"the three-body problem"

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The Three-Body Problem (novel)

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For other uses, see Three-body problem (disambiguation).
"San Ti" redirects here. For the martial arts stance, see Xing Yi Quan.

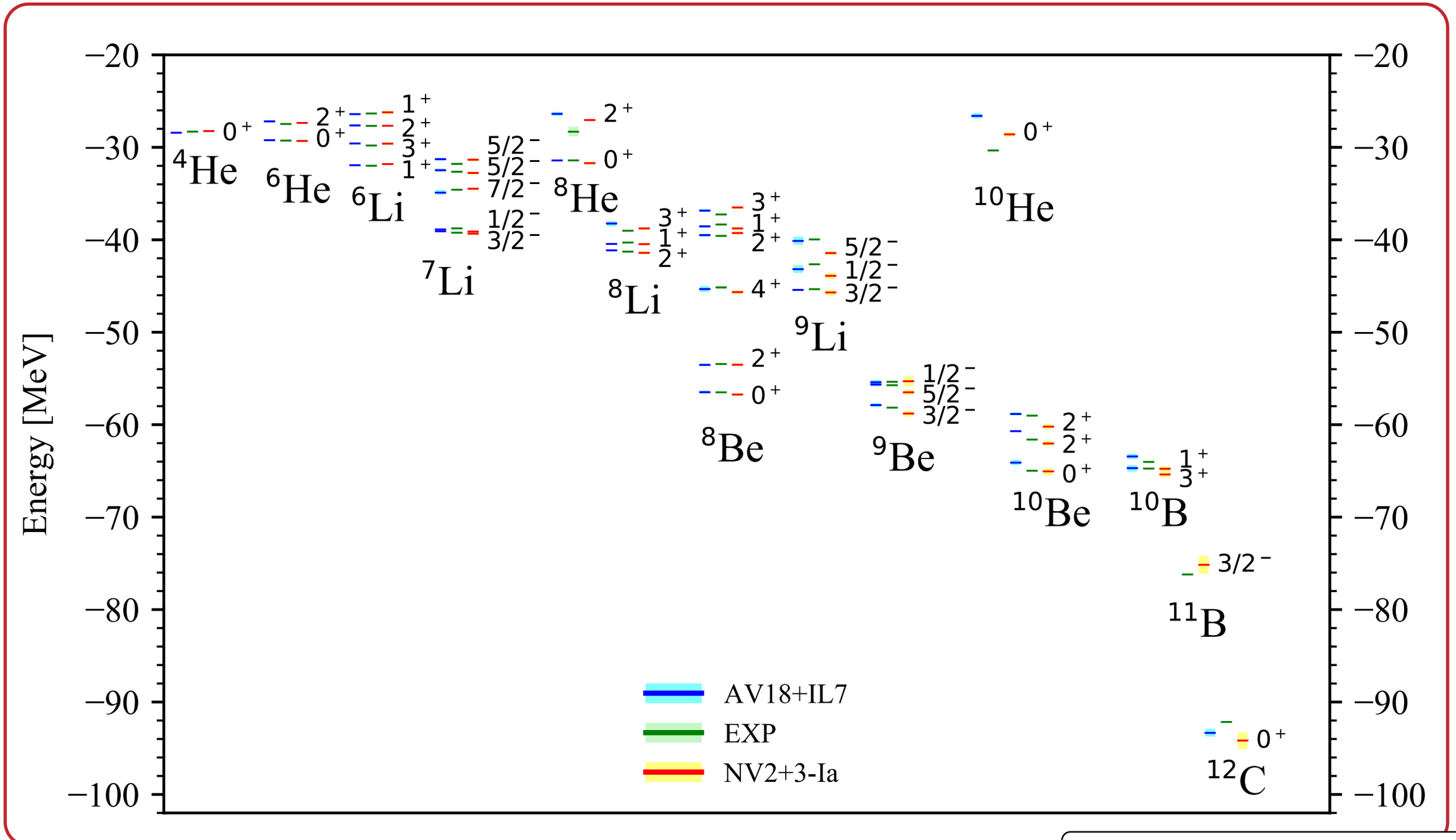
The Three-Body Problem (Chinese: 三体; literally: 'Three-Body'; pinyin: *sān tǐ*) is a science fiction novel by the Chinese writer Liu Cixin. It is the first novel of the *Remembrance of Earth's Past* (Chinese: 地球往事) trilogy, but Chinese readers generally refer to the whole series by the title of this first novel.[1] The second and third novels in the trilogy are titled *The Dark Forest* and *Death's End*. The title of the first novel refers to the three-body problem in orbital mechanics.

The work was serialized in *Science Fiction World* in 2006, published as a book in 2008 and became one of the most popular science fiction novels in China.[2] It received the Chinese Science

The Three-Body Problem

the Trisolaran invasion force has departed, but will not reach Earth for 450 years, as they are traveling at 1/100-th of lightspeed….society splits into factions, with the Adventists seeking complete destruction of humanity by the Trisolarans, and the Redemptionists seeking to help the Trisolarans to find a computational solution to the three-body problem, which plagues their home planet…

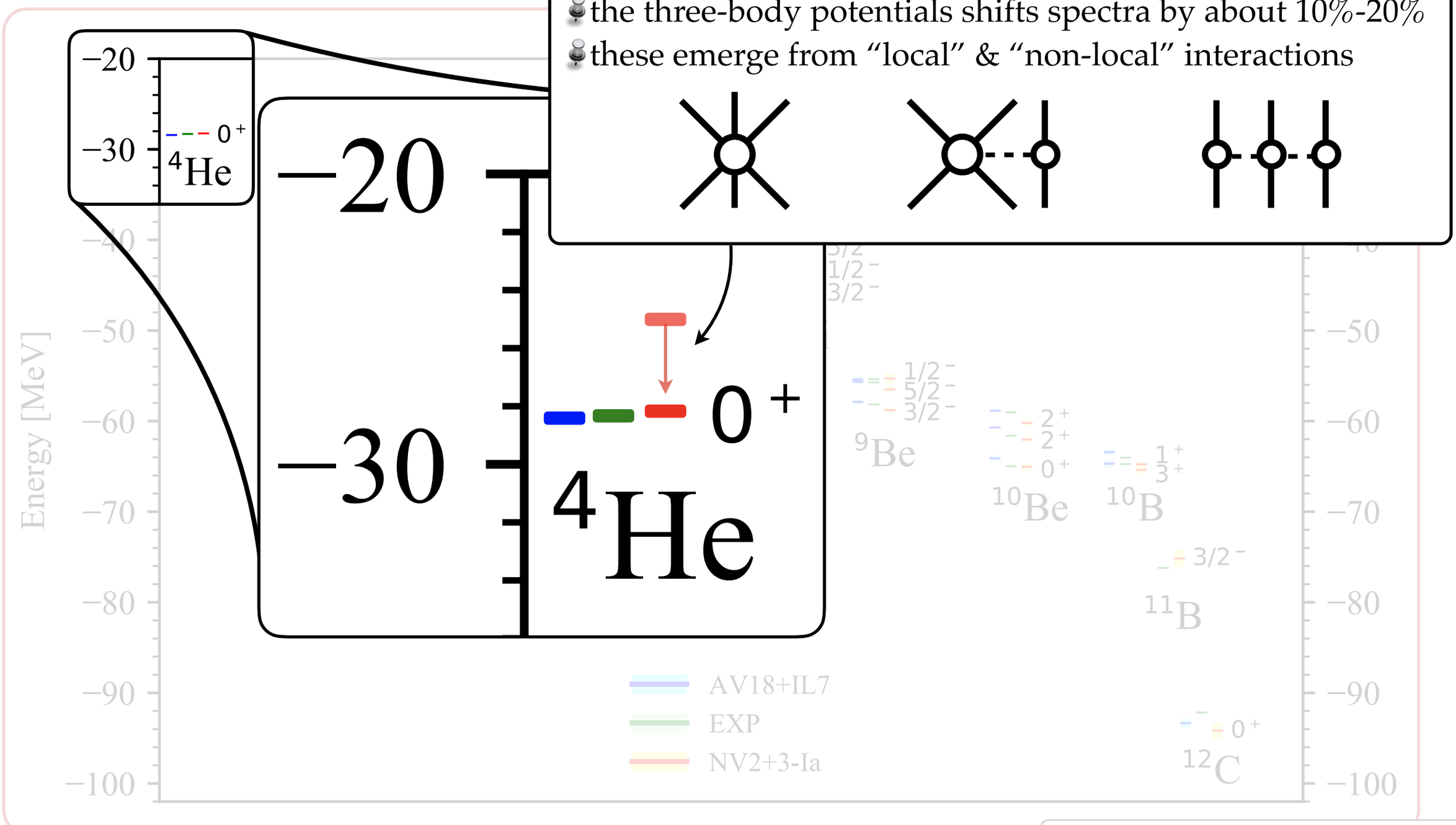
the three-body problem: nuclear physics



Piarullia, Baroni, Schiavilla et al. [2017]

the three-body problem: nuclear physics

- the three-body potentials shifts spectra by about 10%-20%
- these emerge from "local" & "non-local" interactions



Piarullia, Baroni, Schiavilla et al. [2017]

the three-body problem: from nuclei to atoms - *Efimov physics*

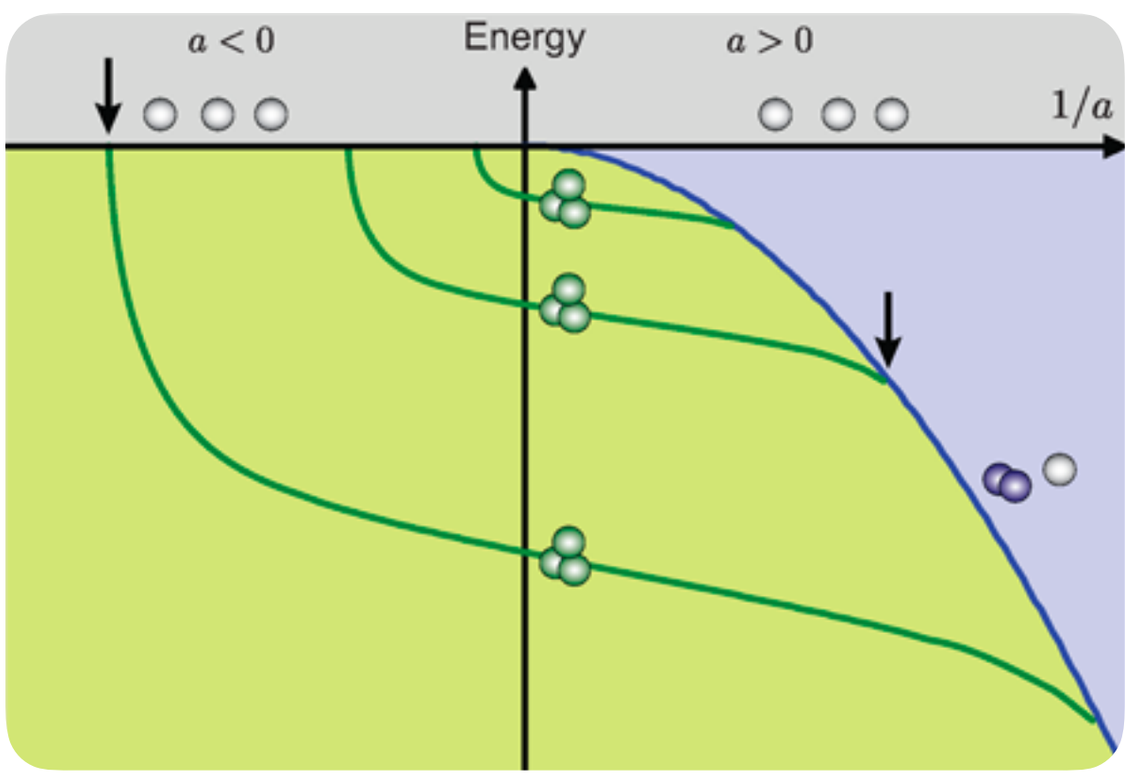
Consider two non-relativistic bosons in the unitary limit: $p\cot\delta = -\frac{1}{a} + \frac{rp^2}{2} + \cdots = 0$

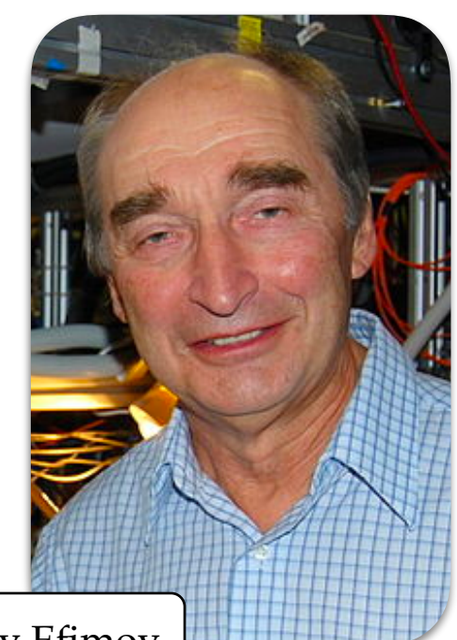
The two-body scattering amplitude will have a pole, right at threshold:

$$\mathcal{M} \sim \frac{1}{p\cot\delta - ip} = \frac{1}{ip}$$

Efimov [1970] predicted an infinite tower of geometrically-separated three-body bound states:

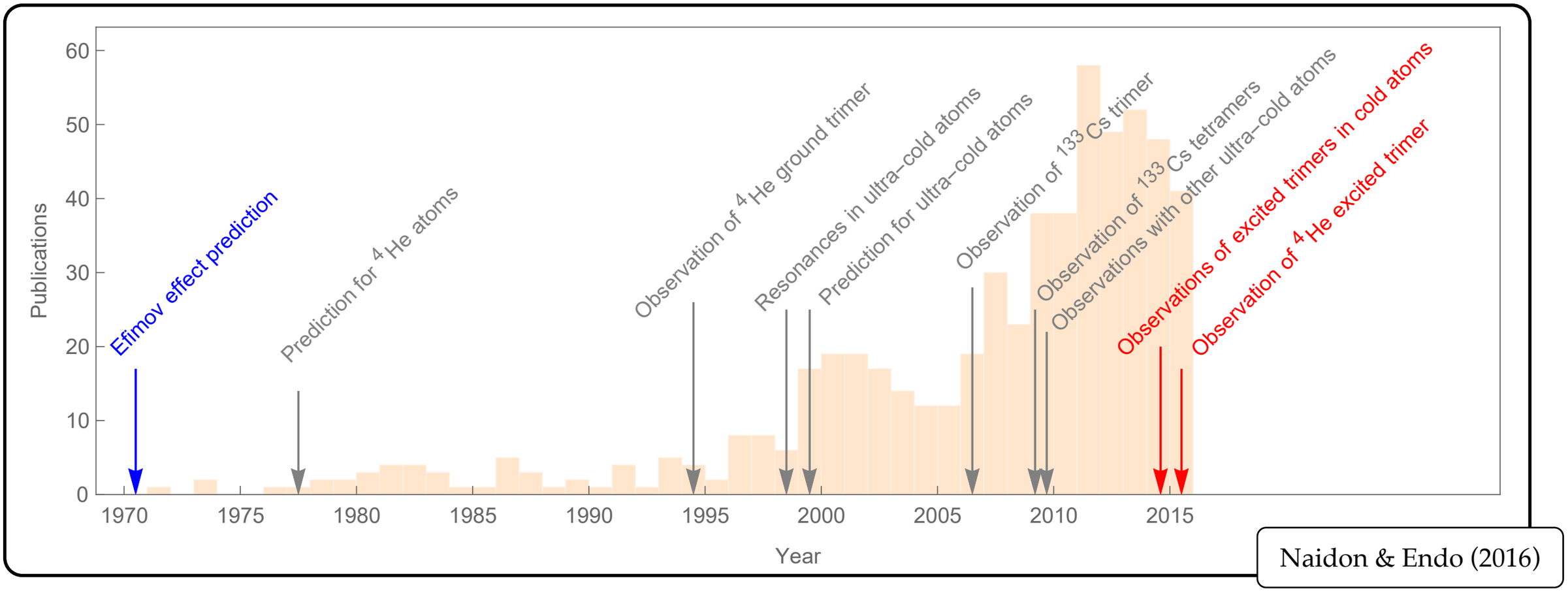
$$\boxed{E_N = E_0\lambda^{-2N}, \qquad \lambda = 22.69438\ldots}$$





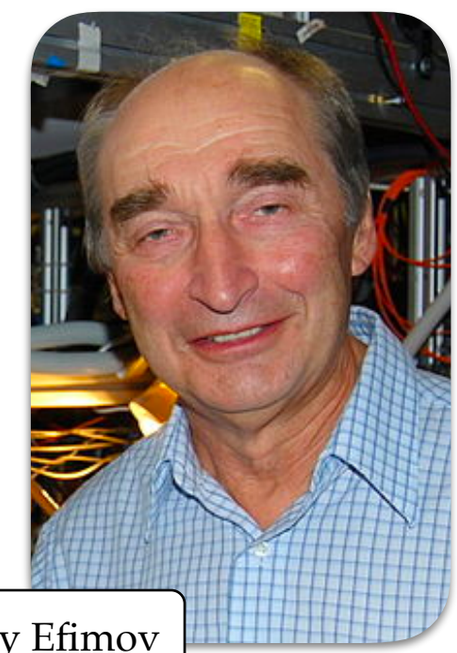
Vitaly Efimov

the three-body problem: from nuclei to atoms - *Efimov physics*



Naidon & Endo (2016)

a somewhat curious observation lead to a wide range of prediction and strong overlap between two seemingly disconnected fields: atomic and nuclear physics



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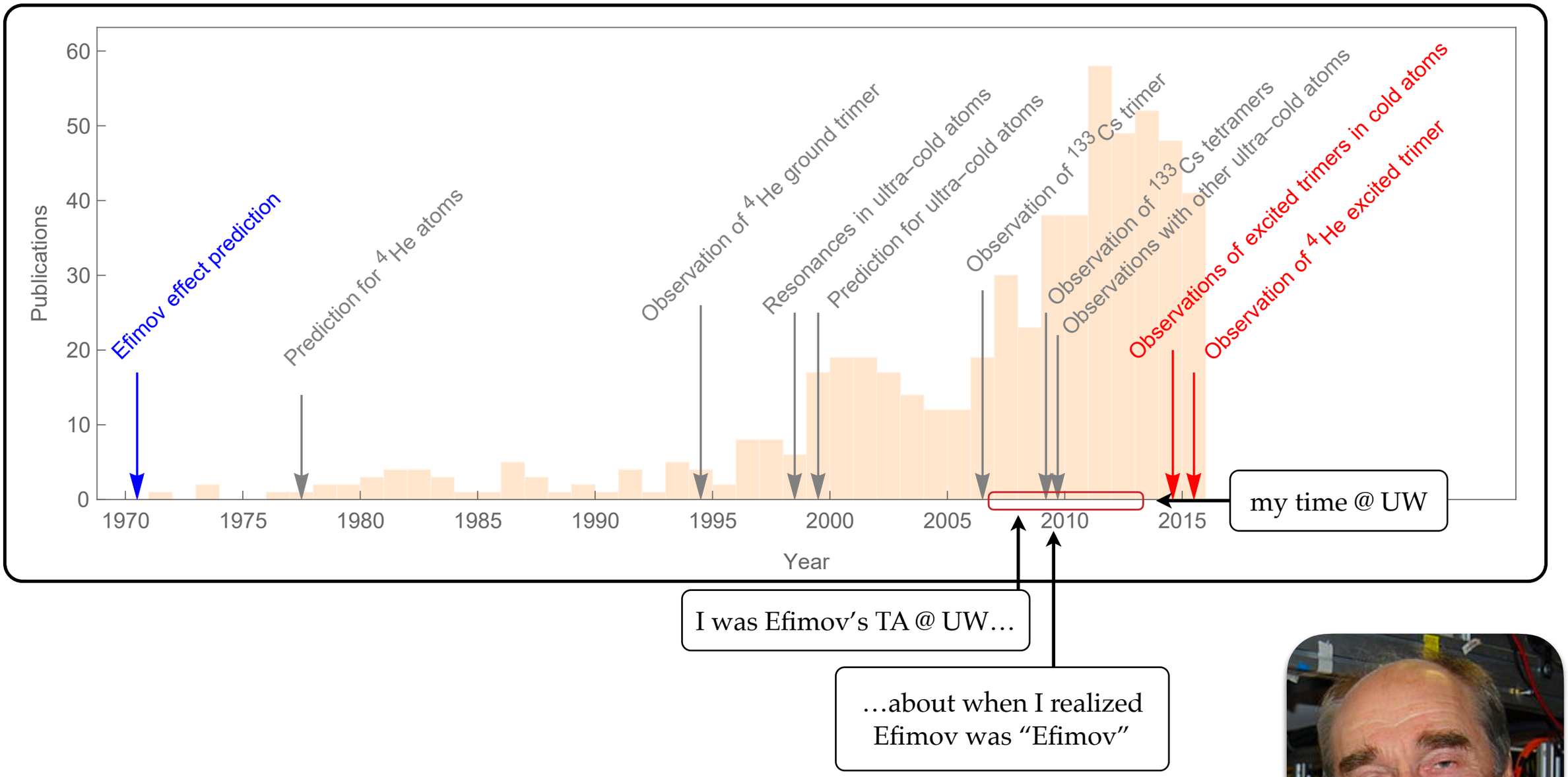
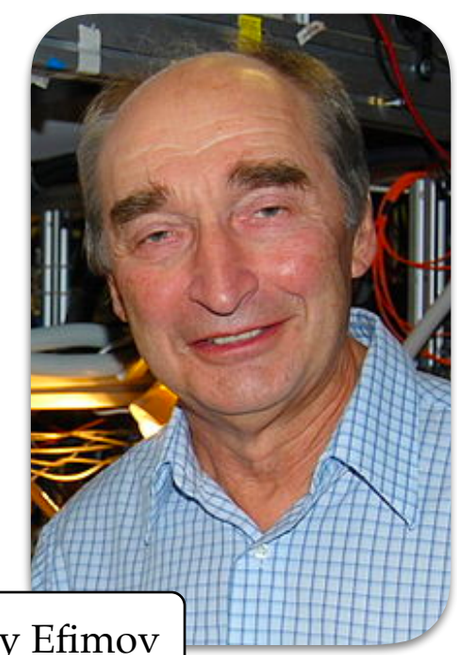
the three-body problem: from nuclei to atoms - *Efimov physics*



a somewhat curious observation lead to a wide range of prediction and strong overlap between two seemingly disconnected fields: atomic and nuclear physics

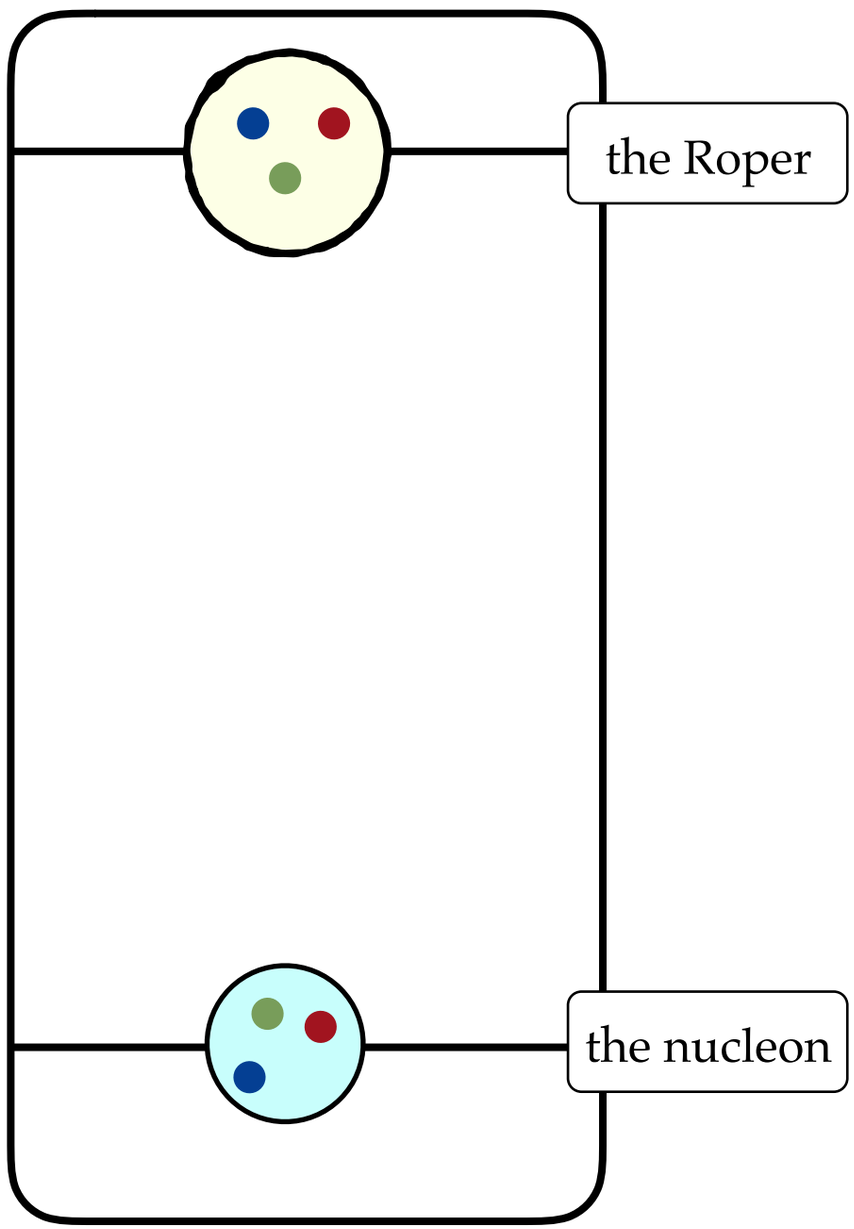


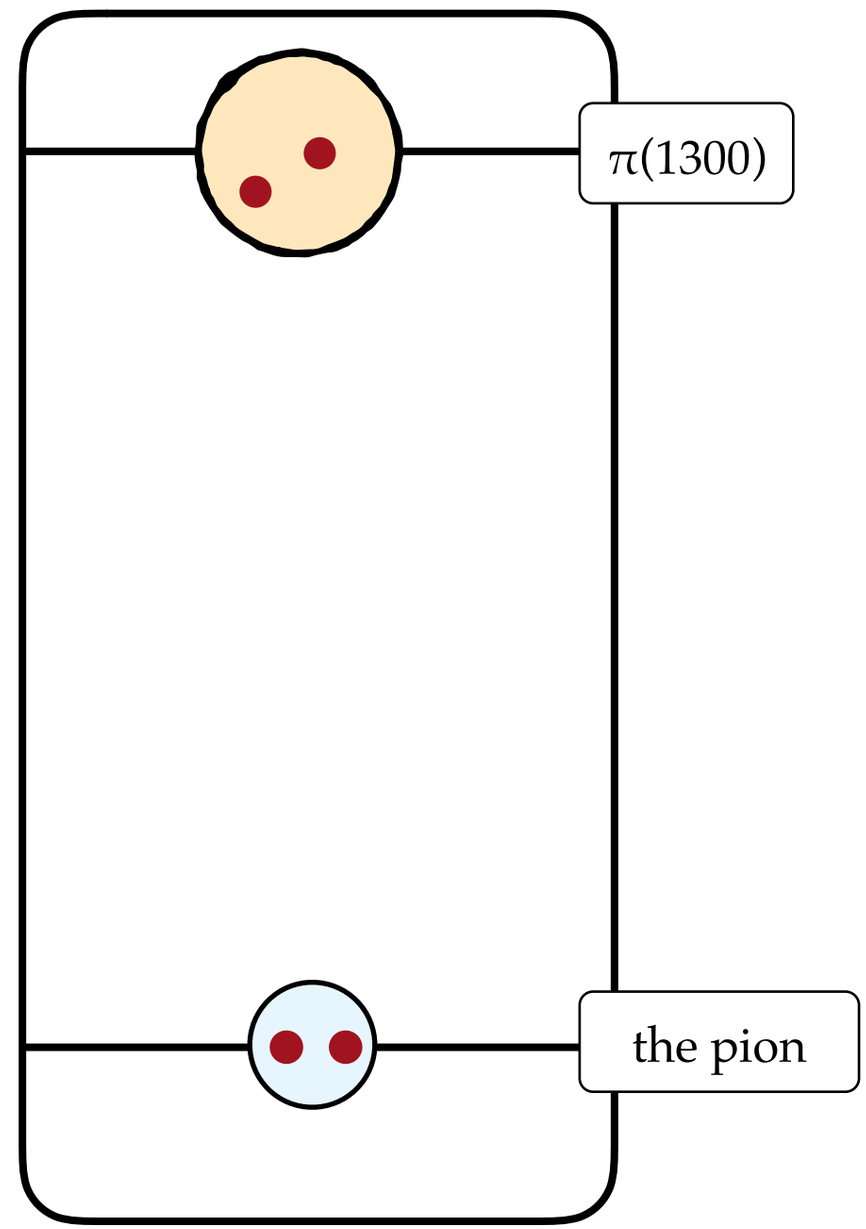
Vitaly Efimov

the three-body problem: hadron spectroscopy

Most excited lie above three-particle thresholds and couple strongly onto these states…

