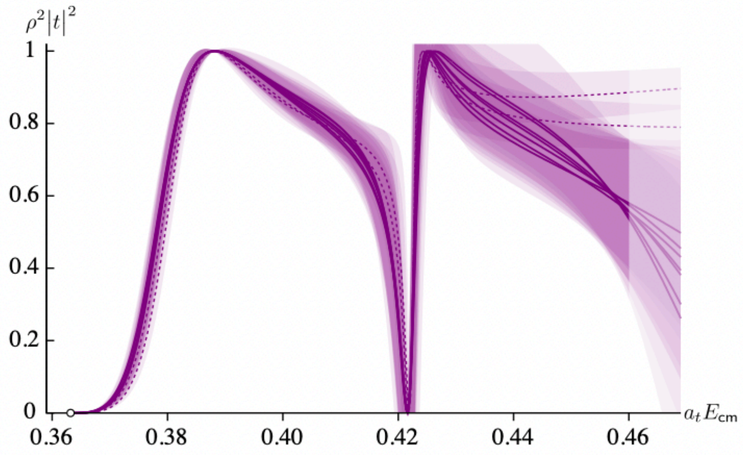


Right-most part of additional cuts at $a_t\sqrt{s} = 0.299$ compared to threshold of $a_t\sqrt{s} = 0.3632$

Additional Singularities

Physical sheet pole at $a_t\sqrt{s} = 0.278(26)$ wrong residue.

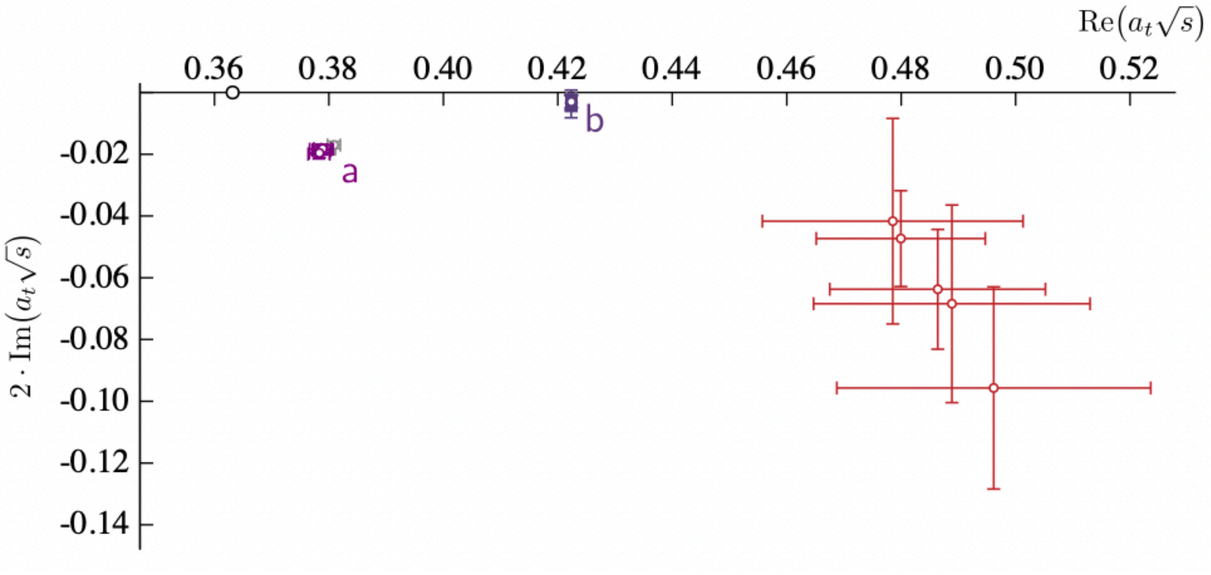
⇒ asses this as a "ghost" occurring from improper treatment of the LHC