…just look at the first excited states of the simplest QCD states





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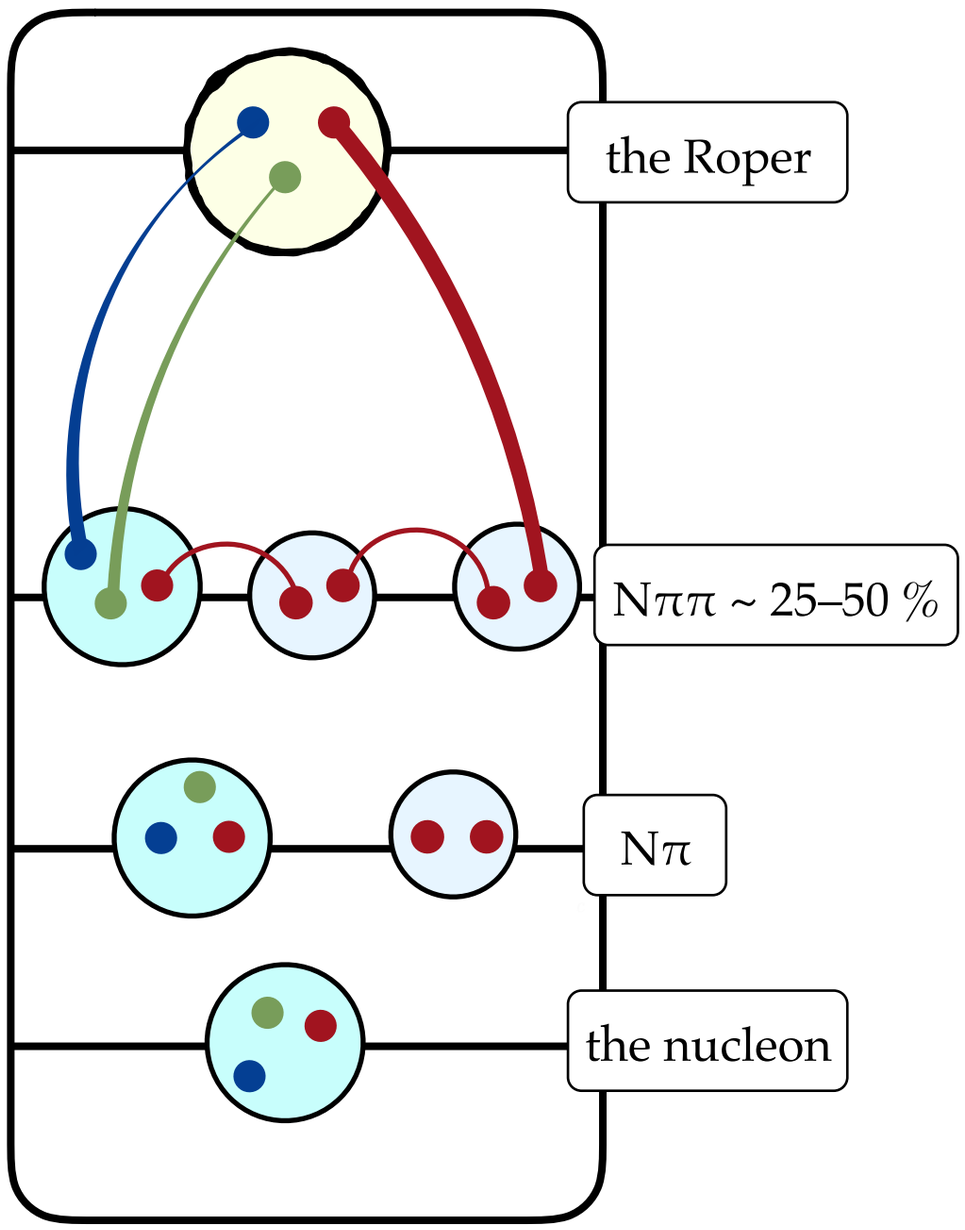
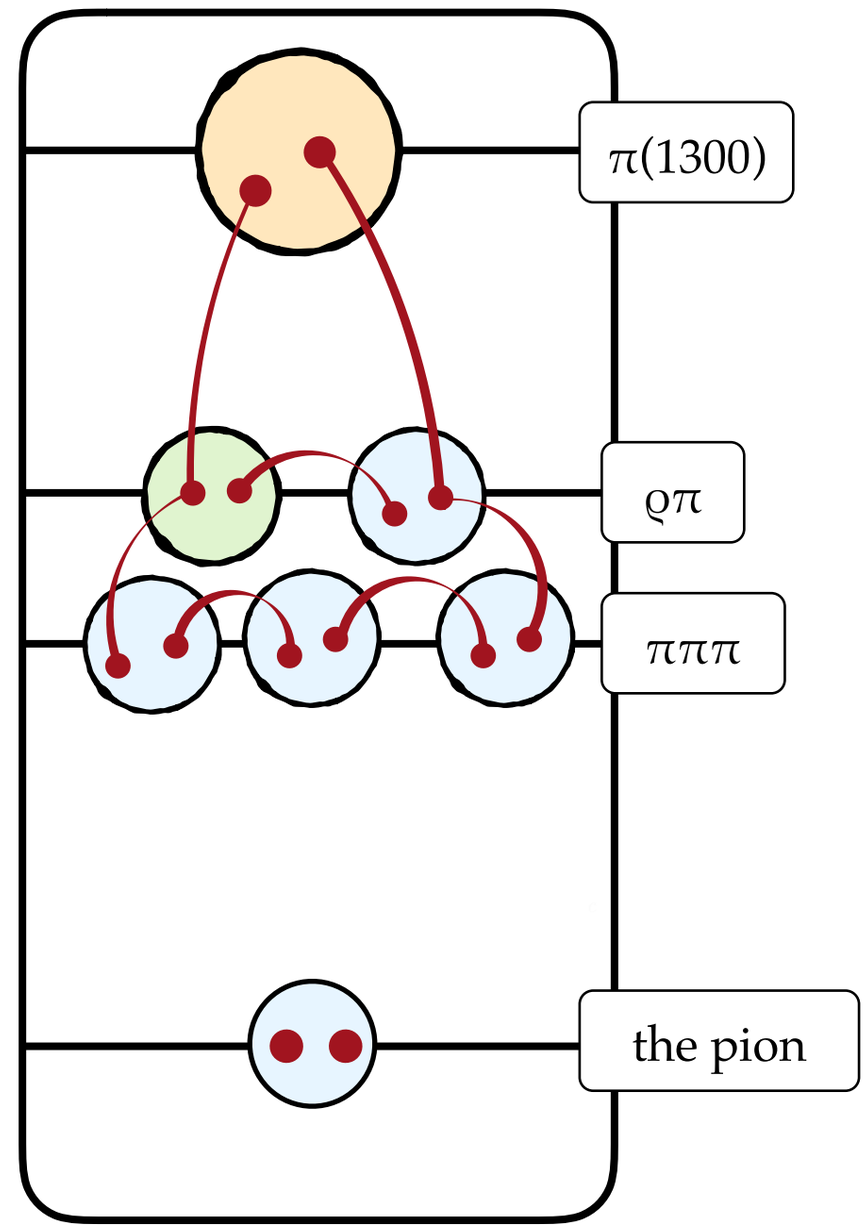
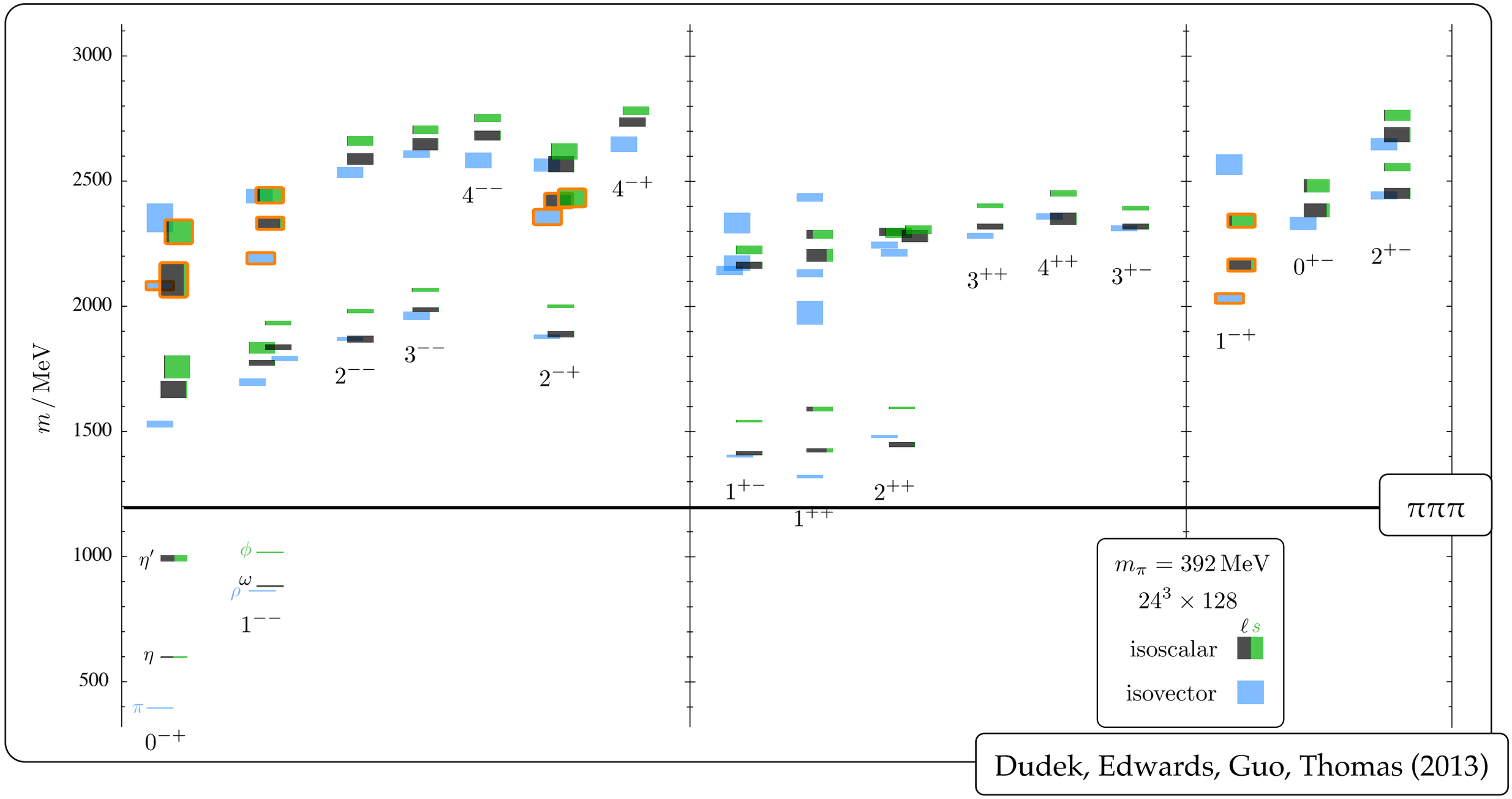




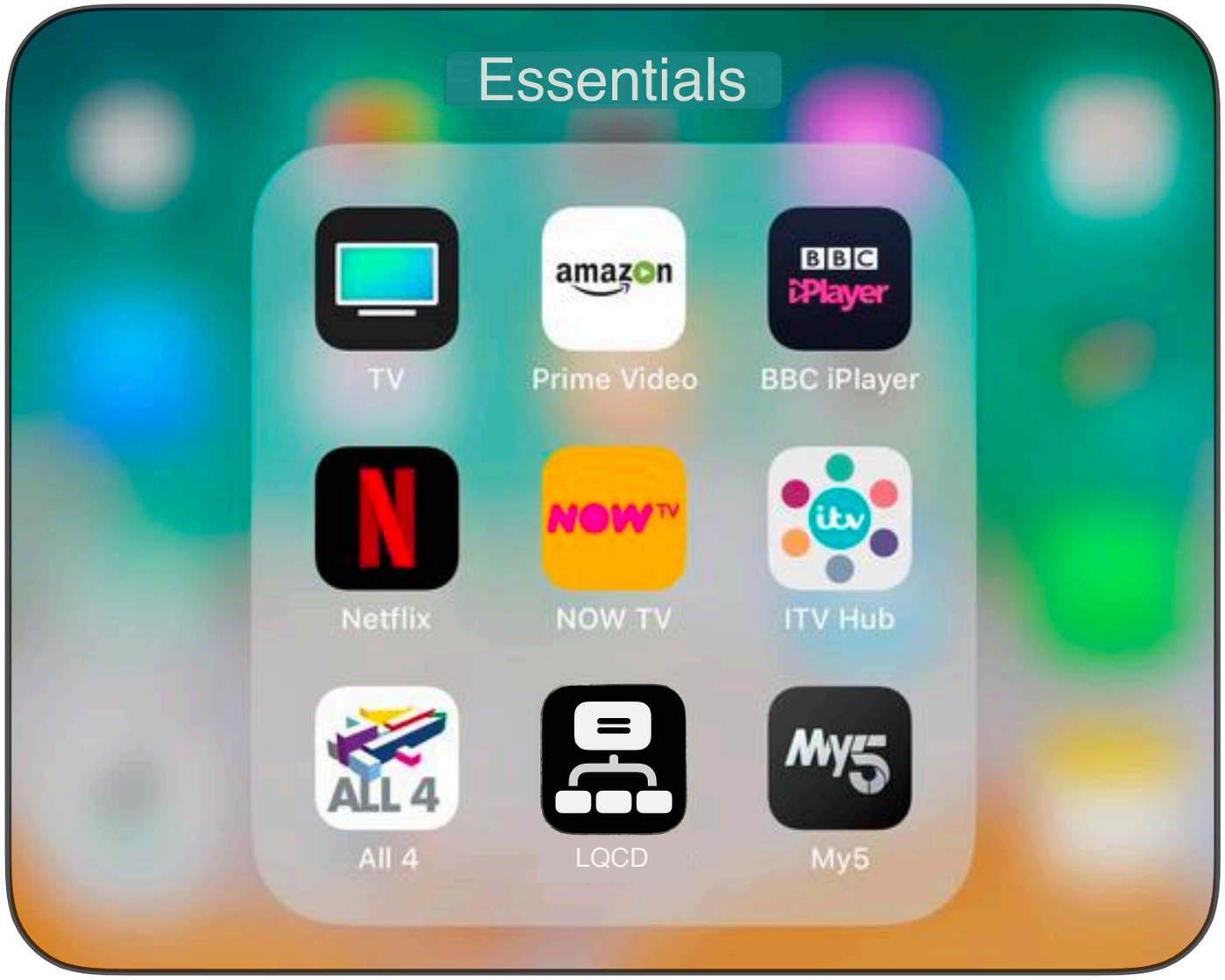
the three-body problem: lattice QCD

How does the three-body problem creep its head in lattice QCD?



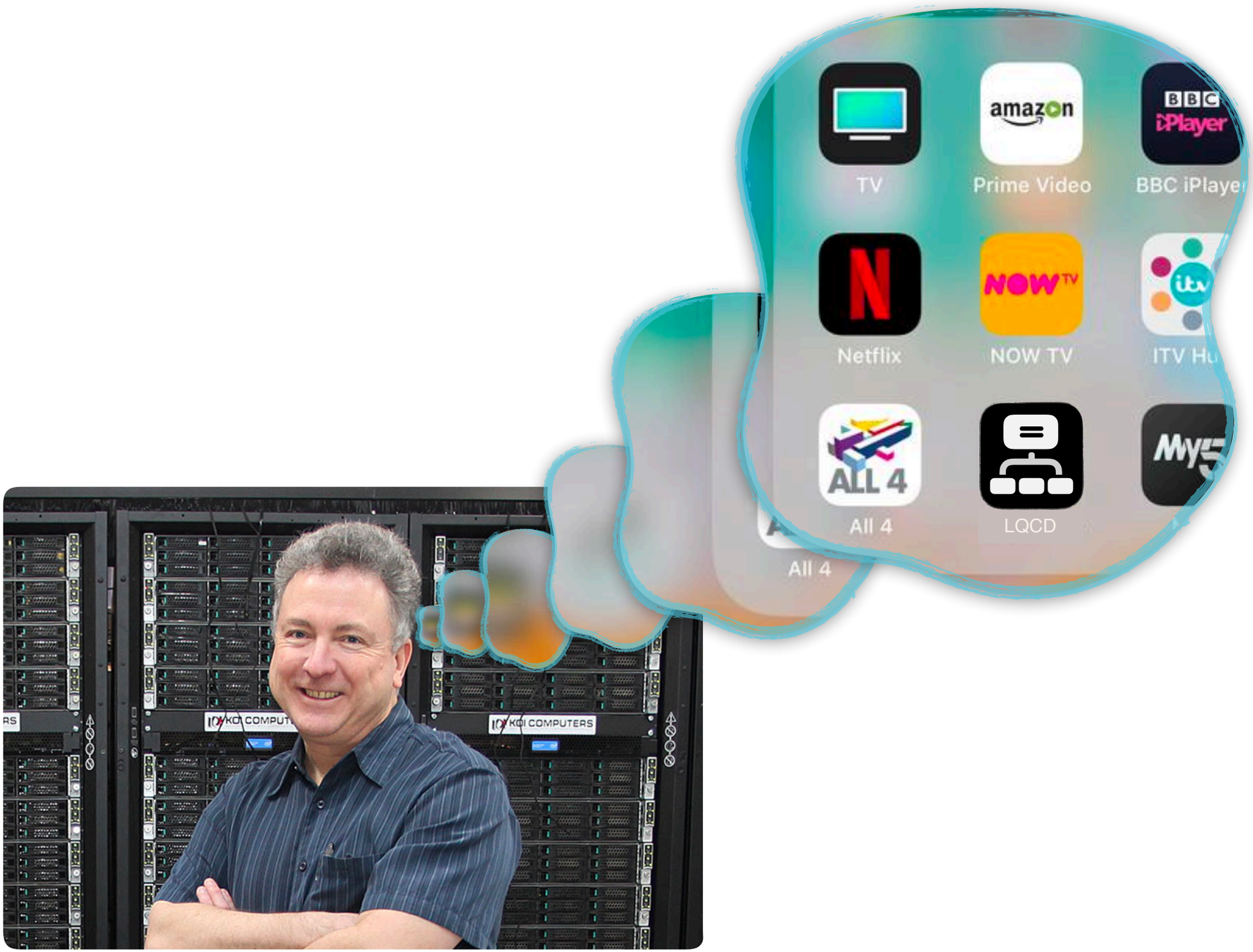
Dudek, Edwards, Guo, Thomas (2013)

Lattice QCD in not *"brute force"*



maybe this is people's idea of lattice QCD….?

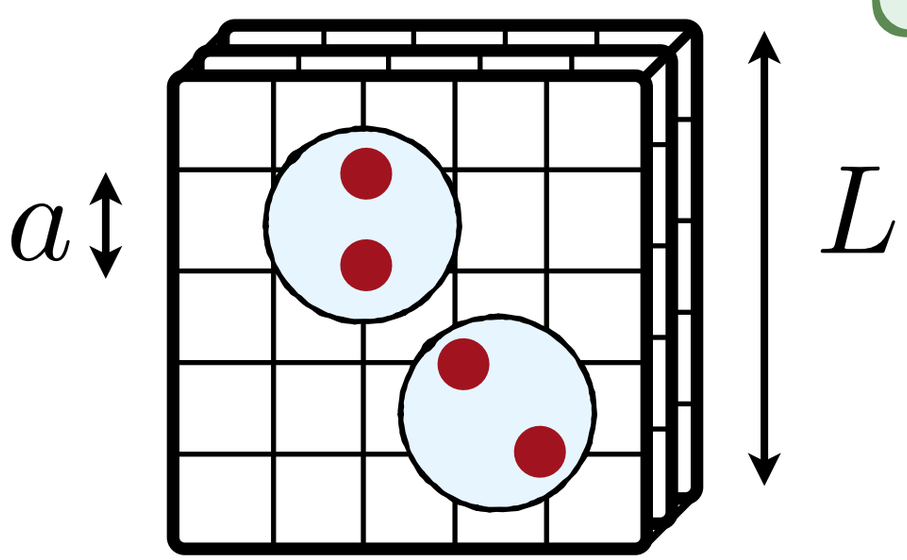
Lattice QCD in not *"brute force"*



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \left(\qquad \right) \Big\updownarrow (L/a)^3 \times (T/a)$$



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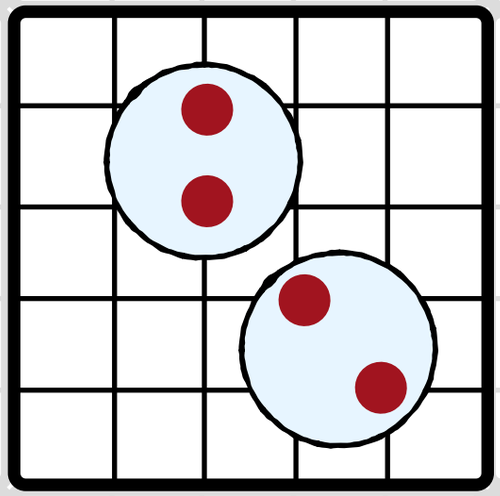
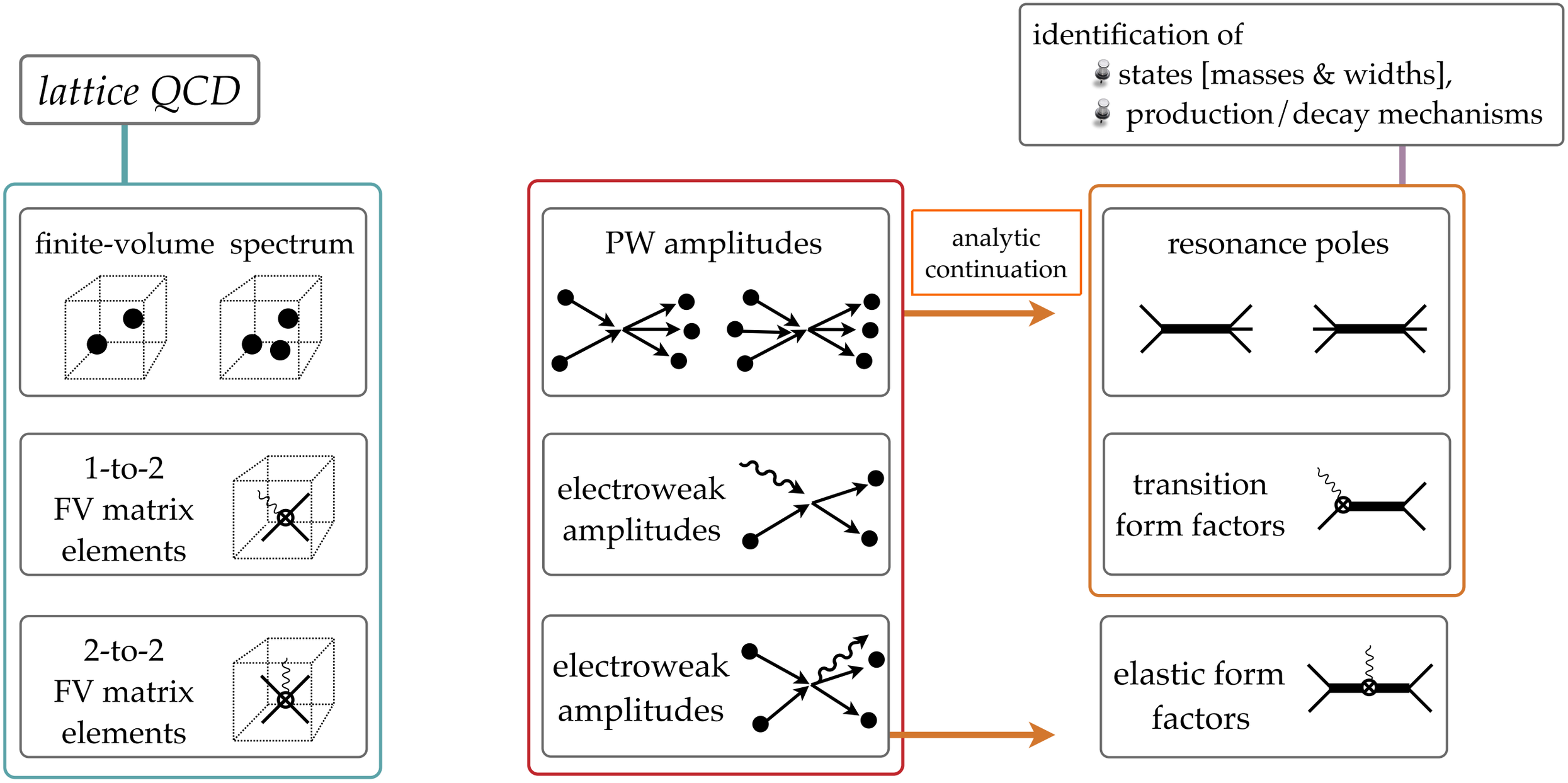


Never free!
No asymptotic states!
No scattering!

in general, you can't separate states by particle number!

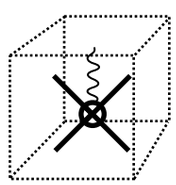
few-body systems in LQCD



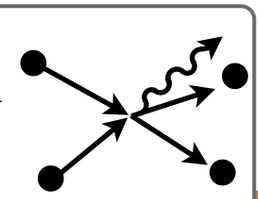
few-body systems in LQCD

lattice QCD

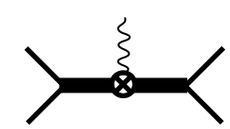
2-to-2
FV matrix
elements



electroweak
amplitudes



elastic form
factors



structure and nature of states

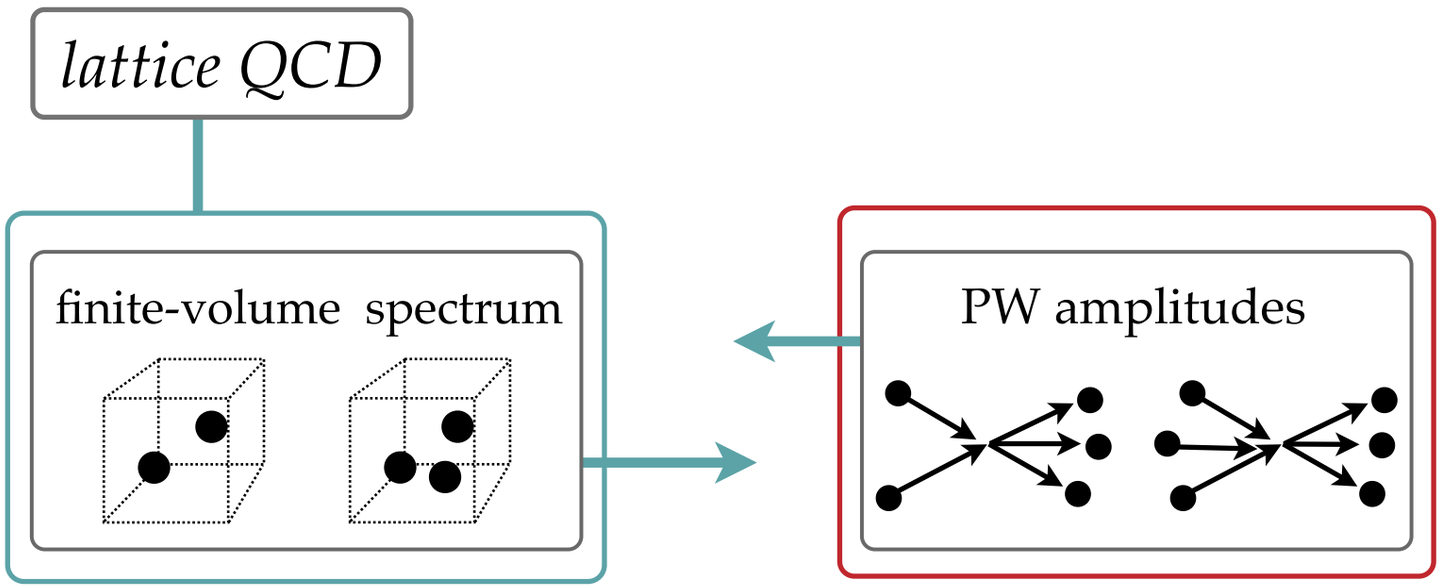
Baroni

Hansen

Ortega-Gamma

RB, Hansen (2015)
Baroni, RB, Hansen, Ortega-Gamma (2018)

few-body systems in LQCD



Alternative approaches

Perturbative - Beane, Detmold, Savage, Tan

EFT driven - Rusetski, Hammer, et al.

Unitarity driven - Mai & Doring

Alternative approaches

Perturbative - Beane, Detmold, Savage, Tan

PT is not good enough for most channels or kinematics of interest

EFT driven - Rusetski, Hammer, et al.

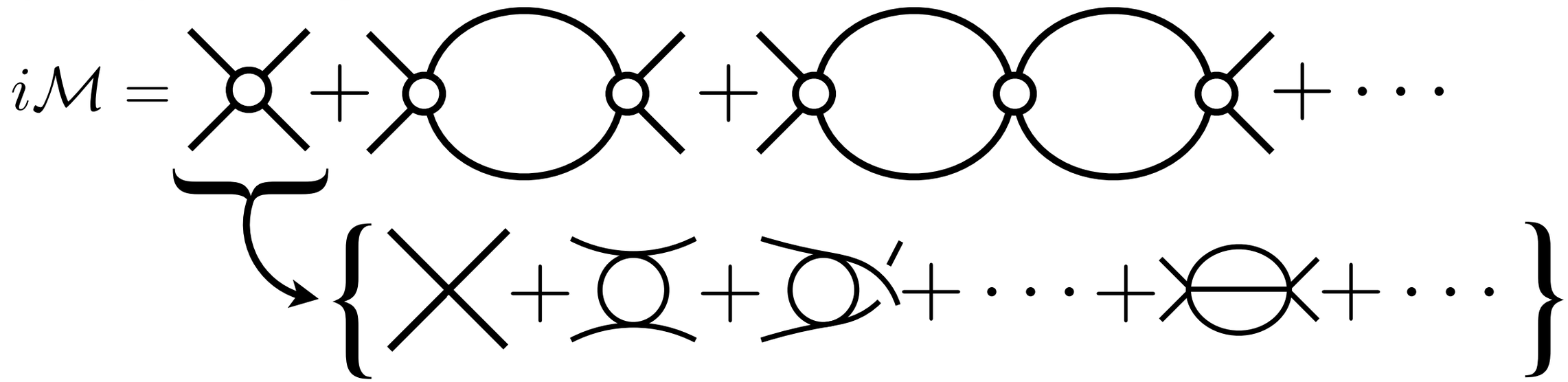
requires having a well defined and convergent EFT for kinematics and channel desired…rarely true.

Unitarity driven - Mai & Doring

Requires first deriving unitary conditions in an infinite-volume, then replacing unitarity cuts with finite cuts. Unfortunately, lots of information is lost in the infinite-volume limit [e.g. partial wave mixing]

Two-body scattering

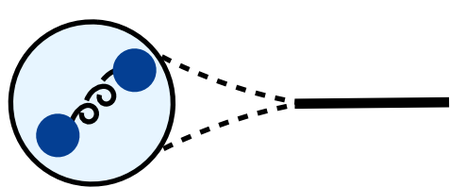
Unitarity using all orders perturbation theory:



non-perturbative kernel including all diagrams not shown…

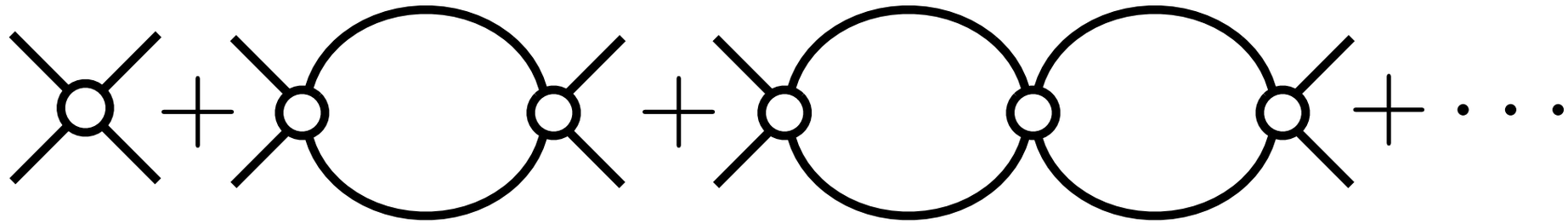
"yep, the left hand cut is there"

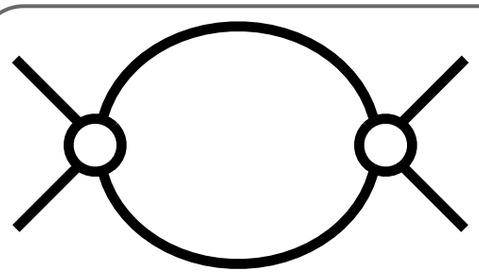
IR limit of QCD, only interested in hadronic d.o.f.



Two-body scattering

Unitarity using all orders perturbation theory:

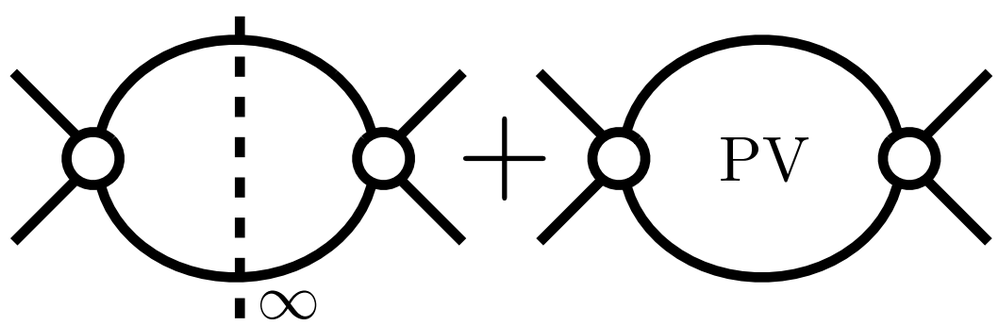
$i\mathcal{M} =$ 



$$= \int \frac{d^4k}{(2\pi)^4} [iB(k,P)]^2 \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{[iB(k,P)]^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} + \text{“smooth”}$$

$$= [iB_{on}]\rho\,[iB_{on}] + \text{“PV integral”}$$

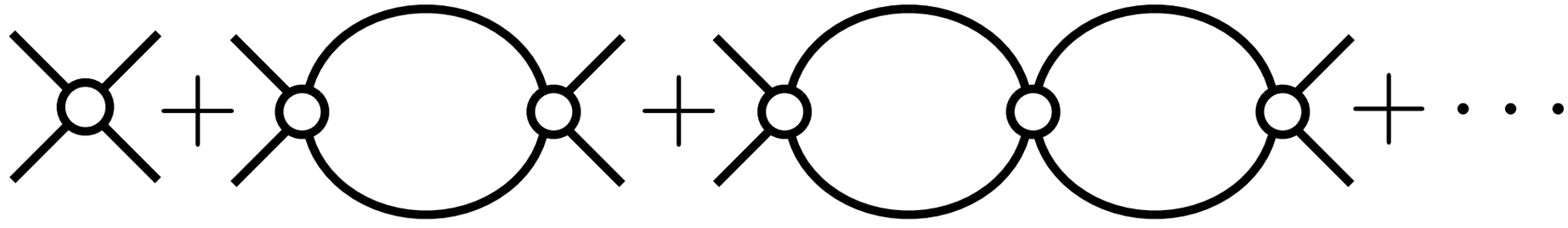
$=$ 

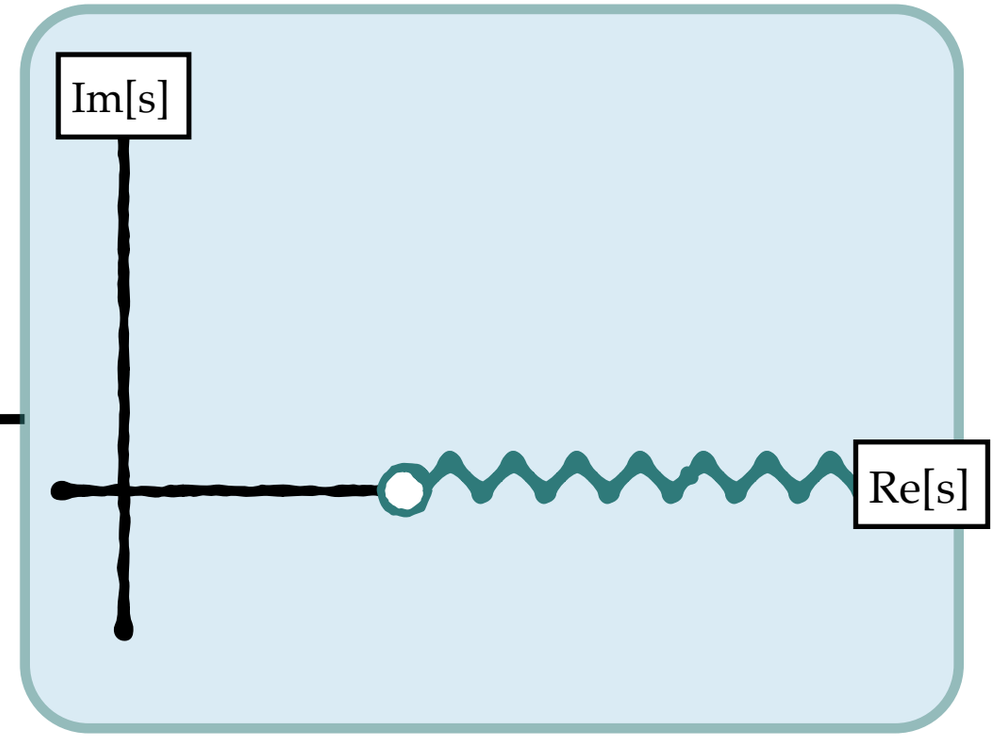
$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

Unitarity using all orders perturbation theory:

$i\mathcal{M} =$ 



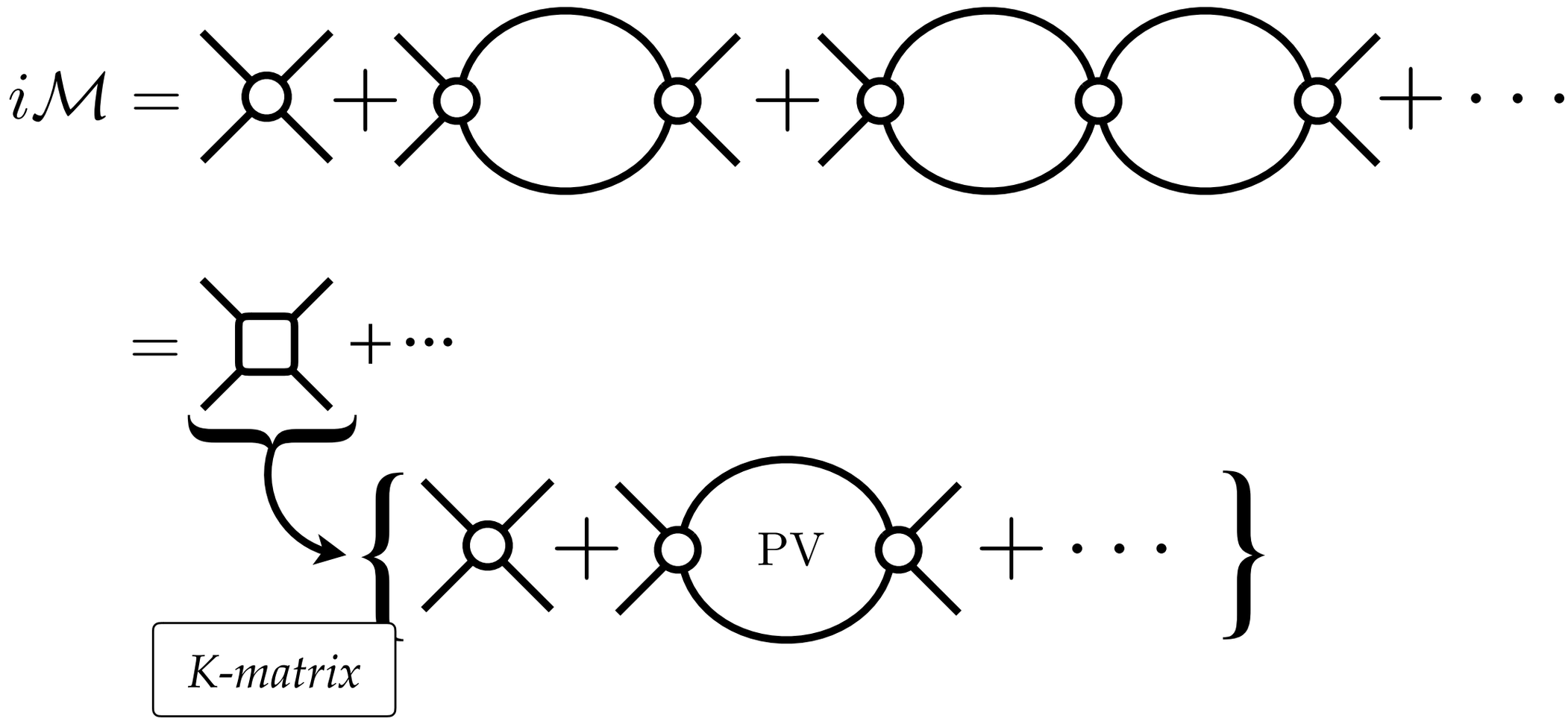
2^N sheets for N open channels

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$

square root singularity.

Two-body scattering

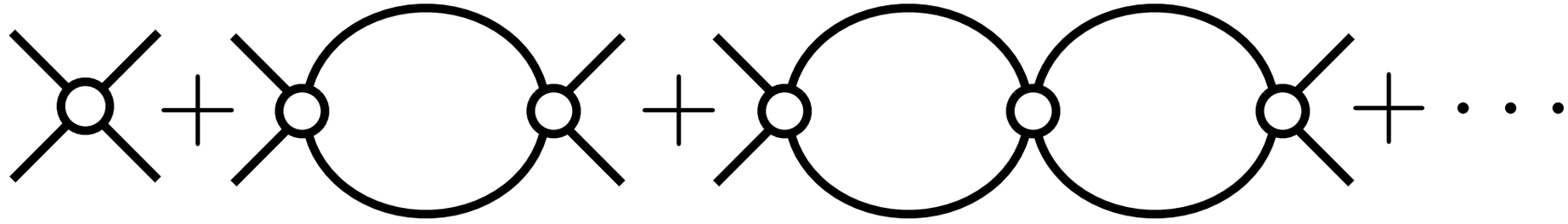
Unitarity using all orders perturbation theory:



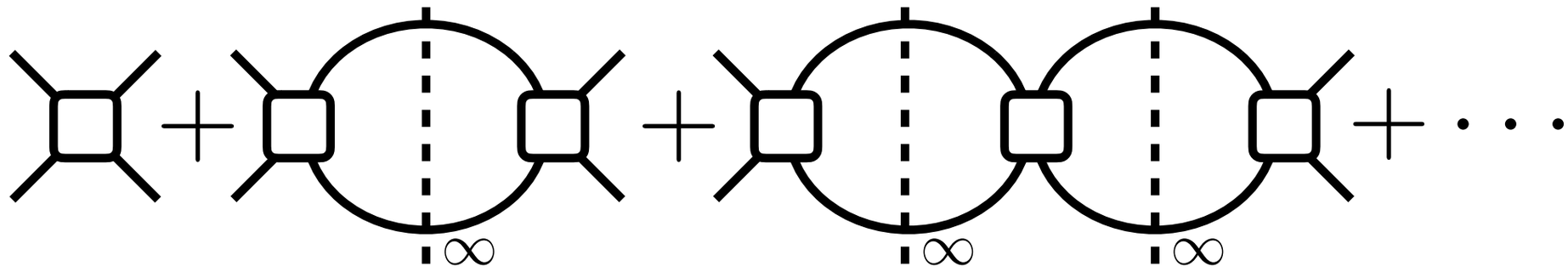
Two-body scattering

Unitarity using all orders perturbation theory:

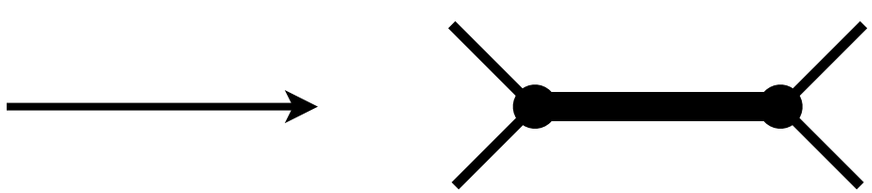
$$i\mathcal{M} =$$

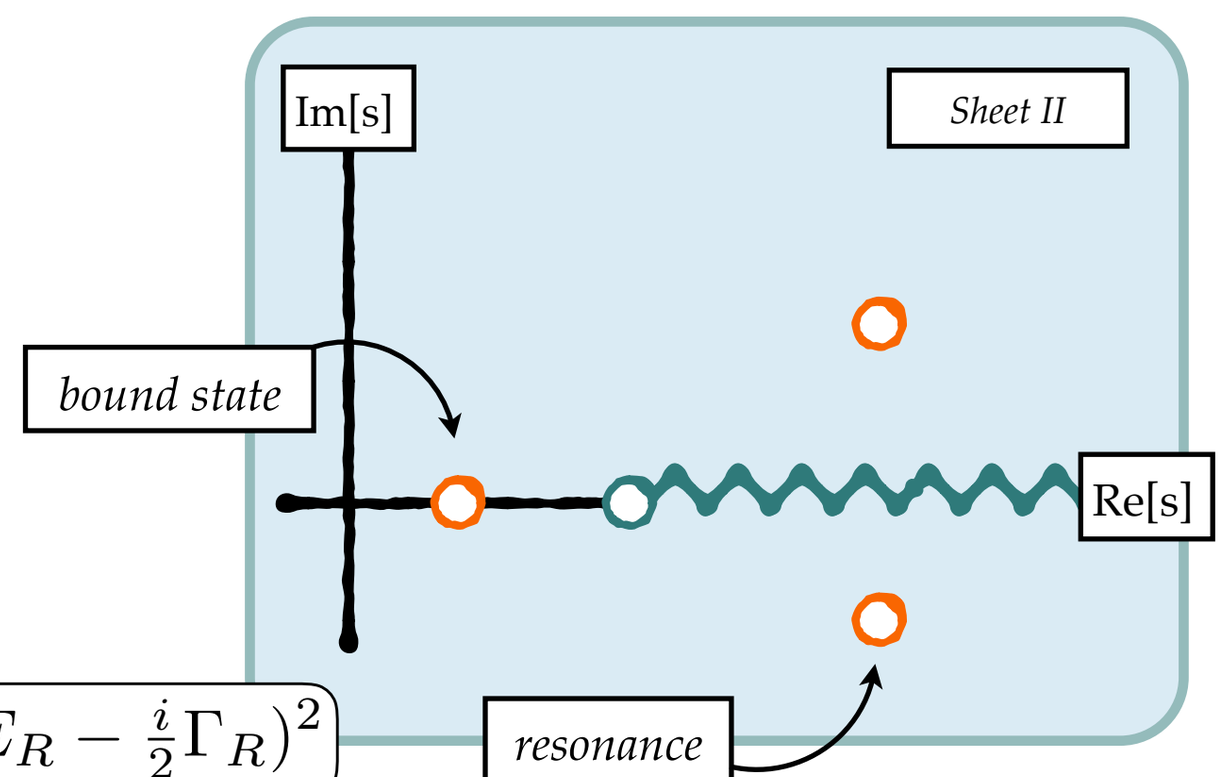


$$=$$



$$= i\mathcal{K}\,\frac{1}{1 - i\rho\mathcal{K}}$$

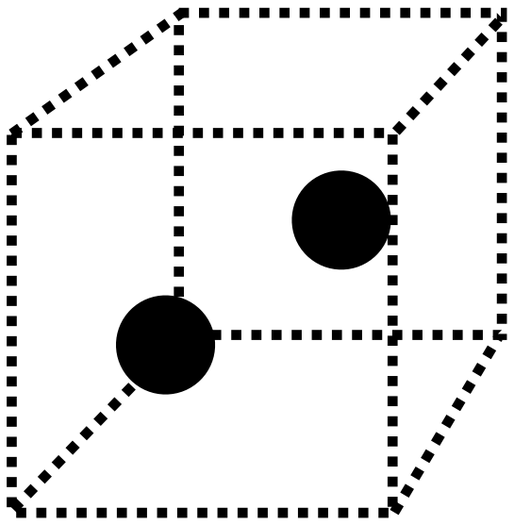




$$s_R = (E_R - \tfrac{i}{2}\Gamma_R)^2$$

Two-particle in finite volume

Next, we want to consider two particles in a box, but before we do this we need to understand how one particle behaves.

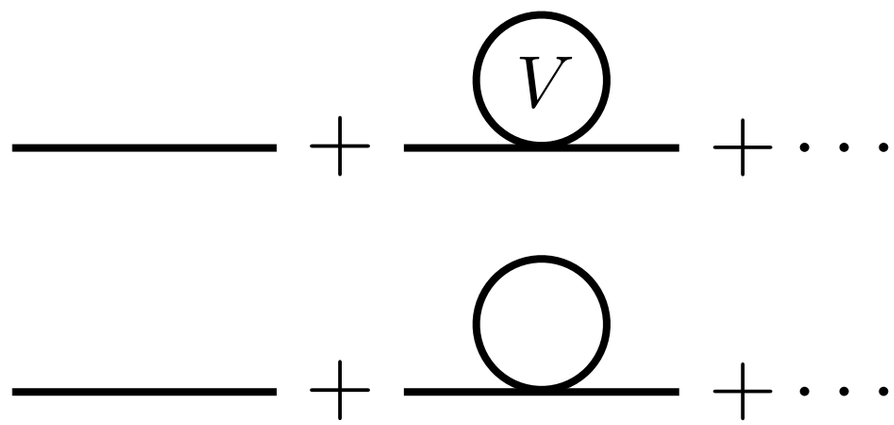


Two-particle in finite volume

Next, we want to consider two particles in a box, but before we do this we need to understand how one particle behaves.