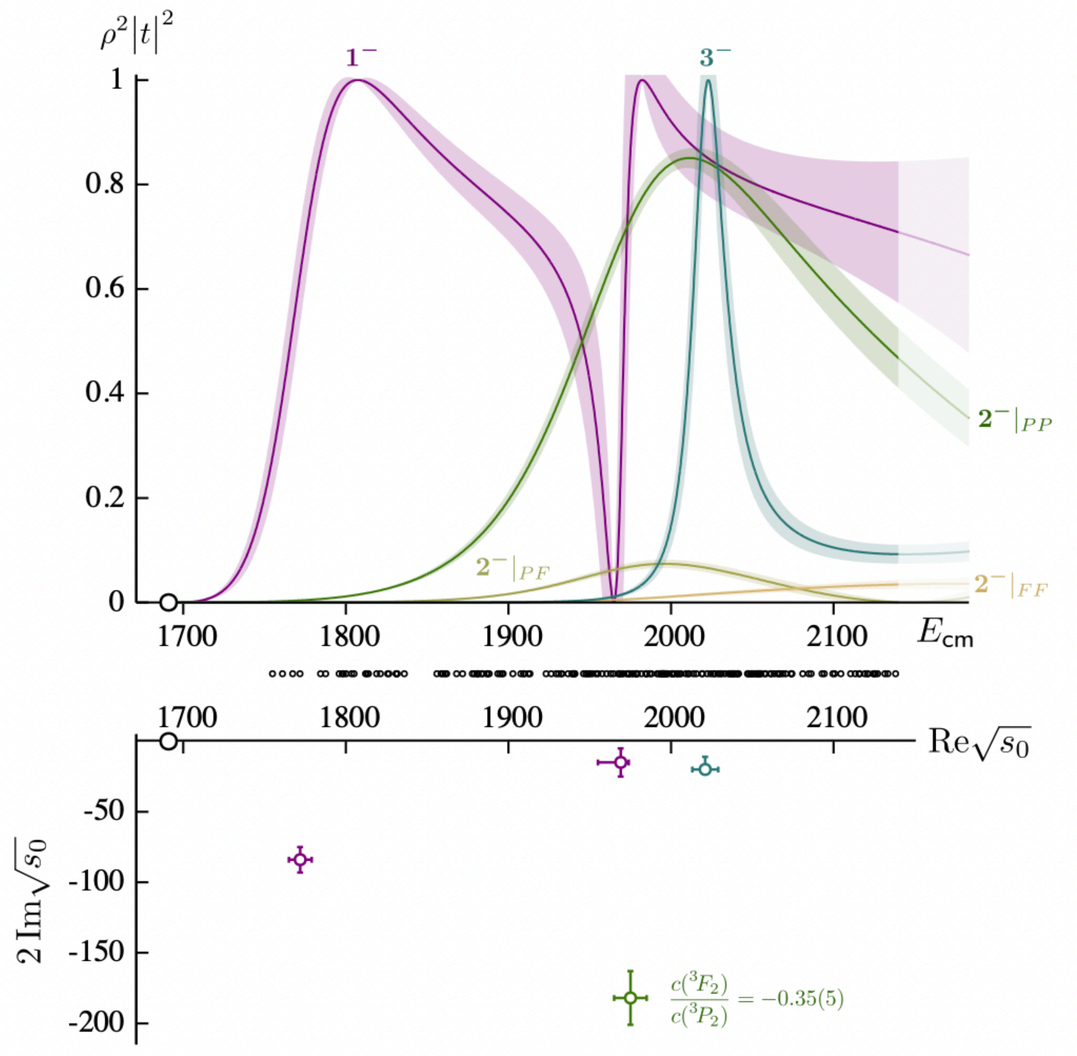
Noisy third unphysical sheet pole lies beyond region of constraint $a_t E \sim 0.46$.

⇒ artifact not present in all parameterizations

⇒ could be feeling presence of a hybrid 1^{--} meson we expect in that region



Summary



We extract 4 resonances consistent with the quark model prediction.

1^{--}: broader lighter first resonance and heavier narrower second resonance

2^{--}: broad resonance coupled mostly to P-wave

3^{--}: narrow F-wave resonance

Finite volume formalism can handle multiple resonances in same partial wave and nearly degenerate resonances in same irrep.

Future

Calculation of the octet is underway:

$\Rightarrow$ more channels

$\Rightarrow$ identical particles $\eta^8\eta^8, \omega^8\omega^8$

$\Rightarrow$ nearly degenerate thresholds in $\eta^8\omega^8, \eta^8\omega^1$

Would like to be able to study the hybrid candidate that lies slightly above in 1^{--}

$\Rightarrow$ likely requires three-particle formalism