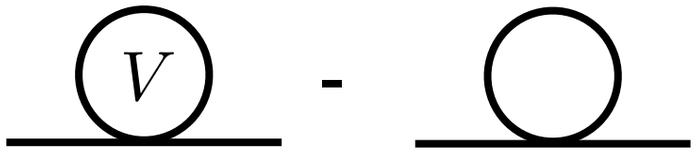
Masses of stable states are exponentially close

$$\frac{iZ_L(p^2)}{p^2+m_L^2} =$$



$$\approx \frac{iZ(p^2)}{p^2+m^2}$$

Exponentially suppressed effects in 1+1D



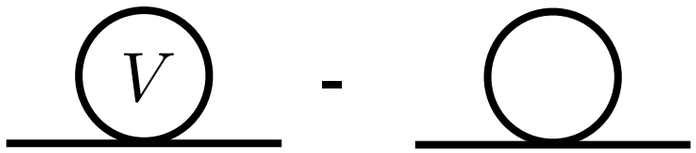
$$\sim \left[\frac{1}{L}\sum_{k_z} - \int \frac{dk_z}{2\pi}\right] \int \frac{dk_0}{2\pi} \frac{i}{k_0^2 - k_z^2 - m^2 + i\epsilon}$$

$$= \left[\frac{1}{L}\sum_{k_z} - \int \frac{dk_z}{2\pi}\right] \frac{1}{2\sqrt{k_z^2 + m^2}}$$

$$= \int \frac{dk_z}{2\pi} \left[\sum_n e^{ik_z L n} - 1\right] \frac{1}{2\sqrt{k_z^2 + m^2}}$$

$$= \sum_{n \neq 0} \int \frac{dk_z}{2\pi} \frac{e^{ik_z L n}}{2\sqrt{k_z^2 + m^2}}$$

Exponentially suppressed effects in 1+1D



$$\sim \left[\frac{1}{L}\sum_{k_z} - \int \frac{dk_z}{2\pi}\right] \int \frac{dk_0}{2\pi} \frac{i}{k_0^2 - k_z^2 - m^2 + i\epsilon}$$

$$= \left[\frac{1}{L}\sum_{k_z} - \int \frac{dk_z}{2\pi}\right] \frac{1}{2\sqrt{k_z^2 + m^2}}$$

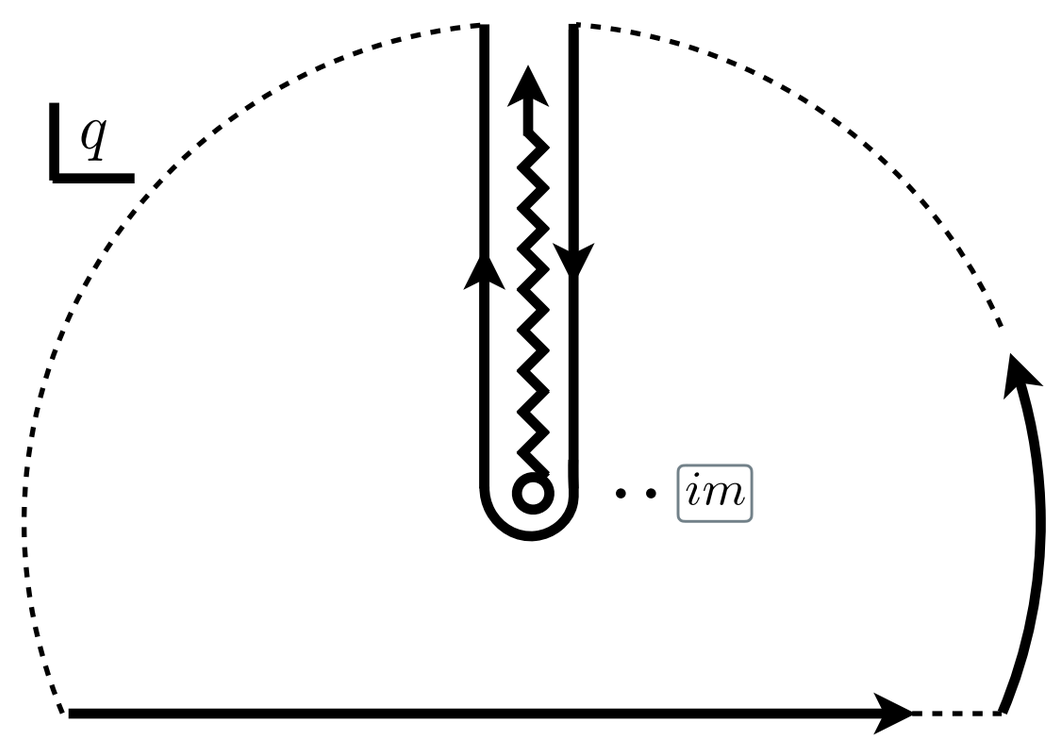
$$= \int \frac{dk_z}{2\pi} \left[\sum_n e^{ik_z L n} - 1\right] \frac{1}{2\sqrt{k_z^2 + m^2}}$$

$$= \sum_{n\neq 0} \int \frac{dk_z}{2\pi} \frac{e^{ik_z L n}}{2\sqrt{k_z^2 + m^2}}$$

Only the integral along the cut contributes…
changing variables ($k_z = iqm$)…

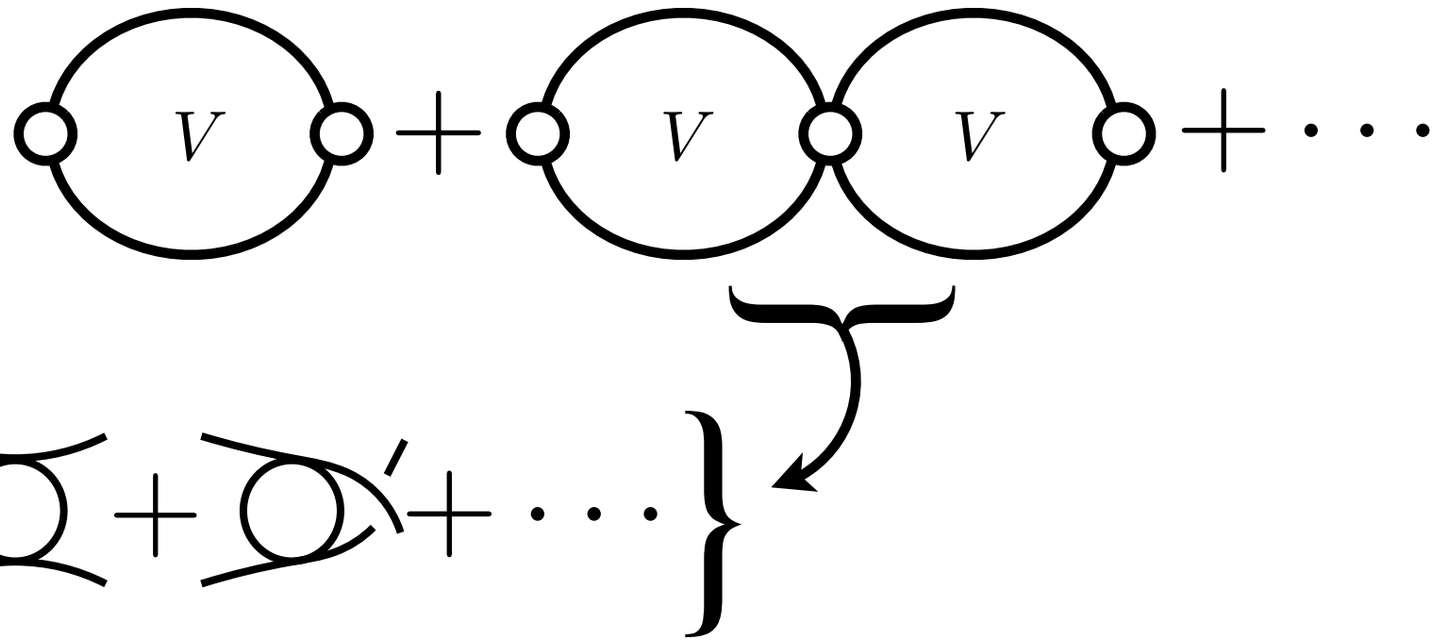
$$\sim \sum_{n\neq 0} \int_1^\infty dq \frac{1}{\sqrt{q^2 - 1}} e^{-q\, mL|n|} \sim \frac{e^{-mL}}{\sqrt{mL}}$$

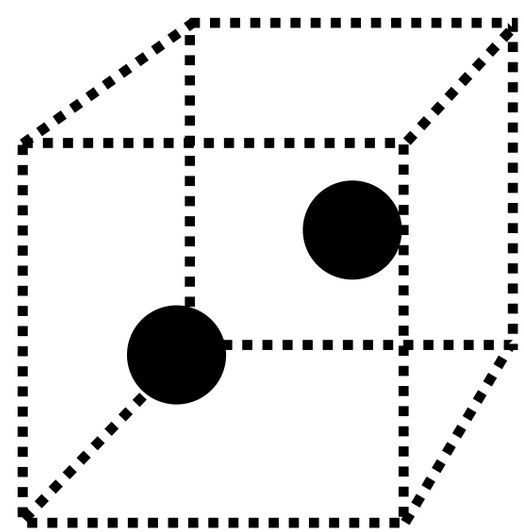
if mL >>1



Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

$C_L^{2pt.}(P) =$ 



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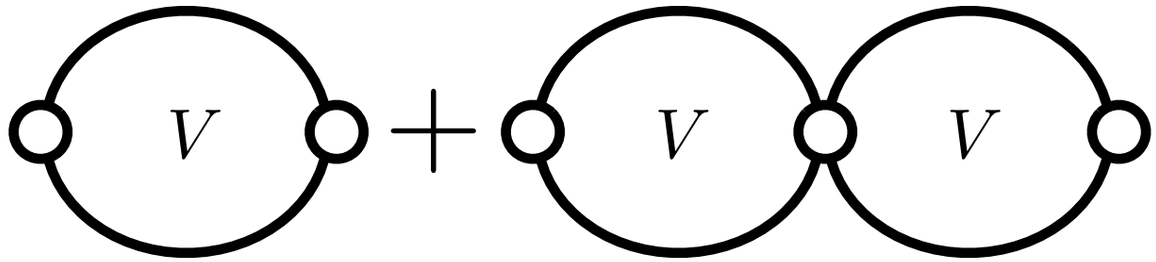
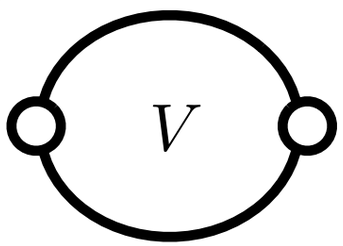


$$+ \cdots$$



$$= \frac{1}{L^3} \sum_{\mathbf{k}} \frac{iB_{\ell' m'}\, iB_{\ell m}}{(2\omega_k)^2} \frac{i\, 4\pi\, Y_{\ell m}(\hat{k})\, Y^*_{\ell' m'}(\hat{k})}{E - 2\omega_k} + \text{“smooth”} \qquad [\mathbf{k} = 2\pi \mathbf{n}/L]$$

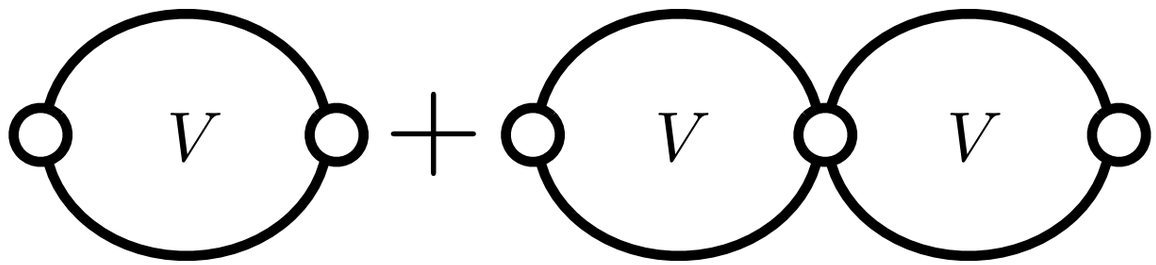
on-shellness: $E = 2\sqrt{k^2 + m^2}$

fixes the magnitude but not the direction of the momentum

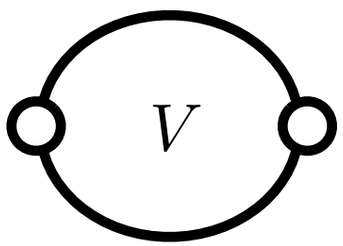
Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

$$C_L^{2pt.}(P) =$$



$$+ \cdots$$



$$= \frac{1}{L^3} \sum_{\mathbf{k}} \frac{iB_{\ell' m'}\, iB_{\ell m}}{(2\omega_k)^2} \frac{i\,4\pi\, Y_{\ell m}(\hat{k})\, Y^*_{\ell' m'}(\hat{k})}{E - 2\omega_k} + \text{“smooth”} \qquad [\mathbf{k} = 2\pi \mathbf{n}/L]$$

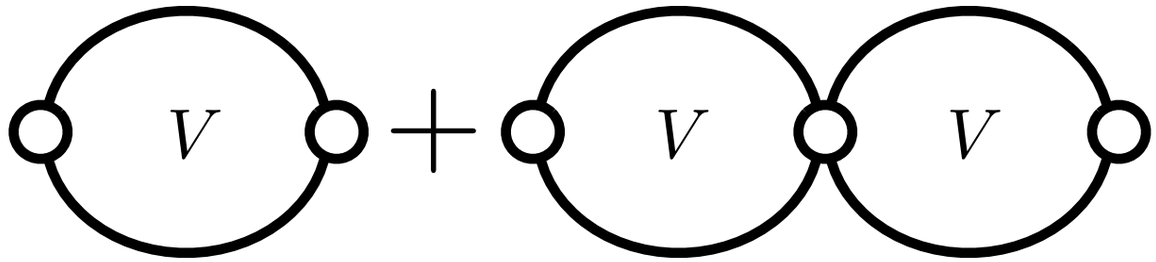
$$= (iB_{\ell' m'}) \left(\boxed{\left[\frac{1}{L^3} \sum_{\mathbf{k}}^{\text{UV}} - \int_{\mathbf{k}}^{\text{UV}} \right]} \frac{1}{(2\omega_k)^2} \frac{i\,4\pi\, Y_{\ell m}(\hat{k})\, Y^*_{\ell' m'}(\hat{k})}{E - 2\omega_k + i\epsilon} \right) (iB_{\ell m}) + i\epsilon \text{ integral}$$

cut-off dependence is exponentially suppressed

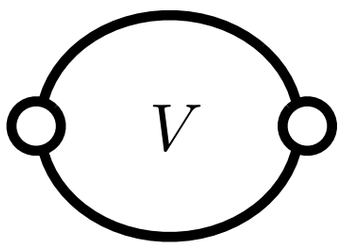
freedom: one could subtract the PV integral,

Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

$C_L^{2pt.}(P) =$ 

$+ \cdots$



$$= \frac{1}{L^3}\sum_{\mathbf{k}} \frac{iB_{\ell' m'}\, iB_{\ell m}}{(2\omega_k)^2} \frac{i\,4\pi\, Y_{\ell m}(\hat{k})\, Y^*_{\ell' m'}(\hat{k})}{E - 2\omega_k} + \text{“smooth”} \qquad [\mathbf{k} = 2\pi\mathbf{n}/L]$$

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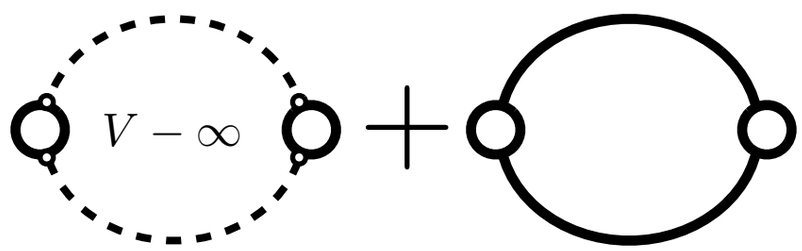
$$\equiv [iB]\, iF\, [iB] + i\epsilon \text{ integral}$$

$=$ 

$+$

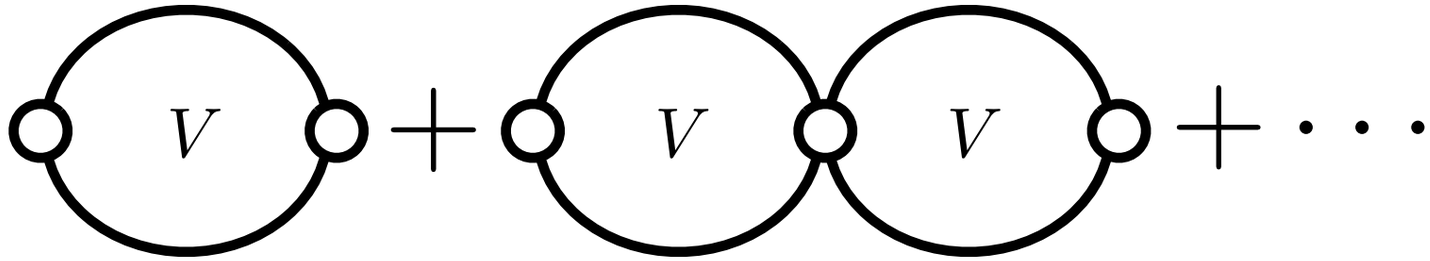
F replaces ρ

“a simple square root singularity is replaced by a function that has both simples poles and the square root singularity”

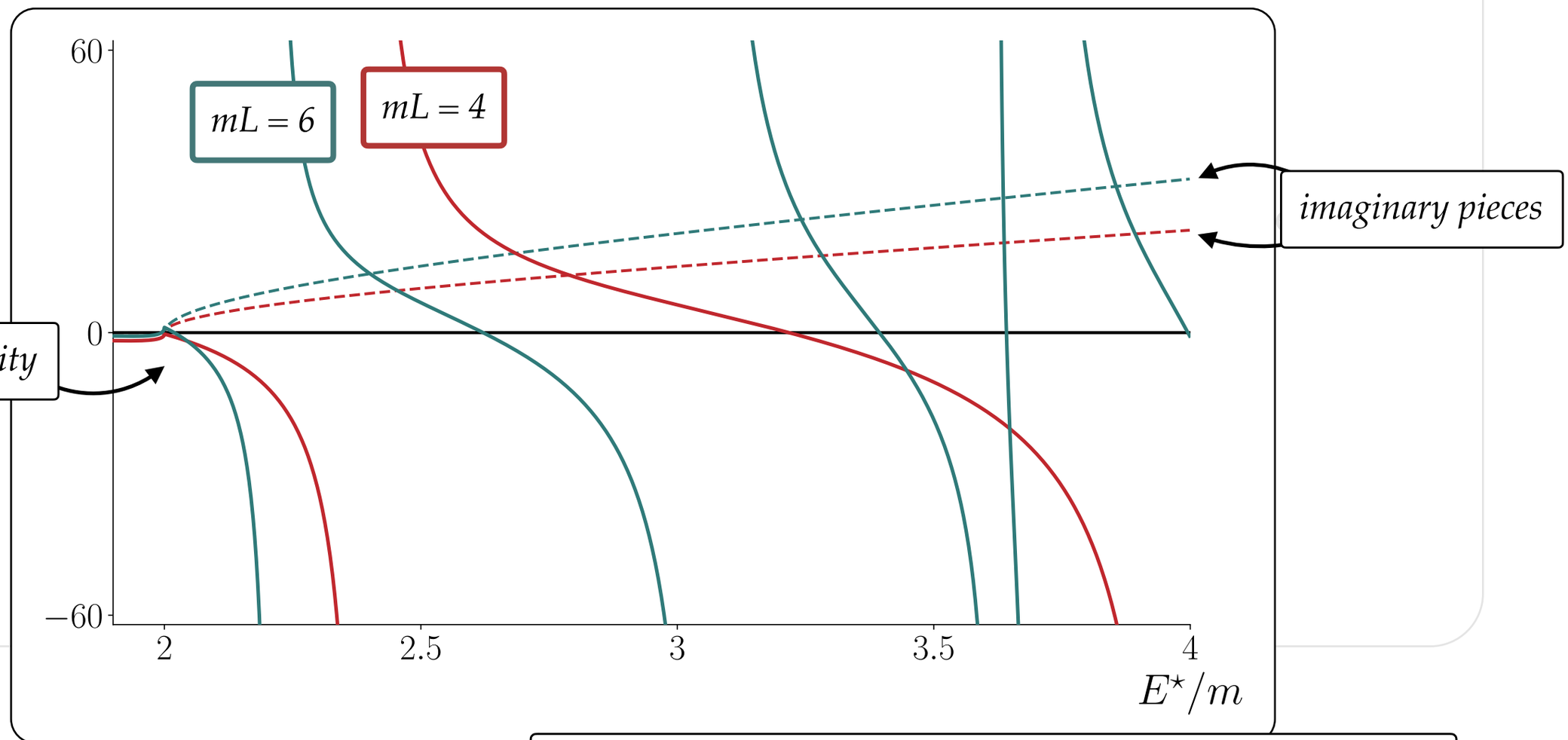
Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m*):

$$C_L^{2pt.}(P) =$$



$$+ \cdots$$



"a simple square root singularity is replaced by a function that has both simples poles and the square root singularity"

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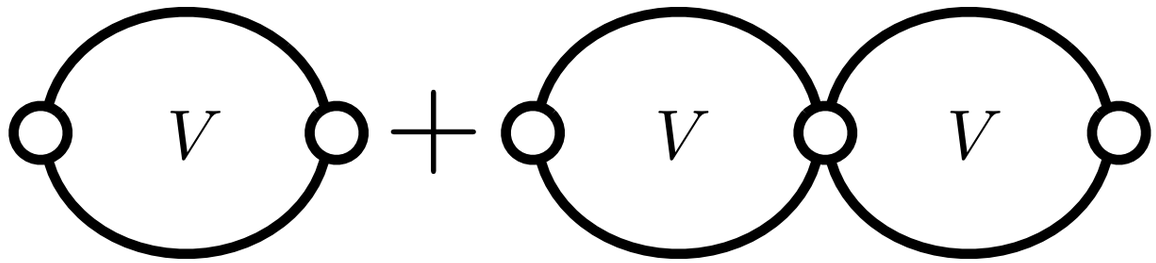

$$+ \cdots$$

$$= C_\infty(P) + \cdots$$

Two-particle in finite volume

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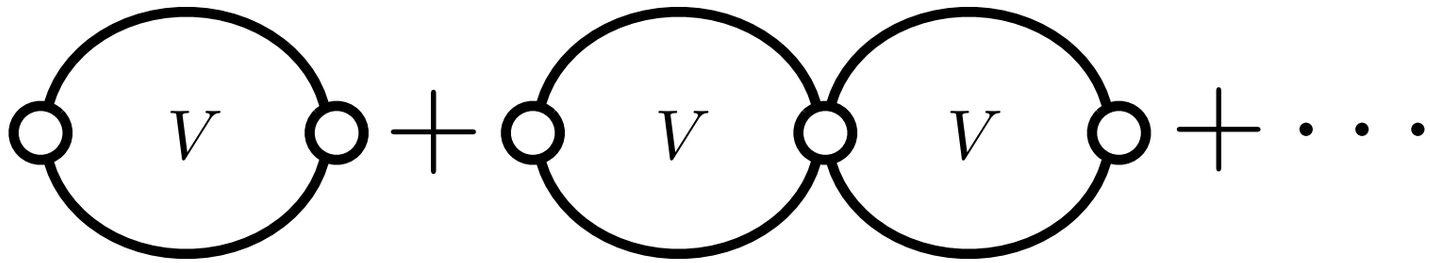
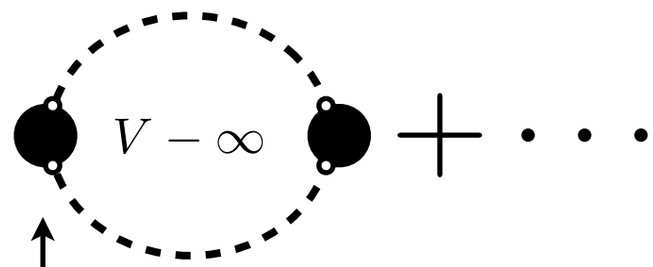


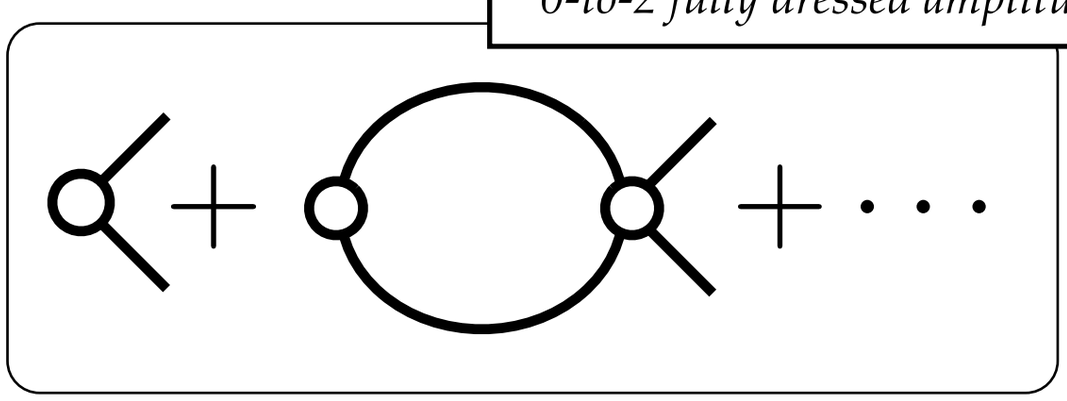
$$= C_\infty(P) +$$



$$+ \cdots$$

"F cut"

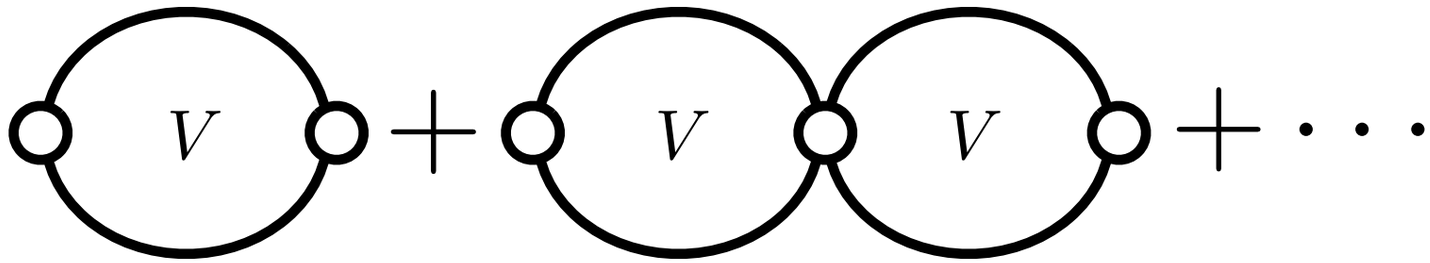
0-to-2 fully dressed amplitude



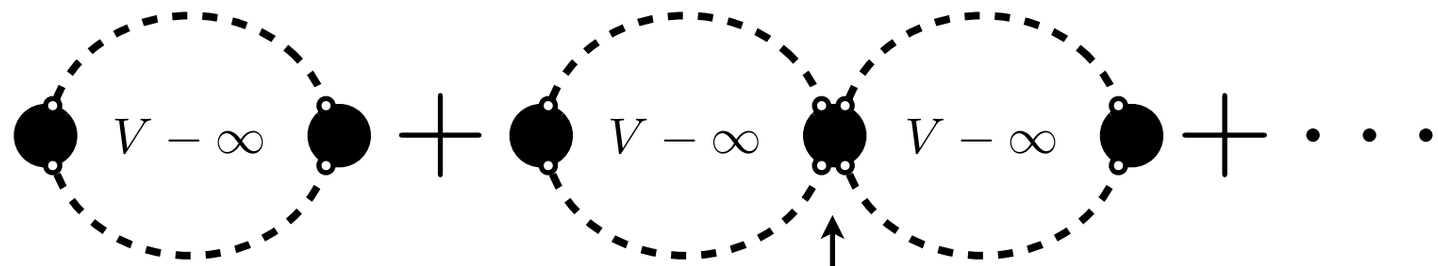
Two-particle in finite volume

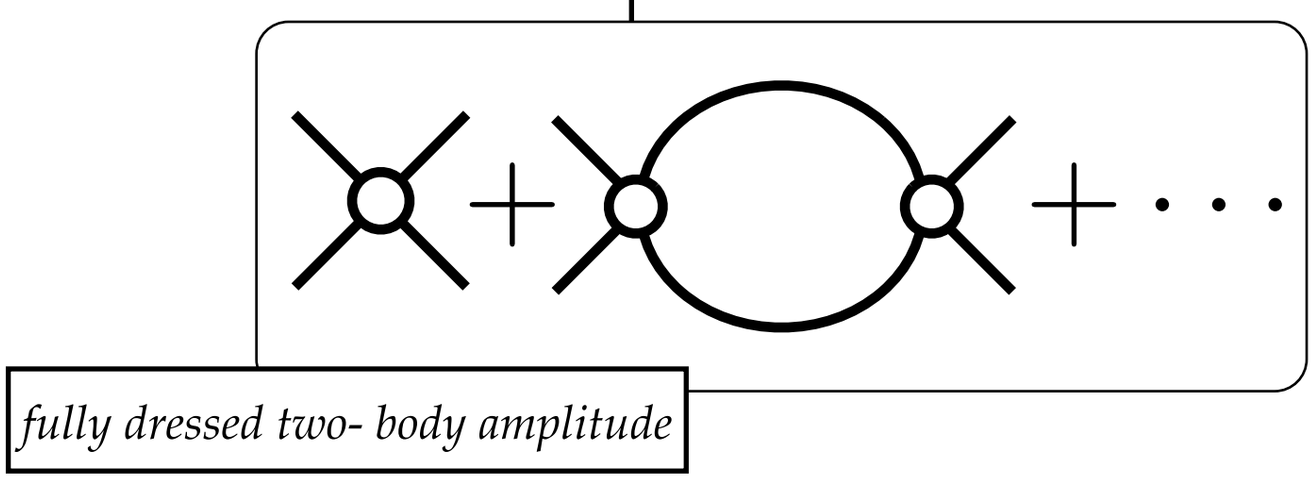
Consider the finite-volume two-particle correlator (*E~2m):*

$$C_L^{2pt.}(P) =$$



$$= C_\infty(P) +$$



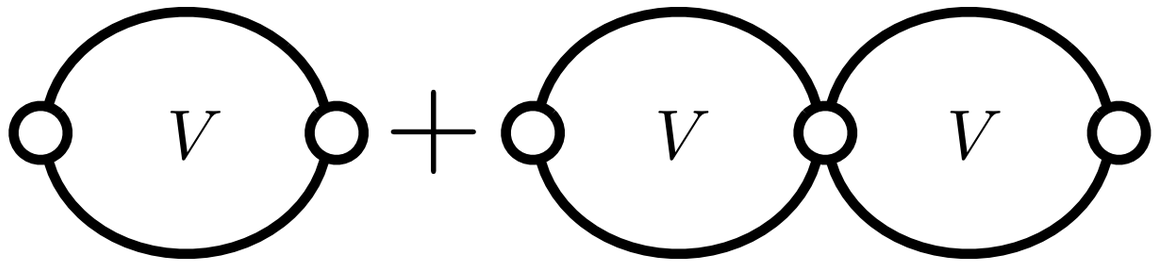


fully dressed two- body amplitude

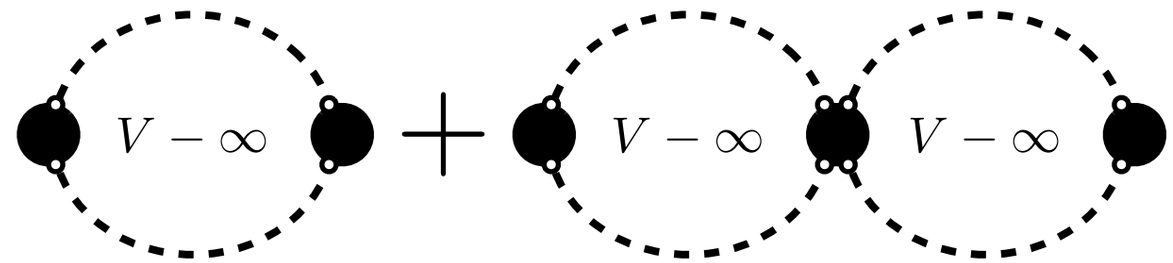
Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E~2m):*

$$C_L^{2pt.}(P) =$$



$$= C_\infty(P) +$$

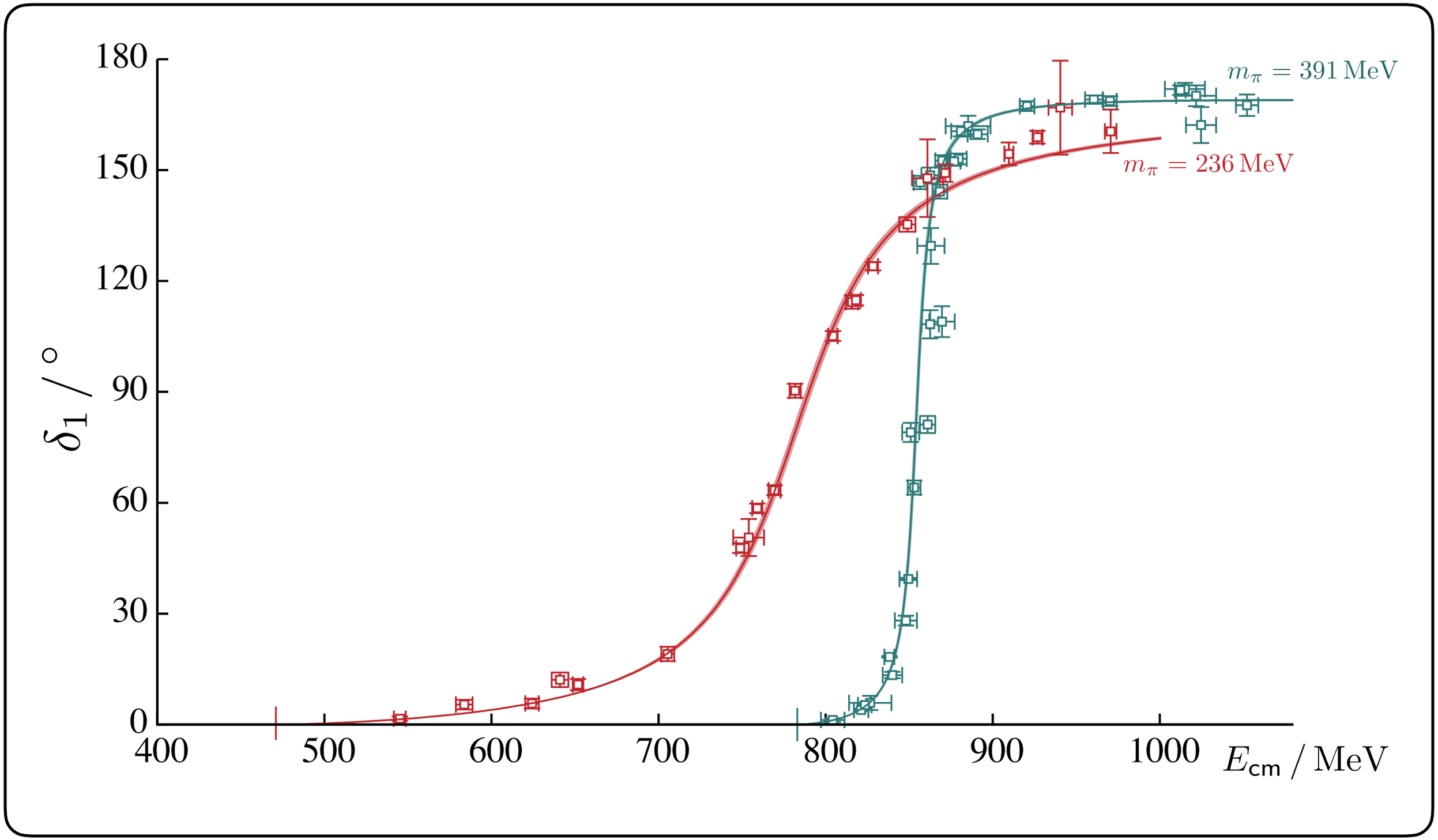


$$= \text{“smooth”} + A \frac{i}{F_2^{-1} + \mathcal{M}_2} B^\dagger$$

poles satisfy: $\det[F_2^{-1}(P, L) + \mathcal{M}_2(P^2)] = 0$

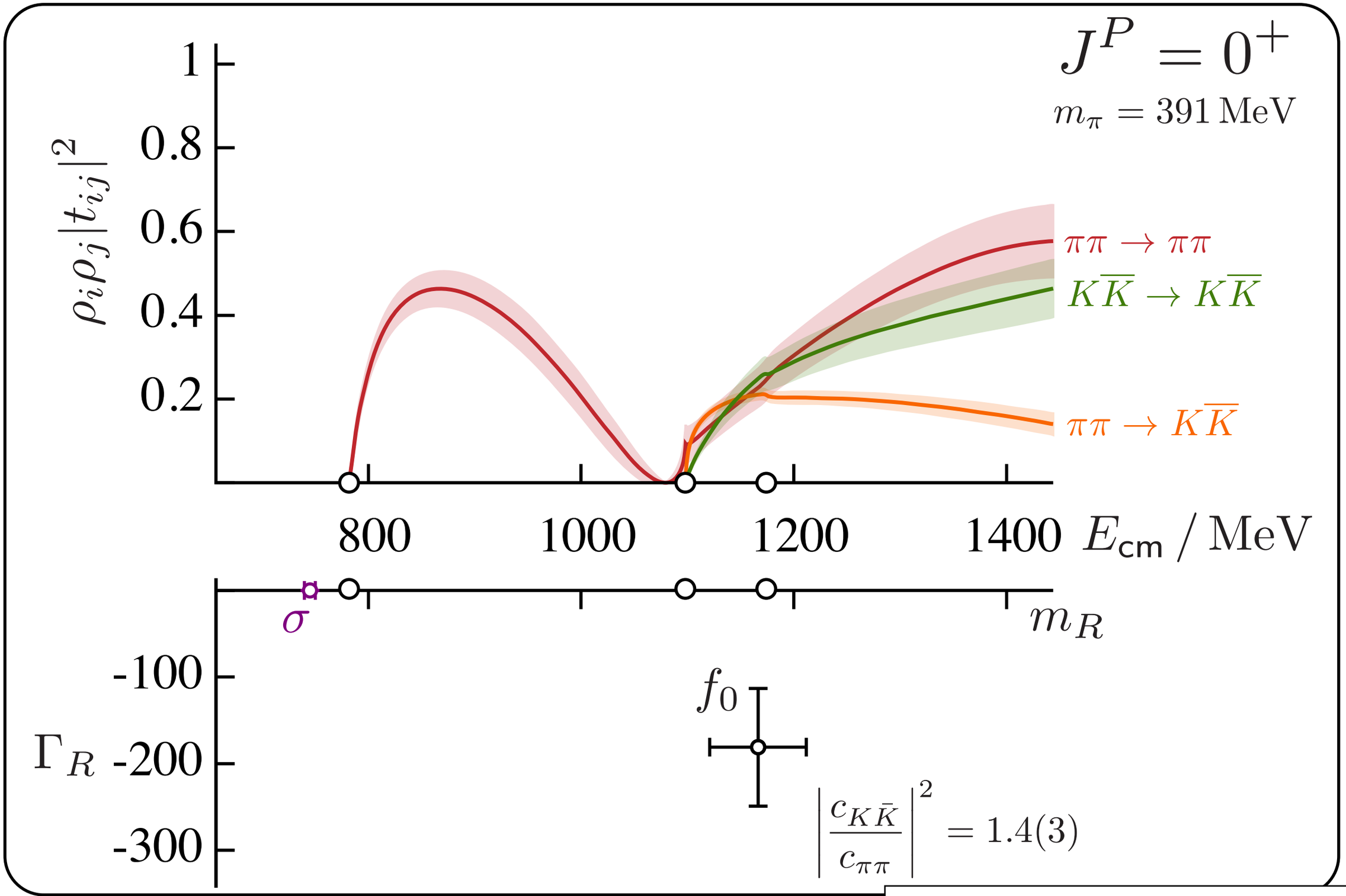
- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

$\pi\pi$ scattering - (I=1 channel)



Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

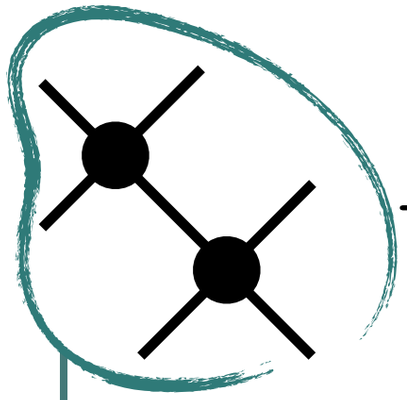
Coupled-channel scattering - (I=0 channel)



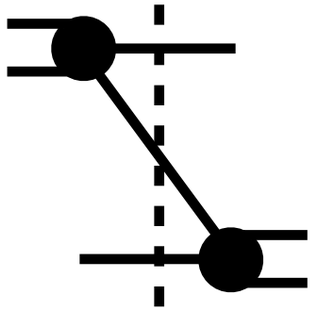
Wilson, RB, Dudek, & Edwards (2017)

Three particles in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.
Sum over all connected 3-to-3 diagrams…

$i\mathcal{M}_{3,\text{con.}} =$  $+ \cdots$

"exchange cuts"



$\sim i\mathcal{K}_2 \, iG \, i\mathcal{K}_2$

motivates the definition of dynamical quantities that are "free" of these simple divergences

Three particles in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.
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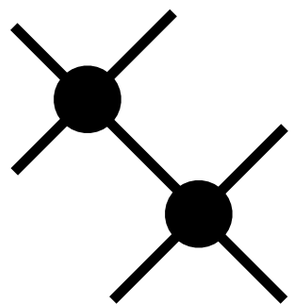
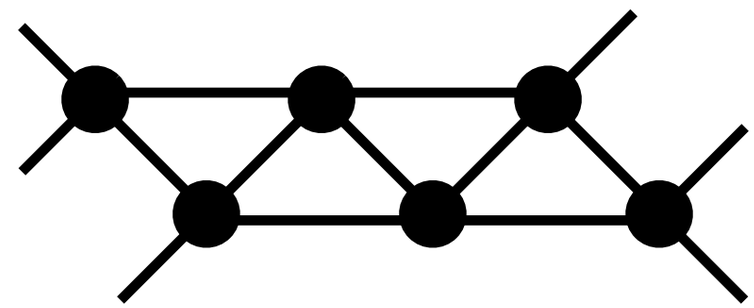
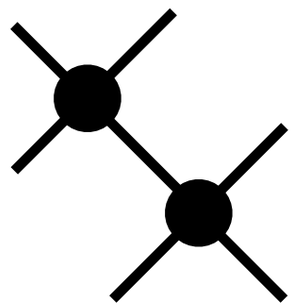
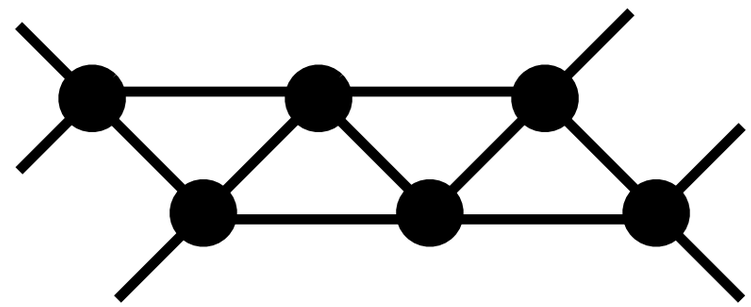
$i\mathcal{M}_{3,\text{con.}} =$  $+ \cdots +$  $+ \cdots$

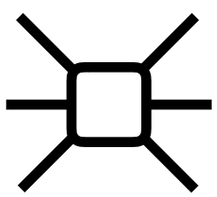
satisfies an integral equations

$$i\mathcal{D} = i\mathcal{M}_2 i G i\mathcal{M}_2 + \int i\mathcal{M}_2 i G \, i\mathcal{D}$$

Three particles in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.
Sum over all connected 3-to-3 diagrams…

$i\mathcal{M}_{3,\mathrm{con.}} =$  $+ \cdots +$  $+ \cdots$

$+$  $+ \cdots$

$K_{\mathrm{df},3}$: the closest thing to a model ind. 3body quantity

Three particles in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.
Sum over all connected 3-to-3 diagrams…

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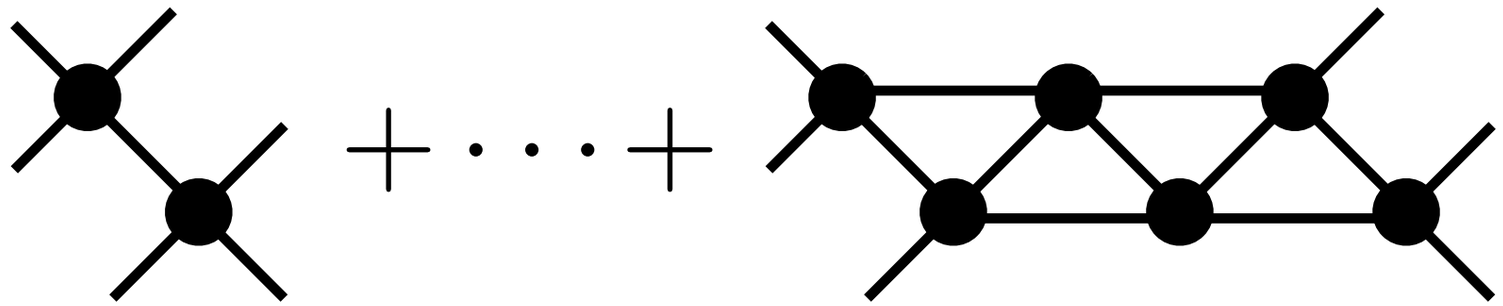
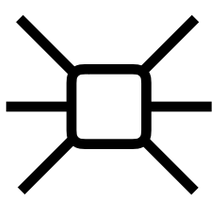


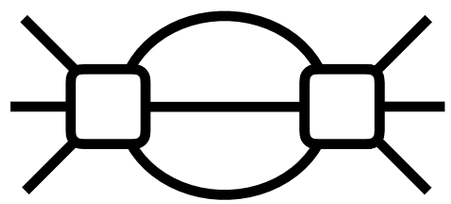
$+ \cdots +$

$+ \cdots$

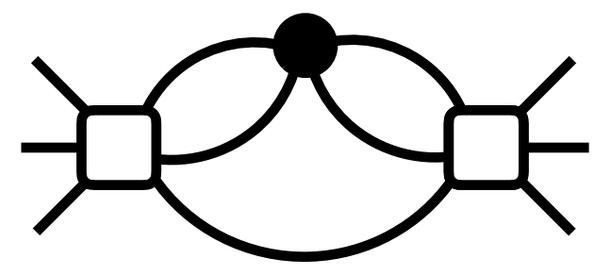
$+$



$+$



$+ \cdots +$



$+ \cdots$

any number of insertions leads to another integral equation

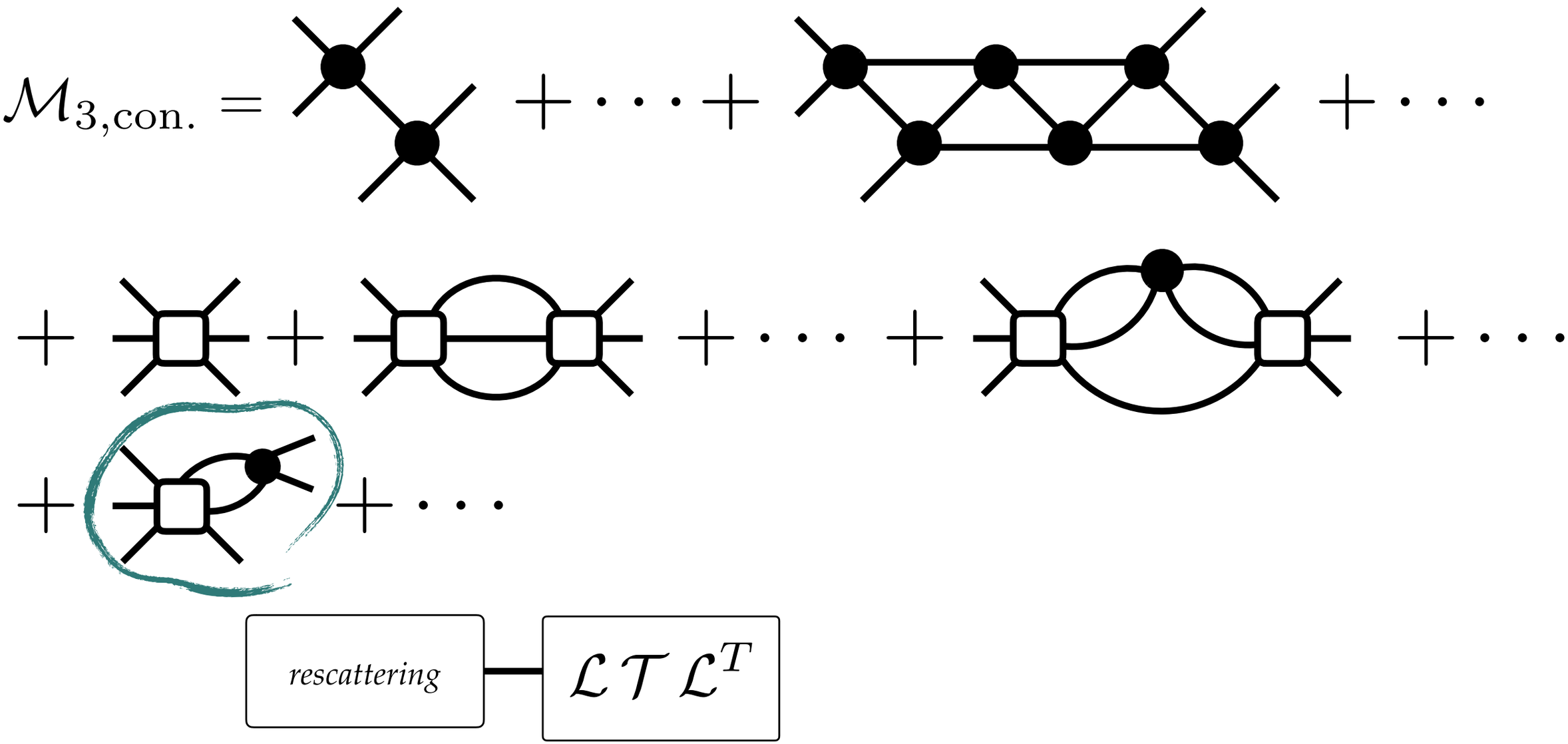
$$\mathcal{T} = \mathcal{K}_{\mathrm{df},3} - \int \mathcal{K}_{\mathrm{df},3}\, \rho\, \mathcal{L}\, \mathcal{T}$$

$$\mathcal{L} = \frac{1}{3} + \mathcal{M}_2 \rho - \mathcal{D}\rho$$

Three particles in an infinite volume

The three-body scattering amplitude using all orders perturbation theory.
Sum over all connected 3-to-3 diagrams…

$i\mathcal{M}_{3,\text{con.}} =$



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Sum over all connected 3-to-3 diagrams…

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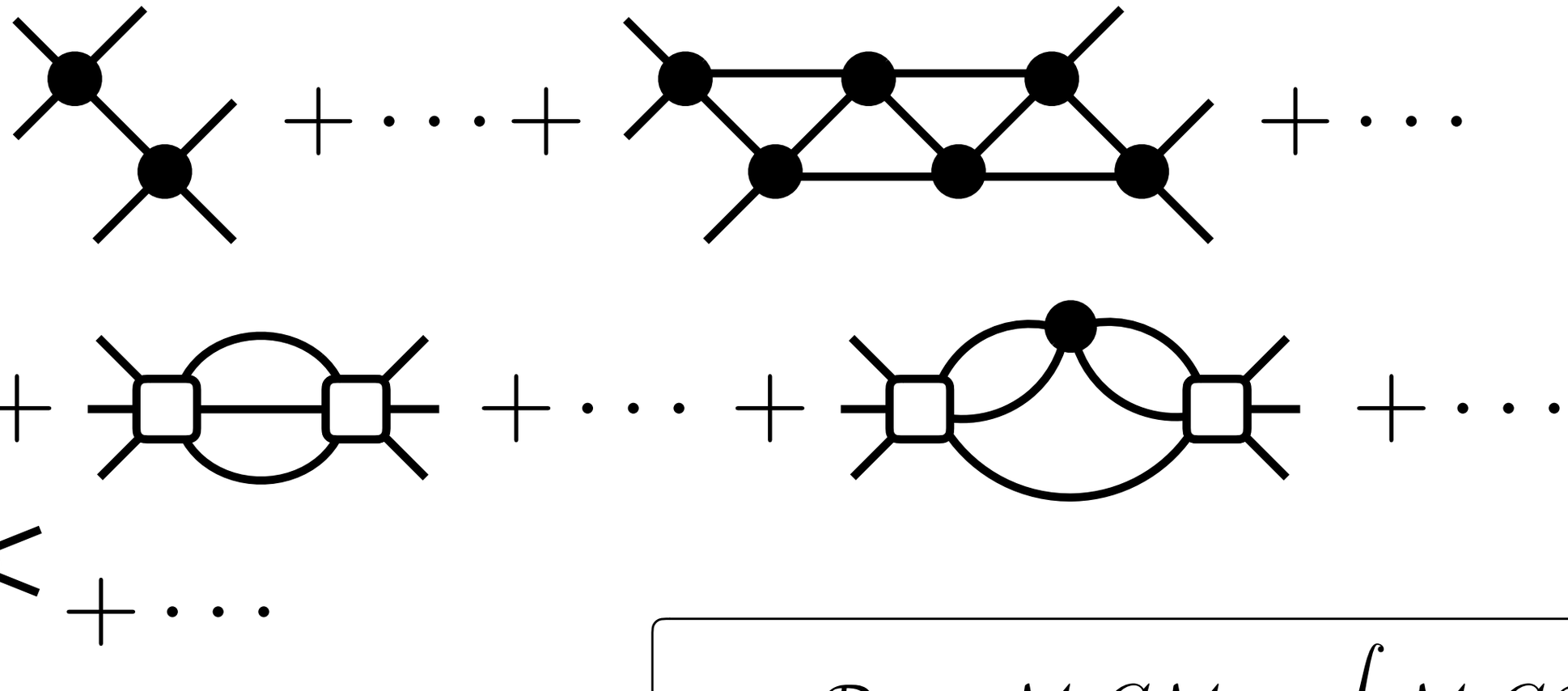


$$\mathcal{D} = -\mathcal{M}_2 G \mathcal{M}_2 - \int \mathcal{M}_2 G\, \mathcal{D}$$

$$\mathcal{L} = \frac{1}{3} + \mathcal{M}_2 \rho - \mathcal{D}\rho$$

$$\mathcal{T} = \mathcal{K}_{\text{df},3} - \int \mathcal{K}_{\text{df},3}\, \rho\, \mathcal{L}\, \mathcal{T}$$

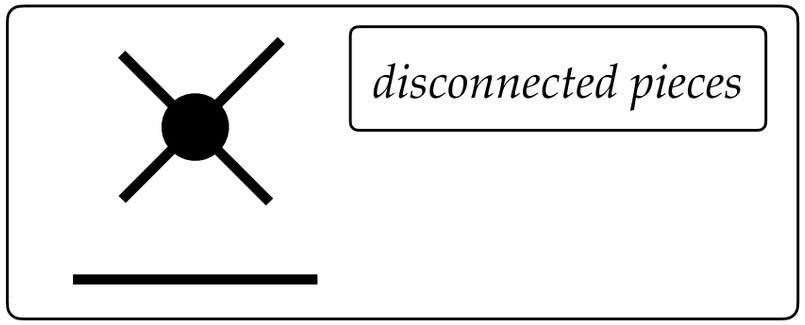
$$\mathcal{M}_{3,\text{con.}} = \mathcal{S}\left\{\mathcal{D} + \mathcal{L}\,\mathcal{T}\,\mathcal{L}^T\right\}$$

two integral equations and you're done!

Unitarity

Unitarity implies that the full amplitude satisfies: $2\,\mathrm{Im}\mathcal{M} = \mathcal{M}^\dagger \mathcal{M}$

If add the disconnected pieces…



…it works…trust me…or trust these guys…

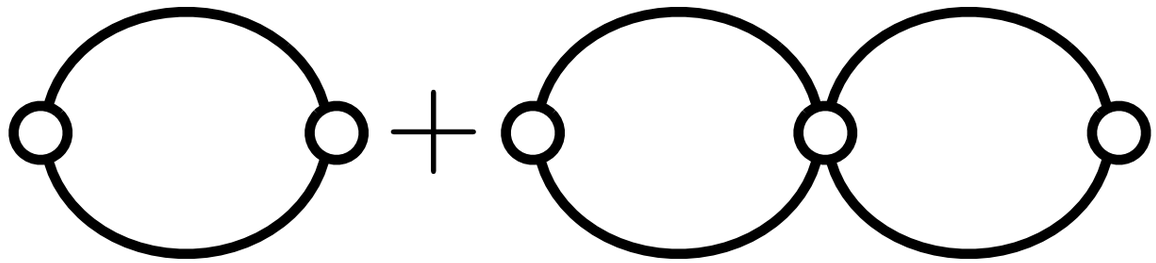


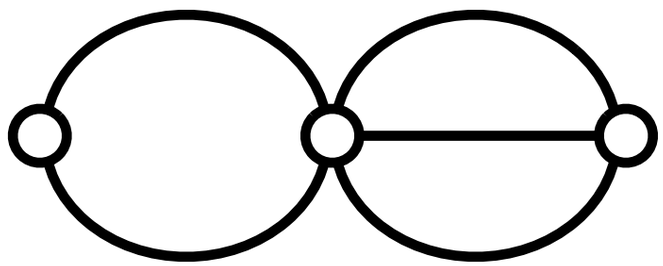
…or don't…do it yourself!

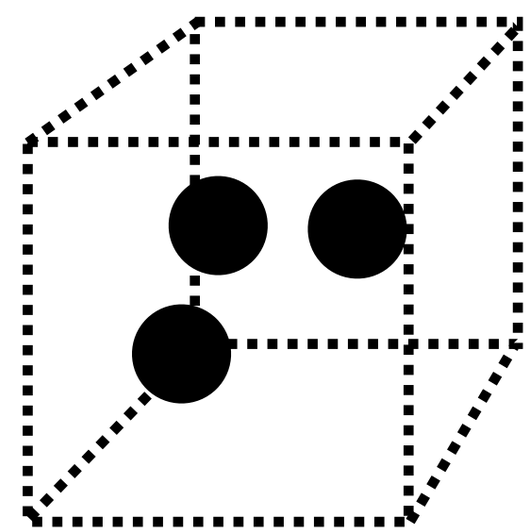
RB, Hansen, Sharpe & Szczepaniak (March 2019?)

Three particles in a finite volume

Moving on to energies where two and three particles can go on-shell…

$C_L^{2pt.}(P) =$  $+ \cdots$

$+$  $+ \cdots$



Three particles in a finite volume

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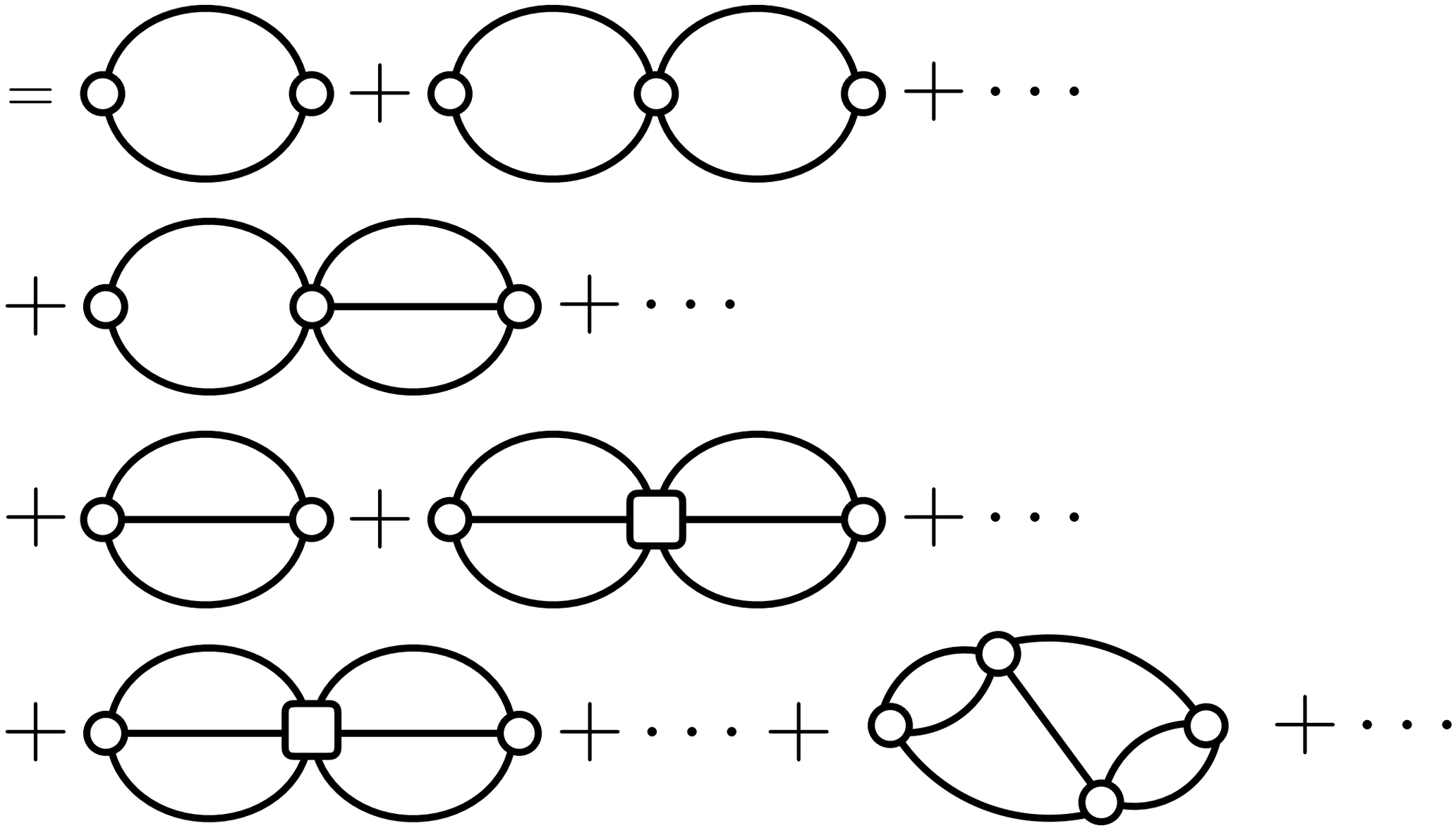

Three particles in a finite volume

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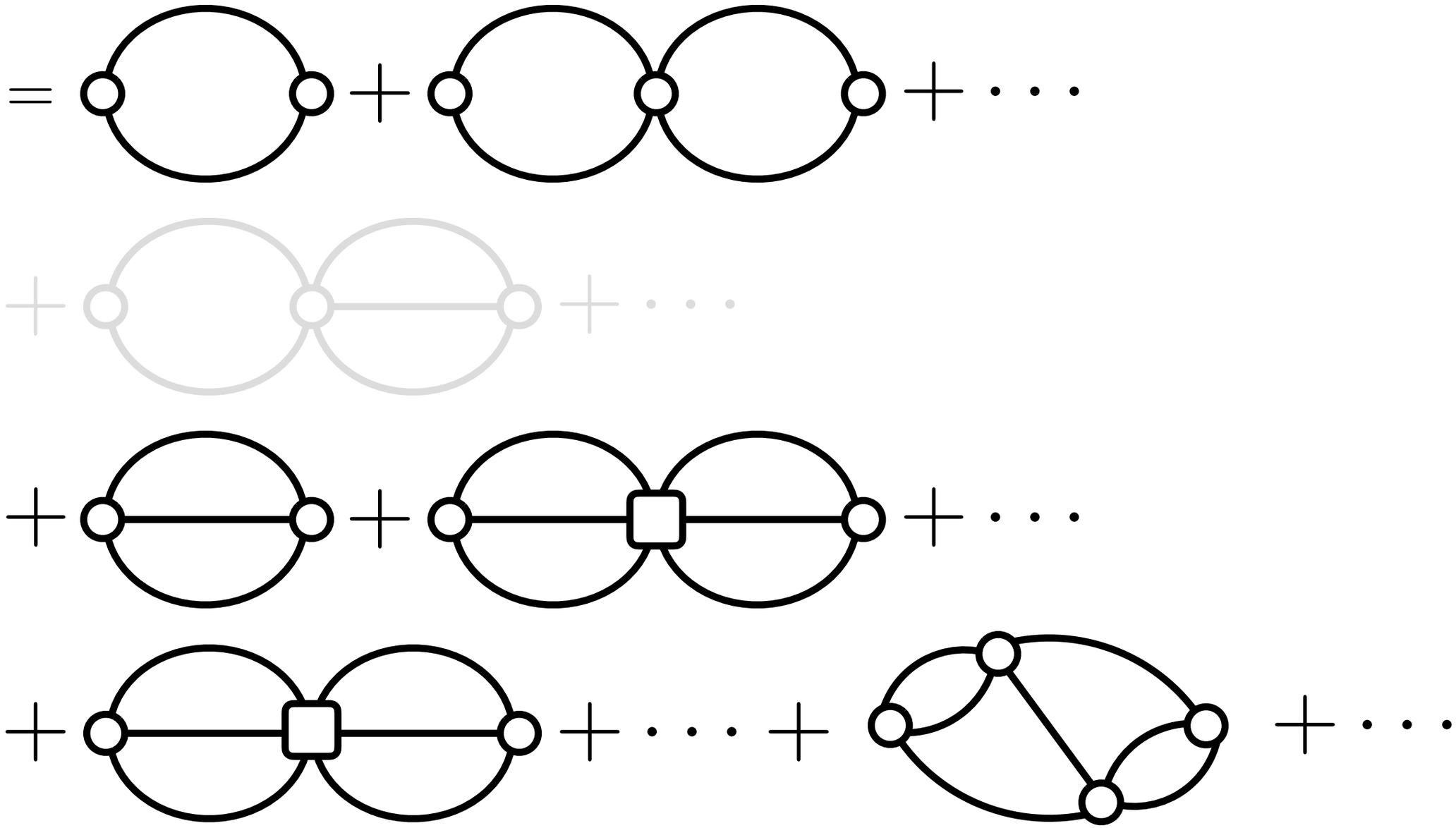
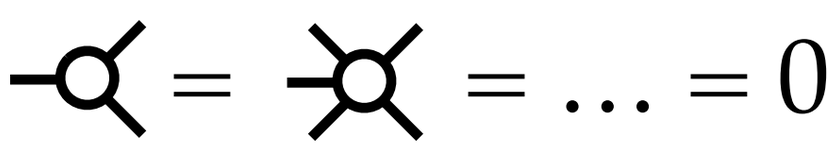
$C_L^{2pt.}(P) =$ 

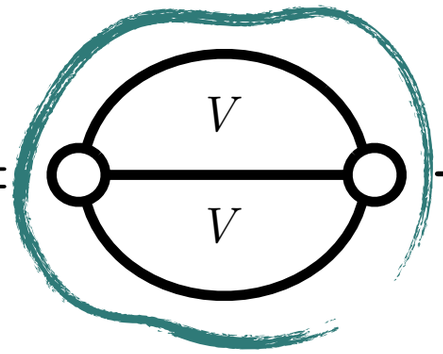
consider first a case with Z2 symmetry



the spectrum factorizes for even and odd number of particles

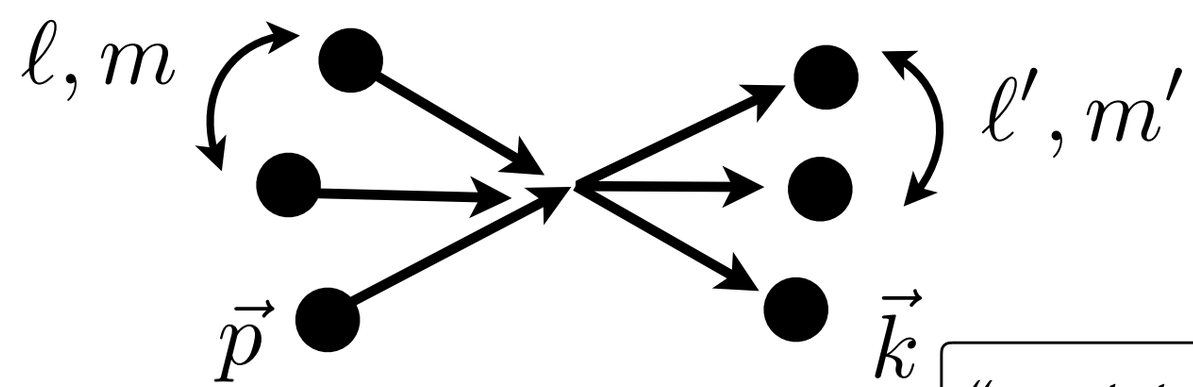
Three particles in a finite volume

Near three-particle states

$C_L^{2pt.}(P) =$ 

$+ \cdots$

the *klm* basis:



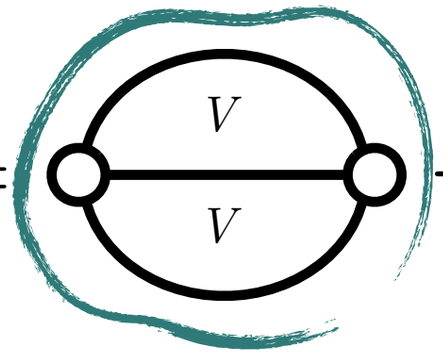
"spectator"

not a natural basis in a infinite volume…that's ok!

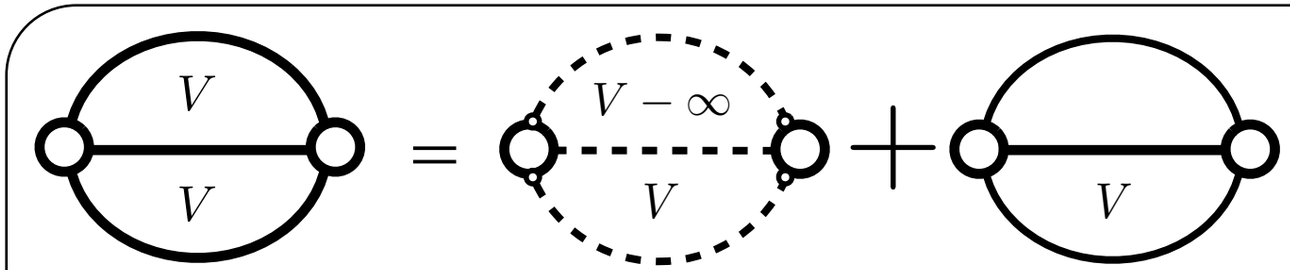
More degrees of freedom.
In infinite volume = (6 particles with 3 unfixed momenta) - (10 Poincare generators) = 8 d.o.f.
The *klm* basis has 1 + 4 + 6 = 11 d.o.f. …also ok!

Three particles in a finite volume

Near three-particle states

$C_L^{2pt.}(P) =$ 

$+ \cdots$



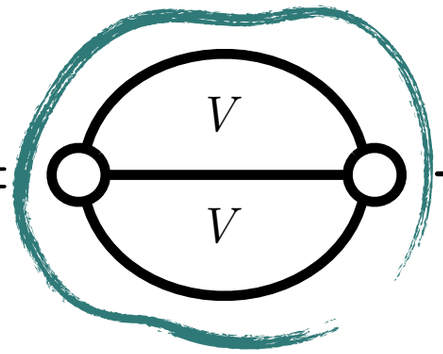
$$F_{00k;00p} = \delta_{kp} \frac{\boxed{H(\vec{k})}}{4\omega_k} \left[\frac{1}{L^3} \sum_{\vec{a}}^{\mathrm{UV}} - \int_a^{\mathrm{UV}} \right] \frac{4\pi\, Y_{\ell m}(\hat{a})\, Y^*_{\ell' m'}(\hat{a})}{4\omega_a \omega_{ka} (E - \omega_k - \omega_a - \omega_{ka})}$$

cut-off

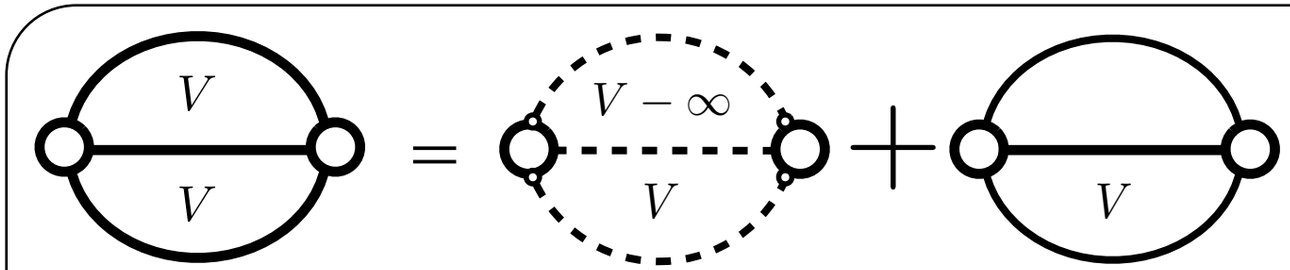
to avoid threshold singularities,
we use PV prescriptions

Three particles in a finite volume

Near three-particle states

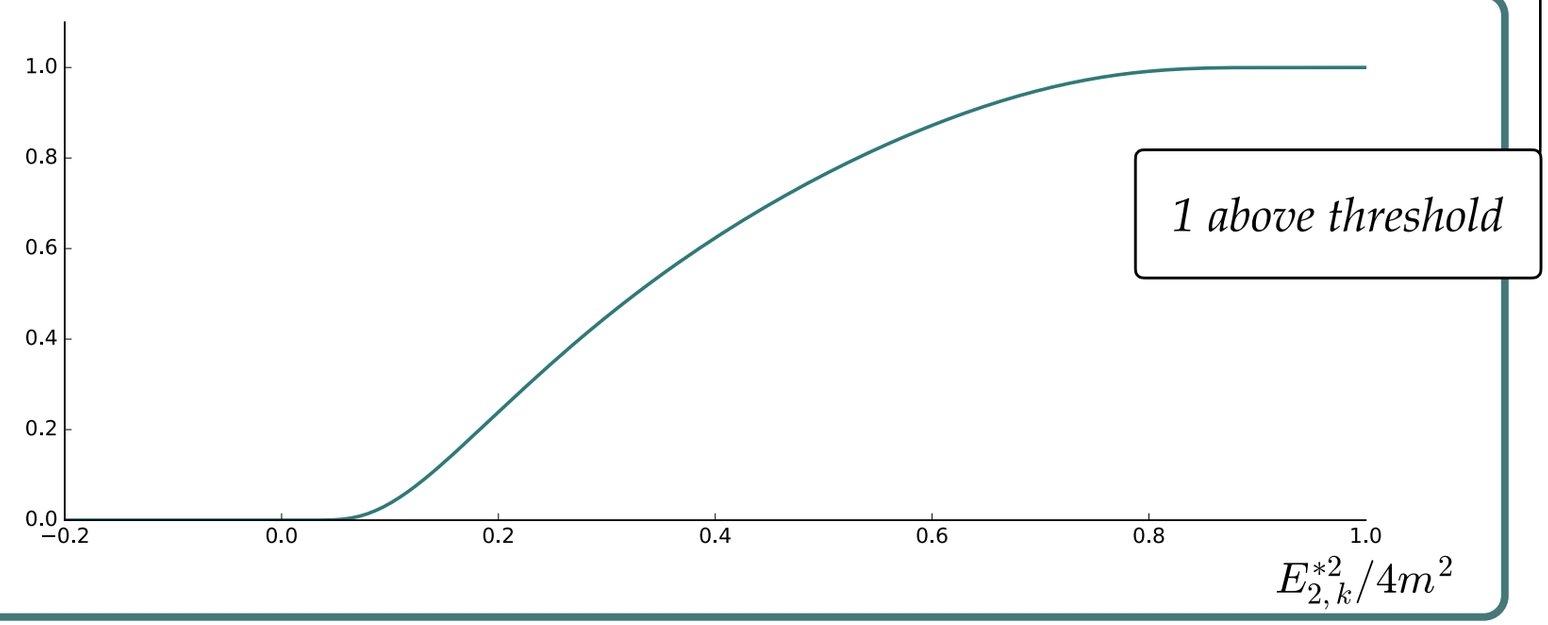
$C_L^{2pt.}(P) =$ 

$+ \cdots$



cut-off

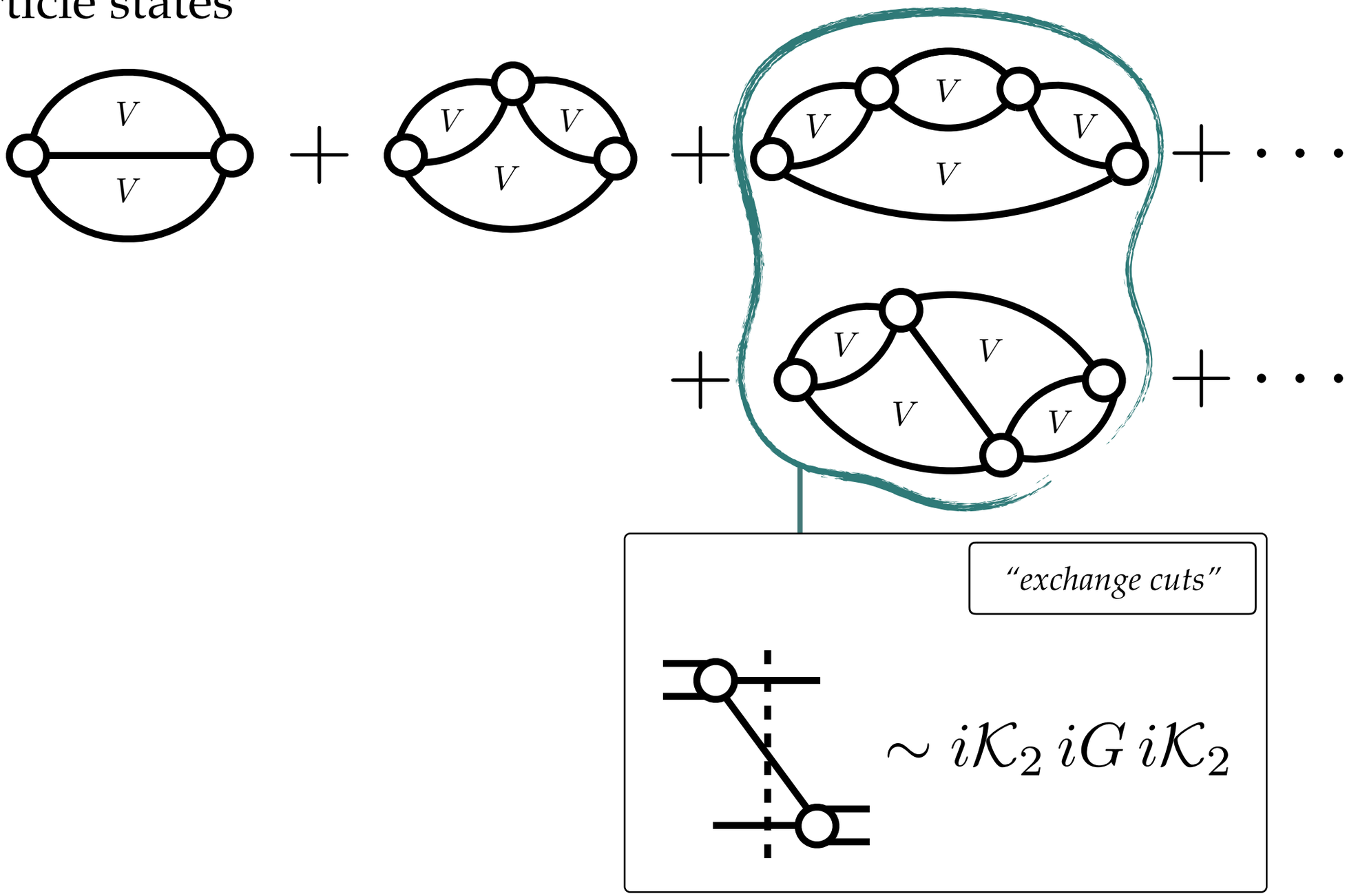
$$F_{00k;00p} = \delta_{kp} \frac{H(\vec{k})}{4\omega_k}$$



1 above threshold

Three particles in a finite volume

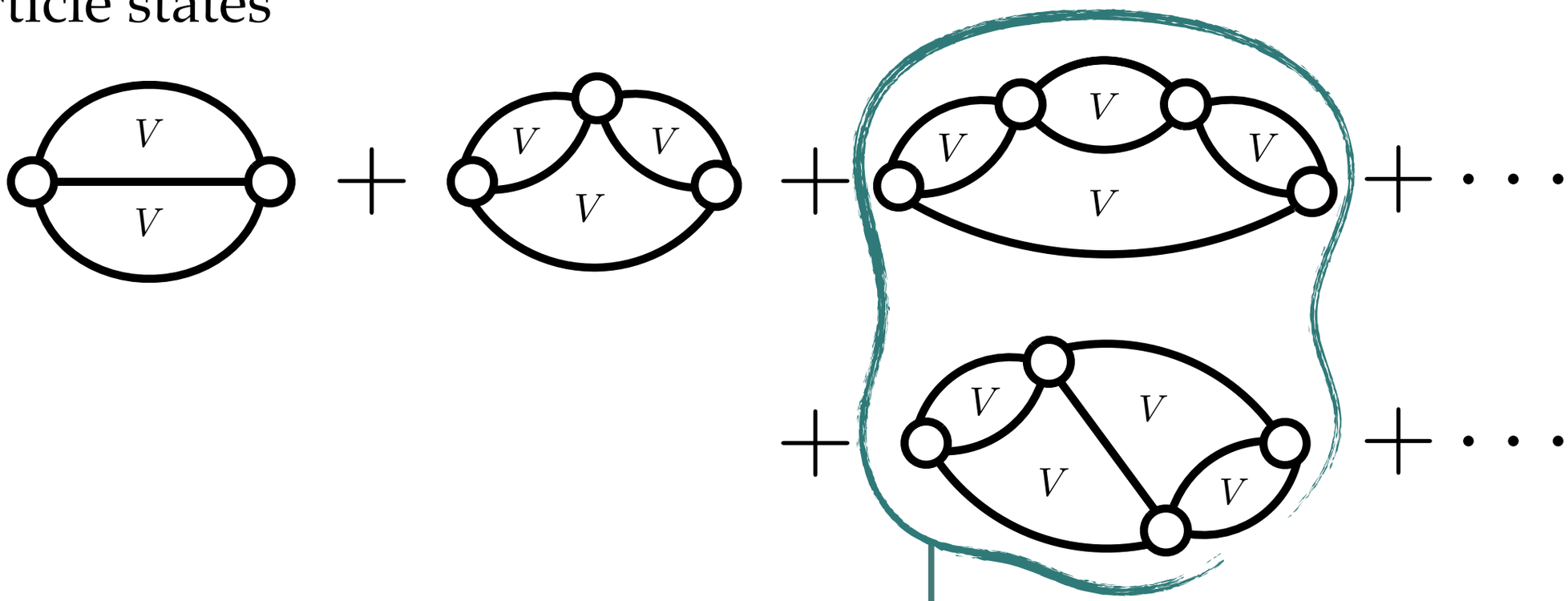
Near three-particle states

$C_L^{2pt.}(P) =$ 

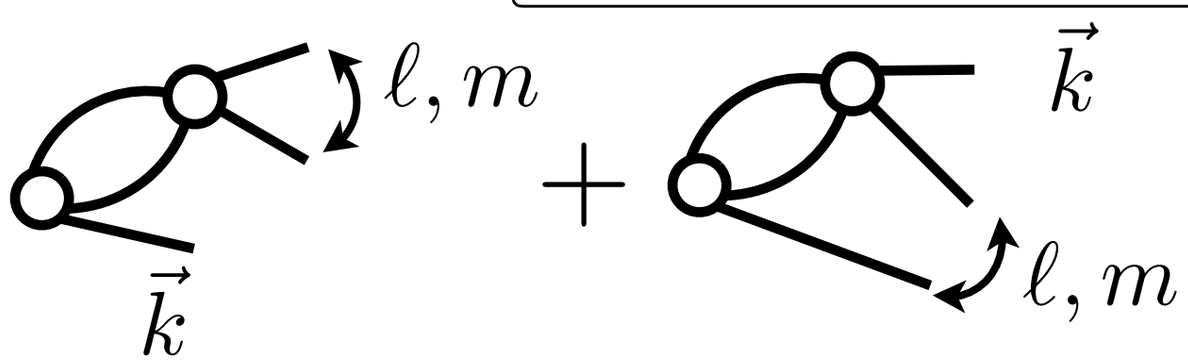
Three particles in a finite volume

Near three-particle states

$$C_L^{2pt.}(P) =$$



exchange symmetry restoration

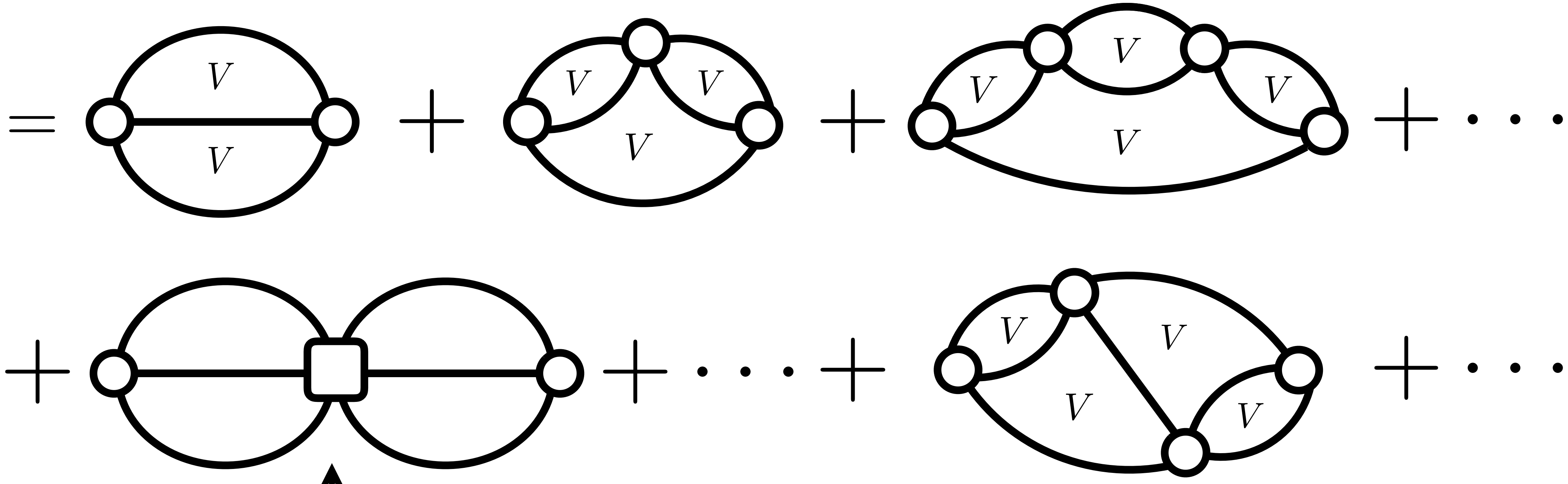


different expansions of the same underlying function

Three particles in a finite volume

Near three-particle states

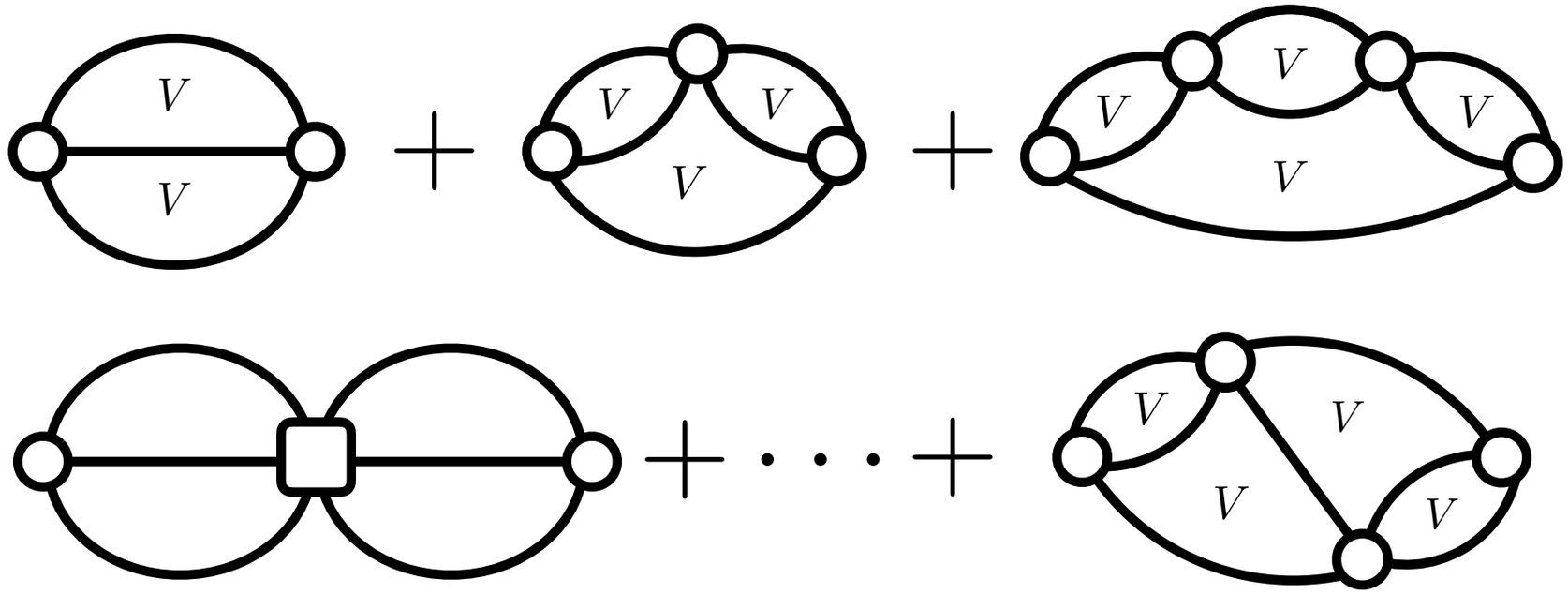
$C_L^{2pt.}(P) =$

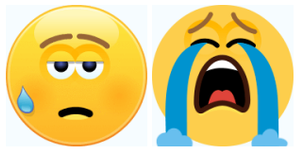


don't forget about $K_{\mathrm{df},3}$

Three particles in a finite volume

Near three-particle states

$C_L^{2pt.}(P) =$ 

…a couple of hundred equations later… 

$$= \text{“smooth”} + A_3 \frac{i}{F_3^{-1} + \mathcal{K}_{\mathrm{df},3}} B_3^\dagger$$

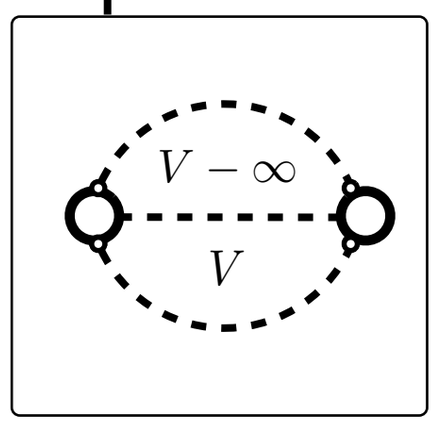
poles satisfy: $\det[F_3^{-1} + \mathcal{K}_{\mathrm{df},3}] = 0$

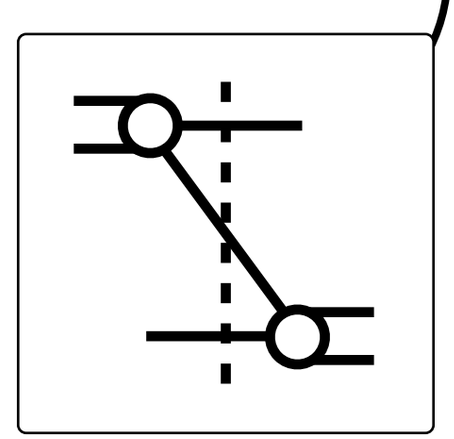
- Hansen & Sharpe (2014)
- RB, Hansen & Sharpe (2017)
- RB, Hansen & Sharpe (2018)

Numerical checks - *"isotropic approximation"*

Consider the case where two-body system is an S-wave

$$[F_3^s]_{kp} = \frac{1}{L^3}\left[\frac{F^s}{3} - F^s \frac{1}{1/(2\omega\mathcal{K}_2^s) + F^s + G^s} F^s\right]_{kp}$$





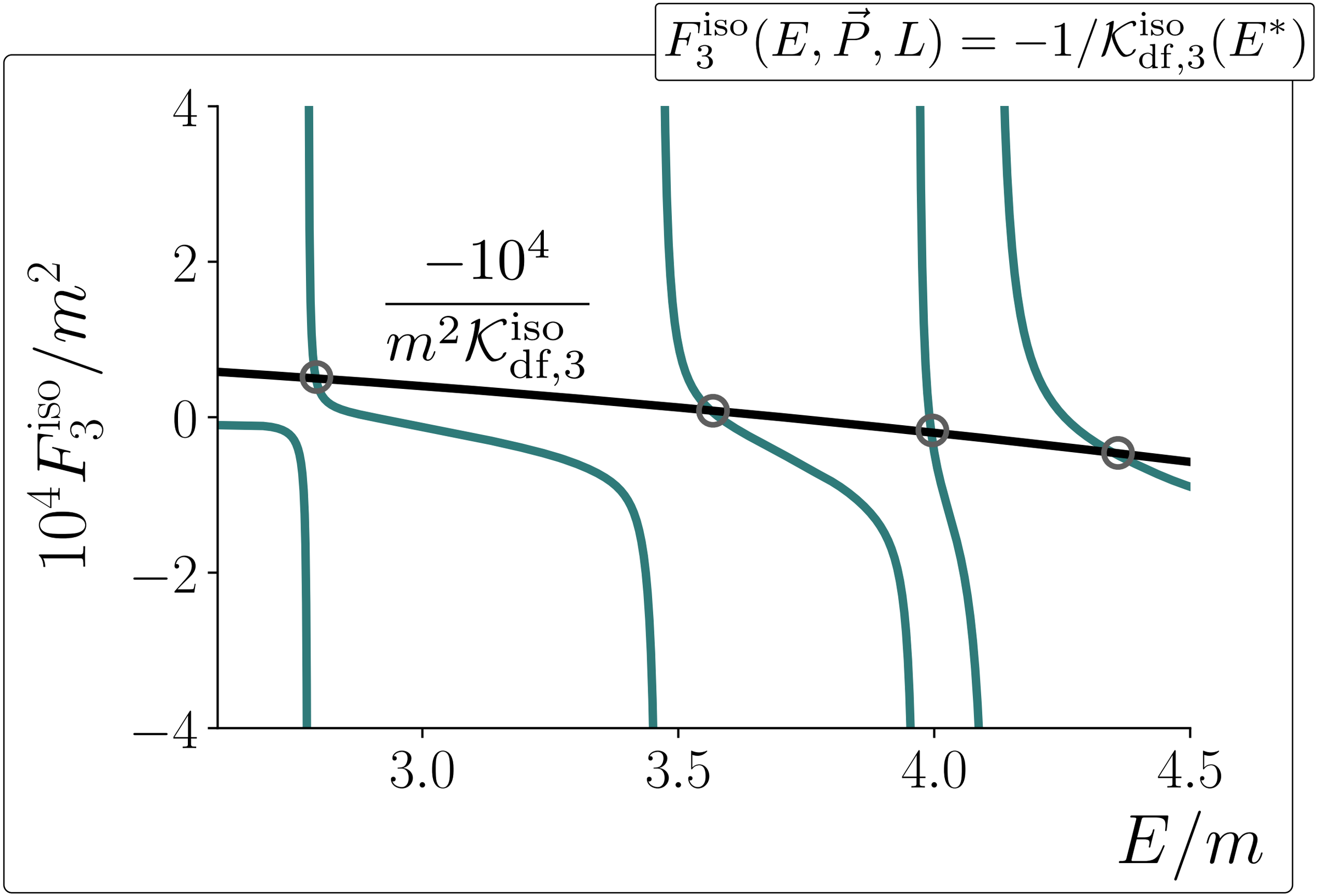
Furthermore, consider the case where the K-matrix does not depend on the spectator momentum. Then the quantization condition reduces to:

$$F_3^{\mathrm{iso}}(E, \vec{P}, L) = -1/\mathcal{K}_{\mathrm{df},3}^{\mathrm{iso}}(E^*)$$

Where:

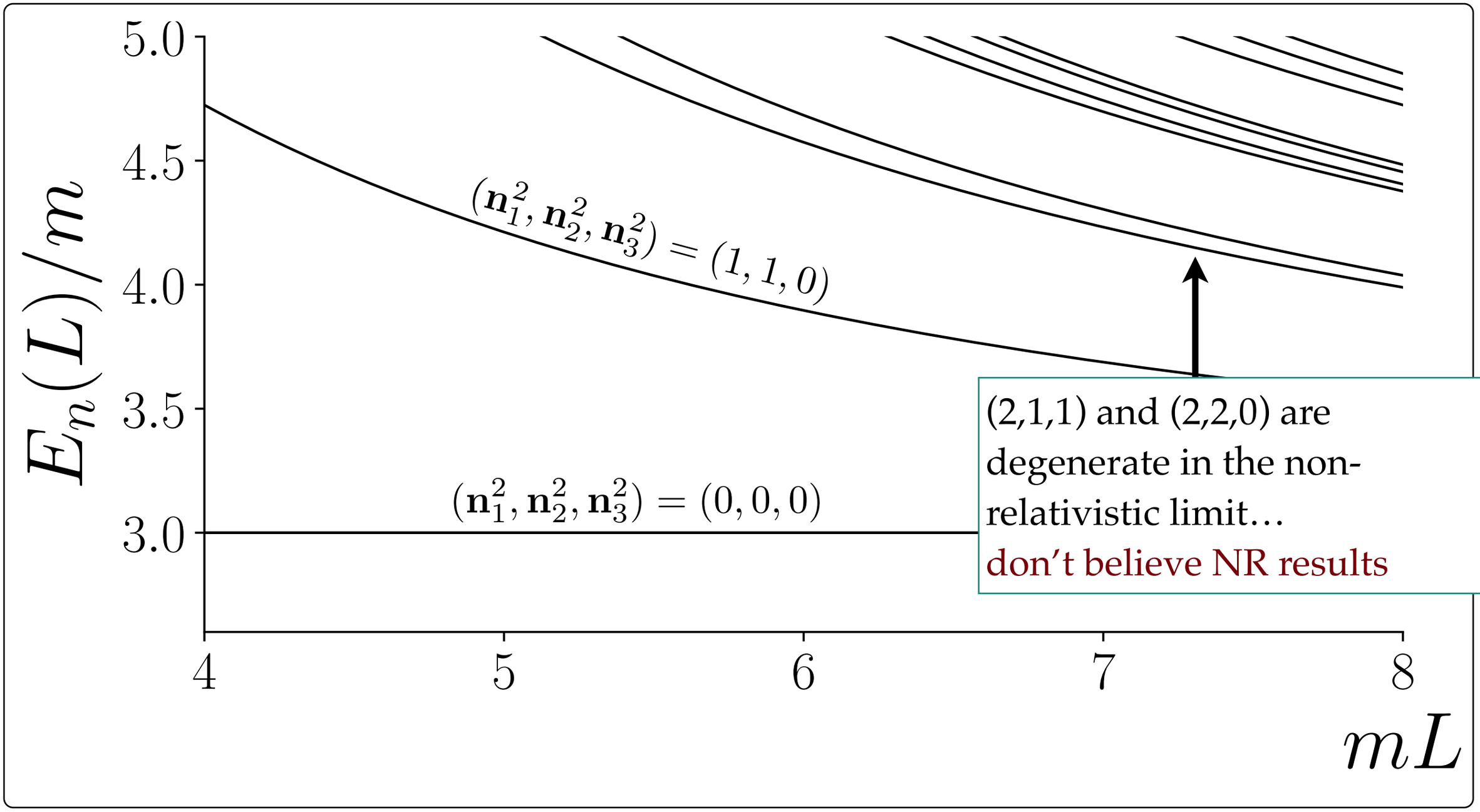
$$F_3^{\mathrm{iso}}(E, L) = \sum_{k,p} [F_3^s]_{kp}$$

Numerical checks - *"isotropic approximation"*



Free states

$$E_n(L) = \sqrt{m^2 + (2\pi/L)^2 \mathbf{n}_1^2} + \sqrt{m^2 + (2\pi/L)^2 \mathbf{n}_2^2} + \sqrt{m^2 + (2\pi/L)^2 \mathbf{n}_{12}^2}$$

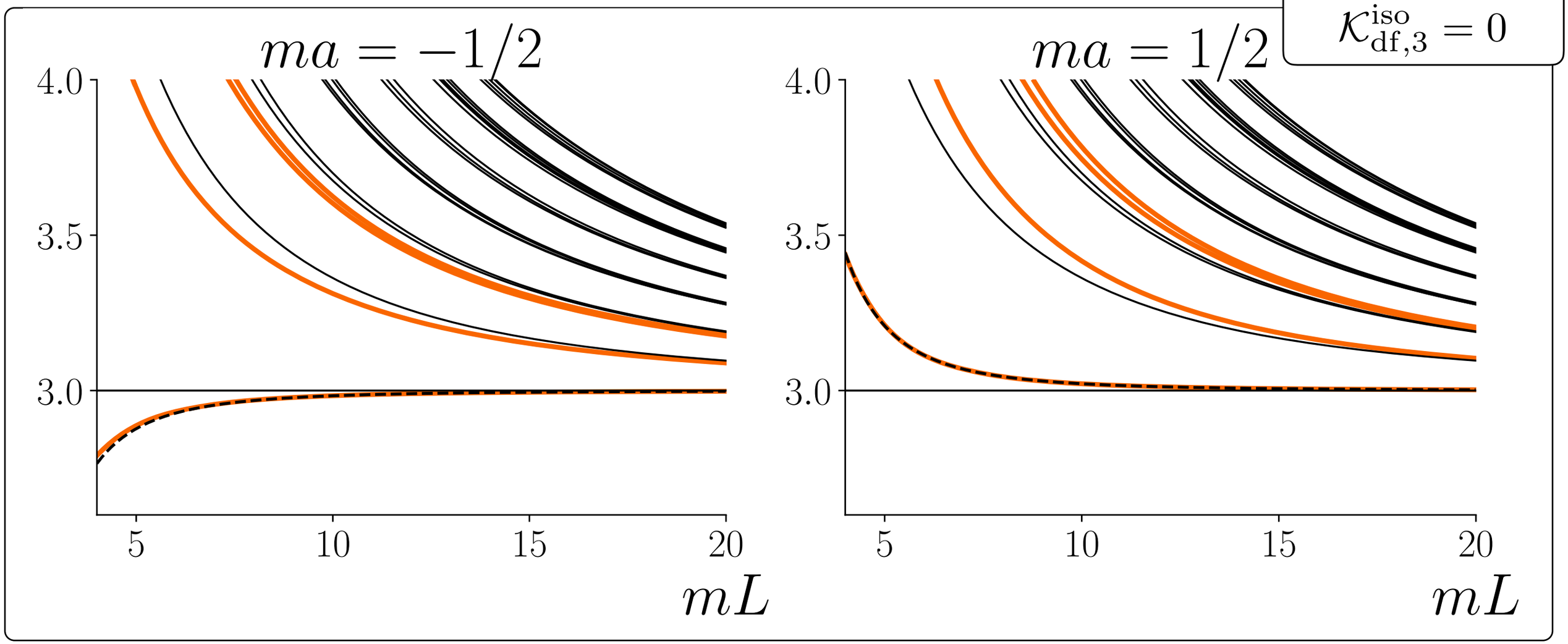


(2,1,1) and (2,2,0) are degenerate in the non-relativistic limit…
don't believe NR results

Perturbative systems

$$p \cot \delta_2 = -\frac{1}{a}$$

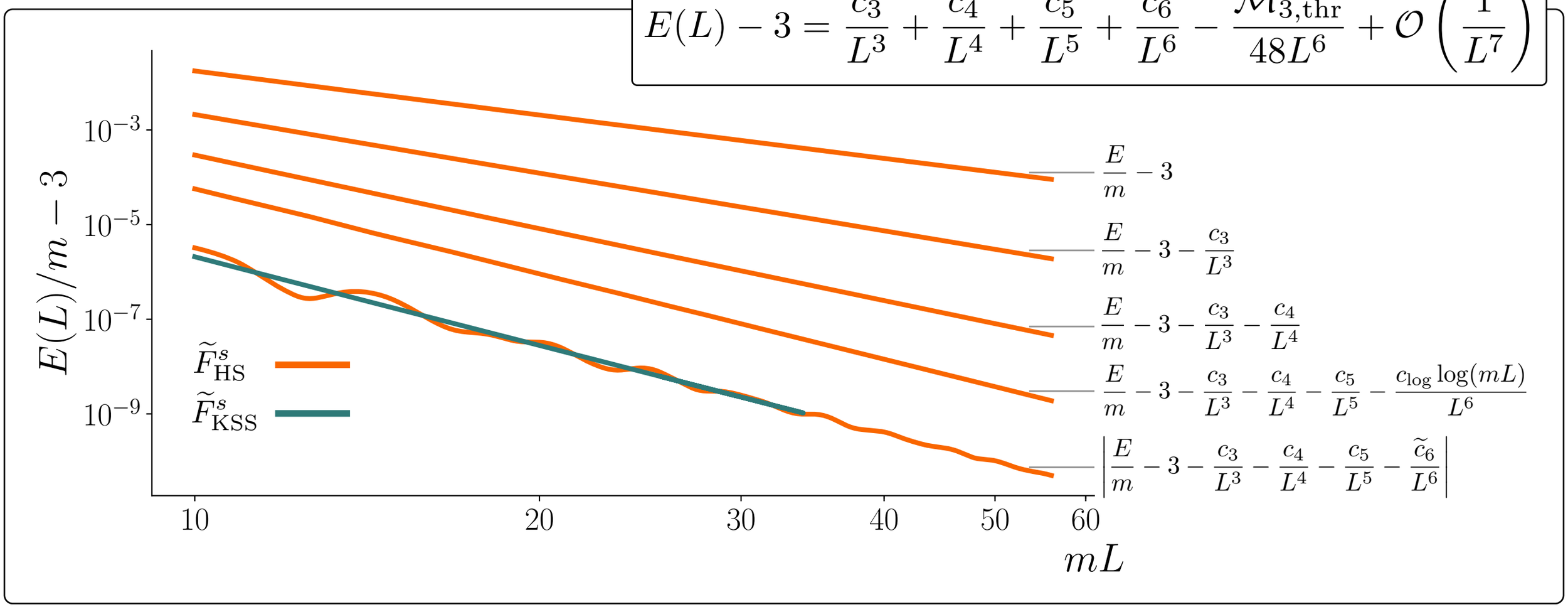
$$\mathcal{K}^{\mathrm{iso}}_{\mathrm{df},3} = 0$$



Perturbative systems

$$ma = 0.41315$$

$$E(L) - 3 = \frac{c_3}{L^3} + \frac{c_4}{L^4} + \frac{c_5}{L^5} + \frac{\tilde{c}_6}{L^6} - \frac{\mathcal{M}_{3,\mathrm{thr}}}{48L^6} + \mathcal{O}\left(\frac{1}{L^7}\right)$$



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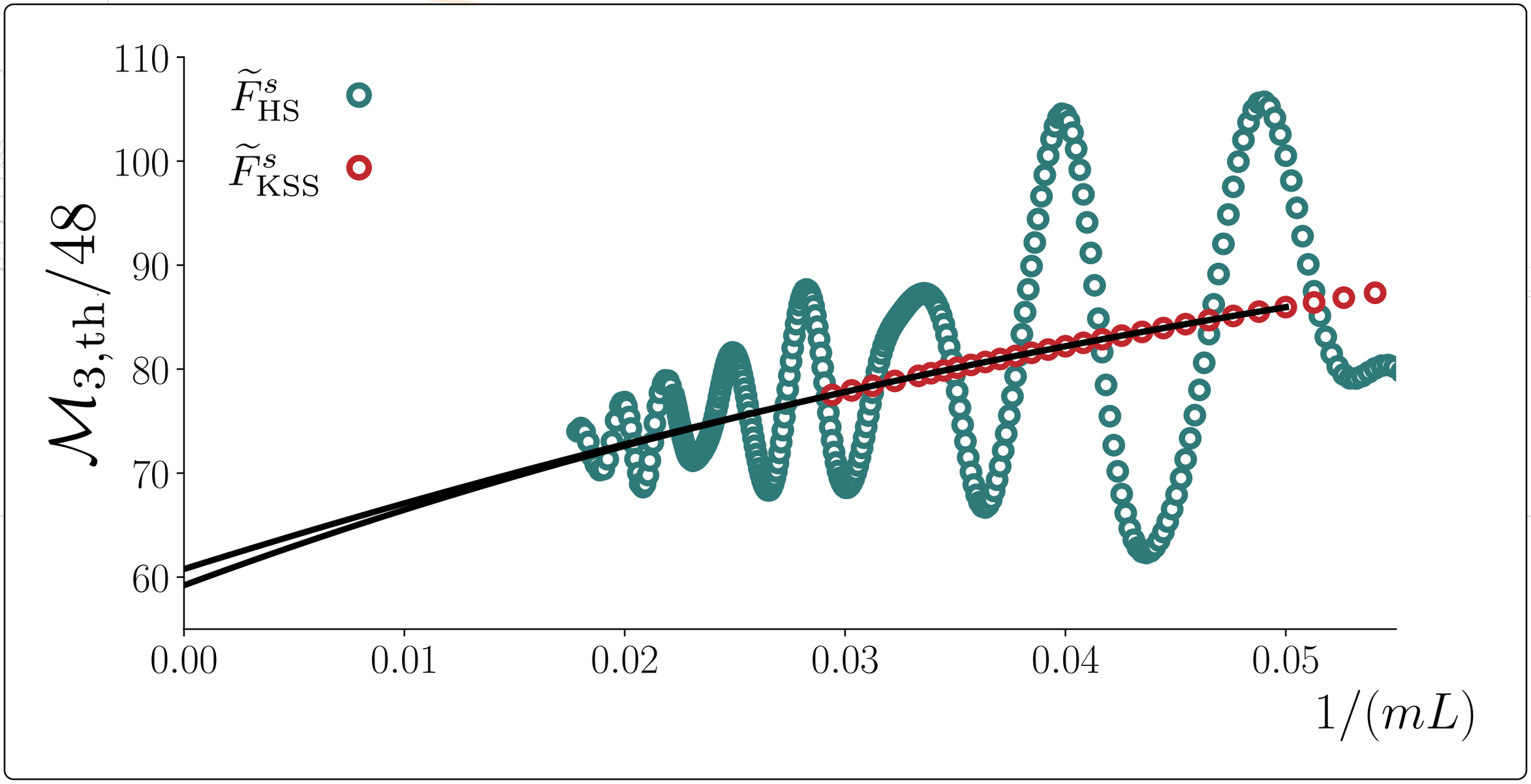
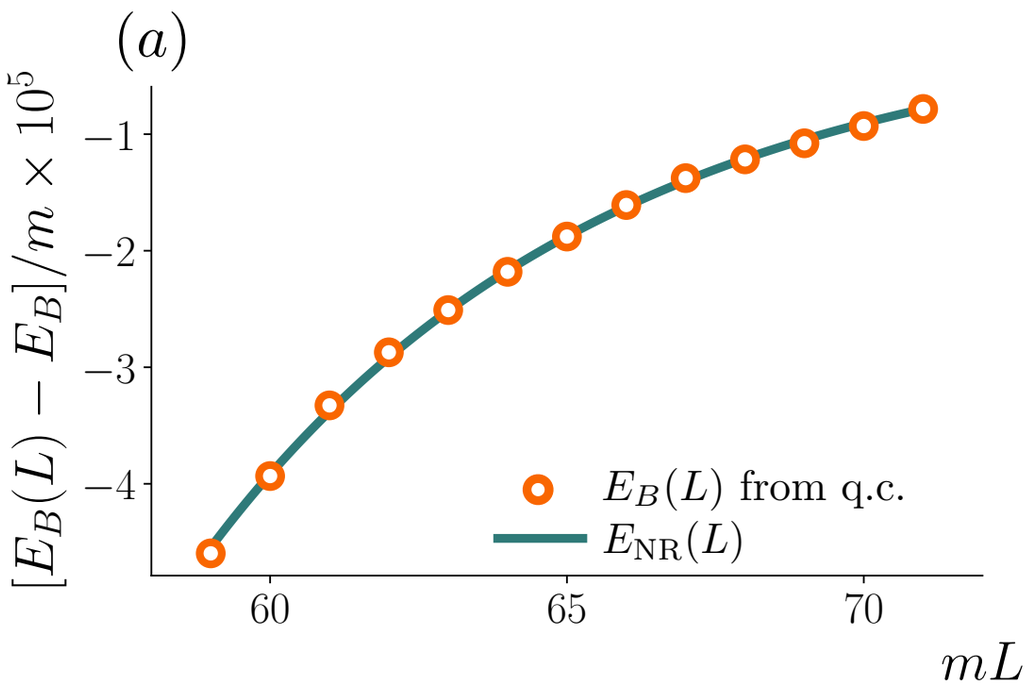
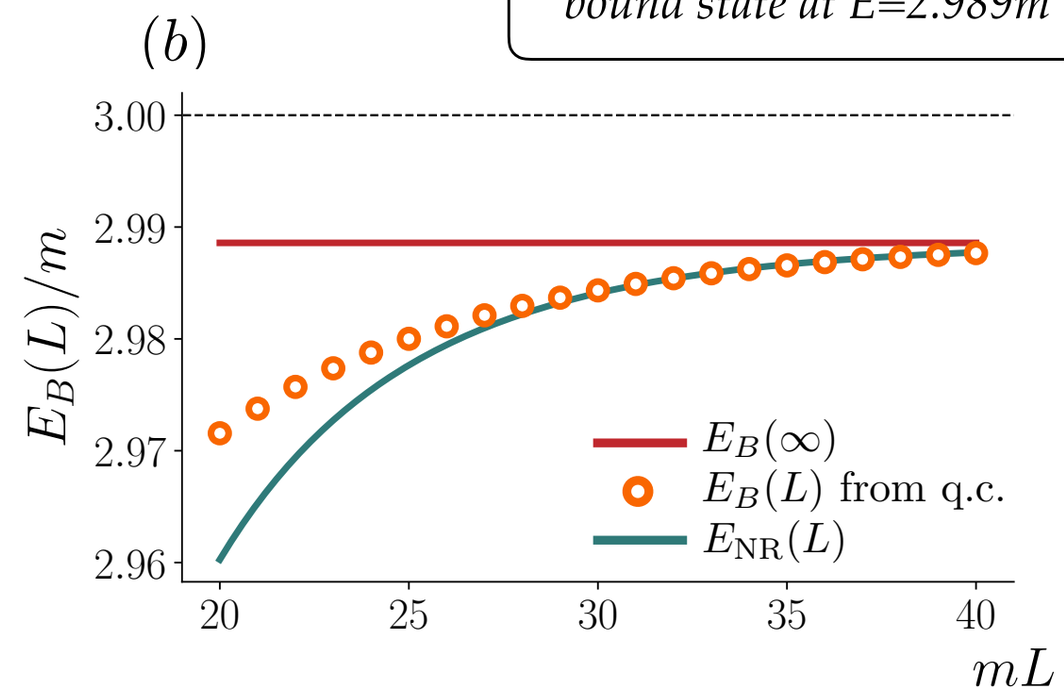


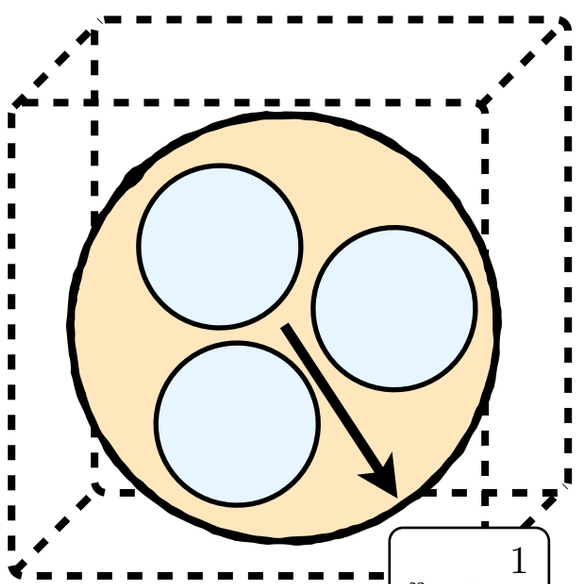
Efimov-like physics

$$p \cot \delta_2 = -\frac{1}{a}$$

- *ma=-10⁴*
- *K_{df} is tuned to have a bound state at E=2.989m*







$$r_B \sim \frac{1}{\kappa}$$

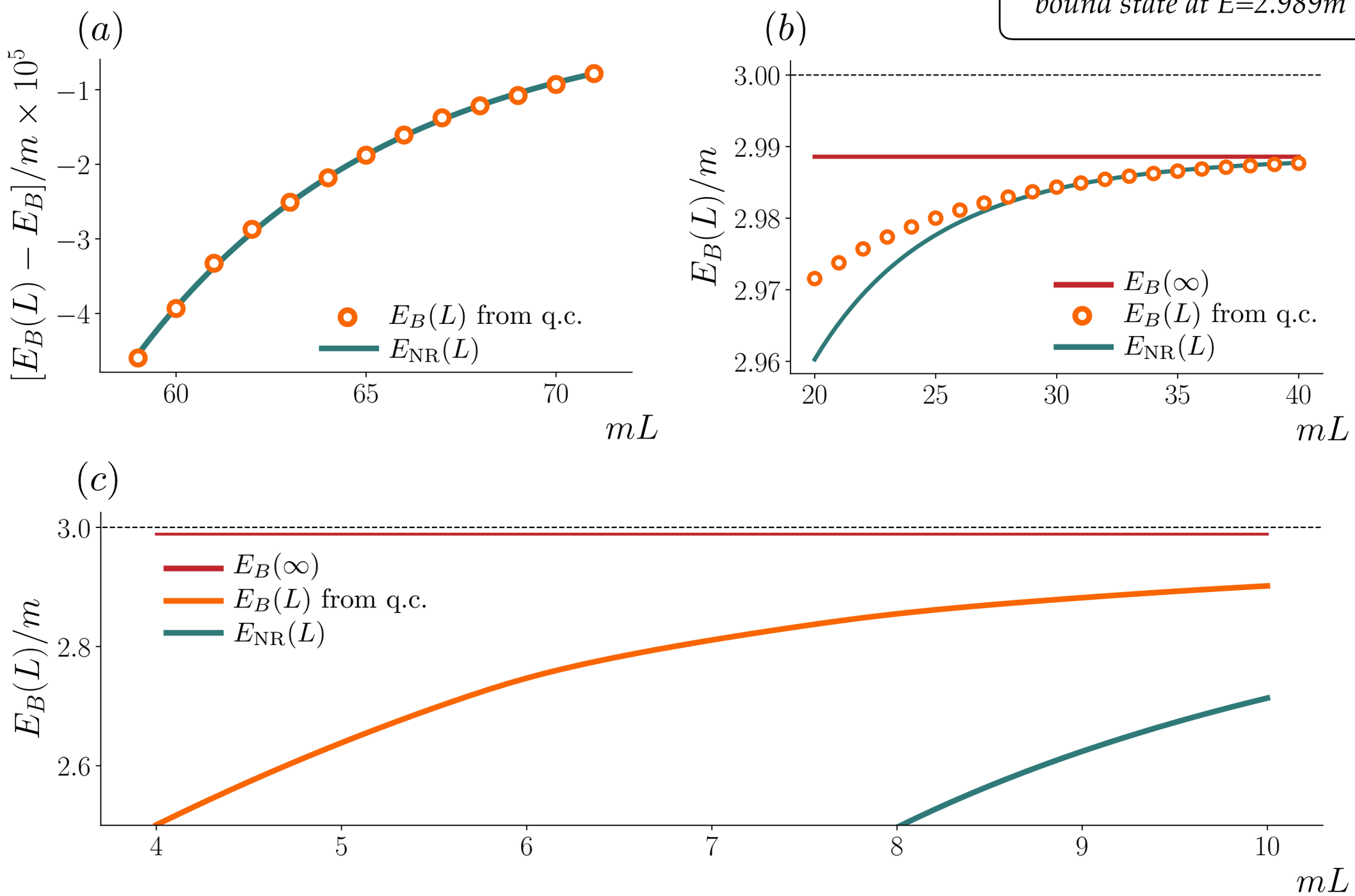
$$E_B(L) = 3 - \kappa^2 - 96.35|A|^2\kappa^2 \frac{e^{-2\kappa L/\sqrt{3}}}{(\kappa L)^{3/2}}$$

Meißner, Ríos, & Rusetsky (2015)

Efimov-like physics

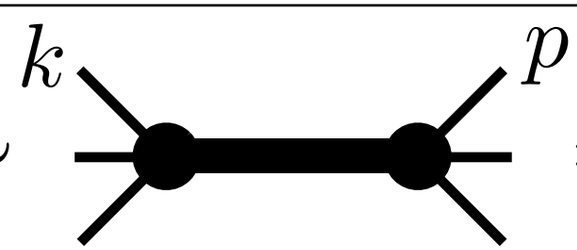
$$p \cot \delta_2 = -\frac{1}{a}$$

- *ma=-10⁴*
- *K_{df} is tuned to have a bound state at E=2.989m*

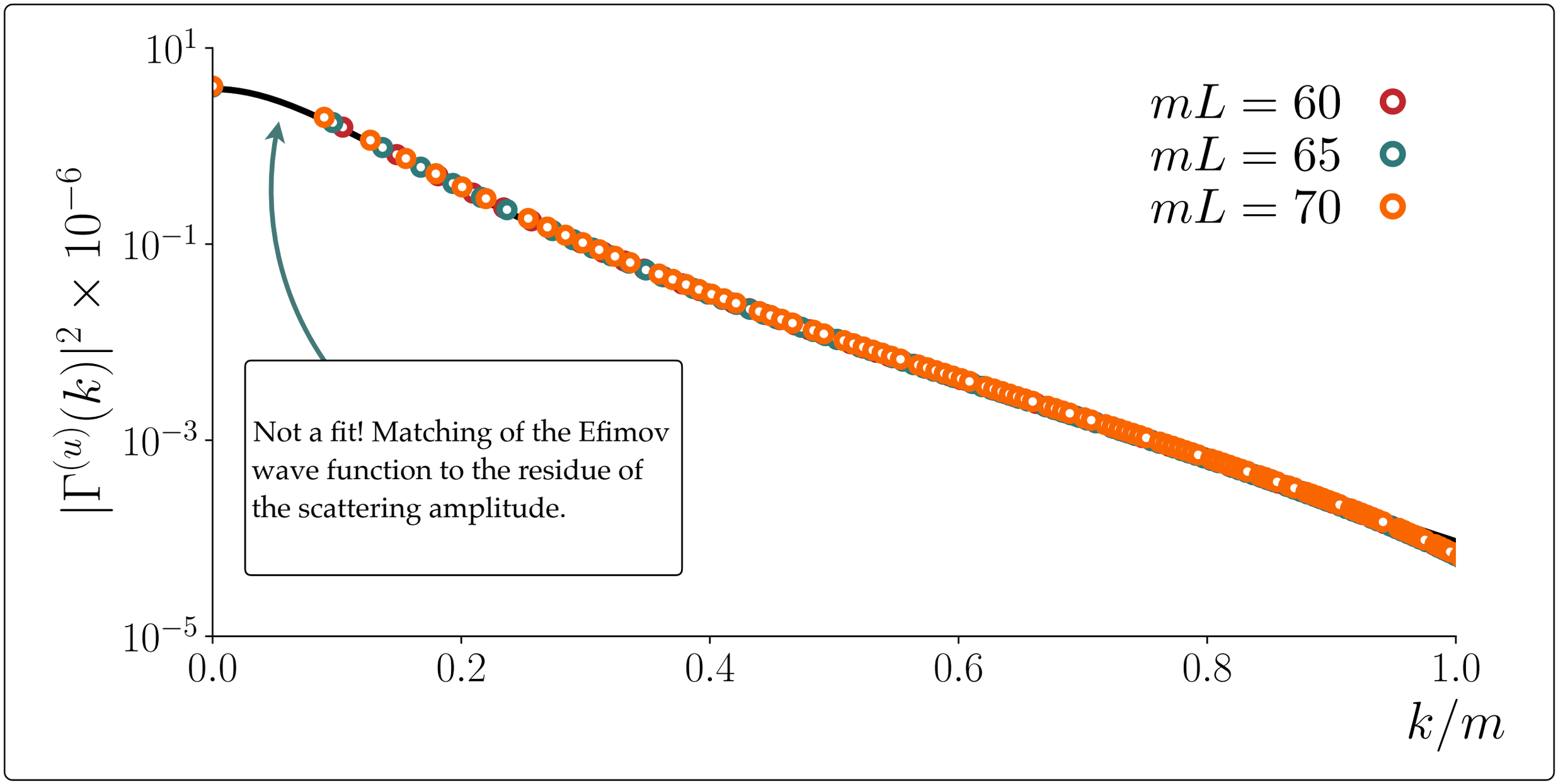


Efimov-like physics

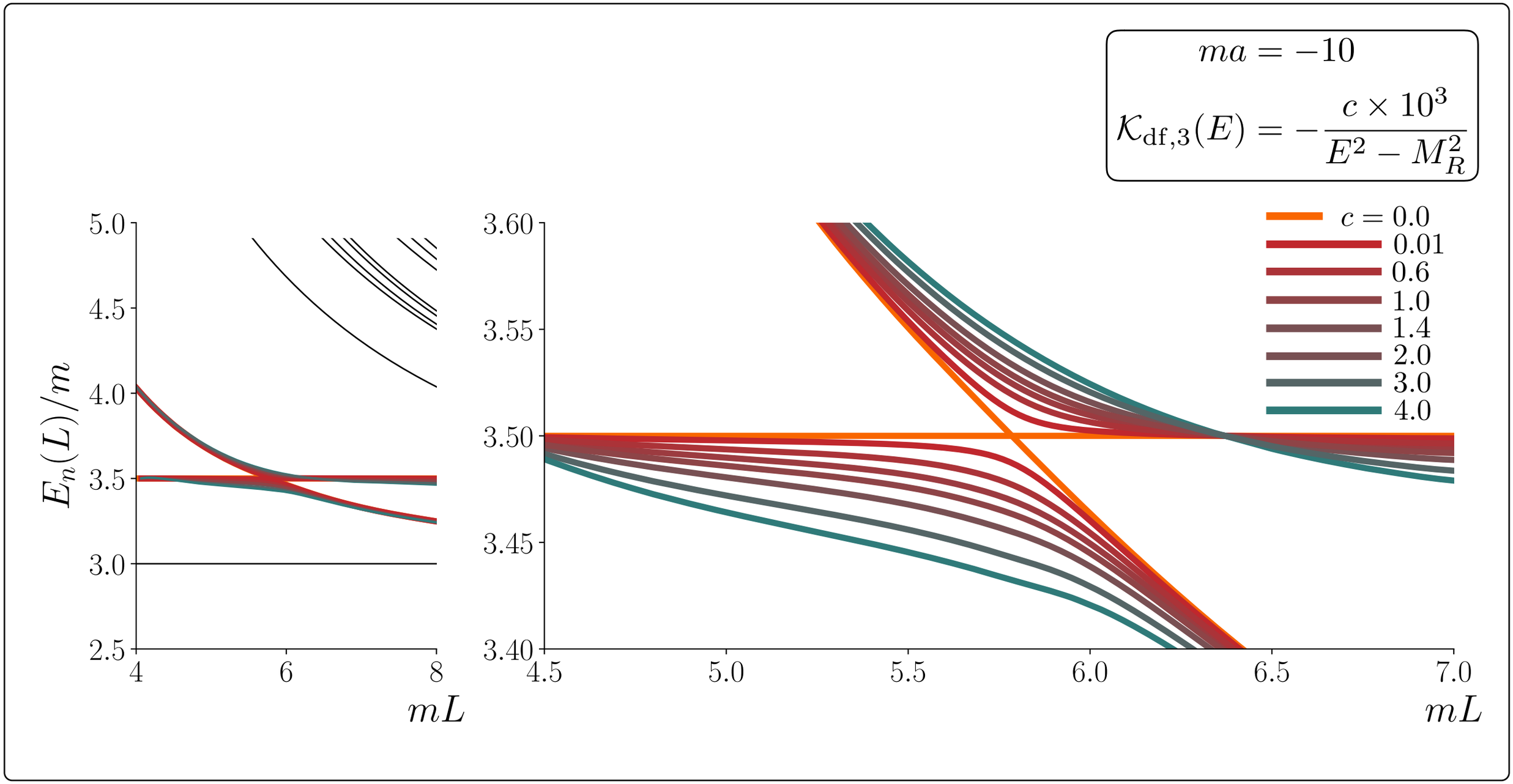
$$i\mathcal{M}_3(k,p) \sim$$



$$= i\Gamma(k)\,\frac{i}{s-s_B}\,i\Gamma(p)$$



Resonant systems

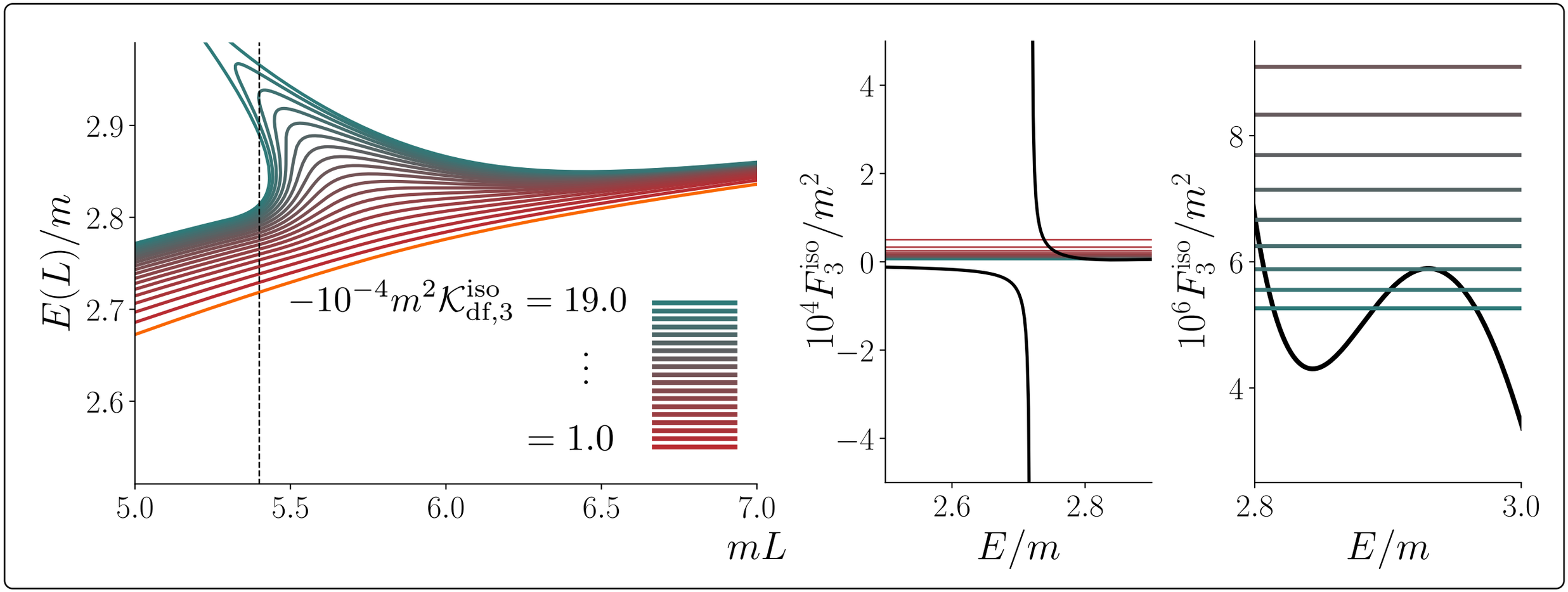


Unphysical solutions

Requiring states to be positively normed, imposes….

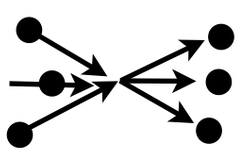
$$\left[\frac{\partial F_3^{\rm iso}(E,L,a)}{\partial E} + \frac{\partial 1/\mathcal{K}_{{\rm df},3}^{\rm iso}(E)}{\partial E}\right]_{E=E_n(L)} < 0$$

We found cases that clearly violate this

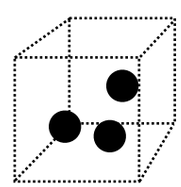


…we have ideas on possible explanations and solutions…

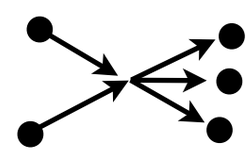
Unitarity in 3Body systems



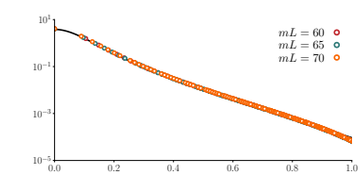
FV formalism for spinless states



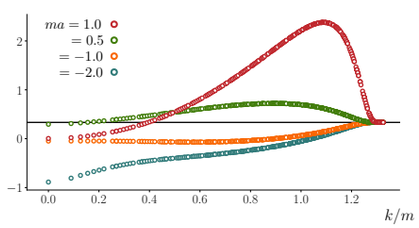
coupled 2/3B systems



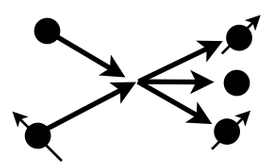
analytic and numerical checks



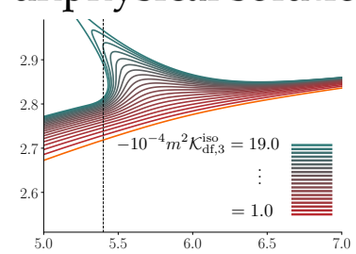
integral equations



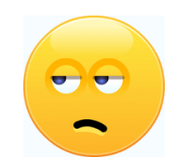
Spin, non-degenerate masses, multichannels



unphysical solutions



do some calculations already…



- Hansen & Sharpe (2014, '15)
- RB, Hansen & Sharpe (2017,'18)
- RB, Hansen, Sharpe & Szczepaniak (coming!)

inside the box

Sharpe

Hansen

outside the box

Szczepaniak